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Scenario Planning for Long-term Economic Growth and the Role of the Oil Sector in Kazakhstan

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ABSTRACT
We provide a framework for conjectural projections of economic growth for economies that have a high oil income to output ratio. We develop the steady-state implications of a cointegration model that distinguishes between endogenous and weakly exogenous variables, which is an appropriate distinction for small open economies. We validate a recursive structure in which the oil price has a key role, and not only output but also the real exchange rate are endogenised. We draw upon consensus oil price projections to form conjectural projections of output considering a long-term scenario in which the oil price may take paths designated as ‘low’, ‘central’ and ‘high’. The conjectural framework recognises both flow and stock constraints in drawing down the exhaustible reserves of oil and shows how policy decisions can be informed by model-based projections.

Keywords: Economic growth, oil sector, conjectural projections, scenario planning, oil depletion; policy analysis.

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1. INTRODUCTION
There are a number of practical policy issues of particular relevance for small, open economies with significant oil resources. For example, given that oil reserves are exhaustible, even if that is thirty or forty years hence, how should the drawdown of such resources be managed? What might be the effect on GDP growth of having a managed path to the exhaustion of oil resources, rather than one of responding to market-based incentives? These scenarios require looking past the immediate future to steady states that might emerge in the longer term of fifteen or twenty or more years? This form of scenario planning is conjecture rather than forecasting, but it can nevertheless provide guidance on the implied policy trade-offs given possible paths of the conditioning variables and an empirically validated model-based structure that links relevant variables.

The central idea behind this study is to provide scenarios that may aid planning for the long term and, at the very least, indicate where decisions might be focussed. That this kind of projection is considered useful is illustrated by the substantial interest in oil price projections provided by a number of government and non-governmental agencies. For example, oil price projections are published by the Energy Information Agency (EIA), the International Energy Agency (IEA), OPEC, the UK’s Department for Energy and Climate Change (DECC), the World Bank and Oxford Economics¹. In turn, these projections, and the particular importance of the projected path of oil prices, are a valuable input into projections of output, as illustrated in, for example, the IMF World Outlook and the OECD outlook².

The DECC review is of particular interest as publishes an annual review of energy projections, which in part summarises other major reviews, and distinguishes a number of scenarios. In its most recent review (DECC, 2015), the key cases are designated as conjectural ‘high’, ‘central’ and ‘low’. The broad idea is to capture the inherent uncertainty in the path of the oil price and provide plausible limits to the variations in the oil price based on a consensus view.

Our analytical framework develops the steady-state implications of the cointegration model due to Esfahani et al., (2014, hereafter EMP), and applies the analysis to Kazakhstan. The framework distinguishes between endogenous and weakly exogenous variables, which is appropriate for relative small, open economies that trade in the world oil market. The premise is that such economies have little or no influence over their exports of oil, from both the volume (barrels per day, bpd) and price perspective and, similarly, they trade with major trading partners, whose output is effectively outside their influence. The distinction between endogenous and weakly exogenous variables natural in this context but it is, however, an empirical one, which can be tested as in Harbo (1998).

Whilst the model suggested by EMP (2014), embodies a single cointegrating vector interpreted as an output (real GDP) equation, which depends on overseas output, the real exchange rate and oil exports (in USS), we find that the real exchange rate can be endogenised through a second cointegrating vector, which depends on oil exports (barrels per day, bpd) and the global oil price. We decompose oil revenue in domestic currency units (dcu) into its component parts, namely the real exchange rate, the volume exports of oil and the oil price and this decomposition has an important role in scenario planning. The weakly
exogenous variables are modelled by a recursive structure in which the highest level variable is the oil price which feeds through to overseas output, which in turn feeds through to oil exports.

The case study for our analysis is of Kazakhstan, which is an oil producing economy and became an independent state after the collapse of the Soviet Union in 1991, (Kalyuzhnova 1998). Kazakhstan developed its oil fields such that oil revenues have become an important part of its overall economic development, with oil exports which increased from approximately 0.19mn bpd in 1994 to 1.5mn bpd in 2014 and reserves estimated to be in the region of 30 bn barrels, see TOGY (2014) and Kalyuzhnova (2008). Even so, Kazakhstan is a ‘small’ player in the world oil market and takes decisions on the oil price and ‘overseas’ output as given, so fitting the relevant variables into an endogenous/weakly exogenous dichotomy. Moreover, in looking to the longer term, supply constraints will become effective, where these constraints relate to the a) flow and b) stock positions. In the context of the former, whilst the stock of an exhaustible resource may be below its ultimate limit, there may nevertheless be a flow constraint due to infrastructure developments; and, as to the latter, some scenarios involving a projection of twenty or more years ahead may result in the exhaustion of the exploitable oil fields. Both need to be taken into account in scenario planning.

Moreover, there are policy issues related to the speed at which reserves should be made available on the world market\(^3\). An example being whether to follow a market-driven

\(^3\)For example, some conjectural situations related to oil resources are considered by Clô (2000) and Akuru and Okoro (2011) consider an oil depletion scenario for Nigeria. More generally, the impact of resource reliance, especially a reliance on oil extraction, on long-term welfare development has been a concern in the literature, see for example, Auty (2001), Karl (1997), Van der Ploeg, (2011) and Wood (1999).
response, that is let the market decide subject only to a flow constraint, until the stock is exhausted, which we term below a ‘go for growth’ strategy (policy 1). An alternative to this strategy is referred to as ‘managed growth’ (policy 2), which sets a flow rate maximum with the intention of a planned ‘harvesting’ of the stock. This contrast suggests the possibility of a trade-off, the cost or benefit or which will depend on the projected oil price and its impact on GDP, which will be larger under policy 1 compared to policy 2 for at least some part of the future, but lower thereafter. In this case, the comparison involves an inter-temporal comparison, which will therefore depend on the government’s rate of time preference.

This study is organised as follows. Section 2 summarises the underlying model. Section 3 describes an appropriate estimation framework and reports the empirical results. Section 4 considers a number of projections in the context of different scenarios and concluding remarks are provided in Section 5.

2. OUTPUT AND THE REAL EXCHANGE RATE

2.1 Long-term Economic Growth

The modelling framework we develop is due to EMP (2014), who developed a theoretical framework and applied it with some success to eight countries in which oil income is important (six OECD and two OPEC countries) as in the case of Kazakhstan. In this section, we briefly review the EMP model in order to define the relevant variables.

In the EMP model, the assumed production technology is Cobb-Douglas, with constant returns to scale:
\[ Q_t = A_t K_t^{\alpha} L_t^{(1-\alpha)} \]  

(1)

Where \( Q_t \) is real output, \( K_t \) is the capital stock, \( L_t \) is the labour stock and, in each case, the flows of services are assumed proportional to the stocks. \( A_t \) is labour augmenting technical progress. \( A_t \) and \( L_t \) are assumed to be linear, stochastic functions of time, with growth rates of \( g \) and \( n \), respectively; hence, \( g^n = g + n \).

We next define the real value of oil exports in dcu: \( o_t X_t^o \equiv RER_t X_t^{os} \), where \( RER_t \equiv E_t/P_t \) is the ‘real’ exchange rate, \( E_t \) is the nominal exchange rate expressed as the number of domestic currency units per US$ and \( P_t \) is a domestic price index; further, the value of exports in US$ is \( X_t^{os} \equiv X_t^{bo} P_t^o \), where \( X_t^{bo} \) is the volume exports of oil in thousands of bpd and \( P_t^o \) is the global oil price (here the Brent spot per barrel). The steady-state growth rate of \( X_t^o \) is denoted \( g^o \) and throughout a lower case letter indicates the logarithm of the corresponding upper case variable.

A key distinction in the modelling framework is the relationship between \( g^o \) and \( g^n \), the case of interest here being when \( g^o \geq g^n \), so that steady state oil income grows at least as fast as the natural rate of growth, which leads to the following steady-state equation for output (GDP):

\[ q_t = \psi_{1,1} q_t^* + \psi_{1,2} X_t^o + \gamma t + \epsilon_{q,t} \]  

(2)

Where \( q_t \) and \( q_t^* \) are the logs of domestic and overseas output, respectively. Note that if \( g^o < g^n \), then the term in \( X_t^o \) is absent from equation (2) and ‘oil does not matter’. Further
‘co-trending’ occurs if $\gamma = 0$ and corresponds to an annihilation of the trends from the component long-run variables especially the domestic and overseas output variables. EMP further decompose $x_t^o$ into the real exchange rate and the US$ value of oil exports, thus $x_t^o = \text{rer}_t + x_t^{os}$, where $x_t^{os} = p_t^o + x_t^{bo}$ and consequently write (2) as:

$$ q_t = \psi_{1,1} q_t^* + \psi_{1,2} \text{rer}_t + \psi_{1,3} x_t^{os} + \gamma t + \varepsilon_{y,t} \quad (3) $$

If $\psi_{1,2} = \psi_{1,3}$, then (3) reduces to (2).

For scenario planning, which involves projections based on the oil price, it is more convenient to use all components of the decomposition, that is $x_t^o = \text{rer}_t + x_t^{bo} + p_t^o$, resulting in:

$$ q_t = \psi_{1,1} q_t^* + \psi_{1,2} \text{rer}_t + \psi_{1,3} p_t^o + \psi_{1,4} x_t^{bo} + \gamma t + \varepsilon_{y,t} \quad (4) $$

If $\psi_{1,3} = \psi_{1,4}$, then (4) reduces to (3). In the context of equation (4), co-trending corresponds to $\gamma = 0$, a hypothesis for which EMP (2014) find empirical support with data from a number of oil-producing countries. As we do not reject the co-trending hypothesis for Kazakhstan, we consider equation (4) without the time trend, thus:

$$ q_t = \psi_{1,1} q_t^* + \psi_{1,2} \text{rer}_t + \psi_{1,3} p_t^o + \psi_{1,4} x_t^{bo} + \varepsilon_{y,t} \quad (5) $$

The division of the variables depends on their putative status as either endogenous or weakly exogenous. For this purpose we define the following vector of variables:

$$ z_t = (q_t, \text{rer}_t, x_t^{bo}, q_t^*, p_t^o)' \quad (6) $$
\[
\begin{bmatrix}
y_t \\
x_t
\end{bmatrix}
\]

The dichotomy suggests the following endogenous and weakly exogenous variables, \( y_t = (q_t, rer_t) \) and \( x_t = (x_t^{bo}, q_t^*, p_t^0) \), respectively. The endogenous variables, \( y_t \), are then modelled conditional on the I(1) exogenous variables, \( x_t \), see Harbo et al., (1998) and Pesaran et al., (2000). In general, \( z_t \) is of dimension \( m \times 1 \), \( y_t \) is \( n \times 1 \) and \( x_t \) is \( k \times 1 \), hence \( k \equiv m - n \). This division allows the potential dichotomy between variables that are weakly, or ‘structurally’ exogenous in the sense that none of the long-run multipliers are present in the determination of the equations for these variables, which is a testable restriction; for further details, see Harbo et al., (1998).

Although the empirical evidence reported by EMP (2014) favours either a single cointegrating vector, \( r = 1 \), for six countries and no cointegrating vector, \( r = 0 \), for two of the countries in their sample, we find, as detailed below, that the results for Kazakhstan favour two cointegrating vectors, that is \( r = 2 \). This is a sensible finding in the following sense. With just a single cointegrating vector interpreted as an equation for output, the implied equation for the other endogenous variable, the real exchange rate, is just a rearrangement of the output equation. However, we find that the second cointegrating vector can be interpreted as an equation for the real exchange rate as a function of the volume exports of oil and the oil price, which moreover provides good in-sample fit and diagnostic statistics.
3. MODELLING THE SYSTEM

3.1 A VARX model

The estimation framework is a specialized case of a VARX model, a vector autoregressive model with exogenous variables. The partial or conditional modelling approach starts from the general VECM specification given by:

\[
\Delta z_t = \mu_z + \delta_z t + \Pi_z z_{t-1} + \sum_{j=1}^{p-1} \Psi_j \Delta z_{t-j} + \varepsilon_{z,t}
\]  

(7)

Where \( \Pi_z = \alpha \beta' \) is the long-run multiplier matrix, combining the matrices of the error correction coefficients \( \alpha \) and the steady-state (cointegration) coefficients \( \beta \), which are \( m \times r \) and \( m \times r \), respectively, where in this case \( r \leq 5 \). Cointegration implies that \( \Pi_z \) is of reduced rank, see Johansen (1992, 1996) and Pesaran and Pesaran (2009). The constant and trend can be decomposed into those parts that are included and those that are excluded from the cointegration space; the relevant case here is where if a (deterministic) trend is present then it is restricted to the cointegration space and does not, therefore, cumulate to a quadratic trend in the level of \( z_t \). As noted above, we find that the trend is absent from the cointegration space, that is the co-trending restriction is satisfied and we, therefore, consider (7) without the deterministic trend.

The system in (7) can also be represented as a conditional model for \( y_t \) plus a marginal model for \( x_t \), as follows:
The conditional model

\[
\Delta y_t = \mu_y + \Lambda \Delta x_t + \Pi_y z_{t-1} + \sum_{j=1}^{p-1} \Psi_j \Delta z_{t-j} + \varepsilon_{y,t} \tag{8}
\]

The marginal model

\[
\Delta x_t = \mu_x + \Pi_x z_{t-1} + \Gamma_y(L) \Delta y_{t-1} + \Gamma_x(L) \Delta x_{t-1} + \varepsilon_{x,t} \tag{9}
\]

Where \(\Pi_y = \alpha_y \beta^\prime\) and \(\Pi_x = \alpha_x \beta^\prime\). The vectors of error correction coefficients \(\alpha_y\) and \(\alpha_x\) are of dimensions \(n \times r\) and \(k \times r\), respectively, where \(r \leq k\), so that there can be at most \(k\) common trends amongst the \(m\) variables in \(z_t\). The hypothesis of weak exogeneity corresponds to the testable restrictions \(\alpha_x = 0\), implying \(\Pi_x = 0\), and thus the marginal model provides no information for estimation and inference on the conditional model (Johansen 1992, Harbo, 1998, Pesaran et. al., 2000).

The idea of modelling the system (7) as a model conditional on a subset of \(z_t\) regarded as weakly exogenous for the remaining variables, and a marginal model for the weakly exogenous variables, is motivated by the idea that the variables of interest, defined by \(z_t\), fall into two groups where one is a group of ‘global’ variables that are not likely to be affected by those in the second group, the ‘local’ variables, and the former are ‘forcing’ variables for the latter. An archetypal situation is just the kind of situation considered here, that is of a small open economy where trade depends on overseas output and the prices of goods set in international markets. In the case considered here, the putative weakly exogenous variables are overseas output, the price of oil and (volume) exports of oil. The hypothesis of weak
exogeneity involves a simple test that there are no error correction terms in the marginal model.

### 3.2 The marginal model

#### 3.2.1 Weak exogeneity and recursivity

The marginal model for the conditioning variables provides the structure for the weakly exogenous variables and has a key role in conjectural projections. It also provides the basis of a test for the weak exogeneity of the variables \( x_t = (x_{t,0}^b, q_t^*, p_t^0)' \) and other simplifying reductions. We start from the general marginal model given by:

\[
\Delta x_t = \mu_x + \alpha_x (\hat{\beta} z_{t-1}) + \Gamma_y (L) \Delta y_{t-1} + \Gamma_x (L) \Delta x_{t-1} + \hat{\epsilon}_{x,t}
\]

(10)

Where \( \hat{\beta} \) is obtained from estimation of the conditional model (8) and serves to define the \( r \) cointegrating relationships \( \hat{\beta}' z_{t-1} \); and \( \Gamma_y (L), \Gamma_x (L) \) are matrix lag polynomials. Weak exogeneity corresponds to the reduction \( \alpha_x = 0 \), a set of \( n \times k \) restrictions; and if this holds, and more explicitly in terms of the variables considered here, the model reduces to:

\[
\begin{bmatrix}
\Delta x_{t,0}^b \\
\Delta q_t^* \\
\Delta p_t^0
\end{bmatrix}
= 
\begin{bmatrix}
\mu_{x,0}^b \\
\mu_{q,t}^* \\
\mu_{p,t}^0
\end{bmatrix}
+ 
\begin{bmatrix}
\Gamma_{1,1}^y (L) & \Gamma_{1,2}^y (L) \\
\Gamma_{2,1}^y (L) & \Gamma_{2,2}^y (L) \\
\Gamma_{3,1}^y (L) & \Gamma_{3,2}^y (L)
\end{bmatrix}
\begin{bmatrix}
\Delta q_{t-1} \\
\Delta \text{rer}_{t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
\Delta x_{t-1}^b \\
\Delta q_{t-1}^* \\
\Delta p_{t-1}^0
\end{bmatrix}
+ 
\begin{bmatrix}
\varepsilon_{x,t} \\
\varepsilon_{q,t} \\
\varepsilon_{p,t}
\end{bmatrix}
\]

(11)
Where $\Gamma^y_{ij}(L)$ and $\Gamma^x_{ij}(L)$ are lag polynomials on the endogenous and exogenous variables of orders $s_1$ and $s_2$, respectively. We denote this general model as a marginal VAR, MVAR($s_1$, $s_2$). As an example, the following model is an MVAR(1, 1):

$$
\begin{align*}
\begin{pmatrix}
\Delta x^{bo}_t \\
\Delta q^*_t \\
\Delta p^0_t
\end{pmatrix}
&= \begin{pmatrix}
\mu^{bo} \\
\mu^q \\
\mu^{po}
\end{pmatrix}
+ \begin{pmatrix}
\Gamma^y_{1,1} & \Gamma^y_{1,2} \\
\Gamma^y_{2,1} & \Gamma^y_{2,2} \\
\Gamma^y_{3,1} & \Gamma^y_{3,2}
\end{pmatrix}
\begin{pmatrix}
\Delta q_{t-1} \\
\Delta \text{rer}_{t-1}
\end{pmatrix}
+ \begin{pmatrix}
\Gamma^x_{1,1} & \Gamma^x_{1,2} & \Gamma^x_{1,3} \\
\Gamma^x_{2,1} & \Gamma^x_{2,2} & \Gamma^x_{2,3} \\
\Gamma^x_{3,1} & \Gamma^x_{3,2} & \Gamma^x_{3,3}
\end{pmatrix}
\begin{pmatrix}
\Delta x^{bo}_{t-1} \\
\Delta q^*_{t-1} \\
\Delta p^0_{t-1}
\end{pmatrix}
+ \begin{pmatrix}
\varepsilon_{x1,t} \\
\varepsilon_{x2,t} \\
\varepsilon_{x3,t}
\end{pmatrix}
\end{align*}
$$

To anticipate the empirical results, we consider two further reductions of the marginal model. First, the endogenous variables, $\Delta y_{t-1}$, do not affect the weakly exogenous variables, $\Delta x_t$, which implies that the ‘global’ variables, overseas output and the price of oil, are unaffected by the ‘local’ variables, domestic output and the exchange rate; and that the volume of oil exports (in barrels) is similarly unaffected. This corresponds to $\Gamma^y_L = 0$ in (10), and is a set of $v = n \times k \times s_1$ restrictions, where in the first order case $v = 6$.

We also consider a recursive or causal structure, where the underlying key variable is the (growth rate of the) price of oil, $\Delta p^o_t$, over which a small country in the oil market, such as Kazakhstan, will not have any influence. In particular, Kazakhstan’s exports of oil respond to rather than influence the global oil price. Next, $\Delta y^*_t$ is related to $\Delta p^o_t$, but not to $\Delta x^{bo}_t$; finally, $\Delta x^{bo}_t$ is related to $\Delta y^*_t$ and $\Delta p^o_t$. This results in the following recursive specification of the marginal model, again illustrated for the first order case.
\[
\begin{pmatrix}
\Delta x_{t}^{bo} \\
\Delta q_{t}^* \\
\Delta p_{t}^o
\end{pmatrix}
= \begin{pmatrix}
\mu_{t}^{bo} \\
\mu_{t}^{q*} \\
\mu_{t}^{po}
\end{pmatrix}
+ 
\begin{bmatrix}
\Gamma_{1,1} & \Gamma_{1,2} & \Gamma_{1,3} \\
0 & \Gamma_{2,2} & \Gamma_{2,3} \\
0 & 0 & \Gamma_{3,3}
\end{bmatrix}
\begin{pmatrix}
\Delta x_{t-1}^{bo} \\
\Delta q_{t-1}^* \\
\Delta p_{t-1}^o
\end{pmatrix}
+ 
\begin{pmatrix}
\varepsilon_{x,1,t} \\
\varepsilon_{x,2,t} \\
\varepsilon_{x,3,t}
\end{pmatrix}
\tag{13}
\]

More generally for recursivity in higher order systems, if necessary with a suitable reordering of the weakly exogenous variables, all coefficients in the lower triangular block are zero. As in the case of weak exogeneity, such a reduction can be tested.

### 3.2.2 Weakly exogenous variables: steady state

The dynamic system in the growth rates given by equation (13) has an implied steady state, \( \Delta g_{x}^{ss} \), given by:

\[
\Delta g_{x}^{ss} = \Lambda \mu_{x} , \text{ where } \Lambda \equiv (I - \Gamma^{-1})
\tag{14}
\]

The steady state exists provided that the roots of \( \Lambda \) have modulus less than unity.

Generalising (12) and (13) we have:

\[
\Delta x_{t} = \mu_{x} + \sum_{j=1}^{s_j} \Gamma_{j}\Delta x_{t-j} + \varepsilon_{x,t}
\tag{15}
\]

\[
\Delta g_{x}^{ss} = \Lambda \mu_{x} , \text{ where } \Lambda \equiv (I - \sum_{j=1}^{s_j} \Gamma_{j})^{-1}
\tag{16}
\]

### 3.3. Estimation
3.3.1 Cointegrating vectors

Distinguishing between the endogenous and weakly exogenous variables, the putative cointegrating vectors without normalisations or restrictions are:

\[
\begin{bmatrix}
\beta_{1,1} & \beta_{1,2} \\
\beta_{2,1} & \beta_{2,2}
\end{bmatrix}
\begin{pmatrix}
q_t \\
rer_t
\end{pmatrix}
- \begin{bmatrix}
\mu_q \\
\mu_{rer}
\end{bmatrix}
+ \begin{bmatrix}
\beta_{1,3} & \beta_{1,4} & \beta_{1,5} \\
\beta_{2,3} & \beta_{2,4} & \beta_{2,5}
\end{bmatrix}
\begin{pmatrix}
x_{t}^{bo} \\
q_t \\
p_t^o
\end{pmatrix}
\]

(17)

The initial estimation results strongly suggested that we could concentrate on the situation where a deterministic trend was not included in the cointegration space, so the co-trending restriction was not rejected, as in a similar context by EMP et al., (2014), and for the underlying theory see Park (1992) and Pesaran et al., (2000, section 4); moreover the intercept could be restricted to the cointegration space without loss, see Pesaran et al., (ibid) for details.

The estimation period was 1999q4 to 2014q4 which is a relatively homogenous period in the recent economic development of Kazakhstan; prior to that the necessary financial and physical infrastructure to exploit the oil fields was incomplete; further in early 1999 there was a substantial devaluation of the Kazakhstan tenge (Kzt) in 1999, which started the year at 85Kzt:US$ and ended at 140Kzt:US$. Overall, there was a general lack of output growth in the 1990s, compared to the ‘take-off’ in 1999. On other data details, \(Q_t\), the measure of output was GDP (deflated by the CPI), \(E_t\) is the exchange rate of the Kazakhstani tenge against the US$, \(P_t\) was the CPI (as in EMP, 2014), \(P_t^o\) is the Brent spot price per barrel in US$ and \(X_t^{bo}\) is the volume exports of oil in thousands of bpd. The proxy for ‘overseas’
output, $Q^*_t$, was based on Kazakhstan’s main trading partners, Russia, the EU and China, which together account for approximately 80% of trade (imports and exports); weights of 0.4, 0.4 and 0.2, respectively, were based on the mid-point of the sample.

The VARX results to determine the cointegrating rank are shown in Table 1. Both the Maximum eigenvalue and Trace test statistics indicate $r = 2$, with the results significant at better than 5%.

**Table 1 Cointegration test statistics**

<table>
<thead>
<tr>
<th></th>
<th>$H_0$</th>
<th>Test value</th>
<th>5% cv</th>
<th>10% cv</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max eigenvalue</td>
<td>$r = 0$</td>
<td>55.91</td>
<td>42.18</td>
<td>40.00</td>
</tr>
<tr>
<td></td>
<td>$r = 1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$r = 2$</td>
<td>31.32</td>
<td>28.80</td>
<td>25.10</td>
</tr>
<tr>
<td>Trace</td>
<td>$r = 0$</td>
<td>87.24</td>
<td>58.71</td>
<td>52.66</td>
</tr>
<tr>
<td></td>
<td>$r = 1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$r = 2$</td>
<td>31.32</td>
<td>28.80</td>
<td>25.10</td>
</tr>
</tbody>
</table>

VARX(1, 6), restricted intercept; bootstrapped critical values.

The over-identification restrictions are tested by reference to a just-identified system, which imposes two restrictions on each cointegrating vector in (17). The set of restrictions distinguished into three types is as follows:

Normalisations: $\beta_{1,1} = -1, \beta_{2,2} = -1$. 
Equality: $\beta_{1,2} = \beta_{1,3} = \beta_{1,5}$; $\beta_{2,3} = \beta_{2,5}$.

Exclusions: $\beta_{2,1} = 0$, and $\beta_{2,4} = 0$.

Imposing these restrictions, results in the following specification:

$$
\begin{bmatrix}
1 & -\beta_{1,2} \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
q_t \\
\sigma_t
\end{bmatrix}
= 
\begin{bmatrix}
\mu_q \\
\mu_{\sigma_t}
\end{bmatrix}
+ 
\begin{bmatrix}
\beta_{1,2} & \beta_{1,4} & \beta_{1,2} \\
\beta_{2,3} & 0 & \beta_{2,3}
\end{bmatrix}
\begin{bmatrix}
x_t^{bo} \\
q_t^* \\
p_t^o
\end{bmatrix}
$$

The test of over-identification has three degrees of freedom, that is the total number of restrictions minus the number of just-identifying restrictions, with a resulting test statistic, distributed as $\chi^2(3)$ under the null, of 3.49 [95% cv = 16.30, bootstrapped critical values are used throughout]. The restrictions are, therefore, safely not rejected. An additional hypothesis of interest is that the elasticity of domestic output with respect to overseas output is unity, which corresponds to $\beta_{1,4} = 1$, resulting in a test statistic of $\chi^2(4) = 3.73$ [95% cv = 20.17] or $\chi^2(1) = 0.247$ when tested as a single additional restriction and, hence, this restriction is also not rejected.

Imposing the set of restrictions, including $\beta_{1,4} = 1$, provides the following estimated cointegrating vectors, with asymptotic standard errors in (.) parentheses, which indicate that the coefficient estimates are well-determined:
\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
- 
\begin{bmatrix}
0 & 0.332(0.032) \\
0 & 0
\end{bmatrix}
\begin{pmatrix}
q_{t}^{ss} \\
\text{rer}_{t}^{ss}
\end{pmatrix}
= 
\begin{bmatrix}
\hat{\mu}_{q} \\
\hat{\mu}_{\text{rer}}
\end{bmatrix}
+ 
\begin{bmatrix}
0.332(0.032) & 1 & 0.332(0.032) \\
-0.377(0.012) & 0 & -0.377(0.012)
\end{bmatrix}
\begin{pmatrix}
x_{t}^{\text{bo,ss}} \\
q_{t}^{*,ss} \\
p_{t}^{o,ss}
\end{pmatrix}
\]  
(19)

where the superscript ss refers to the equilibrium (steady state) solution. The cointegrated system can be written simply as:

\[
q_{t}^{ss} = \hat{\mu}_{q} + q_{t}^{*,ss} + 0.322(\text{rer}_{t}^{ss} + x_{t}^{\text{bo,ss}} + p_{t}^{o,ss})
\]

\[
= \hat{\mu}_{q} + y_{t}^{ss} + 0.322x_{t}^{o,ss}
\]  
(20)

\[\text{ecm}_{t}^{q} = q_{t} - q_{t}^{ss}\]

\[
\text{rer}_{t}^{ss} = \hat{\mu}_{\text{rer}} - 0.377(x_{t}^{\text{bo,ss}} + p_{t}^{o,ss})
\]

\[
= \hat{\mu}_{\text{rer}} - 0.377x_{t}^{o,ss}
\]  
(21)

\[\text{ecm}_{t}^{\text{rer}} = \text{rer}_{t} - \text{rer}_{t}^{ss}\]

From equation (20), the point estimate of \(\hat{\beta}_{1,2}\) is 0.322, which is within the range, 0.11 (Iran) to 0.51 (Nigeria) reported by EMP (2014); the interpretation of equation (21), is that the real exchange rate equation depreciates as oil revenues (US$) increase, which is consistent with the pattern of sustained depreciation of the domestic currency, Kzt, from 140 to 180 against the US$. 


Note that the cointegrated system is recursive in the endogenous variables $\text{rer}_t$ and $q_t$: in steady state, the real exchange rate is determined by oil revenues in US$ given the equality of the coefficients on the weakly exogenous variables, that is the volume exports of oil and the global oil price; output is then determined by overseas output and oil revenues, with the real exchange rate converting oil revenues in US$ into domestic currency units.

### 3.3.2 Error correction equations

We denote the estimated long-run multiplier matrix restricted by the over-identifying restrictions by $\hat{\Pi}_y = \hat{\alpha}_{y,t} \hat{\beta}_t'$, where $\hat{\beta}_t$ refers to the cointegrating vectors from (19); their related error correction coefficients are, $\hat{\alpha}_{y,t}$, and the error correction terms are $\hat{\Pi}_y z_{t-1}$ for the conditional model. The estimated error correction components are as follows:

$$
\hat{\alpha}_{y,t} z_{t-1} = \begin{bmatrix}
-0.267(0.072) & 0.324(0.083) \\
-0.102(0.035) & -0.237(0.039)
\end{bmatrix}
\begin{bmatrix}
\text{ecm}_q^{i-1} \\
\text{ecm}_{\text{rer}}^{i-1}
\end{bmatrix}
$$

The diagnostics for the error correction equations, that is the conditional model equations, are reported in Table 2. The diagnostic test statistics indicate that the equations are well specified and the $\hat{\alpha}_{y,t}$ are plausible and well-determined; for example, the own adjustment coefficients are negative, indicating annual adjustment of approximately 27% and 24% for $q_t$ and $\text{rer}_t$, respectively.

### Table 2 Diagnostic test statistics for the estimated conditional model
<table>
<thead>
<tr>
<th></th>
<th>Serial Correlation (SC)</th>
<th>Functional form (FF)</th>
<th>Normality (N)</th>
<th>Heteroscedasticity (HS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta q_t$</td>
<td>1.1242[0.365]</td>
<td>0.035461[0.852]</td>
<td>0.75063[0.687]</td>
<td>0.54885[0.462]</td>
</tr>
<tr>
<td>$\Delta r_{er_t}$</td>
<td>1.3045[0.292]</td>
<td>0.97266[0.332]</td>
<td>2.6588[0.265]</td>
<td>0.043615[0.835]</td>
</tr>
</tbody>
</table>

Notes: p value in [.]; tests for SC, FF and HS in F form; N test is $\chi^2(2)$.

### 3.3.3 The marginal model: Estimation and testing

Initial reductions steps supported the simplification from an unrestricted MVAR(4, 4) to MVAR(2, 2), which formed the maintained regression. The empirical reductions then supported weak exogeneity and no feedback from the endogenous variables, with $\chi^2(18) = 18.12 [0.448]$ and neither set of restrictions individually significant. We next consider the proposed recursive structure which, with an appropriate ordering of the variables, translates as an upper triangular structure to each component of $\Gamma_x(L)$, see (10); in the context of the MVAR(0, 2) with three variables, this corresponds to an ordering such that $\Gamma_{x,s}^{i,j} = 0$ for $i > j$, for $s = 1, 2$, resulting in 6 restrictions, with $\chi^2(6) = 7.82 [0.252]$, which supports the proposed recursive structure. The complete set of reductions resulted in a test statistic of $\chi^2(24) = 23.91 [0.467]$, which strongly supports the more parsimonious specification.

The recursive structure means that not all the regressors enter all equations and, hence, the equations are in a form that can be estimated more efficiently, (at least asymptotically), by SURE. The triangular version of $\Gamma_x(L)$ can be further reduced by deleting jointly insignificant coefficients (test statistic $\chi^2(5) = 6.62 [0.750]$), resulting in:
\[
\begin{pmatrix}
\Delta x_{t}^{bo} \\
\Delta y_{t}^{*} \\
\Delta p_{t}^{0}
\end{pmatrix} =
\begin{pmatrix}
0.0200 \\
0.0179 \\
0.0284
\end{pmatrix} +
\begin{pmatrix}
0.467 \\
0.430 \\
-0.103
\end{pmatrix}
\begin{pmatrix}
\Delta x_{t-1}^{bo} \\
\Delta y_{t-1}^{*} \\
\Delta p_{t-1}^{0}
\end{pmatrix} +
\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
\Delta x_{t-2}^{bo} \\
\Delta y_{t-2}^{*} \\
\Delta p_{t-2}^{0}
\end{pmatrix} +
\begin{pmatrix}
\hat{e}_{x_{1,t}} \\
\hat{e}_{x_{2,t}} \\
\hat{e}_{x_{3,t}}
\end{pmatrix}
\tag{23}
\]

These estimates from (19) imply the following estimated steady-state growth rates, which are the empirical counterpart of (16):

\[
\begin{pmatrix}
\hat{g}_{bo}^{ss} \\
\hat{g}_{q}^{ss} \\
\hat{g}_{po}^{ss}
\end{pmatrix} =
\begin{pmatrix}
0.0173 \\
0.0186 \\
0.0261
\end{pmatrix}
\tag{24}
\]

These figures relate to fractional growth per quarter, p.q, and where annual rates, pa, are used they are calculated as \(g_{a} = 100((1 + g_{q})^{4} - 1)\), where \(g_{q}\) is the quarterly rate. Thus estimates of the steady-state annual growth rates for oil exports, overseas output and the oil price are, respectively, 7.1%, 7.65% and 10.16%; and the estimated steady-state annual growth rate for oil revenue in US$ is, therefore, \(g_{o}^{ss} = (7.1 + 10.16)\% = 17.26\%\). These figures compare to the historical averages of 7.13%, 7.62%, 11.27% and 18.4%, respectively.

4. Scenario Projections

4.1. Background
To project GDP growth, we consider some scenarios that have been of interest to a number of international agencies including OPEC and the IMF. The key variable in this context is the US$ price of oil and we distinguish here between the period of overall (almost) steady growth from 2000q1 to 2014q3, with average growth of approximately 11% p.a. and the subsequent decline dating from 2014q4, with the oil price finishing the year at about 58US$, having peaked most recently to that date at 128US$ in March, 2012, and prior to that at 143US$ in July 2008.

Such recent movements in the oil price resulted in consequent reductions in the projected oil price. For example, annual projections published by the UK Department of Energy and Climate Change (DECC, 2014, 2015), which summarise a number of sources, including the Institute for Energy Analysis (IEA) and EIA (US Energy Information Administration), were updated with significant reductions comparing the oil price projections at 2014 with those at 2015. Taking the DECC (2015) report as the latest available, we consider their three scenarios, namely: ‘low’, ‘central’ and ‘high’.

The ‘central’ projection for 2015, the ‘short term’, and 2016-2019, the ‘medium term’, are based on the Brent futures curve (DECC, 2015, p.8). The projections are then extended to 2020 and beyond, with the ‘long term’ based on projections of longer-term supply and demand factors informed, inter-alia, by the World Energy Outlook (2014) and Rystad Energy (DECC, 2015, Annex A). The low and high variations start from points that are two standard deviations from the central projection, respectively, than the central case and fan out based on the same demand and supply principles as for the central case, see DECC (2015, pps.6, 10, 11).
The DECC projections are framed in 2015 prices, US$2015, whereas for our purposes, projections in nominal prices per barrel are required and we apply the IEA default rate of 2.3% p.a in the numeraire price index. These projections are shown in Figures 1 and 2 for the levels and logs of the series, respectively, the latter being clearer for the rate of growth implications. On the basis of the central projection, in real terms the oil price is projected to grow from 2016 to 2025 at an annual average rate of 6% p.a, and a nominal rate of 8.3% p.a, and is assumed to be flat thereafter in real terms.

From 2019 to 2025, the high projection has a flatter rate of growth compared to the central case, but a steeper rate of growth until 2019; it also starts from a higher level and continues to grow in real terms from 2025 to 2030. The low projection starts from below the central projection and projects a decline in the real oil price until 2019 and then grows until 2025.
4.2. Projections: the unconstrained case

In this section we apply some long-term scenarios to assess the growth potential in real GDP. Our approach is as follows: initially no constraints are applied arising from flow and stock limitations, whereas in the next section the impacts of these constraints are allowed to affect the projections. The comparison allows an assessment of the impact of the constraints.

As dynamic adjustments are transient, we focus on long-term projections rather than short-term forecasting and, hence, it is relevant to consider projections using the steady state equations. The estimated marginal model (23) provides the basis to project the growth in volume exports of oil and overseas output, $\Delta x_t^{bo}$ and $\Delta q_t^*$, given projections of the growth of the oil price, $\Delta p_t^0$. The projections of these weakly exogenous variables are then inputs to the cointegrated equations for output and given by equations (20) and (21)

4.2.1 Oil variables (unconstrained case)
The projected paths for the levels of $x_t^{bo}$ according to the low, central and high scenarios for the oil price are shown in Figure 3. Notice that a low oil price results in higher projections for the volume exports of oil, given the negative relationship between these two variables in the marginal model, but lower revenues whether in US$ or dcu. The implied oil revenues in US$, $x_t^{os}$, and in domestic currency units, $x_t^o$, are shown in Figures 4 and 5, respectively.

Figure 3 Projection of Oil Exports (logs, bpd)
To illustrate the differences implied by the different oil price projections, the oil export ratios relative to the central case are summarised in Table 3 for 2025q4 and 2030q4; for example, taking 2025q4, the implied level of the volume exports of oil, $X_{t}^{bo}$, is 5% higher in the low
price regime compared to the central case but, because prices are lower, oil revenues in US$ and dcu are respectively 64% and 78% of the central case.

**Table 3 Oil exports: ratio differences relative to the central case**

<table>
<thead>
<tr>
<th></th>
<th>$X_t^{bo}$ exports barrels</th>
<th>$X_t^{oS}$ exports US$</th>
<th>$X_t^o$ exports dcu</th>
</tr>
</thead>
<tbody>
<tr>
<td>2025q4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low price regime</td>
<td>1.05</td>
<td>0.64</td>
<td>0.78</td>
</tr>
<tr>
<td>High price regime</td>
<td>0.97</td>
<td>1.39</td>
<td>1.20</td>
</tr>
<tr>
<td>2030q4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low price regime</td>
<td>1.05</td>
<td>0.62</td>
<td>0.77</td>
</tr>
<tr>
<td>High price regime</td>
<td>0.96</td>
<td>1.55</td>
<td>1.28</td>
</tr>
</tbody>
</table>

Note: table entries are ratios relative to the central case.

### 4.2.1 Output growth (unconstrained case)

The projected outputs according to the three oil price scenarios are shown in Figure 6 and the average growth rates for the medium-term projections of output are shown in Table 4. Note that GDP is highest for the high oil price scenario because although volume exports of oil are higher in the low price regime, oil revenues increase less than in the central and high cases, both in US$ and dcu (see Table 4), which has an impact on GDP via equation (16) for $q_t^{oa}$. 
The reference historical rate for the period 2000q1 to 2014q4 is 0.0288 (that is 2.88% p.q) and relative to this rate, the oil price projections result in projected average GDP growth below the historical average for all but the period 2015q1 to 2025q4 for the high oil price scenario, although the difference for the central case is slight at 0.0287. Thereafter, the general picture is the same for all scenarios, with GDP growth slowing between 2026 and 2030. The differences in growth rates imply a difference in levels and the implied ratio in the end period levels for the low and high oil price scenarios, compared to the central case, are given in {} parenthesis; for example, comparing the low and central cases, then by 2025q4, GDP in the low price scenario is projected to be 2.7% (= 1 – 0.973)% below GDP in the central case.

Table 4 Historical and projected growth rates for oil and GDP: unconstrained case
### 4.3. Supply considerations: flow and stock constraints

#### 4.3.1. The flow constraint

Notwithstanding projections based on the estimated model, practical consideration has to be given to supply constraints reflecting the ability of Kazakhstan’s oilfields to meet the projected growth in oil production. There are two such constraints: the first is the flow constraint; that is the extraction flow, which is limited by the capacity of existing and planned oilfields, associated infrastructure developments and, for the purposes of exports, the demand from domestic sources; the second constraint is the stock constraint, the overall limiting capacity of proven oil reserves.

As to the flow constraint, based on sustained production from the five main oilfields in Kazakhstan and outlook summaries from the Asian Development Bank, the projections in Doi (2010a, b) suggest a limit of approximately 3.6mn bpd for export purposes in the mid-1920s. Starting at the realised 2014q4 figure for oil production of 1.5mn bpd, and taking the capacity boundary to be 3.6mn bpd by 2030q4, the limiting annual rate of growth for exports of oil that this would allow is 5.64% p.a, with a corresponding implied quarterly rate of 1.38% p.q, which is below the unconstrained projected rates of growth for oil exports for all

<table>
<thead>
<tr>
<th></th>
<th>2000q1-2014q4</th>
<th>2015q1-2025q4</th>
<th>2026q1-2030q4</th>
<th>2015q1-2030q4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historical</td>
<td>0.0288</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>0.0281 {0.973}</td>
<td>0.0258</td>
<td>0.0274 {0.968}</td>
<td></td>
</tr>
<tr>
<td>Central</td>
<td>0.0287 {1.000}</td>
<td>0.0261</td>
<td>0.0279 {1.000}</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>0.0292 {1.024}</td>
<td>0.0265</td>
<td>0.0284 {1.031}</td>
<td></td>
</tr>
</tbody>
</table>

Note: figure in {} is the ratio of GDP relative to central case at the end of the corresponding period.
three oil price scenarios. It is, therefore, relevant to consider whether the flow constraint is met by the projections. Reference to Figure 3 indicates that the flow constraint will be reached in the mid-1920s, (given the measurement units, the relevant point on Figure 3 corresponds \( \ln(3,600) = 8.189 \)), the precise timing depending on the oil price scenario; specifically, 2024q2, 2025q4 and 2026q3 for the low, central and high oil price scenarios, respectively, and hence these paths are not feasible. We, therefore, impose the flow constraint at these points and, as the differences in timing are slight, the effects are illustrated with the central case where the constraint is assumed to become binding in 2025q4.

The first scenario we consider in this context is one in which growth in oil exports (bpd) proceeds as determined by the market until the flow limit is reached, with implications for oil revenues in US$ and dcu, the real exchange rate and GDP. The recursive structure in the marginal model implies that constraining \( x_{t}^{bo} \) does not affect either \( p_{t}^{o} \) or \( y_{t}^{*} \), but does affect \( q_{t}, \text{ref}_{t}^{ss}, x_{t}^{o}, \) and \( x_{t}^{os} \). We illustrate the effects on output with the central case in Figure 7, which shows unconstrained and constrained \( q_{t} \) (GDP). The constraint ‘bites’ in 2025 and constrained GDP is thereafter below the unconstrained case; by 2030q4 the ratio of the two is 0.92, implying an 8% trade-off due to the flow constraint, which by 2035q4 is projected to increase to 15.1%.
The prospect of a constraint in the medium term raises the policy issue of whether to let the market decide or manage oil exports in line with a projected limiting growth rate based on a binding constraint. Looking further ahead to the long term, this is an issue that also arises in the context of the eventual exhaustion of the oil reserves, which we consider in greater detail next.

4.3.2. The stock constraint: intertemporal allocation of oil reserves

The stock constraint for Kazakhstan, given by proven reserves, was estimated at 30 billion barrels in 2013, see TOGY (2014) and WEO (2015); thus an issue of importance for long-term projections is how to approach the drawing down of these reserves and when, as a result, they will be exhausted. To highlight the issues, we consider two stylised policies. Policy 1 is a market-driven policy with no constraint on the extraction rate, a ‘go for growth’ policy; and
Policy 2 is an extraction policy designed to manage the lifetime of the reserves, a ‘managed growth’ policy.

In this context an important issue is whether taking the relative output gain of policy 1 over policy 2 in the medium term continues to give an overall gain in the longer term? Under policy 2, slowing the rate of growth implies that the stock constraint, that is the exhaustion of the oilfields, will be reached later rather than sooner compared to policy 1. In our projections we are interested in a conditional policy analysis that addresses the questions: i) when will the stock constraint be met? ii) What is the trade-off output effect comparing the two policies?

**Policy 1: market driven growth**

In considering the first of these questions in the context of policy 1, the differences between the three oil price scenarios are slight in terms of the projected exhaustion of the reserves, (here taking the situation where the flow constraint is met in the mid-1920s), being 2038q3, 2039q1 and 2039q2, for the low, central and high oil price scenarios, respectively. Thus, to illustrate, we consider the central case, for which the average (projected) growth rate of $p_t^o$ over the period 2015q1 to 2040q4, is 0.0133 (1.33% p.q) whilst the average for $x_t^{bo}$ over this period is 0.0085 (0.85% p.q); the overall period is split into two periods, the first prior to the flow constraint where the average is 0.0190 (1.9% p.q) and the second where the average is zero thereafter. The corresponding average growth rates for oil revenue in US$ are, therefore, 0.0323, (3.23% p.q) and 0.85% p.q, for the two periods, respectively. The implied rates of growth of GDP depend on oil revenues in dcu (and overseas output), with a projected average
growth rate in the first period of 2.883% p.q, which is almost identical to the historical rate of 2.885% p.q; the projected growth rate falls to 2.21% p.q in the second period.

**Policy 2: managed drawdown of reserves**

To illustrate the calculations, the managed policy option is taken to be to draw down the exhaustible reserves later than the exhaustion date without management, where the latter is 2039q1. For example, the government may judge a further five or so years later than for the unmanaged central case. Such a policy implies rationing the volume of oil exports given market conditions, one possible implementation being to impose an average growth rate that exhausts the reserves, subject to not exceeding the flow constraint. In the case of an approximate five year extension, the implied average growth rate for $x_{t}^{bo}$ (2015q1 to 2044q4) is 0.0077 (0.77% p.q), with the corresponding average rate of growth for output being 0.0245 (2.45% p.q) over the extended period 2015q1 to 2044q4.

The projected exhaustion of the oil reserves for policies 1 and 2, with the flow restriction, is shown in Figure 8.
Policy 2 has some interesting implications and we first compare oil revenue for the two policies in US$ and then dcu. Figure 9 shows that oil revenue for policy 1 exceeds that for policy 2 until 2039q1 but then, by construction, declines to zero once stocks are exhausted; in contrast, under policy 2, oil revenue is managed to last until the end of 2044. The attenuation of the difference comparing oil revenue in US$ and dcu is due to the endogenisation of the real exchange rate, which depreciates less under policy 2 because of the negative relationship between the real exchange rate and oil revenues.
The corresponding projections of GDP are shown in Figure 10. These assume that under policy 1 of market-led growth, that part of growth engendered by overseas output is maintained, but otherwise no replacement sources are assumed. This assumption is neutral between the policies provided that a symmetry of assumptions was used for the two policies; for example it could be assumed that oil revenues were employed to diversify the service and industrial base as part of both policies.
An evaluation of the merits of policies 1 and 2 requires a comparison of quantities such as oil revenue and output over time. In these projections, both are higher under policy 1 until 2039q1, but lower thereafter; hence which policy is preferred will depend upon the government’s time preference.

The framework for such an evaluation is as follows. Define a generic variable of interest, $Y_{t,j}^p$, to be compared over a particular time period, where the $p$ superscript refers to a projected value and $j$ indexes a policy, for example $j = 1, 2$; the upper case notation refers to the level of a variable which will usually have been modelled in logarithms indicated by lower case letters, hence: $Y_{t,j}^p = \exp(y_{t,j}^p)$; next define the policy differences in outcomes, $DY_{t,1,2}^p \equiv Y_{t,1}^p - Y_{t,2}^p$ and the cumulative discounted sum of differences, that is:

$$\Theta(Y_{t,1,2}^H) \equiv \sum_{t=1}^H \delta^{t-1} DY_{t,1,2}$$  \hfill (25)
where \( \delta \equiv (1 + \rho)^{-1} \) and \( \rho \) is the rate of time preference (see for example Creedy and Guest, 2008, and in an energy context see Henriet et. al., 2014). Thus, \( DY_{t,1,2}^p \) is the difference in output (in levels not logs) comparing policies 1 and 2 and \( \Theta(Y_{t,1,2}^H) \) is the corresponding discounted cumulative difference in output over the projection period, \( t = 1, \ldots, H \).

The discounting factor reflects the government’s time preference; for example \( \delta = 1 \) implies no discounting, a gain or loss in the future is the same as a gain or loss now, in which case the preferred option is simply the one with the greater cumulated GDP. Values of \( \delta < 1 \) imply a time preference in favour of near-dated gains with, for example, \( \delta = 0.99 \) corresponding to a rate of time preference of \( \rho = 4.1\% \) p.a; whereas \( \delta = 0.9 \) is a practical the extreme, where future output is of little importance, with \( \rho = 52.4\% \) p.a. For example one unit of output in 25 years is worth 0.37 and 0, respectively. The policy 1 option leads to higher output in the medium to long term, but hits both the flow and stock boundaries sooner than the policy 2 option, which whilst losing out in the medium term has far-dated gains before then also hitting the stock boundary. Comparing oil revenue is relatively straightforward, since by hypothesis under policy 1, the oilfields become exhausted and it is just a question of when that occurs. Comparing GDP is essentially about the difference due to the loss of oil revenue, so we condition the evaluation on the data-consistent assumption that domestic output grows at the same rate as overseas output after the loss of oil.

The evaluation of \( \Theta(Y_{t,1,2}^H) \) for different discount factors, \( \delta \), is shown in Figures 11 and 12 where \( Y_{t,1,2}^H \) is alternately oil revenues in dci and GDP. Figure 11 shows the conjectured
cumulative difference in oil revenues differing according to the different discount factors. The bold line indicates the unweighted cumulated gain or loss, which increases until the point at which the stock constraint is met in 2039q1; thereafter, the complete loss of oil revenue erodes the cumulative gain, which then becomes negative in 2042q2. Next consider the effect of discounting on the cumulative total. Given that any particular future year will have less weight with $\delta < 1$ compared to $\delta = 1$, the cumulative discounted gain also declines; however, the post-exhaustion gains are also discounted, and the switch-over from a positive to negative cumulative gain is postponed, with a higher discount factor leading to a longer postponement in the switch. If $\delta = 0.99$, then the switchover does not occur until 2044q3 and the gain remains positive for the other values of $\delta$ considered here.

Figure 11 Policy Impacts: Discounted Oil Revenue (dcu)

Figure 12 follows this analysis through to consider the impact of the loss of oil revenue on GDP. As with oil revenue, the decline in the discounted cumulative gain occurs when the oil reserves are posited to be exhausted in 2039q1; however, the effect of discounting is to bring
forward the switch-over point, compared to oil revenues. The switch-over from positive to negative is postponed for just one quarter for $\delta = 0.99$, four quarters for $\delta = 0.98$, eight quarters for $\delta = 0.97$ and sixteen quarters for $\delta = 0.96$. Only, in the relatively extreme case with $\delta = 0.95$, that is $\rho = 22.8\%$ p.a, does the gain remain positive until 2050. At realistic discount rates, there is a policy trade off that depends on the government’s intertemporal perspective. To give an idea of the scale of the gain or loss, the right-hand vertical axis uses 2014q4 GDP as the unit of measurement and the conjectural gains or losses are clearly very large in this metric.

Note: right-hand vertical axis units, $10^7$; left-hand vertical axis units, GDP (2014q4).

One purpose of these conjectures is to encourage consideration of other policies involving a development framework designed to avoid a sharp decline in GDP after the exhaustion of the oilfields and serves to focus on the simple but important question: ‘what happens when there is no more oil’? The scenarios considered here indicate that there are important policy

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**Figure 12 Policy Impacts: Cumulated Weighted Difference in GDP**

- Above zero favours policy 1
- Below zero favours policy 2

Note: right-hand vertical axis units, $10^7$; left-hand vertical axis units, GDP (2014q4).
considerations related to the way in which the path to exhaustion of the oil reserves is approached.

5. CONCLUDING REMARKS

In this study we have extended the modelling framework relating output growth inter-alia to oil revenue, to explicitly endogenise the real exchange rate to depend on oil revenues in US$; the weakly exogenous variables in this analysis, oil exports in bpd, overseas output and the price of oil, were linked with a marginal model which supported a recursive structure in which the oil price was the ‘lead’ variable.

The conditional and marginal models were then used to consider some policy based conjectures as to the kind of options facing a country with a present abundance of a particular resource that is, however, exhaustible in the very long run. This has necessarily involved conjectures rather than forecasts. Whilst the modelling frameworks of these two tasks are similar and indeed may be the same, the forecasts are intended to be compared with outturns to assess accuracy, whereas conjectures are not. We might equally have termed the comparison as one of comparing different ‘scenarios’ along the lines of a ‘what if’ comparison; for example, what would happen if the oil reserves were exhausted in 2040? Would it be better, in a well-defined sense, to allow the market to determine the rate of exhaustion of oil reserves or should there be a policy of a managed draw down? Forming and evaluating different policy options is an essential part of government and takes on particular importance in countries presently reliant on exhaustible natural resources, in order to aid the formation and implementation of policies to avoid possible undesirable long-term outcomes.
Underlying such scenarios and comparisons are a number of ceteris paribus assumptions that may or may not hold. This is impossible to avoid, but it is important to have a framework that allows the sensitivity of the outcomes to these assumptions to be assessed and variations thereof to be evaluated. Of course, this relies heavily on the underlying estimated econometric model based on historical data, but this best summarises the main players in constructing a view of the future to evaluate different policy perspectives.
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