Variational data assimilation for morphodynamic model parameter estimation

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Variational data assimilation for morphodynamic model parameter estimation

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Outline

• Background/ motivation
  - what is morphodynamic modelling?
  - why do we need morphodynamic models?

• A simple 1D morphodynamic model

• Data assimilation and parameter estimation
  - how can we use data assimilation to estimate uncertain model parameters?
  - how do we model the background error covariances?

• Results

• Summary
Terminology

- **Bathymetry** - the underwater equivalent to topography
  - coastal bathymetry is dynamic and evolves with time
  - water action erodes, transports, and deposits sediment, which changes the bathymetry, which alters the water action, and so on

- **Morphodynamics** - the study of the evolution of the bathymetry in response to the flow induced sediment transport

- **Morphodynamic prediction**
  - why?
  - how?
Kent channel

- Channel movement
  - impacts on habitats in the bay
  - affects access to ports
  - has implications for flooding during storm events

1960s
1970s
1997

Picture courtesy of Nigel Cross, Lancaster City Council
Operational coastal flood forecasting is limited near-shore by lack of knowledge of evolving bathymetry
  - but it is impractical to continually monitor large coastal areas

Modelling is difficult
  - longer term changes are driven by shorter term processes
  - uncertainty in initial conditions and parameters

An alternative approach is to use data assimilation
Parameter estimation

- Model equations depend on parameters
  - exact values are unknown
  - inaccurate parameter values can lead to growth of model error
  - affects predictive ability of the model

- How do we estimate these values \textit{a priori}? 
  - theoretical values
  - calibration

or ...

- data assimilation
  - choose parameters based on observations
  - state augmentation: model parameters are estimated alongside the model state
Simple 1D model

Based on the sediment conservation equation

\[ \frac{\partial z}{\partial t} = - \left( \frac{1}{1 - \varepsilon} \right) \frac{\partial q}{\partial x} \]

where \( z(x,t) \) is the bathymetry, \( t \) is time, \( q \) is the sediment transport rate in the \( x \) direction and \( \varepsilon \) is the sediment porosity.

For the sediment transport rate we use the power law

\[ q = A u^n \]

where \( u(x,t) \) is the depth averaged current and \( A \) and \( n \) are parameters whose values need to be set
If we assume that water flux \((F)\) and height \((H)\) are constant

\[ F = u(H - z) \]

we can rewrite the sediment conservation equation as

\[ \frac{\partial z}{\partial t} + a(z, H, F, \varepsilon, A, n) \frac{\partial z}{\partial x} = 0 \]

where \(a(z, H, F, \varepsilon, A, n)\) is the advection velocity or bed celerity.
Can we use data assimilation to estimate the parameters $A$ and $n$?
model run with incorrect parameters & without data assimilation

red line = correct parameters
blue line = incorrect parameters (A over estimated, n under estimated)
State augmentation

- Dynamical system model
  \[ z_{k+1} = f(z_k, p_k) \]
  (discrete, non-linear, time invariant)

- Parameter evolution
  \[ p_{k+1} = p_k \]

- Augmented system model
  \[
  w_{k+1} = \begin{pmatrix} z_{k+1} \\ p_{k+1} \end{pmatrix} = \begin{pmatrix} f(z_k, p_k) \\ p_k \end{pmatrix} = \tilde{f}(w_k)
  \]
Observations

\[ y_k = h(z_k) \]

in terms of the augmented system ...

\[ y_k = \tilde{h}(w_k) \]

where

\[ \tilde{h}(w) = \tilde{h}\left(\begin{array}{c}z \\ p\end{array}\right) = h(z). \]
3D Var

Cost function:

\[ \tilde{J}(w) = (w - w^b)^T \tilde{B}^{-1} (w - w^b) + (y - \tilde{h}(w))^T R^{-1} (y - \tilde{h}(w)) \]

\( \tilde{B} \) and \( R \) are the covariance matrices of the background and observation errors.

\[ \tilde{B} = \begin{pmatrix} B_{zz} & B_{zp} \\ (B_{zp})^T & B_{pp} \end{pmatrix}. \]

\( B_{zz} \) state background error covariance
\( B_{pp} \) parameter background error covariance
\( B_{zp} \) state parameter error cross covariance
augmented gain matrix:

\[
\begin{align*}
\tilde{K} & = \tilde{B}\tilde{H}^T \left[ \tilde{H}\tilde{B}\tilde{H}^T + R \right]^{-1} \\
& = \begin{pmatrix} B_{zz}H^T \\ B_{zp}^TH^T \end{pmatrix} \left[ HB_{zz}H^T + R \right]^{-1} \\
\text{def} & \equiv \begin{pmatrix} K_z \\ K_p \end{pmatrix}
\end{align*}
\]

state & parameter updates:

\[
\begin{align*}
\mathbf{z}^a & = \mathbf{z}^b + K_z(y - h(\mathbf{z}^b)) \\
\mathbf{p}^a & = \mathbf{p}^b + K_p(y - h(\mathbf{z}^b))
\end{align*}
\]
State-parameter cross covariances

The Extended Kalman filter (EKF)

State forecast:

\[ \mathbf{w}_{k+1} = \hat{\mathbf{f}}_k(\mathbf{w}_k^a) \]

Error covariance forecast:

\[ \mathbf{P}_{k+1}^f = \mathbf{F}_k \mathbf{P}_k^a \mathbf{F}_k^T \]

where

\[ \mathbf{F}_k = \left. \frac{\partial \tilde{\mathbf{f}}}{\partial \mathbf{w}} \right|_{\mathbf{w}_k^a} = \left. \begin{pmatrix} \frac{\partial f(z,p)}{\partial z} & \frac{\partial f(z,p)}{\partial p} \end{pmatrix} \right|_{z_k^a,p_k^a} = \begin{pmatrix} \mathbf{M}_k & \mathbf{N}_k \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \]
Error covariance forecast:

\[
P^f_{k+1} = \begin{pmatrix}
M_k P^a_{zz_k} M_k^T + N_k P^a_{pp_k} N_k^T & N_k P^a_{pp_k} \\
P^a_{pp_k} N_k^T & P^a_{pp_k}
\end{pmatrix}
\]

a new hybrid approach ...
for our simple 2 parameter model

\[ B_{zp_k} = N_k B_{pp} \]

\[ = \left( \begin{array}{cc} \frac{\partial f_k}{\partial A} & \frac{\partial f_k}{\partial n} \end{array} \right) \left( \begin{array}{cc} \sigma_A^2 & \sigma_A n \\ \sigma_A n & \sigma_n^2 \end{array} \right) \]

\[ = \left( \sigma_A^2 \frac{\partial f_k}{\partial A} + \sigma_A n \frac{\partial f_k}{\partial n} \right) \left( \sigma_n^2 \frac{\partial f_k}{\partial n} + \sigma_A n \frac{\partial f_k}{\partial A} \right) \]
Model setup

• Assume perfect model and observations

• Identical twin experiments
  - reference solution generated using Gaussian initial data and parameter values $A = 0.002 \, ms^{-1}$ and $n = 3.4$

• Use incorrect model inputs
  - inaccurate initial bathymetry
  - inaccurate parameter estimates

• 3D Var algorithm is applied sequentially
  - observations taken at fixed grid points & assimilated every hour
  - the cost function is minimized iteratively using a quasi-Newton descent algorithm

• Covariances
  - $B_{zz}$ fixed
  - $B_{zp}$ time varying
without data assimilation

with data assimilation …
without data assimilation

with data assimilation …
$A_{\text{true}} = 0.002, A_{\text{est}} = 0.02$

$n_{\text{true}} = 3.4, n_{\text{est}} = 2.4$
$A_{\text{true}} = 0.002$, $A_{\text{est}} = 0$

$n_{\text{true}} = 3.4$, $n_{\text{est}} = 4.4$
The top graph shows the estimated values of a variable $A$ over time (hours). The true value $A_{\text{true}}$ is 0.002, and the estimated value $A_{\text{est}}$ is 0.02.

The bottom graph shows the estimated values of another variable $n$ over time (hours). The true value $n_{\text{true}}$ is 3.4, and the estimated value $n_{\text{est}}$ is 2.4.
Summary

• Presented a novel approach to model parameter estimation using data assimilation
  - demonstrated the technique using a simple morphodynamic model

• Results are very encouraging
  - scheme is capable of recovering near-perfect parameter values
  - improves model performance

• What next …?
  - can our scheme be successfully applied to more complex models?
  - can we say anything about the convergence of the system?
Questions?
Simple Models of Changing Bathymetry with Data Assimilation
Department of Mathematics, University of Reading
Numerical Analysis Report 10/2007*

Data Assimilation for Parameter Estimation with Application to a Simple Morphodynamic Model
Department of Mathematics, University of Reading
Mathematics Report 2/2008*

Variational data assimilation for parameter estimation: application to a simple morphodynamic model
Submitted to Ocean Dynamics PECS 2008 Special Issue*

*available from http://www.reading.ac.uk/maths/research/
\begin{align*}
A_{\text{true}} &= 0.002, \quad A_{\text{est}} = 0.02 \\
\end{align*}

\begin{align*}
n_{\text{true}} &= 3.4, \quad n_{\text{est}} = 2.4 \\
\end{align*}