NOTES
UNIQUENESS IN CHESS STUDIES

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ABSTRACT
Van der Heijden’s ENDEGAME STUDY DATABASE IV, HhdbIV, is the definitive collection of 76,132 chess studies. In each one, White is to achieve the stipulated goal, win or draw: study solutions should be essentially unique with minor alternatives at most. In this second note on the mining of the database, we use the definitive Nalimov endgame tables to benchmark White’s moves in sub-7-man chess against this standard of uniqueness. Amongst goal-compatible mainline positions and goal-achieving moves, we identify the occurrence of absolutely unique moves and analyse the frequency and lengths of absolutely-unique-move sequences, AUMSs. We identify the occurrence of equi-optimal moves and suboptimal moves and refer to a defined method for classifying their significance.

1. INTRODUCTION

In a Chess Study, White is required to achieve a stipulated goal, win or draw: the solution should be essentially unique, if not absolutely unique as in a crossword or a Sudoku puzzle. Variants of the solution are duals and range from ignorable through shades of significance to fatal. Sub-7-man (s7m) errors of stipulation or mainline solution in HhdbIV were reviewed in our first note (Bleicher et al., 2010). Ignoring positions and moves failing to achieve the goal, this note reviews the moves from s7m wtm mainline positions where the Nalimov Depth To Mate endgame tables, DTM EGTs, provide the definitive truth (Nalimov et al., 2000).

In such positions, two key questions arise:

a) is there a unique optimal move (UOM) or a choice of equi-optimal moves (EOMs), and
b) in Win Studies, are DTM-suboptimal moves also available?

These two questions define a two-dimensional partition of the positions into four sets, see Figure 1:

1) AUM: there is an absolutely unique successful move,
2) UOM+: there is exactly one DTM-optimal and at least one DTM-suboptimal move,
3) OEOm: there are at least two DTM-equi-optimal moves and no others, and
4) EOM+: there are at least two DTM-equi-optimal moves and at least one DTM-suboptimal move.

![Figure 1](image-url)

**Figure 1.** Four sets of s7m wtm positions, partitioned by availability of optimal and DTM-suboptimal moves.

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Figure 2. Illustrative positions and longest s7m drawing/winning AUMSs

Absolutely unique moves are the most satisfying aesthetically but essentially unique moves come more than an honourable second as they meet the technical requirements of studies. When suboptimal moves are available, the question is whether they are merely time-wasting moves (TWMs) or not, e.g., moves allowing Black to force a goal-oriented White back to a previous position. If so, they may be considered unambiguously inferior to a move which makes progress towards the study’s goal. In Figure 2a (bottom right), at position 8w (White’s 8th move) of the 1924 Réti-Mandler KNPKPP study, the moves Kd1 and Ne5/Ng1/Ng5/Nh4 are DTM-suboptimal and therefore TWM candidates. Their status as TWMs is easily determined.²

More subtly, if there are non-TWM options into sidelines, one might ask whether these are significantly different from the mainline. In Figure 2a, are 1.Qb7# and 1.Qe8# significantly different? It is a matter of taste but with an extension of chess notation, there would at least be only one line to write: 1.(Qb7/Qe8)#. Can other lines be forced to converge quickly with the mainline by a Black wishing to do so? One might argue that the sooner they converge, the less significant they are. Still in Figure 2a, are the lines 1.Nf8 g5 2.Ng6# and 1.Ne5 g5 2.Nf7/Ng6# significantly different? Again, extending chess notation with [...] meaning any available move, one line suffices: 1.(Ne5/Nf8) g5 2.(Nf7/Ng6)#.

Table 1. A statistical profile of the four sets of positions.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Length (s7m wtm positions compatible with the goal)</th>
<th>Wins</th>
<th>Draws</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUM</td>
<td>150,649</td>
<td>59,409</td>
<td>13,186</td>
<td>234,634</td>
</tr>
<tr>
<td>UOM</td>
<td>25.3%</td>
<td>4.9%</td>
<td>5.6%</td>
<td>100.0%</td>
</tr>
<tr>
<td>OEOM</td>
<td>64.2%</td>
<td>7.2%</td>
<td>100.0%</td>
<td></td>
</tr>
<tr>
<td>EOM+</td>
<td>9.832</td>
<td>75.918</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>150,649</td>
<td>59,409</td>
<td>13,186</td>
<td>234,634</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th># of 'sideline' DTM-equi-optimal moves</th>
<th>26,718</th>
</tr>
</thead>
<tbody>
<tr>
<td>in Win Studies</td>
<td>17,509</td>
</tr>
<tr>
<td>in Draw Studies</td>
<td>22,580</td>
</tr>
<tr>
<td># of DTM-suboptimal moves (win studies only)</td>
<td>65,235</td>
</tr>
<tr>
<td>DTM-suboptimal move played</td>
<td>1,665</td>
</tr>
</tbody>
</table>

Table 2. Counts of Absolutely-Unique-Move Sequences of various lengths.

2. A STATISTICAL PROFILE OF UNIQUENESS

Table 1 provides some statistics about both the positions in the four position-sets defined above and the moves from those positions.

Some 24,576 wtm positions offer a total of 44,227 DTM-equi-optimal moves off the authors’ mainlines, and 70,799 wtm positions offer a total of 320,579 DTM-suboptimal moves into sidelines. These moves may be further analysed by an as yet unimplemented algorithm (Haworth, 2009) to see if the equi-optimals are essentially equivalent to the mainline move, and if the suboptimal moves are merely TWMs. The total of 364,806 moves is not particularly formidable: many can be classified in seconds: we look forward to a production attack on this challenge. The algorithm defines appropriate chess variants Chess(P) to define the impact of deeming the positions in set P to be worth a ½-point less to White than they actually are.

In 9,832 Win Study positions, White plays a goal-preserving but DTM-suboptimal move which may nevertheless be the best move. Most but not all of these studies must be considered unsound because of the DTM-optimal alternative move. Many of these moves had not been found before HHdbIV.

Table 2 gives the number of AUM-sequences of various lengths in Win and Draw Studies. Perhaps not surprisingly, the Draw Studies’ longest AUMSs are longer than the Win Studies’ AUMSs, and there are at least as many drawing AUMSs as winning AUMSs for all lengths greater than five. All have an astonishing fascination and one has to wonder at their composers’ ingenuity. Finding these tightrope lines in EGTs (Bleicher, 2011) is a challenging task and clearly, many did not even have such a source.

The longest AUM sequence in a draw study is in Van diest (1981), HHdbIV#49882, q.v. Figure 2b. KQPKQB, 2K5/8/1k3P2/2q4/8Q4/8Q3/8Q3/8Q2/8K3/2Rb3 w:


Helmut Conrady mined some 5-man EGTs in 2003 specifically to find the longest AUM sequences: he published eight, q.v., HHdbIV#70230-7, including the longest known, HHdbIV#70232, q.v., Figure 2c. KRKPN, 8/7/p7/7/7/2K5/3R4 w:


References


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3 For example, a class of moves quickly seen as TWMs include many where White safely reverses its previous move.

4 e.g., Q7/8/5K/7p/7p6k1 w, position 7w in Aloni Study HHdbIV#31073: White and Black are in effect minimaxing moves to KQK rather than to mate or conversion of force. Thus, White plays 7.Qa1+ rather than 7.Qg8+.

5 As DTM is not the only metric, there will be positions where DTM-suboptimal but DTx-optimal moves are best.

6 With format (HHdbIV study index, AUMS length and start of AUMS), they are: (49882, 43, 8), (71205, 37, 1), (71968, 36, 12), (69257, 35, 5), (75809, 34, 1), (75715, 33, 2), (68418, 26, 1), (38012, 25, 1), (62681, 23, 6), … (70232, 23, 1), (70230, 20, 1), (75845, 18, 1), (17426, 15, 1), (69287, 15, 1), (72829, 15, 3), (76116, 15, 1), …