Portfolio Size and the Reduction of Dispersion:
The Case of the United Kingdom Commercial Real Estate Market

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Abstract

This paper investigates the potential benefits and limitations of equal and value-weighted diversification using as the example the UK institutional property market. To achieve this it uses the largest sample (392) of actual property returns that is currently available, over the period 1981 to 1996.

To evaluate these issues two approaches are adopted; first, an analysis of the correlations within the sectors and regions and secondly simulations of property portfolios of increasing size constructed both naively and with value-weighting.

Using these methods it is shown that the extent of possible risk reduction is limited because of the high positive correlations between assets in any portfolio, even when naively diversified. It is also shown that portfolios exhibit high levels of variability around the average risk, suggesting that previous work seriously understates the number of properties needed to achieve a satisfactory level of diversification.

The results have implications for the development and maintenance of a property portfolio because they indicate that the achievable level of risk reduction depends upon the availability of assets, the weighting system used and the investor's risk tolerance.

1.0 Introduction

The general benefits of diversification have been well established in Modern Portfolio Theory (MPT) since the seminal work of Markowitz (1952). Later work has demonstrated that the value of diversification is limited or bounded by the systematic risk of the market as a whole (see for example Evans and Archer, 1968). The application of MPT to the real estate market is more recent, with the original focus on assessing the place of real estate in the mixed-asset portfolio. Lately the emphasis has moved to investigating the implications of MPT for the real estate portfolio itself (see Lee and Byrne, 1998). Also, a number of studies have used simulation to examine the effects of spreading risk across assets within a real estate portfolio (Jones Lang Wootton (JLW), 1986; Barber, 1991 and Brown, 1988, 1991). The results of these studies show that portfolios with smaller numbers of properties have a higher volatility than larger portfolios. Although there is some disagreement about the optimal number of properties to hold, there is general agreement that the number needed to achieve a satisfactory level of diversification is substantially greater than that held by most institutional investors, confirming findings from other markets; see for example, Evans and Archer, 1968; Wagner and Lau, 1971; Johnson and Shannon, 1974; Elton and Gruber, 1977; Tole, 1982; Bird and Tippett 1986; Lloyd, Hand and Modani, 1987,and Stratman, 1987.

This work has generally assessed the impact on risk of increased portfolio size assuming equal weighting within the portfolios. For most funds however, it is not possible generally to obtain equal weighting and at the same time be represented in key market segments. Equal weighting is therefore probably not a realistic or even desirable goal for fund managers. As a consequence fund managers will seek to hold value-weighted rather than equal-weighted portfolios (Morrell, 1993). Later work has concentrated on the impact that non-equal weighting, or value skewness, has on portfolio risk (Brown, 1988, 1991; Morrell, 1993 and Schuck and Brown, 1997). Results here show that in comparison to equal-weighted portfolios, value-weighting leads to sub-optimal results because of increases in specific risk within the portfolio, in other words the portfolios have higher total risk. However, for an individual portfolio, especially of small size, the reduction in risk could be much higher (lower) than the average, depending on the actual inter-correlation between the subset of assets. This would be especially so if the portfolio is not equally weighted and the highest holdings are in the least (highest) risky assets (Schuck and Brown, 1997). Thus, value-weighting by itself does not always lead to higher portfolio risk in comparison to an equal-weighted portfolio of the same size; see for example Cullen (1991).

Taken as a whole, these studies have tended to use only a small sample of properties with only a few years' data. At the same time real estate professionals have begun to adopt a portfolio construction strategy that has focused, not at the individual property level, but at the town or urban area level (McNamara, 1990). This suggests that fund managers need to know the extent to which a portfolio construction process based on a sector and/or regional/town level analysis leads to a reduction in portfolio risk, especially for value-weighted portfolios. Such issues have not been addressed in previous work, although Brown (1988, 1991) and Morrell (1993) have looked at the impact of risk reduction in sector portfolios using individual property data from different geographical locations.

By replicating and extending experimental methods used previously in equity markets and more recently to a limited extent in real estate markets, this paper examines these issues using a very large database of actual real estate data from 392 'locations' across the United Kingdom (UK) with annual data covering the period from 1981 to 1996. This has important implications for assessing the general effect of diversification. Beyond that, however, it is important for the process of developing and maintaining a real estate portfolio.

2.0 The reduction in portfolio risk

Markowitz (1952) showed that the variance of a portfolio of *N* assets is given by:

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij} \qquad (1)$$

where:

σ_p^2 = portfolio variance

σ_i^2 = the variance of asset *i*;

$\rho_{i,j}$ = the correlation between assets *i* and *j*;

N = the number of assets.

Equation 1, in the special case where σ_i equals the average standard deviation $\bar{\sigma}$ and ρ_{ij} equals the average correlation coefficient $\bar{\rho}$ for all i, becomes:

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \bar{\sigma}^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} w_i w_j \bar{\sigma}^2 \bar{\rho} \qquad (2)$$

For any given w_i, the sum of all w_j for $j \neq i$ must equal $(1 - w_i)$. Substituting into the last term of (2) produces:

$$\sigma_p^2 = \bar{\sigma}^2 (1 - \bar{\rho}) \sum_{i=1}^{N} w_i + \bar{\sigma}^2 \bar{\rho} \qquad (3)$$

In addition if we assume equal weighting then (3) simplifies to:

$$\sigma_p^2 = \bar{\sigma}^2 (1 - \bar{\rho}) \frac{1}{N} + \bar{\sigma}^2 \bar{\rho} \qquad (4)$$

Equation 4 shows that the total risk (variance) of an individual investment can be broken down into two components. This has important implications for the amount of risk reduction that can be achieved by naïve diversification. The first component, as represented by the first term on the RHS of equation 4, can be eliminated by increasing the number of investments in a portfolio. Indeed the elimination of this part of total risk will be achieved rapidly, since as *n* increases *1/n* approaches zero very quickly. In contrast the second component of total risk, the second term on the RHS of equation 4, cannot be eliminated by increasing the number of investments in a portfolio. This risk component is represented by the average correlation (covariance) across all investments under review. Such a component of risk is therefore common to all investments and is consequently called systematic or market risk, see Elton and Gruber (1977), Brown (1991). The first component of risk is then labelled unsystematic or specific risk. This is the risk inherent in an individual investment which is independent of the market and is unique to the individual asset. It is this part of total risk that is eliminated by naive diversification, leaving the systematic component in the portfolio. Consequently the level of risk reduction that can be achieved in a portfolio is limited or bounded by the risk of the market, which is represented by the average correlation of risk across the investments. The lower the correlation the higher the risk reduction.

Schuck and Brown (1997) show that the percentage reduction in total risk, as measured by the standard deviation of returns (the square root of the variance), for any value of *N*, relative to that in an individual asset holding, can be found by:

$$RR\%_N = 1 - \left[\frac{1 - \overline{r}}{N} + \overline{r} \right]^{1/2} \qquad (5)$$

This shows that if the average correlation between assets ($\bar{\rho}$) is 1 there is no reduction in portfolio risk. If $\bar{\rho}$ is zero, then as *N* increases the first term in the bracket on the right hand side of (5) tends to zero and the reduction in risk tends to 100%, i.e. a portfolio of zero risk. Therefore increasing the number of assets in a portfolio will not by itself result in a fall in portfolio risk, unless $\bar{\rho}$ is less than 1. The reduction in risk will be greatest if $\bar{\rho}$ is zero. Hence the closer the average correlation between assets is to one the lower is the reduction in risk.

Following this kind of argument, a number of studies have investigated the impact on portfolio volatility (measured by the standard deviation) of spreading risk across an ever increasing number of properties within the UK real estate market. The first study, by JLW (1986), based on annual data from 1980 to 1985 for about 300 properties and using the averaged results of 100 randomly constructed portfolios, showed that as the number of properties within a portfolio increased, portfolio volatility declined. Their results indicated that portfolios with smaller numbers of properties have a higher volatility than larger portfolios. Beyond 20 properties however, JLW argued that about all the reduction in risk had been achieved. Myer, Webb and Young (1997) have conducted a similar exercise on 57 individual properties in the US, using quarterly data from December 1993 through September 1996. Based on 500 equal-weighted randomly constructed portfolios (with replacement) Myer et al. find that as the portfolio size is increased up to 50 properties portfolio standard deviation decreases, the most rapid fall occurring with the addition of the first few properties. Beyond 10 to 20 properties the marginal decrease in portfolio risk was minimal. In contrast Barber (1991), using the same methodology on 10 years' annual data, found that the majority of diversification was achieved once 40-45 properties were included in a real estate portfolio. Brown (1988), using monthly data and a sample of 135 properties, found that after about ten properties the reduction in portfolio risk by adding further properties diminished dramatically. Holding about 30 properties resulted in portfolios diversified down to the systematic risk level. Brown acknowledged that in order for the portfolio to be highly diversified, that is to have the market explain over 95% of the variability in returns, 200+ properties would be needed. Cullen (1991) confirmed this result using a larger number of actual real estate funds with 10 years of annual data from 1981 to 1990. He concluded that a portfolio of four to five hundred properties is needed to achieve a satisfactory level of diversification. Such a large number, when compared with the average size of just over 50 properties for a UK real estate portfolio (IPD, 1996(b)), indicates that the vast majority of UK funds are not diversified and are subject to high levels of specific risk.

Of course, increasing the number of properties will not be the sole determinant of the degree of diversification in a portfolio. If investors were to follow the rules suggested for optimal diversification advocated by Markowitz, portfolio risk could be significantly reduced with a few well-chosen holdings. However, there is no evidence to suggest that investors (even institutional investors) follow the rules of optimal diversification. Indeed the opposite is more likely to be the case. For example, Rydin, Rodney and Orr (1990) in a survey of UK real estate funds, found that only one fund had taken up the tenets of MPT, with the majority following a naïve investment strategy. Real estate fund managers seem to rely both explicitly and implicitly on the law of large numbers to reduce the portfolio risk.

Morrell (1993) argued that that it is generally not possible for most funds to obtain equal-weighting and at the same time be represented in key market segments. Fund managers typically desire to maintain weights similar to a benchmark portfolio and to try and outperform that benchmark's index of performance by over-weighting in attractive markets and under-weighting undesirable sectors and/or regions. Equal-weighting is therefore probably not a realistic, or even a desirable goal of fund managers. A consequence of the desire to hold weights similar to a benchmark is that it will lead fund managers to hold value-weighted rather than equal-weighted portfolios.

Since (3) shows the risk of a portfolio that is non-equal-weighted (*new*) and (4) shows equal-weighting (*ew*), the difference between 3 and 4 indicates the incremental change in risk due to non-equal (value) weighting. The difference is given by:

$$\sigma^2_{new} - \sigma^2_{ew} = \bar{\sigma}^2(1-\bar{\rho})\left[\sum_{i=1}^{N} w_i^2 - \frac{1}{N}\right] \qquad (7)$$

In the trivial case when $\bar{\rho}$ is 1 there is no difference in risk between a non-equal and an equal-weighted portfolio. Schuck and Brown (1997) show that in all other cases, for $\bar{\rho}$ less than 1, the difference between the two variances will depend on the sign of the term in the square brackets. For all values of $w_i > 0$, $w_i \neq \frac{1}{N}$, the following is true:

$$\sum_{i=1}^{N} w_i^2 - \frac{1}{N} > 0 \text{ or } \sum_{i=1}^{N} w_i^2 > \frac{1}{N} \qquad (8)$$

That is, the sign of the term in the square brackets on the right hand side of (7) is positive. A departure from equal-weighting leads to an increase in total portfolio risk relative to that of an equal-weighted portfolio. Value-weighted portfolios are therefore sub-optimal by comparison with equal-weighted portfolios.

Brown (1988, 1991), Morrell (1993) and more recently Schuck and Brown (1997) have investigated the consequences of value-weighting. Brown found that equal-weighted portfolios have less variability of returns, by comparison with value-weighted portfolios, even for funds with 200+ properties. As a consequence, value-weighting within a portfolio can have a serious impact on the number of properties needed to reduce portfolio volatility. Brown suggested however, that as the portfolio size is increased the impact of large value properties would be diversified away. In contrast Morrell (1993) argues that an uneven distribution of property values is an important source of high levels of specific risk in real estate portfolios. The potential benefits of diversifying specific risk by holding a large number of properties can be swamped by the existence of differential weights within the portfolio. Simply holding a large number of properties is not sufficient in itself guarantee a reduction in specific risk as the distribution of properties within the portfolio is an equal, or perhaps a more important determinant of portfolio specific risk. Schuck and Brown (1997), however, argue that the simplification suggested by Morrell to derive his results - a zero correlation between assets - meant that Morrell drew the wrong conclusions. After deriving the more general equation, which includes the effect of inter-correlations between assets, Schuck and Brown (1997) argue that although:

> "….an uneven distribution of values *can* have a significant impact on the specific risk level within property portfolios……However, the actual effect of value skewness will be a function of the number of properties, their individual total risks, and the correlation structure between returns."

Thus, the actual risk of a value (or non-equal) weighted portfolio depends on the individual risk characteristics of the assets in the portfolio and the actual inter-correlation between the assets. This will mean that the risk level of such a portfolio will be dominated by the characteristics of the highest value properties. Depending on the individual risk characteristics of the properties, the portfolio could have a significantly higher or lower volatility than the average, especially for small sized portfolios. As a consequence, the sub-optimality that results from the value (or non-equal) weighting shown in (7) is valid only under the restrictive assumptions that s_i and ρ_{ij} equal the average standard deviation $\overline{\sigma}$ and the average correlation coefficient $\overline{\rho}$ for each *i*. The increase in portfolio risk due to non-equal weighting is true only for a special case, which is unlikely to be observed in practice. Barber (1991), for example, notes that in some cases volatility could be reduced very quickly with only a limited number of properties. Cullen (1991) observed that for small real estate funds (i.e. with less than 100 properties) such portfolios could exhibit very high as well as very low volatility.

These studies show that, in general, increasing the number of properties in a portfolio leads to a reduction in total portfolio risk and that, as is to be expected, the level of risk reduction is dependent on the average correlation between assets. However, it seems reasonable to assume that most, if not all, fund managers hold sector and regional weights similar to some benchmark portfolio, and that they follow some kind of value-weighted, rather than equal-weighted, diversification strategy. Morrell (1993) and Schuck and Brown (1997) both suggest that value-weighted portfolios need a larger number of properties to reach a satisfactory risk level compared with an equal-weighted portfolio, but that value-weighted portfolios should display a higher variability around the average risk reduction profile, as *n* increases, especially for portfolios of a small size. The simulations of Barber (1991) and the results of Cullen (1991), for actual portfolios, would tend to suggest that this is the case.

More recently fund managers have in any case started to follow a more structured top-down approach. Here the focus is not on the individual property, but at the town or urban area level, and is intended to provide stronger controls on the portfolio construction and management process (McNamara, 1990). This would enable portfolios to be constructed with greater cognisance of the risks to which the investor is exposed. It also suggests however that fund managers need to know the extent to which a portfolio construction process based on a sector and/or regional/local level analysis leads to a reduction in portfolio risk, especially for value-weighted portfolios.

The potential benefits and limitations of risk reduction in a sector/regionally based portfolio strategy using towns or urban areas have not been studied. Brown (1987, 1991) compared the risk reduction that could be achieved for a small sample of individual properties in the three sectors; Retail, Offices and Industrials, using both equal and non-equal weighting. Morrell (1993) undertook a similar exercise using 562 properties, but with only four years' data. The impact on portfolio volatility of increasing the number of real estate holdings within a region across real estate types, however, has not been studied. This is in spite of the fact that previous studies of portfolio diversification report that diversification across sectors within a region offers greater risk reduction than portfolios spread across regions within a real estate type (see Lee and Byrne (1998) for an extensive review). The large database used here permits just such an analysis.

3.0 The potential for risk reduction in UK real estate portfolios

In order to investigate these issues a very large database of actual real estate data in 392 'locations' across the UK with annual data covering the period from 1981 to 1996 is used.

3.1 Data

The data are derived from the Investment Property Databank (IPD) *Local Markets Report* (IPD, 1998). The data in the *Local Markets Report* are in turn drawn from a total database of 13,721 properties with an aggregate value of £65,394m at the end of 1997. The sample data consist of the total returns on properties in three sectors, Retail, Office and Industrial at various locations in the UK over the period 1981 to 1996, to give a total of 392 asset possibilities. To protect confidentiality no data are published for locations containing fewer than four properties in any year. The locations are based for the most part on local authority boundaries drawn under the 1992 Local Government Act. As a result asset descriptions might be, for example, 'Westminster Offices'; 'Manchester Retail' or 'Walsall Industrial'. For Offices and Industrials the *Local Markets Report* results are based on *all* Offices and Industrial properties covered by IPD in each location. For Retails, given the breakdown in the divisions between the real estate sectors with the advent of Retail warehouses and the like, only standard shops, that is standard 'high street' Retail stores in each location, are covered in order to provide a Retail sample on a comparable basis. In terms of coverage however, the Retail data represent over 95% of all Standard shops covered by the IPD data base. The comparable figures for Offices and Industrial are 72% and 70% respectively. The *Local Markets Report* data consequently represent almost a complete coverage of commercial real estate performance in the UK.

The *Local Markets Report* data are also classified into the Standard Economic Regions of the UK but with the South East subdivided into London and the Rest of the South East because London represents a dominant area of institutional real estate investment. As a consequence a number of the regions have only a few data points. In order to have enough data points to conduct the analysis these regions have been amalgamated here into fewer, contiguous, areas. The South West, East Anglia, the East and West Midlands were amalgamated with Wales, to form a South and West region. The Yorkshire and Humberside region was combined with the North West, the North, and Scotland to form a Northern region. Table 1 summarises the resulting real estate sector and regional breakdown of the data.

Table 1: The Number of Data Points in Each Sector/Region

Regions	------------Sectors------------- Retail	Office	Industrial	Total
London	27	13	17	57
South East	61	45	44	150
South and West	63	21	19	103
Northern	55	14	13	82
Total	206	93	93	392

The data in Table 1 are spread unevenly across both the sectors and regions. Of the three sectors most of the data (54%) are in the Retail sector, evenly spread across most of the UK outside London. Offices in comparison are more represented in locations in London and the South-East, as is to be expected. The spread of Industrial locations is more uneven, with little in London, an even spread across the South-East and South West but little representation in the North. The data reflect the North-South divide observed by both Jackson (1996, 1997) and Hoesli and MacGregor (1995) because of institutional bias towards the South of England (IPD, 1996(b)).

3.2 Descriptive statistics

Table 2 shows the descriptive statistics for the sector and regional data over the period 1981 to 1996. Panel A of Table 2 shows that the best performing sector on average was Industrials, while the Office sector performed worst. The best performing region was the South West and the worst the South East. However, Panel B of Table 2 shows that on average high returns are not necessarily associated with higher levels of risk (standard deviation). Although the highest risk was associated with the sector showing highest return (Industrials) the lowest level of sector risk was in Retail, which offered the second best returns. The regional data confirm this observation since the region with the lowest average returns (South East) showed the second highest level of risk, because of the dominance, by value, of Offices within the region. The Northern region, which showed the second best average returns, had the lowest risk level by a considerable margin. These results can be explained by examination of the average correlation coefficients across regions and sectors in Panel C. Panel C of Table 2 shows the overall average correlation between the 392 ‘assets’ is 0.608. This suggests that the amount of risk reduction will be rather small. There is a difference however, between correlation coefficients across the sectors and regions. In general regional portfolios diversified across sectors show the lowest average correlation, while sector portfolios diversified across the regions have marginally higher values. This implies that the best risk reduction will occur within a regional portfolio spread across the sectors. In addition the average correlation of the Retail properties declines further away from London which explains the lower level of risk for these areas. Industrials show a higher level of risk and a greater uniformity of average correlation coefficients across the regions except again for the North although the average correlation is still higher than for the Retail and Office sectors. By comparison the pattern in the Office sector first rises further away from London and then falls sharply in the Northern region.

Table 2: Descriptive Statistics

Sectors/Regions	Retail	Office	Industrial	Overall
Panel A		**Average Mean Returns**		
London	11.60	7.94	12.36	10.68
South East	10.88	8.59	11.98	10.53
South and West	11.33	11.30	12.86	11.63
Northern	10.87	10.81	13.12	11.25
Overall	**11.11**	**9.34**	**12.40**	**10.99**
Panel B		**Average Standard Deviation**		
London	11.67	14.13	12.67	12.63
South East	11.20	12.06	14.02	11.97
South and West	9.68	12.73	13.08	11.12
Northern	9.06	11.18	10.71	9.80
Overall	**10.22**	**13.01**	**13.21**	**11.38**
Panel C		**Average Correlations**		
London	0.688	0.772	0.825	0.673
South East	0.733	0.761	0.811	0.678
South and West	0.693	0.811	0.832	0.584
Northern	0.675	0.605	0.771	0.518
Overall	**0.688**	**0.711**	**0.778**	**0.608**

3.3 The average reduction in portfolio risk

Using the data in Table 2 for the average correlation coefficients and equation 5, the potential reduction in portfolio risk of the sectors and regions is shown in Table 3. Figure 1 shows the decrease in risk as the number of assets in the portfolio increases up to a portfolio of 50 'assets'.

Table 3: Average Reduction in Risk for Equal Weighted Sector/Regional Portfolios

		Reduction in Risk %		
Sectors/Regions	**Retail**	**Office**	**Industrial**	**Overall**
London	17	12	9	**18**
South East	14	13	10	**18**
South and West	17	10	12	**24**
Northern	18	22	12	**28**
Overall	**17**	**16**	**12**	**22**

As can be seen in Table 3 the largest reduction in portfolio risk occurs in regional portfolios diversified across the three sectors. Figure 1 also clearly shows that the percentage reduction in risk is always greater for real estate types diversified across a region. The greatest benefits occur the further the region is away from London. This result is in accord with Eichhholtz et al. (1995), who found that the further away from London the lower the correlation and hence the greater the reduction in risk.

Diversification within a sector across regions is therefore less beneficial than within a region across sectors. This indicates that two properties in the same sector, but in different regions, are closer substitutes than two different real estate types in the same region. The potential for portfolio risk reduction is therefore greater by diversifying across sectors. Consequently real estate fund managers need to pay closer attention to the sector allocation of their portfolios than the regional spread.

Figure 1 shows that using equation (5) most of the reduction in risk occurs with the addition of the first 20-25 assets (representing perhaps about 100 properties), after which the reduction in risk is only marginal (see also Cullen, 1991 and Brown 1987, 1991). Table 3 and Figure 1 show that the potential reduction in risk for UK real estate portfolios diversified across local authorities is severely limited however. At the limit the reduction in risk is, on average, only 22% and no more than 28% in the best region, the North, and could be as little as 12% in the Industrial sector. The figures for the sector/regional portfolios are even less satisfactory with a percentage risk reduction of no more than 22% and as little as 9%. The results support Young and Graff (1995), who suggest that because of the non normality of real estate returns, reducing the risk in a real estate portfolio requires thousands if not millions of properties, well beyond the capabilities of any institutional investor.

It can be argued however, that earlier approaches are deficient because an individual investor owns only one portfolio and results based on averages are not really relevant to his/her particular case, which may be substantially different from the average (Newbould and Poon, 1993). Thus previous studies disguise the variability around the average, which could be large. Investors who base their diversification strategies on average results could be leaving themselves open to unpleasant results. This is similar to the argument of McDonald (1975) who suggests most investors see diversification as designed to reduce the "probability of ex post returns being an adverse surprise". Therefore those investors who wish to avoid such adverse surprises from an unfortunate selection, would be better off looking at the upper bound rather than the average standard deviation or variance when considering the effect of diversification from increasing sample sizes. The descriptive statistics in Appendix Table A suggest that this could be a serious problem here, because they indicate a wide variation in correlation coefficients within and between sectors and regions. The lowest correlation between any pair of assets of -0.684 indicates that it would be possible for a portfolio of two assets to achieve extremely low levels of risk if the tenets of Markowitz were followed. By the same token a portfolio showing no reduction is also possible if an investor were to choose to invest in the two assets which show the highest correlation of 0.977.

Given that the sampling variation in the correlation coefficients is quite large, the results in Table 3 do not fully convey the actual impact that increasing the portfolio size has on total risk. More importantly, equation (5) cannot assess the effects of sampling without replacement. This effect can only be tested by simulation. Of course, as the sample size increases, the variance of a portfolio approaches the average covariance between all properties. This relationship might suggest that simulation is unnecessary, but this may not be the case. Newbould and Poon (1993) argue that sampling variation can be quite large. In addition, in situations of sampling without replacement, the average variance and covariance are constantly changing as properties leave the population. When using equation (5) stable values of the average variance and covariance between the parameters are required. In such a circumstance the safest thing to do is to conduct the Monte Carlo experiment, at least to understand the way in which these parameters are performing. The next part of the paper therefore uses Monte Carlo simulation to investigate the risk reduction potential in sector and regional portfolios in the UK.

4.0 Experimental design

The simulations were carried out in two stages.

In the simulations, the portfolio variance was used as the risk measure (Whitmore, 1970; Evans, 1975). An initial asset was randomly selected from the data set and its volatility (variance) was calculated. A second asset was then randomly selected and added to the first and the volatility of the equal-weighted naïve portfolio derived. This process continued until a 50-asset portfolio was achieved. This choice of this cut-off portfolio size was determined by a number of criteria. First, the limitation on the number of data points within a region. Second, as each real estate location in the *Key Centres* data contains a minimum of four properties, to protect confidentiality, a portfolio of 50 locations across the UK must actually contain, at a minimum, 200 properties. These are effective portfolio sizes far in excess of the average for funds in the UK of just over 50 properties (IPD 1996(b)). Increasing the simulated portfolio size beyond 50 would serve little purpose in the practical sense (although some simulations were made using much larger 'samples' for specific reasons). The earlier observation that for portfolio sizes of about 25 and above the average portfolio risk is almost identical to that of the limiting case when the number of assets held is infinite might also be noted, as this indicates that smaller simulated portfolios might be satisfactory for analytical purposes.

Experiments were conducted by sampling randomly from a uniform distribution with and without replacement. It is of course necessary to sample a sufficient number of times to obtain statistically acceptable output distributions and statistics. Initially a sample size of 500 was used, but smaller sample sizes were tested, and finally a sample size of 100 was felt to be acceptable, since it did not show any significant effect on the output statistics. Each experiment was thus repeated 100 times for portfolios with 1 to 50 assets. An asset could appear more than once in the sample when the 'with replacement' option was used.

Morrell (1993), suggests that value-weighted portfolios are likely to be more representative of the actual portfolio structure of investors who wish to be represented in key market segments and hold a portfolio with weights close to that of the benchmark portfolio. To be able to consider this, a set of capital values is necessary. For this, IPD made available the aggregate capital value for each asset/location in each period. While these are still aggregate figures, they represent the closest to individual values yet used for such a large data set. Appendix Table B shows the average values over the whole period within and between sectors and regions. Not surprisingly Offices, especially in London had the highest average value while Retail in the North showed the lowest. London Offices were almost five times as large by value as Northern Retail. The implication of this might be that a value-weighted portfolio which consisted of these two asset classes would be strongly influenced in risk terms by the characteristics of the larger value asset(s). A second set of simulations was undertaken here based on a value-weighted scheme that used these individual annual capital values, to test the effect, which it was expected should be more pronounced. An asset was chosen at random and its value noted. A second asset was then randomly chosen, again noting its value. The two-asset portfolio's value was then calculated and the percentage allocation of the portfolio, between the two properties, was estimated using the ratio of the individual asset values to that of the overall portfolio, on a period by period basis. The portfolio weights were then used in the calculation of the portfolio's return and risk. This process was continued as more assets were selected, until, as before, a 50 asset portfolio was reached.

Overall therefore, the following simulations were carried out to generate portfolios from size one to size fifty:

Equal-weighted		**Value-weighted**	
With Replacement	**Without Replacement**	**With Replacement**	**Without Replacement**
All	***All***	***All***	***All***
Sector	***Sector***	***Sector***	***Sector***
Retail	Retail	Retail	Retail
Office	Office	Office	Office
Industrial	Industrial	Industrial	Industrial
Region	***Region***	***Region***	***Region***
London	London	London	London
South East	South East	South East	South East
South & West	South & West	South & West	South & West
Northern	Northern	Northern	Northern

Since risk is the principal concern here, the mean portfolio variance of the 100 simulations for each portfolio size was computed, as well as the maximum, minimum and standard deviation. More than 200,000 individual simulations were made.

5.0 Simulation results

Previous work has identified several issues that need to be looked at in this area. First, are value-weighted portfolios riskier than equal-weighted portfolios of the same size? The studies by Brown and Morrell would suggest that this is the case. Second, all the other studies have used simulations with replacement, and the effect of non-replacement on simulation results has not been studied. Thirdly, instead of using average values, it has been argued that investors are more interested in the consequences of diversification on *their* portfolio rather than the average. In this case it is worth looking at the confidence that investors can have in the performance of their portfolio. These issues are examined in the following sections.

5.1 Value versus equal-weighted portfolios with and without replacement

Table 4 shows the ratio of value-weighted to equal-weighted portfolio mean risks for portfolios of various sizes, sampled from all assets, with and without replacement. The most notable feature of the table is that there is apparently only a small difference in the levels of portfolio risk (variance) between equal and value-weighting. Value-weighted portfolios do tend to display higher average risk levels than equal-weighted portfolios.

The other notable feature of Table 4 is the lack of difference between mean risk levels when equal and value weighted portfolios are sampled with and without replacement.

Table 4: Ratio of Value-weighted to Equal-weighted Mean Portfolio Risk Selected Portfolio Sizes: Sampled With and Without replacement

					Test of Equality of Mean Risk			
Portfolio	Equal	Value	With	Without	Equal v.'s Value		With v.'s Without	
Size	With	Without	Equal	Value	With	Without	Equal	Value
1	101.92	103.29	88.23	89.42	0.17*	0.49*	1.35	1.16
2	115.81	116.33	95.92	96.36	1.91	2.63	0.69	0.51
3	123.44	116.03	102.80	96.63	3.99	3.28	0.61	0.67
4	121.94	116.04	104.21	99.17	4.40	3.62	1.02	0.19
5	121.31	115.37	104.39	99.29	4.79	3.69	1.17	0.17
6	119.90	112.85	103.88	97.76	4.74	3.30	1.10	0.58
7	119.81	111.45	104.52	97.22	5.08	3.07	1.34	0.76
8	119.23	114.29	102.11	97.88	5.14	4.13	0.69	0.61
9	119.71	114.31	101.62	97.03	5.64	4.28	0.55	0.91
10	121.44	114.26	102.47	96.41	6.31	4.56	0.90	1.16
15	120.11	114.22	103.75	98.65	7.23	5.36	1.55	0.53
20	118.83	113.72	102.12	97.73	8.06	5.96	1.05	1.03
25	118.47	114.36	101.97	98.43	8.86	7.19	1.06	0.83
30	117.99	115.19	101.96	99.54	9.47	8.22	1.15	0.26
40	117.05	114.67	101.18	99.12	10.73	9.21	0.80	0.59
50	116.19	114.29	99.88	98.24	11.25	9.95	0.09	1.33

Note 1. In the case of equal v.'s value-weighted portfolios all t statistics **are significant** at the 5% level except *.
2. In the case of with and without replacement simulations all results are insignificant.

The statistical difference between these results can be tested by means of a t-test at some predetermined significance level. The test is whether the mean variance of the value-weighted portfolio is significantly different from the average portfolio risk of the equal-weighted portfolio for each portfolio size. Because the assumption is that the mean portfolio risk of value-weighted portfolios is higher than the mean portfolio risk of the equal-weighted portfolio the appropriate test is based on the one tailed significance level. Similarly a test can be made as to whether there is a significant difference between portfolios, both equal and value-weighted, when the analysis is undertaken with and without replacement. The tests were made assuming equal and unequal variance, with similar results. The results reported assume unequal variance. There is a statistically significant difference between the equal and value-weighted mean portfolio risks at the 5% significance level, except for portfolios of one asset, due to the extreme variability around the average risk of the one asset portfolios. At all portfolio sizes, other than size 1 however, this difference is statistically significant at the 5% level, and becomes increasingly so as the number of assets in the portfolio grows. The reason for the lack of a significant difference at portfolio size 1 is a consequence of the high variability in portfolio variances around the average. With increasing portfolio sizes, this variability rapidly declines, leading to increases in statistical significance.

In practice therefore more assets are needed to achieve the same level of portfolio risk as an equal-weighted strategy. To obtain the same average level of portfolio risk as a 2 asset portfolio a value-weighted strategy requires 5 assets. To have the same level of risk as a 3 asset portfolio the number for the value-weighted portfolio increases to 17.

In contrast there is no significant difference between the average risks of portfolio constructed with or without replacement. At all portfolio sizes there is no significant difference between these measures. The implication of this is that simulations do need to be based upon both value and equal-weighted strategies, but little more is to be gained from an analysis using sampling without replacement as an option (even though sampling without replacement might be thought to be more representative).

The significance or non-significance of a given increase in portfolio size may not, however, indicate the desirability of such an increase. Investors are interested in the marginal reduction in either the mean portfolio risk level and the standard deviation around the mean relative to their cost function.

The impact on the mean risk of increasing the portfolio size, and on the variability around the mean risk, is demonstrated in Table 5. This shows that initially there is a large percentage fall in the mean risk level but only up to portfolio sizes of four to five assets (20 properties). After this point, the reduction in mean risk tends to diminish and the mean level shows little or no decline. This result is comparable with previous research which showed little worthwhile reduction in mean risk levels beyond a few assets. Indeed, using t tests, as described above, the mean risk level shows a significant fall only for portfolio sizes up to about five assets.

Table 5: Stepped Percentage Reduction in Risk and Test of Equality Equal and Value-weighted Portfolios

	Percentage Reduction in Risk				*Tests of Equality (Sampled With Rep.)*			
Increase	Mean		Standard Deviation		Mean		Variance	
From - To	Equal	Value	Equal	Value	Equal	Value	Equal	Value
1-2	25.13	14.93	52.71	39.12	2.97*	1.57	7.53*	2.62
2-3	12.14	6.35	36.98	37.96	2.13*	0.93	4.27*	4.29*
3-4	5.64	6.78	13.08	23.80	1.23	1.40	0.37	2.51
4-5	3.85	4.35	13.15	15.05	0.91	1.06	1.15	1.11
5-6	0.39	1.54	9.38	3.50	0.10	0.40	0.27	0.07
6-7	1.34	1.41	4.15	11.21	0.37	0.39	0.10	0.60
7-8	0.16	0.64	5.73	2.92	0.04	0.19	0.27	0.03
8-9	0.90	0.50	6.17	8.56	0.27	0.15	0.26	0.57
9-10	1.24	0.19	8.22	0.90	0.40	0.06	0.15	0.02
2-5	20.29	16.51	52.43	59.85	3.79*	2.64*	10.64*	14.37*
5-10	2.28	0.95	18.82	12.02	1.12	1.10	4.62*	4.13*
10-15	1.36	2.61	6.02	15.07	1.17	1.55	3.39*	3.15*
15-20	0.25	0.37	6.75	11.54	0.24	0.70	2.19	1.64
20-25	0.83	0.08	2.83	6.16	0.23	0.39	0.11	2.05
25-30	0.04	0.57	5.62	6.07	0.29	0.53	1.90	1.12
30-40	0.18	0.98	14.97	17.59	0.11	0.61	0.86	1.66
40-50	0.56	0.18	6.77	10.74	0.39	0.13	0.12	2.29

Note: * indicates significant at the 5% level.

In contrast, the standard deviation around the mean risk level shows much larger percentage reductions and continues to do so up to larger portfolio sizes. The standard deviation around the mean declines quite rapidly up to size 15/20. Beyond this point, the reduction in variability continues, but at a lower rate. The significance of these falls in the variance around the mean portfolio risk is tested by the Brown-Forsythe modified Levene test (Brown and Forsythe, 1974). This appears to be a superior test in terms of robustness and power compared to other tests; Conover et al (1981). The test is distributed as an F statistic with G-1 degrees of freedom in the numerator and N-G degrees of freedom in the denominator, where G is the number of groups (G=2) and N is the number of observations.

The null hypothesis is that there is no difference in the equality of the variances. Using this approach the results in Table 5 show that there is a significant fall in the variability around the mean only for portfolios moving from 2 assets to 3 or from 2 to 5, or 5 to 10 asset portfolio sizes after which the marginal decline in variability shows no significant reductions.

Consequently, even for portfolios with large numbers of assets, there remains a good deal of variability around the mean risk level. The mean risk may therefore be insufficient as an indicator of the 'optimum' portfolio size for a particular fund manager. Investors need to be concerned not only about the rate at which the average risk declines with increased portfolio size but also with the degree of confidence regarding the expected mean portfolio risk level at portfolios of different sizes.

5.2 Confidence

Figure 2 shows that the ratio of the maximum and minimum risk levels relative to the average for a given portfolio size is large even in a 50 asset portfolio (±22%). On the same basis, an investor with a portfolio of say 25 assets (at least 100 properties) relying on average risk levels is left open to the possibility that the risk for their portfolio could be more than 40% different from that expectation. In order to consider this, rather than report the average risk of the simulated portfolios, following Newbould and Poon (1993), Table 6 shows the 95% confidence level as a percentage of the average value. In this table the average risk is assigned a value of 100 and the 95% upper bound is presented as a percentage of this. Thus in the case of a one asset equal-weighted Office sector portfolio (with replacement) there is a 5% chance of being 87 percent above the average. The comparable value for the one asset value-weighted portfolio is 89 percent. At the five-asset level there is still a 5% chance of being about 50/43 percent above the average. The figures for the Retail and Industrial sectors show similar results. Such a wide variability suggests that reporting the average value may be misleading.

Table 6: Risk values at the 95% Confidence level Upper Boundary, as a proportion of Mean risk (100)
Simulation Results – Sampled with and without replacement – Equally-weighted and Value-weighted portfolios of selected sizes

	Sampled WITH Replacement									Sampled WITHOUT Replacement							
		Sector			Region						Sector			Region			
No. of	Equally Weighted Portfolios									Equally Weighted Portfolios							
Assets	**ALL**	Retail	Office	Industrial	London	SE	S&W	Northern	Mean Risk	**ALL**	Retail	Office	Industrial	London	SE	S&W	Northern
1	**249.9**	271.8	186.9	190.7	219.2	238.8	201.4	200.5	100	**192.2**	186.8	198.3	177.9	171.3	189.3	197.0	187.3
2	**194.7**	199.6	174.1	166.2	179.9	172.0	173.5	165.3	100	**168.6**	160.1	177.1	150.9	144.2	164.5	183.5	168.7
3	**167.9**	171.8	162.8	153.7	167.5	154.2	163.7	157.2	100	**157.9**	160.3	165.5	144.7	126.4	158.1	168.9	156.4
4	**162.5**	153.3	156.5	147.2	158.3	152.4	161.8	150.8	100	**149.1**	151.2	157.0	135.7	118.9	146.6	159.1	151.8
5	**156.5**	146.0	150.1	139.7	149.4	147.1	155.4	145.7	100	**145.1**	147.1	151.3	137.5	115.9	141.4	153.5	148.7
6	**151.4**	140.5	145.5	136.2	143.6	143.2	146.4	143.3	100	**144.0**	142.4	144.1	134.0	115.4	141.9	145.1	146.2
7	**149.9**	137.0	139.1	135.1	140.3	137.5	144.0	138.2	100	**141.5**	139.5	144.7	133.7	115.4	138.8	144.4	145.8
8	**147.1**	135.7	138.4	132.1	138.0	133.4	137.2	135.5	100	**136.6**	135.3	140.0	131.5	114.3	132.8	142.6	141.2
9	**144.6**	133.8	136.1	131.0	136.6	130.4	135.6	132.2	100	**135.0**	133.1	139.6	128.0	113.6	130.8	139.0	138.7
10	**141.5**	131.5	134.9	128.7	135.7	130.3	136.2	131.6	100	**133.8**	134.7	137.6	126.6	112.4	128.8	134.7	135.7
15	**135.4**	126.5	130.4	124.3	129.4	122.2	131.1	127.7	100	**130.6**	127.5	127.0	121.2	111.4	121.8	131.2	126.4
20	**129.4**	122.0	125.5	120.5	125.9	122.3	126.7	123.2	100	**126.1**	121.9	123.7	116.6	110.5	120.0	123.8	121.8
25	**127.5**	119.6	120.3	119.3	122.4	120.5	123.2	121.4	100	**123.3**	119.1	119.7	114.6	110.6	118.2	118.9	120.1
30	**124.1**	118.3	119.4	117.2	120.8	117.4	121.4	120.4	100	**122.8**	116.7	117.1	114.3	110.2	116.0	117.3	117.6
40	**120.5**	115.0	117.3	115.1	118.3	115.6	119.8	119.3	100	**119.9**	114.9	114.1	111.8	109.3	113.1	114.5	113.9
50	**119.0**	113.2	116.5	114.1	116.0	113.4	117.3	119.1	100	**117.8**	112.6	112.2	108.8	108.9	111.4	111.4	110.6
	Value Weighted Portfolios									Value Weighted Portfolios							
1	**258.9**	174.5	188.7	183.3	204.4	241.8	213.9	188.3	100	**191.4**	193.0	189.3	182.0	197.4	234.6	199.5	191.2
2	**213.7**	160.9	170.2	156.2	185.6	185.1	185.9	172.9	100	**186.3**	165.0	181.6	154.6	162.1	165.9	177.6	181.9
3	**175.3**	150.0	160.9	152.5	170.0	153.2	165.1	165.9	100	**165.8**	161.9	160.1	143.6	158.9	163.5	167.8	167.6
4	**161.6**	144.6	152.3	142.4	157.7	150.6	164.4	162.0	100	**161.8**	150.2	154.6	138.6	155.9	149.0	156.5	160.9
5	**154.7**	137.1	143.2	138.0	149.3	150.7	161.6	154.1	100	**159.1**	149.4	147.6	134.4	151.0	143.2	160.0	158.0
6	**153.6**	133.9	140.9	133.9	142.7	143.3	156.2	157.5	100	**155.2**	142.8	140.4	133.7	148.3	137.5	149.7	147.1
7	**148.3**	132.8	138.4	132.4	141.3	138.4	153.8	155.9	100	**154.0**	141.4	136.7	131.3	143.5	134.0	146.8	144.2
8	**147.2**	132.1	133.8	130.9	138.5	134.3	152.3	155.4	100	**149.9**	138.9	135.6	128.3	138.9	130.6	146.4	141.9
9	**143.4**	132.8	132.4	129.0	136.5	131.9	152.3	149.5	100	**148.5**	138.7	132.5	126.2	135.4	129.2	145.1	139.1
10	**142.9**	130.8	129.4	127.2	134.3	129.2	149.0	146.1	100	**144.8**	137.6	132.0	125.8	131.8	127.2	143.5	133.9
15	**134.6**	126.2	123.7	120.2	128.6	124.5	139.2	134.9	100	**136.8**	128.7	126.6	122.0	123.6	122.1	136.3	132.4
20	**129.6**	122.1	120.9	119.1	124.0	121.6	136.1	130.3	100	**132.3**	123.0	122.3	118.0	119.9	117.1	130.0	127.0
25	**125.5**	120.4	118.3	116.7	120.9	121.3	132.3	128.7	100	**127.5**	120.4	119.6	115.7	116.6	115.7	126.5	123.9
30	**124.1**	117.9	117.0	115.0	119.3	120.1	128.3	126.3	100	**124.5**	118.0	118.0	113.7	113.7	113.8	123.1	121.4
40	**120.0**	114.6	115.5	114.2	116.5	118.0	125.7	122.6	100	**121.0**	115.0	113.1	111.2	108.8	111.8	118.0	117.3
50	**117.9**	113.1	113.8	112.9	115.7	116.3	123.2	118.0	100	**119.1**	112.5	110.6	108.7	104.4	111.3	114.8	113.2

After Newbould and Poon (1993)

In general, the deviation of the upper bound from the average depends on the spread of the correlation coefficients around the average. For example, the correlation coefficient of the Office sector has the largest standard deviation of the three sectors (0.191). Offices displays the greatest variability around the average risk level. The same is true for the Northern region. However, the *actual* spread also depends on the individual risks of the assets and the number of assets under consideration. For example, the correlation coefficients of the Retail sector display the lowest standard deviation. This would imply that it would have the lowest upper bound. Initially this turns out not to be the case. It is the Industrial sector that displays the lowest variability, for the one asset case. This is due to the number of properties that can be sampled. In the case of the Retail sector the number is more than double that of the Industrial sector (see Table 1). When running a simulation it is more likely that a poor combination can be formed in the Retail sector than the Industrial sector and so the Retail sector can show high variability, at least for small sample sizes. However, the low risk displayed in the Retail sector quickly becomes dominant and beyond the one asset case the Retail sector shows the lowest variability, at least for the with-replacement simulations. In the case of the simulations without replacement the number of assets that can be sampled from becomes still more important. In sectors with only 70 assets when a simulation without replacement is run on portfolios of size 50 the choice of assets to be included is successively more constrained, hence variability around the average will be small. Both Offices and Industrials show lower variability than Retails in portfolios of 40 assets or more.

Previous studies have suggested that as few as 20 properties are sufficient to eliminate most of the unsystematic risk with in a real estate portfolio. Beyond this level the marginal reduction in portfolio risk might be thought to be too small to justify the increased costs of purchasing additional properties. The arguments of Newbould and Poon (1993) and McDonald (1975) suggest though that this is not necessarily the case. Investors are concerned about their portfolio and the possibility of an unpleasant surprise occurring from holding their portfolio, not the average portfolio. The results in Table 6 indicate that at the five authority (asset) level, equivalent to a minimum of 20 properties, there is still a 5% chance of having a risk level approximately 62-16 percent above the average, depending on the sector or region chosen and the weighting scheme employed. At the 10-authority (up to 40-property level) the deviation from the average is still 49-12 percent. Even at the 50 asset (200+ property) level the portfolio variance may not be small enough to be ignored. This implies that a portfolio size of over 200 properties is the bare minimum level that investors need in order to feel comfortable with the risk level of the real estate portfolio. This confirms the results of Brown (1988, 1991) and Cullen (1991).

The importance of increasing the portfolio size is therefore not only related to the accompanying reduction in the mean portfolio risk level but also to the confidence that an investor can have in achieving such a reduction. In order to get some idea of the number of assets needed to reduce the upper 95% confidence limits to some “acceptable” level for an investor the value-weighted simulations (in this case without replacement) were extended up to the full sample size, 392. This is seen graphically in Figure 3. This simulation is representative of the ‘true’ distribution of funds and permits us to determine the likely number of assets needed to achieve a given percentage level of deviation of the upper 95% confidence limit from the average portfolio risk level. Accordingly this is done for deviation levels of 20, 15, 10, 5 and 1 percent above the mean variance level. The results, in Table 7, for the value-weighted case, with replacement, show that the upper 95% confidence limit requires 45 authority level assets (at least 180 properties) to be 20% above the average result. An investor wishing to be no more 1% above the average risk with a confidence 95%, would need to hold more than 97% of all the available assets! Such an investor basically needs to hold the ‘market’, which would be unrealistic for even the largest fund. Consequently for the majority of funds, especially small funds, it seems reasonable to suggest that individually they will display an actual risk level considerably higher or lower than the average risk. The results of the simulations reported here clearly show that a small portfolio is almost as equally likely to have high *or* low risk. The observations of Cullen (1991) based on actual real estate fund portfolios indicate that this is the case. Therefore all that can be said is that as the number of properties within a portfolio increases, portfolio volatility declines, with smaller portfolios in general having a higher risk on average than larger portfolios and with greater variability around the average.

Table 7: The Number of Assets Needed to Achieve a Given Deviation from the Mean Portfolio Risk Level in a Value-weighted Portfolio

Deviation	20%	15%	10%	5%	1%
Value Weighted	45	66	113	268	382

6.0 Conclusions

Using a large dataset for 392 locations (local authorities) this paper generally confirms results of many previous studies that a diversification strategy across sectors offers lower risk levels than by region (see Lee and Byrne, 1998). Based on an equal-weighting strategy, the reduction in risk is limited to 22% on average and no more 28% at best.

However, the simulations show some small but significant differences in outcomes between value-weighted and equal-weighted portfolios using the overall data. This is in line with the work of Morrell (1993). The true impact of value skewness still may be understated and cannot be fully explained since the analysis used averaged data for the local authorities, rather than the values of individual properties.

There is little difference in the results when the analysis is conducted with and without replacement, although the latter may be thought to be more realistic.

The reduction in mean portfolio risk is initially quite rapid but only shows a significant fall up to about portfolios of size 5 (at least 20 properties). In contrast the variability around the mean displays a rapid and significant decline for portfolios up to size 15-20 (60-80 properties), after which any reduction in variability around the mean falls at a much reduced and insignificant rate. This leaves the investor facing a good deal of variability. The averaged results can hide a large amount of variability within the data and the simulations show that the actual volatility within a portfolio can be much higher or lower than the average, especially for small sized portfolios. As a consequence an individual investor who follows the advice contained in previous studies which are based on the results of average portfolios may be exposing themselves to greater risk than they intended. The results in Table 6 suggest that the previous recommendations that 20-40 assets are needed to achieve a satisfactory level of risk reduction would seem to underestimate the actual number of assets needed. The actual number of assets held is dependent on the actual universe of properties under consideration; the weighting scheme used; the desired confidence level and the individual's attitude to risk.

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Appendix

Table A: Maximum, Minimum and Standard Deviation of Correlations

	Max. Correlation			
Sectors/Regions	**Retail**	**Office**	**Industrial**	**Overall**
London	**0.961**	**0.968**	**0.969**	**0.969**
South East	**0.973**	**0.964**	**0.976**	**0.977**
South and West	**0.977**	**0.959**	**0.973**	**0.973**
Northern	**0.946**	**0.946**	**0.973**	**0.976**
Overall	**0.977**	**0.976**	**0.976**	**0.977**
	Min. Correlation			
Sectors/Regions	**Retail**	**Office**	**Industrial**	**Overall**
London	**0.207**	**0.320**	**0.497**	**-0.155**
South East	**0.114**	**0.205**	**0.389**	**-0.257**
South and West	**0.057**	**0.603**	**0.371**	**-0.547**
Northern	**0.109**	**-0.188**	**0.368**	**-0.091**
Overall	**-0.010**	**-0.481**	**-0.296**	**-0.684**
	SD of Correlation			
Sectors/Regions	**Retail**	**Office**	**Industrial**	**Overall**
London	**0.143**	**0.139**	**0.115**	**0.176**
South East	**0.125**	**0.128**	**0.100**	**0.224**
South and West	**0.137**	**0.079**	**0.106**	**0.264**
Northern	**0.139**	**0.308**	**0.144**	**0.161**
Overall	**0.143**	**0.191**	**0.149**	**0.212**

Table B: Average Capital Values 1981-1996

	Retail	Office	Industrial	Overall
London	£1,083,269	£4,753,403	£3,416,252	£2,709,954
South East	£1,036,100	£2,939,733	£2,683,319	£2,088,665
South and West	£1,069,053	£2,028,087	£1,771,824	£1,389,246
Northern	£1,102,195	£2,223,856	£1,623,766	£1,369,068
Overall	£1,070,007	£2,984,945	£2,420,448	£1,844,701

Source IPD *Local Markets Report*

Table C: Ratio of Value-weighted to Equal-weighted Average Portfolio Risk Selected Portfolio Sizes

PANEL A								
REGION								
With Replacement				No. of Assets	Without Replacement			
London	South East	South & West	Northern		London	South East	South & West	Northern
96.05	102.71	102.86	95.54	1	96.99	105.73	102.04	102.27
101.91	103.41	104.17	102.07	2	94.05	100.29	98.42	103.89
101.79	99.76	100.14	103.18	3	98.50	101.36	101.27	104.32
100.68	99.56	100.86	104.18	4	101.12	100.79	99.94	104.08
100.71	101.15	102.69	103.24	5	102.68	100.65	102.88	103.56
100.26	100.01	104.71	105.09	6	104.12	98.61	101.94	100.46
100.72	100.21	104.52	106.81	7	104.24	98.50	101.19	99.84
100.43	100.34	106.44	107.45	8	103.80	99.31	101.74	100.19
100.09	100.67	106.94	106.61	9	102.61	99.56	102.63	100.05
99.52	99.63	105.36	105.91	10	102.58	99.56	103.51	99.24
99.66	101.01	103.46	102.52	15	101.28	100.30	102.23	102.67
99.03	99.80	104.15	102.87	20	101.42	98.84	102.82	102.32
99.16	100.46	104.04	103.05	25	101.05	98.99	103.53	101.61
99.24	101.26	103.18	102.44	30	100.45	98.95	102.72	101.69
99.08	101.04	102.64	101.37	40	99.93	99.35	101.63	101.63
99.87	101.31	102.59	99.40	50	99.65	99.93	101.62	101.29

PANEL B								
SECTOR								
	Asset						Asset	
Retail	Office	Industrial				Retail	Office	Industrial
82.69	100.30	102.32		1		97.83	87.37	91.04
87.13	103.27	98.61		2		100.93	100.24	102.15
93.44	104.93	96.90		3		102.80	98.12	101.79
94.72	103.07	100.11		4		101.70	98.12	97.60
98.26	102.16	99.17		5		100.63	95.64	97.30
98.02	103.03	98.14		6		101.18	95.17	98.38
97.78	103.12	99.03		7		101.64	95.50	97.29
97.59	103.51	98.55		8		99.69	96.57	98.60
98.42	103.21	99.63		9		100.22	96.70	98.66
97.47	102.36	100.12		10		100.19	97.06	97.86
97.20	102.18	100.94		15		99.52	97.65	99.31
96.99	102.46	100.06		20		98.28	99.15	98.92
97.91	102.39	100.83		25		97.54	98.74	100.16
98.47	102.08	100.24		30		98.58	98.49	100.02
97.57	101.06	100.64		40		98.59	99.14	99.52
97.98	100.55	100.90		50		98.54	98.81	99.24

Figure 1: Percentage Risk Compared with that of a One Asset Portfolio

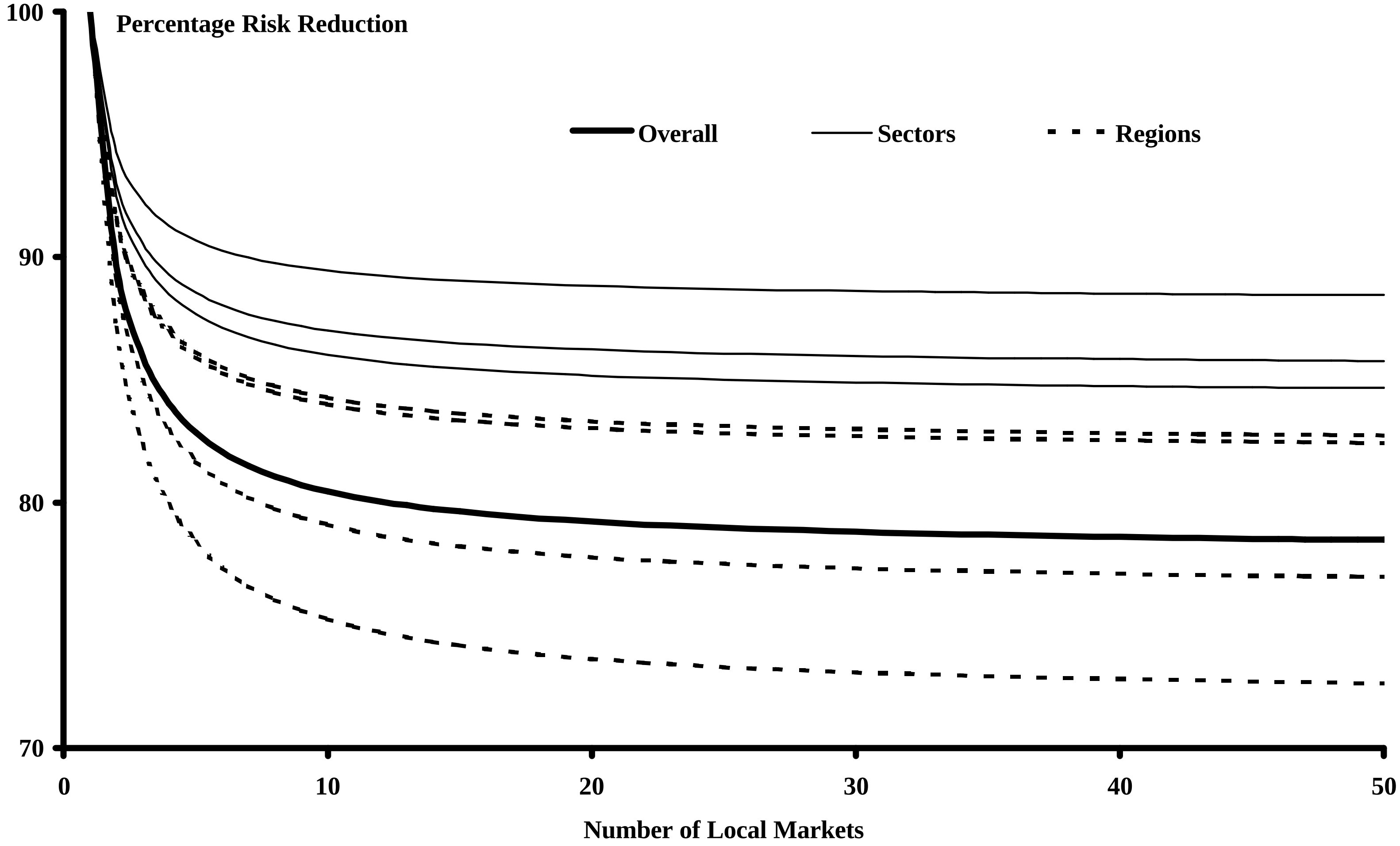

Figure 2: Ratios of Maximum and Minimum SDs to Mean
Portfolio sizes 1 to 50

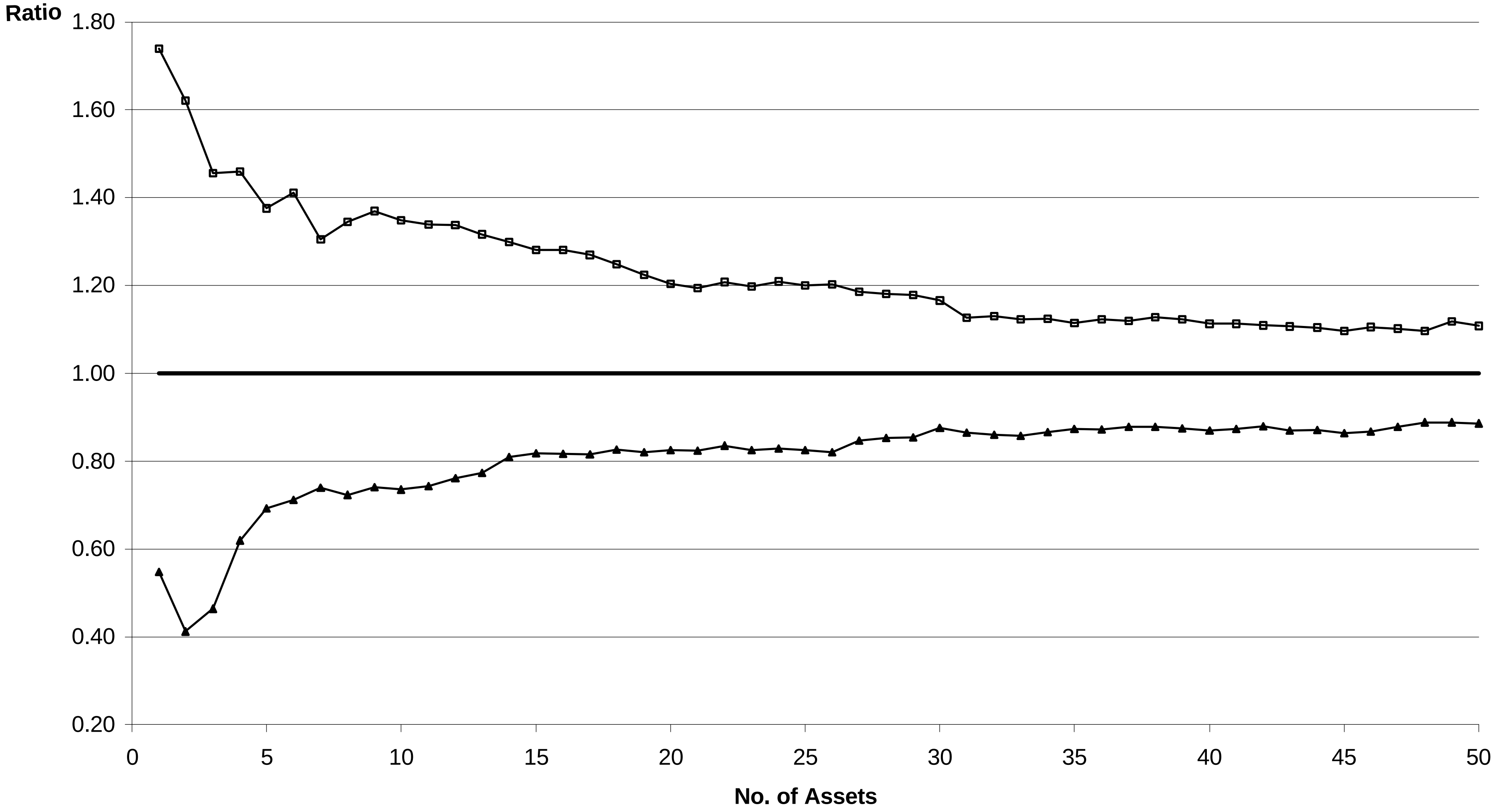

Figure 3: Upper 95% confidence level of Risk as a Proportion of Mean Risk
Value-weighted without replacement: All assets