Issues in High Resolution Data Assimilation

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Met Office Website
NCEO: Delivering world-class science by unlocking the full potential of Earth Observation to monitor, diagnose and predict environmental and climate change, and ensuring that scientific advances translate into public good.
T1: EO for climate diagnosis and prediction (Integrating Theme)
- Exploiting EO to improve national capability for climate prediction over timescales from months to decades
- Reducing and quantifying uncertainty

Leaders: Prof K Haines (Reading)
Prof A Slingo (Reading)
Dr S Laxon (UCL)
Prof P Cox (Exeter)

T2: Monitoring, diagnosis, re-analysis and prediction of the global carbon cycle
- Understanding the feedbacks between physical and biological processes involving the carbon cycle, in order to predict changes in carbon fluxes at the Earth’s surface

Leaders: Prof S Quegan (Sheffield)
Prof J Allen (RIHU)

T3: Atmospheric composition: air quality and climate
- Developing an integrated approach to the analysis of satellite measurements, to provide new information on atmospheric composition and aerosols for air-pollution forecasting and testing climate models

Leaders: Dr B Kerridge (RAL)
Prof M Chipperfield (Leeds)

T4: High resolution predictions of hazardous weather, floods and water resources
- Developing capability to forecast hazardous weather and hydrological consequences
- Understanding multiscale dynamics
- Developing novel assimilation techniques for highly non-linear processes

Leaders: Prof R Gurney (UoF Reading)
Dr S Dance (UoF Reading)

T5: Cryosphere and polar oceans
- Using new EO data to quantify changes in the mass balance of the cryosphere and to develop new models to represent the relevant processes in coupled climate prediction models
- Determining the impact of polar melt on the circulation of the ocean

Leader: Dr S Laxon (UCL)

T6: Dynamic Earth and geo-hazards
- Using global satellite measurements of the Earth’s surface and volcanic gas emissions to advance knowledge of processes responsible for earthquakes, tsunamis and volcanoes, and hence developing better warning systems

Leader: Prof B Parsons (Oxford)

T7: Data assimilation and treatment of uncertainty (Cross Cutting Theme)
- Developing the theory of data assimilation, including methods to test data and model uncertainty, to underpin applications in NCEO and partner agencies
- Promoting collaboration with groups funded by the EPSRC on research into the underpinning theory

Leader: Prof I Roulstone (Surrey)

EO informatics (Underpinning Theme)
- Exploiting and developing e-science and new data informatics technologies in order to make EO data and derived products from multiple data centres more easily accessible, and to ensure proper archiving of data in line with NERC policy

Leaders: Dr B Lawrence (RAL)
Dr G Robinson (Reading)
T1: EO for climate diagnosis and prediction (Integrating Theme)
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P. A. Stingo (Reading)
Dr. S. Laxon (UCL)
Prof. P. Cox (Bristol)

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Outline

• Motivation
• Challenges
• Multi-scale Modelling
• Summary and Outlook
Hazardous weather and flooding

- Observations
- Data assimilation
- Hydrological model
- NWP model
• New observation types providing detail on required scales

• Operational storm-scale (1.5 km) limited area models now expected – possibly higher resolution in future.

• Improvements in hydrological models, including increased interest in the use of more sophisticated data assimilation techniques.
Data assimilation on convective scales is a **NEW** problem – very different in character from assimilation on synoptic scales.

What are the challenges?
Challenges

1. Observations
2. Background Covariances
3. Multi-scale Dynamics / Coupled Systems
4. Nonlinearity and Uncertainty
5. Model Reduction
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1. Observations
2. Background Covariances
3. Multi-scale Dynamics / Coupled Systems
4. Nonlinearity and Uncertainty
5. Model Reduction
Multi-Scale Dynamics

Strong dynamical forcings and feedback exist between synoptic and storm-scale systems. In high resolution convective models:

• need to update fine-scale information while preserving large scale information

• need lateral boundary conditions for nested limited area models from synoptic-scale data

• need to retain rapid convergence of all important scales in the optimization algorithm
Question:

How are different scales treated in a LAM?

- Study aliasing problems in limited area models: examine how different wave lengths are projected onto the limited area analysis, using a simple nested advection-diffusion model.

- Examine methods for combining longer wave-lengths from the global model with shorter wave-lengths from the LAM.
Model

The 1D linear advection-diffusion equation

\[ u_t + c u_x = \partial \ u_{xx} \]

with periodic boundary conditions for the parent model and the parent analysis for the LAM boundaries. Discretization is explicit time, up-wind advection and centred diffusion.

In buffer zone:

\[ u_{i}^{new} = (1 - \omega)u_{i}^{L} + \omega u_{i}^{G}, \quad \omega = 1 - [(i-1)/b] \]
Assimilation System

- Uses 4DVar
- Transforms to spectral space using double-sine control variable transform
- Perfect observations at all points
- LAM boundary conditions from parent analysis
- Davies Relaxation at LAM boundaries
- High Resolution LAM = 4 x parent
- High Resolution truth = 2 x LAM
Experiment 1: Long and short waves

truth: \(2\sin(x/4) + 2\sin(2x) + \sin(8x) + \sin(16x)\)

LAM domain

Power spectrum

- o observations
- --- truth
- --- parent analysis
- --- LAM analysis
Summary:

• Higher resolution allows higher wave-numbers to be captured by the LAM

• A large proportion of the “long wave” information is aliased onto wave-number $k=1$

• Some “long wave” information is aliased onto higher wave-numbers

• These conclusions can be shown to hold mathematically for a general case using discrete Fourier transforms
Assimilation in Spectral Space

\[ J(x_0) = \frac{1}{2} (x_0 - x_b)^T B^{-1} (x_0 - x_b) \]

\[ + \frac{1}{2} \sum_{k=0}^{t} (y_k - h_k(x_k))^T R_{k}^{-1} (y_k - h_k(x_k)) \]

sine transform: \( x = Uz \), \( \Sigma^{-1} = U^T B^{-1} U \)

\[ J(z_0) = \frac{1}{2} (z_0 - z_b)^T \Sigma^{-1} (z_0 - z_b) \]

\[ + \frac{1}{2} \sum_{k=0}^{t} (y_k - h_k(Uz_k))^T R_{k}^{-1} (y_k - h_k(Uz_k)) \]
Background Matrix B

\[ \Sigma_1 = \text{diag}\{1.0, 0.5, 0.1, 0.01, 0.005\} \]

\[ \Sigma_2 = \text{diag}\{0.005, 0.01, 0.1, 0.5, 1.0\} \]

Correlation structure

Red = \( B_1 \)
Blue = \( B_2 \)
Different weightings in spectral space on background

$LAM$ domain

Power spectrum

o observations  --- truth  --- parent analysis  --- LAM analysis
Conclusions

- Wave-lengths shorter than the resolution of the global models can be analysed in the LAM, but longer wave-lengths may be incorrectly represented due to aliasing.

- Weighting global background differentially in spectral space can affect scales analysed in LAM model.

Further Work - ???
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- Test methods for combining long wave information from Global models with high frequency information from the Lam via control variable transforms more generally
- Improved treatment of boundary conditions
- Scale-dependence of 4DVar convergence
Multi-scale systems (2)

- **Convergence** of the inner loop of the Met Office incremental 4DVar data assimilation system at different Fourier scales has been analysed. Multi-level optimization methods are planned for development.

- **Conditioning** of the linearized minimization problem as a function of the length-scales in the background covariances and as a function of the observation variances.
Fourier spectrum of pressure increment at lowest model level as inner loop converges.

- Faster convergence at large and small scales
- Slower convergence at intermediate scales
Conditioning of 3DVar

Condition Number of \((B^{-1} + HR^{-1}H^T)\) vs Length Scale

Periodic Gaussian Exponential

\[
B_{ij} = \sigma_b^2 \exp \left( \frac{-r_{i,j}^2}{2L^2} \right)
\]

Blue = no obs  Red = with obs variances 0.1 / 0.2

Laplacian 2\textsuperscript{nd} Derivative

\[
B^{-1} = \gamma^{-1} \left( I + \frac{l^4}{2\Delta x^4} (L)^2 \right)
\]
Results:

• The Met Office inner loop converges more slowly at mid-wave-lengths. Multigrid approach might improve rates.

• Conditioning of inner linear system decreases with the length scales in the background error covariance matrix.

• Conditioning is improved by the addition of the observations

Haben et al., *Internal Reports*, 2009
The Discrete Fourier sine Transform of a function $f_j$ is

$$f_k = \sum_{j=1}^{N-1} f_j \sin(\pi j k/N)$$

where $k$ is the wavenumber and $N$ is the number of gridpoints.