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Chaos in Foreign Exchange Markets: A Sceptical View

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Abstract

This paper tests directly for deterministic chaos in a set of ten daily Sterling-denominated exchange rates by calculating the largest Lyapunov exponent. Although in an earlier paper, strong evidence of nonlinearity has been shown, chaotic tendencies are noticeably absent from all series considered using this state-of-the-art technique. Doubt is cast on many recent papers which claim to have tested for the presence of chaos in economic data sets, based on what are argued here to be inappropriate techniques.

Keywords: Chaos, Nonlinearity, Lyapunov Exponents, Correlation Dimension, Exchange Rates

I. Introduction

Economists have searched long and hard for chaos in financial, macro- and micro-economic data, with very limited success to date. Booth *et al.* (1990), and Frank *et al.* (1988), for example, claim to have tested for and rejected the possibility of deterministic chaos in various data sets. The motivation behind this endeavour is clear: a positive sighting of chaos implies that while, by definition, long term forecasting is futile, short-term forecastability and controllability (Ott *et al.*, 1990, Shinbrot *et al.*, 1993) are possible, at least in theory, since there is some deterministic structure underlying the data, if only we knew what it was. Varying definitions of what actually constitutes chaos can be found in the literature, but the definition which will be used here is that a system is chaotic if it exhibits sensitive dependence on initial conditions (SDIC). This is a definition which is frequently used, although many others are possible (Brock *et al.*, 1991). The concept of SDIC embodies the fundamental characteristic of chaotic systems that if an infinitesimal change, $\delta x(0)$ is made to the initial conditions, then $\delta x(t)$, the corresponding change iterated through the system until time t , will grow exponentially with t (Ruelle, 1990). We can write

$$|\delta x(t)| \approx |\delta x(0)| e^{\lambda t} \quad (1)$$

Two statistics which are commonly used to test for the presence of chaos are the correlation dimension and the largest Lyapunov exponent. The correlation dimension of Grassberger and Procaccia (1983a, 1983b) is a computationally simplified variant of the information or Hausdorff dimension (Ruelle, 1990, p244), which measures the amount of m -dimensional space which is filled by the reconstructed attractor. Dimension can also be viewed as a measure of complexity of a system (Mizrach, 1992). The largest Lyapunov exponent (LE) measures the rate at which information is lost from a system, and is usually given in units of base 2, so that the measure can be interpreted as the information loss in bits per iteration. A positive largest Lyapunov exponent implies sensitive dependence, and therefore that we have evidence of chaos. This has important implications for the predictability of the underlying system, since the fact that all initial conditions are in practice estimated with some error (either measurement

error or exogenous noise), will imply that long term forecasting of the system is impossible as all useful information is likely to be lost in just a few iterations.

Even within the foreign exchange literature, numerous recent publications have developed theoretical models which, for some values of the parameters, could generate time series which behave chaotically. DeGrauwe and Dewachter (1992) and DeGrauwe *et al.* (1993), for example, propose a chaotic model of the exchange rate based upon the structural model of Dornbusch. The model is a generalisation of DeGrauwe and Vansanten (1990) which removes the implausible limiting assumption that the chaotic dynamics are generated by the presence of a *J*-curve effect. The possibility of chaos in the revised model arises from the differing expectations of two heterogeneous groups of traders in the market, namely fundamentalists, who base their expectations of future exchange rate movements upon the economic fundamentals, and chartists, who base their expectations on previous patterns in the exchange rate. An even simpler chaotic model of exchange rates is suggested by Ellis (1994), who argues that “if such a model can demonstrate chaos, the phenomenon must surely be a possibility in much more complex systems” (p195). But in empirical applications, the presence of deterministic chaos in economic series has been elusive to say the least.

The intention of this paper is not to provide a “chaos primer”; numerous excellent review papers of varying technical complexity exist elsewhere (Ramsey *et al.*, 1990; Ruelle, 1990; Frank and Stengos, 1988a; Hsieh, 1991, to name but a few¹), but the purposes of this paper are twofold. First, a brief theoretical derivation of the concepts of the correlation dimension and Lyapunov exponents will be given, and the techniques applied to a set of foreign exchange rates. Second, the results obtained will be examined in the context of related work which has employed these methodologies. It will be argued that too much attention has so far been

¹ LeBaron (1994) and Scheinkman (1994) represent up-to-date surveys of the theoretical and empirical issues related to testing for and modelling chaos in economic and time series more generally, respectively.

focused upon dimension calculations, which cannot, when used in isolation, be viewed as a test for chaos.

II.1 The Data and Preliminaries

The analysis presented here is based on just over twenty years of daily mid-price spot exchange rate data, denominated in Sterling, taken from *Datastream*. The sample period taken covers the whole of the post-Bretton Woods era until the present day, specifically from 2 January 1974 until 1 July 1994 inclusive, a set of 5191 observations. A set of ten currencies are analysed, namely the Austrian Schilling/Pound (hereafter denoted A), the Canadian Dollar/Pound (C), the Danish Krone/Pound (D), the French Franc/Pound (F), the German Mark/Pound (G), the Hong Kong Dollar/Pound (H), the Italian Lira/Pound (I), the Japanese Yen/Pound (J), the Swiss Franc/Pound (S), and the U.S. Dollar/Pound (U). The raw exchange rates were transformed into log-returns which can be interpreted as a series of continuously compounded daily returns (Brock *et al.*, 1991. One possible justification for using returns rather than raw data is that the raw data is likely to be nonstationary (see, for example, Corbae and Ouliaris)². Brock (1986) shows that linear processes with near unit roots will generate low dimension estimates.

II.2 The Use of Surrogate Data

When testing for chaos, it is often useful to have a standard set of data with the same distributional properties and possibly the same autocorrelation structure as the raw data, but with any nonlinear dependence removed. The results of the tests on the raw data can then be directly compared with those on the “randomised” data. Scheinkman and LeBaron (1989) suggest the use of a “shuffle diagnostic”, where the original data is sampled randomly with replacement to form a new random data series. Successive applications of this procedure should yield a collection of data sets with the same distributional properties, on average, as the raw

²The data were tested for the presence of unit root nonstationarity using the Dickey Fuller (Dickey and Fuller, 1979; Fuller, 1976), Phillips Perron (Phillips, 1987; Phillips and Perron, 1988) and Sargan

data. Although the shuffle diagnostic has been widely employed in economic applications (e.g. Blank, 1991; Mayfield and Mizrach, 1992), in this study it was considered preferable to instead employ the method of surrogate data (e.g. Theiler, 1991; Rapp *et al.*, 1993). This technique is similar in many respects to the shuffle algorithm, but the new data set is not sampled directly from the original, but rather a randomised data set with the same distribution and autocorrelation structure is created. Technically, this is achieved by taking the Fourier transform of the original series, randomising the phases, and taking the inverse Fourier transform. This technique has the advantage over the Scheinkman and LeBaron technique³ that the autocorrelation structure is preserved, so that the surrogate data set has the same level of linear dependence as the original, but all traces of nonlinear dependence have been removed. Furthermore, if a linearly independent series is required with a similar distribution to the raw data, this can easily be achieved by linearly filtering the data - that is, fitting the “best” $AR(p)$ model to the data (according to some criterion), and then running all subsequent tests on the residuals from the estimated linear model. Brock (1986) has shown that dimension and Lyapunov exponent estimates are unaltered by linear filtering. If the positive result is still apparent in a test on the surrogate data, then the result is likely to be due to linear dependence in the data, but if the results between the two data sets differ, this must be due to nonlinear and possibly chaotic dependence in the raw data which by definition is not present in the surrogate data.

III.1 The Correlation Dimension

The Grassberger Procaccia (GP) correlation dimension (Grassberger and Procaccia, 1983a, 1983b) is a general characteristic statistic used to distinguish deterministic systems from

Bhargava (Sargan and Bhargava, 1983, Bhargava, 1986). The levels data and the log-levels data were found in all cases to be strongly $I(1)$, but there was no evidence of nonstationarity in the returns series.

³Frank and Stengos (1989) find the shuffle diagnostic useful for dimension calculations, but misleading for estimating the Kolmogorov entropy, a measure which is defined as the sum of all positive Lyapunov exponents

random noise. The formulation of the test statistic is as follows, and is based on the calculation of the correlation integral. First, the “ m -histories” of the series, $x_t^m = (x_t, x_{t+\tau} \dots, x_{t+\tau(m-1)})$ are computed for time $t = 1, \dots, T-m$, for embedding dimension m , and for time delay τ . This step is known as phase space reconstruction, and is due to Takens (1984), who shows that we can test if a system is chaotic simply by observing the behaviour of one series from within that system. Takens shows that for $m \geq 2dim + 1$ (where dim is the dimension of the attractor), the phase space spanned by the m -history will be an embedding, i.e. it will be topologically equivalent to the original unknown set of equations of motion of the system. The reconstructed m -vector will therefore have the same correlation dimension and set of Lyapunov exponents. Define the correlation integral as

$$C_m(\varepsilon) = \frac{1}{(T-m+1)(T-m)} \sum_{\forall t,s} I_\varepsilon(x_t^m, x_s^m) \quad (2)$$

where I_ε is an indicator function that equals one if $\|x_t^m - x_s^m\| < \varepsilon$, and zero otherwise. $\|\cdot\|$ denotes the supremum norm, which is the most widely used distance measure. Although the usual Euclidean norm is equivalent, it is computationally more intensive and hence is rarely used in practical applications. The correlation integral thus measures the proportion of points that are within a distance ε of each other in m -dimensional space. Next calculate the log of the correlation integral divided by the log of the distance, ε , and take the limit as ε is made progressively smaller. Denoting this limit by v_m , we have that

$$v_m = \lim_{\varepsilon \rightarrow 0} \frac{\log(C_m(\varepsilon))}{\log(\varepsilon)} \quad (3)$$

If the process generating the data is characterised by white noise, v_m will scale with m , i.e. $v_m = m \forall m$, but if a deterministic process underlies the system, v_m will cease to increase at some value $v = v_{dim}$, ($dim \ll m$ in order to sufficiently reconstruct the phase space) as m is increased. v_{dim} gives the correlation dimension of the system. In practice the correlation dimension is estimated from actual data by plotting $\log(C_m(\varepsilon))$ against $\log(\varepsilon)$ and by taking the slope of a judiciously chosen linear region where ε is as close to zero as is feasibly possible given the

number of data points. If ε is chosen too large, the slope of the line will be nearly horizontal, while if it is chosen very small, the plot becomes rather jagged (Dechert, 1992). The time delay, τ , can be chosen in at least two ways (Casdagli *et al.*, 1991): set τ as the first zero (i.e. the first value that is not significantly different from zero) of the autocorrelation function, or set τ as that value which minimises the mutual information function between the past and the future (Fraser and Swinney, 1986). Both methods were computed here, and the “optimal” delay times selected using each technique are shown in the following table

Table 1: Optimal Delay Times for Calculation of the Correlation Dimension

Currency	A	C	D	F	G	H	I	J	S	U
1 st zero of autocorrelation function	4	2	6	1	3	3	2	2	3	2
1 st min. of mutual information function	2	1	3	2	2	3	3	1	2	1

There is little to choose between these methods, aside from the point that the first minimum of the autocorrelation function should ensure linear independence between x_t and $x_{t+\tau}$, while minimising mutual information should ensure general independence. Employing both methods yields almost identical results, and hence only those for the latter criteria are shown in the tabulated results below.

III.2 Estimation of Lyapunov Exponents

Arguably the only test explicitly formulated for chaos is the computation of the largest Lyapunov exponent. The spectrum of Lyapunov exponents can be defined as follows (Wolf *et al.*, 1985). Consider an infinitesimally small hypersphere of radius ε . If we monitor the evolution of the sphere, it will become deformed into an ellipsoid as the system evolves over time. The Lyapunov exponent is then measured by the extent of the deformation, and is given by

$$\lambda_i = \lim_{t \rightarrow \infty} \lim_{\varepsilon(0) \rightarrow 0} \left[\frac{1}{T} \log_2 \left(\frac{\varepsilon_i(t)}{\varepsilon_i(0)} \right) \right] \quad (4)$$

where $\varepsilon_i(t)$ is the length of the i^{th} principal axis of the ellipsoid at time t . In this paper, two separate algorithms for the calculation of Lyapunov exponents are employed. The first is an implementation of the Wolf *et al.* (1985) algorithm, and the second is a more recent technique due to Dechert and Gencay (1990, 1992). The Wolf *et al.* algorithm was the first method proposed for estimating Lyapunov exponents in time series data. However, only the largest exponent is calculated, and a number of authors (Brock and Sayers, 1988 for example) have found that the results of the estimation are highly sensitive to noise, which is particularly problematic in economic data where noise is more prevalent and data series are typically much shorter than in the physical sciences. The results given in tables in the appendix are for a delay time of one and are given in base 2; a largest Lyapunov exponent in base 2 can be interpreted as the loss of information in bits per iteration.

The new technique of Gencay and Dechert seems more promising in that, potentially, the whole spectrum of Lyapunov exponents can be estimated. Furthermore, according to simulations on known chaotic data sets by the authors of the test (Dechert and Gencay, 1992), the algorithm is more powerful in the presence of noise than the earlier technique, although the signal-to-noise ratio used in their simulations was high, probably much higher than would be the case for actual economic data. Thus even a noisy chaotic system (particularly if the noise is dynamic noise which will propagate through the system, rather than additive noise) could lead to estimated Lyapunov exponents which are negative⁴. The method uses a similar technique to the more frequently cited work by Ellner *et al.* (1991), in that, unlike the Wolf *et al.* method which directly finds similar pairs of state vectors within the series and estimates how the subsequent trajectories diverge, the new procedures use Jacobian methods. These estimate the exponents through the intermediate step of estimating the individual Jacobian matrices. Using the

terminology of Nychka *et al.* (1992), let \hat{J}_t be the estimate of the Jacobian and $\hat{T}_m = \hat{J}_m \dots \hat{J}_1$.

The estimate of the Lyapunov exponents is given by

$$\lambda = \frac{1}{2m} \log \left| \hat{\nu}_1(m) \right| \quad (5)$$

where $\hat{\nu}_1(m)$ is the largest eigenvalue of $(\hat{T}_m' \hat{T}_m)^{m/2}$. In practice, the method of Gencay and Dechert uses a single hidden layer feed-forward neural network to model the dynamics of the series and the spectrum of Lyapunov exponents are then calculated from the derivative matrices of the network models. With this technique, the user does not have to choose a value for the delay time (usually denoted τ), but one does have to select the number of inputs to the network (equivalent to the embedding dimension) and the number of hidden units in the intermediate layer, N . The inputs were selected as own lagged values of the series from $t-1$ to $t-m$, where m is the number of inputs. The network is given by

$$\hat{x}_{N,m}(X; \beta, w, b) = \sum_{j=1}^N \beta_j \phi \left(\sum_{i=1}^m w_{ij} Z_i + b_j \right) \quad (6)$$

where \hat{x} is a vector of fitted values, Z is the input, β represents the hidden to output weights, and w and b represent the input to hidden weights. Let

$$x_t^m = (x_{t+m-1}, x_{t+m-2}, \dots, x_t) \quad (7)$$

The multivariate nonlinear least squares minimisation problem is then given by

$$\min_{\beta, w, b} \sum_{t=0}^{T-m-1} [x_{t+m} - \hat{x}_{N,m}(x_t^m; \beta, w, b)]^2$$

and the activation function for the hidden layer is the sigmoid

$$\phi(p) = \frac{1}{1 + \exp(-p)} \quad (8)$$

The number of inputs was varied from 1 to 6, and the number of hidden layers from 1 to 10. These values were severely constrained by available CPU time, since the estimation is so data intensive. To calculate the spectrum of Lyapunov exponents with just six inputs and 1-10 hidden layers took over 50 hours of C.P.U. time per series on a SparcCentre 2000 with 8

⁴ I am grateful to an anonymous referee for making this point clear.

50Mhz processors. Thus it is impractical to use bootstrapping in this case to construct confidence intervals or to undertake tests of significance. The “best fit” from among all combinations of alternative models can be chosen using Schwarz’s (1978) information criterion (SBIC) or by minimising the in sample-mean square prediction error. The number of inputs, m , and the number of hidden units, N , which minimise SBIC and the mean square prediction error are shown in tables 2 and 3 respectively:

Table 2: Number of Inputs, m , and Hidden Units, N that Give the First Minimum of SBIC

Series	A	C	D	F	G	H	I	J	S	U
m	1	1	2	2	1	1	1	1	2	1
H	1	1	1	2	1	3	2	1	1	1

Table 3: Number of Inputs, m , and Hidden Units, N that Give the First Minimum of Mean Squared Prediction Error

Series	A	C	D	F	G	H	I	J	S	U
m	3	2	5	4	5	1	3	3	3	3
H	4	5	6	6	5	5	4	2	4	3

The values of Schwarz’s information criterion and of the mean squared prediction error may indicate the optimal number of inputs and hidden layers. The latter criterion should be interpreted with caution, since there is much evidence that good in-sample prediction may be achieved by over-parameterising the network to yield a good fit to the data, at the expense of poor out-of-sample performance. In any case, the conclusion is not qualitatively altered by this, but in general, Schwarz’s criterion suggests that only one or two inputs and one or two hidden units are required for the real and surrogate data. This in itself is indicative of a lack of dynamic structure in the data since the “optimal” models according to this metric are very small ones. Hence even with such long data series, the improvement in model fit is insufficient to compensate for the increase in the penalty term.

IV.1 Results of Correlation Dimension Estimation

The results of an application of the GP algorithm are given in table 4 below.

Table 4: Grassberger and Procaccia Correlation Dimension Estimates

	Embedding Dimension														
Series	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Returns data															
A	0.96	1.92	2.90	3.90	4.42	5.49	6.10	6.44	7.09	7.51	8.54	7.94	8.29	8.89	8.62
C	0.99	2.01	2.96	3.96	4.82	5.20	5.80	5.82	5.86	6.10	6.59	6.77	6.80	7.52	7.54
D	0.99	1.84	2.92	3.95	4.84	5.38	5.98	6.69	6.81	7.20	7.82	7.54	8.29	8.87	8.05
F	0.97	1.97	2.99	3.98	4.39	4.94	5.19	5.44	5.58	5.56	5.60	5.68	5.81	5.89	6.02
G	0.94	1.96	2.97	3.93	4.75	5.35	5.82	6.10	6.52	6.51	7.27	7.30	7.37	7.73	7.80
H	0.99	2.00	2.99	3.71	4.34	4.62	4.76	4.93	5.04	4.90	5.23	5.33	5.20	5.34	5.41
I	0.99	1.97	2.90	3.25	3.64	3.26	4.10	4.20	3.81	4.53	4.63	4.47	4.89	4.79	4.78
J	0.92	1.96	2.96	3.94	4.71	5.22	5.88	6.17	6.52	7.01	7.32	7.56	7.87	7.98	8.18
S	0.93	1.94	2.99	3.97	4.81	5.50	6.32	6.48	7.16	7.59	7.95	8.05	9.07	8.49	8.64
U	0.98	1.99	2.86	3.16	3.19	3.40	3.22	3.72	3.89	3.47	4.12	4.30	3.74	4.24	4.40
Surrogate Data															
A	0.96	1.92	2.96	3.98	4.97	5.64	6.55	6.94	7.61	8.17	8.64	9.04	9.56	9.45	10.05
C	0.99	1.99	2.97	3.99	4.90	5.62	6.42	6.89	7.53	8.02	8.51	9.10	9.47	9.78	10.38
D	0.98	1.99	2.99	3.99	4.86	5.57	6.39	6.93	7.43	7.94	8.28	8.91	9.38	9.80	9.92
F	0.97	1.96	2.96	3.92	4.89	5.55	6.45	7.00	7.53	7.86	8.27	8.50	9.24	9.62	9.67
G	0.98	1.97	2.90	3.99	4.87	5.61	6.19	6.79	7.22	7.74	8.22	8.58	8.82	9.28	9.43
H	0.99	2.00	2.98	3.99	4.72	5.34	6.10	6.58	7.04	7.60	8.00	8.56	8.74	9.07	9.63
I	0.99	1.99	3.00	3.99	4.63	5.40	6.11	6.68	7.32	7.84	8.17	8.33	8.85	9.05	9.50
J	0.92	1.96	2.96	4.00	4.86	5.52	6.30	7.04	7.56	7.29	8.60	8.95	9.32	9.93	10.29
S	0.94	1.91	2.96	3.99	4.99	5.77	6.44	7.11	7.60	8.26	8.80	9.36	9.51	10.09	10.40
U	0.99	2.00	2.95	3.97	4.84	5.52	6.13	6.67	7.18	7.71	8.19	8.81	9.14	9.83	10.28

As detailed above, a saturation in the estimate relative to that of the corresponding surrogate data, at a given embedding dimension, is taken as evidence of deterministic behaviour underlying the series. This dimension estimate provides a lower bound on the number of independent variables which would be required to model the series. As table 4 shows, all the returns series show some degree of saturation which is not present in their surrogate counterparts. This may be indicative that there is some degree of determinism underlying all the series, and in many applications, which will be described below, the test has been assumed to give *prima facie* evidence for chaos. Although the degree of saturation is evidently stronger in some series than others, it is often around 5-6. This ties in well with the results of Scheinkman and LeBaron (1989), and a number of other authors, who find this order of magnitude common across many financial markets. The lowest and most stable correlation dimension estimates come from the Italian Lira / Pound and U.S. Dollar / Pound returns, which give estimates of between 4 and 5 for embedding dimensions up to 15. However, it has been suggested (Ramsey

et al., 1990) that this provides only limited evidence for deterministic dynamics, since even completely randomly generated data samples will appear to saturate⁵.

⁵. This appears to be the case, since even the dimension estimate for artificial data generated as pure Gaussian noise (not shown) slows to just over 10 as the embedding is increased to 15.

IV.2 Results of Lyapunov Exponent Estimation

The results of Lyapunov exponent estimation, using both the Wolf algorithm and the neural network technique of Gencay and Dechert, are shown in the following tables:

Table 5: Estimation of Largest Lyapunov Exponents using the method of Wolf *et al.*

Series	Embedding Dimension									
	1	2	3	4	5	6	7	8	9	10
Returns data										
A	1.497	1.284	0.969	0.603	0.360	0.222	0.166	0.116	0.088	0.068
C	1.847	1.107	0.856	0.723	0.431	0.264	0.198	0.136	0.108	0.075
D	1.247	1.270	0.965	0.641	0.405	0.247	0.159	0.127	0.101	0.076
F	1.233	1.131	0.870	0.572	0.369	0.235	0.164	0.118	0.097	0.068
G	1.364	1.376	1.023	0.659	0.398	0.255	0.170	0.138	0.098	0.078
H	1.250	0.976	0.792	0.554	0.366	0.232	0.169	0.104	0.078	0.073
I	1.112	1.053	0.701	0.531	0.346	0.223	0.148	0.114	0.092	0.062
J	1.591	1.368	1.024	0.669	0.413	0.252	0.180	0.124	0.094	0.064
S	1.531	1.475	1.054	0.686	0.409	0.256	0.185	0.134	0.091	0.075
U	1.614	1.440	1.085	0.658	0.402	0.256	0.174	0.128	0.089	0.079
Surrogate Data										
A	1.425	1.131	0.923	0.710	0.421	0.281	0.184	0.131	0.098	0.072
C	2.087	1.660	1.270	0.776	0.479	0.313	0.220	0.172	0.133	0.119
D	1.851	1.305	1.056	0.720	0.445	0.289	0.199	0.142	0.099	0.080
F	1.608	1.425	1.147	0.748	0.465	0.302	0.202	0.154	0.118	0.095
G	1.545	1.439	1.140	0.749	0.455	0.302	0.200	0.144	0.109	0.088
H	1.405	1.296	1.037	0.726	0.437	0.268	0.197	0.136	0.102	0.077
I	1.577	1.146	0.911	0.660	0.439	0.278	0.213	0.139	0.106	0.077
J	1.779	1.542	1.217	0.765	0.463	0.296	0.205	0.154	0.118	0.097
S	1.821	1.575	1.196	0.787	0.462	0.308	0.214	0.158	0.124	0.104
U	2.048	1.515	1.168	0.770	0.469	0.288	0.208	0.154	0.110	0.090

The salient feature of the data is that there is no evidence of chaotic dynamics in any of these Pound exchange rates. Whilst the largest Lyapunov exponent estimated by the Wolf algorithm is in every case positive, as shown in table 5, this cannot be taken as evidence for sensitive dependence, since the corresponding estimates for the surrogate data are also positive and are of the same order of magnitude. One would expect the Lyapunov exponents for the surrogate data to be significantly negative since any chaotic structure which was present in the original series should have been destroyed by the randomisation. Calculation of the whole spectrum of Lyapunov exponents can potentially be achieved using the method of Dechert and Gencay,

although only the largest are shown since it is only the largest Lyapunov exponent which is of direct interest⁶.

Table 6: Estimation of Lyapunov Exponents for the German Mark Returns using the method of Dechert and Gencay (1990), only largest Lyapunov exponent shown

	Number of hidden units									
<i>m</i>	1	2	3	4	5	6	7	8	9	10
1	-5.3325	-4.1221	-2.5603	-3.9918	-3.1416	-3.0514	-2.7805	-2.6132	-2.5713	-3.0728
2	-3.6197	-2.1553	-1.7933	-1.7618	-1.3154	-1.3417	-1.1881	-1.3038	-1.1326	-1.1011
3	-0.2341	-1.4485	-0.9230	-0.8256	-0.8870	-0.8566	-1.0082	-0.6362	-0.8545	-0.6659
4	-0.4345	-0.6858	-0.7449	-0.5844	-0.5963	-0.4703	-0.4785	-0.5741	-0.4511	-0.5217
5	-1.1297	-1.1275	-0.4560	-0.6140	-0.4517	-0.3759	-0.3860	-0.3383	-0.2886	-0.2271
6	-0.8743	-0.7197	-0.3360	-0.4435	-0.3113	-0.3081	-0.2671	-0.2242	-0.2278	-0.2526

Table 7: Estimation of Lyapunov Exponents for the Japanese Yen Returns using the method of Dechert and Gencay, only largest Lyapunov exponent shown

	Number of hidden units									
<i>m</i>	1	2	3	4	5	6	7	8	9	10
1	-2.8543	-2.4349	-2.2309	-3.2254	-2.2154	-3.0422	-1.9149	-1.6613	-1.3588	-1.2613
2	-2.2560	-1.9439	-2.2819	-1.7703	-1.2669	-1.0833	-1.0269	-0.9783	-0.7011	-0.8103
3	-1.1907	-0.7417	-1.1974	-0.9897	-0.8506	-0.7242	-0.9406	-0.6902	-0.5629	-0.7053
4	-1.0013	-1.0057	-1.1144	-0.7051	-0.5578	-0.4751	-0.6011	-0.4293	-0.3879	-0.3857
5	-0.7379	-0.7679	-0.5536	-0.5608	-0.3806	-0.4705	-0.4809	-0.3503	-0.2614	-0.2448
6	-0.6211	-0.4922	-0.3639	-0.3838	-0.3032	-0.2267	-0.1786	-0.2491	0.2210	-0.2149

⁶ Only those results for the three largest trading-volume currencies investigated are shown, since results for the other series are qualitatively identical. A full appendix containing these results and additional statistics for the surrogate and a set of artificially generated data, is available upon request from the author.

Table 8: Estimation of Lyapunov Exponents for the US Dollar Returns using the method of Dechert and Gencay, only largest Lyapunov exponent shown

	Number of hidden units									
m	1	2	3	4	5	6	7	8	9	10
1	-2.8197	-5.7844	-2.4788	-2.1211	-2.4536	-2.2030	-2.3111	-2.4493	-2.3102	-1.5110
2	-2.1872	-2.0325	-2.8629	-1.8652	-1.5275	-1.3440	-1.0610	-0.9746	-0.9330	-0.9288
3	-1.6316	-1.1715	-1.4562	-0.8225	-0.6339	-0.6084	-0.7491	-0.7449	-0.5414	-0.5123
4	-1.2344	-0.7019	-0.8316	-0.6124	-0.5163	-0.4560	-0.3455	-0.2696	-0.3684	-0.2652
5	-0.6803	-0.5439	-0.4440	-0.3146	-0.4045	-0.3750	-0.3891	-0.2507	-0.2828	-0.2388
6	-0.2859	-0.3701	-0.4042	-0.3821	-0.2936	-0.2053	-0.3139	-0.2684	-0.2231	-0.1912

Once again, the results shown in tables 6 to 8 give no support to the presence of chaotic dynamics in the series considered. Thus it appears overall that the possibility of chaotic dynamics is rejected for all the series considered, and this has been the conclusion of almost all studies of economic data (Scheinkman, 1994).

V. Relation to other work

It is important to note that saturation of the correlation dimension estimate is a necessary, but by no means sufficient condition for the existence of a chaotic attractor (Hsieh, 1993). In fact, an infinite number of nonlinear, but non-chaotic data generating mechanisms would also lead to a low and stable dimension estimate. Osbourne and Provenzale (1989) and Scheinkman and LeBaron (1989) have shown that nonlinear stochastic systems are capable of exhibiting this property⁷. In other words, correlation dimension estimation has power against non-chaotic as well as chaotic alternatives. Indeed, the dimension estimates calculated here stabilise at low values relative to their randomised surrogate counterparts, and yet the largest estimated Lyapunov exponent is almost invariably below zero.

Many authors claim to have tested for chaos using only an application of the Grassberger Procaccia technique (Liu *et al.*, 1992; Yang and Brorsen, 1992; Booth *et al.*, 1990 & 1994; DeGrauwe *et al.*, 1993; Peters, 1991; Willey, 1992; Varson and Jalivand, 1994). Of these, the

last four claim to have found chaos, but all claim to have tested for its presence. Willey (1992), for example, states that

“The increase in dimensionality estimate of the shuffles compared to the residuals of the whitened data indicates that both series are chaotic” (p71).

Varson and Jalivand (1994) also claim to have found chaos in some series based on low dimension estimates, and DeGrauwe *et al.* (1993) argue that they have “presented empirical evidence on the existence of chaos” in foreign exchange rates. It is also possible, however, that these low dimension estimates are the product of some nonlinear stochastic (non-chaotic) data generating mechanism or “some smoothness in the original data” as Ruelle (1990, p246) puts it.

Frank *et al.* (1988), Barnett *et al.* (1992), Dechert and Gencay (1992) and Blank (1991) do compute the largest Lyapunov exponent of various financial and macroeconomic time series. Of these, only Blank finds a positive exponent in agricultural futures price series which is robust to changes in the value of the user-adjustable parameters, such as the embedding dimension and delay time. This conjecture is, however, based on an average of only 336 observations, and uses the direct and less stable algorithm of Wolf *et al.* (*op cit.*).

VI. Conclusions

The correlation dimension and spectrum of Lyapunov exponents have been calculated for a set of daily Sterling exchange rates. In common with many other studies, the dimension estimates in some cases stabilised as the embedding dimension increased, but in all cases, the largest Lyapunov exponent was calculated to be negative. This indicates the presence of some kind of nonlinear determinism in the data (of a form which cannot be identified from the tests used here, but see Brooks (1996), for a consideration of the form of nonlinearity that may be present), but makes the stronger possibility of deterministic chaos unlikely.

⁷Moreover, Scheinkman and LeBaron (1989) show that even data generated by ARCH or GARCH models will lead to saturation in dimension estimates.

It has been shown that the correlation dimension estimate of Grassberger and Procaccia has been much abused by economists in recent years. These estimates are likely to be biased downwards for random noise (Ramsey *et al.*, 1990), and are at best unreliable for small data sets, with no finite sample distribution theory available as yet (Ramsey *et al.*, 1990). Furthermore, the use of a low-pass filter to reduce noise can give finite, non-integer dimension estimates (Rapp *et al.*, 1993). Mitschke *et al.* (1988) argue that this will not be the case for the largest Lyapunov exponent, since only those exponents less than zero will be affected by the filtering process. Mizrach (1992) argues that "...entropy should be regarded as the defining characteristic" (p188), and Ruelle (1990) notes that "there are quantities other than the correlation dimension that one may try to compute, and that may be better behaved..."(p246). Given these facts, that more researchers do not compute LE measures is puzzling, but may be a consequence of their relative computational complexity and the huge CPU requirement mentioned above. Moreover, Lyapunov exponent estimation from observed time series is highly unstable to dynamic noise. The problem is likely to be even more troublesome than for calculation of the correlation dimension.

Armed with this evidence, a re-examination of the literature has revealed no papers which find chaos in economic series based on what may be considered robust techniques (i.e. calculation of LE by an indirect method over a long data series). It appears, then, that we are left with the likelihood that chaos is decidedly not present in any economic data, or at least cannot be detected by the best of the tools which are currently available. Nearly a decade ago, Brock and Sayers (1988) argued that "evidence of chaos is weak, but our tests may be too weak to detect it..." (p71). Although the Dechert and Gencay technique is a clear step in the right direction, it is likely that the levels of system noise present in economic and financial time series would render any chaotic structure undetectable, even with the large sample sizes available for the

latter. Thus unless a further quantum leap is made in the robustness of tests to the presence of dynamic noise, it is likely that chaotic dynamics in economic data will remain elusive.

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