Two-Way CSI-Assisted AF Relaying with HPA Nonlinearity

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Abstract—In this paper, we investigate half-duplex two-way dual-hop channel state information (CSI)-assisted amplify-and-forward (AF) relaying in the presence of high-power amplifier (HPA) nonlinearity at relays. The expression for the end-to-end signal-to-noise ratio (SNR) is derived as per the modified system model by taking into account the interference caused by relaying scheme and HPA nonlinearity. The system performance of the considered relaying network is evaluated in terms of average symbol error probability (SEP) in Nakagami-m fading channels, by making use of the moment-generating function (MGF) approach. Numerical results are provided and show the effects of several parameters, such as quadrature amplitude modulation (QAM) order, number of relays, HPA parameters, and Nakagami parameter, on performance.

I. INTRODUCTION

Cooperative relaying stands as a promising mechanism for future wireless communication networks, which can enhance transmission reliability, enlarge system capacity, broaden network coverage, and ensure quality of service (QoS). The nodes in the network could not only act as source node to issue information, but also perform as relay to forward the signals received from other nodes. In this paper, we investigate the half-duplex two-way amplify-and-forward (AF) relaying, where source nodes simultaneously transmit their information to the relay nodes in the first phase, and then, in the following phase, the relay nodes simply amplify the received signals and forward them to the source nodes. The two-way AF relaying can cope with the drawback of the one-way relaying and obtain higher spectrum efficiency [1], [2].

However, the practical performance of the dedicated two-way relaying is crucially affected by radio frequency (RF) impairments, which bring distortions to the network and then degrade the system performance [3], [4]. Herein, we consider one type of RF impairments, namely, high-power amplifier (HPA) nonlinearity. The HPA may operate in its nonlinear region, which causes in-band and out-of-band distortions [5], [6]. The nonlinear HPAs can be characterized by two kinds of models: memoryless models with frequency-flat responses, and memory models with frequency-selective responses. The former include Saleh model for travelling wave tube amplifier (TWTA), solid state power amplifier (SSPA) model, soft-envelope limiter (SEL) model, and memoryless polynomial model. Memory models contain Volterra, Wiener, Hammerstein, Wiener-Hammerstein, and memory polynomial models. Recent research effort has studied the effect of HPA nonlinearity on the performance of orthogonal frequency-division multiplexing (OFDM)-based cooperative systems using the Bussgang’s theorem in [7]. On the other hand, one-way dual-hop AF relaying with HPA nonlinearity in semi-blind relays was investigated by making use of memoryless polynomial model for HPAs in the authors’ previous work [8].

In this paper, we study the half-duplex two-way dual-hop channel state information (CSI)-assisted AF relaying system in the presence of HPA nonlinearity at relays. The updated model has been built up by taking into account the factors caused by nonlinear HPAs. The expression for the end-to-end signal-to-noise ratio (SNR) has been modelled, and followed by two asymptotic expressions for low and high SNR regions, respectively. Then, by making use of the moment-generating function (MGF) approach, the expression for the average symbol error probability (SEP) of the considered system is derived in Nakagami-m fading channels. Afterwards, numerical results and comparison are provided and show the effects of several parameters, such as number of relays, HPA parameters, quadrature amplitude modulation (QAM) order, and Nakagami parameter, on the system performance.

The remainder of this paper is organized as follows: Section II introduces the half-duplex two-way dual-hop CSI-assisted AF relaying system model in the presence of HPA nonlinearity at the relays. The system performance metrics, in terms of end-to-end SNR and average SEP, are derived in Section III. Numerical results and comparisons are then presented in Section IV, followed by the conclusion in Section V.

II. SYSTEM MODEL

We consider a dual-hop half-duplex two-way AF cooperative relaying system allocated with two source nodes, $S_1$ and $S_2$, and $N$ relay nodes, $R_n$ ($n = 1, 2, \ldots, N$), which are all equipped with a single antenna. No direct path between $S_1$ and $S_2$ exists so that the source nodes communicate through the relays. The transmission works in two phases, i.e., multiple-access (MAC) phase and broadcast (BC) phase. In the first
phase, $S_1$ and $S_2$ simultaneously transmit their information to the $N$ relay nodes. Then, in the second phase, the relays amplify the received signals and broadcast them to the source nodes.

In practice, the performance of the two-way relaying system may get affected by HPA nonlinearity, which causes in-band and out-of-band distortions. Herein, we only consider in-band distortion caused by nonlinear HPAs implemented at the relay nodes. In particular, the memoryless polynomial model \cite{9, 10} is used here to describe the characteristics of nonlinear HPA. In a general way, the input-output relationship of the nonlinear HPA can be expressed as

$$y_{out} (t) = x_{in} (t) \sum_{k=0}^{K} a (2k + 1) |x_{in} (t)|^{2k},$$  \hspace{1cm} (1)

where $x_{in} (t)$ and $y_{out} (t)$ denote the baseband-equivalent input and output signals, respectively, $K$ is the order of the polynomial function, and \{a (2k + 1)\} refers to a set of coefficients. Note only odd-order product terms in (1) are considered due to the fact that the signals generated from the even-order terms are outside the frequency band of interest \cite{10}. For simplicity of the analysis, the HPA coefficients are assumed to be perfectly known.

In the MAC phase of the two-way relaying system, the received baseband-equivalent signal at the $n$-th ($n = 1, 2, \ldots, N$) relay node is given by

$$y_{r_n} (t) = \sqrt{P_{s_1}} x_1 (t) h_n (t) + \sqrt{P_{s_2}} x_2 (t) g_n (t) + n_{r_n} (t),$$  \hspace{1cm} (2)

where $x_1 (t)$ and $x_2 (t)$ denote the normalized baseband-equivalent transmitted signal from the source nodes $S_1$ and $S_2$, respectively, $P_{s_1}$ and $P_{s_2}$ denote the transmit power at $S_1$ and $S_2$, respectively, $h_n (t)$ and $g_n (t)$ are the channel coefficients from the sources $S_1$ and $S_2$ to the $n$-th relay node, respectively, $n_{r_n} (t)$ refers to the noise term at the $n$-th relay, belonging to $\mathcal{C}\mathcal{N} (0, N_0)$ and uncorrelated with the transmitted signals, and $\otimes$ denotes the convolution product.

In the BC phase, the received signal is multiplied by the gain of the relay, $G_n (t)$, which can be expressed as

$$G_n (t) = \frac{P_n}{\sqrt{P_{s_1} |h_n (t)|^2 + P_{s_2} |g_n (t)|^2 + N_0}},$$  \hspace{1cm} (3)

where $P_n$ represents the power of the $n$-th relay. Then, the signal at the output of HPA is given by

$$x_{r_n} (t) = G_n (t) y_{r_n} (t) \sum_{k=0}^{K_n} a_n (2k + 1) |G_n (t) y_{r_n} (t)|^{2k},$$  \hspace{1cm} (4)

where $K_n$ and \{a_n (2k + 1)\} represent the order and coefficient set of the polynomial model for the HPA at the $n$-th relay node, respectively.

It is assumed that the channels between the sources and relays are reciprocal, and remain constant during the MAC and BC phases. Without loss of the generality, we consider the received signal at the source $S_2$. In the BC phase, using the CSI-assisted AF relaying, the received signal at the source node $S_2$ from the $i$-th relay is given by

$$y_{i,s_2} (t) = x_{r_i} (t) \otimes g_n (t) + n_{i,s_2} (t),$$  \hspace{1cm} (5)

where $n_{i,s_2} (t)$ denotes the noise at the source node $S_2$ from the $n$-th relay, belonging to $\mathcal{C}\mathcal{N} (0, N_0)$.

Remark 1: Unlike the case with linear HPAs, the self-interference presenting the source nodes cannot be removed totally due to the nonlinear characteristics of the HPAs at the relays, even though the perfect CSI is assumed to be known at the sources, which consequently degrades the performance of two-way relaying.

It is assumed that perfect sampling is performed in order to profit from the property of the raise-cosine rolloff and root-raised cosine rolloff pulses. Based on this, substituting (4) into (5) and omitting the time index, the received signal at the source node $S_2$ from the $i$-th relay can be expressed as

$$y_{i,s_2} = n_{i,s_2} + g_n G_n \left( \sqrt{P_{s_1} x_1 h_n} + \sqrt{P_{s_2} x_2 g_n} + n_{r_n} \right) \times \sum_{k=0}^{K_n} a_n (2k + 1) \left| G_n \left( \sqrt{P_{s_1} x_1 h_n} + \sqrt{P_{s_2} x_2 g_n} + n_{r_n} \right) \right|^{2k}.$$  \hspace{1cm} (6)

By further subtracting the self-interference properly, we can obtain the expression as

$$y_{i,s_2}^F = a_n (1) \sqrt{P_{s_1} h_n x_1} + n_{i,s_2} + a_n (1) G_n g_n n_{r_n} + g_n G_n \left( \sqrt{P_{s_1} h_n x_1} + \sqrt{P_{s_2} g_n x_2} + n_{r_n} \right) \times \sum_{k=1}^{K_n} a_n (2k + 1) \left| G_n \left( \sqrt{P_{s_1} h_n x_1} + \sqrt{P_{s_2} g_n x_2} + n_{r_n} \right) \right|^{2k},$$  \hspace{1cm} (7)

where the interference term comes from both the HPA nonlinearity and AF relaying. The self-interference could be removed for $k = 0$, while it cannot be mitigated for the terms with $k \geq 1$. The effective end-to-end SNR via the $i$-th relay, $\gamma_{i,s^2}$, is given by (8), where $E[\cdot]$ denotes the expectation operator. It is not straightforward to derive the SNR expression for the case with large values of the HPA parameter $K_n$ in closed-form. Herein, for simplicity of the analysis, we study the case with $K_n = 1$, i.e., 3-order polynomial model for the nonlinear HPAs. Thus, the effective end-to-end SNR becomes (9), where $\Re[\cdot]$ refers to the real part of a complex number.

III. IMPACT OF HPA NONLINEARITY

In this section, we investigate the impact of HPA nonlinearity on the performance of the considered two-way CSI-assisted AF relaying system in Nakagami-$m$ fading environment.

In a generic way, the probability density function (PDF) of the received SNR in Nakagami-$m$ fading channels can be expressed as

$$f_{\gamma} (\gamma) = \frac{m^m}{\Gamma (m)} \bar{\gamma}^{m-1} \exp \left( \frac{-m}{\bar{\gamma}} \right),$$  \hspace{1cm} (10)

where $\bar{\gamma}$ refers to the average received SNR, $\Gamma(\cdot)$ denotes
\[
\begin{align*}
\gamma_n^2 &= E \left[ a_n(1) \sqrt{P_s h_n g_n x_1}^2 \right] / \{ N_0 + E \left[ |a_n(1) G_n g_n n_{r_n} + g_n G_n \left( \sqrt{P_s h_n x_1} + \sqrt{P_{s_2} g_n x_2} + n_{r_n} \right) + g_n G_n \sum_{k=1}^{K_n} a_n(2k+1) G_n \left( \sqrt{P_s h_n x_1} + \sqrt{P_{s_2} g_n x_2} + n_{r_n} \right)^{2k} \right]^2 \} \right]. \tag{8}
\end{align*}
\]

\[
\gamma_n = P_{s_1} |a_n(1) G_n g_n n_{r_n}|^2 / \{ N_0 + E \left[ |a_n(1) G_n g_n n_{r_n} + a_n(3) g_n G_n n_{r_n} | n_{r_n} |^2 \right]^2 \} \right]. \tag{9}
\]

\[
\begin{align*}
\gamma_{n,1} &= \frac{P_{s_1} P_n N_0^2 |a_n(1) h_n g_n|^2}{\left( P_{s_1} |h_n|^2 + P_{s_2} |g_n|^2 + N_0 \right) N_0^3 + P_n E \left[ |a_n(1) N_0 g_n n_{r_n} + a_n(3) g_n P_n n_{r_n} | n_{r_n} |^2 \right]^2 } \\
&= P_{s_1} P_n N_0^2 |a_n(1) h_n g_n|^2 / \left\{ \left( P_{s_1} |h_n|^2 + P_{s_2} |g_n|^2 + N_0 \right) N_0^3 \right. \\
&\quad \left. + P_n \left[ N_0 |a_n(1) g_n|^2 + |a_n(3) g_n P_n|^2 \right] (|n_{r_n}|^6) + 2Re \left( a_n(1) (a_n(3))^* P_n N_0 |g_n|^2 \right) E \left( |n_{r_n}|^4 \right) \right\}. \tag{11}
\end{align*}
\]

the Gamma function, and \( m \geq 1/2 \) is the Nakagami parameter representing the average fading power. Denote \( \gamma_{s_1, r_n} = P_{s_1} |h_n|^2 / N_0 \), \( \gamma_{s_2, r_n} = P_{s_2} |g_n|^2 / N_0 \), and \( \gamma_{r_n, s_2} = P_{r_n} |g_n|^2 / N_0 \) as the received SNRs from the source nodes \( S_1 \) and \( S_2 \) to the \( n \)-th relay, and from the \( n \)-th relay to the source \( S_2 \), whose PDFs are of the form (10), with Nakagami parameters \( m_{s_1, r_n}, m_{s_2, r_n}, m_{r_n, s_2} \) and expectations \( \bar{\gamma}_{s_1, r_n} = m_{s_1, r_n} P_{s_1} / N_0 \), \( \bar{\gamma}_{s_2, r_n} = m_{s_2, r_n} P_{s_2} / N_0 \), \( \bar{\gamma}_{r_n, s_2} = m_{r_n, s_2} P_{r_n} / N_0 \), respectively. Note that \( m_{s_2, r_n} = m_{r_n, s_2} \) by assuming the channel reciprocity.

As noticed from (9), the end-to-end SNR expression for the case with \( K_n = 1 \) remains complicated for the purpose of expression derivation in closed-form. In order to achieve appropriate analytical expressions, we investigate two asymptotic scenarios.

**Scenario 1:** The noise dominates the interface term, i.e., \( |n_{r_n}| \gg |\sqrt{P_{s_1} h_n x_1} + \sqrt{P_{s_2} g_n x_2}| \). In this case, the system works in the low SNR region. Accordingly, the end-to-end SNR can be expressed as (11), where the even-order moments of the Gaussian-distributed random variable \( |n_{s, r_n}| \) are given by

\[
E \left[ |n_{r_n}|^p \right] = N_0 \frac{2}{p}(p-1)! \quad \text{if } p \text{ is even}, \tag{12}
\]

where \((q)!!\) denotes the double factorial, that is the product of every odd number from \( q \) to 1, if \( q \) is odd. Then, we have

\[
\begin{align*}
E \left[ |n_{r_n}|^2 \right] &= N_0, \\
E \left[ |n_{r_n}|^4 \right] &= 3N_0^2, \\
E \left[ |n_{r_n}|^6 \right] &= 15N_0^3. \tag{13}
\end{align*}
\]

Substituting (13) into (11), the end-to-end SNR via the \( n \)-th relay in the low SNR region can be further expressed as (14).

**Scenario 2:** We consider the case that the system works in the high SNR region. The noise could be neglected compared to the interface term caused by the transmitted signal, i.e., \( |n_{r_n}| \ll |\sqrt{P_{s_1} h_n x_1} + \sqrt{P_{s_2} g_n x_2}| \). In this case, the end-to-end SNR can be expressed as

\[
\gamma_{n,2} = P_{s_1} |a_n(1) h_n|^2 \left\{ E \left[ |a_n(1) n_{r_n} + a_n(3) G_n^2 \right. \right. \\
&\quad \left. \cdot \left( \sqrt{P_{s_1} h_n x_1} + \sqrt{P_{s_2} g_n x_2} \right) \right] \right\}^2 \\
&\quad \cdot \left. \left( \left( \sqrt{P_{s_1} h_n x_1} + \sqrt{P_{s_2} g_n x_2} \right)^2 \right) \right\}^2. \tag{15}
\]

By further performing the approximation, the expression could be further derived as (16), which could be further approximated as

\[
\gamma_{n,2} \approx |a_n(1)|^2 P_{s_1} |h_n|^2 \left( P_{s_1} |h_n|^2 + P_{s_2} |g_n|^2 \right)^2 \\
&\quad \div \left\{ |a_n(1)|^2 N_0 \left( P_{s_1} |h_n|^2 + P_{s_2} |g_n|^2 \right)^2 \\
&\quad + |a_n(3)|^2 P_n \left( \left( \sqrt{P_{s_1} h_n} \right)^6 \right) \right\}. \tag{17}
\]

At the destination node, the maximal ratio combining (MRC) is used to utilise the signals received at the from the source and the \( N \) relays. As per the combination scheme, the end-to-end SNR at the output of the MRC detector in the presence of HPA nonlinearity is given by

\[
\gamma_{tot} = \sum_{i=1}^{N} \gamma_n. \tag{18}
\]

Applying the MGF approach, the average SEP of \( M \)-QAM
\[
\gamma_{n,1} = \frac{|a_n(1)|^2 \gamma_{s_1, r_n} \gamma_{r_n, s_2}}{1 + \gamma_{s_1, r_n} + \gamma_{s_2, r_n} + \gamma_{r_n, s_2} \left\{ |a_n(1)|^2 + 15 |a_n(3) P_n|^2 + 6 \Re \left[ a_n(1)(a_n(3))^* P_n \right] \right\}}. \\
\gamma_{n,2} \approx \frac{P_{s_1} |a_n(1) h_n|^2}{|a_n(1)|^2 N_0 + |a_n(3) G_n^2| E \left\{ \left[ \sqrt{P_{s_1}} h_n x_1 + \sqrt{P_{s_2}} g_n x_2 \right]^6 \right\}} \frac{\left| a_n(1) g_n^2 N_0 \left( P_{s_1} |h_n|^2 + P_{s_2} |g_n|^2 \right)^2 + |a_n(3)|^2 P_n^2 E \left[ \left( \sqrt{P_{s_1}} h_n x_1 + \sqrt{P_{s_2}} g_n x_2 \right)^6 \right] \right|}{|a_n(1) g_n|^2 N_0 \left( P_{s_1} |h_n|^2 + P_{s_2} |g_n|^2 \right)^2 + |a_n(3)|^2 P_n^2 E \left[ \left( \sqrt{P_{s_1}} h_n x_1 + \sqrt{P_{s_2}} g_n x_2 \right)^6 \right]}. 
\]

Fig. 1: Average SEP versus average SNR for different modulation formats.

Fig. 2: Average SEP versus average SNR for different values of HPA coefficients \(\{a_n(2k + 1)\}_{k=0}^{N-1}\).

In the two-way CSI-assisted AF cooperative system with HPA nonlinearity at the relays can be expressed as

\[ P_s = \frac{4 a_{QAM}}{\pi} \int_0^{\pi/2} \prod_{i=1}^N M_{\gamma_n} \left( \frac{-g_{QAM}}{\sin^2 \theta} \right) d\theta \]

\[ - \frac{4 a_{QAM}^2}{\pi} \int_0^{\pi/4} \prod_{i=1}^N M_{\gamma_n} \left( \frac{-g_{QAM}}{\sin^2 \theta} \right) d\theta, \]

where \(a_{QAM} = 1 - \frac{1}{\sqrt{M}}\), \(g_{QAM} = 3/[2 (M - 1)]\), and \(M_Z(s) = E \left[ \exp \left( sZ \right) \right] \) denotes the MGF of random variable. Note that deriving the expression, \(M_{\gamma_n}(s)\), in closed-form is not straightforward. The expression for the average SEP of \(M\)-QAM in the two-way CSI-assisted AF cooperative relaying system in the presence of HPA nonlinearity, over Nakagami-\(m\) fading channels, is obtained using numerical integration method.

IV. NUMERICAL RESULTS

In this section, we demonstrate numerical results to show the effect of nonlinear HPAs at the relays on the performance of half-duplex two-way dual-hop CSI-assisted AF relaying system in Nakagami-\(m\) fading environment, in terms of average SEP. It is assumed that the power at the source nodes are equal to those at the relays, i.e., \(P_{s_1} = P_{s_2} = P_n \) (\(n = 1, 2, ..., N\)). We define the average SNR as \(\bar{\gamma} = \frac{P_n}{N_0}\). Unless otherwise specified, we set the Nakagami parameters at all links to be 2, i.e., \(m_{s_1, r_n} = m_{s_2, r_n} = m_{r_n, s_2} = 2 (n = 1, 2, ..., N)\), and the HPA parameters at all relays to be \(\{a_n(2k + 1)\}_{k=0}^{N-1} = \{1, 0.15\} \) (\(n = 1, 2, ..., N\)).

First, we evaluate the average SEP versus average SNR for 16QAM and 64QAM with \(N = 3\) relays. Fig. 1 shows that there is a huge degradation of the average SEP in the case with HPA nonlinearity, especially in the average SNR region 10dB-25dB, which occurs due to the nonlinear distortion. In this specific range, the average SEP cannot be reduced by increasing the average SNR, where the deconstructive effect of larger SNR, i.e., higher distortion, outperforms the constructive impact. On the other hand, in the SNR region higher than 25dB, the average SEP will decrease along with the enlarged average SNR. Furthermore, as observed from the figure, the asymptotic performance at low and high SNR regions, plotted
average SEP by increasing the number of relays in the case with HPA nonlinearity. However, it does not help a lot, especially in the high SNR region due to the severe distortion caused by HPA nonlinearity. The diversity order cannot be effectively achieved by increasing the number of relays unless appropriate compensation scheme for HPA nonlinearity is applied in the system.

Finally, in Fig. 4, we illustrate the average SEP versus average SNR for different cases of Nakagami parameters in the presence of HPA nonlinearity, with 16-QAM and $N = 3$. We consider three cases for the Nakagami parameters: (1) $m_{k_1} = m_{k_2} = m_{n} = 1$ $(n = 1, 2, ..., N)$; (2) $m_{k_1} = 2, m_{k_2} = m_{n} = 4$ $(n = 1, 2, ..., N)$; (3) $m_{k_1} = 4, m_{k_2} = m_{n} = 2$ $(n = 1, 2, ..., N)$. It can be observed that the average SEP of the two-way relaying system becomes lower in the case with higher values of Nakagami parameters. Moreover, the link from the other source node to the relays plays a more crucial role on the performance at the targeted source node, that is, high values of $m_{k_1}$ bring a better system performance, which is validated by comparing cases (2) and (3).

V. CONCLUSION

In this paper, half-duplex two-way dual-hop CSI-assisted AF relaying systems in the presence of nonlinear HPAs at the relays were studied. We calculated the expression for the end-to-end SNR and derived closed-form expression for two asymptotic cases at low and high SNR regions, respectively. The performance of the considered two-way relaying system was evaluated in terms of average SEP in Nakagami-$m$ fading environment, by making use of the MGF scheme. Numerical results and comparison were also provided and show the effects of the HPA parameters, numbers of relays, QAM order, Nakagami parameters, on the system performance. Ongoing research is to propose an iterative compensation mechanism for HPA nonlinearity in two-way relaying systems.

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