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Published Version

GEOPHYSICAL RESEARCH LETTERS, VOL. 34, L23103, doi:10.1029/2007GL031317, 2007

Self-consistent wave-particle interactions in dispersive scale long-period field-line-resonances

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Received 19 July 2007; revised 6 September 2007; accepted 19 October 2007; published 4 December 2007.

[1] Using 1D Vlasov drift-kinetic computer simulations, it is shown that electron trapping in long period standing shear Alfvén waves (SAWs) provides an efficient energy sink for wave energy that is much more effective than Landau damping. It is also suggested that the plasma environment of low altitude auroral-zone geomagnetic field lines is more suited to electron acceleration by inertial or kinetic scale Alfvén waves. This is due to the self-consistent response of the electron distribution function to SAWs, which must accommodate the low altitude large-scale current system in standing waves. We characterize these effects in terms of the relative magnitude of the wave phase and electron thermal velocities. While particle trapping is shown to be significant across a wide range of plasma temperatures and wave frequencies, we find that electron beam formation in long period waves is more effective in relatively cold plasma.
Citation: Rankin, R., C. E. J. Watt, and J. C. Samson (2007), Self-consistent wave-particle interactions in dispersive scale long-period field-line-resonances, *Geophys. Res. Lett.*, *34*, L23103, doi:10.1029/2007GL031317.

1. Introduction

[2] At short perpendicular spatial scale, parallel electric fields and large wave Poynting flux in dispersive shear Alfvén waves are speculated to produce electron acceleration up to the keV range [*Chaston et al.*, 2003]. There is strong observational evidence for this in the case of short parallel wavelength inertial or kinetic scale Alfvén waves propagating on the PSBL [*Keiling et al.*, 2001; *Wygant et al.*, 2002] and polar cusp [*Su et al.*, 2001]. It is also known that low frequency (long parallel wavelength) Pc5-range shear Alfvén waves are associated with modulations of the optical aurora [*Samson et al.*, 1991]. This is suggestive of a common mechanism for electron acceleration operating in SAWs over a wide range of wave phase velocities. However, an important question is whether the precipitation observed under short perpendicular scale Pc-5 field line resonances is produced by the wave itself. While we cannot provide closure on this issue, we consider this particular aspect in this paper.

[3] Many authors have considered mechanisms that are speculated to produce parallel electric fields of large enough magnitude to explain electron acceleration by dispersive Alfvén waves [*Wei et al.*, 1994; *Lu et al.*, 2003]. The existence of a parallel electric field is clearly not sufficient, as explaining electron acceleration involves understanding the response of the electron distribution function to the wave [*Hui and Seyler*, 1992; *Thomson and Lysak*, 1996; *Kletzing and Hu*, 2001; *Watt et al.*, 2005], and vice versa. If the wave phase velocity is comparable to the electron thermal speed, linear Landau damping can be expected on the finite slope of the electron velocity distribution function. Electrons moving slower than the wave gradually pick up energy from the wave as they surf on wave phase fronts, whereas those electrons moving faster than the wave will lose energy. A net energy gain for electrons is therefore expected when the distribution function has a negative velocity gradient at the wave phase velocity. On the other hand, at long parallel wavelengths characteristic of low frequency Pc-5 field line resonances, the wave phase velocity can be much smaller than the thermal speed over a significant extent of geomagnetic field lines, implying that electrons in linear Landau resonance with the wave are in the part of the distribution function that has essentially zero slope. In this case, the net energy gain of the electrons will be essentially zero. This simple conceptual framework neglects, however, nonlinear trapping of electrons in the wave, and the effect of the wave on current carrying electrons at both high and low altitude.

[4] In cold plasma with $\omega/k_{||} \sim \nu_A > \nu_{th}$ field-aligned electron currents in long period Pc-5 waves strongly perturb electron orbits. The distribution function must shift appreciably in velocity space to accommodate the wave current. This implies an increase in the number of electrons moving at or near the wave phase velocity. The nonlinear interaction of waves and particles thus offers the possibility of producing beams of accelerated electrons. In reality, long-period field line resonances [*Chen and Hasegawa*, 1974; *Southwood*, 1974; *Kivelson and Southwood*, 1986] are on geomagnetic field lines that encompass field-aligned variations in temperature, magnetic field strength and density, particularly at low altitude below $\sim$1–2 R_E. Geomagnetic field line convergence also implies a varying perpendicular scale of the wave. In spite of this complexity, it is nevertheless useful to investigate self-consistent nonlinear wave-particle interactions in field line resonances, in order to test some basic concepts. In the material that follows, we present Vlasov drift-kinetic modeling of plasma having a fixed temperature, density and magnetic field strength. These restrictions may seem severe, but nevertheless they reveal some interesting and informative aspects of wave-particle interactions that can be tested in more complete models.

2. 1D Self-Consistent Nonlinear Kinetic Model

[5] We have developed a 1D (in space) Vlasov-Maxwell computer model [*Watt and Rankin*, 2007] that is applicable to field line resonances. The electrons are considered in the

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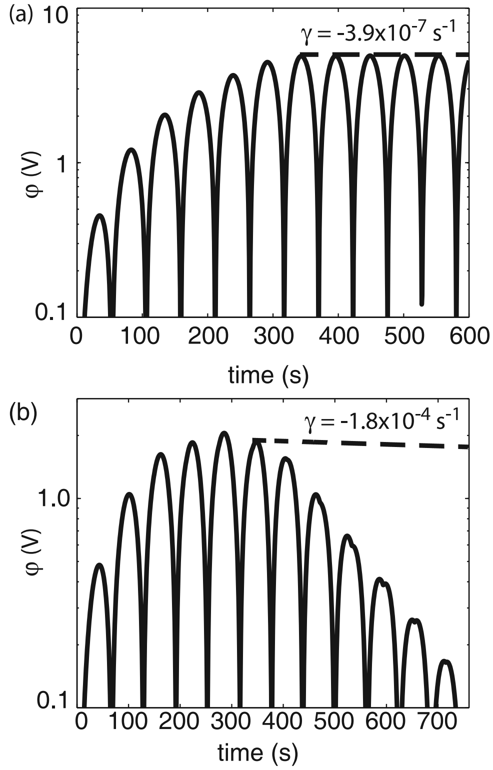

Figure 1. Growth of perpendicular electric field as a function of time in cold plasma with T_e = 0.5 eV. The wave frequency is approximately 10 mHz. The perpendicular wavenumber corresponds to $k_\perp = 2 \times 10^{-5}$ m^{-1} in (a) and $k_\perp = 1.5 \times 10^{-4}$ m^{-1} in (b). In both (a) and (b), $\nu_A = 1.42 \times 10^6$ ms^{-1}, and $\nu_{th} = 4.19 \times 10^5$ ms^{-1}.

zero drift approximation, and are gyro-averaged on their orbits. Under uniform plasma assumptions described above, we solve numerically the nonlinear Vlasov equation,

$$\frac{\partial f}{\partial t} + v_{\|}\frac{\partial f}{\partial z} + \frac{q}{m}\left(v_{\|}\frac{\partial A_{\|}}{\partial z} - \frac{\partial \psi}{\partial z}\right)\frac{\partial f}{\partial p_{\|}} = 0 \tag{1}$$

together with wave equations for the parallel component of the magnetic vector potential $A_{\|}$ and the electric scalar potential ψ,

$$\begin{aligned} \frac{\partial \psi}{\partial t} + v_A^2\frac{\partial A_{\|}}{\partial z} &= 0 \\ A_{\|} &= \frac{\mu_0 q F_{df}}{k_\perp^2 + \mu_0 q^2 Z_{df}/m}. \end{aligned} \tag{2}$$

Here, $p_{\|} = \nu_{\|} + qA_{\|}/m$is the canonical momentum per unit mass along the field line, while F_{df} and Z_{df} are first and zeroth order moments of the electron distribution function with respect to the canonical momentum coordinate, respectively.

[6] Note that in equation (1) the mirror force is absent due to the assumption of straight field lines, and that the Vlasov equation is expressed in the momentum coordinate for numerical stability reasons. In solving (1) and (2), we take an electron number density equal to 1 particle per cubic cm, and a magnetic field strength of 65 nT. The length of the field line, the perpendicular wave number, and the electron temperature determine the frequency of the wave. In exciting waves in the system, we add a constant driving term to the right hand side of equation (2) for the electric scalar potential. The driving term is of the form $\psi_d = \omega R\sin(\omega z/\nu_A)$ where R is a constant such that the excited scalar potential grows at the rate ωRt and ω is determined from solutions to the linear dispersion relation, equation (3) below, for fixed $k_\perp$and $k_{\|} = 2\pi/L_z$ (L_z is the length of the simulation domain). We turn the driving term off after three wave periods and use simple periodic boundary conditions for all three simulation variables.

3. Electron Trapping in Dispersive SAWs

[7] In discussing particle trapping, it is useful to consider the kinetic dispersion relation supported by the linearized form of equations (1) and (2). This is identical to the small ion gyroradius limit of equation (6) in [*Lysak and Lotko*, 1996]:

$$\frac{\omega^2}{k_{\|}^2 v_A^2} = 1 + \frac{k_\perp^2\rho_s^2}{1 + \xi Z(\xi)}. \tag{3}$$

Here, ρ_s is the ion acoustic gyroradius, ν_A is the Alfvén speed, $Z(\xi)$ is the plasma dispersion function, $\xi = \omega/k_{\|}\nu_{th}$, and $\nu_{th} = \sqrt{2T_e/m}$ is the electron thermal speed. The hot electron limit of equation (3) leads to the dispersion limit $\omega \sim k_{\|}\nu_A\sqrt{1 + k_\perp^2\rho_s^2}$, while the cold electron, zero ion acoustic gyroradius limit leads to $\omega \sim k_{\|}\nu_A/\sqrt{1 + k_\perp^2\lambda_e^2}$, where $\lambda_e = c/\omega_{pe}$. Note that in cold plasma the dispersion relation and parallel electric field are temperature independent, i.e., $E_{\|} = k_{\|}k_\perp\lambda_e^2 E_\perp$, whereas in warm plasma, $E_{\|} = k_{\|}k_\perp\lambda_e^2\nu_{th}^2/\nu_A^2 E_\perp$ and it is obvious that the parallel electric field increases with plasma temperature [*Chaston et al.*, 2003].

[8] Landau damping of dispersive SAWs has been considered by *Lysak and Lotko* [1996]. Here, we demonstrate that nonlinear trapping of electrons in long period waves can be much more significant, even at small electron temperatures. Trapping is important when significant numbers of electrons exist at the trapping velocity defined by $\nu_{tr} \sim \sqrt{2eE_{\|}/mk_{\|}}$. In the limit of very small electron temperature, or at high phase velocities satisfying $\omega/k_{\|} \sim \nu_A \gg \nu_{th}$, electron trapping is not so effective because very few electrons are in the trapping region. In long period field line resonances, on the other hand, temperature has a very strong effect due to the increase in the parallel electric field strength with electron temperature.

[9] By way of illustration, Figure 1 shows the growth of a field line resonance that is excited by driving the electric potential for three wave periods in plasma with T_e = 0.5 eV. The ratio of ν_A:ν_{th} is 3.4:1. In Figure 1a, when $k_\perp = 2 \times 10^{-5}$ m^{-1}, the electric potential saturates at 5 Volts, but damps strongly due to particle trapping when $k_\perp = 1.5 \times 10^{-4}$ m^{-1} (Figure 1b). The period of the wave is 104 s in this example, which is higher than is typical of auroral zone field line resonances. However, particle trapping in cold plasma is not strongly affected by the wave frequency, since

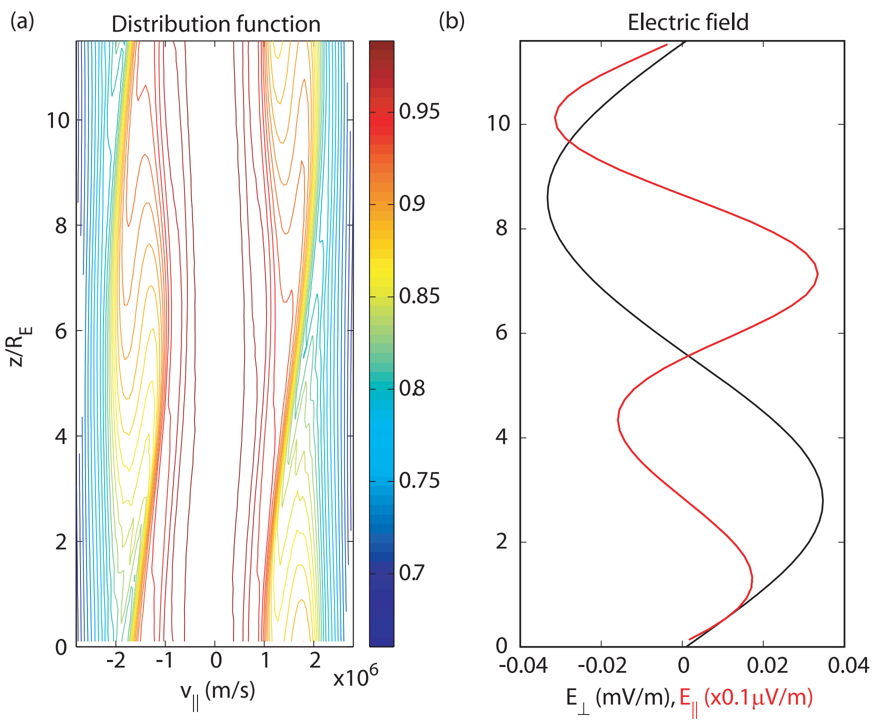

Figure 2. The left panel shows the electron distribution function as a function of $\nu_{\|}$ and the spatial coordinate along the field line. The right panel shows the corresponding wave electric field components perpendicular and parallel to the geomagnetic field. The electron temperature corresponds to T_e = 60 eV, with $\nu_A = 1.42 \times 10^6$ ms^{-1} and $\nu_{th} = 4.59 \times 10^6$ ms^{-1}, and $k_\perp = 5 \times 10^{-5}$ m^{-1}.

the phase velocity is on the order of the Alfvén speed, and $\nu_{tr} \propto \sqrt{k_\perp \lambda_e^2}$ in cold plasma depends only on the perpendicular length scale. The dependence of $E_\|$ and $\nu_\|$ on $k_\perp$explains why nonlinear wave damping is significant at short perpendicular length scales. Interestingly, Figure 1 shows that in the inertial limit the wave damps strongly when the perpendicular wavelength is on the order of 8 λ_e, i.e., comparable to one of the characteristic arc scales that *Borovsky* [1993] discussed based on numerous auroral arc observations.

[10] Similar results to Figure 1 are obtained when T_e = 30 eV, and all other parameters are the same. However, unlike Figure 1, when $k_\perp = 2 \times 10^{-5}$ m^{-1} the SAW in this case is already very strongly damped due to the increase in $E_\|$ with temperature. In this case, the perpendicular wavelength is 26 ρ_s (60 λ_e), implying dominance of thermal effects over inertial effects. The simulations reveal that damping remains strong until the wavenumber is reduced to $k_\perp \sim 2 \times 10^{-5}$ m^{-1}. Since in reality, the perpendicular scale of the wave and the plasma temperature (and hence $E_\|$) both increase with altitude, this suggests that trapping will be an important mechanism in damping the wave along a significant part of the field line. Indeed, computer studies covering a range of temperatures up to 1 keV, and frequencies up to 40 mHz confirm that nonlinear trapping is very effective at damping dispersive standing SAWs. Note also that particle trapping is a saturating effect on the excited waves since it induces a frequency shift that decouples the excited wave from the driver. Moreover, in the trapping regime it is known that Landau damping is stopped because of phase mixing of trapped particles [*O'Neil*, 1965].

4. Electron Response to SAW Current System

[11] In order to discuss the electron response to standing SAWs, we present two results. The left panel of Figure 2 shows contours of the electron distribution function at different times during the evolution of a driven SAW. The right panel shows the corresponding perpendicular and parallel electric fields along the field line. Here, T_e = 60 eV, placing the SAW in the kinetic regime, with a wave period of 45 s. In this case, $\nu_{th} = 4.59 \times 10^6$ ms^{-1}, which can be compared with $\nu_A = 1.42 \times 10^6$. Over the time interval shown, the maximum wave perpendicular electric field occurs earlier in time than is shown, and corresponds to $E_\perp \sim 0.1$ mVm^{-1}. The maximum parallel electric field strength corresponds to $E_\| \sim 0.1$ μVm^{-1}, which compares well with the warm plasma limit discussed above. This gives a potential drop $E_\| / k_\|$ on the order of 1–2 eV and a trapping velocity $\nu_{tr} \sim 10^6$ ms^{-1}, which agrees well with the width of the phase space vortices shown in Figure 2a. Note that the vortices are centered on the wave phase velocity. The simulations reveal that later in time, after the wave driver is turned off, phase mixing in velocity space smears out the phase space vortices. Thus, particle trapping is important in thermalizing wave energy into the plasma.

[12] Since particle trapping is a nonlinear effect, it is interesting to consider the effect of increasing the amplitude of the excited SAWs, which has the effect of increasing the trapping width in velocity space. This is accomplished in

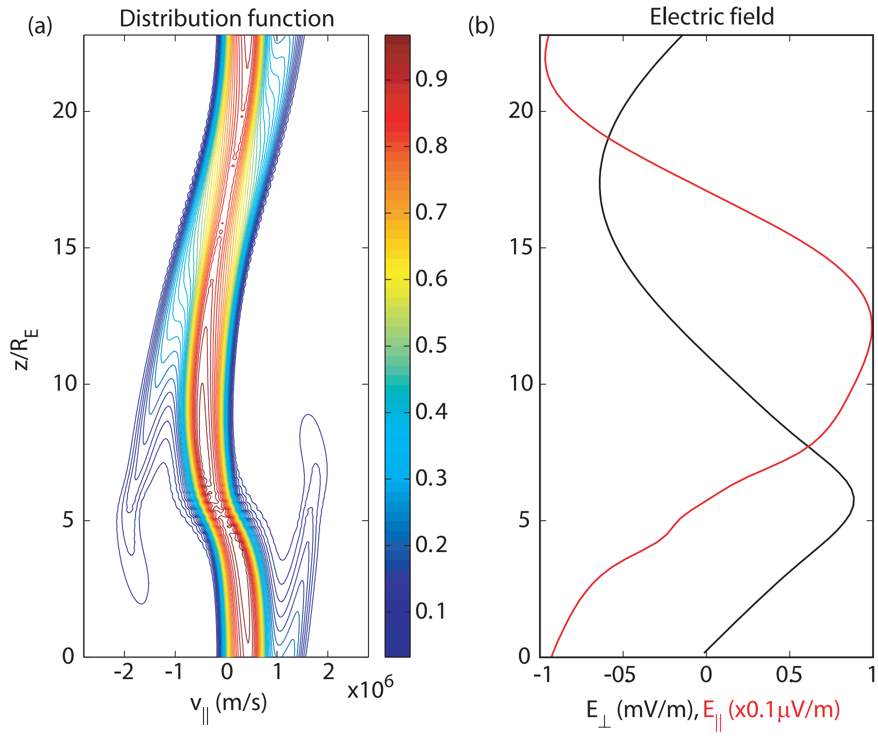

Figure 3. The left panel shows the electron distribution function as a function of $\nu_{\|}$ and the spatial coordinate along the field line. The right panel shows the corresponding wave electric field components perpendicular and parallel to the geomagnetic field. The electron temperature corresponds to $T_e = 0.5$ eV, with $\nu_A = 1.42 \times 10^6$ ms^{-1} and $\nu_{th} = 4.19 \times 10^5$ ms^{-1}.

the simulations by increasing the strength of the driver by a factor of five. We place the excited wave in the inertial regime by reverting to parameters relevant to Figure 1. In Figure 3, the same basic behavior observed in Figure 2 is present, but as the wave amplitude grows, beams of electrons appear that become detached from the wave. This is because in cold plasma, and at high wave amplitude, the distribution function shifts significantly in velocity space to accommodate the SAW field-aligned current. This moves larger numbers of electrons into the trapping regime centered at the wave phase velocity. In simulations using temperatures above 100 eV or so, it is found that the SAW field aligned current does not cause the distribution function to shift appreciably in velocity space. At the same time, the trapping width is also increased. Formation of electron beams in warm plasma at high altitudes in field line resonances seems, therefore, to be more difficult. In contrast, geomagnetic field convergence (large $E_{\|}$) and relatively low plasma temperature at auroral altitudes appear to favour electron beam formation. Nonlinear trapping, on the other hand, is very effective in thermalizing wave energy at both small and large perpendicular wavelengths.

5. Conclusions

[13] In long period field line resonances, nonlinear electron trapping in quasi-static parallel electric fields is very effective at dissipating wave energy. The electrons form phase space vortices that provide a sink for wave energy, damping the waves in as little as a wave period when the electron temperature is on the order of a few 100 eV. We find in particular that when the thermal speed is above the Alfvén speed, the increase in the parallel electric field with temperature makes nonlinear trapping more effective, leading to efficient damping of the waves. Perhaps surprisingly, nonlinear damping in the kinetic regime remains effective at perpendicular wavelengths that can be much larger than the ion acoustic gyroradius, for example when $k_{\perp}\rho_s \sim 0.1$ and ν_A:ν_{th} is in the ratio 1:2.3. The damping rates are also much larger than for linear Landau damping. This is illustrated by the dashed lines in Figure 1, which indicate the amplitude decay predicted by solutions to equation (3). In cold plasma, at electron temperatures of a few eV, simulations reveal that electron trapping is effective when the perpendicular wavelength is on the order of 8–10 electron skin depths, **or** $k_{\perp}\lambda_e \sim 0.6$–$0.8$. The electron distribution function is strongly perturbed by the wave, bringing larger numbers of electrons into the trapping regime. The trapping width is defined by $\nu_{tr} \sim \sqrt{2eE_{\|}/mk_{\|}}$, and depends on the strength of the parallel electric field and parallel wavelength.

[14] At large wave amplitude, kinetic simulations of cold plasma ($T_e = 0.5$ eV) reveal that electrons moving at the wave phase velocity become detached from the wave, forming beams of electrons with an energy that is proportional to the wave amplitude. This effect will therefore be most pronounced on low-altitude auroral field lines, suggesting perhaps that field line resonances may offer an explanation for optical features observed in all-sky camera data that is collocated with Pc-5 field line resonances. Although we have ignored finite ion gyroradius, this is likely not important as it does not affect the strength of parallel electric fields. In our relatively simple model, finite ion gyroradius will affect the absolute magnitude of the

wave phase velocity, which can be accounted for by a simple rescaling of the ambient geomagnetic field strength.

[15] **Acknowledgments.** The authors acknowledge the support of the Natural Sciences and Engineering Research Council of Canada (NSERC) and the Canadian Space Agency. The authors also acknowledge useful discussions with V. T. Tikhonchuk.

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