Depth to mate and the 50-move rule


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NOTE

DEPTH TO MATE AND THE 50-MOVE RULE

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1. INTRODUCTION

The first author’s DTM\(^{50}\) ‘EGT’ endgame tables (Huntington, 2013; Haworth, 2014a/b) provide ‘DTM’ Depth to Mate information as moderated by the FIDE (2014) ‘50mr’ 50-move rule and the ply-count \(pc\). This note puts that achievement in the context of earlier DTM computations (Nalimov et al., 2000/2001; Wu and Beal, 2001a/b; Bleicher, 2015) and data from previous studies of 50mr-impact (Tamplin and Haworth, 2004; Bourzatschky et al., 2005; Tamplin, 2015). It compares some DTM\(^{50}\) statistics with the intrinsic, unmoderated DTM and DTZ\(^{50}\) data.\(^3\) Datasets supporting these results are available (Huntington and Haworth, 2015) and include a pgn file, its annotation, and the fuller statistics which cannot be accommodated here.

When considering 50mr impact, the ply-count \(pc\) or rather the ply-remaining count \(pr = 100 - pc\) must be borne in mind. A position’s \(dmt_{50,pc}\) may increase as \(pc\) increases until the win becomes a ‘frustrated win’, a ‘50mr-draw’. Even a mate in two ply will be frustrated if 99 ply have been expended, e.g., in KNNKP. These ‘EM\(^{50}\)’ DTM\(^{50}\) EGts are the first to provide depths for any value of \(pc\). Clearly, max\(DTM_{50,pc<0} \geq \max\(DTM_{50,pc=0}\).

The FIDE 50-move rule is not an intrinsic rule of the game but a ‘rule of play’ introduced by Ruy López (1561) for the convenience of professional coffee house players. The results here show that it has major impact on KBBKN and KNNKP, and therefore on the upstream KBBKNN, KNNKNP and KNNKPP. For s6m, s7m and s8m chess, max\(DTZ\) is greater than 80, 240 and 510 moves respectively (Haworth, 2014b). The 50mr, now backed by the 75-move-rule, will apparently frustrate ever more subtle wins as the number of men increases.

The perspective here is that these extreme cases of extended wins, rather than being denied by rules of play, should become part of the culture, experience, record and history of chess, at least when an EGT-armed computer-engine demonstrates infallible play.

2. COMPUTING DTM: ALGORITHM, LANGUAGE AND PROGRAM

When generating any EGT, the fundamental principles are that (a) successor endgames’ EGts are computed first, and (b) a position can be given a depth:

- temporarily when one of its immediate successors has been given a depth, but
- definitively only after enough of its successors have been given their definitive depth.

Any computation begins by identifying ‘mated’ (in 0 ply) positions. Positions which can be assigned a depth may be found by repeated, linear sweeps of the whole endgame and the first sweeps efficiently net many positions. Shortly though, the more selective and efficient method is to ‘unmove’ from the ‘frontier’ of positions which have just been given a depth in the last cycle of the algorithm.

The best known DTM computations are those of Nalimov (2000/2001) which created EGts for the whole of sub-7-man (s7m) chess (Bleicher, 2015). Nalimov employed linear sweeps rather than the more retrograde ‘unmove’ algorithm, giving a position a depth at the earliest opportunity but lowering it later as required. As evidenced by the results here, this occurred many times in generating the KNNKP EGT. In contrast, Wu (2001a/b) worked exclusively in unmove mode. His key idea was to defer identifying ‘mates in \(m+1\)’ until all ‘mates in \(m\)’ had been identified. As each \(dtm\) is associated with one cycle of the algorithm, it only required two bits per position rather than one- or two-bytes, economising on memory by a factor of four or eight.

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\(^3\) DTZ = minimaxed depth to the zeroing of the ply-count, i.e., to a pawn-push, capture and/or mate which end the current phase of play.
Huntington reverts to the principle of assigning depths as soon as possible. Each algorithm cycle corresponds to a specific number, \( pr \), of ply remaining: \( pr \) effectively increases from 0 to 100 during the computation. Lower values of \( DTM_{50} \) become possible when more ply are available. Table 3 shows that the highest variability of \( DTM_{50} \) seen so far occurs in KNNKP with 25 different mate-depths. By noting on what cycle a \( DTM_{50} \) value is set, the various ply-ranges corresponding to a specific \( dtm_{50} \) can be identified for each chess position.

EGT generation, as a computation task, is challenging because the results are not self-evidently correct: the subtlety and toxicity of errors is well known (Hurd and Haworth, 2010). Where clarity of coding and correctness are particularly at risk, as has been argued in classic texts elsewhere (Hughes, 1990; Bird, 2014), ‘FPL’ functional programming languages, without side effects, may be preferred to imperative languages. The declarative style of FPLs, using higher levels of abstraction, creates programs which are more readable and understandable. Research in FPLs created an embarrassment of riches and ideas, and the best of these were brought together in the language HASKELL, ‘standardised’ in HASKELL 98 (Peyton-Jones, 2002) and then in HASKELL 2010 (Marlow, 2010). HASKELL is most liked for its elegance as a language, for its type system and for the confidence it creates that compiled code is likely to be correct code (MacIver, 2015) and was the choice of the first author here.

FPLs once had a reputation for inefficiency but modern compilers have largely offset this and performance is competitive today. For example, FPLs allow new methods of whole-program optimisation made possible by the promise of purity. Also, code can be written much more compactly due to the expressive power of treating functions as ordinary values that can be built at runtime. Certainly, there are new kinds of pitfalls which programmers must contend with: the challenge of balancing use of space and time remains. Small changes can have a large effect on performance by, e.g., causing an optimization to become inapplicable. Worse, unexpected memory leaks are common, and measures must be taken to prevent overzealous time optimisation that causes excess space usage. Such snags plagued earlier versions of the \( DTM_{50} \) EGT generator, and in general avoiding them is an active area of compiler research. The EGT itself consists internally of an array where each cell value stored the possible mate lengths for various ranges of PC, and an ‘overflow’ hash table to store values that do not fit in an array cell. The cell’s size has to be fixed in advance and chosen with care. Too small, and the overflow structure becomes heavily used, which is much less space/time efficient than an array; too large, and much memory is wasted on unused array space.

\[ \text{Figure 1. 5-man endgame wins, 1-0 and 0-1, at first apparently DTZ-susceptible to the 50-move rule.} \]

3. **THE RESULTS**

DTM endgame tables presume a ply-count of \( pc = 0 \) and therefore the \( DTM_{50,pc} \) results with \( pc = 0 \) provide a natural first focus. The later consideration of a free-ranging \( pc \) places emphasis on the cost of \( pc > 0 \) and the variability of \( dtm_{50,pc} \).

3.1 **DTM_{50,pc} with zero ply-count pc**

For sub-5-man chess, no endgames are affected as \( \text{maxDTM} < 100 \) ply. Figure 1 shows those 5-man endgames, ranked by number of pawns, which appear at first to be susceptible to the 50mr, 1-0 and/or 0-1 wins\(^4\) perhaps being converted to 50mr-draws or lengthened in some sense. Tamplin and Haworth (2004) identified the actual 50mr impact in DTZ terms and this is annotated under four headings in Figure 1 and Table 1.

\(^4\) In fact, the 50mr only affects both 1-0 and 0-1 wins in the endgames KBBKP, KNNKP, KRPKQ and KRKP of Figure 1.
The four-way DTZ-based taxonomy is indicated as follows:

- obviously unaffected wins (7) (bracketed ‘lower case’), e.g., KBPKR (1-0), KPPKR(Q/P) (0-1):
  \[ \text{maxDTM} \leq 100 \text{ ply} \Rightarrow \text{EM}_{50} = \text{EM} \Rightarrow \text{there need be no 50nr-effect here or downstream}, \]

- perhaps affected wins (6) (unbracketed lower case), e.g., K/B(N)PK (1-0 and 0-1):
  \[ \text{the DTZ50 and DTZ EGTs}, \text{i.e., } \text{EZ50} = \text{EZ}, \text{are identical but perhaps } \text{dtm50} > \text{dtm} \text{ somewhere,} \]

- affected wins (14) (upper case), e.g., KBKKP and KRPPK (both, 1-0 and 0-1), KPPKR(P/Q) (1-0):
  \[ \text{maxDTZ} \leq 100 \text{ ply but there are } 50\text{nr draws, i.e., } \text{EM}_{50} \neq \text{EM}, \]

- obviously affected wins (10) (underlined, upper case), e.g., KNNKP (1-0) and KRKPKQ (0-1):
  \[ \text{maxDTZ > 100 ply } \Rightarrow \text{50nr-draws } \Rightarrow \text{EM}_{50} \neq \text{EM}. \]

Beyond the scope of Figure 1, Table 1 provides DTZ_{50} data for a number of other endgames which are affected. A second DTZ/Z_{50} review (Bourzutschky et al., 2005) provides further context for Painless 6-man endgames.

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**Table 1. Data for some endgame wins, the first 37 being those of Figure 1.**

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  \[ \text{maxDTM} \leq 100 \text{ ply} \Rightarrow EM_{50} = EM \Rightarrow \text{there need be no 50nr-effect here or downstream}, \]

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- affected wins (14) (upper case), e.g., KBKKP and KRPPK (both, 1-0 and 0-1), KPPKR(P/Q) (1-0):
  \[ \text{maxDTZ} \leq 100 \text{ ply but there are } 50\text{nr draws, i.e., } EM_{50} \neq EM, \]

- obviously affected wins (10) (underlined, upper case), e.g., KNNKP (1-0) and KRKPKQ (0-1):
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\[ ^5 \text{maxDTZ}_{50} \text{is compared with maxDTM}. \text{maxDTM}_{50,p} \text{is compared with maxDTM} \text{and then maxDTZ}_{50,p}, \downarrow, \uparrow, \downarrow \text{and } \uparrow \text{mean, respectively, } "\text{if anything, less than}, \text{" identical”, } "\text{if anything, more than} \text{" and } \text{"considering wtm/btm, both less than and more than}," \text{"rather than } \text{" indicate that the number of maximal positions has changed.} \]
With increasing likelihood, maxDTM_{50,pc} is greater than, less than or equal to maxDTM as indicated in Table 1:

maxDTM_{50,pc} > maxDTM:
- (1-0) KPPPK, KPQQQ, KQPPK btm, KQPPK, KRPQK btm
- (1-0) KB(B/N)K(N/P), KBKNN, KNKNN, KNQPK, KQRPQ btm, KQRPQ(P/Q), KRBBK, KRPPK, KRPQK wtm (and 6-man) KBKNNN, KNKNNP, KNQPKQ, KRQPKQ, KRQPKQ

maxDTM_{50,pc} < maxDTM:
- (1-0) KBK(P/Q), KNKN(P/Q), KNNKP, KRPQK

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Table 2. Positions p01-p37: example maxDTM_{50} wins and 50-moves.\(^6\)

<table>
<thead>
<tr>
<th>Value</th>
<th>depths in ply</th>
</tr>
</thead>
<tbody>
<tr>
<td>id</td>
<td>Endgame</td>
</tr>
<tr>
<td></td>
<td>FEN</td>
</tr>
</tbody>
</table>

Notes: annotations to the DTZ_{50,pc} metric
- maxDTM_{50,pc,pm} is less than equal to maxDTM_{50,pc,pm}
- maxDTM_{50,pc,pm}, any-penalty winning for \(pc \geq 0\)
- maxDTM_{50,pc,pm}, any-penalty winning for \(pc \geq 0\)

3.2 DTM_{pc} with free-ranging ply-count

With \(pc\) being allowed to range over all values 0-99, each endgame will have:
- a maxDTM_{50,pc} and a set of positions and a set of \(pc\)-values for which \(dtm_{50,pc} = maxDTM_{50,pc,pm}\)
- 'maximum penalty' positions for which ply-count \(pc\) produces max \(dtm_{50,pc,dtm}\)
- 'maximum variety' positions for which varying \(pc\) gives most values of \(dtm_{50,pc}\)

Five endgames have maxDTM_{50,pc} > maxDTM and sixteen more have maxDTM_{50,pc} > maxDTM:
- within Figure 1: (1-0) KBPPK(P/R), KNNKK, KNKKP, KQPPK, KQRPQK wtm, KRBBK, KRQPQK wtm, KRPQK btm, KRPKR and (0-1) KB(B/N)P/KQ, KRPQK.
- beyond Figure 1: (0-1) KPQKN, KRBBK and (6 man) KNNKPP.

\(^6\) \(m\) = absolutely unique value-preserving move; \(m\) = unique metric-optimal move; \(m\) = metric-equivalent move
Tables 2 and 3 provide a list of positions illustrating 50mr-impact:

- p01-p06: zonal maxDTM50, pc=0 and maxDTM50, pc>0 positions,
- p07-p12: endgame-specific maxDTM50, pc=0 and maxDTM50, pc>0 positions,
- p13-p37: ‘50mr-draw’ frustrated wins or saved losses,
- p38-p46: DTM-lengthened wins or losses with pc=0,
- p47-p50: zonal ‘maximum pc/DTM-penalty’ positions,
- p51-p56: endgame-specific ‘maximum pc/DTM-penalty’ positions,
- p57-p60: zonal maximum of the number of pc-determined mate-depths DTM50, pc,

<table>
<thead>
<tr>
<th>id</th>
<th>Endgame</th>
<th>FEN</th>
<th>1-0? 5-way</th>
<th>Value depths in ply</th>
</tr>
</thead>
<tbody>
<tr>
<td>38</td>
<td>KNNKP</td>
<td>6k/P7/6k/8/7/7/5/5w - - 0.1</td>
<td>1-0 2</td>
<td>180 181 223 32 58 58</td>
</tr>
<tr>
<td>39</td>
<td>KPPKP</td>
<td>8/P8/6k/4p4v/P7/7b - - 0.1</td>
<td>1-0 2</td>
<td>248 282 2 2 2</td>
</tr>
<tr>
<td>40</td>
<td>KPPKP</td>
<td>4/P8/6k/4q6qK/P7/7b - - 0.1</td>
<td>1-0 2</td>
<td>214 275 1 1 1</td>
</tr>
<tr>
<td>41</td>
<td>KPPKP</td>
<td>8/K7/7/7/6q2q3/7/6b - - 0.1</td>
<td>1-0 2</td>
<td>220 240 274 100 100</td>
</tr>
<tr>
<td>42</td>
<td>KPPKP</td>
<td>1/4R1P/7/8/9/16q6qP - - 0.1</td>
<td>1-0 2</td>
<td>412 114 4 4 4</td>
</tr>
<tr>
<td>43</td>
<td>KBBKN</td>
<td>8/6/6k/4/1B7/2/6/5K5/8 - - 0.1</td>
<td>1-0 2</td>
<td>133 149 1 55 55</td>
</tr>
<tr>
<td>44</td>
<td>KBBKP</td>
<td>8/6/6k/4/4/8/1K5/8 - - 0.1</td>
<td>0-1 2</td>
<td>123 125 1 13 13</td>
</tr>
<tr>
<td>45</td>
<td>KBBKP</td>
<td>8/5/5/6/4/6/1K7/6 - - 0.1</td>
<td>0-1 2</td>
<td>159 163 1 11 11</td>
</tr>
<tr>
<td>46</td>
<td>KBBKP</td>
<td>8/4/8/3/8/8/8/8/8 - - 0.1</td>
<td>0-1 2</td>
<td>113 163 167 3 41</td>
</tr>
<tr>
<td>47</td>
<td>KPPKP</td>
<td>8/4/8/3/8/2/K9/5/3/8 - - 99 1</td>
<td>1-0 2</td>
<td>17 275 1 1 1</td>
</tr>
<tr>
<td>48</td>
<td>KKKKN</td>
<td>7/3/4/8/8/8K/4/6c w - - 97 1</td>
<td>1-0 2</td>
<td>45 81 1 1 4 4</td>
</tr>
<tr>
<td>49</td>
<td>KNNKN</td>
<td>5/2/5/2/8/8/8/8/8 - - 98 1</td>
<td>0-1 2</td>
<td>3 73 1 1 1 1</td>
</tr>
<tr>
<td>50</td>
<td>KRRKP</td>
<td>8/8/8/2/K6/4/3/8/8/6 - - 88 1</td>
<td>0-1 2</td>
<td>13 13 29 191 7 7</td>
</tr>
<tr>
<td>52</td>
<td>KRRKP</td>
<td>7/2/2Kp5/4/8/8/8/8/8 w - - 91 1</td>
<td>1-0 2</td>
<td>17 17 207 8 8 8</td>
</tr>
<tr>
<td>53</td>
<td>KRPKP</td>
<td>8/1/6/6/8/9/2/4P3/4K3w - - 96 1</td>
<td>1-0 2</td>
<td>7 33 275 1 1 1</td>
</tr>
<tr>
<td>54</td>
<td>KRRKR</td>
<td>8/8/8/1/6K/3/8/6/1K7/6 - - 90 1</td>
<td>1-0 2</td>
<td>9 11 39 7 9 9</td>
</tr>
<tr>
<td>55</td>
<td>KRRKP</td>
<td>5/2/8/7/8/5/6/8/1K7/6 - - 98 1</td>
<td>0-1 2</td>
<td>1 3 57 1 1 1</td>
</tr>
<tr>
<td>56</td>
<td>KBBKRN</td>
<td>4/8/4/8/3/8/8/8/8/8 - - 98 1</td>
<td>1-0 2</td>
<td>1 3 161 1 1 1</td>
</tr>
<tr>
<td>57</td>
<td>KNNKP</td>
<td>8/K2/4/8/8/8/8/8/8 - - 1</td>
<td>1-0 2</td>
<td>120 121 223 42 44 44</td>
</tr>
<tr>
<td>58</td>
<td>KNNKP</td>
<td>7/4/4/8/8/8/8/8/8 - - 1</td>
<td>1-0 2</td>
<td>11 29 66 11 11 1</td>
</tr>
<tr>
<td>59</td>
<td>KNNKP</td>
<td>7/8/4/6/8/8/8/8/8 - - 1</td>
<td>0-1 2</td>
<td>8 42 142 6 8 8</td>
</tr>
<tr>
<td>60</td>
<td>KRPKP</td>
<td>8/4/8/3/K9/4/2P5/8/7/8 w - - 1</td>
<td>0-1 2</td>
<td>58 104 164 31 31</td>
</tr>
<tr>
<td>61</td>
<td>KBBKN</td>
<td>8/2/5/8/8/8/8/8/8 - - 1</td>
<td>1-0 2</td>
<td>17 37 17 17</td>
</tr>
<tr>
<td>62</td>
<td>KQKPQ</td>
<td>8/6/8/4/8/8/8/8 - - 1</td>
<td>0-1 2</td>
<td>99 115 159 29 29</td>
</tr>
<tr>
<td>63</td>
<td>KBBKP</td>
<td>4/8/5/5/6/2/8/1K7/6w - - 1</td>
<td>1-0 2</td>
<td>23 33 41 23 23 23</td>
</tr>
</tbody>
</table>

Table 3. Positions p38-p63: example elongated wins, and maximum penalty/variety positions.

4 SUMMARY AND VIEW FORWARD

The impact of the 50-move rule on sub-6-man chess, including the effect of the ply count pc, has been identified. Given a shortage of remaining ply, the winning strategy is more manageable in some endgames than in others. The 50mr impact increases as the number of men increases, and has been observed in key 6- and 7-man endgames of interest. This impact should be measured as the 50mr, now backed up by a mandatory 75-move rule, truncates chess as experienced over the board, whether played by man or machine. The evolving data files associated with this note (Huntington and Haworth, 2014) provide:

- extended versions of Tables 1-3,
- (annotated) pgn files illustrating various positions and lines mentioned here,
- a set of spreadsheets of the first author’s complete data.

The maxDTZ50 figures inferred from these DTM50 EGs confirm previous figures (Tamplin and Haworth, 2004). Some maxDTM50, pc positions and lines have been published in an evolving collection of chess records (Haworth, 2014b) which facilitates the comparison of DTM, DTM50, DTZ and DTZ50 lines.

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7 The 5-way value-scale is ±2 for unconditional ‘50mr-wins/losses’, ±1 for ‘50mr-draws’ and 0 for unconditional draws.
8 The list includes a frustrated win but not necessarily a DTM - elongated win for every affected endgame.
Acknowledgements. Thanks first to ‘Kronsteen’ (2012) who inspired this work on DTM50 and shared data from his own work. Thanks also to Eiko Bleicher (2014) and John Tamplin (2014) for previous collaborations and for their long-standing EGT query services.

References


