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Conference or Workshop Item

Published Version

Nishiyama, D. and Radosavljevic, M. (2009) Mathematical modelling of decision making processes in construction projects. In: 25th Annual ARCOM Conference, 7-9 September, Nottingham, UK, pp. 85-94. Available at http://centaur.reading.ac.uk/8132/

MATHEMATICAL MODELLING OF DECISION MAKING PROCESSES IN CONSTRUCTION PROJECTS

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Many different individuals, who have their own expertise and criteria for decision making, are involved in making decisions on construction projects. Decision-making processes are thus significantly affected by communication, in which a dynamic performance of human intentions leads to unpredictable outcomes. In order to theorise the decision making processes including communication, it is argued here that the decision making processes resemble evolutionary dynamics in terms of both selection and mutation, which can be expressed by the replicator-mutator equation. To support this argument, a mathematical model of decision making has been made from an analogy with evolutionary dynamics, in which there are three variables: initial support rate, business hierarchy, and power of persuasion. On the other hand, a survey of patterns in decision making in construction projects has also been performed through self-administered mail questionnaire to construction practitioners. Consequently, comparison between the numerical analysis of mathematical model and the statistical analysis of empirical data has shown a significant potential of the replicator-mutator equation as a tool to study dynamic properties of intentions in communication.

Keywords: mathematical modelling, decision making, communication.

INTRODUCTION

The notion of decision making appears in a wide range of arguments on construction research, such as construction contracts (Murdoch and Hughes 2008), ethics in construction activities (Fewings 2009), project management (Fewings 2005, Walker 2007), and human resource management (Loosemore, Dainty and Lingard 2003). Their common arguments are that many different individuals, who have their own expertise and criteria for decision making, are involved in making decisions on construction projects. From a historical perspective, the arguments regarding optimisation and rationalisation have been popular in the construction management research to find an optimal solution by computers (e.g. Hua 2008). Such arguments are in line with the development of decision support systems.

Does an optimum solution found by computers become a final decision in the construction world? Even if an optimum solution is obtained, it is unavoidable to communicate with people involved in order to make a final decision. Cairns (2008) made a similar point:

It is acknowledged that, at the level of the local, discussion between involved actors will elicit a variety of perspectives that are informed by different theoretical and

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Nishiyama, D. and Radosavljevic, M. (2009) Mathematical modelling of decision making processes in construction projects. *In:* Dainty, A.R.J. (Ed) *Procs 25th Annual ARCOM Conference,* 7-9 September 2009, Nottingham, UK, Association of Researchers in Construction Management, 85-94.

conceptual standpoints, underpinned by different beliefs and values, and influenced by the power relations and the politics of those involved.(Cairns 2008)

This suggests that the communication among individuals who have different intentions is arguably an important element of decision making in construction.

Can a holistic model of decision making processes including communication be built? It is a research aim to provide such a mathematical model of decision making. In order to realise this aim, one of the mathematical formula in evolutionary dynamics was borrowed here, namely the replicator-mutator equation (e.g. Komarova 2004). Then, the questionnaire survey was conducted for the validation of the mathematical modelling.

EVOLUTIONARY DYNAMICS

Evolutionary dynamics is the mathematical description of evolution, which is based on selection and mutation (Nowak 2006). In light of history, the theory of evolution has become more mathematical over time. This trend in evolution can mean that verbal approaches to evolutionary perspectives have become unsatisfactory. In fact, evolutionary dynamics has been applied to a variety of academic topics, which are summarised in Table 1.

Table 1: The contemporary applications of evolutionary dynamics

	Outline of argument
Infectious diseases and evolution of cancer	The mechanism of disease progression and cancer progression is explained by evolutionary dynamics. For instance, the following questions have been argued: how the balance of power between virus and the immune system can impact the outcome and how long it takes for a population of reproducing cells to inactivate a tumour suppressor gene.
Games and evolution of cooperation	Games including the Prisoner's Dilemma are represented by introducing a payoff matrix in evolutionary dynamics. An important question is how natural selection can lead to altruistic interactions. The nature of cooperation and defection is analysed. The evolutionary game dynamics is explained with "the replicator equation."
Evolution of human language	A theory describing the deterministic evolutionary dynamics of grammar is constructed by means of combining three different worlds: formal language theory, learning theory, and evolutionary dynamics. It has been argued about how children learn grammar, and how a population can evolve grammar. The fundamental equation is "the replicator-mutator equation."

Note: based on Nowak (2006)

In particular, applications of evolutionary dynamics to the evolution of cooperation and language are of the essence because of the arguments on the social phenomena. Furthermore, the arguments are based on a deterministic differential equation (i.e. the replicator-mutator equation), which can not only reach an equilibrium, but also demonstrate oscillations and chaos (Nowak and Sigmund 2004). The equation is given as follows:

$$\dot{x}_i = \sum_{j=1}^{n} x_j f_j(\vec{x}) q_{ji} - \phi x_i \quad i = 1, \ldots, n \tag{1}$$

There are n types in a population. The frequency of type i is given by x_i and its fitness by f_i, which generally depends on all frequencies, $f_i(\vec{x})$, representing selection dynamics. Meanwhile, the mutation probability from type i to type j is denoted by q_{ij}, which are entries of the mutation matrix. The average fitness of the population is

$$\phi = \sum_{i=1}^{n} x_i f_i \ . \tag{2}$$

Fitness values are often linear functions,

$$f_i(\vec{x}) = \sum_{j=1}^{n} x_j a_{ij} \ . \tag{3}$$

The coefficients a_{ij} are entries of the payoff matrix. The equation (1) is the most generalised deterministic description of evolutionary dynamics. Interestingly, it can be transformed into adaptive dynamics and the Price equation and also can be used for models of language evolution (Nowak and Sigmund 2004).

In the model of language development, the dynamics is based on a communicative sense that individuals who communicate successfully are more likely to influence language acquisition of others. The interactions within the linguistic community (i.e. children learn the language of their parents) are described by communicative functions based on the payoff matrix, a_{ij}, while the language acquisition is given by the mutation matrix, q_{ij}.

Meanwhile, another important application of the replicator-mutator equation is the evolution of cooperation. In this case, where mutation is negligible, the replicator-mutator equation becomes the replicator equation which describes selection only. The nature of cooperation is condensed in the payoff matrix, which considering the game between two types of option, A and B, is given by

$$\begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \end{array} .$$

(4)

In general, there are four outcomes (i) Dominance: B disappears, if A is the best reply to both A and B (i.e. a>c and b>d). (ii) Bistability: Either A or B disappears, depending on the initial condition, if each option is the best reply to itself (i.e. a>c and b<d). (iii) Coexistence: A and B coexist in stable equilibrium, if each option is the best reply to the other (i.e. a<c and b>d). (iv) Neutrality: The frequencies of A and B do not change, if each option fares as well as the other for any composition of the population (i.e. a=c and b=d) (Nowak and Sigmund 2004).

From the two applications of the replicator-mutator equation, it becomes clear that the payoff matrix is related to the feature of social interaction in the population, while the mutation matrix has a connection with the competence of success in the interaction. For example, the hierarchical relationship between hawks and doves can be represented in the payoff matrix (Hofbauer and Sigmund 1998). Then, the probability of change from one intended matter into another can be incorporated into the mutation matrix.

APPLICATION TO DECISION MAKING

Communication in decision making processes

Communication is dynamic and unpredictable in terms of uncertain outcomes. In general, communication is defined as any process where orders, information, and advice are transmitted from one agency to another (Simon 1997). Despite of the media of communication, whether it is oral or written, the transmittal of information affects people's deliberations. Philosophically speaking, deliberation is structured by intentions and also ends up in actions (Bratman 1987). Therefore, intentions in communication are regarded as a basic element in decision making processes.

Human intention is inseparable from the individual action, which is the basic building block of explanation for social science, such as management theories (Elster 1983, Ghoshal 2005). Of course, construction management is no exception in that individual action is guided by intentions. For instance, decision-making based on morality and ethics, which are related to intentions and are articulated in the difference of concepts between good/bad or right/wrong, is comprehensively argued in the field of construction management with an increase in closer relationships in construction projects (Fewings 2009).

Originally, the concept of intention, which has been argued by philosophers (e.g. Anscombe 1963, Davidson 1980), characterises both people's actions and their minds (Bratman 1990: 15). For example, the expression of doing something intentionally is concerned with actions, while the description of intending to do something or having an intention concerns the sense of minds. Philosophically, intention can be defined as an element of deliberation on deciding to do something (Bratman 2007).

Strictly speaking, intention is different from decision, but essentially influences decision. Decisions, which are the upshot of deliberation between options, are made in a specific context, while intentions are independent of context (Bratman 1999). Namely, intention has more general sense than decision. In theory, a wide variety of decisions, which have been made in different contexts, result in the formation of intentions. Such mutual interactions give decisions the pressure to be compatible with intentions themselves. That is why the agents have to coordinate various activities over time. From this perspective of intrapersonal conflict, intentions essentially influence every decision through coordination.

Moreover, in a social setting to make a decision, coordination is required in interpersonal relationship, which is characterised by communication (Bratman 1999). Suppose two participants in a construction project intend to lay a pavement together. One intends to make it all asphalt, another intends to make it all concrete. They know each other's intentions, but they are not willing to compromise. In this case, it is usual that they communicate with each other in order to decide which pavement is good and right for the project. Apparently, the communication will lead to a decision, but it can depend on their relationship such as colleagues, rivals, professional and amateur, or superior and inferior. If they are colleagues or rivals, then the decision can rely only on rational comparison between asphalt and concrete in terms of value for money. If they have a relationship between professional and amateur, then the professional can refute the amateur's intention by persuading him to give up his idea. If they have a clear hierarchical relationship between superior and inferior, then the inferior can have no choice but to obey the superior, regardless of his intention. In this way, interpersonal relationship involving communication affects a decision vitally.

Analogy of decision making processes with evolutionary dynamics

Decision-making processes are similar to evolutionary dynamics in terms of interaction-based phenomena. The similarities are summarised in Table 2. In evolutionary dynamics, there are three variables: the initial frequency, $x_i(0)$, the payoff matrix, a_{ij}, and the mutation matrix, q_{ij}. These variables can be interpreted in the decision making processes as follows: the initial support rate, the authority, and the power of persuasion. The initial support rate can be interpreted as a function of rational comparison between options. Authority is a power in which a subordinate would accept his behaviour to be guided by the decision of a superior, and the superior would just obtain his acquiescence regardless of convincing him (Simon 1997). In contrast, persuasion is a kind of power to make others act in an intended way by confidence, which makes sense as mutation. Interestingly, it is argued that power can exist without authority, and that authority works without competence (Keltner, Gruenfeld and Anderson 2003, Anderson and Kilduff 2009). As argued, there are various combinations of the variables in addition to the number of options. In the case of two options, there are at least 48 possible patterns, as shown in Table 3.

Table 2: Similarities between decision making processes and evolutionary dynamics

Decision-making processes	Evolutionary dynamics
Initial frequency of individuals who have the same intention in a group	Initial frequencies of types in population
↓	↓
Communication between different individuals with business hierarchy and persuasiveness	Interaction with selection (Game) and mutation on the basis of reproduction
↓	↓
Group Decision (e.g. agree/disagree)	Equilibrium (e.g. survival/extinction), oscillation, or chaos

Table 3: Patterns of variables in decision making processes

Initial support rate	Authority (Selection)	Power of persuasion (mutation)
X1(0)>X2(0) Option 1 has a greater initial support among participants.	Dominant Either option has higher fitness, due to higher-ranking people's supports.	q11=q22=1 Both options have strong persuasion. Each supporter is convinced in his selection.
X1(0)<X2(0) Option 1 has a lower initial support among participants.	Neutrality Both options have the same fitness because participants hold equivalent positions.	q11>q22 Option 1 is more convincing.
X1(0)=X2(0) Both options are equally supported among participants.	Bistability Each option is the best reply to each supporter.	q11<q22 Option 1 is less convincing.
	Coexistence Each option is the best reply to the other supporter.	q11=q22=const. Both options have similar persuasion, although it is not strong.
3 patterns	4 patterns	4 patterns

METHODS AND SAMPLE

In order to evaluate the patterns of decision making, self-administered mail questionnaires were distributed to 550 practitioners of construction works. The questionnaires include demographic questions and hypothetical questions concerning some patterns of decision making, which are identified in Table 3. The design of hypothetical questions consists of seven critical scenarios, where the factors considered were the initial support rate (greater/even/lower), the business hierarchy (higher/equivalent/lower), and the power of persuasion (more/even/less). All scenarios include two options to be selected on a 7-point Likert scale (1: extremely unlikely; 7: extremely likely). The data of the scale have identified to what extent each option would have potential to be agreed in the scenario, as well as whether the scenario itself has occurred. An example of the questions is as follows:

Imagine a problem-solving situation where the colleagues involved hold equivalent positions, one of the possible solutions has a MUCH GREATER initial support among colleagues, but the other solution with LOWER initial support is MORE convincing (e.g. better solution).

The situation has occurred ☐ Never ☐

If so, please answer ↓

What is the likelihood that in such a problem-solving situation you all would agree on going for the solution with MUCH GREATER initial support?

extremely unlikely: 1 2 3 4 5 6 7 : extremely likely

What is the likelihood that in such a problem-solving situation you all would agree on going for the solution with MUCH LOWER initial support?

extremely unlikely: 1 2 3 4 5 6 7 : extremely likely

Statistical analysis including two-sample t-test, which is aimed at clarifying effects of the three factors in the scenarios, has been performed on the data by using Instat+ for Windows; Version 3.036 (2006, The University of Reading, UK).

Consequently, 78 people responded within 45 days after commencing the distribution at the end of February 2009 (i.e. the response rate was 14%). The general information from the respondents is summarised in Table 4. The majority of respondents belong to local authorities in the UK (59%) and work as senior managers (53%).

Table 4: Characteristics of respondents

Location	Profession	Age	Gender	Sector	Employee number
London 10.26%(8)	Director 12.82%(10)	0-20 years 1.28%(1)	Male 90.67%(68)	Construction 23.08%(18)	20 or less 8.97%(7)
England 74.36%(58)	Senior manager 52.56%(41)	21-30 years 7.69%(6)	Female 9.33%(7)	Consultancy 15.38%(12)	21-50 7.69%(6)
Wales 1.28%(1)	Middle/junior manager 23.08%(18)	31-40 years 23.08%(18)		Local Authority 58.97%(46)	51-200 23.08%(18)
Scotland 14.10%(11)	Others 11.54%(9)	41-50 years 33.33%(26)		Others 2.56%(2)	201-500 24.36%(19)
Northern Ireland (0)		51+years 34.62%(27)			501 or more 35.90%(28)
Total (78)	(78)	(78)	(75)	(78)	(78)

RESULTS AND DISCUSSION

The results of the seven scenarios to be investigated are summarised in Table 5. Apart from this table, the practicality of all scenarios is supported by the majority of respondents (around 70%) in terms of whether scenarios have occurred in real-life situations. With respect to each scenario, there were significant differences between the two options in five scenarios (i.e. scenario 1, 2, 5, 6, and 7), while the other two did not make significant differences (i.e. scenario 3 and 4).

For example, scenario 2; where colleagues involved hold equivalent hierarchical positions, showed a significant difference that option 2 which has lower initial support among colleagues but is more convincing, is more likely to be agreed among people than option 1 which simply has a much greater initial support (i.e. mean probability of 4.5 as opposed to 4.0). Conversely, there was no significant difference between the two options (i.e. mean probability of 4.0 each other) in scenario 3; where the people involved hold different hierarchical positions, are all 100% convinced in their selected options, and the higher-ranking people prefer option 2.

Furthermore, comparison of some options beyond scenarios has been conducted in order to clarify the effects of the three variables on decisions, as shown in Table 6. It is largely interpreted that the business hierarchy is more effective than the power of persuasion, and the power of persuasion is more influential than the initial support rate.

On the other hand, aside from a statistic point of view, meaningful changes have been seen in the comparison between scenario 1, 2, 5, and 7. In scenario 1, option 1, which has greater initial support among people, tends to become agreed, while in scenario 2, where the power of persuasion exists, the more convincing option (option 2) slightly prevails over option 1. Then in scenario 5, where the business hierarchy appears, option 1 that higher-ranking people like prevails over option 2 again. Finally, for scenario 7, the result that option 1 prevails remains as was the case for scenario 5, regardless of the difference in initial support rate.

These meaningful outcomes above can be described by manipulating the variables of the decision making based on equation (1), as illustrated in Figure 1. Of course, the values of variables used in the figure are all assumption, but surely they follow the qualitative categories of patterns of variables (e.g. neutrality or dominant). Perhaps, it is another research to identify the specific values of variables.

Nevertheless, the figure depicts an interesting characteristic that even though an optimum option in terms of rational comparison is greatly supported among people, it is not necessarily the final decision among people. The unreasonable feature of decision making absolutely relies on communication, which is given by the three variables: initial support rate, business hierarchy, and power of persuasion. Therefore, it is considered that unpredictable outcomes (e.g. bribery or collusion) can appear. This is how the change of intentions through communication can shed light upon complex decision making.

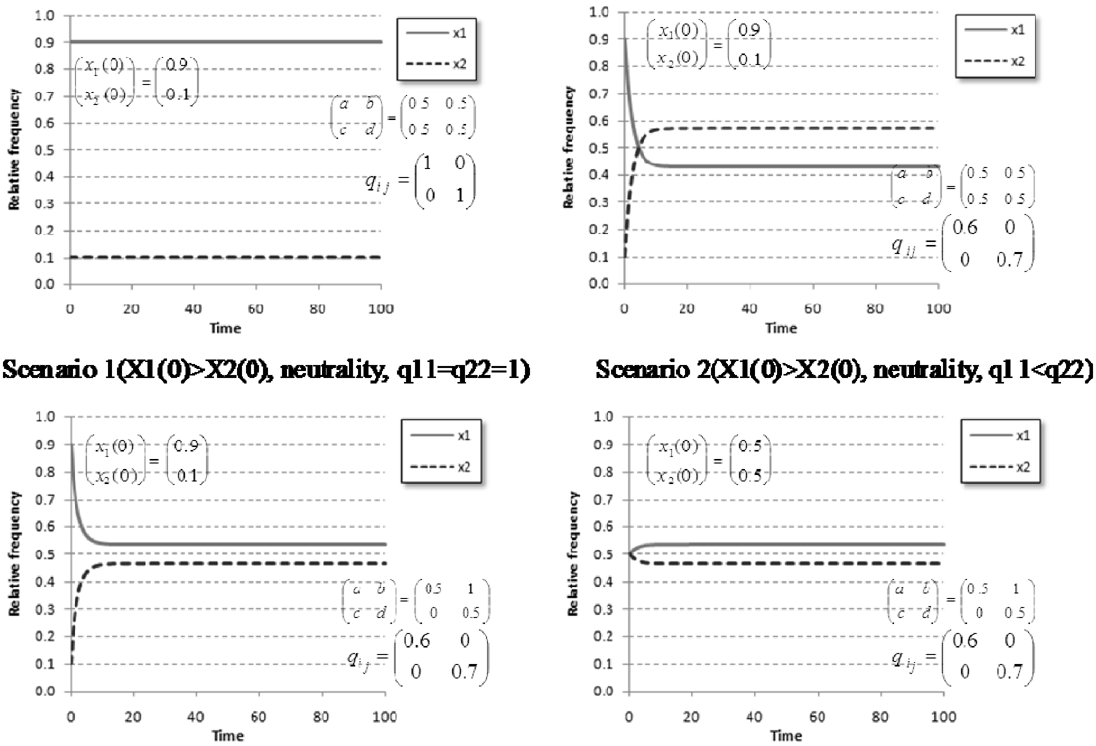
Table 5: Results of the seven scenarios

Scenario	Option	Variables* Initial support	Business hierarchy	Persuasion	Means (1 to 7)	Differences (Op.1-Op.2)	Standard errors of difference	Statistical significance (two-sided t-test)
1	1	+			5.000	1.9310	0.1998	Significant
	2	-			3.069			(P=0.0000)
2	1	+		-	3.984	-0.4831	0.2160	Significant
	2	-		+	4.467			(P=0.0272)
3	1	+	-		4.000	-0.0377	0.2289	NS
	2	-	+		4.038			(P=0.8694)
4	1	+	-	+	4.204	0.3148	0.2438	NS
	2	-	+	-	3.889			(P=0.1994)
5	1	+	+	-	4.691	1.0909	0.2194	Significant
	2	-	-	+	3.600			(P=0.0000)
6	1		+		5.322	2.4255	0.1922	Significant
	2		-		2.897			(P=0.0000)
7	1		+	-	4.259	0.4317	0.2098	Significant
	2		-	+	3.827			(P=0.0420)

‘Greater, higher or more: + /even or equivalent: "blank" /lower or less: -’ are given respectively.

Table 6: Interpretations of the results of the seven scenarios

Scenario	Option	Probability to be a final decision	Interpretations
1	1	Likely	Initial support works
	2	Unlikely	
2	1	Unlikely	Persuasion > Initial support
	2	Likely	
3	1	Could be	Initial support = Business hierarchy
	2	Could be	
4	1	Could be	Initial support + Persuasion = Business hierarchy
	2	Could be	
5	1	Likely	Initial support + Business hierarchy > Persuasion
	2	Unlikely	
6	1	Likely	Business hierarchy works
	2	Unlikely	
7	1	Likely	Business hierarchy > Persuasion
	2	Unlikely	



Scenario 1(X1(0)>X2(0), neutrality, q11=q22=1)

Scenario 2(X1(0)>X2(0), neutrality, q11<q22)

Scenario 5(X1(0)>X2(0), dominant, q11<q22)

Scenario 7(X1(0)=X2(0), dominant, q11<q22)

Figure 1: Numerical analysis of decision making processes

CONCLUSIONS

Communication is an inevitable process in making a decision. Based on this view, it is argued here that the decision making processes between two options resemble evolutionary dynamics in terms of taking into account interactions, which can be simply expressed by the replicator-mutator equation. The application of evolutionary dynamics to decision making processes is partly validated by empirical data collected from questionnaire surveys. A good example is that even though an optimum solution in terms of rational comparison is greatly supported among people, it does not necessarily become the final decision among people because a solution which higher-ranking people like is likely to be employed in construction projects. Of course, the development of decision support systems is important for the construction world, but it does not mean that an optimum solution found by computers is always a final decision among people.

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