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Real Option Pricing in
Mixed-use Development Projects

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Real Option Pricing in Mixed-use Development Projects

Abstract

The application of real options theory to commercial real estate has developed rapidly during the last 15 Years. In particular, several pricing models have been applied to value real options embedded in development projects. In this study we use a case study of a mixed-use development scheme and identify the major implied and explicit real options available to the developer. We offer the perspective of a real market application by exploring different binomial models and the associated methods of estimating the crucial parameter of volatility. We include simple binomial lattices, quadranomial lattices and demonstrate the sensitivity of the results to the choice of inputs and method.

1. Introduction

Several pricing models have been applied to real estate developments to value embedded real options. All models, looking either at the valuation of development projects or at the interaction of players in the development market, have suggested that several factors may determine a higher/lower value of these embedded options. In this paper we specifically apply a real options framework to a case study setting and discuss the problems and the *de facto* solutions that might be adopted to solve them. We use three main real option models: the first includes only one stochastic variable (i.e. selling price); the second one extends it to two stochastic variables (i.e. selling price and construction costs) through a quadranomial tree with correlation between the component variables; finally the third model combines the two previous approaches using the net present value of selling price and construction costs as the only stochastic variable. For each model we compute sensitivity factors to changes in interest rates, volatility estimates, correlation between selling price and construction costs, number of steps within each period and maturity of the project.

Specifically, we are examining a case of mixed-use development in which the explicit option is to defer development. We also consider the embedded put option to sell the land. This option only adds value when the put arises from a guaranteed price agreed between the developer and the local authority granting planning permission and it differs from the potential market price of the land. If there is no asymmetric cash flow the value of the put option would be equal to zero because it would be already embedded in the initial valuation (reflecting the residual value of the land and the building at completion).

The article is structured as follows: in the following section we discuss some of the issues involved in adopting a real options approach to investment decision making in real estate. The detail of the case study is discussed in section 3, while section 4 presents five different valuation models. Finally, in section 5, 6 and 7 we discuss main results, suggest five different models to estimate volatility parameters for development projects, and conclude suggesting some possible extensions of our work.

2. Literature Review

Since the early research by Titman (1985), Grenadier (1992), Williams (1991, 1993), researchers have developed several pricing models to determine the implied value of real estate investments with embedded real options. The literature has been extensively reviewed by other authors and we do not present details of the individual papers but instead focus on the some topics which are particularly relevant when real option models are applied to development projects: interest rates, volatility estimates, correlation between selling price and construction costs, time to maturity and number of steps within each year (to increase the number of steps makes the binomial lattice more similar to a continuous time framework.

Firstly, Fernandez (2001) argues that applying models like the Black-Scholes' (1973) model to real projects requires that the option can be replicated. If that is the case, the option can be valued a contingent claim approach by using the risk free rate as both the discount rate and the total return expected on the underlying asset. In the absence of replication the option must be valued by a risk-adjusted discount rate and, even worse, the underlying asset return should be the return expected by the investor – giving rise to the possibility of different valuations by different investors. This is a serious concern for those using the options approach as it is usually rounded in arbitrage arguments and is used precisely because it minimizes investor's forecast returns and yields a value that is invariant to the expected return on the underlying asset.

An alternative view taken by Copeland and Antikarov (2001) is to assume that the basic investment project can be traded[4]. This approach is convenient because it not only provides the base case appraisal, but it can also be applied to generate option values using either a risk-neutral approach or by explicit replication. In practice, it would be impossible to use this approach to create portfolios but Copeland and Antikarov argue that it is a practical solution to the theoretical challenges of real options analysis. Indeed, in many of the more applied discussions of real options applications, it is argued that the most important result comes more from a correct framing of the problem as against a precise formulation and model valuation.

Several studies present a simulation of different option values obtained with different volatility estimates. It is common practice to assume that the underlying asset can be represented by traded securities such as equities. Patel and Paxson (2001) use daily returns data from the call options of the equity of two companies, the first – Land Securities representing the volatility of an office property investment and the second, Tarmac, representing the volatility of construction contracts. In their model, they required not only the different volatilities but also the correlation between the activities. Using daily returns to generate estimates of correlations would in our view tend to produce lower estimates than the correlation measured over longer intervals over the periods (1999-2009) covered. This bias would seem to magnify the option value of compound options since the value would be positively correlated with volatility and negatively with the correlation between the two factor variables. As Fernandez (2001) observes, many applications of real options assume a high volatility and because of the mathematical property that increasing volatility increases the value of options (*ceteris paribus*), the assumption leads to the odd result that the best investments would be shown to be associated with the

[4] *"…we make the Marketed Asset Disclaimer assumption that we can estimate the present value of the underlying without flexibility by using traditional net present value techniques"* (p. 111).

greatest uncertainty. This is further discussed below in covering the methods of estimating volatility.

Somerville (2004) finds that planning permissions, starts and completions are cross-predictable. Consequently, once planning permissions are granted, there is a high likelihood of development projects to be started. Since the investment project in our analysis had already planning permission and the split between single uses had been defined, we do not consider the impact of the correlation between different sectors allowing for switches between different uses as in Childs et al (1996). Our choice is consistent with Childs et al's results which showed that the switching option would not necessary lead developers to change the planned composition of the project. Furthermore a switching decision would not be consistent with the planning system in the UK in which, for a mixed use development project, planning permissions would normally be granted only if a specified mix of uses were specified.

The immediate choices of the stochastic processes of the series lie between arithmetical and geometrical Brownian motion. Although it is true that the variables in real estate such as rent and yields – and their combination, total returns, do not behave in a way consistent with Brownian motion, it does not follow that all real option models should seek to use more exotic stochastic processes. Extensive discussion by Copeland and Antikarov (2001) of Samuelson's proof, that the *"properly anticipated prices fluctuate randomly…"* even when the cash flows exhibit serially correlation or seasonal patterns, re-iterate the position that in an efficient market, prices and returns do not reflect the autoregressive character of cash flows. The problem that arises in real estate private markets therefore arises because of the absence of trading and the consequential dampening of apparent market volatility causing returns to retain some element of seasonality and autoregressive characteristics. In this paper we distinguish between the stochastic behavior of variables such as rent and construction costs, which may exhibit large positive auto-regressive characteristics, and the returns from an assumed-publicly traded asset on such cash flows, by assuming that the traded assets would exhibit no serial correlation in returns.

Malchow-Moller and Thorsen (2005) argue that with repeated real options (as for options to defer development), the value of waiting is smaller and less sensitive to parameter change. In contrast, we find that the value of embedded options in real estate development project is relatively high and the smaller sensitivity is only true for some parameters included in our model – e.g. interest rates are not significant as found by Capozza and Li's (2001).

Finally, we recognize that a decision to delay the project may have consequences falling from actions taken by competitors. Considering the work of Grenadier (2000) and Smith-Ankum (1993) on game theory, Grenadier (1999) on information revelation when the option is exercised and Trigeorgis (1991) and Grenadier (1996, 2002) and Childs et al (2002) on pre-emption risk, we leave the discussion about the impact of competition on development appraisal to another paper.

3. Case study

We take an undeveloped town centre site of approximately 6 acres adjacent to a major public transport interchange. The site is in Croydon, a large commercial centre about 10 miles south of the centre of London. A comprehensive mixed use scheme has been granted planning permission comprising: a supermarket (83,455 sq ft); retail units (68,348 sq ft); restaurants and bar (83,110 sq ft); health club and swimming pool (48,355 sq ft); Night Club (40,006 sq ft); Casino (25,867 sq ft); Offices (135,791 sq ft); and car parking (500 spaces);

An investment fund acquired some of the site as part of a portfolio acquisition at a cost of $8m (reflecting the development potential). It also inherited option agreements with other landowners (to assemble the site) which would show a total site acquisition cost of $12.75m to be able to implement the scheme;

There are some costs involved in holding the property and keeping the options open with the other landowners of approximately $150,000 p.a., but these are counterbalanced by continuing income from car parking on the site. Since the margin is relatively small, we assume that the underlying project neither generates nor costs money in deferment (other than the financial costs represented by discounting).

The local authority wishes to see the site comprehensively developed for the scheme and have granted permission for the specific development. They also have a long held objective of developing a public arena or exhibition space in the centre of Croydon. Under an agreement with the investor in conjunction with granting the planning permission, the local authority has said it would acquire the land at a fixed price of $8m at any time up to 5 years from grant of planning permission should the investor wish to sell i.e. not implement the scheme. Thereafter the local authority would acquire the site using compulsory purchasing order powers if the development were not implemented. Compensation to the fund in such circumstances has been calculated at $5m.

4. Valuation models

Three approaches are used to value the above development: a traditional approach as adopted by UK professional real estate investors, a conventional DCF analysis and a real options approach.

4.1 UK "professional" valuation

In the UK the traditional approach to development appraisal combines a cash flow model (with financing cash flows) with a compounded future value calculation. Thus, the initial model of the case study starts with the initial costs of assembling and preparing the site for construction. Since these cash flows are assumed to be easily audited, finance costs are assumed at a low rate of interest (in the initial case, 6%). A traditional mark-up is then added to the land cost as a "normal profit" component of cost. In this example, 10% was used. The construction is then costed, quarter by quarter, and cash flows are then identified over the life of the development along with associated finance costs (using a higher rate of 7% to reflect higher risk). On completion, the "normal profit" margin for the developer (17.5% of the total construction costs) is then added to the accumulated construction plus finance cash flows to arrive at the overall ("normal profit") costs of the development. Mention should also be made of a contingency margin of 2.5% on construction costs to allow some additional slack in the budgeted cost plan. The selling price is derived using forecasted market variables: space utilization, rental values and cap rates for different uses. The difference between the selling price and the compounded overall (interest "normal profit") cost is then calculated and in economic terms can be identified as either an economic rent for the project or as a contingency reserve for the construction. In financial terms, this is equivalent to finding the net future value of the project, applying a flow-to-equity approach[5].

One important feature of this calculation is that the profit, the selling price and the accumulated costs are calculated as at the completion of the project. In contrast, a more conventional financial analysis would use a present value perspective and discount the cash flows to the start of the project rather than compound them to the date of completion.

[5] To financially literate investors, this approach suffers from the lack of identifying the required cost of equity. It is by no means clear that the added margins on land and construction costs are sufficient to satisfy the required return on a project that is very highly levered.

4.2 DCF valuation model

In order to arrive at consistent estimates of the unlevered returns on the project we recast the professional model in a more conventional financial framework. The above mixed-used development is appraised with a "traditional" valuation using discounted cash flow modeling. The analysis takes a present value view and discounts the expected cash flows back to the start of the project using a weighted average cost of capital (i.e. WACC) – in this case, 9% was assumed. The cash flows used did not include the financing costs, or the “normal profit” margins used in the traditional professional approach. The contingency margin of 2.5% was included on the grounds that it might be assumed to be expected as an actual cash flow.

Of course, the initial result of this exercise is to reveal that there is no obvious translation between the numbers arrived using the professional approach and those derived from rigorous financial practices. Close correspondence would be unexpected because of the use of multiple and different discount rates in the professional approach, although some approximate coherence could be shown by discounting the appropriate cash flows from the professional approach to a present value at the project start date. Since the figures at the completion date computed with the professional approach used a flow-to-equity method, to obtain the corresponding present value at time zero, we should discount both the selling price and compounded costs at the cost of equity. In our case study, we show that the same NPV would be produced by using an equity rate of 20.2%.

In the financial model the value of the overall project is computed through the following equation:

$$Value = \sum_{t=1}^{n} \frac{NCF_t}{(1+WACC)^t} + \frac{SP_n}{(1+WACC)^n}$$

where NCF_t is the cash flow generated in each period, *WACC* is the appropriate discount rate and SP_n is the selling price of the property at time n, computed as capitalizing the estimated sale value of the property. The figure at the numerator represents the net cash flow and it is obtained as follows:

$$NCF_t = OTINC_t - LANDC_t - DEVC_t$$

where $OTINC_t$, $LANDC_t$ and $DEVC_t$ refer respectively to Income generated, land acquisition costs and development (construction) costs, all at time *t*.

In this approach, financing costs are ignored on the grounds that without tax shields, there is no added value from project leverage. Questions of budget viability and cash draw-down facilities are best left to cash flow analysis and the treasury function. Inclusion of the financing issues into the investment appraisal causes the project analysis to be distorted by cash management issues.

There is an added benefit of adopting the financial analytical approach in that it is the basis of one method of calculating the volatility of returns using the approach advocated by Copeland *et al*. briefly discussed above. We will return to this topic in discussing different approaches to deriving estimates of the volatility of returns on the underlying asset.

4.3 Real option models

McDonald and Siegel (1986) were among the earliest advocates of a real-option approach to project investment appraisal. But applications of real options expanded greatly with the recognition that the binomial approach facilitated a greater intuitive understanding of real options without a huge sacrifice in rigor. As Trigeorgis (1996, p. 337) writes, *"Lattice approaches emulate the dynamics of the underlying stochastic processes and are generally simpler, more intuitive and* *practically more flexible in handling different stochastic processes …option payoffs early exercise or other intermediate decision and optimal policies, several underlying variables, etc."*

Within the Cox, Ross and Rubinstein (1979) framework, we define the risk-neutral probability within the lattice:

$$p = \frac{e^{(r-q)(\delta t)} - d}{u - d} \tag{3}$$

where *u* and *d* represent respectively the up and down movements on the lattice, *r* and *q* are the risk free rate of return[6] and "dividend payment"[7]; *σ* is the volatility of returns on the underlying asset; and *t* is the time period to expiration. The above can then be applied to create a lattice with which the decisions to defer development or to sell the land back to the local authority can be integrated and valued.

Adjustment for risk can be achieved in two ways: either the cash flows can be discounted by a risk adjusted discount rate or the expected cash flows can be adjusted by using risk-neutral probabilities and discounting by the risk-free rate. In our model we adopt the second approach. This methodology is also preferred in options analysis because it reduces the risk of estimation errors of a risk-adjusted rate (Mun, 2002).

The essence of market replication underlying financial option theory is that there are no arbitrage possibilities as assets (derivatives) are freely traded (and therefore liquid). In real options the assets are by definition "real", firm specific, illiquid (as in the case of real estate) and as a result complex to replicate and

[6] Within the assumption of project duplication, the rate of interest is the risk-free rate of interest. Without that assumption, it should reflect the cost of capital for the underlying asset.

[7] It represents leakages or the opportunity cost of maintaining the option open.

mathematically model. The risk neutral method adjusting cash flows, however, assumes that investors have access to assets with the same risk characteristics (i.e. beta) as the capital investment being evaluated. The real option prices of (not necessarily existing) "other assets" are identically to that being evaluated. Consequently the composition of an arbitrage portfolio made by a proportion of the same-beta assets and of lending/borrowing may be difficult to obtain. The pragmatic solution advocated by Copeland and Antikarov (2001) is to value the static base case project using the appropriate risky discount rate, then to assume that the value constitutes a market-traded asset which will then change in value assuming GBM, with returns forming an ABM process.

Having decided to use a binomial model, there are still two main ways of creating the model. One is to separate the two types of cash flows (construction costs and selling price) and model them separately in the binomial lattice, thereby arriving at the value of instantaneous development at each point in time. The second is to calculate the present value of the project at each time period using an estimated volatility of the return from the static project, following the procedures outlined in Copeland and Antikarov (2001) [8].

Within the first approach we can adopt two different procedures regarding the modeling of the cost cash flows. For both procedures we build a binomial model by creating four linked binomial lattices. The first comes from calculating the present value of the project if started at time 0. This is then expanded into a lattice (A) by assuming a volatility of returns on the underlying asset (development value) and the assumption of the opportunity costs of waiting expressed in the form of dividend on the underlying value – if the land is left undeveloped, its value would fall by the expense of taxes and maintenance but be increased by the generation of income from the use of the undeveloped land (car park). In the present case, it is assumed that these two effects are

[8] As Holland et al (2000) notice, *"changes in price volatility more quickly summarize information […] than do changes in observed price levels"*.

approximately the same in the first instance thus no dividend is assumed. The second lattice (B) is based on the construction costs. Within the first and simplest assumption (1.a), the cost function was deterministic over time. It then followed that the third binomial lattice (C) could reflect the difference between each node of Lattice A and the corresponding node of Lattice B to derive the net value of development at any point. The fourth lattice then compared the net value of development at any point with the alternative of either waiting a further period (holding the call option) or selling the land back to the local authority (exercising the put option).

The more realistic variation (1.b) on the first procedure is to assume a stochastic cost function that evolves with an assumed correlation with the value of development (a reasonable assumption given the influence of common factor variables such as inflation), This procedure involved building a quadranomial lattice, using the approach of Clewlow and Strickland (1998), in which the three-dimensional lattice uses the Independent volatilities of the development values and the cost function as well as the correlation between the two values. Using this approach allows us to demonstrate how sensitive the option values are to underlying factors of interest as well as to explore more finely detailed lattice constructions.

The second approach is to model the volatility of the returns on the base case project using the Monte Carlo method advocated by Copeland and Antikarov. The advantage of this approach is that the binomial lattice is relatively simple although the preparation required is slightly greater since it involves running Monte Carlo simulations of the base case project in order to generate plausible estimates of the returns on the project.

Cash flows and project value

If project cash flows (i.e. CFs) follow a geometric Brownian motion (GBM) and the value of the project (V) is proportional to the cash flows, then V also follows a GBM with the same parameters of CF.

Consider for example the following risk-neutral GBM for cash flows and the equation for V:

$$dP = \left(r - d\right) P \; dt + s \; P \; dz$$
$$V = qBP$$

By applying the Itô's Lemma – Dixit & Pindyck (1994) – to V (P, t):

$$dV = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial P} dP + \frac{1}{2} \frac{\partial^2 V}{\partial P^2} \left(dP\right)^2$$

However,

$$\frac{\partial V}{\partial t} = 0 \, ; \qquad \frac{\partial V}{\partial P} = qB \, ; \qquad \frac{\partial^2 V}{\partial P^2} = 0$$

Hence,

$$dV = qB\left(dP\right) = qBP\left(r - \delta\right)dt + qBP\sigma \; dz$$

Finally, we obtain a similar risk-neutral equation for the stochastic process of V:

$$dV = \left(r - \delta\right) \; V \; dt + \sigma \;\; V \; dz$$

If (Copeland and Antikarov) the contingent claim is expressed in terms of the log of the underlying asset, then we can derive an expression for the change in the value of the contingent claim, thus

$$dC = \left(\mu + \frac{\sigma^2}{2} \right) dt + \sigma \, dz$$

This expression defines the growth rate or percentage change in the value of the contingent claim – which is in this instance equal to the growth rate of the value of the underlying asset. Thus the growth in the value of the marketed project is normally distributed with a mean of $\left(\mu + \frac{\sigma^2}{2} \right)$ and a standard deviation of $\sigma\sqrt{t}$.

We use this result in deriving the probabilities for the quadranomial model described below.

5. Main results

In this section we present the main results of our analysis. Firstly we report the estimates obtained with the three different real option models. Finally we discuss the sensitivity of these models to different assumptions.

5.1 Consistency of results for different binomial models

The first question which might be asked of different models and approaches to quantitative problems concerns the consistency of the approaches. Obviously, it is difficult to compare simple one-factor stochastic models with the quadranomial model but the following table summarises the results of comparable models.

Table 1: Comparison of the different binomial model option valuations

	Quadranomial		Copeland-Antikarov			One factor	
Volsale	20%	30%	60%	65%	70%	20%	30%
Volcost	5%	5%					
Corr Sales/Costs	0	0					
TotalOption	43%	70%	59%	64%	69%	52%	107%
Put	10%	59%	11%	12%	12%	9%	9%
Call	33%	11%	48%	53%	57%	43%	99%

The table shows the comparison of the three real option models showing the results obtained with two different asset volatilities, 20% and 30%. In the case of the Copeland-Antikarov's (2001) model, the volatility input refers to the volatility of the returns generated using a simulation based on cash flow volatility of 20% and 30% volatility of respectively for the two 65% and 70% volatilities reported in the table.
Legend:
Volsale = Standard deviation of sale price return.
Volcost = Standard deviation of cost growth.
Corr Sales/Costs = Correlation used in quadranomial between sales and costs.
Total option = Proportion of static NPV represented by the option value.
Put and Call show the components as proportion of static NPV.

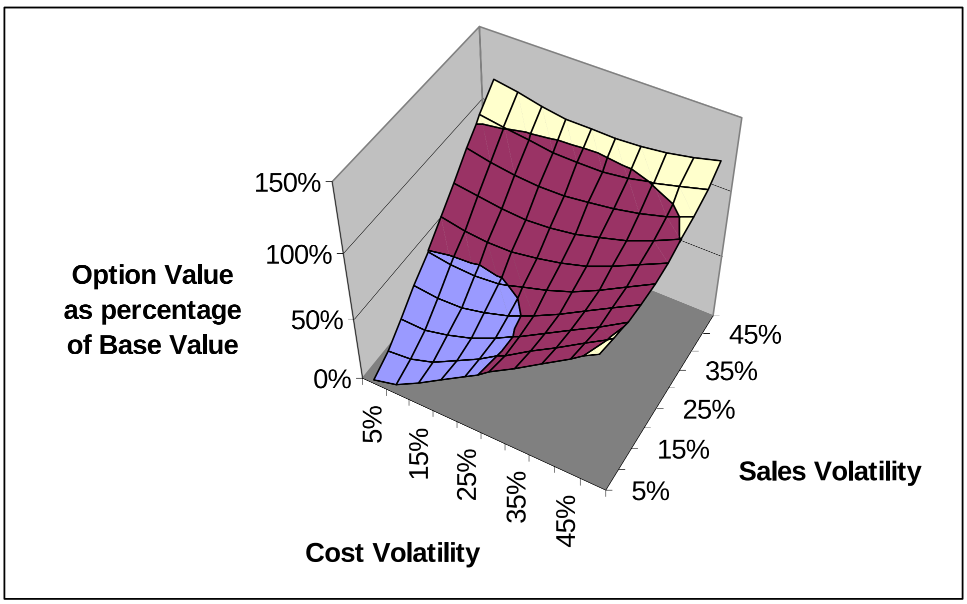
As can be seen in table 1, the major differences are (a) the quadranomial is less sensitive to changes in volatility than the simple one-factor model, (b) the Copeland model, uses a higher volatility because its asset is defined slightly differently and the volatility is estimated from a Monte Carlo simulation using the base case project, (c) the one-factor model is extremely sensitive to the volatility of the returns on the underlying asset.

To explore the sensitivity of the models more closely, we concentrate on the quadranomial approach which facilitates investigation of the component variables.

5.2 Effect of volatility on the option value

Figure 1 shows the sensitivity of the Option value to the base NPV for different values of volatility. Since we have estimated the revenue volatility to lie between 25% and 50%, the graph shows how sensitive the option value is to this range. In a real estate project, the revenue would come from the sale of the finished project and would in magnitude be much larger than the individual construction cash flows so it is not surprising that the option is more sensitive to changes in what is effectively a forecasted selling price of the completed property in several years time.

Figure 1: The effect of Volatility on Option Value



The option value is calculated using the quadranomial model as a percentage of the static value of the inflexible project. The assumed correlation between costs and revenue is 0.5.

This dependence on sale price volatility is also revealed using the combined factor (Copeland et al) approach where we build the project and simulate the returns from the project before measuring the volatility. By conducting a number of Copeland valuations for different values of volatility and auto-correlations for the construction costs, we built a small sample of observed option values. In all cases, we assumed that there was little or no correlation between the selling price of the property and the individual cost cash flows. We then regressed the estimated standard deviation of the returns against the standard deviation of the sales, costs and the auto-correlation of the construction costs. The regression is reported in Table 2 and shows that the volatility of the selling price effectively determines the standard deviation of the project returns. The effect of assuming that the variations in construction costs are auto correlated has an insignificant effect on the volatility – and thus on the option value. The sales volatility is more than twice as important as the cost volatility and this emphasises that for real estate developments, this relationship is bound to dominate.

Table 2 Regression of the project return volatility against sales price volatility, cost volatility and auto correlation of the cost cash flows.

	Parameter	Std Error	T-stat	P-value
Intercept	0.34	0.10	3.44	0.01
VolSale	1.69	0.33	5.15	0.0003
VolCost	0.83	0.40	2.06	0.06
Autocorrel	0.07	0.06	1.23	0.25
Adj-R2 = 0.65				

The table reports the parameter estimates (along with standard errors, t-statistics and p-values) of the following equation:

$VolRet = \alpha + \beta_1 * VolSale + + \beta_2 * VolCost + \beta_3 * Autocorrel + \varepsilon$,

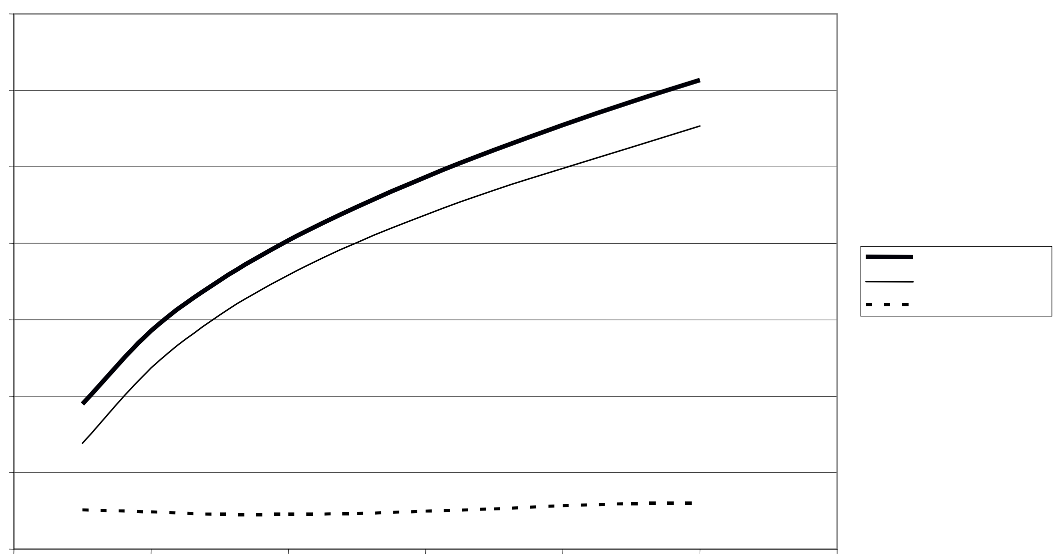
Legend:
VolRet = Standard deviation of returns from the Copeland-Antikarov’s procedure.
Volsale = Standard deviation of sale price return.
Volcost = Standard deviation of cost growth.
Autocorrel = Autocorrelation coefficient of the cost cash flows.

5.3 Effect of lengthening the life of the option

Lengthening the life of an option should increase the value of the option; if planning permission could be extended or renewed, the investor might seek negotiate a longer period in which to delay the start of the project.

Figure 2 shows that the option does indeed gain value as the maturity increases albeit at a slower rate. It is clear that the increase stems from the call option and that the effect of the put is comparatively small and insensitive.

Figure 2: Sensitivity to time to expiry.

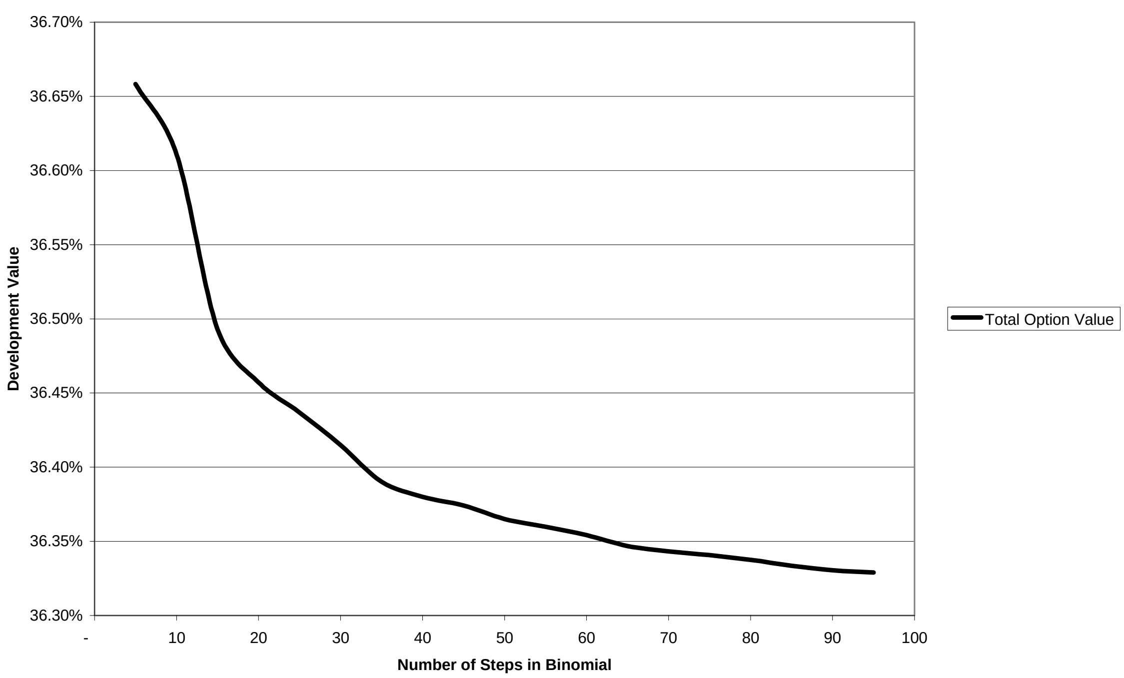


The table shows the effect of increasing the time to expiry of the option on the value of the option to defer (call) and to sell back to the local authority (put)

5.4 Effect of increasing the number of steps in the binomial lattice.

One criticism levelled at binomial lattices in valuing options and real options is that they are slow to converge and less efficient than other methods such as finite differences. In particular, a quadranomial lattice is computationally more complex than a normal binomial lattice and might be expected to cause more concern. In this paper we constructed quadranomial lattices of different number of steps from 5 (annual) to 100. The results are reported in Figure 3 and they show that the pattern of convergence is, as expected, not a smooth nor linear path, but in magnitude, increasing the number of steps does not significantly affect the valuation.

Figure 3: Sensitivity to the number of steps.



The figure shows the effect on option value of increasing the number of steps in a quadranomial lattice.

5.5 Effect of changing the interest rate on the option value

Table 3 shows that the overall effect of varying the interest rate is comparatively small but there is some change in the relative importance of the two types of option. The put option becomes less important and the call option increases in value.

Table 3: Sensitivity to interest rates.

Interest rate	**2.5%**	**5.0%**	**7.5%**	**10.0%**
Proportion of both Options	51%	51%	50%	48%
Proportion of Call Option	40%	40%	41%	41%
Proportion of Put Option	12%	11%	9%	8%

The table reports the effect of changes in interest rates on the option values of development. The figures represent proportions of the static net present value.

6. Volatility estimation

As our main results show, real option values are very sensitive to the volatility estimation. In this section we present three different models we used to estimate the volatility input in a real option model. We also differentiate between the different models and suggest a practical way to arrive at a plausible figure.

6.1 Multi-period returns

If we consider a development project and we assume that the development period is equal to three years, we can also assume that the investor will look at the “3 year ahead” return to determine if it is worth buying the land on sale. When we try to identify the risk of achieving that return, we should then refer to the same time horizon.

Consequently to estimate the volatility of the asset price in our real option model, we compute the annualized standard deviation of multi-period returns. We use IPD capital appreciation rates from 1971 to 2006 and split the sample into sub-periods of 3 years each. We then compound the annual rates within each 3 year interval and we finally obtain the standard deviation of the 3 year returns. Since our result may be sensitive to the starting year within the first interval (i.e. either 1971, 1972, 1973), we run the same procedure three times having as a start date one of the three years. We then obtain the estimate of the volatility from the average standard deviation of the three computed measures. Finally we compute the annualized volatility (i.e. *AnnVol*) by dividing the previous multi-period standard deviation (which refers to a 3 year return, i.e. *3YrVol* below) by the square root of 3 (no. of years within each interval) as follows:

$$AnnVol = \frac{3YrVol}{\sqrt{3}}$$

Since we may believe that IPD returns are not reflecting the true movement of market prices, we also run the same procedure on unsmoothed IPD capital appreciation rates.

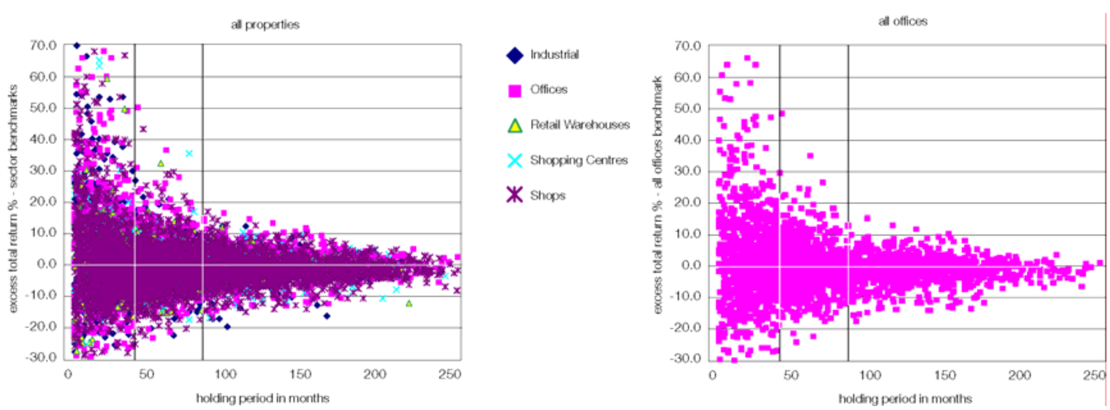
The estimated volatility using original IPD data would be equal to 13.5%, while the one obtained with unsmoothed data is equal to 16.3%.

6.2 Alpha returns

Another possible extension of this model considers market efficiency and assumes that market risk can be diversified away. Particularly with the development of real estate derivatives, hedging positions can be taken by selling real estate swaps. So, assuming developers behave rationally, they will achieve a reward only if they take on specific, but not systematic risk. The volatility associated with development activities may then be proxied by the standard deviation of alpha returns achieved by an investor with a three year investment horizon.

As in Fourt et al (2006) we use the IPD/Gerald Eve *transactions database*, containing over 21,000 properties bought and sold during the period 1983-2005. For each property we compute the annualized return considering capital expenditures between the purchase date and the sale date. We then group the properties into 3 main categories depending upon the holding period: less than 4 years, between 4 and 7 years, more than 7 years. Since developments are short-term type of investments (i.e. the investment horizon is around 3 years), we only consider properties that falls within the first category. Since we are interested in the alpha, we then subtract the market performance (by market segment to distinguish between different sectors) and compute the standard deviation of the alpha performance[9].

Figure 4: Alpha estimates by market segments.



The figure reports the alpha estimates for all properties (left hand side) and offices (right hand side) included in the transactions database. The alpha estimates are extra-return achieved above (or below if negative) the market return.

[9] It is worth noticing that in this case we already obtain an annualised volatility because we used annualised – instead of multi-period – returns

The ranges of excess returns relative to their own sector benchmarks are shown in the two scatter plot diagrams in Figure 4 for both all properties (left) and the office sector (right).

The results above show that, for holding periods within 4 years excess returns are symmetrically distributed although with a slightly bigger tail on the upside for very short holding periods.

Table 4 sets out the number of observation and provides volatility results for the five major sub sectors.

Table 4: Alpha estimates by UK market segments.

Sector						**1989-2005**	
		All Observations	Traded Observations <4 years				
		Acquired	Total Traded	% of Acquired	Av. pa	Min(yr)	Max(yr)
Offices		12,132	3,510	28.93%	206	69	367
Retail							
	Shops	16,924	4,648	27.46%	273	84	455
	SC	859	151	17.58%	9	2	28
	Retail WH	3,814	1,053	27.61%	62	4	133
Industrial		8,624	2,242	26.00%	132	40	290
Total		42,353	11,604	27.40%	683	199	1,273

Sector		Standard Deviation			
		Av.%	Min%	Max%	Spread
Offices		16.1	10.3	25.8	15.57
Retail					
	Shops	13.1	7.9	19.8	11.90
	SC	13.3	3.2	29.4	26.20
	Retail WH	9.1	4.3	19.5	15.22
Industrial		17.2	10.1	27.7	17.68
Total					17.31

The first part of the table (block above) describes the total sample of 42,353 transactions used in the analysis. The second part of table (block below) reports the main estimates of alpha volatilities for different market segments, along with both minimum and maximum values. The spread is computed as the difference between maximum and minimum values.

6.3 Copeland and Antikarov's method

The third method follows the Copeland and Antikarov (2001) procedure. We first construct the DCF valuation model of the project (see section 4.2). We then define the stochastic properties of the quarterly construction costs and the selling price of the developed property. We experimented with various distributions of the costs throughout the project. We also used different rates of serial correlation between successive cost expenditures to see how sensitive the estimated volatility was to changes in construction cost dynamics. We then used Crystal Ball to generate Monte Carlo forecasts of the present values and returns from the project[10].

$$ret = \frac{NPV_1}{NPV_0} - 1$$

Table 5 shows the estimated volatilities of returns on the base case project for selected input parameters.

The estimated volatilities appear very high relative to the other methods but in this case, our estimate is used in the binomial lattice of the combined cost and revenue functions, therefore it reflects the profit and profit margin which will be much more sensitive than the volatility of the components when modeled separately – as used in the quadranomial lattice. We have shown in the previous section, the consistency of modeling the whole project with one stochastic variable with the quadranomial approaches.

[10] We used Latin Cube sampling, with 10,000 simulations using Crystal Ball

Table 5: Copeland and Antikarov's method of estimating volatility.

VolSale	VolCost	AutoCorr	VolRO
10%	10%	0	67%
10%	20%	0	70%
10%	10%	0.9	71%
10%	20%	0.9	85%
20%	10%	0.9	72%
20%	10%	0	67%
20%	20%	0.9	85%
20%	20%	0	69%
30%	10%	0	100%
30%	20%	0.9	113%
30%	20%	0	102%

The table reports the estimates of volatility (last column) used in the Copeland and Antikarov's real option model, along with the underlying assumptions (columns 1 to 3) used in the Monte Carlo simulation.
Legend:
Volsale = Standard deviation of sale price return.
Volcost = Standard deviation of cost growth.
Corr Sales/Costs = Correlation used in quadranomial between sales and costs.
VolRO = Volatility estimate used in the binomial lattice to value the real option.

One by-product of this estimation procedure is the demonstration of the effect raised by Fernandez that increased uncertainty about the forecasts has a counter-intuitive effect of increasing the option value of the project. Copeland and Antikarov suggest that one technique of assessing the volatility of cash flows is to ask the managers involved for the 95% confidence limits of component cash flows of the project. Clearly, the less certain managers are about their expected project cash flows, the wider they would set the limits, with the consequence that the volatility of the project would tend to increase, relative to the static NPV. This would have the effect of increasing the option values for decisions based on the project – perhaps an unforeseen product of the method used in constructing the volatility.

7. Conclusions

In this paper, we have used three different lattice option pricing models to value a specific real estate development in outer London, UK. The analysis of the models shows some consistency and coherence in the results. The estimates do not appear to be particularly sensitive to the rate of interest, nor to the number of steps used in calculating the lattice. The correlation between the negative and positive cash flows involved in the project does not seem important and the sensitivity of the options value to further deferment of the project seems plausible.

Furthermore, we have also used three different ways of estimating the volatility of underlying assets and found that they can give rise to substantial differences in the resultant important factor required in all options applications – the volatility. Sensitivity analysis in the paper confirms that this variable is indeed the most important input and that substantial error could result from naïve attempts at estimation. All of the techniques used have their drawbacks. In the case of the three-year market index, even unsmoothed, there is likely to be some under-estimate of the volatility of individual developments since idiosyncratic returns are diversified away. In the case of the alpha approach, cross-sectional estimation overcomes this but may, by ignoring market movements, again underestimate the volatility of development value over 3 or 4 years. Finally, using the Copeland and Antikarov approach still requires some input into the volatility of cash flows – and in particular the final selling price. The suggested solution of asking for subjective assessment of confidence intervals would seem to introduce the possibility of bias towards over-valuing projects that had a large degree of uncertainty.

Taking a broad view of the various estimates shown and discussed in this paper, we conclude that taking into account the option of deferring the start of the project and the option of selling the land to the local authority could add anything in the range of 40% to 60% to the value of the project. Such a result seems counter-intuitionally high but might be counter-balanced by the negative options that could be exercised by competitive developers. If real options analyses are to prove important in applications to real estate development, researchers must now explore both the perceived and the actual threats of competitive development. There is plenty of academic research on how competitive development might reduce the value of deferment options, but much less on the assessment of the threat and the practical consequences.

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