

Parameterization of directional absorption of orographic gravity waves and its impact on the atmospheric general circulation simulated by the Weather Research and Forecasting Model

Article

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1 **Parameterization of Directional Absorption of Orographic Gravity Waves and Its**
2 **Impact on the Atmospheric General Circulation Simulated by the Weather**
3 **Research and Forecasting Model**

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Abstract

37 In this work, a new parameterization scheme is developed to account for the directional
38 absorption of orographic gravity waves (OGWs) using elliptical mountain wave theory. The
39 vertical momentum transport of OGWs is addressed separately for waves with different
40 orientations through decomposition of the total wave momentum flux (WMF) into individual
41 wave components. With the new scheme implemented in the Weather Research and Forecasting
42 (WRF) model, the impact of directional absorption of OGWs on the general circulation in boreal
43 winter is studied for the first time. The results show that directional absorption can change the
44 vertical distribution of OGW forcing, while maintaining the total column-integrated forcing. In
45 general, directional absorption inhibits wave breaking in the lower troposphere, producing
46 weaker orographic gravity wave drag (OGWD) there and transporting more WMF upwards. This
47 is because directional absorption can stabilize OGWs by reducing the local wave amplitude.
48 Owing to the increased WMF from below, the OGWD in the upper troposphere at midlatitudes is
49 enhanced. However, in the stratosphere of mid-to-high latitudes, the OGWD is still weakened
50 due to greater directional absorption occurring there. Changes in the distribution of midlatitude
51 OGW forcing are found to weaken the tropospheric jet locally and enhance the stratospheric
52 polar night jet remotely. The latter occurs as the adiabatic warming (associated with the OGW-
53 induced residual circulation) is increased at midlatitudes and suppressed at high latitudes, giving
54 rise to stronger thermal contrast. Resolved waves are likely to contribute to the enhancement of
55 polar stratospheric winds as well, because their upward propagation into the high-latitude
56 stratosphere is suppressed.

57 **1 Introduction**

58 Mountains can generate gravity waves capable of transporting momentum upward from
59 the troposphere to the middle atmosphere (Fritts and Alexander 2003; Alexander et al. 2010).
60 Momentum transport by these orographically forced gravity waves (OGWs) or mountain waves
61 has an important impact on the general circulation of the middle atmosphere where gravity
62 waves tend to break. However, small-scale OGWs cannot be fully resolved by even high-
63 resolution climate models like the Community Earth System Model (CESM, Hurrell et al. 2013).
64 The effects of unresolved OGWs need to be parameterized in these models (Kim et al. 2003).

65 The parameterization of OGWs within numerical weather prediction (NWP) models dates
66 back to the early 1980s. Palmer et al. (1986) and McFarlane (1987) established the first-
67 generation OGW parameterization schemes according to the Eliassen-Palm (EP) flux theorem
68 (Eliassen and Palm 1961) and the wave saturation hypothesis (Lindzen 1981). Later, these
69 schemes were revised to better represent the momentum transport of gravity waves forced by
70 large-amplitude mountains (Kim and Arakawa 1995; Lott and Miller 1997; Scinocca and
71 McFarlane 2000; Webster et al. 2003). The major improvements were the inclusion of the effects
72 of low-level wave breaking and flow blocking, which can respectively cause resonant
73 amplification and reduction of gravity wave drag at the surface. Subgrid-scale orographic (SSO)
74 properties, such as orographic asymmetry, convexity and anisotropy, were also considered (Kim
75 and Doyle 2005).

76 The parameterization of OGWs is considered a necessary component in climate models
77 given their relatively coarse horizontal resolutions. The parameterization can help reduce
78 systematic model biases, such as the cold-pole bias associated with too strong westerlies in the
79 mid and high latitudes, and delayed breakdown of the polar vortex in Antarctica (Palmer et al.

80 1986; Shin et al. 2010; McLandress et al. 2012; Pithan et al. 2016; Garcia et al., 2017; Garfinkel
81 and Oman 2018). Furthermore, medium- and short-range weather predictions can also benefit
82 from OGW parameterization (Hong et al. 2008; Zhong and Chen 2015; Choi and Hong 2015;
83 Choi et al. 2017).

84 Like many other subgrid-scale processes however, gravity-wave drag is still not well
85 represented in models. Biases in modelled atmospheric circulation that may result from an
86 inaccurate representation of this drag are still a significant source of uncertainty in climate
87 change projections (Shepherd 2014). According to the recent inter-comparison exercise proposed
88 by the WMO Working Group for Numerical Experimentation (WGNE), parameterized
89 orographic stresses have a considerable spread among models (Sandu et al. 2016). These large
90 uncertainties have been attributed to the lack of observational constraints, so that parameters
91 controlling the strength of OGWs are often tuned subjectively. A variational data assimilation
92 technique was developed by Pulido and Thuburn (2005), aiming to estimate gravity wave forcing
93 in the middle atmosphere and thus optimize parameterization. The parameters, although
94 estimated for non-orographic gravity waves, have been shown to be helpful in simulating the
95 splitting/breakup of the Antarctic polar vortex (Scheffler and Pulido, 2017).

96 Uncertainties of OGWs also result from misrepresentation of their physics in the model
97 due to simplifying assumptions. For example, parameterized OGWs are assumed to propagate in
98 the vertical only, but in reality they propagate both vertically and horizontally, i.e., they have a
99 three-dimensional propagation (Alexander and Teitelbaum 2011; Kalisch et al. 2014; Ehard et al.
100 2017). Horizontal propagation of OGWs can reduce the local wave amplitude and thus affect
101 wave breaking (Eckermann et al. 2015). Another process influencing the momentum transport of
102 OGWs but missing in existing OGW parameterizations is the directional absorption (or, selective

103 critical-level absorption, Shutts 1995) of wave momentum flux (WMF). Hereafter, the term
104 “WMF” will denote the momentum flux of subgrid-scale OGWs unless otherwise stated. In the
105 case of mean flows turning with height (i.e., directionally sheared wind), there exist an infinite
106 number of critical levels at different heights (Broad 1995) such that OGWs are continuously
107 absorbed during propagation (Teixeira and Miranda 2009; Teixeira and Yu, 2014; Xu et al. 2012,
108 2013). Unlike the orographic gravity wave drag (OGWD), directional absorption of gravity
109 waves exerts a lift force on the mean flow, i.e., an orographic gravity wave lift (OGWL), which
110 is perpendicular to the mean flow (Xu et al. 2012).

111 Recently, Xu et al. (2018) designed an OGW parameterization scheme taking into
112 account the directional absorption of OGWs (hereafter, the X18 scheme). *Offline* evaluation
113 using reanalysis data in X18 showed that the scheme can produce weaker (stronger) OGWD in
114 the lower stratosphere (upper stratosphere and lower mesosphere) because directional absorption
115 tends to a transfer of wave breaking to higher levels. Although offline evaluation can provide
116 some insights into the effects of directional absorption, it is yet unknown how this effect would
117 affect large-scale circulations within actual numerical models. In principle, this can be examined
118 by applying the X18 scheme within a numerical model that enables wave-mean flow interactions.
119 However, the X18 scheme uses a high-order ray tracing method known as the Gaussian Beam
120 Approximation (GBA, Pulido and Rodas 2011; Xu et al. 2017a). Although the GBA solution can
121 be applied to OGWs forced by both idealized and realistic mountains, the wave fields are
122 obtained by superposition of a number of Gaussians. This procedure is computationally very
123 expensive, and hence limits its practical use for OGW parameterization within actual NWP or
124 climate simulation models.

125 In this paper, a computationally more efficient parameterization scheme is proposed for
126 the directional absorption of OGWs by assuming elliptically-shaped mountains. This assumption
127 enables the use of analytical mountain wave solutions (Phillips 1984) within the parameterization
128 scheme and removes the need for expensive ray tracing. Elliptical mountain wave theory has
129 been used in previous OGW parameterization schemes (e.g., Lott and Miller 1997, hereafter
130 LM97), yet the effect of directional absorption of OGWs, based on the theoretical approaches of
131 Teixeira and Miranda (2009), Xu et al. (2012, 2013) and Teixeira and Yu (2014), was never
132 considered. The scheme proposed in this work, which implements those approaches, is therefore
133 used to revise the OGW parameterization scheme in the Weather Research and Forecasting
134 (WRF) model, which was developed by Kim and Arakawa (1995, hereafter KA95) and Kim and
135 Doyle (2005, hereafter KD05). With the original and revised parameterization schemes, global
136 WRF simulations are conducted to examine the impact of directional absorption of OGWs on the
137 large-scale atmospheric circulation.

138 This paper is organized as follows: Section 2 describes the new parameterization scheme
139 and its implementation in the WRF model. The setup of numerical experiments performed using
140 the WRF model is also introduced. In section 3, the effects of parameterized directional
141 absorption on the vertical momentum transport of OGWs and large-scale atmospheric circulation
142 are studied. A summary is given in section 4, including additional discussion.

143 **2 Parameterization of OGWs in directionally sheared winds**

144 2.1 Theoretical framework

145 Gravity waves forced by isolated obstacles are made up of wave components with
146 different orientations. In current operational parameterization schemes, the ambient wind is

147 always assumed to be unidirectional, with all wave components treated as a whole for their
 148 upward propagation and breaking. In the case of winds with directional shear, different wave
 149 components are selectively filtered at different heights. Therefore, they should be addressed
 150 separately (see section 2.3 in Xu et al. 2018).

151 While there appears to be no simple way to represent the shape of the realistic SSO,
 152 previous schemes often assume an elliptical-shaped mountain of the form

$$153 \quad h(x, y) = \frac{h_m}{\left[1 + \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2\right]^\mu}, \quad (1)$$

154 where h_m is the mountain amplitude, a and b are the mountain half-widths in the x and y
 155 directions respectively, and μ denotes the mountain sharpness. In this work, μ is set to 3/2, for a
 156 bell-shaped mountain which has been widely used before (e.g., Teixeira and Miranda 2006). For
 157 hydrostatic and nonrotating airflow over an elliptical bell-shaped mountain, the WMF at the
 158 surface can be readily obtained according to linear wave theory

$$159 \quad \boldsymbol{\tau}_0 = (\tau_x, \tau_y) = 0.5\rho_0 N |V_0| a h_m^2 \gamma \int_{-\pi/2}^{+\pi/2} (\cos\varphi, \sin\varphi) \cos(\varphi - \psi_0) [\gamma^2 \cos^2\varphi + \sin^2\varphi]^{-\frac{3}{2}} d\varphi, \quad (2)$$

160 where ρ_0 is the Boussinesq flow base-state density, N is the Brunt-Väisälä frequency, $|V_0|$ and ψ_0
 161 are the speed and direction of the horizontal wind at the surface, $\gamma = \frac{a}{b}$ is the horizontal aspect
 162 ratio of the mountain, and φ is the azimuthal direction of the horizontal wave number. The
 163 detailed derivation of the above equation is given in the Appendix. It is important to notice that
 164 Eq. (2) is derived in a frame of reference aligned with the main axes of the elliptical mountain,
 165 i.e., τ_x and τ_y are parallel to the two principal axes of the elliptical mountain respectively. For
 166 practical use in the OGW parameterization (such as its implementation in the WRF model

167 presented herein), this WMF needs to be remapped to the model coordinates by rotation of the
 168 coordinate system.

169 Assuming a simple case with $\psi_0 = 0$, i.e., the surface wind is along one of the principal
 170 axes of the elliptical mountain, the above equation reduces to

$$171 \quad \boldsymbol{\tau}_0 = 0.5\rho_0 N |V_0| a h_m^2 \int_{-\pi/2}^{+\pi/2} (\cos\varphi, \sin\varphi) F_{GW}(\varphi, \gamma) d\varphi, \quad (3)$$

$$172 \quad F_{GW}(\varphi, \gamma) = \gamma \cos\varphi [\gamma^2 \cos^2\varphi + \sin^2\varphi]^{-\frac{3}{2}}. \quad (4)$$

173 In our implementation of this expression in the WRF OGWD parametrization scheme, to be
 174 tested for the first time in this paper, the above assumption is made. This is because in the KD05
 175 OGWD scheme adopted in WRF the effective orography widths (i.e., the principal axes of the
 176 elliptical mountain) are only defined in the *along-wind* and in the *across-wind* direction (Fig. 1,
 177 see also Fig. 7 in KD05). Therefore, the low-level wind may be understood as being by
 178 definition along one of the mountain's principal axes. In the $X'OY'$ coordinate defined by the
 179 mountain (Fig. 1), the total WMF at the surface ($\boldsymbol{\tau}_0$) is then simply in the x' direction, owing to
 180 the symmetry of the orography elevation. For any wave component φ_i in the azimuthal range
 181 $(-\frac{\pi}{2}, +\frac{\pi}{2})$, the corresponding WMF is along the direction of φ_i with its magnitude given by

$$182 \quad |\boldsymbol{\tau}_0(\varphi_i, \gamma)| = |\boldsymbol{\tau}_0| \frac{F_{GW}(\varphi_i, \gamma)}{\int_{-\pi/2}^{+\pi/2} F_{GW}(\varphi, \gamma) d\varphi} = |\boldsymbol{\tau}_0| R(\varphi_i, \gamma). \quad (5)$$

183 Evidently, $R(\varphi_i, \gamma)$ only depends on the anisotropy of the assumed elliptical mountain. Thus for
 184 *practical use*, it is feasible to build a look-up table of $R(\varphi, \gamma)$ for a number of discrete wave
 185 components at different orographic anisotropies. Figure 2 presents a few examples of $R(\varphi, \gamma)$. In
 186 the case with $\gamma > 1$, i.e., when the horizontal wind is parallel to the mountain ridge, the surface
 187 WMF is mainly represented by the along-wind wave components. By contrast, when the

188 horizontal wind is normal to the mountain ridge (i.e., $\gamma < 1$), the cross-wind wave components
189 carry more WMF. For isotropic mountains with $\gamma = 1$, $R(\varphi, \gamma)$ is qualitatively similar to the
190 case with $\gamma > 1$ but the WMF is more evenly distributed about φ .

191 As mentioned above, in the presence of directional wind shear, the upward propagation
192 and OGW momentum deposition should be addressed separately for different wave components.
193 At each model level, the parameterization follows a two-step procedure.

194 (i) Directional absorption check. This is to remove the wave components (if any) from the
195 wave packet which are selectively filtered between the current level and the level below.
196 For example, the wave components between the azimuths of φ_1 and φ_2 are removed when
197 the horizontal wind experiences a rotation from $\mathbf{V}(z_1)$ to $\mathbf{V}(z_2)$, as shown in Fig. 1. The
198 directionally absorbed waves produce a lift force (i.e., OGWL) pointing to the left (right)
199 of a mean flow that backs (veers) with height (Xu et al. 2012);

200 (ii) Wave breaking check. This step is similar to that in previous parameterization schemes.
201 The wave components not directionally filtered are taken as a whole. Airflow stability is
202 checked according to the wave-modulated Richardson number Ri_m . If Ri_m falls below a
203 critical value Ri_c (typically 0.25), wave breaking occurs and produces a drag force (i.e.,
204 OGWD) that is principally in the direction opposite to the flow, with the residual wave
205 amplitude controlled by the saturation hypothesis.

206 The remaining WMF is passed on to the next model level, with the above procedure
207 repeated until the WMF is totally attenuated or encounters the model top. Readers are referred to
208 LM97 and KD05 for more details about this second step.

209 2.2 Implementation in WRF

210 The KD05 scheme in the WRF model actually includes two kinds of orographic drag,
 211 namely, flow blocking drag (FBD) and gravity wave drag. FBD occurs as the incident flow is
 212 blocked by the mountain when it does not have enough kinetic energy to go over it (LM97). In
 213 contrast, gravity wave drag is related to the breaking of vertically propagating OGWs which
 214 usually occurs at upper levels. For OGWs forced by large-amplitude mountains, this drag can
 215 also occur in the lower troposphere as a result of low-level wave breaking (KA95). In this study
 216 we mainly focus on the parameterization of gravity wave drag, including that due to directional
 217 wind shear which, as mentioned previously, we will call OGWL.

218 In the KD05 scheme, the WMF at the reference level (i.e., the effective mountain surface
 219 height in the model) is along the direction of the mean low-level wind, with its magnitude given
 220 by

$$221 \quad \tau_{\text{ref}} = \rho_0 E \frac{m}{\lambda_{\text{eff}}} G \frac{|V_0|^3}{N}, \quad (6)$$

222 with

$$223 \quad E = (OA + 2)^{C_E \frac{Fr_0}{Fr_c}}, m = (1 + L_x)^{OA+1}, G = \frac{Fr_0^2}{Fr_0^2 + C_G OC^{-1}}, \quad (7)$$

224 where E is the enhancement factor accounting for the drag enhancement by low-level breaking
 225 and/or lee wave trapping, m is the number of mountains within the model grid cell, and G is the
 226 asymptotic function providing a smooth transition between non-blocking and blocking flow.
 227 These parameters are controlled by both the incident flow properties and SSO statistics, e.g.,
 228 orographic asymmetry (OA), orographic convexity (OC) and effective orographic length (L_x)
 229 *defined in the direction of the low-level wind* (cf. Fig. 7 of KD05). The Froude number is given
 230 by $Fr_0 = \frac{Nh_m}{|V_0|} OD$ where $OD = \frac{L_x^\perp}{L_x}$ is the orographic direction, with L_x^\perp the effective orography

231 length normal to L_x , i.e., in the cross-wind direction. Moreover, λ_{eff} is the effective grid length
232 used as a tunable coefficient; $C_E = 0.8$ and $C_G = 0.5$ are constants calibrated according to
233 mesoscale simulations (KA95). A critical Froude number of $Fr_c = 1$ is used to determine the
234 level of flow blocking.

235 Our new scheme can be readily implemented in the WRF model through modifications to
236 the KD05 scheme. While one can readily obtain the magnitude of the surface WMF according to
237 linear elliptical mountain wave theory, that quantity is simply set to τ_{ref} in Eq. (6). This is to be
238 compatible (and comparable) with the original scheme which also takes into account the effect of
239 nonlinear mountain waves. The anisotropy of SSO is represented by OD , i.e., $\gamma = \frac{L_x}{L_x^\perp} = OD^{-1}$.
240 Given τ_{ref} and $R(\varphi_i, \gamma)$, it is straightforward to obtain the reference-level WMF for each wave
241 component. Note again that the wave components are in the coordinate system defined by the
242 elliptical mountain, which should be rotated relative to the model coordinates. The upward
243 transport of WMF in the KD05 scheme is also modified, following the above two-step procedure.

244 2.3 Setup of numerical experiments

245 Three sets of numerical experiments are conducted in this work by using the global
246 version of the WRF model (GWRF). GWRF is an extension of the mesoscale WRF and a variant
247 of Planet WRF (Richardson et al. 2007). Latitude-longitude horizontal coordinates are employed
248 and Fourier spectral filtering is applied in the polar regions to avoid numerical instabilities near
249 the poles. The first set of simulations is run without OGW parameterization (named CTL
250 experiment), while the other two sets are run with the existing KD05 scheme (OLD experiment),
251 and the revised scheme (NEW experiment). Each set consists of six simulations which are run
252 from 00Z January 1 to 00Z February 1 from the year 2013 to 2018. The GWRF model is

253 configured with a horizontal resolution of $1^\circ \times 1^\circ$ and 41 levels in the vertical, with the model top
254 located at 10 hPa. Initial conditions come from the $1^\circ \times 1^\circ$ Global Forecast System (GFS)
255 analyses produced by the National Center for Environmental Prediction (NCEP). The available
256 levels of the GFS data limit the choice of the WRF model top to 10 hPa. A sponge layer is placed
257 at the top 5 km of the model domain, which aims to minimize the influence of waves reflected
258 from the domain top. In this regard, only the numerical results below 20 hPa are studied in this
259 work. The WRF single-moment 3-class scheme (Hong et al. 2004) is used for microphysics.
260 Other model physics include the RRTMG longwave and shortwave radiation schemes (Iacono et
261 al. 2008), the Yonsei University (YSU) Planetary Boundary Layer (PBL) scheme (Hong et al.
262 2006), the MM5 similarity scheme for the surface layer (Beljaars 1994), the new Tiedtke
263 cumulus parameterization scheme (Zhang et al. 2011) and the Noah land surface model (Tewari
264 et al. 2004).

265 **3 Results**

266 3.1 Zonal wind structure

267 Figure 3a shows the zonal-mean zonal winds in January averaged over the six years from
268 2013 to 2018 from the $2.5^\circ \times 2.5^\circ$ NCEP reanalysis (R2) in both the Southern and Northern
269 Hemisphere (SH and NH). The most prominent features are the two tropospheric jets located in
270 the subtropical upper troposphere. The NH tropospheric jet is more intense and occurs at higher
271 altitude. Easterlies are found to prevail in the tropical lower-to-middle troposphere and SH
272 stratosphere. Contrastingly, the NH stratosphere is dominated by westerlies, with another upper-
273 level jet found in the high latitudes. This jet is actually the lower portion of the polar-night jet
274 which is separated from the tropospheric jet.

275 Figures 3b-3d are the corresponding zonal-mean zonal winds obtained from the three
276 experiments. In general, the WRF simulations can capture the structural features of the zonal
277 winds well, including the two tropospheric jets and easterlies in the SH stratosphere. At first
278 sight, the CTL experiment appears to best reproduce the zonal wind, for example, in terms of the
279 maximum wind speed of the NH tropospheric jet. However, closer examination reveals that there
280 are considerable discrepancies between the CTL simulation and reanalysis. As can be seen in Fig.
281 4a, westerly wind biases are found in the NH midlatitudes which extend vertically from the
282 surface to the stratosphere. The largest bias occurs near 70 hPa, i.e., above the tropospheric jet.
283 At both low and high latitudes, there are even stronger easterly wind biases, especially in the NH
284 polar stratosphere where the negative wind biases exceed 8 m s^{-1} . By contrast, zonal winds in the
285 SH are much better simulated than in the NH, with generally weaker biases (of less than 3 m s^{-1}).

286 In the OLD experiment (Fig. 4b), the aforementioned westerly biases in NH midlatitudes
287 are reduced significantly, showing little difference ($< 1 \text{ m s}^{-1}$) from reanalysis. The deep column
288 of positive bias at $\sim 40^\circ\text{N}$ in CTL (Fig. 4a) is mostly gone (Fig. 4b). The zonal winds at low
289 latitudes are also improved, although there are still notable biases in the upper troposphere.
290 However, zonal winds are simulated worse at high latitudes, where the negative biases exceed 10
291 m s^{-1} in the stratosphere at $\sim 60^\circ\text{N}$ (Fig. 4b). In the NEW experiment (Fig. 4c), the midlatitude
292 westerly biases are also markedly reduced. The NH tropospheric jet intensity is slightly
293 underestimated which, as will be shown later, is due to greater OGW forcing there. Nevertheless,
294 there is an overall enhancement of stratospheric winds at high latitudes that reduces the negative
295 biases compared to OLD (Fig. 4d). The negative biases in the high-latitude stratosphere are
296 comparable to those in CTL (Figs. 4a, 4c), whereas the position of the polar night jet agrees

297 better with reanalysis (centered around 65°N , see Fig. 3). In this regard, among the three
298 experiments the polar night jet is best reproduced in the NEW case.

299 Next, we will focus on the parameterized OGWs and their influence in the NH, because
300 the SH is mainly covered by ocean, including few mountain ranges. Nonetheless, there is still
301 strong OGW activity in the SH, especially during austral winter (Geller et al. 2013; Hindley et al.
302 2015), which has an important influence on the general circulation of the SH (McLandress et al.
303 2012). The effect of the revised parameterization scheme on the SH will be the object of a future
304 study.

305 3.2 Distribution of WMF and OGW forcing in the NH

306 Figures 5a and 5b show the vertical distribution of zonal-mean WMF in the NH obtained
307 from the OLD and NEW experiments, respectively. Similarly, Figs. 5c and 5d depict the vertical
308 distribution of WMF normalized by the surface WMF. Significant WMF is found between 30°N
309 and 50°N where the main mountain ranges exist in the NH, along with a secondary WMF
310 maximum between about 60°N and 70°N . In the OLD and NEW experiments, the surface WMF
311 is very similar because no changes were made to the reference-level WMF in the revised scheme.
312 Accordingly, the column-integrated OGW forcing (or the total WMF divergence in the vertical
313 column) in the two cases agree well with each other, with the largest forcing found in
314 midlatitudes (Fig. 6). Hereafter, the total body force exerted on the mean flow by parameterized
315 OGWs will be called OGW forcing unless explicitly stated. This is because OGWs in the NEW
316 experiment produce both OGWD (by wave breaking) and OGWL (due to directional absorption)
317 while there is only OGWD in the OLD case.

318 There are remarkable differences for the upward propagation of WMF. In the mid and
319 high latitudes, on average more than 30% of the WMF originating at the surface can be

320 transported to the stratosphere, in particular between about 45°N and 75°N, which appears to be
321 an atmospheric window for topographically forced gravity waves (Figs. 5c, 5d). This is because
322 the steadily increasing wind speeds from the surface up to the polar-night jet at these latitudes
323 allow for the vertical propagation of OGWs without encountering critical layers. The WMF in
324 OLD decreases more rapidly with height than in NEW. Taking the WMF between 45°N and
325 60°N as an example, about 60% of the WMF is transported to above 100 hPa in the NEW
326 experiment (Fig. 5d), while only ~40% reaches that level in OLD (Fig. 5c). This means that the
327 revised scheme allows more WMF to be transported to upper levels. In the latitudes south of
328 about 20°N, the WMF generally cannot be transported to the stratosphere, showing a rapid drop
329 by more than 80% in the upper troposphere. In the OLD case, this is due totally to wave breaking
330 in the lower troposphere (Fig. 7a) where the zonal wind is reversed from easterlies to westerlies,
331 forming a critical layer for OGWs (Booker and Bretherton 1967). In NEW, while low-level wave
332 breaking still plays a dominant role (Fig. 7b), directional absorption of OGWs also makes a
333 contribution, especially in the middle and upper troposphere (Fig. 7c).

334 In Fig. 7, wave attenuation at a given level is evaluated as the ratio between the
335 attenuated WMF (due to either wave breaking or directional absorption) and the local WMF. As
336 seen in Fig. 7c, directional absorption of OGWs is much weaker in midlatitudes than in low and
337 high latitudes. This meridional variation largely depends on the rotation of the horizontal wind
338 with height. As noted by Xu et al. (2012), the more the horizontal wind rotates with height, the
339 more WMF is directionally absorbed. In winter, the rotation of tropospheric winds is weak in the
340 midlatitudes of the NH (Xu et al. 2018), due to the strong westerly jet.

341 Given that the vertical gradient of WMF denotes the body force exerted on the mean flow
342 by OGWs, the above results suggest different vertical distributions of total OGW forcing in the

343 two experiments. Figures 8a and 8b depict the zonal-mean zonal OGW forcing due to wave
344 breaking (i.e., OGWD) in OLD and NEW respectively, with their difference (NEW minus OLD)
345 shown in Fig. 8c. In both cases, significant OGWD is found in the lower troposphere as well as
346 in the upper troposphere and stratosphere at midlatitudes, with the forcing maxima located just
347 above the tropospheric jet core. There is a natural increase of OGWD with altitude due to the
348 exponential reduction in air density, which results in an increase of wave amplitude. The weak
349 winds in the lower stratosphere also favor breaking of mountain waves. This weak-wind layer
350 has been named mountain-wave “valve layer” by Kruse and Smith (2016) since it controls the
351 transport of wave momentum through it. Low-level OGWD is also found at low and high
352 latitudes but is much weaker and less extensive than its midlatitude counterpart.

353 It is clear that the westerly biases in the midlatitudes of the CTL experiment are
354 satisfactorily reduced because of the westward OGWD. However, there are apparent differences
355 between the midlatitude OGWD in the two cases (Fig. 8c). The revised scheme generally
356 produces weaker OGWD in the lower troposphere and stratosphere than the OLD experiment.
357 On the contrary, OGWD is increased in the upper troposphere and lower stratosphere between
358 about 200 hPa and 70 hPa. This behavior is closely related to the directional absorption of
359 OGWs. As shown in Fig. 7, there is widespread suppression of wave breaking in the NEW
360 experiment. For example, in the upper troposphere above the tropospheric jet, wave breaking in
361 the OLD experiment causes attenuation of local WMF by up to 30%, while the WMF only
362 attenuates by about 15% to 20% in the NEW experiment. This is because directional absorption
363 is able to stabilize the OGWs by reducing the local wave amplitude. Due to such inhibition of
364 wave breaking, there is a weakening of OGWD in the lower troposphere. But, at the same time,
365 this allows more upward propagation of OGWs to the upper troposphere and stratosphere, as

366 evidenced by the greater WMF found there (Fig. 5). As such, the *upper-level* OGWD is
367 determined by two opposite effects, i.e., the increased WMF which tends to enhance OGWD and
368 directional absorption which is prone to suppress wave breaking and thus OGWD. In the upper
369 troposphere, it is the former effect that dominates, giving rise to increased OGWD. By contrast,
370 directional absorption has a larger impact in the stratosphere, with the stratospheric OGWD
371 being in general decreased.

372 Meanwhile, considerable OGWL is produced in the stratosphere, showing a magnitude
373 comparable to the OGWD difference (see Figs. 8c, 8d). As the zonal OGWL is mostly westward,
374 it can to a certain degree compensate for the weakening of stratospheric OGWD. As seen in Fig.
375 8f, the total OGW forcing in the NEW experiment (i.e., sum of OGWD and OGWL, Fig. 8e) is
376 strengthened in the stratosphere north of $\sim 50^\circ\text{N}$, compared to the OLD experiment (in which the
377 total OGW forcing is simply the OGWD).

378 The zonal-mean meridional OGW forcing was also studied and shown to increase in the
379 upper troposphere. Nonetheless, the meridional OGW forcing (not shown) is much weaker than
380 its zonal counterpart.

381 3.3 Physical interpretation

382 The revised parameterization scheme produces more intense OGW forcing in the upper
383 troposphere at midlatitudes, which correctly produces a weaker tropospheric jet in the NEW case.
384 On the other hand, there is more notable enhancement of stratospheric winds at high latitudes
385 (Fig. 4d), leading to a better representation of the polar night jet. What is responsible for the
386 increase of polar stratospheric winds given the rather small direct OGW forcing found there?

387 Previous studies have showed that the momentum sink due to westward OGW forcing
388 can induce a meridional circulation, with downward (upward) motion on the poleward
389 (equatorward) flank of the forcing, which subsequently leads to adiabatic warming (cooling)
390 (e.g., Palmer et al., 1986). The zonal mean temperature difference between OLD and CTL
391 experiments (OLD minus CTL) is depicted in Fig 9a. There exists widespread warming in the
392 upper troposphere of high latitudes (i.e., north of the maximum OGW forcing), with the warming
393 center located near 65°N at 200 hPa. The meridional temperature gradient north of ~70°N is thus
394 increased in the upper troposphere, which would enhance the stratospheric winds aloft according
395 to the thermal wind relation. However, this effect appears to be largely cancelled out by the
396 decrease of meridional temperature gradient in the stratosphere (i.e., cooling in midlatitudes and
397 warming in polar regions). Stratospheric winds actually are decreased in OLD compared to the
398 CTL experiment, leading to a worse simulation of the polar night jet. Figure 9b is similar to Fig.
399 9a but for the NEW experiment. Significant warming also occurs in the upper troposphere at mid
400 and high latitudes. Compared to OLD, stronger warming is found between around 40°N and
401 60°N (Fig. 9c), in association with an equatorward displacement of the warming center to about
402 60°N, creating a larger gradient between the pole and ~60°N. Meanwhile, warming in the high
403 latitudes north of about 70°N is suppressed in both the upper troposphere and stratosphere.
404 Therefore, the meridional temperature gradient increases considerably in the polar region,
405 resulting in stronger stratospheric winds than in CTL and OLD (Fig. 4).

406 In the above analysis, warming at high latitudes is attributed to the adiabatic sinking of
407 the OGW-forced residual circulation. In accordance with the “downward control” principle
408 (Haynes et al. 1991), the magnitude of the residual circulation (and hence of adiabatic warming)
409 at a given level is proportional to the meridional gradient of OGW forcing above that level. For

410 the warming center existing at 200 hPa, Fig. 10a presents the integrated OGW forcing above 200
411 hPa for the OLD and NEW experiments, respectively, with their difference given in Fig. 10b. In
412 the NEW experiment, the integrated OGW forcing is notably increased (i.e., more negative)
413 between about 30°N and 40°N, primarily owing to its enhancement in the upper troposphere (Fig.
414 8f). Meanwhile, a relatively small reduction is found between about 42°N and 52°N, in response
415 to the weakened stratospheric OGW forcing. In consequence, the meridional gradient of
416 integrated OGW forcing is enhanced between about 30°N and 50°N, giving rise to the intensified
417 warming found in Fig. 9c. Similarly, the suppression of warming at high latitudes can be
418 ascribed to the decreased meridional gradient of integrated OGW forcing between about 50°N
419 and 60°N. The integrated OGW forcing above 50 hPa was also studied. It is mainly decreased
420 between about 25°N and 45°N but increased poleward (not shown), causing a decrease of
421 warming in the stratosphere at high latitudes.

422 Besides parameterized OGW forcing, the zonal winds in the stratosphere at high latitudes
423 can also be affected by resolved waves, for example, vertically propagating Rossby waves (e.g.,
424 McLandress and Shepherd 2009). The modification of the large-scale flow by parameterized
425 OGWs can influence the propagation of resolved waves and thus their forcing (McLandress et al.
426 2012; Sandu et al. 2016; van Niekerk et al. 2017). Indeed, previous studies have suggested a
427 compensation between parametrized and resolved wave drag in the stratosphere (e.g., Cohen et al.
428 2013; Sigmond and Shepherd 2014). Resolved-wave forcing (which may include both Rossby
429 waves and resolved inertia-gravity waves) can be quantified by the divergence of their EP flux
430 (Andrews 1987). Following Edmon et al. (1980), the zonal-mean EP flux associated with
431 resolved waves was calculated and is shown in Fig. 11 for the OLD and NEW experiments
432 respectively. Resolved waves are found to propagate upward from the lower troposphere at

433 midlatitudes and separate into two branches in the upper troposphere, with one branch
434 propagating equatorward and the other propagating upward into the stratosphere (Figs. 11a, 11b).
435 The latter branch appears to diverge above about 100 hPa, showing both equatorward and
436 poleward propagations. Nonetheless, the westward resolved-wave forcing in the stratosphere (i.e.,
437 EP flux convergence) indicates that the horizontal divergence of EP flux is overwhelmed by
438 vertical convergence there. In the NEW case, the vertical propagation of resolved waves is
439 reduced, yielding weaker resolved wave forcing in the high-latitude stratosphere (Fig. 11c). Thus
440 the resolved wave forcing may also act to produce stronger polar stratospheric winds in the NEW
441 experiment, although the difference between the resolved wave forcings of the two experiments
442 is not significant at the 99% level (not shown).

443 **4. Discussion and conclusions**

444 Internal gravity waves forced by mountains have long been considered an important
445 process in the coupling between the lower troposphere and middle atmosphere, given their ability
446 to transport momentum from source regions at the surface to the upper levels where the waves
447 break. Vertical momentum transport by orographic gravity waves (OGWs) is affected by
448 directional shear of the mean flow, which is known as directional wave absorption. In such a
449 case, OGWs can produce a lateral lift force (i.e., OGWL) on the mean flow, in addition to the
450 commonly-known orographic gravity wave drag (OGWD) induced by wave breaking. However,
451 this effect is not considered in existing OGW parameterization schemes (at least operational
452 ones), and it is an important source of error in subgrid-scale OGW parameterization.

453 In this study, a new parameterization scheme is developed which explicitly deals with the
454 directional absorption of OGWs. By assuming an elliptical shape for subgrid-scale orography,
455 the wave momentum flux (WMF) carried by each wave component can be easily obtained by

456 using elliptical mountain wave theory (Phillips 1984). Therefore, the new scheme is
457 computationally efficient and acts only within the vertical column, meaning it could be easily
458 adopted operationally. Nonetheless, since the momentum transport by each wave component is
459 handled separately in the new scheme, this increases the computational cost depending on the
460 number of wave components used. In the current study we use 60 wave components evenly
461 distributed in the azimuthal angle interval $(-\pi/2, \pi/2)$, leading to $\sim 30\%$ more CPU time. The
462 vertical propagation and momentum deposition of different wave components is handled
463 separately (rather than as a full spectrum, as is the case with normal OGWD that does not
464 consider directional absorption). The new scheme is implemented in the WRF model, as an
465 addition to the existing OGW drag parameterization scheme, to investigate the impact of
466 parameterized directional absorption of OGWs on the general atmospheric circulation. Three sets
467 of numerical experiments are conducted, containing six one-month-long global simulations from
468 January 2013 to 2018. The first experiment, CTL, is run without an OGW parameterization. The
469 other two experiments separately employ the original OGWD parameterization scheme of KD05
470 and the improved scheme proposed herein, i.e., the OLD and NEW experiments, respectively.

471 The structure of the simulated zonal wind is compared with the NCEP reanalysis data
472 (R2). The CTL experiment shows pronounced westerly wind biases in the midlatitudes of the
473 Northern Hemisphere (NH), with salient easterly wind biases present in the low-latitude
474 troposphere and high-latitude stratosphere. On the contrary, the zonal wind structure in the
475 Southern Hemisphere (SH) is much better reproduced and, moreover, is less sensitive to the
476 parameterization of OGWs. In the OLD experiment, the westerly biases in midlatitudes are
477 significantly reduced, due to the presence of OGW forcing there. The simulated tropospheric jet
478 is brought to a good agreement with reanalysis. The NEW experiment also achieves a

479 satisfactory reduction of midlatitude westerly biases, although the tropospheric jet is slightly
480 underestimated because of the stronger OGW forcing in the upper troposphere at midlatitudes.
481 On the other hand, the stratospheric winds at high latitudes are simulated worse in the OLD
482 experiment, whereas there is an overall enhancement in NEW, with the stratospheric polar night
483 jet being best reproduced among the three experiments.

484 The vertical momentum transport of OGWs and the resulting OGW forcing are studied,
485 with particular attention paid to the NH midlatitudes, where the strongest orographic forcing is
486 present. The OLD and NEW experiments show very similar surface WMF and column-
487 integrated OGW forcings, but the vertical distributions of the OGW forcing are quite different,
488 which is caused by the directional absorption of OGWs. The directional absorption has a
489 tendency to inhibit wave breaking, producing weaker OGWD in the lower troposphere. On the
490 other hand, the suppressed low-level wave breaking allows for more upward transport of WMF
491 to the upper troposphere and stratosphere, which can induce stronger OGWD there via wave
492 breaking. Therefore, the upper-level OGWD is jointly determined by two competing effects, i.e.,
493 increased WMF from below and local directional absorption. In the upper troposphere, the
494 former effect dominates, thus increasing the OGWD above the tropospheric jet. Conversely, this
495 effect is overwhelmed by the enhanced directional absorption in the stratosphere at mid-to-high
496 latitudes, with the OGWD being reduced there. Nevertheless, the total OGW forcing (i.e., sum
497 of OGWD and OGWL) in NEW is still improved in the stratosphere north of $\sim 50^\circ\text{N}$, because the
498 weakened stratospheric OGWD is compensated by the considerable OGWL present there.

499 It is noteworthy that the OGWL studied in this work is different from the mountain lift
500 mentioned by Lott (1999). The latter is a lateral force exerted by the subgrid-scale orography
501 caused by the pressure gradient associated with geostrophic balance of the incoming flow. It is

502 therefore proportional to the Coriolis parameter. This mountain lift associated with the Earth's
503 rotation can significantly affect the pattern of steady Rossby waves (Lott 1999). Conversely, the
504 OGWL discussed herein is the same as that studied in Martin and Lott (2007), which can cause
505 synoptic-scale disturbances.

506 Possible links between the changes of midlatitude OGW forcing and stratospheric winds
507 at high latitudes are explored, which are summarized schematically in Fig. 12. In the NEW
508 experiment, the increased OGW forcing in the midlatitude upper troposphere and more
509 widespread weakening of stratospheric OGW forcing jointly enhance upper-tropospheric
510 adiabatic warming (associated with the wave-induced vertical residual circulation) between
511 about 40°N and 60°N. Meanwhile, adiabatic warming is suppressed in the upper troposphere and
512 stratosphere at high latitudes north of ~70°N, due mainly to the reduced stratospheric OGW
513 forcing. Such changes enhance the meridional temperature gradient at high latitudes, which in
514 turn strengthens the polar stratospheric winds according to the thermal wind relation. In addition
515 to parameterized OGWs, the role played by resolved waves is also addressed. In the NEW
516 experiment, there is an increase of equatorward propagation of EP flux in the upper troposphere,
517 whereas the vertical propagation of resolved waves is reduced, leading to a weakening of
518 resolved-wave forcing in the stratosphere. This might also contribute to the intensification of
519 stratospheric winds. Nonetheless, the relative importance of parameterized and resolved wave
520 forcing requires further quantitative diagnostic study.

521 The result that directional absorption can redistribute the OGW forcing and affect the
522 large-scale circulation both directly and indirectly seems to be rather robust. An additional
523 sensitivity experiment (NEW1) was conducted, similar to NEW but with the directional
524 absorption of OGWs included only above the PBL. This can be viewed as a case with “weak

525 directional absorption”, as it omits the rotation of the horizontal wind within the boundary layer.
526 The results in NEW1 are qualitatively similar to those in NEW but show weaker differences in
527 the zonal winds, OGW forcing, etc. (Figs. 13, 14), consistent with the weaker effect of
528 directional absorption of OGWs. This suggests an interaction between parameterized OGWs and
529 the PBL (Kim and Hong 2009).

530 For the implementation of the new scheme in the WRF model, the low-level wind is
531 assumed to be aligned with one of the principal axes of the elliptical mountain. The main reason
532 for considering only this incidence angle is to be consistent with the KD05 scheme within WRF,
533 which is extended in this study to include the additional effect of directional absorption. In the
534 LM97 scheme, which also uses elliptical mountain wave theory, the incoming flow can be
535 oblique to the principal axes of the mountain, allowing a misalignment between surface WMF
536 and wind. Further development is needed to relax the former assumption, which will be a topic
537 for future research.

538 The propagation of resolved waves and their forcing are influenced by the modification
539 of the large-scale circulation. This is one component of the problem of the complicated
540 interactions involving parameterized wave drag, resolved wave drag and mean flow, which has
541 important implications for both present-day climate and projections of future climate change
542 (McLandress and Shepherd 2009; Sigmond and Scinocca 2010; Calvo et al. 2017) as well as for
543 NWP. As shown by Smith et al. (2017), the variability of the troposphere can be transported to
544 the mesosphere and lower thermosphere by gravity waves. Given the limitation that the model
545 top is at 10 hPa in the present study, it is not possible to know how directional absorption of
546 OGWs will affect the general circulation in the middle atmosphere. According to Xu et al.
547 (2018), the OGWD in the upper stratosphere and lower mesosphere are in general increased

548 under the influence of directional absorption of OGWs. This will be studied in more detail in the
549 future by implementing the new scheme in a more comprehensive climate model.

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554 **Appendix A: Derivation of surface WMF of OGWs forced by elliptical mountains**

555

556 For linear mountain waves, the momentum flux at the surface $\boldsymbol{\tau}_0$ is given by

557
$$\boldsymbol{\tau}_0 = -\rho_0 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathbf{v}_0' w_0' dx dy, \quad (\text{A1})$$

558 where $\mathbf{v}_0' = (u', v')$ and w_0' are the horizontal and vertical velocity perturbations of gravity
559 waves at the surface. Using two-dimensional Fourier transforms, i.e.,

560
$$A'(x, y, z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \hat{A}(k, l, z) e^{i(kx+ly)} dk dl, \quad (\text{A2})$$

561
$$\hat{A}(k, l, z) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} A'(x, y, z) e^{-i(kx+ly)} dx dy, \quad (\text{A3})$$

562 with $A'(x, y, z)$ and $\hat{A}(k, l, z)$ being a generic field in physical and spectral space, respectively,
563 Eq. (A1) can be written as

564
$$\boldsymbol{\tau}_0 = -4\pi^2 \rho_0 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \hat{\mathbf{v}}_0' \hat{w}_0^* dk dl, \quad (\text{A4})$$

565 where \hat{w}_0^* is the complex conjugate of \hat{w}_0 and k and l are the components of the horizontal
566 wavenumber vector $\mathbf{K} = (k, l)$.

567 In a situation of spatially uniform hydrostatic flow past an isolated obstacle, the vertical
568 velocity in spectral space can be easily obtained by solving the Taylor-Goldstein equation (cf. Eq.
569 (9) in Xu et al., 2012), yielding

570
$$\hat{w} = i(Uk + Vl) \hat{h} e^{i \frac{NK}{Uk+Vl} z}, \quad (\text{A5})$$

571 where $\mathbf{V} = (U, V)$ is the spatially uniform horizontal wind vector, and \hat{h} is the Fourier transform
572 of the terrain elevation. In accordance with the polarization relations of internal gravity waves (cf.
573 Eqs. (5) and (6) in Xu et al., 2017b), the horizontal velocity in spectral space is

574
$$\hat{u} = i \frac{k}{K^2} \frac{\partial \hat{w}}{\partial z} = -\frac{k}{K} \frac{N}{Uk+Vl} \hat{w}, \quad (\text{A6})$$

575
$$\hat{v} = i \frac{l}{K^2} \frac{\partial \hat{w}}{\partial z} = -\frac{l}{K} \frac{N}{Uk+Vl} \hat{w}. \quad (\text{A7})$$

576 Substituting Eqs. (A5)-(A7) into (A4) yields

$$577 \quad \boldsymbol{\tau}_0 = 4\pi^2 \rho_0 N \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\mathbf{K}}{K} (Uk + Vl) |\hat{h}|^2 dk dl. \quad (\text{A8})$$

578 For simplicity, polar coordinates are introduced, i.e., $\mathbf{K} = K(\cos\varphi, \sin\varphi)$ with K being the
579 magnitude of the horizontal wavenumber vector, such that the above equation can be rewritten as

$$580 \quad \boldsymbol{\tau}_0 = 8\pi^2 \rho_0 N |V_0| \int_{-\pi/2}^{+\pi/2} \int_0^\infty (\cos\varphi, \sin\varphi) \cos(\varphi - \psi_0) |\hat{h}|^2 K^2 dK d\varphi, \quad (\text{A9})$$

581 where $|V_0|$ and ψ_0 are the speed and direction of the horizontal wind at the surface, and φ is the
582 azimuthal direction of the horizontal wavenumber vector.

583 For the elliptical bell-shaped mountain given by Eq. (1), the Fourier transform is

$$584 \quad \hat{h}(K, \varphi) = \frac{h_m ab}{2\pi} e^{-Kb\sqrt{\gamma^2 \cos^2\varphi + \sin^2\varphi}}, \quad (\text{A10})$$

585 where $\gamma = \frac{a}{b}$ is the horizontal aspect ratio of the mountain. Substituting Eq. (A10) into Eq. (A9)

586 results in

$$587 \quad \boldsymbol{\tau}_0 = 2\rho_0 N |V_0| (h_m ab)^2 \int_{-\pi/2}^{+\pi/2} (\cos\varphi, \sin\varphi) \cos(\varphi - \psi_0) G(\varphi) d\varphi, \quad (\text{A11})$$

588 where

$$589 \quad G(\varphi) = \int_0^\infty e^{-2Kb\sqrt{\gamma^2 \cos^2\varphi + \sin^2\varphi}} K^2 dK = 4b^{-3} (\gamma^2 \cos^2\varphi + \sin^2\varphi)^{-\frac{3}{2}}, \quad (\text{A12})$$

590 with the latter equality in Eq. (A12) being obtained from $\int_0^\infty e^{-qx} x^2 dx = 2q^{-3}$ (Gradshteyn and

591 Ryzhik 2007). Finally, the WMF at the surface takes the form

$$592 \quad \boldsymbol{\tau}_0 = 0.5\rho_0 N |V_0| h_m^2 a \gamma \int_{-\pi/2}^{+\pi/2} (\cos\varphi, \sin\varphi) \cos(\varphi - \psi_0) [\gamma^2 \cos^2\varphi + \sin^2\varphi]^{-\frac{3}{2}} d\varphi. \quad (\text{A13})$$

593 Note that the foregoing derivation was performed in a coordinate system with an arbitrary
594 orientation up to Eq. (A9), but from Eq. (A10) to (A13) (which is identical to Eq. (2)), it was
595 assumed that the principal axes of the elliptical mountain (which are by design chosen to be
596 aligned with the incoming wind) are in the x and y directions respectively. In the general case of

597 an incoming wind and mountain that are oblique relative to the zonal-meridional directions (used
598 in WRF), the transformation from one coordinate system to the other may be made
599 straightforwardly by applying an appropriate horizontal rotation.

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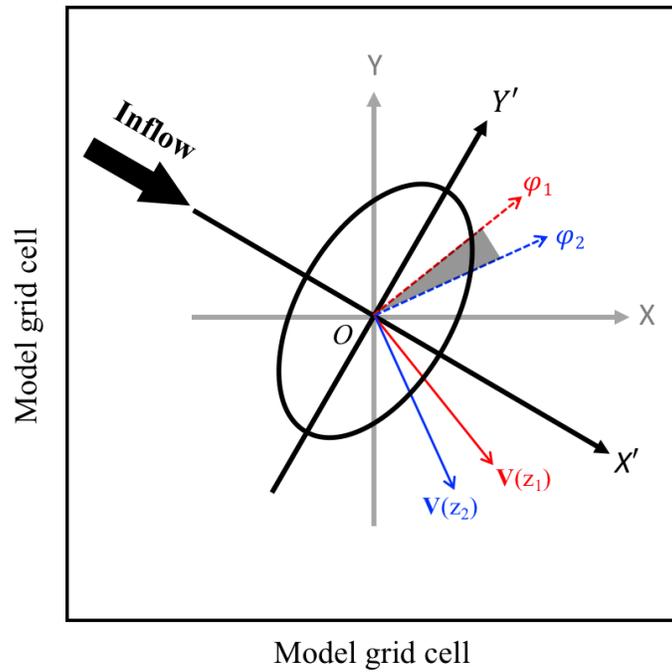
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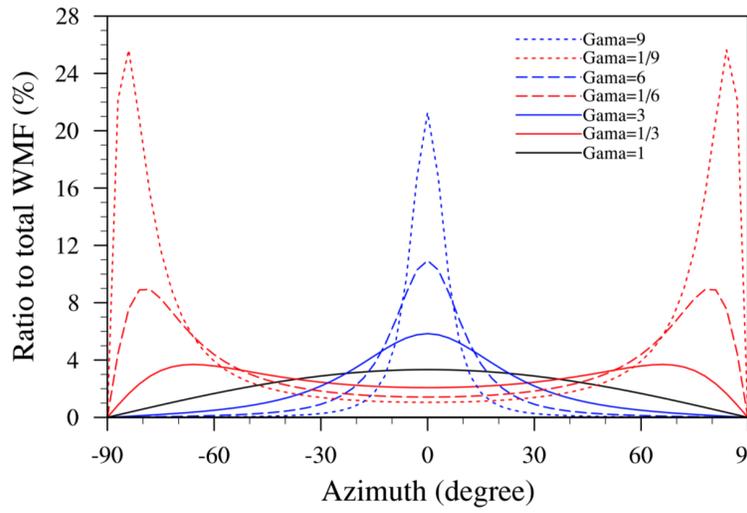
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809 Fig. 1. Schematic of elliptical SSO within a model grid cell. The principal axis of the SSO is (by
810 design) along the direction of low-level inflow. Solid red and blue arrows indicate the horizontal
811 winds at heights z_1 and z_2 respectively. Dashed red and blue arrows are perpendicular to their
812 solid counterparts. Due to rotation of the horizontal wind with height, the wave components
813 between the azimuths φ_1 and φ_2 (grey shading) are selectively absorbed.

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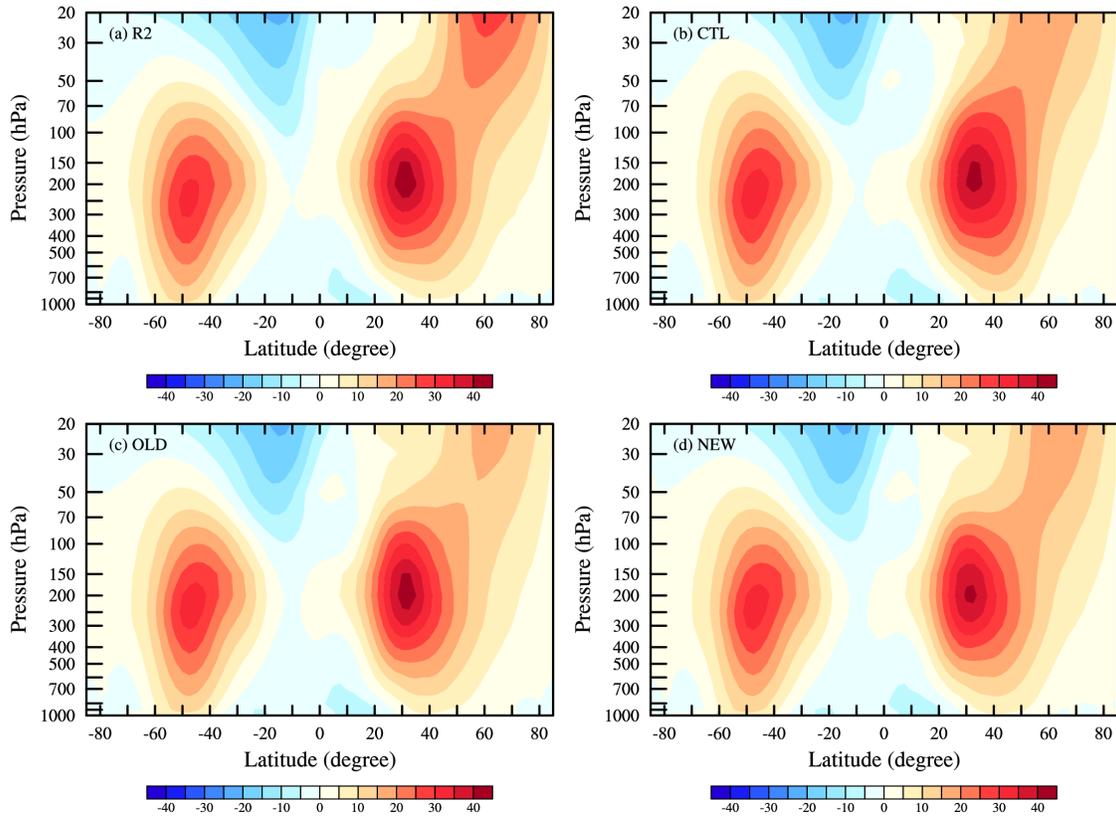
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Fig. 2. Distribution of WMF about the orientation of the horizontal wave number for gravity waves forced by elliptical mountains of different horizontal aspect ratios of $\gamma = 1$ (black), $1/3$ (solid red), 3 (solid blue), $1/6$ (dashed red), 6 (dashed blue), $1/9$ (dotted red), and 9 (dotted blue).

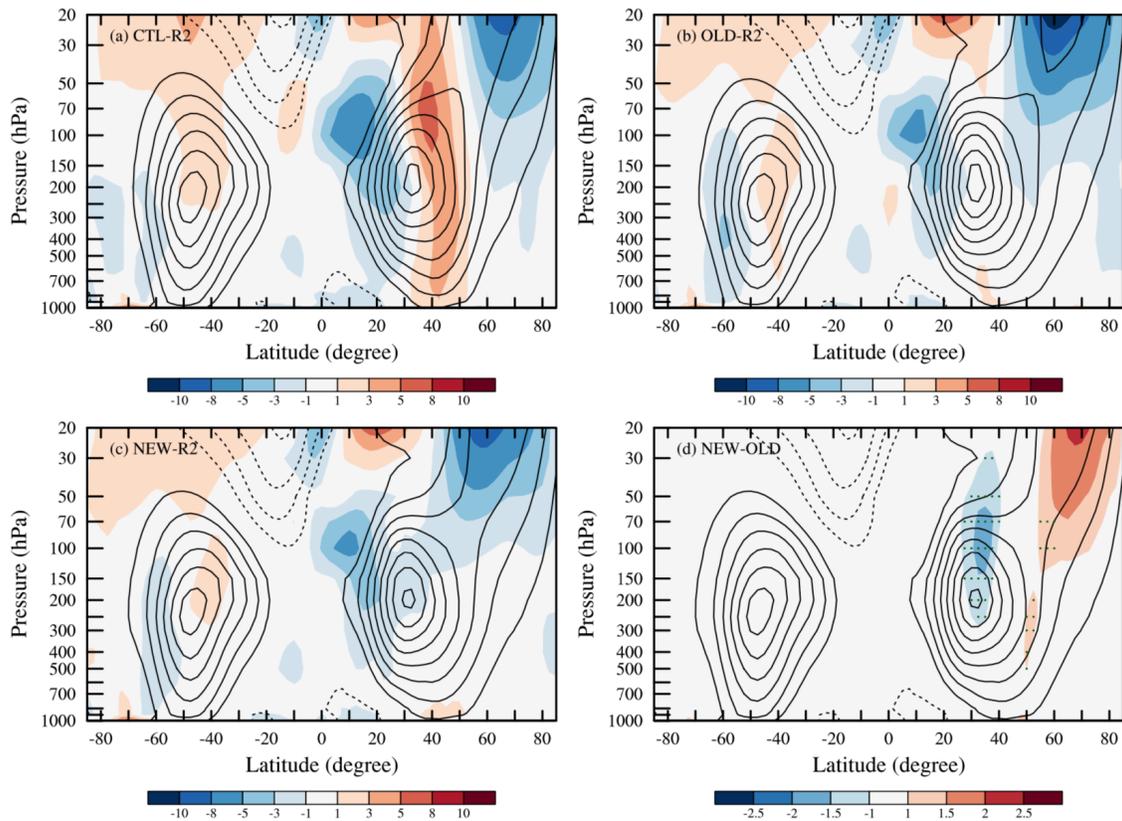
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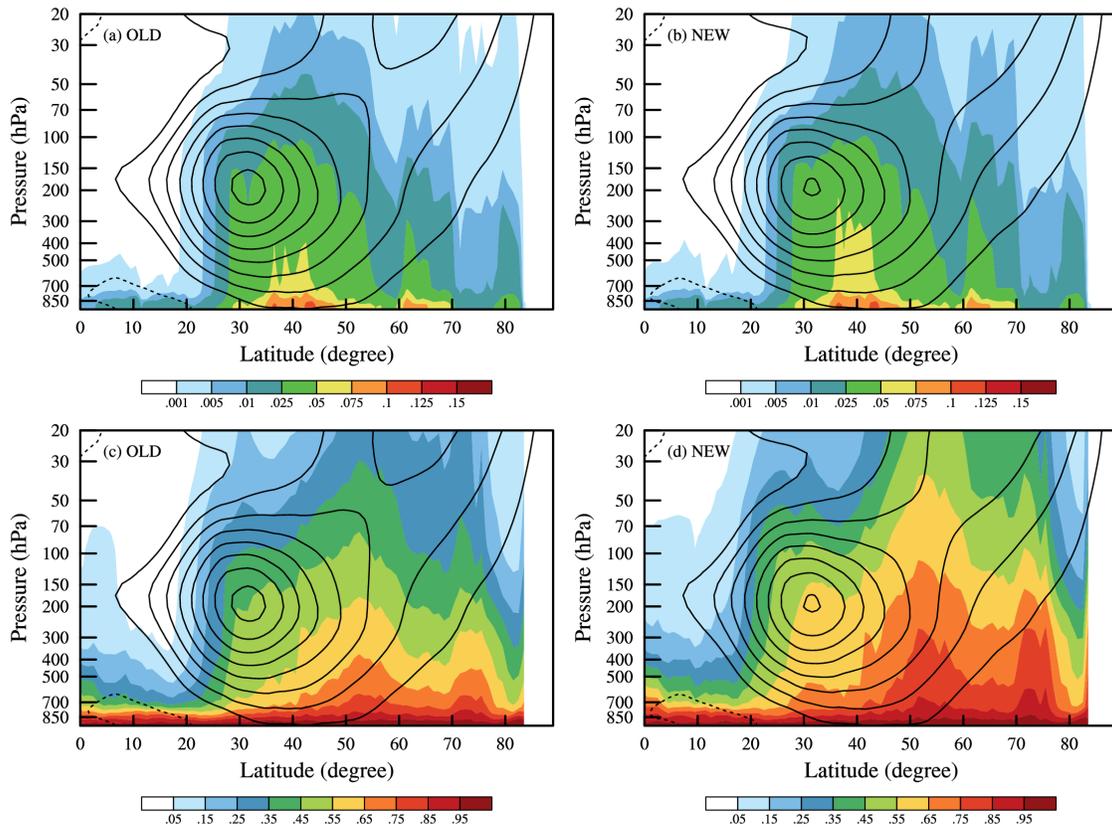
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826 Fig. 3. Zonal-mean zonal winds (units: m s^{-1}) in January averaged from 2013 to 2018 from the (a)
827 NCEP Reanalysis (R2) and global WRF simulations of the (b) CTL, (c) OLD, and (d) NEW
828 experiments.
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832 Fig. 4. Zonal wind difference (shading, units: m s^{-1}) between the NCEP Reanalysis and WRF
 833 simulations averaged for January of 2013-2018. (a) CTL, (b) OLD, and (c) NEW. (d) Difference
 834 between the zonal winds in OLD and NEW. Contour lines are the corresponding zonal-mean
 835 zonal winds (units: m s^{-1}). Statistical significance at the 99% level using the student t test is
 836 indicated by green dots in (d).
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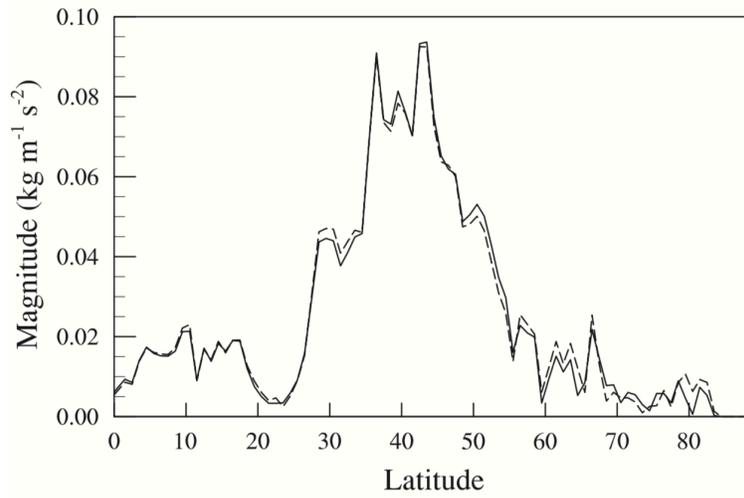


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840 Fig. 5. Vertical distribution of zonal-mean WMF (shading, units: $\text{kg m}^{-1} \text{s}^{-2}$) in the Northern
 841 Hemisphere averaged for January of 2013-2018 in the (a) OLD and (b) NEW experiment. (c) and
 842 (d) are similar to (a) and (b) but for the scaled WMF (units: %) normalized by surface WMF.
 843 Contours are the corresponding zonal-mean zonal winds (units: m s^{-1}).
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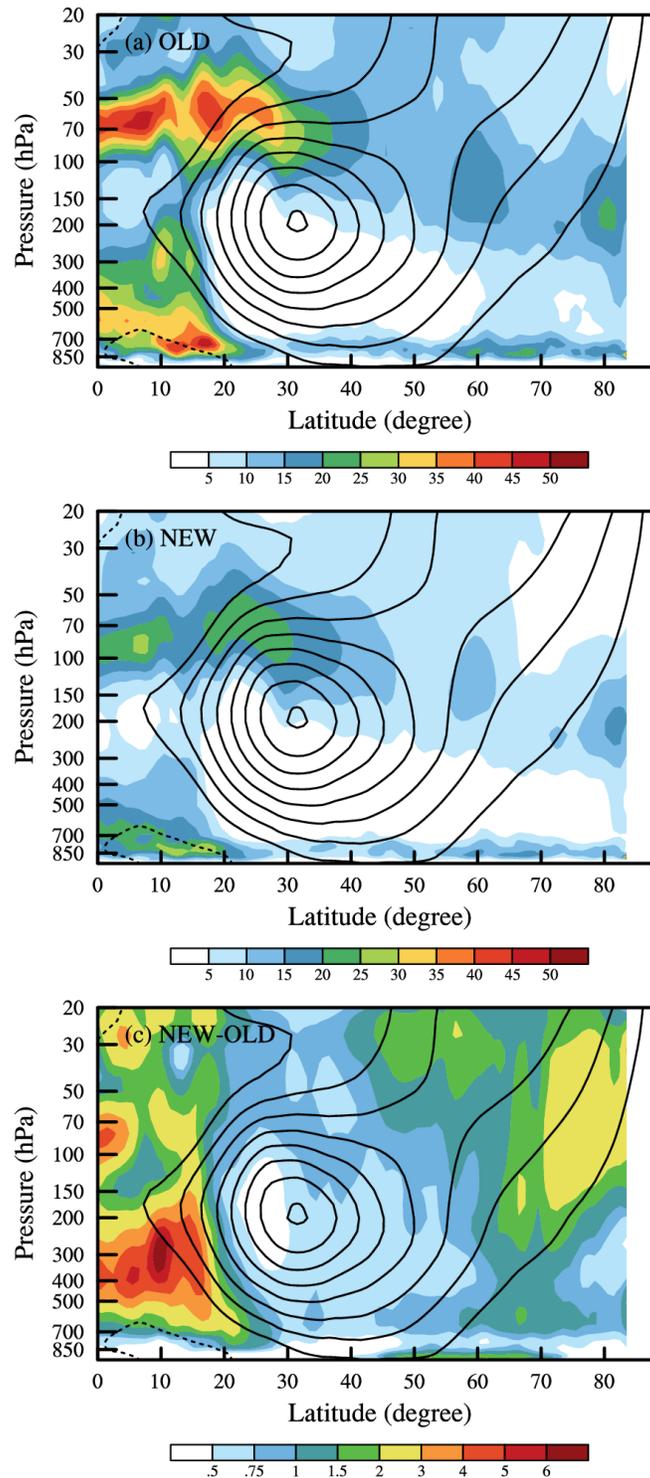
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848 Fig. 6. Zonal-mean column-integrated OGW forcing in the OLD (solid) and NEW (dashed)
849 experiments in the Northern Hemisphere averaged for January 2013-2018.

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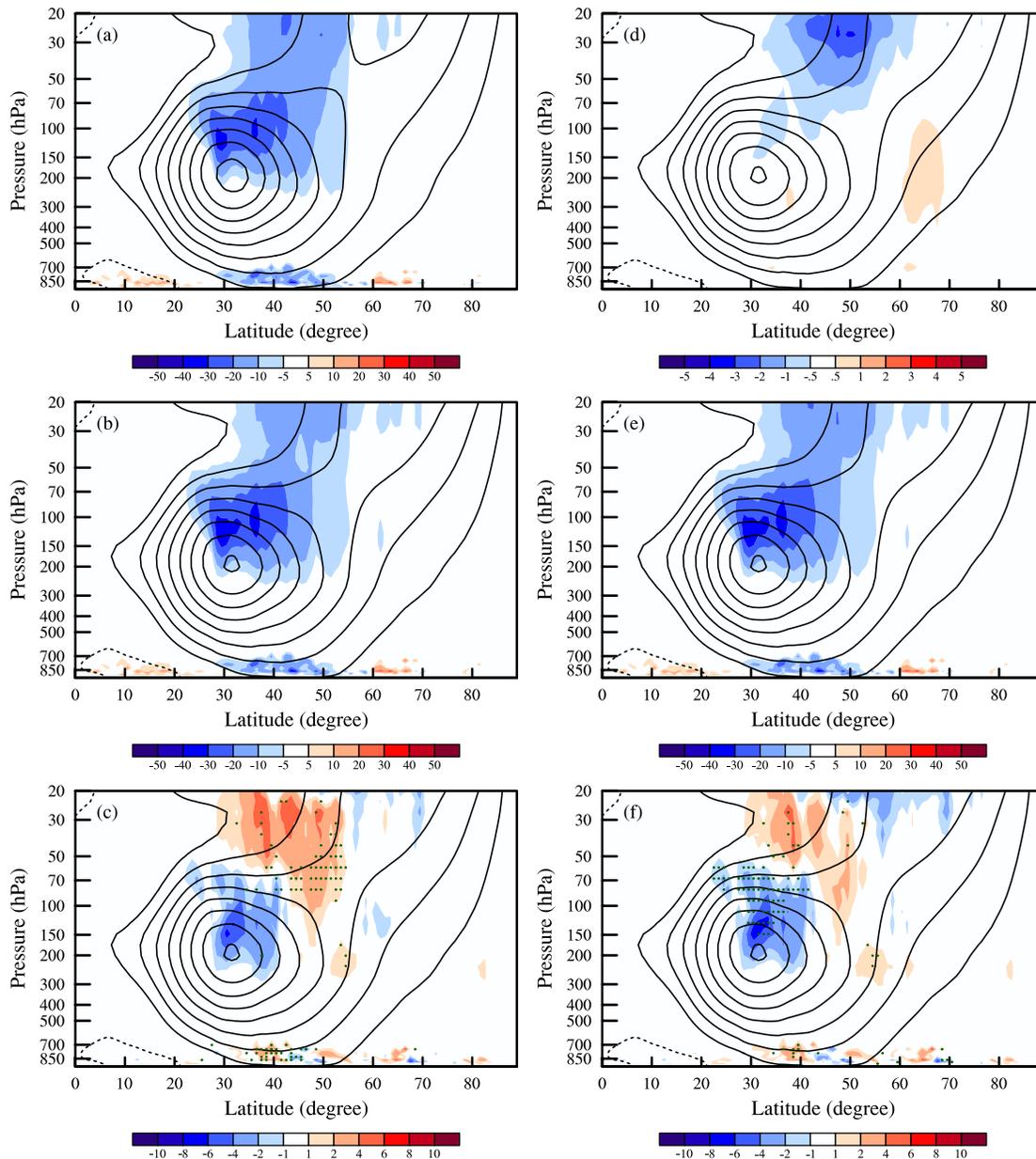
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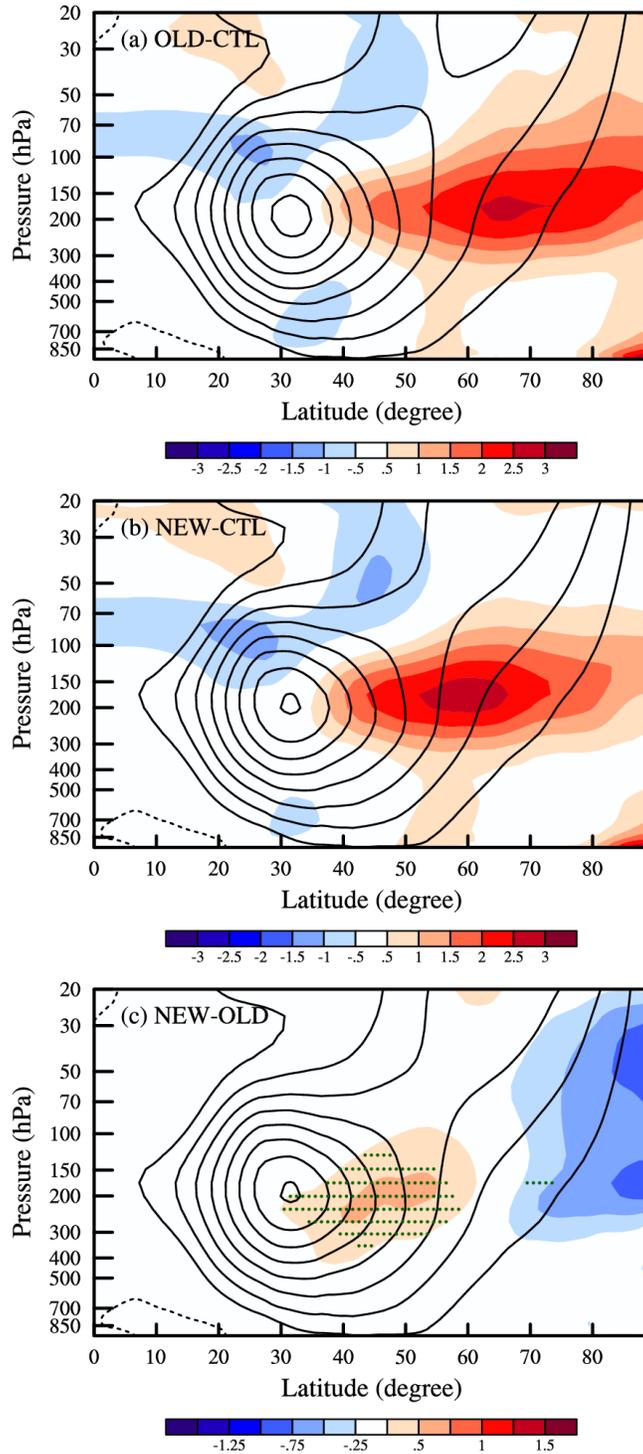
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855 Fig. 7. WMF attenuation (shading, units: %) due to wave breaking in the (a) OLD and (b) NEW
 856 experiment in the Northern Hemisphere averaged for January 2013-2018. (c) is similar to (b) but
 857 for the WMF attenuation due to directional absorption. Contours are the corresponding zonal-
 858 mean zonal winds (units: m s^{-1}).



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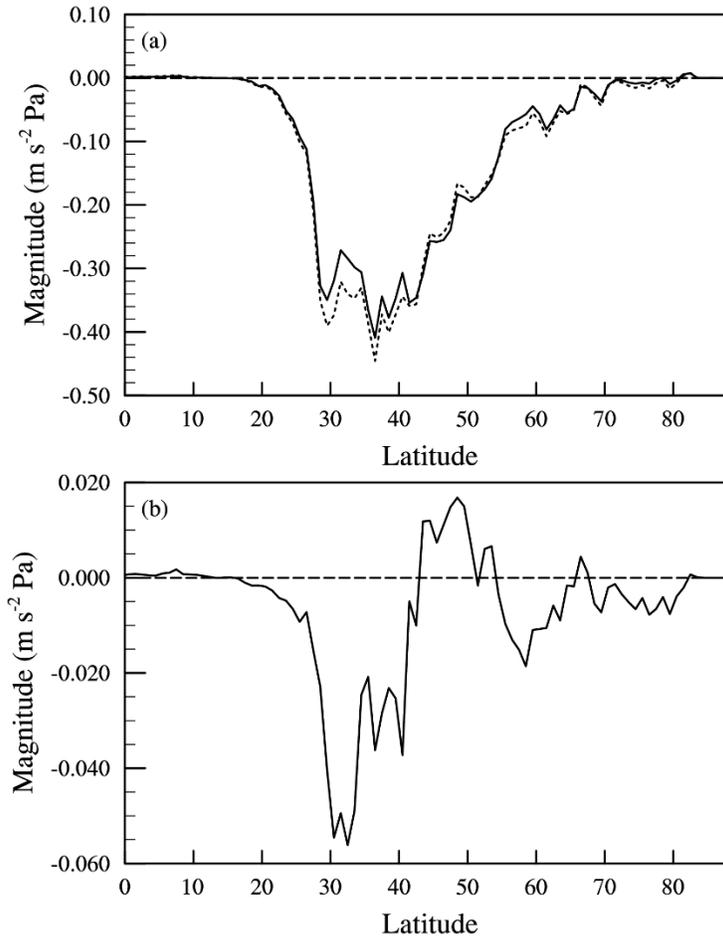
861 Fig. 8. Vertical distribution of zonal-mean OGWD due to wave breaking (shading: units: m s^{-2})
 862 in the (a) OLD and (b) NEW experiment in the Northern Hemisphere averaged for January 2013-
 863 2018, with their difference (NEW minus OLD) shown in (c). (d) is similar to (b) but for zonal-
 864 mean OGWL due to directional absorption in the NEW experiment. (e) Sum of OGWD and
 865 OGWL (i.e., total OGW forcing) in NEW. (f) Difference between the total OGW forcing in the
 866 OLD and NEW experiment, i.e., (e)-(a). Contour lines are the corresponding zonal-mean zonal
 867 winds (units: m s^{-1}). Statistical significance at the 99% level using the student t test is indicated
 868 by green dots in (c) and (f).
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871 Fig. 9. Vertical distribution of zonal-mean temperature difference (shading, units: K) between (a)
 872 CTL and OLD (OLD minus CTL), (b) CTL and NEW (NEW minus CTL), and (c) OLD and
 873 NEW (NEW minus OLD) in the Northern Hemisphere averaged for January 2013-2018. Contour
 874 lines are the corresponding zonal-mean zonal winds (units: m s^{-1}). Statistical significance at the
 875 99% level using the student t test is indicated by green dots in (c)

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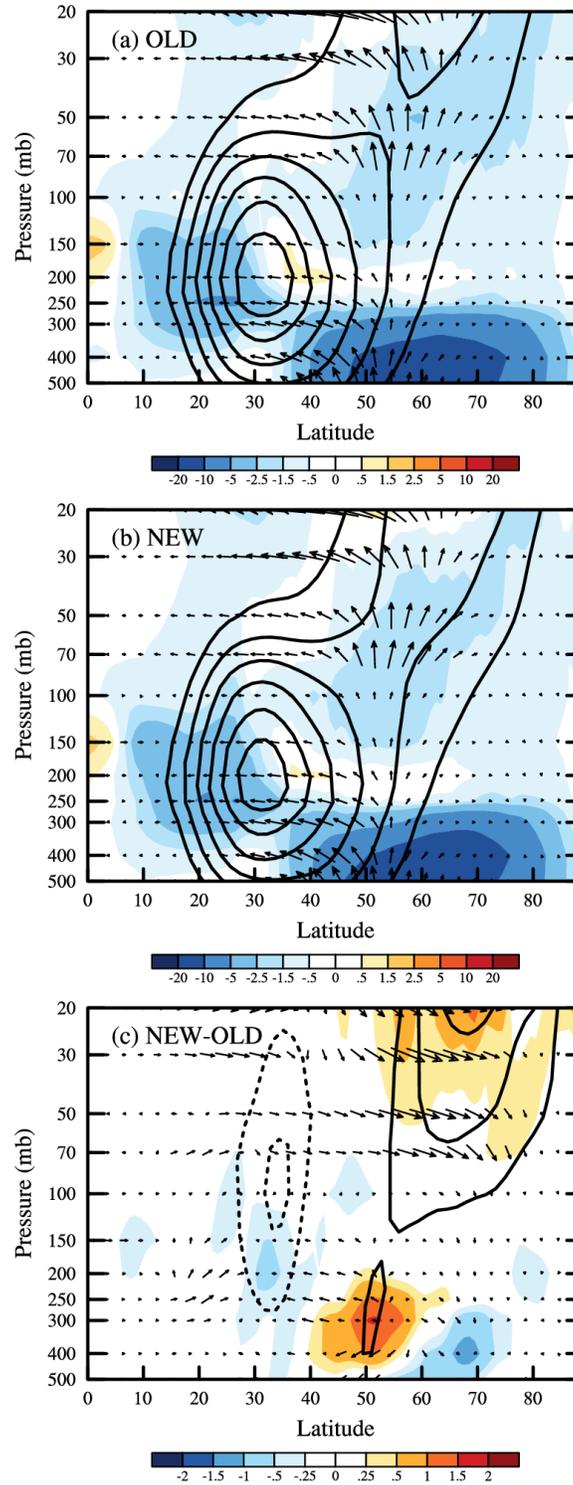


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878 Fig. 10. Zonal-mean (a) OGW forcing integrated between 200 hPa and the model top (units: m s⁻² Pa) in the OLD (solid) and NEW (dotted) experiment and (b) their difference (NEW minus
879 ² Pa) in the OLD) in the Northern Hemisphere averaged for January 2013-2018.
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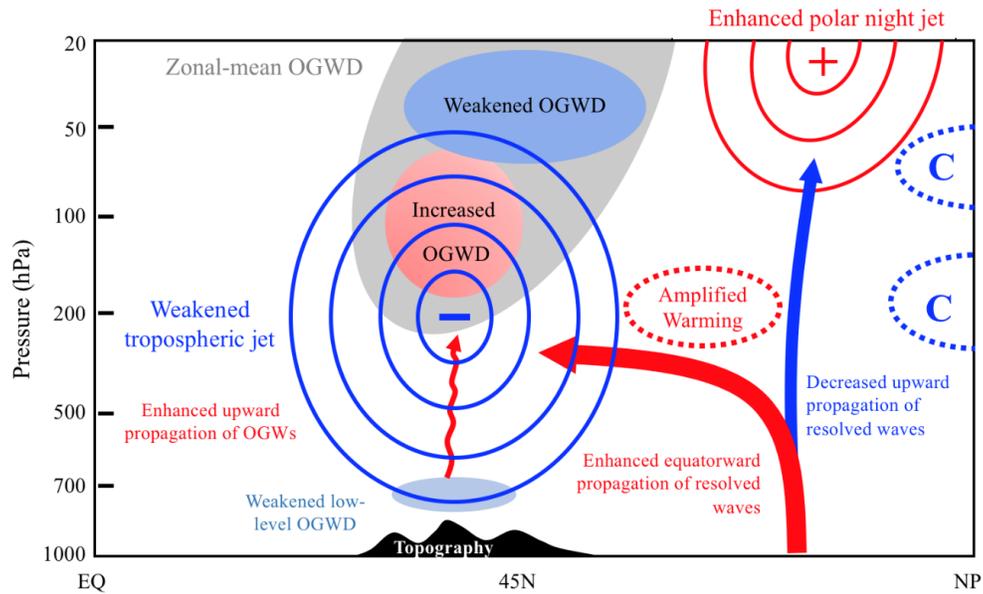
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884 Fig. 11 Vertical distribution of zonal-mean EP flux (vectors) and acceleration (shading, units: m
 885 s^{-2}) due to resolved waves in the Northern Hemisphere averaged for January 2013-2018 in the (a)
 886 OLD and (b) NEW experiment. Contour lines are the corresponding zonal-mean zonal winds
 887 (units: m s^{-1}). (c) Difference between (a) and (b) (b minus a). The EP flux above 100 hPa is
 888 exaggerated by a factor of 5 for clarity.

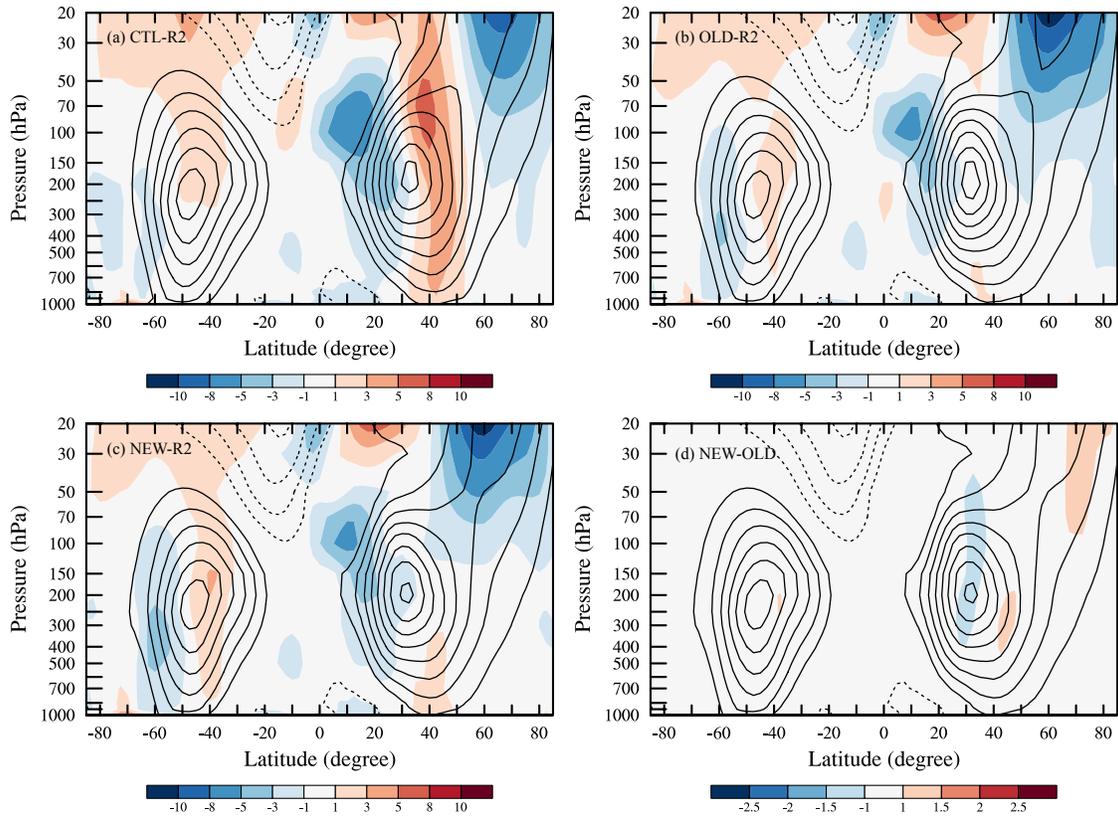
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Fig. 12. Schematic illustration of the impact of directional absorption of OGWs on the large-scale atmospheric circulation in boreal winter. Solid blue and red contours in the midlatitude troposphere and high-latitude stratosphere denote the weakened tropospheric jet and enhanced stratospheric polar night jet, respectively. Dashed red and blue contours indicate increased and suppressed warming in the mid and high latitudes respectively. Grey shading represents the zonal mean OGWD, with blue (red) shadings denoting reduced (increased) OGWD in the lower troposphere and stratosphere (upper troposphere) of midlatitudes. The thick red arrow denotes enhanced equatorward propagation of resolved waves, whereas the thin blue arrow indicates decreased upward propagation of resolved waves into the stratosphere.

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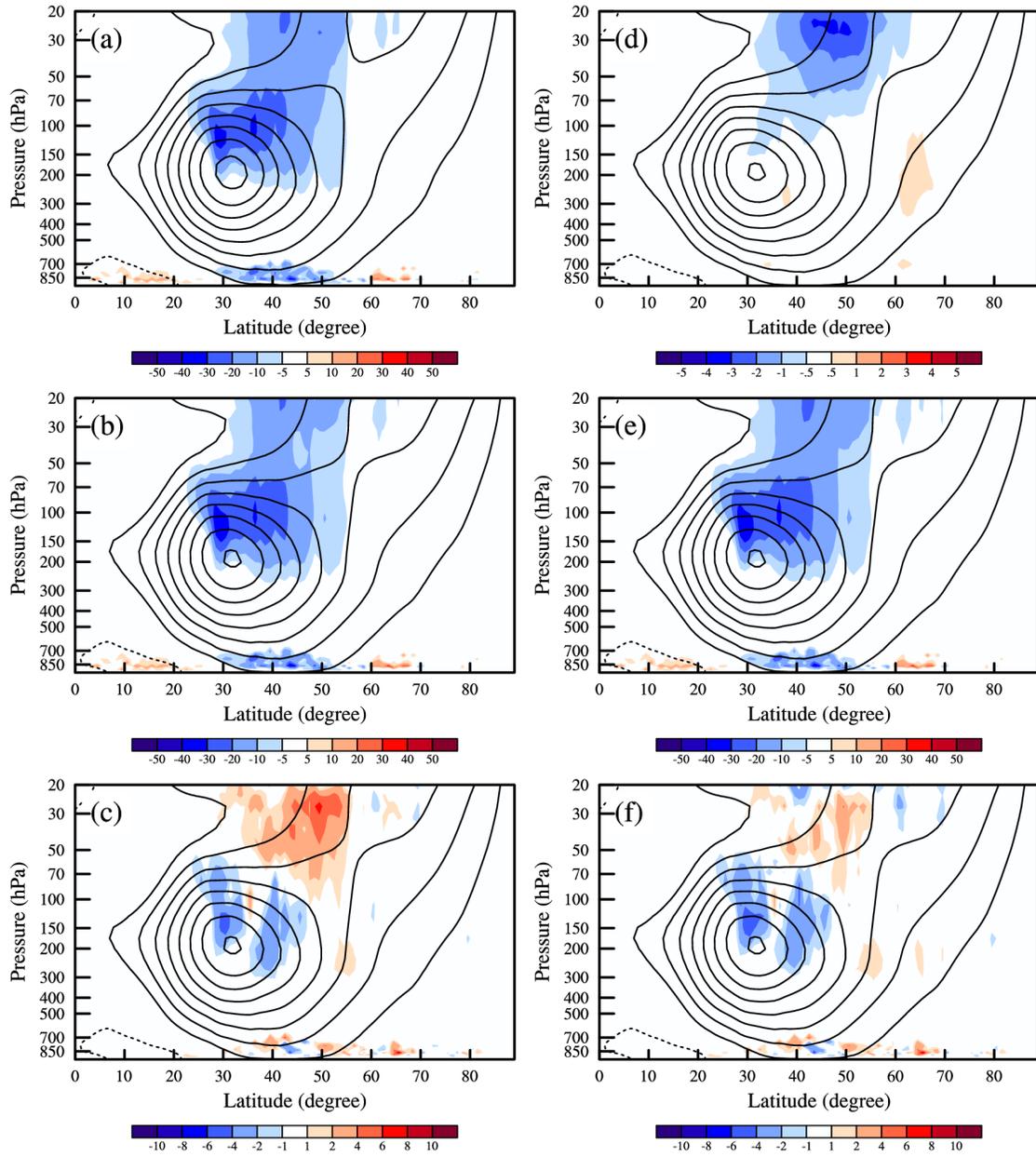


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904 Fig. 13. Similar to Fig. 4 except for the “weak directional absorption” case (NEW1 experiment).

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Fig. 14. Similar to Fig. 8 except for the “weak directional absorption” case (NEW1 experiment).