# *Angular momentum conservation and gravity wave drag parameterization: implications for climate models*

# Angular Momentum Conservation and Gravity Wave Drag Parameterization: Implications for Climate Models

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ABSTRACT

The robustness of the parameterized gravity wave response to an imposed radiative perturbation in the middle atmosphere is examined. When momentum is conserved and for reasonable gravity wave drag parameters, the response to a polar cooling induces polar downwelling above the region of the imposed cooling, with consequent adiabatic warming. This response is robust to changes in the gravity wave source spectrum, background flow, gravity wave breaking criterion, and model lid height. When momentum is not conserved, either in the formulation or in the implementation of the gravity wave drag parameterization, the response becomes sensitive to the above-mentioned factors—in particular to the model lid height. The spurious response resulting from nonconservation is found to be nonnegligible in terms of the total gravity wave drag–induced downwelling.

## 1. Introduction

The importance of gravity waves in determining the large-scale structure of the middle atmosphere is well established (Fritts and Alexander 2003). Gravity waves transport angular momentum from their source regions in the troposphere and exert a torque where they dissipate in the middle atmosphere. The angular momentum transfer due to small-scale gravity waves [gravity wave drag (GWD)] is not explicitly resolved and thus must be parameterized in general circulation models (GCMs); not parameterizing this process leads to unacceptable climate biases such as the cold pole problem (Garcia and Boville 1994). Unfortunately, GWD parameterizations are very poorly constrained by current observations (Fritts and Alexander 2003). This results in tuning of GWD parameterization parameters to obtain a reasonable mean climate.

Climate perturbation experiments are a standard way to explore climate sensitivity and compare different models. Because physical parameterizations represent the largest uncertainty in atmospheric models, it is important to understand the feedbacks from these parameterizations under such perturbations. For middle atmospheric applications, in particular, it is necessary to confirm that the parameterized GWD responds in a physically correct manner to perturbations in the resolved wave drag or radiative forcings. The fact that a GWD parameterization can be tuned to obtain the current mean climate does not mean that it responds correctly to climate perturbations. Moreover it is important to know the extent to which the response to a climate perturbation is robust to this tuning.

GWD feedbacks to zonal wind perturbations induced by stratospheric sudden warmings were first elucidated by Holton (1983). Zonal wind perturbations lead to a filtering effect (filtering of gravity wave momentum flux), which, according to the principle of downward control (McIntyre 1989; Haynes et al. 1991), results in adiabatic temperature changes through changes in the amount of gravity wave induced downwelling. Applying Holton's (1983) reasoning to a polar cooling (e.g., from polar ozone depletion) predicts enhanced GWD induced downwelling above the polar cooling, which leads to adiabatic warming. Indeed, there exists evidence of a warming above the polar cooling due to the Antarctic ozone hole in both radiosonde observations and in the National Centers for Environmental Prediction (NCEP) reanalysis (Randel and Wu 1999). Moreover Manzini et al. (2003) reported a stratospheric polar warming from GWD feedbacks in GCM experiments studying the climate sensitivity of the middle at-

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mosphere to ozone depletion and changes in greenhouse gases, which provide a radiative cooling.

An important question is the robustness of the above effect to modeling choices such as the gravity wave breaking criterion and source spectrum, given the poorly constrained nature of GWD parameterization. The response in the real atmosphere respects all relevant physical laws, including conservation of angular momentum, which therefore should also be respected by the parameterization in both its formulation and implementation. Rayleigh drag, which is still used in many middle atmosphere models as a surrogate for GWD (e.g., Shine et al. 2003; Dameris et al. 2005), is nonconservative in its formulation. While most current middle atmosphere models use a flux-based GWD parameterization that is conservative in its formulation, the momentum flux at the top of the model is typically allowed to escape to space, in which case the parameterization is nonconservative in its implementation. Here we examine the effect of angular momentum conservation on the robustness of the GWD response to an imposed polar cooling. Shepherd and Shaw (2004, hereafter SS04), showed that GWD feedbacks to a stratospheric polar cooling could lead to physically spurious downward influence when momentum conservation was violated. However, they did not explicitly explore issues of robustness.

In the current state of understanding, angular momentum conservation by parameterized gravity waves is believed to apply, to good approximation for current GCM resolutions, within each vertical column and hence within each latitude band. [This is also assumed by the Holton (1983) filtering mechanism.] In this case, angular momentum conservation is equivalent to zonal momentum conservation, and for simplicity we use the term momentum hereafter.

In this paper we show that when momentum is conserved the physically correct GWD-induced response of adiabatic warming above an imposed polar cooling, with no response below, is robust to changes in the gravity wave source spectrum, basic state, gravity wave breaking criterion, and model lid height. Therefore, tuning of GWD parameters does not alter the basic features of the response provided momentum is conserved (although details will depend on those parameters). This robustness follows from the constraints of downward control, and is illustrated via quantitative calculations using the zonal mean model used by SS04. When conservation of momentum is violated, on the other hand, the GWD response to an imposed radiative cooling is no longer robust. The response then becomes sensitive to the gravity wave breaking criterion, gravity wave source spectrum, and, especially, model lid height. Examples are provided showing how nonconservation can even lead to a reversal in sign of the adiabatic temperature change above the imposed cooling. Thus, conserving momentum in GWD parameterization has important implications for ensuring a robust model response to climate perturbations, and reduces the sensitivity to poorly constrained aspects of GWD parameterization.

## 2. Idealized model calculations

As in SS04 we focus on the zonal mean response to a switch-on radiative cooling over the pole (e.g., from polar ozone depletion) in the long time limit calculated from the quasigeostrophic (QG) approximation to the transformed Eulerian mean (TEM) equations (Andrews et al. 1987). The imposed polar cooling is centered at 20 km with a maximum magnitude of 20 K. We consider the model response spun up from a state of rest (section 3) as well as in the context of an idealized polar jet (sections 4 and 5). The GWD is parameterized using the Alexander and Dunkerton (1999, hereafter AD99), parameterization without back-reflection, unless otherwise stated. AD99 is a spectral parameterization that assumes that each gravity wave propagates vertically, conserving momentum flux until its breaking height where the combined lapse rate of the wave and background mean state is convectively unstable (Lindzen 1981). The breaking height is ultimately reached, irrespective of the basic state, due to the decrease in density with altitude, but may occur much lower in the presence of a critical layer. Unless stated otherwise, the gravity wave source spectrum is symmetric, $F_0(c) = 0.25\ \mathrm{m^2\,s^{-2}}\ \mathrm{sgn}(c)$ for $5\ \mathrm{m\,s^{-1}} \leq |c| \leq 40\ \mathrm{m\,s^{-1}}$, with a net momentum flux (of each sign) of $2 \times 10^{-3}$ Pa and a launch level of 14 km. These settings differ from those used in SS04, but are more typical of those used in GCMs. We enforce momentum conservation within each latitude band by depositing any momentum flux at the model lid in the top 10 km of the model domain. This generates the same vertical motion within the domain that would have resulted had the breaking region been resolved (SS04). The GWD has been confined to midlatitudes for numerical reasons and because downward control does not hold in the Tropics. Furthermore, we cannot expect a steady solution in the Tropics where a steady gravity wave forcing can lead to oscillating zonal mean zonal winds (Campbell and Shepherd 2005). The model includes a frictional boundary layer and Newtonian cooling (see SS04).

## 3. Robustness to changes in the gravity wave source spectrum and model lid height

The symmetric source spectrum used in the quantitative calculation performed by SS04 was simply chosen for illustration. In reality the source spectrum is not well constrained by current observations and is likely not symmetric. Here we show that the details of the source spectrum are unimportant in terms of the robustness of the GWD feedback to a radiative perturbation and the absence of spurious downward influence, provided momentum is conserved. Figure 1 compares the circulation response for symmetric and asymmetric gravity wave source spectra. Figure 1a shows the time mean streamfunction response spun up from a state of rest without the polar cooling, Fig. 1b with the polar cooling, and Fig. 1c the difference, all for the symmetric source spectrum. The absence of contours in Fig. 1a reflects the fact that a basic state at rest combined with a symmetric source spectrum results in zero net GWD (the eastward and westward components are equal) for all heights and hence zero vertical residual velocity. The exact cancellation of the GWD at each level is also apparent from the symmetry of the gravity wave breaking height as a function of gravity wave phase speed (Fig. 2, dotted lines).

With a symmetric source spectrum, a zonal wind is required to induce nonzero GWD and hence a net circulation. The structure of the zonal wind induced by the polar cooling according to thermal wind balance is plotted in Fig. 2 (thin solid line). The eastward jet Doppler shifts the eastward component of the gravity wave spectrum toward zero intrinsic phase speed ($|\overline{u} - c|$ is reduced, where $\overline{u}$ is the zonal mean zonal wind and $c$ is the wave phase speed) and so these waves encounter their breaking levels at lower heights. The westward component, on the other hand, is shifted further away from zero intrinsic phase speed ($|\overline{u} - c|$ is increased), so the waves break higher up. The shifting of the breaking heights with the polar cooling is shown in Fig. 2 (thick solid lines). The result is negative drag driving poleward motion above positive drag, which drives equatorward motion, with corresponding downwelling over the pole (Fig. 1b). The downwelling leads to adiabatic warming. This response is analogous to the filtering mechanism described by Holton (1983) for the case of a stratospheric sudden warming, which leads to a GWD-induced cooling of the mesosphere. The difference between Figs. 1a and 1b (Fig. 1c) representing the GWD feedback is identical to Fig. 1b. Figure 1c is comparable to Fig. 2c of SS04 even though the parameter settings have changed. Conservation of momentum (via deposition of momentum flux at the model lid above 40 km) ensures that the drag anomalies balance, leading to a closed circulation above the polar cooling.

Figure 1d shows the steady streamfunction response for an asymmetric gravity wave source spectrum without the polar cooling. The symmetry in the gravity wave spectrum is broken by including more eastward momentum flux through a broader range of phase speeds. There is a nonzero circulation because the vertical integral of the density-weighted GWD is nonzero and hence, according to downward control, results in a net vertical residual velocity. Figure 1e shows the circulation response with the polar cooling. The changes in the circulation are due to Doppler shifting of the gravity wave phase speeds, which leads to a corresponding shift of the drag regions in the vertical, as was described above for the symmetric spectrum. The difference (Fig. 1f) is in qualitative agreement with the response for the symmetric source spectrum (Fig. 1c). Because the vertical integral of the density-weighted GWD has not changed with the cooling (the surface conditions remain unchanged), and because there was no change to the zonal wind below the radiative perturbation, and hence no change in the GWD below, the residual circulation below the perturbation is unchanged; and thus the GWD response to the perturbation, shown in Figs. 1c,f, is qualitatively similar in the two cases. An analogous response would occur for any kind of symmetry breaking of the source spectrum (increased momentum flux of one sign, etc.). The source spectrum determines the strength of the circulation response, but changing the source spectrum does not alter the structure of the physical response or induce spurious downward influence so long as momentum is conserved.

As discussed by SS04, the effect of depositing parameterized momentum flux within the top few model levels to enforce conservation of momentum is to preserve the correct physical response and avoid spurious downward influence. Although the deposition of momentum flux in the upper levels distorts the drag profile there, it preserves the physically correct response further below as it effectively parameterizes the vertical residual velocity induced by the gravity waves breaking above the model lid height (in this case above 50 km). If the model lid were higher then all the parameterized gravity waves would break within the model domain and the circulation would be in qualitative agreement with the circulation in the low lid model.

Figure 3 illustrates this point by considering the effect of raising the model lid. Figure 3a shows the steady streamfunction response with the polar cooling for the model lid at 80 km; in this case all the parameterized gravity waves break within the model domain. The circulation is confined to levels above the region of the

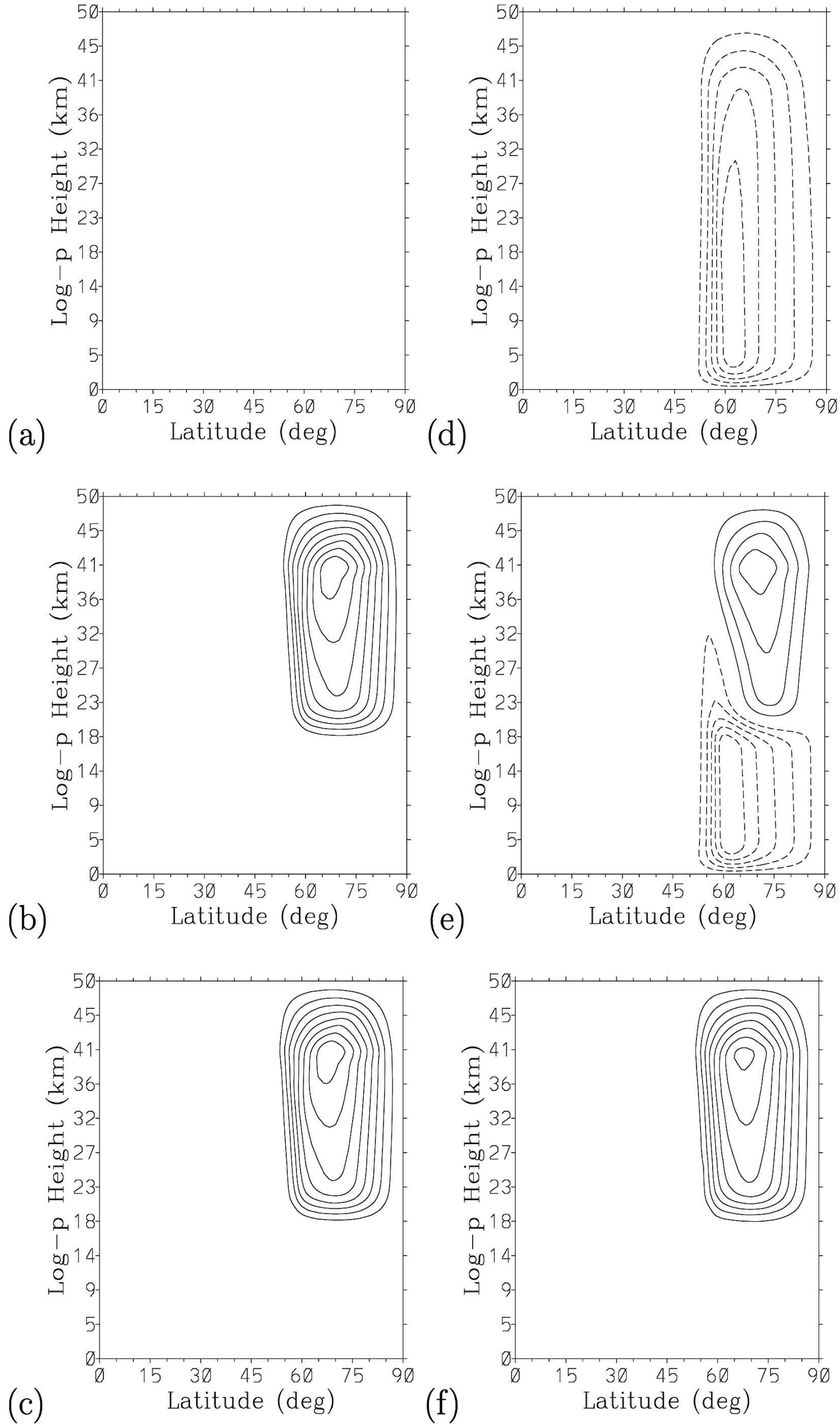


FIG. 1. The steady streamfunction response to an imposed radiative perturbation (polar cooling) centered at 20 km, calculated with a zonally symmetric QG version of the TEM equations spun up from a state of rest. The streamfunction response, with the same contour interval (0.01 kg $m^{-1}$ $s^{-1}$) in all panels, for a momentum-conserving implementation of the AD99 gravity wave drag parameterization with a symmetric source spectrum: (a) without the radiative perturbation, (b) including the radiative cooling, (c) the difference, and (d)–(f) corresponding streamfunction responses for an asymmetric source spectrum. Positive contours correspond to clockwise circulations.

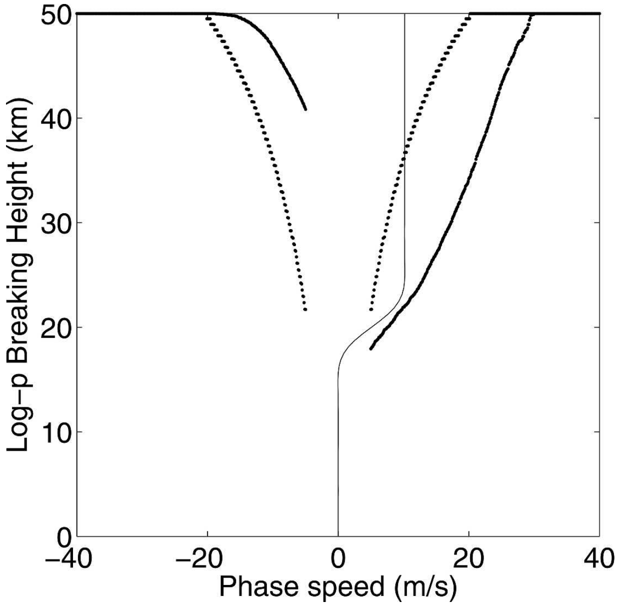


FIG. 2. Wave breaking height as a function of phase speed for the AD99 parameterization for a symmetric gravity wave source spectrum $F_0(c) = 0.25\ \mathrm{m^2\ s^{-2}}\ \mathrm{sgn}(c)$ for $5\ \mathrm{m\ s^{-1}} \leq |c| \leq 40\ \mathrm{m\ s^{-1}}$ averaged between 60°N and the pole. The thick dotted lines correspond to the breaking heights without the polar cooling, and the thick solid lines the breaking heights with the cooling. The zonal wind perturbation associated with the cooling is shown averaged between 60°N and the pole (thin solid line). The corresponding streamfunction responses are shown in Figs. 1a–c.

imposed cooling with adiabatic warming over the pole. This is the physical response discussed previously. Figure 3b shows the corresponding circulation with a truncated model domain (i.e., model lid at 50 km), which is exactly Fig. 1c with the plotted region extended to 80 km for comparison purposes. The difference between Figs. 3b and 3a is shown in Fig. 3c and is confined to the levels where the momentum flux at the model lid in the low lid case was deposited (above 40 km). Below 40 km, the difference vanishes. This calculation justifies the deposition of excess momentum flux in the upper model levels as one means of conserving momentum.

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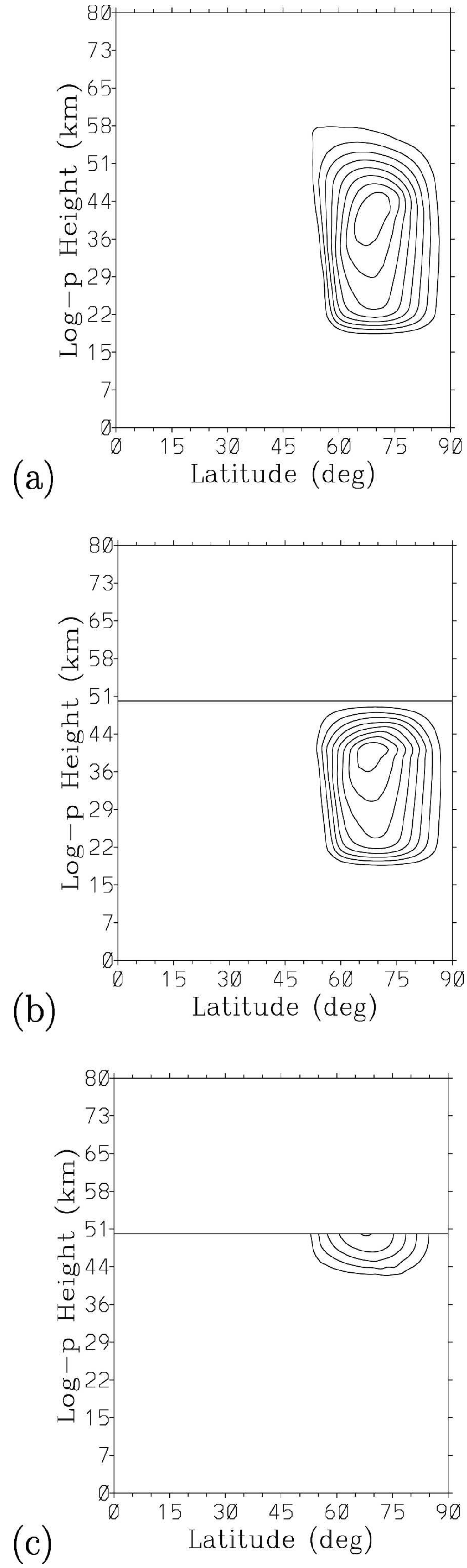


FIG. 3. The steady streamfunction response, with the same contour interval ($0.01\ \mathrm{kg\ m^{-1}\ s^{-1}}$), to a polar cooling for a momentum-conserving implementation of the AD99 parameterization with a symmetric source spectrum when (a) the model lid is raised to ensure all parameterized wave dissipation occurs in the model domain, and (b) the model lid is truncated and momentum flux at the model lid is deposited above 40 km. (c) The difference between (a) and (b). Positive contours correspond to clockwise circulations.

## 4. Robustness to a nonresting basic state

A more realistic assessment of the GWD feedback to a polar cooling occurs in the presence of a polar jet. Figure 4 shows various aspects of the steady streamfunction response for the case of a polar cooling imposed in the presence of an idealized polar jet (as in Scott and Haynes 1998). Once again the AD99 GWD parameterization is implemented with a symmetric source spectrum.

Figure 4a shows the steady streamfunction response without the polar cooling. As in Fig. 1b, the circulation reflects the structure of the eastward polar jet; there is a region of positive drag due to the Doppler shifting of eastward propagating gravity waves toward zero intrinsic phase speed (essentially critical layer filtering by the imposed eastward jet) below a region of negative drag due to the saturation of the westward portion of the gravity wave spectrum (due to the decrease in density). The drag distribution results in a circulation with adiabatic warming over the pole. The magnitude of the induced downwelling is in agreement with Garcia and Boville (1994; Fig. 5) and is thus realistic in terms of the total parameterized momentum flux currently used in climate models to alleviate the cold pole problem. (The temperature response depends on the Newtonian cooling coefficient, which is somewhat arbitrary, so we focus on the circulation.)

Figure 4b shows the net circulation; that is, the difference between the responses with and without the polar cooling. The polar cooling enhances the eastward shear near the pole. This results in a lowering of the positive drag region due to a lowering of the critical levels. The negative drag region is shifted to higher altitudes due to the Doppler shifting of the intrinsic phase speeds further from zero. The circulations induced as a result of these shifts of the drag regions (shown in Fig. 6a) are clearly visible in Fig. 4b. The drag dipole resulting from the shifting of the eastward component of the gravity wave spectrum around 20 km occurs below that due to the shifting of the westward component of the spectrum. This is the physically robust response discussed previously. In the real atmosphere the gravity wave–induced feedbacks to a polar cooling are closed localized drag dipoles because of the shifting of the individual drag components (eastward and westward) above the region of the imposed cooling. In these plots the model lid is at 80 km to ensure all the parameterized wave breaking occurs within the model domain.

The effect of truncating the model domain to 50 km is shown in Figs. 4c,e. Figure 4c shows the circulation response when the model lid is truncated to 50 km and momentum is conserved. The plotted domain is extended to 80 km for comparison with Fig. 4b. The difference in the circulation is shown in Fig. 4d. The effects of depositing momentum flux in the top few model levels are seen to be confined to levels above 40 km. A truncated model domain is evidently not detrimental in terms of altering the GWD response to the imposed cooling.

However a violation of momentum conservation in a truncated model domain, by allowing momentum flux to escape to space, distorts the physical response leading to adiabatic cooling over the pole and spurious downward influence (Fig. 4e). The difference between Figs. 4c,e (shown in Fig. 7) is a circulation cell that extends from just below the model lid to the surface, and is associated with the loss of the negative drag, which in Fig. 4e has been allowed to escape to space. (The fact that one contour does not reach the surface is a feedback effect, which is discussed in section 6.) According to downward control, the strength of the spurious downward influence is proportional to the momentum flux reaching the model lid, which obviously depends on the source spectrum and model lid height. The cancellation of the missing drag above 50 km in midlatitudes in Fig. 4e is somewhat fortuitous. If the model lid were placed at a level where the drag in midlatitudes did not cancel there would be spurious downward influence to the surface extending from the pole to midlatitudes. The sensitivity of the strength of spurious downward influence to the model lid height is discussed further in section 6.

The density-weighted vertical component of the residual circulations at 80° (solid) and 85°N (dashed) for Figs. 4a,c,e are shown in Fig. 8. The GWD-induced downwelling with consequent adiabatic warming over the pole in the basic state is shown by the lines labeled a. The downwelling is enhanced due to GWD feedbacks from the polar cooling (lines c), as described previously. It is clear from Fig. 8 that the GWD feedback represents a nonnegligible fraction of the downwelling in the climatological state. There is a striking change in the density-weighted downwelling when the momentum flux at the model lid is allowed to escape to space (lines e). The physical response of the downwelling over the pole is completely distorted. There is instead upwelling over the pole at 80°N, representing an error of more than 100%. Also, the vertical component of the residual circulation does not go to zero below the level of the imposed cooling when momentum is not conserved (cf. lines e with lines c), which reflects the spurious downward influence seen in Fig. 4e.

Neglecting momentum flux at the model lid is only one way to violate momentum conservation. Figure 4f

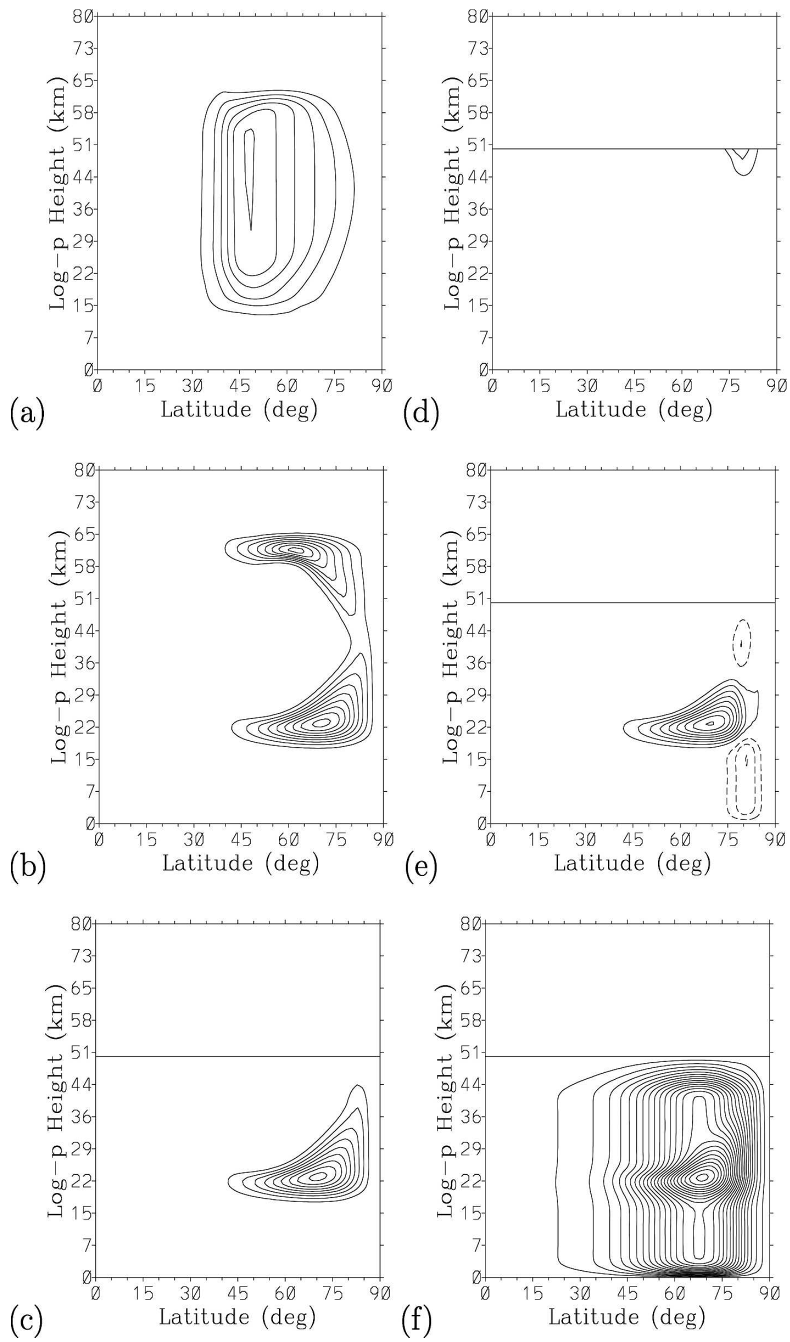


FIG. 4. The steady streamfunction response to a polar cooling for the AD99 parameterization with a symmetric source spectrum imposed in the presence of a polar jet: (a) the response prior to turning on the polar cooling, (b) the net circulation after taking the difference between before and after the imposed perturbation, (c) the response when the model lid is truncated and momentum is conserved, (d) the difference between (b) and (c), (e) the response when the model lid is truncated and momentum flux at the model lid is allowed to escape to space, and (f) the response when the model lid is truncated and momentum is conserved in the presence of a Rayleigh drag sponge layer above 40 km. Contour intervals are 0.09 kg $m^{-1}$ $s^{-1}$ for (a) and 0.0125 kg $m^{-1}$ $s^{-1}$ for (b)–(f). Positive contours correspond to clockwise circulations.

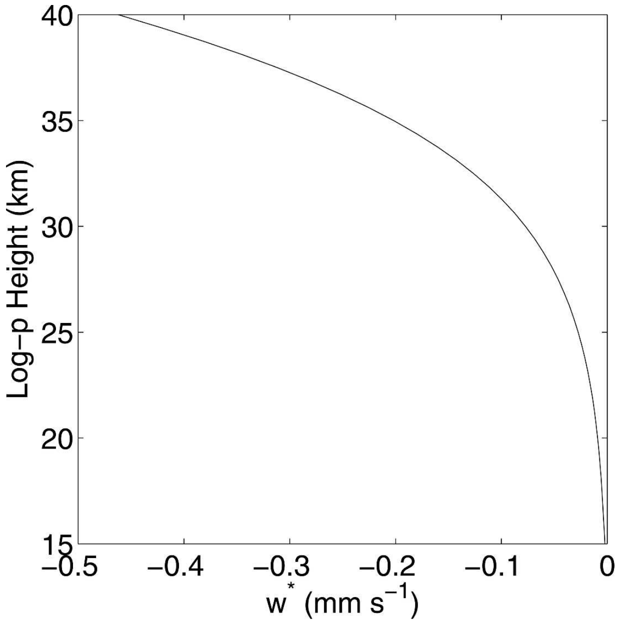


FIG. 5. The GWD-induced downwelling from Fig. 4a at 80°N, which is seen to be comparable to Fig. 4 of Garcia and Boville (1994).

shows the streamfunction response when the AD99 parameterization is implemented conserving momentum as in Fig. 4c, but in the presence of a Rayleigh drag (RD) sponge layer above 40 km. The RD is a linear drag that does not represent the divergence of a flux, is not constrained by a conservation principle, and is known to induce spurious climate sensitivity (Shepherd et al. 1996). Although momentum is conserved in the flux-based GWD parameterization (the circulation induced by AD99 is visible), imposing an RD sponge layer nevertheless leads to a severe distortion of the expected response including spurious downward influence. The reason for the spurious downward influence is clear when one compares the physical response involving shifting of the drag regions to the response under RD (Figs. 6a,b), as in the thought experiment depicted in Fig. 1 of SS04. Since the zonal wind anomaly induced by the cooling is single signed (eastward) so too is the RD response (negative). There is no balance of the RD-induced drag within the vertical column and hence there is a net vertical residual velocity resulting in spurious downward influence to the surface. This further emphasizes that in order to obtain a physically correct GWD response to a zonal wind perturbation, there must be no zonal mean sponge layer. (An RD sponge layer acting only on the zonal waves, to avoid reflection of upward-propagating waves at the model lid, is fine with regard to the meridional circulation.)

## 5. Robustness to the gravity wave breaking criterion

The circulation response to the polar jet (Fig. 4a) is inherently parameterization-dependent because different parameterizations make different assumptions concerning gravity wave saturation and breaking. There exist numerous parameterizations of GWD based on different breaking criteria; for a detailed review see Fritts and Alexander (2003). Here we compare the responses from the AD99 parameterization (Fig. 4) to those obtained from the Hines (1997a,b), both hereafter H97, parameterization, performing analogous model experiments to those in Fig. 4.

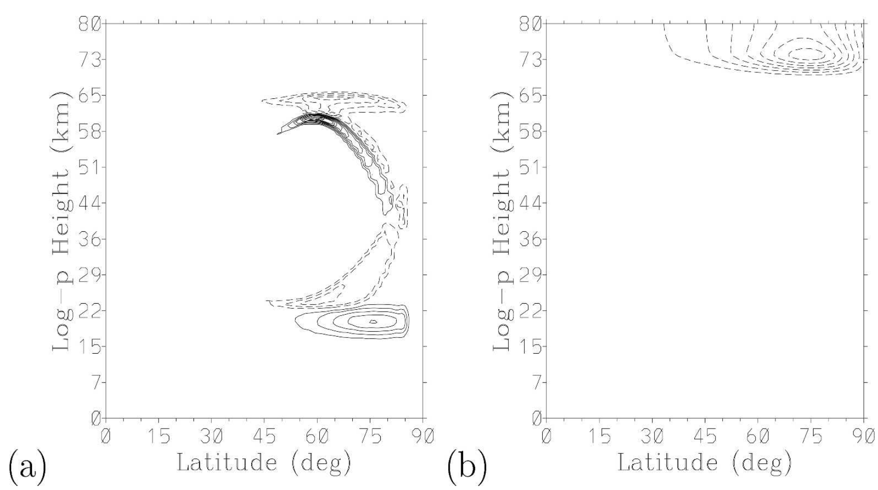


FIG. 6. Density-weighted gravity wave drag response to the radiative cooling in the presence of a polar jet using (a) AD99 with a symmetric source spectrum with contour interval 0.00025 kg $m^{-2}$ $s^{-1}$ $day^{-1}$, and (b) Rayleigh drag with contour interval 0.025 kg $m^{-2}$ $s^{-1}$ $day^{-1}$.

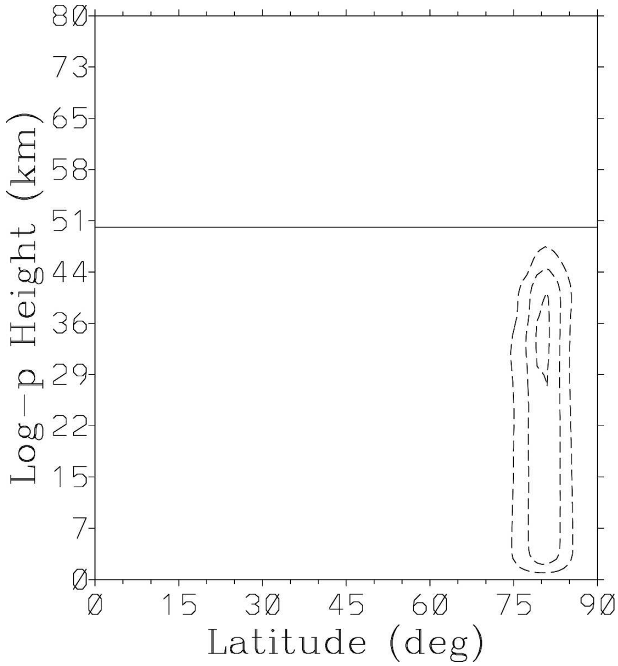


FIG. 7. The difference between Fig. 4c and Fig. 4e. Contour interval 0.0125 kg m$^{-1}$ s$^{-1}$.

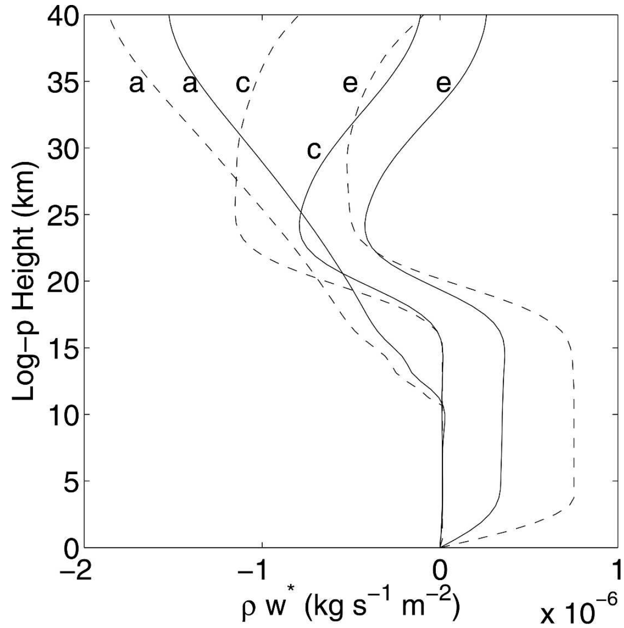


FIG. 8. The density-weighted downwelling corresponding to Figs. 4a,c,e. The solid lines are for 80°N and the dashed are for 85°N.

Because different parameterizations generally have different ways of specifying the source spectrum, it is not only the breaking criteria that differ but also the source spectra. To compare the breaking criteria alone, one would need to use a framework such as that of McLandress and Scinocca (2005). Instead we choose to implement the AD99 and H97 parameterizations with a different source spectrum, as is typically done in practice. This adds to the potential nonrobustness of the GWD response resulting from the different parameterizations.

According to the Doppler spread parameterization of H97, the gravity waves behave in a linear fashion until the point where the winds induced by the entire gravity wave spectrum (quantified by the rms wind) become comparable to the smallest horizontal phase speed of the imposed spectrum. At that point nonlinear effects begin to alter the spectrum and cause it to spread beyond the cutoff vertical wavenumber (specified at the source level), which is associated with the minimum phase speed. Waves with vertical wavenumbers greater than the cutoff value are assumed to be completely dissipated.

Figure 9 shows the steady streamfunction response in the format presented in Fig. 4 for the H97 parameterization implemented with a symmetric source spectrum that is a linear function of the vertical wavenumber (McLandress 1998). The minimum vertical wavenumber is (2 km)$^{-1}$ and the characteristic horizontal wavenumber is $k = 1.75 \times 10^{-5}$ m$^{-1}$ similar to values used in McLandress (1998). Figure 9a shows the circulation response without the polar cooling. The qualitative aspects of the circulation are comparable to those for AD99, reflecting the structure of the polar jet, although the circulation is stronger. Figure 9b shows the difference in circulation response between with and without the cooling, reflecting once again the shifting of the drag regions. However, the upward shift of the westward drag in the upper part of the domain leads to a stronger circulation than before. Nevertheless it is clear that irrespective of the quantitative details of the circulation, which could be altered by changing the detailed settings of the H97 parameterization, one obtains the physical response of adiabatic warming above the imposed cooling and zero response below.

Figure 9c shows the circulation response for a truncated model domain when momentum is conserved. The circulation is both qualitatively and quantitatively similar to that in Fig. 4c (note the contour intervals in Fig. 4c are the same). As for AD99, the difference between the low and high lid model domains (Fig. 9d) is confined above 40 km. When momentum is not conserved (Fig. 9e), however, the physical response is once again distorted with spurious downward influence. As for AD99 there is a fortuitous cancellation of the missing drag, except here the cancellation extends closer toward the pole and so the spurious downward influence for a 50-km lid height is below the contour inter-

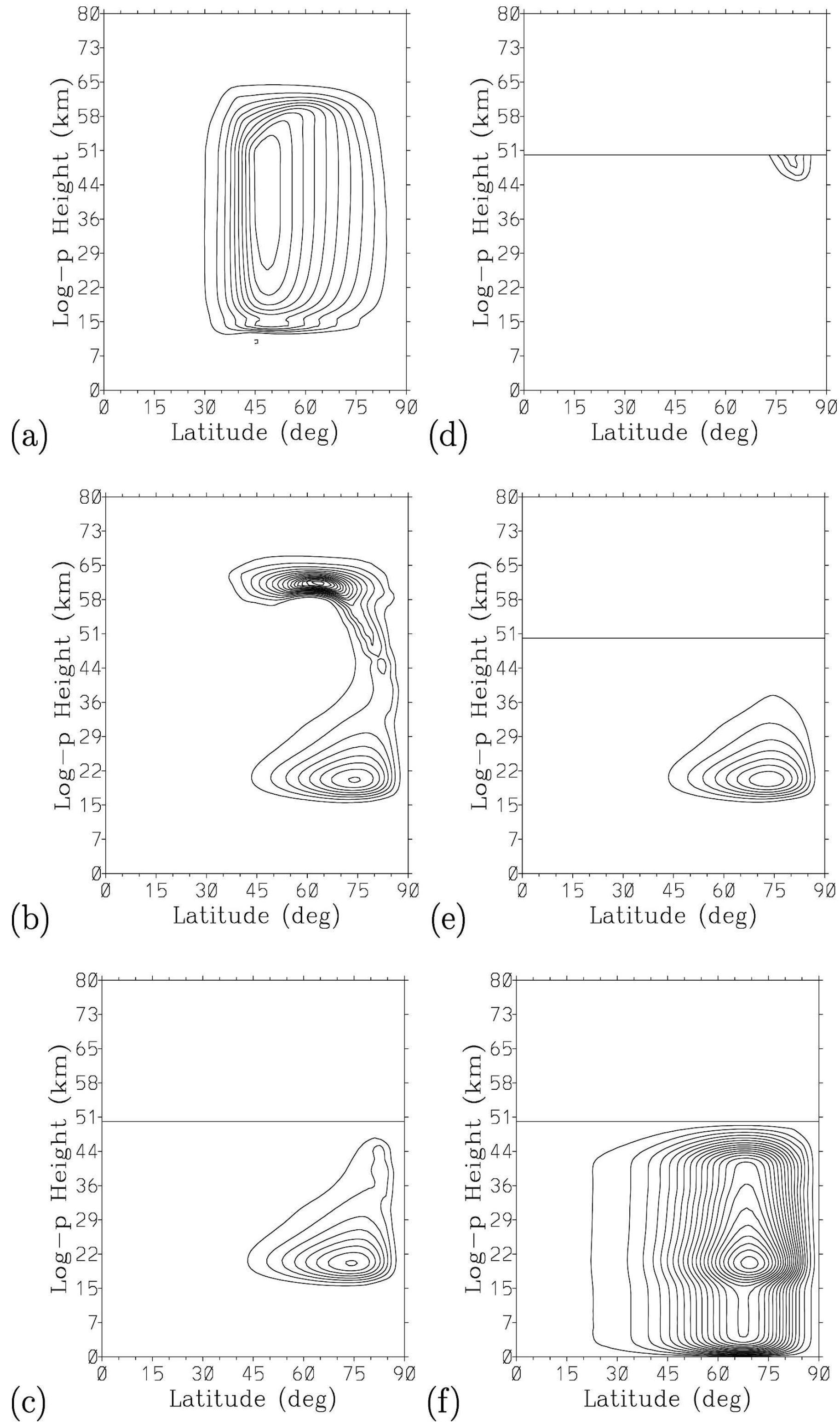


FIG. 9. As Fig. 4 but for H97.

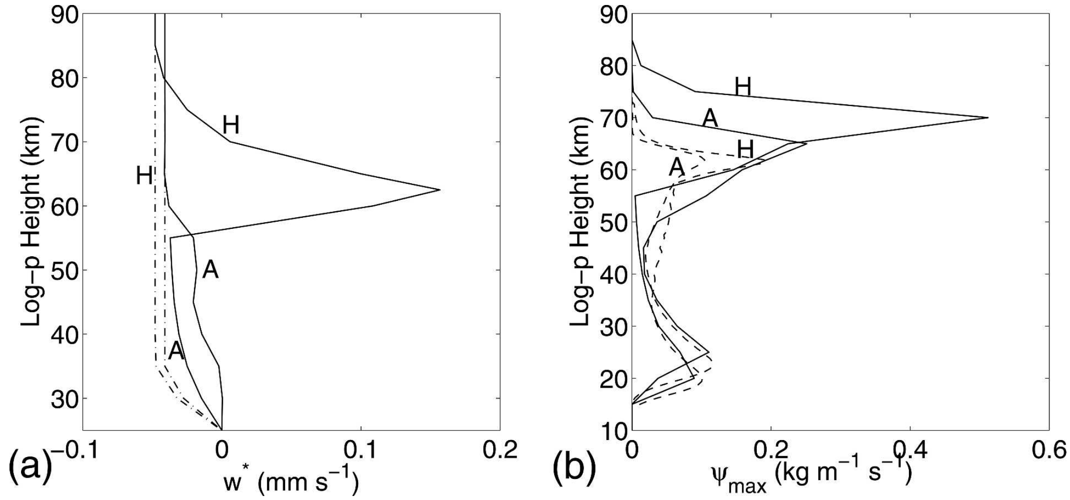


FIG. 10. (a) Downwelling at 85°N and 25 km as a function of model lid height for AD99 (A) and H97 (H). Dash–dotted lines are used when momentum is conserved and solid lines are for nonconservation. (b) Maximum mass flux at 7 km as a function of model lid height when momentum is not conserved, from direct computation (solid) and inferred from the downward control diagnostic applied to the high lid model (Figs. 4b and 9b, dashed).

val. The dependence of spurious downward influence on model lid height is explored in section 6. Figure 9f shows the result of implementing a conservative version of H97 in the presence of an RD sponge layer. The circulation driven by the momentum-conserving H97 parameterization is evident but the RD-induced circulation nevertheless results in spurious downward influence.

## 6. Effect of model lid height, revisited

Figures 4e and 9e (and Fig. 8) show the spurious sensitivity of the GWD response when gravity wave momentum flux is allowed to escape to space, for a model lid height at 50 km. This sensitivity is the result of drag regions being shifted into or out of the domain by the perturbation, and is very worrisome from a modeling perspective. The effect of nonconservation on the physical response at 85°N and 25 km when momentum is conserved and not conserved for AD99 and H97 is shown in Fig. 10a. In the momentum-conserving case the effect of depositing momentum flux near the model lid is of course seen for lid heights between 25 and 35 km. Beyond 35 km the downwelling at 25 km is constant, independent of the model lid height. However, nonconservation leads to a distortion of the physical response (errors of more than 100%) that is very sensitive to model lid height.

Figure 10b shows the maximum (negative) value of the mass streamfunction at 7 km—representing the spurious tropospheric circulation response to the imposed stratospheric cooling—as a function of model lid height for AD99 and H97 from exact calculations (solid) and from a downward control diagnostic applied to the high-lid model (dashed), when momentum is not conserved. (When momentum is conserved, the value is zero for both AD99 and H97 for all model lid heights.) It is clear from comparing the solid and dashed lines in Fig. 10b that when momentum is not conserved one cannot accurately infer sensitivity to model lid height using the downward control diagnostic applied to the high-lid model. From the latter, one would predict that the maximum sensitivity occurs for a model lid height around 60 km for both AD99 and H97 (cf. Figs. 4b and 9b). However, from Fig. 10b the maximum sensitivity is actually found somewhat higher, around 65 and 70 km for AD99 and H97, respectively. Furthermore, it is clear from Fig. 10b that the downward control diagnostic underpredicts the maximum value by more than 50%. Lawrence (1997), in GCM experiments using H97 without momentum conservation, also found that the sensitivity of polar downwelling to model lid height was much greater than would be inferred from the downward control diagnostic applied to the high-lid model. This is because of a feedback whereby letting the negative drag region at high altitudes escape leads to less downwelling, a stronger vortex, and the negative drag region moving up in altitude so that even more drag escapes.

The sensitivity to model lid height in this calculation (without momentum conservation) depends on the initial GWD response to the idealized jet (Fig. 11). In the case of the spurious tropospheric response, which reflects the total amount of missing westward drag, the

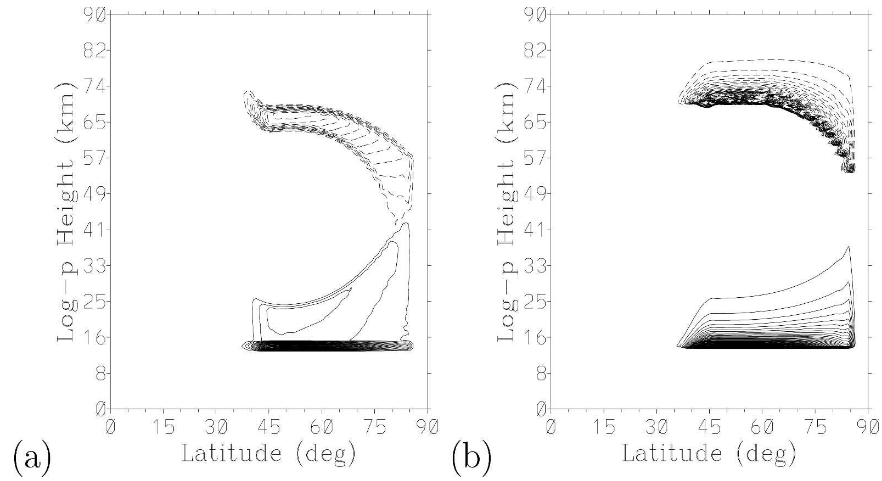


FIG. 11. Initial density-weighted gravity wave drag response (no evolution) to the idealized polar jet for (a) AD99 and (b) H97, with contour interval 0.000 25 kg $m^{-2}$ $s^{-1}$ $day^{-1}$.

peak sensitivity at 65 km for AD99 and 70 km for H97 (Fig. 10b) corresponds to the bottom of the midlatitude westward drag region in Fig. 11. In the case of the polar downwelling, the extent of the vertical separation of the polar drag regions (eastward and westward) in Fig. 11 explains why there is a continuous dependence of the polar downwelling on model lid height for AD99 whereas H97 exhibits a sharp transition (Fig. 10a). If the model lid is located below the region of initial westward drag then the westward drag does not evolve (within the domain) and the drag dipole resulting from the shifting of the westward drag in response to the radiative perturbation, as in Fig. 6a, occurs outside the domain. Essentially, the error made by neglecting the westward drag in the basic state subtracts out in the difference. Of course in practice one would presumably tune the parameterization to bring westward drag into the truncated domain. The effect of tuning against the error of nonconservation in the basic state is a subject for future investigation.

## 7. Summary and discussion

Understanding the feedbacks from GWD parameterizations as well as their robustness to poorly constrained aspects of the parameterization (gravity wave source spectrum, choice of GWD parameterization itself) is important for quantifying the middle atmospheric response to climate perturbations. We address this question in the context of the GWD response to a lower stratospheric polar cooling mimicking that due to the ozone hole. Based on results with a comprehensive chemistry–climate GCM, Manzini et al. (2003) argued that the warming observed above the ozone-hole-induced cooling could be attributed to an increase in GWD induced downwelling, according to the filtering effect described by Holton (1983). (While Holton considered a wind anomaly induced by a planetary wave drag anomaly, the same principles apply to a wind anomaly induced by a radiative perturbation.) Here we have shown that the parameterized response of GWD (with reasonable parameter values) to a zonal wind perturbation is robust to changes in the gravity wave source spectrum, choice of gravity wave parameterization, and model lid height, so long as momentum is conserved. This result assuages some of the uncertainty associated with GWD parameterization.

In the two-dimensional model experiments used here to illustrate the robustness of the GWD response, conservation of momentum was achieved by depositing momentum flux at the model lid in the top few model levels. In a two-dimensional model, one could also modify the upper boundary condition to parameterize the induced downwelling or upwelling according to the downward control principle. In a GCM, there is typically a region in the upper part of the domain that is not of interest, where an RD sponge layer or horizontal diffusion is generally applied to the resolved flow. So long as the sponge applies only to deviations from the zonal mean, then momentum conservation and a robust GWD response can be guaranteed in a GCM by depositing the momentum flux at the model lid within the sponge layer.

SS04 showed that violating momentum conservation leads to spurious downward influence. Here we show further that violating momentum conservation leads to nonrobustness of the GWD response—both the spurious response and the response one expects on physical grounds. In particular, when momentum is not conserved by allowing momentum flux to escape the model lid, the parameterized response becomes very sensitive to the details of the gravity wave source spectrum and to the choice of GWD parameterization, especially close to the pole. Examples were provided illustrating how the physical response can be completely distorted, even leading to a change in sign of the adiabatic temperature change above the imposed cooling.

A neglect of momentum flux at the model lid also has consequences for the amount of downwelling over the pole even without the imposed polar cooling anomaly. It is typical that GWD parameterizations are tuned to cure the cold pole problem. When the model domain is truncated and momentum flux is lost to space then there is missing downwelling. If one were then to tune the GWD parameterization to cure the cold pole problem this would be tuning against an error, in this case violating conservation of momentum.

Our results explain the strong sensitivity of polar night temperature to lid height found by Lawrence (1997) in GCM simulations. Lawrence (1997) allowed parameterized momentum flux to escape the model lid, and it is clear from his Fig. 8 that this nonconservation strongly affected his GWD-induced polar downwelling. When momentum is conserved such strong sensitivity disappears (our Fig. 10a).

Sensitivity to the gravity wave source spectrum, choice of parameterization, and model lid height is unacceptable for model intercomparisons since it leads to an inherently ill-posed comparison. Furthermore errors in the gravity wave effects will lead to errors in other aspects of the circulation such as planetary wave drag (e.g., McLandress and McFarlane 1993). When momentum is conserved such sensitivity is greatly reduced—and, for model lid height, disappears entirely—and hence momentum conservation is a point of principle that all current parameterizations should respect. At the very least a model should diagnose the implications of nonconservation. (For models with very high lids where all parameterized gravity waves break within the domain the implications can be expected to be minimal.)

The current calculations concern only the time mean extratropical GWD response to a zonal wind perturbation. In the case of a transient response, as might be relevant in intraseasonal annular mode variability (e.g., Thompson et al. 2006), depositing momentum flux in the top levels of the model would not properly parameterize its effects. However, it seems very likely that letting the momentum flux escape would be much worse. In the Tropics, the GWD response is inherently transient. In the classical quasi-biennial oscillation (QBO) model of Plumb (1977) information is expected to propagate upward, in which case momentum conservation would not seem to be of much importance. However, Campbell and Shepherd (2005) have shown that the GWD response propagates information downward, at least in the AD99 parameterization. In this case the proposed mechanism of ensuring momentum conservation by depositing gravity wave momentum flux near the model lid would have consequences on the semiannual oscillation (SAO) and QBO, as found by Lawrence (2001). While the effects would not be correctly represented in a low-lid model, certainly such deposition would be better than allowing momentum flux to escape to space.

Conservation of angular momentum as formulated here, applying within each latitude band, is based on reasonable assumptions made by all current GWD parameterizations. Of course, conservation of total angular momentum is a general principle that applies to the free atmosphere irrespective of any assumptions. Recent work by Bühler and McIntyre (2003) has shown that when one considers the full three-dimensional basic state, mean zonal forces can be induced even without dissipation. In that case, the local expression of momentum conservation would need to take into account the meridional as well as the vertical flux of angular momentum. The same statement applies to the meridional propagation of gravity waves between grid boxes, an effect that is also not represented in current GWD parameterizations (Fritts and Alexander 2003). Such effects were considered by SS04 when discussing the constraints imposed by conservation of angular momentum on planetary wave drag feedbacks. It would be interesting to explore the robustness of GWD feedbacks when such effects are incorporated.

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