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COMMENTARY
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Key Points:
- Radiation belt models capture subgrid physics through the use of diffusion coefficients that require wave observations for their calculation
- Current empirical wave models do not capture the true temporal variability of magnetospheric waves leading to overestimates of diffusion
- We suggest that methods such as stochastic parameterization of diffusion are superior; there is no evidence that a deterministic wave model exists

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The parameterization of wave-particle interactions in the Outer Radiation Belt

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Abstract We explore the use of mean value empirical wave models in diffusion models of the Outer Radiation Belt. We show that magnetospheric wave power is not normally distributed in time and that geomagnetic activity does not provide a deterministic proxy for the temporal variability of wave activity. Our findings indicate that current diffusion models significantly overestimate the action of wave-particle interactions due to extremely low frequency and very low frequency waves in the magnetosphere. We suggest that other techniques such as stochastic parameterization will lead to a better characterization of subgrid diffusion physics in the Outer Radiation Belt.

1. Introduction

The Earth's Outer Radiation Belt is a region of near-Earth space hosting high-energy electrons that are trapped by the geomagnetic field. The region spans radial distances of $2.5 < r < 7\ R_E$, where R_E is an Earth radius, although the size and location of the Outer Radiation Belt varies dramatically in response to solar wind variability [*Hudson et al.*, 2008]. Identifying the processes controlling the large variability in volume, energy, and number of Outer Radiation Belt electrons is a challenging magnetospheric and plasma physics problem. The Outer Radiation Belt is also the source of critical Space Weather hazards for Low, Medium, and Geosynchronous Earth Orbiting (LEO, MEO, and GEO) spacecraft; thus, the ability to predict its variability is a key goal of the magnetospheric Space Weather community [e.g., *Denton et al.*, 2016].

The plasma in the inner magnetosphere is essentially collisionless; i.e., the mean free path for high-energy electrons in the Outer Radiation Belt is so long that collisions are rare. Changes in the number and energy of electrons at any particular location are governed by (i) large-scale changes in the geomagnetic field topology and electric fields due to solar wind variability and substorm activity and (ii) wave-particle interactions over a range of different frequencies that are analogous to collisions. It is important to note that the analogy is not perfect; wave-particle interactions are resonant processes where electrons with particular momenta are singled out because they are resonant with the wave. Those electrons with momenta such that they are in phase with the wave electric field become preferentially accelerated or decelerated. Hence, the most accurate physical description of electron behavior in the Outer Radiation Belt is kinetic plasma physics. Unfortunately, for computational purposes, a full first-principles kinetic plasma description of the plasma in six phase space dimensions ($x, y, z, p_x, p_y,$ and p_z) is truly intractable, especially for the length and timescales of the electromagnetic waves involved. Instead, we typically turn to quasilinear diffusion theory (QLT) to describe the wave-particle interactions that control the variability of the Outer Radiation Belt. This important theoretical breakthrough [*Kennel and Engelmann*, 1966] follows slow changes in phase space density $f(L^*, \mu, J)$ as a result of the much more rapid wave-particle interactions, where L^* is a location parameter tied to the geomagnetic field topology and μ and J are the first and second adiabatic invariants, respectively [*Schulz and Lanzerotti*, 1974]. QLT allows us to reduce the number of dimensions required in phase space and use much coarser grids in time and space than first-principles modeling would allow.

Quasilinear diffusion theory is the bedrock of the Outer Radiation Belt modeling community. This powerful theory has allowed scientists to model the full radial extent of the Radiation Belts and how they respond to a myriad of different wave-particle interactions in the magnetosphere. For example, radial diffusion due to ultralow-frequency (ULF) waves (1–10 mHz) can result in inward transport of electrons such that they gain energy through conservation of the first adiabatic invariant [e.g., *Walt*, 1971]. At the same time, pitch angle scattering due to extremely low frequency/very low frequency (ELF/VLF) plasmaspheric hiss (0.1–1 kHz) can result in increased loss of energetic particles to the upper atmosphere [*Lyons et al.*, 1972]. Radiation belt models (RBMs) based upon QLT allow the relative contribution of each type of wave-particle interaction to be quantified and, given the number of competing processes that can occur in the Outer Radiation Belt, provide the current best method to probe the underlying physics. Recent advances allow some RBMs to be run as operational predictive models [e.g., *Horne et al.*, 2013a].

Phase space diffusion depends upon the amplitudes of the waves, as well as the resonant details of the interaction. In RBMs, wave amplitudes are prescribed using observational data and so empirical wave models are a key component. Models are constructed from observational data from many different inner magnetospheric missions (e.g., Dynamics Explorer 1, Combined Release and Radiation Effects Satellite (CRRES), Cluster, Double Star TC1, and Time History of Events and Macroscale Interactions during Substorms (THEMIS), as used by *Meredith et al.* [2012]). Typically, the inner magnetosphere is divided into volume elements whose boundaries are determined by the magnetic field; i.e., typically, models are constructed in L^* and magnetic latitude λ, although more sophisticated models are available that include magnetic local time (azimuthal) variations [*Horne et al.*, 2013b]. As spacecraft pass through each cell over the course of many years or decades, observations of wave power, spectral distribution, and wave normal angle are collected. Average (mean) wave properties are then calculated. Where appropriate, average wave properties are collected as a function of global geomagnetic activity. For example, *Meredith et al.* [2012] use the *AE* index [*Davis and Sugiura*, 1966] to parameterize the wave power. Sometimes, the planetary *K* index *Kp* [e.g., *Mayaud*, 1980] is used for an operational model, as it can be predicted from upstream solar wind data [e.g., *Wing et al.*, 2005; *Horne et al.*, 2013a].

In this article we explore in detail whether the theoretical basis of QLT supports the use of empirical wave models such as these. Given the non-Gaussian statistical properties of waves in the magnetosphere, we will question whether averages of wave parameters are appropriate for RBMs. Our findings indicate that current empirical models significantly overestimate the action of wave-particle interactions due to ELF and VLF waves in the magnetosphere. We find no evidence of a deterministic model of wave activity based upon geomagnetic activity, and we suggest alternative parameterization methods that will improve the description of wave-particle interactions in models of the Outer Radiation Belt.

2. Quasilinear Theory and Timescales

We consider here important aspects of QLT germane to the modeling of electron diffusion due to ELF/VLF waves in the inner magnetosphere, focusing particularly on timescales. QLT describes the slow evolution of a spatially averaged phase space density f_0 [*Drummond and Pines*, 1964]. The timescale of quasilinear diffusion τ_{diff} is assumed to be long compared to the wave period T, and the spatial averaging takes place over lengths much larger than the wavelength L. In RBMs, the time step τ_{RBM} is of the order of minutes to hours and the length scale in the radial direction ℓ_{RBM} is of the order of 0.1 – 1 R_E [e.g., *Subbotin and Shprits*, 2009]. The quasilinear diffusion coefficients are used to describe the subgrid physics occurring over the spatial cells in the model. We discuss here whether these scales are appropriate for quasilinear momentum-space diffusion due to ELF/VLF waves. Note that here we will include electromagnetic ion cyclotron (EMIC) waves, typically at the upper end of the ULF range (as defined by the Union Radio Scientifique Internationale - URSI), in our discussion, since they contribute to momentum-space diffusion in RBMs. Empirical wave models and QLT diffusion coefficients are constructed in the same way for EMIC waves as they are for whistler mode waves throughout the magnetosphere, and so it is appropriate to include them here.

Quasilinear diffusion can only occur when and where there are waves present, so we define an interaction time τ_{exist} to describe the length of time during which waves in a particular volume of space exist. In idealized circumstances, this would be the length of time required for the waves to grow, interact with the plasma, saturate, and die away. Numerical experiments can provide an estimate of τ_{exist}, although they are usually of the form of initial-value simulations, and so without further sources of free energy, they likely provide a lower bound for τ_{exist}. Reported observations often take the form of case studies, from a variety of platforms where spatial-temporal ambiguities are difficult to disentangle, but we suggest that they form a useful upper bound for τ_{exist}. Typically, in numerical simulations of whistler mode waves where plasma parameters are chosen to promote fast growth, $\tau_{\mathrm{exist}} \ll 1$ s [e.g., *Fu et al.*, 2014; *Ratcliffe and Watt*, 2017]. In situ observations of whistler mode wave chorus indicate repeating individual packets of waves with short timescales of less than a second within an envelope of activity lasting a few minutes [e.g., *Santolík et al.*, 2004], small pockets of waves in space within a finite source region, or a combination of these spatiotemporal effects.

Observations of whistler mode waves inside the plasmasphere indicate that plasmaspheric hiss can persist for tens of minutes [*Tsurutani et al.*, 2015]. Numerical simulations of EMIC waves [e.g., *Denton et al.*, 2014] show that EMIC waves can persist for tens of seconds. Observations of EMIC waves from ground magnetometers demonstrate that they can persist at the edge of the plasmasphere for many hours [*Mann et al.*, 2014]. For all wave observations used to create empirical wave models of ELF/VLF waves, $T \ll \tau_{\mathrm{exist}}$, as required by QLT. It is important, however, to note that the period of time during which some wave modes exist may be shorter than a single RBM time step (~10 min to 1 h).

From calculations of diffusion coefficients based upon observed average wave amplitudes and spectral properties from an empirical model [e.g., *Meredith et al.*, 2012], diffusion timescales for ELF/VLF waves are $\tau_{\mathrm{diff}} \sim 1 - 100$ h [e.g., *Glauert et al.*, 2014]. The empirical wave models themselves are constructed from mean values obtained over much longer timescales, $\tau_{\mathrm{WM}} \sim$ years or decades. The temporal variability of the waves is assumed to be dependent upon some measure of geomagnetic activity, and so the mean value of observed wave power is used once observations have been collected for similar values of *AE* or *Kp*.

For intermittent waves such as magnetospheric chorus, we therefore have $\tau_{\mathrm{exist}} \lesssim \tau_{\mathrm{RBM}} \ll \tau_{\mathrm{diff}} \ll \tau_{\mathrm{WM}}$. For longer-lived waves such as plasmaspheric hiss, we have $\tau_{\mathrm{RBM}} \ll \tau_{\mathrm{exist}} \ll \tau_{\mathrm{diff}} \ll \tau_{\mathrm{WM}}$. In short, empirical wave models use mean values of wave power obtained over timescales τ_{WM} which are very long compared to the diffusion timescale and compared to the RBM timescale. We discuss below the ramifications of using average wave power over τ_{WM} timescales in RBMs.

3. Distributions and Averages of Whistler Mode Wave Power

We use Cluster Spatio-Temporal Analysis of Field Fluctuations (STAFF) [*Cornilleau-Wehrlin et al.*, 2003] magnetic field data for the period 2001–2013 to accumulate wave observations in a cell defined by $6.1 < L < 7.1$, 6–9 magnetic local time and between $-4°$ and $+4°$ magnetic latitude from the equator. This region exhibits enhanced whistler mode wave power during heightened geomagnetic activity [e.g., *Li et al.*, 2010] and is therefore perfect for our illustration. Note that our results are not sensitive to our choice of cell boundary; the qualitative results hold whether large or small cells are chosen. Similar wave distributions can be seen over a wide region of the magnetosphere for different types of waves [see, e.g., *Orlova et al.*, 2014; *Spasojevic et al.*, 2015; *Murphy et al.*, 2016]. For our analysis, magnetic field power spectral density (PSD) is obtained over the frequency range 0.01–4 kHz. For each PSD data point, the local magnetic field strength is used to calculate the electron gyrofrequency Ω_e, and then PSD is integrated between $0.1\Omega_e$ and $0.9\Omega_e$ to obtain an estimate of the whistler mode wave spectral power S_B at that time. Data intervals are only used where the STAFF instrument covers the whole $0.1\Omega_e$ to $0.9\Omega_e$ frequency range. Note that in an empirical wave model for a RBM, the integrated wave power is often used to normalize a spectral function (e.g., a Gaussian with respect to frequency) to allow for more straightforward calculation of the diffusion coefficients [e.g., *Glauert and Horne*, 2005].

Observations from every available spacecraft that pass through the region are processed as described above, and the statistics of the integrated wave power is shown in Figure 1. Figure 1a shows the occurrence of S_B over the 12 year sample period. The accumulation of power near 10^{-6} nT^2 likely indicates the noise floor of the instrument [e.g., *Spasojevic et al.*, 2015]. The red vertical line indicates the mean value of S_B, and the orange line indicates the median. The first thing to note is that these data are not normally distributed. This is not a new finding [e.g., *Spasojevic et al.* [2015]], but it is important for our subsequent discussion. The mean of all the observations in this volume cell is nearly 2 orders of magnitude greater than the median.

It is typical in an empirical wave model to separate wave observations into bins of geomagnetic activity. Here we use AE^*, the maximum value of *AE* over the previous 3 h [e.g., *Li et al.* [2009]], although our qualitative results do not depend upon the specific details of how we define prior substorm activity using the *AE* index. Figures 1b–1d illustrate the occurrence of wave power for increasing values of the geomagnetic activity proxy AE^*. None of these distributions are normally distributed, the means are very different from the medians, and there is a large spread of values in each geomagnetic activity bin. For each level of activity, the mean is around 1 order of magnitude larger than the median. The mean values are similar to those seen in THEMIS data with increasing geomagnetic activity as shown in *Meredith et al.* [2012]. In common with previous studies, we can see from the Cluster data that larger wave amplitude observations are more likely as the

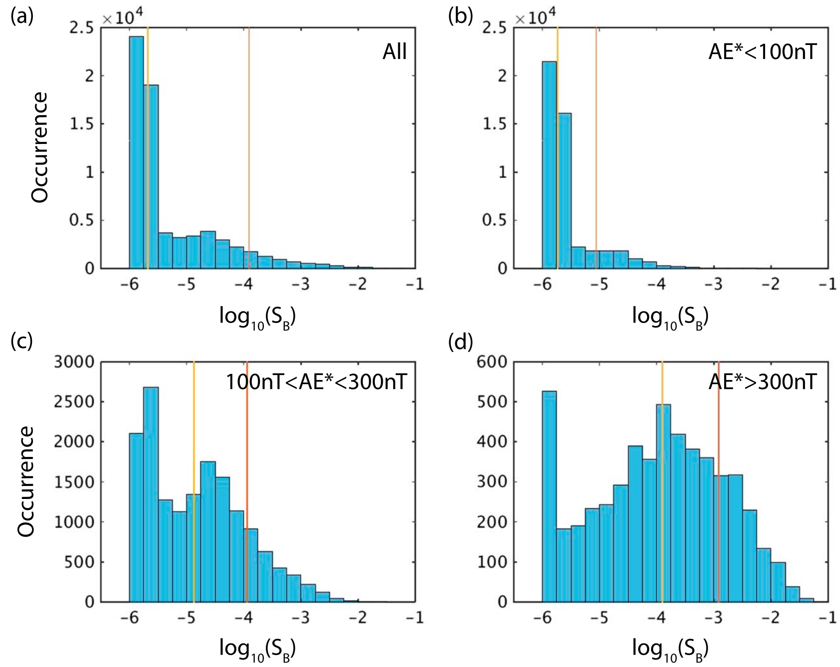

Figure 1. Occurrence of whistler mode wave amplitudes observed by Cluster STAFF magnetic field instrument for period 2001–2013. (a) All data and (b–d) data for geomagnetic activity $AE^* < 100$ nT, 100 nT $< AE^* <$ 300 nT, and $AE^* >$ 300 nT, respectively.

geomagnetic activity increases. This in turn means that the mean wave power increases as the geomagnetic activity increases. However, the mean wave power is not a useful description of the most likely or typical wave power.

The observations in each cell are obtained from sporadic passes of spacecraft through the cell. These passes are not randomly or uniformly distributed in time; there is most likely some systematic resampling frequency due to the spacecraft orbit. However, the occurrence of the passes relative to geomagnetic activity is likely to be random. Over the period $\tau_{WM} = 12$ years, we have effectively randomly sampled the wave activity in each cell as a function of geomagnetic activity. The geomagnetic activity index selected does not adequately describe the temporal variability of the waves, as the spread of possible observed wave power is large. This trend is also seen for other geomagnetic indices, including instantaneous *AE*, *AL*, and *Kp*. Observed wave power across a small volume of the magnetosphere is not well determined by any geomagnetic index, and therefore, geomagnetic index is not a good proxy for the temporal variability of the wave power (additional evidence can be seen in Figure 6 of *Agapitov et al*. [2015]). Previous work has shown that geomagnetic activity does not determine ultralow-frequency (ULF) wave power in a deterministic manner either [*Murphy et al.*, 2016]. We therefore conclude that deterministic models of mean [e.g., *Meredith et al.*, 2012; *Orlova et al.*, 2014; *Spasojevic et al.*, 2015], time-averaged root-mean-square [e.g., *Agapitov et al.*, 2015; *Li et al.*, 2015], or indeed median wave power, based upon any combination of geomagnetic indices, are inappropriate for use in RBMs because the spread of wave power is much larger than the variation in median or mean with geomagnetic activity.

4. Use of Wave Power in a Radiation Belt Diffusion Model

We now explore the use of the mean value as a time average of wave power in a diffusion model. The diffusion equation takes the form [*Schulz and Lanzerotti*, 1974]

$$\frac{\partial f_0(J_i,t)}{\partial t} = g(f_0,J_i,t) = \sum_{i,j} \frac{\partial}{\partial J_i}\left[D_{ij}(t)\frac{\partial f_0(J_i,t)}{\partial J_j}\right] \tag{1}$$

where D_{ij} are the diffusion coefficients and the J_i are the action integrals involving the three adiabatic invariants. The explicit dependence upon time has been added to the right-hand side of the equation for illustrative purposes. For most RBMs, this equation is discretized in time and phase space and integrated forward in time using a numerical algorithm. If, over the period of time being modeled, the values of $g(t)$ were some linear function of the wave power, then the time integration of the diffusion equation using either the mean value of g or a randomly sampled collection from the distribution of $g(t)$ would converge to the same solution as the sampled collection size increases. However, $g(t)$ is significantly more complicated than that.

Most importantly, both the diffusion coefficients and the $\partial f_0(t)/\partial J_j$ change with time. The multiplication of these terms on the right-hand side of equation (1) means that a model where the diffusion equation is time integrated using the mean value of the wave power would not converge to the same solution as a model using a series of diffusion coefficients sampled from a distribution that more realistically reflects the distribution of possible wave power. More simply and more generally, one cannot average the right-hand side of equation (1) in time by averaging only the diffusion coefficient. Even the median of the diffusion coefficients would not necessarily provide the correct response, because the $\partial f_0(t)/\partial J_j$ change with time.

In addition, the calculation of the temporally varying diffusion coefficient is not a straightforward function of the wave power at any particular frequency. Various coordinate transforms can be used to change from the action integrals in equation (1) to a more convenient coordinate system such as momentum or energy, pitch angle, and L^*. For example, momentum-space (p, α) diffusion coefficients can all be calculated from the pitch angle scattering diffusion coefficient $D_{\alpha\alpha}$ [e.g., *Glauert and Horne*, 2005]

$$D_{\alpha\alpha}(L^*, p, \alpha, t) = \frac{e^2}{4\pi} \sum_n \int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta}{\cos\theta} \sum_i \frac{\hat{B}^2_\omega(\omega_i, t) G(\theta) \left|\Phi_{n,k}\right|^2}{\left|v_{||} - \partial\omega/\partial k_{||}\right|_{k_{||,i}}} \left(\frac{\frac{n\Omega_{ce}}{\gamma} - \omega_i \sin^2\alpha}{\cos\alpha} \right)^2 \tag{2}$$

In equation (2), $\hat{B}^2_\omega(\omega_i, t)$ is the wave power at ω_i, which is a simultaneous solution of the resonance equation

$$\omega - k_\parallel v_\parallel = \frac{n\Omega_e}{\gamma} \tag{3}$$

and the dispersion relation for whistler mode waves $D(k, \omega, \theta) = 0$. The other terms are related to the resonance conditions and the relationship between electric and magnetic fields in cold plasma waves and are not related to the amplitude of the wave power and so will not be discussed here. Since there can be more than one resonant frequency for a particular wave mode [*Lyons*, 1974; *Albert*, 1999], the diffusion coefficient is not a simple function of the wave power. The distribution of the diffusion coefficients with time is therefore not guaranteed to have the same distribution as the wave power.

For the observations shown in Figure 1, at least 88% of the observations of wave power are less than the mean value and 80% are less than the time-averaged root-mean-square value. If one constructed a model where the diffusion coefficients were randomly sampled from a distribution that reflected the observed wave power, diffusion coefficients would often be much weaker than those calculated from the mean wave amplitude. We therefore suggest that most RBMs significantly overestimate the diffusion due to whistler mode waves by using the mean wave power to calculate diffusion coefficients, since the mean wave power is much larger than the typical, or median, value.

5. Future Directions

Diffusion models enable scientists to study the different mechanisms at play in the Outer Radiation Belt and provide a framework with which to investigate how geomagnetic activity, in all its forms, influences high-energy electrons in near-Earth space. However, it is clear that current methods for constructing diffusion coefficients from the time average of ELF/VLF wave power overestimate the action of important wave-particle interactions in the Outer Radiation Belt, and alternative methods are required.

First, we suggest that further investigation into the characteristic time and length scales of ELF/VLF diffusive processes is necessary. Given that important wave modes in the magnetosphere such as whistler mode waves can be bursty and/or localized, the average effect of the resulting diffusion across longer time and length scales is currently unknown. Idealized diffusion models across smaller computational domains and over shorter timescales are relatively straightforward to construct and will yield important insight into the subgrid diffusion physics for RBMs. In situ observations can be used to constrain the time and length scales of regions of wave activity. Since many current RBMs are at least bounce averaged [*Shprits et al.*, 2015] if not bounce and drift averaged [e.g., *Su et al.*, 2011; *Glauert et al.*, 2014], it is important to ensure that the averaging time and length scales are appropriately chosen for the physics in the model.

Second, we suggest that diffusion coefficients should be constructed for observed wave spectra *before* a parameterization method is applied. We argue that a deterministic model of wave power as a function of any other geomagnetic property such as activity index, upstream solar wind properties, or local plasma parameters either has not yet been found or does not exist. Other fields (e.g., in numerical weather prediction) have developed techniques to include subgrid physics in large-scale models known as stochastic parameterizations where parameters are chosen randomly from an appropriate distribution of observed or modeled values. In the case of a RBM, the parameter should be the diffusion coefficient, and not the wave characteristics, for the reasons noted above. Note that the introduction of stochastic parameterization has significantly improved numerical weather prediction over short and long timescales [e.g., *Berner et al.*, 2017]. We suggest that modeling of phase space diffusion due to all wave modes important for the Outer Radiation Belt will benefit from such a treatment. Given the wealth of in situ measurements currently at our disposal, we look forward to improved characterizations of subgrid wave-particle interactions that will provide better insight into the physics of the Outer Radiation Belt.

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