KQQKQQ AND THE KASPAROV-WORLD GAME

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USA, Switzerland and the UK

ABSTRACT

The 1999 Kasparov-World game for the first time enabled anyone to join a team playing against a World Chess Champion via the web. It included a surprise in the opening, complex middle-game strategy and a deep ending. As the game headed for its mysterious finale, the World Team requested a KQQKQQ endgame table and was provided with two by the authors. This paper describes their work, compares the methods used, examines the issues raised and summarises the concepts involved for the benefit of future workers in the endgame field. It also notes the contribution of this endgame to chess itself.

1. INTRODUCTION

The contest between Kasparov and the World began on June 21\(^{st}\), 1999 and was essentially a novel correspondence game played at the regulated pace of one ply per day. The World was a web-enabled group led by GM Danny King as moderator and four talented, young coaches, together with a bulletin board of FIDE-rated and rateable analysts and chess enthusiasts. Moves were voted for by the participating public along democratic principles (Marko and Haworth, 1999). The game itself was remarkable and lived up to the occasion in all three phases; innovative moves led to a dynamic, unbalanced position requiring precise play by different forces.

As early as move 10, it was clear that the game was likely to go into a complex ending, and after move 39, this could still have been KRBBKN or KQPKQPP. Soon it was down to the royal pieces and their foot soldiers. Defending a notorious QP-ending, the World team called for that utopia of perfect information, an Endgame Table (EGT), also referred to as a Database (Van den Herik and Herschberg, 1986) and a Tablebase (Edwards and the Editorial Board, 1995).

The first request, for KQPKQPP, was quickly seen to be unrealistic but given its prolonged fight for a draw, the World Team had shown great restraint in not asking for EGT help much earlier. The next priority was for the KQPKQP EGT which needed at least the KQQKQQ, alias 4Q, if not all fifteen KQxKQy EGTs.

Stiller (1992, 1995 and 1996) had created some 41 pawnless 6-man, White win/no-win EGTs including all KQxKQy endgames except those with \(xy\) as QB, QN, RQ, BQ and NQ. Unfortunately, none survived due to a lack of file space and they were sorely missed by the World Team. Nevertheless, Stiller’s remaining summary information usefully underlined the feasibility of computing 4Q with its maximal depth to a subgame of 88 plies and its many clearly illegal positions and shallow wins. Encouraged by this and ever the optimist, Haworth sent requests for 4Q to Nalimov and Wirth, both known to be leading, active contributors in this field.

Both responded quickly to the moment and to the challenge, agreeing to make any results publicly available. To everyone’s surprise except their own, they produced self-consistent 4Q EGTs within days which, under all tests, confirmed each other and Stiller’s results. Each chose to strength-test their code, producing almost incidentally the tables for KNNKNN, KRRKRR and KBBKBB as well.

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Nalimov optimised *depth to mate* (DTM) and Wirth optimised *depth to force-conversion* (DTC), adding to the degree of independence between the two sets of results. The World Team supported and implemented the authors’ *pro bono publico* principles by publicising the existence of the 4Q EGTs and making them freely available. This was done via the game’s bulletin board, the WWW Computer Chess Club (1999) and a public ftp site (Hyatt, 1999) which soon sported an upgraded version of CRAFTY exploiting 6-man EGTs.

Several on the World Team downloaded both the new CRAFTY engine and the 4Q EGT. Kasparov had done the same and this raised the prospect of the game ending with perfect 4Q play. Certainly his official website showed Black dramatically securing a 4Q draw which was both immaculate and bizarre, Black sacrificing both Queens to force stalemate: see Appendix B4.2. Two 4Q-services were set up on the web (Mobley, 1999; Tamplin, 1999) and these added to the unusually significant contribution that EGTs were already making to the analysis of the game.

The production of 4Q on request was notable in itself and re-opened the 6-man chapter of endgame history. It was also the first time that two authors had worked simultaneously on the same endgame. The event suggested a comparative study to Haworth. The following sections therefore describe the 6-man EGT challenge, the two approaches and the sets of results. The paper includes a survey of the concepts involved to suggest some nomenclature and principles for EGT generation in the future.

## 2. THE 6-MAN ENDGAME DOMAIN

A complete EGT must include the value and depth of all legal positions in its scope. It may also cover unreachable positions whose illegality its generator program has not discovered, see Table 1 which includes all positions cited in this paper. The positions are Gödel-numbered using a 1-1 function \( \text{Index}(\text{Pos}) \) that must have an inverse in \( \text{Pos(Index)} \) if a position is to be easily found from its index.

The next sections introduce the key concepts of *notionally considered, indexed, legal and broken positions*.

### 2.1 Considered Positions

The simplest approach to creating \( \text{Index}(\text{Pos}) \) is to number the chessboard squares 0-63, define an order for the \( n \) men, say \([\text{wK, bK, wQ, ...}]\), list the squares \( \{s_i \mid i \in [0, 63]\} \) of the men for position \( \text{Pos} \) and define \( \text{Index}(\text{Pos}) = 64^\alpha \kappa + \sum 64^\xi s_i \) where \( \kappa = 0 \) or 1 for wtm or btm positions respectively.

In effect, the chessboard gives rise to a *hyperboard* or *n-cube* with edges of length 64 where each chess position occupies its own cell. This indexing scheme sufficed for 5-man EGTs (Thompson, 1986; Edwards *et al.*, 1995). For 6-man tables, there are 64\(^6\) or 68,719,476,736 cells and this number is not conveniently manageable by an EGT generator program accessing a 4 Gigabyte address space, the maximum in a 32-bit architecture.

However, many of these positions are equivalent and/or illegal. The required index range may therefore be reduced by indexing only positions in some standard form and by avoiding as many of the illegal ones as possible. This is done by first ensuring that all indexed positions satisfy condition C as defined below:

- \( C_1 \) legality there is at most one piece on any square
- \( C_{KK} \) legality the two Kings are not next to each other
- \( C_L \) legality \( C_1 \land C_{KK} \)
- \( C_{ad} \) symmetry a specified K (e.g., the K of the side to move, stm) must be on one of files a-d
- \( C_C \) symmetry if Wh. and Bl. forces are equal, the position is replaced by its btm equivalent
- \( C_P \) existence Pawns are present but only on ranks 2-7
- \( C_S \) symmetry the specified K is in a specified a-d octant (here, a1-d1-d4)
- \( C_D \) existence the specified K is on the diagonal of the specified octant (here, a1-d4)
- \( C_{TE} \) symmetry the other K is in the *full-board triangular extension* of the chosen octant (here, if the specified King is on a1-d4, the other King is in a1-h1-h8)
- \( C_S \) symmetry \( C_{ad} \land C_C \land \{ C_P \lor \{ C_S \land (\neg C_D \lor C_{TE}) \} \} \)
- \( C \) composite \( C_L \land C_S \)
### Key 4Q? Position

<table>
<thead>
<tr>
<th>Key</th>
<th>Position</th>
<th>ply</th>
<th>ply</th>
<th>ply</th>
<th>Val</th>
<th>DTM</th>
<th>DTC</th>
<th>DTF</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>yes 7K/q4/8/3q4/8/7Q/1k5Q/8</td>
<td>b</td>
<td>1-0</td>
<td>100</td>
<td>88</td>
<td>88</td>
<td>maxDTM/DTC btm 4Q position</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M2</td>
<td>yes 7K/q4/8/3q4/8/7Q/1k6</td>
<td>w</td>
<td>1-0</td>
<td>99</td>
<td>87</td>
<td>87</td>
<td>maxDTM/DTC wtm 4Q position</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M3</td>
<td>yes 8/4q3/6k1/2Q5/2k5/7q7</td>
<td>w</td>
<td>1-0</td>
<td>99</td>
<td>81</td>
<td>81</td>
<td>maxDTM wtm 4Q position</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M4</td>
<td>no 8/q7/P6k/8/8/8k1</td>
<td>w</td>
<td>1-0</td>
<td>235</td>
<td>213</td>
<td>141</td>
<td>maxDTF wtm QKPKQ position .. QKPa6QK</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M5</td>
<td>no 8/2Q4k8/1g6/3P4/8/7k</td>
<td>w</td>
<td>1-0</td>
<td>247</td>
<td>227</td>
<td>?</td>
<td>maxDTM/DTC wtm QKPKQ position .. QKP(d3)QK</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Legal Positions

| L1  | yes 3Q/q1k5/8/2K5/8/8/3qq3/8 | b | 1-0 | 0 | 0 | 0 | Prior move: 1.b8=Q# ... double check by Queens is possible |
| L2  | yes QQ1k4/6qq/3K4/8/8/8 | b | 1-0 | 0 | 0 | 0 | Prior move: 1.axb8=Q# or 1.Qxb8# |

### Illegal Positions

| I1  | no 8/8/8/8/8/8/P1K5/k7 | w | 1-0 | 23 | 11 | 1 | KPK: no anterior move (Haworth and Velliste, 1998) |
| I2  | no 8/8/8/8/8/2K5/1P6/8 | w | 1-0 | 15 | 1 | 1 | KPK: 1. ... Kc3 was illegal (Haworth and Velliste, 1998) |

### Both Kings on the long diagonal

| S1  | yes 7q6/1q8/3Q3Q/2K5/8/8k7 | w | 1-0 | 5 | 2 | 2 | S1 = 7q6/1q8/3Q3Q/2K5/8/8k7; n.b. 1. Kh3+ wins |
| S2  | yes 7q6/1q8/3Q3Q/2K5/8/8k7 | w | 1-0 | 5 | 2 | 2 | S2 = 7q6/1q8/3Q3Q/2K5/8/8k7 |

### The Kasparov-World Game and Analysis

| GK1 | no 7Q/q1p6/3p2k1/6P1/8/8/8/8 | w | 1-0 | 3 | 1 | 1 | Impossible double check by two Knights (cf. L1 above) |
| GK3 | no 8/Q1k4/8/8/8/8/8 | w | 1-0 | 158 | ? | ? | After 58. g6: Bl. played 58. ... Qe4? [Qf5'] |

### Weaknesses of pure F and M strategies

| RR  | no 8/8/8/8/RK/6k1K7/8 | w | 1-0 | 3 | 2 | 2 | 3M-M: Rh2#; Ka1 2, Rgl#: 3.M-M: 1. Qa3 Kxa3 & mt in 17 ply |
| BN1 | no 8/8/8/8/P1P6/8/8/8 | w | 1-0 | 5 | 2 | 2 | 3M-M: 1. Qb3#; Ka1 2, Be5#; Ne3 3. Bxc3# |
| BN2 | no 8/8/8/8/P1p6/8/8/8 | w | 1-0 | 127 | 103 | 103 | From BN1, 1.M-M: 1. Qa2#; Kxa2 [BN2] |
| NP1 | no 8/8/8/8/8/8/8/8/8 | w | 1-0 | 229 | 228 | ? | maxDTM KNKPK(b) pos. (Dekker, 1989): M = C = MC = CM |
| NP2 | no 8/8/8/8/8/8/8/8/8 | w | 1-0 | 181 | 180 | ? | After 24... h4: h3 must be forced before m75 |
| NP3 | no 8/8/8/7p1N1K4/7N/8/8k3 | w | 1-0 | 105 | 104 | ? | After 62. Ke1: Wh. needs Strat F; C = M = M = M = M = M = M = M = M = M |
| NP4 | no 8/8/8/8/8/8/8/8/8 | b | 1-0 | 104 | 103 | 35 | After 63. Nd2: 63... KQf2 forces b3 to at least m80 ... draw |

### Depth-illustrating positions

| NP5 | no 8/8/8/8/8/8/8/8/8 | b | 1-0 | 2 | 1 | 1 | Bl. starts & finishes this; "Conversion in 0" in Wh. moves |

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4 DTF is Distance to FIDE event where FIDE event is defined here as P-push, capture or mate.
The set of legal K-K positions in Pawnless- or Pawn-endgames can be listed and counted as follows:

\[
N_K = |\{ K-K \text{ positions } | C \}| = 33 + 3 \times (30 + 58 + 55) = 462
\]

\[
N_{KP} = |\{ K-K \text{ positions } | C_L \land C_{ad} \}| = 4 \times 60 + 24 \times 58 + 36 \times 55 = 1806 \approx 3.91 \times N_K
\]

The explicit management of these positions in a table generator ensures that \( C_{KK} \land C_{ad} \) is satisfied and also replaces the index’s n-cube with an n-space of less volume and lower dimensionality. The index ranges required for these illustrative 6-man endings can be calculated in the simplest way as:

\[
N_a = N_K \times 62 \times 61 \times 60 \times 59 \times 2 = |\{ \text{wtm/btm KQRKBN positions satisfying } C \}| = 12,370,770,720
\]

\[
N_b = N_a / 2 = |\{ \text{btm KQRKQR positions satisfying } C \}| = 6,185,385,360
\]

\[
N_{aP} = N_{KP} \times 48 \times 61 \times 60 \times 59 \times 2 = |\{ \text{wtm/btm KQRKQP positions satisfying } C \}| = 37,438,813,440
\]

\[
N_{bP} = N_{aP} / 2 = |\{ \text{btm KQPKQP positions satisfying } C \}| = 18,134,425,260
\]

\( N_b \) and \( N_{bP} \) are only 9.00% and 26.39% of \( 64^6 \) respectively, a considerable improvement. The next section introduces three ideas which further reduce the size of the index ranges to be managed.

### 2.2 Indexed positions

Further rules which may be exploited to ease the task of table generation are:

- \( C_{LM} \) symmetry: \( L \) like men of a side are indexed as a set, reducing the index range by a factor of \( L! \)
- \( C_{UC} \) legality: a side-to-move man, Q/R/B/N/P, may not give an unblockable check to the \( sntm \) K

Nalimov and Wirth, but not Stiller, used \( C_{LM} \) to reduce the number of distinct 4Q positions to 1,546,346,340, just 12.5% of \( N_a \). The function \( \text{Index}(Pos) \) must calculate the sequence number of a specific arrangement of the \( L \) like men and \( \text{Pos}(\text{Index}) \) requires a function to calculate the specific arrangement of the \( L \) like men for a specific index.

Suppose now that btm positions are being generated and the Kings have been placed. Depending on the position of the white King, condition \( C_{UC} \) constrains a black Queen from being on 3, 5 or 8 squares. Similar calculations can be done for the R, B, N and P: those for N and P also involve the position of the black King. Condition \( C_{UC} \) in effect replaces the one n-space index by a number of n-space sub-indexes, each tailored to Black’s force profile and the positions of the white King and sometimes the black King. The same argument holds of course for wtm positions. It is then necessary to list the starting point and dimensions of each sub-index n-space in a metatable.

It is always possible to partition an endgame according to the positions of one or more men (Lake, Schaeffer and Lu, 1994) and particularly convenient to do so in P-endings such as KQPKQP which was significant to the outcome of this game. Positions with the same pawn formation may be considered as a sub-endgame of a P-endgame and their part of the EGT can be managed as a logical block split into two blocks by condition \( C_{ad} \). This has a runtime advantage for chess engines; as Steinitz said, Pawns cannot move backwards so certain positions will be unreachable. References to the EGT will be clustered in the remaining subgames which are most relevant.

### 2.3 Legal Positions

Determining whether a position is legal or not can be difficult (Lippold, 1997) as the following positions from Table 1 illustrate. Double check by two Queens is possible and L1-L2 are in fact legal. Positions I1-I4 have no prior move, I4 being merely L1 shifted down one row. Lippold’s I5 has no move before the previous 1. ... b1=B. I6 has the impossible promoted force of a second black-square Bishop, a fact better proved than discovered by a retro-search that can only be contemplated. I7-I8 have the side not to move, \( sntm \), in check.

EGT-generating programs usually do not root out all illegal positions and there is no need for them to do so. Table generators will spot ‘side not to move in check’ but I1-I6 are typically regarded as being legal. The EGT’s prime purpose is to provide the value and depth of all legal positions. Derived statistics will be affected by illegal positions being treated as legal but this is a secondary consideration.

\[\text{For further simplicity, these calculations ignore the fact that the coding of any } \text{en passant}, \text{if not castling, rights is now customarily included (Heinz, 1999; Wirth and Nievergelt, 1999; Nalimov and Heinz, 2000).}\]
2.4 Broken Positions

As discussed above, an EGT generator cannot typically classify positions as legal and illegal. It can only mark some positions as seen to be illegal and the term broken was therefore coined (Edwards et al., 1995) for these. In addition, the broken classification allows a generator to remove from consideration those positions which are redundant or do not satisfy condition C above.

A position with both Kings on the a1-h8 diagonal can satisfy condition C before and after being reflected in that diagonal: the EGT may therefore represent two equivalent positions. Positions S1 to S4 of Table 1 illustrate that the best way to remove one is to mark the higher-indexed position as broken: there is no simpler rule in terms of a sequence of nominated pieces’ positions. This does not however remove the requirement on the generator to standardise any such positions to the chosen form (Van den Herik and Herschberg, 1985) if one has been defined.

With one version of such positions marked as broken, each equivalence class of positions is represented just once. EGT statistics then guarantee to represent distinct positions and the subsequent work of generating the EGT is reduced by some 4.35% (~21/483).

3. SUMMARY OF ALGORITHMS

This section documents the common structure of Nalimov’s and Wirth’s algorithms and the next sections detail the differences at points marked * between them. The common principle is that deeper wins are identified from the shallower wins of their successors. Val[i] becomes the endgame table, holding the value and depth of Position(i). The function Evaluate sets ChangeFlag to true if the depth of any position is changed in array Val.

\{initialise ChangeFlag and endgame table Val\} ChangeFlag ← true;
   determine * index range R required for wtm and btm positions satisfying C ∩ C_{LM}:
   for i ∈ [1, R) do Val[i] ← broken end_for;
   for each Position(i) do if Position(i) is not broken then Val[i] ← InitialValue * end_if end_for;
   \{seed Val with the value and depth of terminal won/lost positions in the endgame chosen\}
   for each unbroken Position(i) do
      if side-to-move is mated then Val[i] ← 0 end_if;
      if side-to-move can convert to win then Val[i] ← Depth1 * end_if;
      if side-to-move must convert to lose then Val[i] ← Depth2 * end_if end_for;
   \{iterative search\}
   while ChangeFlag = true do ChangeFlag ← false;
      for each necessary * i do Evaluate (Val[i], \{Val[j] | Position(j) is a neighbour of Position(i)\}, ChangeFlag);
   end_for end_while;
   \{check self-consistency of endgame table \Val\}
   for each unbroken Position(i) do
      Compatibility-Check(Val[i], \{Val[j] | Position(j) is a successor of Position(i)\}) end_for;
   \{finish - endgame table Val completed\}

3.1 Nalimov’s algorithm

Nalimov introduced condition C_{UC} to endgame indexing, namely the avoidance of unblockable checks by the side to move, stm, as described in section 2.2. Such checks cannot be blocked by placing further men on the board. Nalimov and Heinz (2000) describe the general use of condition C_{UC} in more detail. Here, the constraints on the Queens save some 19.98% of the index space because:

\[ R = \{33 \times 59 \times 58 + [58 \times 57 \times 56 + (30 + 55) \times 54 \times 53] \times 3\} \times (60 \times 59) / (2! \times 2!) = 1,237,357,440 \]

Unless the side-not-to-move is in check, Val[i] is initialised to draw. Both versions of unbroken positions with both Kings on a1-h8 are retained.
The depth of a position about to be converted and won, $Depth_1$, is set to the converted position’s DTM + 1. The depth of a position which must be converted but is lost, $Depth_2$, is set to the converted position’s DTM.

Nalimov’s iterative process is a sequence of forward searches. It is necessary to visit each unbroken Position(i) in each search and $Evaluate$ infers or re-infers its value and depth from those of its neighbours, i.e., its successors. A position is rated an stm win if any successor has been designated a loss for the stntm. Similarly, a position is rated an stm loss if all stntm successors are wins for the stntm. Depths in moves are eventually ascribed to any position which is a win for White or Black. The search making no value/depth changes terminates the process.

3.2 Wirth’s algorithm

Wirth does not use $C_{19}$ but calculates $R$ more simply as $462 \times 62!/(58! \times 4! \times 2! \times 2!)$. The higher-indexed version of a position with both Kings on the a1-h8 diagonal is marked as broken as are positions with the stntm K in check. The $InitialValue$ of Val[i] is the number of successor positions in the target endgame profile, here KQQKQQ. Positions which can be converted to winning positions or must be converted to lost positions have Depth set to 1 ply.

Wirth used Gasser’s (1995, 1996) domain-independent RETROENGINE which executes an iterative process of retro-searches. This minimises the work involved by two techniques (cf. Thompson, 1986). First, the necessary positions are only those whose information changed in the last iteration. Second, the algorithm counts down the number of successors of a position which have not yet been proved to lose. The neighbours of a position here are its predecessors and the function $Evaluate$ changes their Val[i].

Positions which are stm losses back up, via an unmove generator which cannot introduce new men by uncapture, to a set of stntm wins. However, stm wins merely reduce their predecessors’ number of unresolved successors by one, and only if this number is zero is the predecessor position marked as lost in n plies. The losing side is of course trying to avoid mate rather than helping to get itself mated.

$Evaluate$ assigns any positions resolved as wins or losses to the set of necessary positions for the next iteration.

4. The Results

The summary figures are in Table 2; the per-level detail is in Table 3. Derived %-statistics differ slightly because of the different treatments of indexed and broken positions.

<table>
<thead>
<tr>
<th>Type of KQQKQQ Position</th>
<th>Nalimov: DTM</th>
<th>Wirth: DTC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%</td>
<td># of positions</td>
</tr>
<tr>
<td>Notionally considered</td>
<td>NC</td>
<td>1,546,346,340</td>
</tr>
<tr>
<td>Broken: not included in the index</td>
<td>BA</td>
<td>BA/NC</td>
</tr>
<tr>
<td>Indexed</td>
<td>I</td>
<td>1,237,357,440</td>
</tr>
<tr>
<td>Wins by White (to move)</td>
<td>W</td>
<td>417,439,889</td>
</tr>
<tr>
<td>Draws</td>
<td>D</td>
<td>129,060,182</td>
</tr>
<tr>
<td>Losses by White</td>
<td>L</td>
<td>136,739,434</td>
</tr>
<tr>
<td>Broken: found after indexed</td>
<td>BB</td>
<td>BB/I</td>
</tr>
<tr>
<td>Unbroken: W + D = L</td>
<td>U</td>
<td>683,239,505</td>
</tr>
<tr>
<td>Total broken: BA + BB</td>
<td>TB</td>
<td>863,106,835</td>
</tr>
<tr>
<td>Zugzwangs</td>
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</tr>
<tr>
<td>Maximal White wins</td>
<td>wtm</td>
<td>2</td>
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<tr>
<td></td>
<td>btm</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Summary of comparative KQQKQQ statistics.
### Table 3: Detailed KQQKQQ statistics - White wins in \( n \) plies.
It is said (Beasley and Whitworth, 1996, p. 180) that “more complicated positions may be evaluated by ignoring pairs of like pieces”. In fact KQKQ, see Table 5 in Appendix A, has only 41.74% wins and 0.45% losses for the first player. The figures and comparison are affected by shallow wins which are the inevitable result of active play continuing from the previous phase. If pieces are not actually left en prise, there is typically the opportunity for a fork, skewer, discovered check or other equally sharp, winning tactic.

In Table 3, the column $H$ shows the percentage of positions which are in fact won beyond the horizon of a search-engine armed only with 5-man EGTs. The engine may have a full-width search capability of $h$ plies but may only be able to search $q$ plies into the 4Q domain. For example, 37.40% of positions unresolved by a 5-ply search into 4Q are won or lost. This is of course different from the density of wins/losses of greater than $q$ plies in the set of all unbroken positions, or the percentage of wins greater than 5 plies.

Nalimov’s one maxDTM btm loss in Table 2 equates to the ‘2’ 100-ply losses in Table 3 which mirror each other.

5. **THE COMPUTATIONS**

Lewis Stiller’s innovative computation mapped the structure of the EG position set to that of the SIMD (Single Instruction, Multiple Data) Connection Machines, CM-2 and CM-200 (Hillis, 1985). These each had 64k processors running in step at 7MHz and 10MHz respectively. Nalimov created 4Q on a 2GB, 500MHz server, working completely ‘in store’ and avoiding all disc thrashing. The EGT was completed and checked for self-consistency successfully in two days. Wirth used a 1GB, 450MHz G3 PowerMac and sustained a total of approximately 100GB of disc traffic; the computation and integrity-check took five days.

These were colossal number-crunching feats by any standards but computations of this complexity are open to systematic or sporadic errors affecting hardware or software. Cosmic rays, power and electromechanical faults, flawed systems software and human error have all played their part in the past (Gasser, 1995; Wirth and Nievergelt, 1999) although Nalimov reports that no hardware errors have ever occurred on the servers he has used. As the results are not self-evidently correct in themselves, an independent integrity test to underpin blind faith in correctness is vital.

Both EGT authors therefore confirmed that the value and depth of each position was consistent with that of its successors. This was done by a parallelisable forward pass of the EGTs checking, for each position, that:

- a position won by the stm in $p$ plies is succeeded by positions lost in $\leq p-1$ plies,
- a position lost by the stm in $p$ plies is succeeded only by positions won in $\geq p-1$ plies.

This test ensures that any unmove generator is the exact inverse, within the target profile, of the move generator, and that no random errors have occurred. It cannot check any software which is common to itself and the generation process. Both authors also first strength-tested their ‘5-man’ codes in the 6-man domain on the EGs KNNKNN and KRRKRR. Nalimov’s code had also produced DTM EGTs which agreed with previous DTM EGTs (Edwards et al., 1995). Each reproduction of a previous Stiller result suggested that all three EGTs were correct and aligned.

6. **ENDGAME SERVICES**

Nalimov provides practical tables for chess engines and these are now widely used by many top programs including 11 of the 30 WCCC ’99 participants (Beal, 1999; Feist, 1999). Code, data and essential statistics are all available via Hyatt’s public ftp site. Two www services (Mobley, 1999; Tamplin, 1999) were set up to help those users who did not wish to actually download the EGTs themselves.

Tamplin’s established service provides all available moves for a position, indicating whether they are optimal, value-preserving or whether they lose a half or full point. Mobley’s service, now discontinued, allowed one piece to be in free position and returned the set of matching positions in the EGT with their values. Wirth distributed his EGT-access software and 4Q EGT to Haworth and Marko. It provided reassuring second-sourcing of Nalimov’s position evaluations.
7. OBSERVATIONS ON EGT GENERATION

The approaches of Nalimov, Wirth and others provide concepts and guidelines for future work.

<table>
<thead>
<tr>
<th>Goal Description</th>
<th>GV</th>
<th>GF</th>
<th>GC</th>
<th>GM</th>
<th>GR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimax strategy</td>
<td>V</td>
<td>F</td>
<td>C</td>
<td>M</td>
<td>R</td>
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<td>DTF</td>
<td>DTC</td>
<td>DTM</td>
<td>DTR = DTF</td>
</tr>
<tr>
<td>Position depth</td>
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<td>dr ply</td>
<td>dl ply</td>
<td>dn ply</td>
<td>dj ply</td>
</tr>
</tbody>
</table>

| Endgame Table    | EV | EF  | EC | EM  | ER |

Example EG tables by:
- Komissarchik & Futer (1974) none KQPKQ none none none
- Arlazarov & Futer (1979) none none KRPKR none none
- Thompson (1986, pp. 133/5) none none KxPKx, x = Q/R KQKxx, x = B/N ... KQNQ ... 3- and 4-man
- KT: CD & Tamplin (1999) none none 5-man 3- & 4-man 3- and 4-man
- Lake et al. (1994) for checkers 2- to 8-man n/a n/a n/a n/a
- Edwards (1995) none none none any done 3- and 4-man
- Nalimov (2000) none none none 3-, 4- & 5-man 3- and 4-man
- Nalimov (2000) none none none 10 KxKyy none
- Wirth (1999) none none KPPKP, KQKQQQ 3- & 4-man 3- and 4-man

Table 4: Goal-oriented strategies, metrics, depths and EG tables.

The authors chose different metrics in DTC and DTM. Both have their advantages and supporters so it is perhaps not possible to focus on one metric for second-sourcing purposes. Note, e.g., from positions M1-M5 of Table 1, that the choice of metric affects the set of maximum-depth positions. Table 4 lists the various goals that might in general be chosen and reviews the endgame work done to date. It discriminates between the related concepts of Goal, Strategy, Metric, Position Depth and Endgame Table. Its notation facilitates an examination of strategy-specific play; in this paper, for example, M-F indicates White using strategy M, Black using strategy F.

As Nalimov produces tables to support play over the board, the issue of the FIDE 50-move rule (cf. Dekker, Van den Herik and Herschberg, 1989) has to be discussed. The positions M4, BN1-2 and NP1-4 of Table 1 are salutary warnings that strategies M, C and F alone cannot win all positions that can be won in the context of the rule.

Because KQPKQQQ is won in exactly 50 moves, it is not necessary to consider here the production of tables EF and ER. Also because sufficient memory was available, it was not necessary to produce EV tables to support goal GV for KQPKQQQ although there is an argument that such tables are useful for chess engines playing in real time.

Both Nalimov’s and Wirth’s EG tables provide the value - win, draw or loss - and depth of both wtm and btm positions. This is important for two reasons. First, chess engines should not have to search forward one ply from btm positions (Heinz, 1999). Second, although Black is traditionally the weaker side, 6-man endings are showing a higher percentage of wins for Black than previous endings.

If an EM table indicates “side to move wins in m moves”, it is possible to infer that this is 2m-1 plies, assuming the convention is being followed of quoting depth in the moves of the side-to-move. However, if table EC analogously indicates “side to move wins after conversion in m moves”, it is not possible to infer whether this is 2m-1 or 2m plies as position NP5 indicates. The loser may be forced to or wish to make the conversion (cf. Thompson, 1990).

Nalimov’s use of advanced index schemes and compression techniques contribute to the use of his tables in actual play. Work currently in progress, including that on KQPKQP (Karrer, 1999), shows that there are further ways of optimising the runtime performance of these tables. Index ranges may in some cases be reduced further and there are major gains to be had by partitioning EG tables for P-endgames. This means that although P-endgames cannot
be produced definitively before the pawnless endgames have been done, their tables need not be the largest that chess engines have to manage in real time.

The plethora of EG tables now available suggests that standards should perhaps be defined for the provision of these results to chess engines and humans both for play and analysis. A stable, high-level interface between chess engines and endgame services is desirable, the engine merely presenting the position and a definition of the information required. Behind the interface, the endgame service would find the EG table or subtable of the appropriate type, manage the logistics of referencing these tables efficiently and minimise memory requirements.

Theorists also require access to endgame tables, preferably those not moderated by the 50-move rule, and to the statistics about them. Statistics can be published via the web in a way which facilitates the further analysis of the data by the end user.

8. A CHESS PERSPECTIVE

The creation of the 4Q EGT has had an effect on EGT progress and on the Kasparov-World game - and will have an effect on the game, art and science of chess in the future.

8.1 4Q and EGT progress

Stiller (1992, 1995, 1996) was the first to produce EGTs for six fully mobile men and it will clearly take some time before his 41 EGTs are recreated. Both Nalimov and Wirth tested their codes on the somewhat more easily generated KNNKNN and KRRKRR before attempting 4Q: both have lower position fan out and smaller maxDTM than 4Q. With the code proved, Nalimov went on to produce KBBKBB and all six KxxKyy (x ≠ y; x, y ≠ P) even though the latter feature twice as many positions.

Gasser’s RETROENGINE, later developed by Wirth and Lincke, has solved Merrils, created Awari endgame tables to 28 stones, found the deepest 15-puzzle positions and computed the chess endings KPPKP, 4Q and KRRKBN. It is now working on KQRKQB and will no doubt be exercised on other chess endings.

With 4Q available, the World Team turned to the production of KQQKQP≈ and KQPKQP≈ tables (Karrer, 1999), the ‘≈’ denoting the use of the simplifying but approximating no underpromotions heuristic. In the process, they considered the EV table but instead used endgame partitioning (Lake et al., 1994). Following Kasparov’s win (Marko and Haworth, 1999), this work is helping to evaluate the game’s last moves under the P=Q assumption.

8.2 4Q impact on the Kasparov-World game analysis

The Kasparov-World game was extraordinary from many points of view, not least that of the endgame expert. The very fact that EGTs played a significant part made it a rare event. A survey of Fatbase (Monkman, 1999) shows that only 164 games have arrived at any of the 3-3-man endgames with EGTs to date though many more would do so in background analysis. At one point, the World Team entertained the prospect of Kasparov having to win KQPKQP and KQQKQQ endgames against infallible play.

Table 1 shows three key endgame positions, GK1-3, and it is notable that GK1 without the two black Pawns is a draw. Black indeed sought to lose its Pawns safely and at one time over 120 lines of the World Team’s analysis were simultaneously rated theoretical draw by the KQPKQ EGT. The 4Q, KQPKQ and KQKPP EGTs all helped to establish 52. ... Kc1 as best but, critically, this move did not win the vote. Around move 57, Ken Regan researched the safe KQPKQ positions of the game and discovered position AN1 - a 66-move loss to avoid. The 4Q EGT saved the World Team’s computers from evaluating search trees with a potential branching factor of over 50. Peter Karrer reported after an experiment that the 4Q EGT was cutting CRAFTY’s analysis time by 25%.

The preliminary analysis of the 4Q domain did not recommend a 4Q ending as a safe haven for Black. The fact that a draw might turn into a win for White merely by moving one piece one square made 4Q look like a particular dangerous minefield until it was explained that this is true for all endgames. Then the odds of a draw were examined,
assuming Black dropped the game into a ‘random’ wtm 4Q position. The densities of first player wins, 61% with the random assumption and 76% assuming also that the second player cannot win, were not selling points.

Although Black would typically have had some freedom to improve the position before its P-promotion with checking moves, there was a danger that White could eliminate this freedom by some zwischenzug checks before its own P-promotion. Kasparov’s views on this theme would be interesting. Since 4Q draws often involved a forced Q-exchange, the World Team aimed to force such a Q-exchange even before 4Q was reached.

For the future, it is inevitable that endgame tables will attract at least computer play towards guaranteed wins when these are identified by forward search. This phenomenon occurred in WCCC ’99 (Feist, 1999, p. 152) when SHREDDER found and selected a 31-move win in KBBKN.

### 8.3 Mining KQQKQQ

Any new EGT should be mined for higher grade knowledge, both for its contribution to chess lore and for its verdict on past games, theory, problems and studies. Hopefully, some principles of good play (Haworth and Velliste, 1998) will be created or refined and some positions which surprise and delight (Wirth and Nievergelt, 1999) will emerge. A comparison of computer play with and without the EGT would indicate to what extent computers need the perfect information now available.

Only six games featuring 4Q have been recorded. Three were quick wins and two were unavoidable draws. In the remaining game, Black converted a drawn position to a 14-move win which White missed by accepting a draw on the next move, see Appendix B4.1. There are just two studies featuring 4Q in the main line, see Appendix B5. Of these, one (Elkies, 1993) was inspired by Stiller’s discovery of the zugzwang position Z3 in Appendix B2.

The maximal depth positions have been identified and played out as is customary, even if they are in one sense at an extreme of and possibly not typical of the endgame domain as a whole. Appendix B1 lists the one btm and two wtm maxDTM White wins. Without perfect information, the win from position M3 with 16 ‘unique winning moves’ for White would hang by a more slender thread than that from position M2. Both show the extraordinary ability of the two white Queens to combine as a team, ignoring even Black’s Q-interpositions. This play currently looks like Michie’s Martian Chess but will become more meaningful, especially in the shallower won positions.

No searches have been done to find maximal problems or studies (Lindner, 1991) satisfying various criteria but it is worth noting that retro-search, and RETROENGINE in particular, could be exploited to perform this work.

The set of mutual zugzwangs discovered by Stiller has been independently rediscovered, see Appendix B2. All are wtm draws and btm wins for White. They affect the optimal play of adjacent positions: it is an open question as to what their basins of attraction are, i.e., the four sets of positions from which optimal play in some metric necessarily reaches, can be forced by White or Black to reach, or may reach that zugzwang. Denoting these basins in a natural way by $B_{\text{min}}$, $B_{w}$, $B_{b}$ and $B_{\text{max}}$, respectively, they satisfy the set relations $\emptyset \subseteq B_{\text{min}} \subseteq B_{x} \subseteq B_{\text{max}}$, $x = w$ or $b$.

The 40 or so successive and precise Q-checks of the maximal mates raise the question of how common voluntary quiet moves are in forced wins: these are prized generally in the art of chess and seem particularly rare here. Mutual zugzwangs are reached of course with non-checking moves. The eventual winner typically rides out a checking sequence after a forced Q-sacrifice by the other side but these moves are neither voluntary or in 4Q. After Z7, White maintains the Royal Battery with the quiet 1. Qf2, the only winning move.

### 9. SUMMARY

The Kasparov-World game of 1999 was a new mode of play and a new computer contribution to the world of chess. It enabled a worldwide group on the web to work together to great effect on a shared problem against fixed, short-term deadlines. It demonstrated the depth of analysis achievable by the synergy of carbon and silicon intelligence in ‘correspondence’ situations, the value of perfect information in EGTs, and the current feasibility of attacking 6-man endgames, even ‘on demand’.
Inspired by this event, the first two authors produced 4Q EGTs in two independent and entirely successful initiatives. Between them, they highlighted the range of decisions to be made while working in the EGT arena. This paper surveyed their approaches and results, deriving some guidelines for future workers in this field.

10. ACKNOWLEDGEMENTS

First, we recognise the pioneering achievements of previous EGT workers, particularly Ken Thompson, Lewis Stiller and Steve Edwards. We also recognise First USA’s and Microsoft’s innovation in sponsoring and hosting the Kasparov-World game and the combined talents of the World Team - moderator, coaches, analysts and participants. Finally, we applaud Garry Kasparov for taking on yet another form of AI, the Augmented Intelligence of the World Team backed by its computing resources.

Many people contributed to the work and the material in this paper. Lewis Stiller was most helpful in revisiting his 6-man results, both published and unpublished. Noam Elkies assisted with that history and with the analysis of Appendix B. Peter Marko, an outstanding volunteer ringmaster on the game’s bulletin board, co-instigated the production of the 4Q EGT and the 4Q-services on the web. Robert Hyatt hosted Nalimov’s 4Q EGT on his ftp server and promptly upgraded CRAFTY to use it. Peter Karrer provided software for a 4Q service hosted by Carter Mobley. John Tamplin extended his established EGT service and investigated several details here. Francis Monkman scanned Fatbase for 6-man endings, Noam Elkies and Harold van der Heijden contributed on ‘4Q’ endgame studies and Stefan Meyer-Kahlen advised on the recent SHREDDER-REBEL game at WCCC ’99.

Jürg Nievergelt and Thomas Lincke assisted Christoph Wirth at ETH, Zürich. The editor and referees of this journal contributed as ever with their constructive and encouraging comments.

11. REFERENCES


Karrer, P. (1999). Results from KQP(g5+)&KQP(d6)-. Private communication.


**APPENDIX A: KQKQ STATISTICS**

<table>
<thead>
<tr>
<th>Nalimov’s DTM results</th>
<th>Wirth’s DTM results</th>
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<tr>
<td><strong>White wins in n ply</strong></td>
<td><strong>White wins in n ply</strong></td>
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<td>Wh. to move</td>
<td>H%</td>
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<td>25</td>
<td>2</td>
</tr>
<tr>
<td>27</td>
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</tr>
</tbody>
</table>

| Wins | 478,238 | 41.75 | W, W/U | 5,254 |
| Draws | 662,096 | 57.80 | D, D/U |
| Losses | 5,254 | 46.41 | L, L/U |
| Unbroken | 1,145,588 | 73.26 | U, U/I |
| Broken | 418,147 | 26.74 | BB/I |
| Total broken | 601,696 | BA + BB |
| Indexed | 1,563,735 | BA + BB |

<table>
<thead>
<tr>
<th>Wirth’s DTM results</th>
<th><strong>White wins in n ply</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wh. to move</td>
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<td>25</td>
<td>2</td>
</tr>
<tr>
<td>27</td>
<td>0</td>
</tr>
</tbody>
</table>

| Wins | 467,214 | 41.74 | W, W/U | 5,091 |
| Draws | 646,911 | 57.80 | D, D/U |
| Losses | 5,091 | 0.45 | L, L/U |
| Unbroken | 1,119,216 | 64.05 | U, U/I |
| Broken | 628,068 | 35.95 | BB/I |
| Total broken | 628,068 | BA + BB |
| Indexed | 1,747,284 | 1 |

*Table 5: Detailed KQKQ statistics.*
APPENDIX B: EXAMPLE KQQKQQ PLAY

Some notation has been introduced to indicate properties of the lattice of value-preserving moves:

' for only optimal move, " for only value-preserving move, ° for only legal move,

n for one of n unlisted equi-optimal moves, [...] for an equi-optimal move list and {...} for comments.

These additions complement established Chess Informant notation and are hopefully discrete enough not to reduce the readability of the moves themselves.

B1 The maxDTM and maxDTC wins for White

B1.1 Position M1: White mates in 100 plies. Stiller’s optimal DTC line coincides with this chosen ‘M-M’ DTM-minimaxing line until they diverge with Black’s 39th move.


In the 40-move 4Q phase proper here, White has only one winning move 7 times and a two-way choice of optimals 3 times. Black has a two-way choice of optimals 3 times and interposes a Queen 4 times.

After White’s m39, Stiller’s DTC line continued:


B1.2 Position M3: White mates in 99 plies. Again, this is an ‘M-M’ line with play minimaxing DTM:

In the 42-move 4Q phase proper here, White has only one winning move 16 times and a two-way choice of optimals 3 times. Black has one legal move twice, a two-way choice of optimals once and interposes a Queen 14 times.


A DTC line for position M3 branches off after 29. Qcc5+ with 29. ... Kg7’ 30. Qa7+ Q8e7’ 31. Qg1+ Kf6’ 32. Qb6+ Kf7’ 33. Qh6+ Kf7’ 34. Qf2+ Q6f5’ 35. Qh7+ Kf7’ 36. Qb6+ Kg5’ 37. Qhh6+ [Qd8+] Kg4° 38. Qg1’ Kf3’ 39. Qhh1+ [Qgh1+] Kf4’ 40. Qhh2+ Kf3°. A 4Q mate.

B2 Eight Mutual Zugzwangs

Both sides are optimising DTM; on balance, Black sacrifices a Queen earlier than it would with strategy C.

Z1, btm: 1. ... Qh1’ 2. Qg7” Qb8+’ 3. Kxb8” {KQQKQ} Qh2+’ 4. Ka8 [Kc8, Qgg3, Qag3] Qg2+’ 5. Qb7+’ {not 4.Qxg2?=} Qxb7+’ 6. Kxb7° Kc2 7. Qe3’ {and mate on move 13}.

Z2, btm: 1. ... Qe3+’ 2. Kxe3° {KQQKQ} Qg1+’ 3. Kd2’ Qg5+’ 4. Qf6’ Qg2+’ 5. Ke3’ Qh3+ [Qg1+] 6. Qf3 [Kf2] Qe6+ [Qfx3+, Qh6+] 7. Kf2’ Qb6+’ 8. Qf1’ Qc5 [Qa6+, Qb4, Qb5+, Qe3, Qf6, Qg1+] 9. Qab3+ [Qb3+, Qe2+, Qg2+] Kf1’ 10. Qa8+ [Qfd1+, Qf6+] Qa3 [Qa5, Qa7] 11. Qaxa3#.

Z3, btm: 1. ... Qaa6’ 2. Qh1+” Qafl’ [Qff1] 3. Q1xf1+ [Q2xf1+] {KQQKQ} Qxf1+° 4. Qxf1+” Kb2’ 5. Qd3° Ka1 [Kc2, Kc1] 6. Qd2” {and mate on move 12}.


Z5, btm: 1. ... Qe1’ 2. Qb3+’ Ke1’ 3. Qc3+’ Kf1’ 4. Qh1+” Qel1’ 5. Qc1+” Qe1’ 6. Qxe1’ {KQQKQ} Kxe1° 7. Qxg1” Kd2 [Ke2] 8. Qg3” Ke2’ 9. Kc2 [Kc1, Qf4] {and mate on move 13}.


B3 Chess Game: Kasparov-World

The ! and ? comments are from the World Team’s commentary (Krush and Regan, 1999). A QPKQ+ table not recognising Pawn-underpromotion (Karrer, 1999) indicates that, after moves 55. Qxb4, 58. ... Qe4 and 60. ... Qc2, Kasparov had mates in 82, 40 and 30 moves, finishing the game on moves 137, 98 and 90 respectively.


B4 Chess Analysis

B4.1 Künitz-Baez (1992, ECO A21) after 57. ... b1=Q, 6Q4/3K1Q2/k7/8/8/8/8/8+w. White asked the best question with 58. Qf6+ requiring the reply 58. ... Kc7'. Black in fact played 58. ... Qb6?, leaving a 27-move win. White returned the favour with 59. Qf1+? instead of 59. Qa1+, leaving a draw again. The point was halved at this stage. M-optimal play from 58. ... Qb6 is:


B4.2 Kasparov-World (1999, ECO B52). This is an equi-optimal line chosen (Tsaturjan, 1999) for its curiosity value: Black has just one escape route eight times and then forces a QQKQ stalemate. The best choice of the many equi-optimal moves requires an opponent-modelling strategy to leave the greatest difficulties on the next move.

Pos. GK1: 51. Qh7 d5 52. Qxb7+ Ka1 53. Kh6 d4 54. g6 d3 55. Qc1+ 56. Kh7 d2 57. g8Q Qc2+ 58. Kh8 d1Q {4Q: 6Q4/3K1Q2/k7/8/8/8/8/8+w =} 59. Qga8+ Ka2 [Qa4] 60. Qg7+ Ka1 61. Qe4+ Kb2 62. Qgb7+ Ka2 [Kb1] 63. Qb6+ Kb1" 64. Qb5+ Ka1 [Kc1] 65. Qa5+ Qa2" 66. Qee5+ Kb1' 67. Qab5+ Qab3" 68. Qe4+ Kb2 [Ka1, Ka2, Kc1] 69. Qc5+ Ka2' 70. Qa8+ Qa4 71. Qh2+ Kb1' 72. Qxa3' {but now Black forces the draw in two} Qd4+ [Qd8+] 73. Kg8 [Kh7] Qg7+ [Qc4+, Qd5+, Qg4+] 74. Kxg7°= {KQQQ}.

B5 Chess Art

Here are the two known studies (PCCC, 1999) featuring 4Q in the main line and one study featuring 4Q only in sublines. Nalimov confirms the 4Q positions as won and reproduces Elkies’ optimal play. In the second study (Po-spišil, 1976), 8. ... Qd3 and 9. Qc1+ are M-suboptimal and there is an attractive optimal line after 8. ... Qd3. Elkies notes the similarity between this study and the game above.

B5.1 N.D. Elkies, Amer. Chess J. (1993), later reprinted (Rusinek, 1994, #546; Stiller, 1995, p. 107 and 1996, p. 177), 5Q2/5P1b/8/7K/8/4q4k1/1p4B1/8+w:
The author's analysis: 1. Qg7+ (1. Qd6+? Kg2 2. f8=Q Qh3+ 3. Kg5 Qe3+ forcing perpetual check or a Q-trade) 1. ... Kh2 2. f8=Q (2. Qe5+ Kxg2 3. f8=Q Qh3+ 4. Kg5 b1=Q with Kh1 and Be4 draws) 2. ... Qb5+ (not 2. ... b1=Q 3. Qf4+ Kg1 4. Be4+ and mate, or 2. ... Qd1+? 3. Bf3) 3. Kh6 Qb6+ 4. Bc6! (4. Kxh7? b1=Q+ 5. Kh8 Qb8+ =) 4. ... Qxc6+ (4. ... Qe3+ 5. Qf4 Qxg5+ 6. Kgx5 b1=Q 7. Qf2+ mates) 5. Kh7 b1=Q+ {5Q/2/6QK/2Q5/8/8/7k/1q6+w: 4Q, Bl. having checked first!} 6. Kh8 Kh1 [Qg2] (6. ... Qg2 7. Qe7+ Kg1 8. Qxc5+= Kh1 9. Qh5+" wins) 7. Qfg8" {= pos. Z3} Qbc1" 8. Q8h7+" Q1h6 [Q6h6] 9. Qxh6+" and wins

B5.2 J. Pospíšil (1976), study #1476 entered in tourney (1975), 2K5/4P2Q/8/1k2qp2/3p4/8/8/8+w, Wh. to win:


The 4Q EGT therefore cooks the study from move 7 where 7. Qeb5+ also wins even if it is suboptimal by 7 moves. Alternative winning moves in the main line are 8. Qc8+/Qh8+, 9. Qe1+ and 10. Qa1+/Qa4'/Qg1+. The study is best repaired by ending the line at move six unless a subline can be promoted to be the Principal Variation.

B5.2: Pospíšil (1976). wtm and win

B5.3 J. Pospíšil (1973), study #1359, 2nd hon. mention, 5K2/6Q1/3k1P2/3q4/3p4/8/8/8+w, Wh. to win:


Some notes by this paper’s third author:

ii White has a KQPKQ mate in 21. After 8. ... Qg2, 9. Kh7 is optimal but 9. Kf8 adds 11 moves to the win.

iii btm 4Q position but White mates in 10.