

# *Thermodynamic cycles in the stratosphere*

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1 **Thermodynamic cycles in the stratosphere**

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## ABSTRACT

11 Large-scale overturning mass transport in the stratosphere is commonly ex-  
12 plained through the action of potential vorticity (PV) rearrangement in the  
13 flank of the stratospheric jet. Large-scale Rossby waves, with wave activ-  
14 ity source primarily in the troposphere, stir and mix PV and an overturning  
15 circulation arises to compensate the zonal torque imposed by the breaking  
16 waves. In this view, any radiative heating is relaxational and the circulation  
17 is mechanically driven. Here we present a fully thermodynamic analysis of  
18 these phenomena, based on ERA-Interim data. Stream functions in a ther-  
19 modynamic, log-pressure-temperature space are computed. The sign of a cir-  
20 culation cell in these coordinates directly shows whether it is mechanically  
21 driven, converting kinetic energy to potential and thermal energy, or thermally  
22 driven, with the opposite conversion. The circulation in the lower stratosphere  
23 is found to be thermodynamically indirect, i.e. mechanically driven. In the  
24 middle and upper stratosphere thermodynamically indirect and direct circula-  
25 tions coexist, with a prominent semiannual cycle. A part of the overturning  
26 in this region is thermally driven, whilst a more variable indirect circulation  
27 is mechanically-driven by waves. The wave driving does not modulate the  
28 strength of the thermally direct part of the circulation. This suggests that the  
29 basic overturning circulation in the stratosphere is largely thermally driven,  
30 while tropospheric waves add a distinct indirect component to the overturn-  
31 ing. This indirect overturning is associated with poleward transport of anoma-  
32 lously warm air parcels.

## 33 1. Introduction

34 The existence of an overturning circulation in the stratosphere was inferred from the observed  
35 distribution of chemical species above the tropopause, in particular water vapor (Brewer 1949) and  
36 ozone (Dobson 1956). Deduction of Lagrangian velocities from the height-latitude distribution of  
37 such tracers is one approach to diagnose the meridional circulation in the middle atmosphere.  
38 Furthermore, dynamical estimates of stratospheric transport have been developed which helped  
39 understand the physical mechanisms driving motion. The currently accepted paradigm is that the  
40 stratospheric overturning circulation is the result of the Coriolis force of the overturning circulation  
41 compensating for the westward wave torque imposed by breaking Rossby and gravity waves (see  
42 e.g. Cohen et al. 2014). Quasi-geostrophic theory led to this identification of wave-mean-flow  
43 interaction as the main driver of the overturning circulations, but the introduction of Lagrangian  
44 averages in the theory raised awareness of the relation between the actual mass transport in the  
45 stratosphere and diabatic effects (Dunkerton 1978).

46 A successful approximation to the Lagrangian circulation in this context has been provided by  
47 the introduction of the Transformed Eulerian Mean (TEM, see Andrews and McIntyre 1976, 1978;  
48 Andrews et al. 1987). This theory offers a framework to diagnose both the wave forcing on the  
49 mass transport and the diabatic effects. The net impact of the waves on the mean flow can be  
50 explained in terms of the stirring of potential vorticity (PV) by breaking waves which changes the  
51 mean PV gradient, establishing a surf zone (McIntyre and Palmer 1984; Becker and Schmitz 2003;  
52 Scott and Liu 2014) of homogeneous PV in the midlatitude stratosphere.

53 Progress in this area helped explain the distribution of tracers found by Brewer and Dobson  
54 and contributed to the current perspective on the middle-atmosphere overturning circulation that  
55 is summarised by Plumb (2002). Synoptic waves induce a symmetric circulation (with respect

56 to the equator) in the lower-stratosphere with air rising in the tropics and falling in the extra-  
57 tropics. Planetary, quasi-stationary waves perturb the core of the stratosphere, inducing an inter-  
58 hemispheric cell, with upward motion at low latitudes and sinking motion in winter polar latitudes.  
59 Gravity waves drive a global cell in the mesosphere. The stratosphere can be thought of as divided  
60 into 4 regions with specific features, namely the summer extratropics, the tropical stratosphere, the  
61 surf zone and the cold pole (see e.g. Butchart 2014).

62 Overturning circulations in the atmosphere are induced by diabatic heating and by wave-torque  
63 (Schneider 2006). The idealised inviscid limit described by Held and Hou (1980) explains the  
64 tropospheric Hadley cell implied by non-uniform insolation. A stratospheric analogue (Seme-  
65 niuk and Shepherd 2001) of this mechanism explains tropical upwelling even in the inviscid limit  
66 (Dunkerton 1989). Thermodynamically this circulation is maintained by diabatic heating and pro-  
67 duces work, i.e. is thermally driven and thermodynamically direct. Turbulent advection and waves  
68 must be invoked to ensure conservation of angular momentum (see e.g. Plumb and Eluszkiewicz  
69 1999; Schneider 2006). This is particularly relevant to the stratosphere where hemispheric cells  
70 move equatorial air into the poles, across surfaces of constant angular momentum.

71 Evidence and consensus have established the role of extra-tropical planetary Rossby waves as  
72 major driver of the stratosphere overturning circulation. The mechanism behind a wave driven cir-  
73 culation is often named 'extra-tropical pump', after the work of Holton et al. (1995). A meridional  
74 circulation in presence of breaking waves is deducible from the TEM equations, and further clari-  
75 fication in this regard was provided by the formulation of the downward control principle (Haynes  
76 et al. 1991), which showed that any steady, wave-driven vertical motion at a certain level in the  
77 stratosphere can be predicted by the wave-force exerted above that level.

78 In the stratosphere, waves move air masses adiabatically out of radiative equilibrium and diabatic  
79 heating brings the system back to radiative equilibrium, attempting to restore the unperturbed

80 equator-to-pole PV gradient. If the wave-induced adiabatic motion increases the potential energy,  
81 it requires work to be maintained, and is damped by heating and cooling. The circulation is  
82 then mechanically driven and thermally indirect (Shepherd 2007). However, the wave-induced  
83 adiabatic motion may also convert potential and thermal energy to kinetic energy by flattening the  
84 isentropic surfaces. Thermodynamically, such a circulation is thermally driven, even if waves are  
85 a necessary ingredient.

86 In this study a thermodynamic analysis of the stratospheric overturning circulation in  
87 log(pressure)-temperature coordinates is presented. Thermodynamic coordinates have been stud-  
88 ied and applied in oceanography (Zika et al. 2012; Nurser and Lee 2004) and the analysis presented  
89 by Nycander et al. (2007) is an example of a fully thermodynamic description of the global oceanic  
90 circulation. In the atmosphere, thermodynamic coordinates have typically been restricted to their  
91 use as a vertical coordinate, e.g. pressure or potential temperature, while keeping latitude as  
92 horizontal coordinate. For example, isentropic coordinates yield a good estimate of the lagrangian  
93 motion in the stratosphere. Indeed the meridional stream function in potential temperature-latitude  
94 coordinates shows two cells extending from tropical regions to high latitudes (Kållberg et al. 2005)  
95 in agreement with the TEM overturning circulation. Examples of fully thermodynamic descrip-  
96 tions of the atmospheric circulation can be found in scientific literature (e.g. Pauluis et al. 2010;  
97 Kamieniecki et al. 2018), but a thermodynamic analysis specifically designed for the stratosphere  
98 circulation has not been used previously.

99 Here we apply a fully thermodynamic set of coordinates to describe the state of the strato-  
100 sphere and its variability. A major advantage is that this offers a possibility to use a definition  
101 of ‘mechanically’ and ‘thermally’ forced circulations that agrees with fundamental thermodynam-  
102 ics. According to this definition, the circulation is mechanically forced if it converts mechanical  
103 energy to heat, and thermally forced if it converts heat to mechanical energy. Note that “mechan-

104 ically driven” is not necessarily the same thing as “wave driven” in stratospheric research, where  
105 it is common to regard the meridional circulation as wave driven if it is proportional to the eddy  
106 terms in a simplified analytic model. In thermodynamics, a heat engine is thermally forced and a  
107 refrigerator is mechanically forced. By contrast, the meridional circulation in an idealized model  
108 of the atmosphere may be proportional to friction or viscosity, although friction is always a sink of  
109 kinetic energy and cannot be a forcing agent from a thermodynamic point of view. One problem  
110 with the term “wave-driven” is that it is not straightforward to diagnose the connection between  
111 the eddy terms and meridional circulation in a three-dimensional simulation, or in observational  
112 data. It is therefore unclear how to identify which parts of the circulation are wave-driven, and  
113 which parts are not.

114 By using thermodynamic coordinates, we can identify what parts of the stratospheric overturning  
115 circulation are mechanically driven, with a passive adjustment of diabatic effects, and what parts  
116 are thermally driven. In both cases, waves and eddies may be an essential part of the dynamics,  
117 but in the former case their energy is transported from other regions by the waves, while in the  
118 latter case the eddies extract most of their energy from the local mean flow. However, our analysis  
119 does not distinguish between eddies and mean flow.

120 Moreover, the magnitude of this energy conversion is obtained directly by integrating the stream  
121 function over the cell. At the same time we will find that much geometric information is implicitly  
122 maintained. This is due to the simple observation that pressure decreases with altitude, while  
123 temperature, broadly, decreases with latitude.

124 In section 2, the thermodynamic stream function is presented, and its connection to the conver-  
125 sion between different kinds of energy is derived. Section 3 presents the numerical implementa-  
126 tion of the calculations, and the reanalysis data used. In section 4, results are presented, looking  
127 in particular at the comparison of months with strong and weak planetary wave forcing and at the

128 seasonal cycle. Finally, section 5 discusses the main findings of this study focusing on how they  
 129 can enrich the conventional interpretation of stratospheric motion.

## 130 2. Thermodynamic stream function

131 We begin by analysing the energetics of the primitive equations for an ideal gas in pressure  
 132 coordinates. They are

$$\frac{d\mathbf{v}}{dt} + f\hat{z} \times \mathbf{v} + \nabla\phi = \mathbf{F} \quad (1)$$

$$\frac{\partial\phi}{\partial p} = -\alpha \quad (2)$$

$$\nabla \cdot \mathbf{v} + \frac{\partial\omega}{\partial p} = 0 \quad (3)$$

$$c_v \frac{dT}{dt} + p \frac{d\alpha}{dt} = \frac{1}{g} \frac{\partial\Phi_E}{\partial p} + Q \quad (4)$$

$$p\alpha = RT \quad (5)$$

Here  $\mathbf{v}$  is the horizontal velocity,  $f$  the Coriolis parameter,  $\nabla$  the horizontal gradient operator along isobaric surfaces,  $\phi$  the geopotential,  $\mathbf{F}$  the viscous force,  $\alpha$  the specific volume,  $\Phi_E$  the vertical non-advective energy flux by turbulent mixing,  $Q$  represents the internal sources and sinks of energy due to radiation and latent heat,  $T$  and  $p$  are respectively temperature and pressure,  $\omega = dp/dt$  is the vertical velocity in pressure coordinates, and

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \omega \frac{\partial}{\partial p}.$$

Define the mass element  $dm = \rho dV = \mathbf{dr} dp/g$ , where  $\mathbf{dr}$  is a horizontal surface element,  $\rho$  the mass density,  $V$  volume and  $g$  the gravity acceleration. The kinetic energy is then given by

$$K = \int \frac{|\mathbf{v}|^2}{2} dm,$$

the internal energy by

$$I = \int c_v T dm,$$



and the potential energy by

$$U = \int \phi \, dm.$$

The integrals are here taken over the whole atmosphere. Assuming that the bottom surface is flat, with  $\phi = 0$  at the surface, we also have

$$U = - \int p \frac{\partial \phi}{\partial p} \, dm = \int p \alpha \, dm.$$

137 By using eq. (5) we obtain

$$U = \frac{R}{c_v} I. \quad (6)$$

138 We now derive budget equations for the various energy forms. Multiplying eq. (1) by  $\mathbf{v} \cdot$  and  
139 integrating, we obtain

$$\frac{dK}{dt} = C - D, \quad (7)$$

140 where  $D > 0$  is the frictional dissipation, and  $C$  is the conversion from potential energy to kinetic  
141 energy. By using eqs (3) and (2) we can write

$$C = - \int \omega \alpha \, dm. \quad (8)$$

142 Then integrate eq. (4) over the fluid volume. Using the relation

$$\int p \frac{d\alpha}{dt} \, dm = \frac{d}{dt} \int p \alpha \, dm - \int \alpha \frac{dp}{dt} \, dm, \quad (9)$$

143 we obtain

$$\frac{d}{dt} (U + I) = S - C, \quad (10)$$

where  $S$  is the total heat source,

$$S = \int Q \, dm.$$

144 In the terminology of classical thermodynamics,  $U + I$  is the enthalpy of the atmosphere. It is also  
145 sometimes called the 'total potential energy'. As a result of the hydrostatic approximation,  $U$  and

146  $I$  are not independent quantities, as seen from eq. (6). Nevertheless, it is possible to diagnose the  
 147 transfer between  $I$  and  $U$  by using the definition of the thermodynamic work  $W$ :

$$W = \int p \frac{d\alpha}{dt} dm. \quad (11)$$

148 Equation (9) can now be written

$$\frac{dU}{dt} = W - C. \quad (12)$$

149 By combining eqs (10) and (12) we obtain

$$\frac{dI}{dt} = S - W, \quad (13)$$

150 explicitly showing that  $W$  represents the conversion from internal energy to potential energy. The  
 151 budget equations (7), (12) and (13) are illustrated in the box diagram in Fig. 1. As a result of the  
 152 hydrostatic approximation, there is no direct energy transfer between  $I$  and  $K$  without involving  
 153  $U$ . Such a direct transfer is involved in sound waves, which cannot be described by the hydrostatic  
 154 model (1)-(5).

155 We now introduce the thermodynamic stream function for mass with specific volume and pres-  
 156 sure as coordinates,  $\Psi(\alpha, p)$ . By definition,  $\Psi(\alpha, p)$  is the mass flux through that part of the  
 157 isobaric surface given by  $p$  where the specific volume is greater than  $\alpha$ . Equivalently, it is the  
 158 mass flux through the surface where the specific volume is  $\alpha$  and the pressure is greater than  
 159  $p$ . The equivalence requires that the atmosphere is in a statistically steady state, so that that the  
 160 mass distribution on the  $(\alpha, p)$ -plane is stationary. In this case the mass flux on this plane is non-  
 161 divergent, and can be represented by a stream function. The thermodynamic stream function is  
 162 illustrated in Fig. 2.

We will now calculate the thermodynamic work  $W$  defined in eq. (11) by integrating over the  
 $(\alpha, p)$ -plane. We then need to know the mass  $dm$  in the element  $d\alpha dp$  shown Fig. 2. This mass

is given by  $dm = d\Psi \delta t$ , where  $\delta t = d\alpha / (d\alpha/dt)$  is the time it takes for the flow to go from  $\alpha$  to  $\alpha + d\alpha$ , and  $d\Psi = (\partial\Psi/\partial p)dp$  is the difference of  $\Psi$  across the element. We obtain

$$dm = \frac{\partial\Psi/\partial p}{d\alpha/dt} d\alpha dp.$$

Using this, eq. (11) gives

$$W = \int p \frac{\partial\Psi}{\partial p} d\alpha dp = - \int \Psi d\alpha dp,$$

163 where the partial integration was carried out using the fact that  $\Psi \rightarrow 0$  as  $p \rightarrow 0$  and  $p \rightarrow \infty$ .

164 Below, we will use the temperature  $T$  as a coordinate instead of  $\alpha$ , while pressure is replaced  
 165 by ‘log(pressure)’  $z = H \ln(p_0/p)$  as the vertical coordinate. This makes little difference, since  
 166 the contours  $T = \text{const}$  and  $\alpha = \text{const}$  coincide on isobaric surfaces. Using  $dT = (p/R)d\alpha$  and  
 167  $dz = -(H/p)dp$  we obtain

$$W = \frac{R}{H} \int \Psi dT dz. \quad (14)$$

168 The sign convention for  $\Psi$  is here chosen so that a circulation cell with positive  $\Psi$  on the  $(T, z)$ -  
 169 plane is anticlockwise, i.e. thermally direct with  $W > 0$ , with warm air advected upward, to lower  
 170 pressure, and cold air downward. This corresponds to the clockwise circulation in Figure 2 on the  
 171  $(\alpha, p)$ -plane.

The thermodynamic character of the circulation is closely connected to the sign of the vertical  
 advective heat flux  $F$ . The upward advective heat flux through the pressure surface  $p = p_0$  is given  
 by

$$F = -\frac{1}{g} \int_{p=p_0} \omega c_p T dS,$$

172 where  $dS$  is a surface element and  $c_p$  the heat capacity at constant pressure. Consider two isother-  
 173 mal curves on the pressure surface, enclosing the surface element  $dS$ . The mass flux across this  
 174 element is  $d\psi$ , which is the difference between the value of  $\psi$  on the two curves, and the advective

175 heat flux is  $c_p T d\psi$ . We obtain, after a partial integration

$$F = - \int_{p=p_0} c_p T d\Psi = \int_{p=p_0} c_p \Psi dT. \quad (15)$$

176 Thus, the advective heat flux is obtained directly from the thermodynamic stream function, by  
177 integrating  $\psi(T, p)$  along the line  $p = p_0$ . If the circulation is thermally driven, and  $\Psi$  hence  
178 positive, heat is advected upwards by the circulation.

179 If  $\Psi$  is positive,  $W$  and  $C$  in Figure 1 are both positive, i.e. internal and potential energy is  
180 converted to kinetic energy. (Note that  $C = W$  if the circulation is steady.) A thermally indi-  
181 rect circulation cell, with negative  $\Psi$ , on the other hand, converts kinetic energy to internal and  
182 potential energy by adiabatic compression.

183 Thus, the advantage with the thermodynamic stream function is that the sign of  $\Psi$  immediately  
184 shows whether the circulation is thermally forced or mechanically forced, using our definition.

185 The main non-adiabatic effect in the stratosphere is radiation, which relaxes the atmosphere to a  
186 stably stratified radiative equilibrium. Consider a wave that propagates on this background strat-  
187 ification. If a wave crest pushes air upwards, the air cools adiabatically, and is then warmed by  
188 radiation. The result is that the anomalously cold air is advected irreversibly upwards, through an  
189 isentropic surface. Similarly, the air in a wave trough that pushes air downwards is anomalously  
190 warm, then cooled by radiation, and advected irreversibly downwards. The net result is down-  
191 ward heat advection. Thus, radiation damping of waves leads to a mechanically forced, indirect  
192 circulation that converts kinetic energy to internal and potential energy.

193 However, since the radiative equilibrium has sloping isentropic surfaces, waves may instead  
194 extract energy from the background by flattening these surfaces. This is what happens in baroclinic  
195 instability. The result is a direct circulation that converts potential and thermal energy to kinetic

196 energy. From a thermodynamic point of view it is thermally forced, even if it is mediated by the  
197 presence of waves.

198 The circulation of the atmosphere is ultimately driven by solar heating. Thus, the integral of  
199 the thermodynamic stream function over the whole atmosphere must be positive. But there may  
200 still exist separate circulation cells with different sign. The kinetic energy required to sustain the  
201 circulation in an indirect cell must then be generated in another, thermally forced cell (typically in  
202 the troposphere). Such energy transfer between different cells may be achieved by various waves.

203 The purpose of the presented analysis is to provide a definition of thermally-driven and  
204 mechanically-driven from a thermodynamic perspective. For the stratosphere, this approach al-  
205 lows to quantify to what extent waves act as a mechanical forcing of the overturning, as is com-  
206 monly assumed. Indeed, while waves are a necessary ingredient to ensure conservation of angular  
207 momentum, to some extent their role is analogous to the role of friction in a steady gravity cur-  
208 rent down a slope, where the driver is gravitational potential energy and the friction is required to  
209 ensure steadiness.

### 210 **3. Methodology**

#### 211 *a. Circulation in thermodynamic coordinates*

212 Results presented in section 4 are based on a computation with reanalysis data of the flow in  
213 a temperature-log(pressure) ( $T$ - $z$ ) space in the stratosphere. The vertical coordinate is defined by  
214  $z \equiv H \ln(p_0/p)$  where the reference pressure  $p_0$  is chosen as 1000 hPa, and the temperature scale  
215 height  $H$  is chosen as 6 km. The flow is defined by the mass flux densities

$$\Omega(T, z) \equiv \Phi(T, z)\dot{z}(T, z) \quad (16a)$$

$$\tau(T, z) \equiv \Phi(T, z)\dot{T}(T, z) \quad (16b)$$

216 where  $\Phi$  is a mass density in  $(T, z)$ -space and  $\dot{z}$  and  $\dot{T}$  are mass-weighted tendencies.  $\Omega$  is defined  
 217 as

$$\Omega(T, z) = \iint_V \delta(T(\mathbf{r}) - T) \delta(z(\mathbf{r}) - z) \dot{z}(\mathbf{r}) \rho(\mathbf{r}) \, d\mathbf{r}, \quad (17)$$

218 where  $\mathbf{r}$  is integrated over all of the domain  $V$ , and where  $\rho$  is the usual (geometric) mass density.  
 219 Other variables in  $(T, z)$  space are computed analogously; the mass density  $\Phi$  is computed by  
 220 replacing  $\dot{z}(\mathbf{r})$ , above, with unity.  $\tau$  can be computed replacing  $\dot{z}(\mathbf{r})$  in Eq. 17 with the lagrangian  
 221 tendency of temperature ( $\dot{T}(\mathbf{r})$ ).

222 Assuming that the flow defined by  $\Omega$  and  $\tau$  is non-divergent (this is a consequence of the diag-  
 223 nosed flow corresponding to a broadly steady state, so that the mass distribution  $\Phi$  is essentially  
 224 constant in time) a stream function can be defined as the integral of  $d\Psi = \tau \, dz - \Omega \, dt$ . More specif-  
 225 ically, for the Earth's atmosphere,  $\Phi$  can be regarded as the total atmospheric mass at temperature  
 226  $T$  and pressure  $p$ ,  $\dot{z}(T, z)$  and  $\dot{T}(T, z)$  are respectively the mass-weighted, lagrangian tendency of  
 227 the logarithm of pressure and of temperature, again at temperature  $T$  and pressure  $p$ . With these  
 228 premises,  $\Psi$  can be computed everywhere if either  $\Omega$  or  $\tau$  are known.

229 The definition of  $\Omega$  for a discrete, longitude-latitude grid on a pressure level  $p$  is:

$$\Omega(T, z) = \sum_{l,m} \frac{\dot{z}_{lm} M_{lm} \delta_{T_{lm}, T}}{\Delta z \Delta T}. \quad (18)$$

230 where the sum is done over longitudes  $l$  and latitudes  $m$ ,  $M_{lm}$  is the mass represented by the  
 231 gridbox, and

$$\delta_{T_{lm}, T} = \begin{cases} 1, & \text{if } T - \Delta T/2 < T_{lm} < T + \Delta T/2 \\ 0, & \text{elsewhere.} \end{cases} \quad (19)$$

232  $\Delta z$  and  $\Delta T$  in eq. 18 are the spacing in the discretised thermodynamic space.

233 Assuming hydrostatic balance, the mass of the air in each gridbox can be expressed as the  
 234 product of the gridbox area  $\Delta A_{lm}$  and mass per unit area  $\Delta p/g$ . This decomposition is useful

235 because the underlying data are organised along pressure surfaces representing a fixed pressure  
 236 thickness  $\Delta p$ . Further details are written out in the Appendix. Similarly, hydrostatic balance can  
 237 be used to show that for the log-pressure coordinate  $z$  we will have  $\dot{z}/\Delta z = -\dot{p}/\Delta p$ . Substituting  
 238 in equation 18 and multiplying by  $H$  the mass flux can be written as

$$\Omega(T, z) = - \sum_{l,m} \frac{\dot{p}_{lm} \Delta A_{lm} \delta_{T_{lm}, T}}{g \Delta T}. \quad (20)$$

239 The stream function can be computed by integrating over the  $T$ -dimension

$$\Psi(T, z) = - \int_0^T \Omega(T', z) dT'. \quad (21)$$

240 Densities of mass ( $\Phi$ ) and latitude ( $\Theta$ ) have been computed similarly, as reported in the Appendix.  
 241 The operation introduced by Eq. 20 and 21 corresponds to summing up mass-weighted lagrangian  
 242 tendencies (trajectories) corresponding to air masses of similar temperature and pressure, and is  
 243 conceptually analogous to the computation of a stream function in geometric coordinates. In this  
 244 case, the usual geometric coordinates are replaced by thermodynamic variables. Therefore,  $\Omega$  and  
 245  $\tau$  indicate motion along respectively the vertical ( $z$ ) and horizontal ( $T$ ) coordinate of the  $T$ - $z$  space  
 246 that is introduced in Fig. 3.

## 247 *b. Data*

248 The analysis presented in this study is based on reanalysis data from ERA-Interim, the European  
 249 Centre for Medium-Range Weather Forecasts reanalysis (Dee et al. 2011). Fields of temperature,  
 250 lagrangian pressure tendency, zonal wind and meridional wind are obtained 6-hourly, on a  $0.75^\circ \times$   
 251  $0.75^\circ$  regular, longitude-latitude grid on all available pressure levels, for a reference period from  
 252 September 1994 to August 2015. The circulation in the stratosphere is diagnosed in temperature  
 253 and log-pressure coordinates on 14 pressure levels between 225 and 1 hPa. The temperature spac-  
 254 ing  $\Delta T$  is constant and set at 3K, the pressure spacing  $\Delta p$  is equal to the vertical spacing of the

255 dataset, and typically varies with pressure level. Results presented in Figs. 9-13 are computed  
 256 for two subsets of the reference period, namely May and June (MJ) and November and December  
 257 (ND). Since a thermodynamic stream function can only be defined for the global circulation, to  
 258 identify a correspondence between regions of the geometric space and portions of the thermo-  
 259 dynamic space, we have defined 4 sectors, whose boundaries are shown in table 1, and we have  
 260 computed mass densities in  $T$ - $z$  space for each of these limited areas.

261 Because the mass fluxes are not completely steady, nor numerically fully conservative, stream  
 262 functions are computed applying a small correction on the mass flux density. Introducing the  
 263 average  $\langle \dots \rangle = (\Delta T)^{-1} \int_0^\infty \dots dT$ , then a corrected mass-flux is defined by

$$\tilde{\Omega}(T, z) = \Omega(T, z) - \Phi(T, z) \langle \Omega \rangle / \langle \Phi \rangle \quad (22)$$

264 which ensures that  $\langle \tilde{\Omega} \rangle = 0$ . The stream function  $\Psi$  is then computed using eq. 21 with  $\Omega$  replaced  
 265 by  $\tilde{\Omega}$ . For pressure levels between 1 and 175 hPa  $|\langle \tilde{\Omega} \rangle| / |\langle \Omega \rangle| < 0.01$ , so the corrections required  
 266 are of a small magnitude compared to typical diagnosed mass fluxes.

## 267 4. Results

### 268 a. Stratosphere circulation in thermodynamic coordinates

269 Figure 3 shows the annual mean stream function  $\psi$  in the  $T$ - $z$  space. Contours in the figure  
 270 identify streamlines of the motion, with the direction of the motion identified in every point by the  
 271 vector  $(\Omega, \tau)$ . This direction is indicated qualitatively by the arrows. Closed streamlines identify  
 272 thermodynamic cycles, and  $\Psi$  defines the mass per unit time flowing in a certain cycle.

273 At the very bottom, there is a thermally direct cell. This is the uppermost part of the Hadley  
 274 cell in the troposphere, which is driven by diabatic heating from below. We can then identify three  
 275 different altitude regions. In the tropopause and lower stratosphere, roughly 225 hPa - 50 hPa,



276 the entire circulation is thermally indirect, with the warmest air downwelling and the coldest air  
277 upwelling (this is the upwelling through the cold point at the tropopause).

278 In the mid stratosphere, say between 50 hPa and 10 hPa, there are two cells. In the direct cell  
279 warm air rises in the tropics, and cold air downwells. There is also an indirect cell of greater  
280 intensity with downwelling at the highest temperatures. The final region is the upper-stratosphere  
281 (10 hPa and 1 hPa) where the indirect cell fades and the circulation turns into predominantly direct.

282 The integral of the stream function  $\Psi$  over temperature is proportional to the vertical advective  
283 heat flux, as seen from Eq. (15). The heat flux associated with the direct and indirect cells taken  
284 individually is shown in Figure 4. The displayed quantity is  $(c_p/A) \int \Psi dT$ , where  $c_p$  is the air  
285 specific heat at constant pressure and  $A$  is the area of the Earth. The heat flux is strongly positive  
286 in the troposphere and becomes negative above 200 hPa where the circulation is only indirect.  
287 The absolute value of the negative flux in the lower stratosphere reaches a maximum around 125  
288 hPa and then starts decreasing with height, while the direct component starts being detectable at  
289 about 50 hPa. Above 20 hPa, the direct and indirect components broadly compensate, producing  
290 a near-zero heat flux.

291 Air motion in the stratosphere undergoes substantial seasonal variations driven by the seasonal  
292 cycle of radiative forcing and by the seasonal cycle of upward propagating waves. We therefore  
293 investigate the variations of the stream function through the year at three pressure levels in the  
294 middle of the three regions previously identified. They are: the upper-stratosphere (1-10 hPa)  
295 where the direct and indirect cell coexist and have similar magnitudes, the mid-stratosphere (10-  
296 50 hPa) where the two cells coexist but the indirect one is stronger, and the lower stratosphere  
297 (50-225 hPa) where the circulation is only indirect. In Fig. 5 the seasonal cycle of the monthly  
298 mean values of the stream function is diagnosed. At 5 hPa (Figure 5 a–b), seasonal variations  
299 are large (of the order of typical monthly mean values) with a semiannual cycle for both cells: the

300 direct cell peaks around the solstices, as expected from the strong meridional gradient of insolation,  
301 while the indirect cell peaks in late boreal winter and early boreal autumn.

302 Lower in the stratosphere, at 20 hPa (Figure 5 c–d), seasonal variations are weaker and two cells  
303 are always present, although the direct one is remarkably weak in late equinoctial seasons and the  
304 indirect one is weak in boreal summer. The direct cell shows a net semiannual cycle as in the  
305 upper stratosphere, whereas the indirect cell shows an annual cycle (Figure 5 e–f).

306 At 125 hPa the circulation is always thermodynamically indirect, and the seasonal cycle is mod-  
307 erate, with variations of about 15% of the annual mean. The strength of the cell is modulated  
308 from a peak in boreal winter to the minimum in the boreal summer. This finding is in agreement  
309 with the annual cycle of the lower branch of the Brewer-Dobson circulation due to the larger sea-  
310 sonal amplitude in the Northern upwelling (see e.g. Plumb 2002; Seviour et al. 2012; Homeyer  
311 and Bowman 2013). The root-mean-square vertical mass flux per unit Kelvin, in the right panels,  
312 also shows moderate seasonal variations broadly in tandem with the magnitude of the mass stream  
313 function.

314 To quantify the intensity of the circulations associated with the two cells, in Figs 6 and 7 the  
315 maximum and minimum values of the stream function are shown as function of the day of the  
316 year at the same levels used in Fig. 5. The direct circulation (Fig 6) peaks in the surroundings of  
317 the solstices in the mid and upper stratosphere. The intensity of the circulation is comparable in  
318 the mid and upper stratosphere and it undergoes a similar modulation through the seasons. In the  
319 lower stratosphere a small peak is found only in the late boreal summer. The indirect circulation  
320 (Fig 7) instead ranges through three orders of magnitude and shows considerably different features  
321 at different levels. It will be shown, in the following section, that the modulation of the intensity  
322 of the indirect circulation can be explained by the seasonality of upward propagation of Rossby  
323 waves that are also shown to be inadequate to explain the modulation of the direct part.

324 *b. The effect of wave-torque in the thermodynamic space*

325 The traditional view of the stratospheric overturning circulation implies a meridional mass dis-  
326 placement induced by large scale waves generated in the troposphere. Figure 8 shows the eddy  
327 heat flux  $[v^*T^*]$  at 100 hPa, averaged in the mid-latitudes (40°-80° N/S) both in the Northern  
328 Hemisphere (NH) and the Southern Hemisphere (SH) and their sum. This is a measure of the  
329 upward propagation of planetary Rossby waves into the stratosphere, and a strong predictor of PV  
330 rearrangement in the stratosphere (Hinszen and Ambaum 2010). The sign of the SH heat flux in  
331 this figure has been flipped, because the upward wave activity flux is proportional to the poleward  
332 heat flux, and not the northward heat flux (in essence, the corresponding upward Eliassen–Palm  
333 flux has a factor proportional to the Coriolis parameter; note also that the poleward heat fluxes are  
334 dominant in the midlatitudes).

335 In the NH the poleward heat flux peaks in mid boreal winter, whereas in the SH high values are  
336 found in late boreal summer and early autumn. The sum of the northern and southern poleward  
337 heat flux, i.e. the black line in Figure 8, shows an annual cycle with a short window of weak flux  
338 between May and August, and a longer window of high flux peaking in December. Because of the  
339 proportionality with the upward Eliassen–Palm flux, this sum can be interpreted as a measure of  
340 the total zonal wave-induced torque exerted on the global stratosphere.

341 The asymmetry between the two hemispheres allows to compare the global stratospheric cir-  
342 culation under similar radiative forcing (insolation) but with very different mechanical forcing  
343 (wave-induced torque), as the NH generates more wave activity than the SH in their respective  
344 winters. In view of this, May and June (MJ) are used as a reference, unperturbed stratospheric  
345 state, and November and December (ND) are used as a perturbed state, where the stratosphere  
346 is strongly forced by tropospheric waves. In Figure 9 the mass and vertical mass flux densities

347 (respectively  $\Phi$  and  $\Omega$ , defined in section 3) for the two reference periods MJ and ND are shown.  
348 Here shadings indicate how much mass is found in the atmosphere at temperature  $T$  and pressure  
349  $p$ , while contours indicate whether the air mass moves predominantly upward (positive sign) or  
350 downward (negative sign).

351 MJ can be regarded as the state of the stratosphere under weak mechanical forcing by planetary  
352 Rossby waves. The ensuing dynamics is therefore expected to be dominated by direct radiative  
353 forcing, at least in the mid-high stratosphere where the role of these waves is dominant (Plumb  
354 2002). In the lower stratosphere the mass of the atmosphere is distributed over a wide temperature  
355 range between 190 K and 230 K, a fact that is explained by the strong temperature gradients  
356 between the tropics, that contain much of the mass of the atmosphere, and the extra-tropics. At  
357 these levels, cold air rises in the equatorial regions and warmer air sinks. Above 50 hPa most of  
358 the mass is confined in a range of about 10–15 K, with a linear increase with pseudo-height of the  
359 temperature with the highest upward mass flux. Downward mass fluxes span a wider temperature  
360 range, at cooler temperatures.

361 ND is influenced more strongly by wave fluxes. It can be seen that in ND motion in the lower  
362 stratosphere is qualitatively similar to the unperturbed case, perhaps intensified, while in the mid  
363 and upper stratosphere some important changes are found. Air masses are distributed almost sym-  
364 metrically in temperature with respect to the upward and downward mass fluxes, with downward  
365 mass fluxes now also happening at high temperatures.

366 These differences in the mass flux have important implications for the thermodynamics of the  
367 overturning circulation, as confirmed by Figure 10 that shows the stream function  $\Psi$ , in contours,  
368 and the mass-weighted absolute value of latitude  $\Theta$ , in coloured shading, again for MJ and ND. In  
369 this figure contours can be interpreted as in Fig. 3, while shadings provide a qualitative informa-  
370 tion on the angular distance from the equator (with no distinction between the two hemispheres).

371 The circulation in the lower stratosphere is indirect in both cases. Tropical air masses move up-  
372 ward through the cold tropical tropopause region and move downward across a range of higher  
373 temperatures and latitudes. In MJ in the upper levels tropical air moves across isentropes in the  
374 updraft region found in Figure 9, but moves down at cold temperatures in the mid-latitudes and  
375 in the quasi-isothermal polar atmosphere. The motion is predominantly direct, as indicated by the  
376 positive sign of  $\Psi$  which dominates the upper stratosphere.

377 Although the tropical portion of the stratosphere is geometrically large, it is thermodynamically  
378 confined in a narrow temperature region. The figure also shows that polar air is found at both lower  
379 and higher temperatures, but the warm polar air is thermodynamically inactive in the MJ season,  
380 as can be seen from the low streamfunction gradients there, signifying slow thermodynamic trans-  
381 formation rates.

382 The corresponding panel for ND shows that the anomalous downward motion at high temper-  
383 atures is associated with an indirect circulation that in the thermodynamic space appears as an  
384 expansion of the lower tropical indirect branch into the upper levels. In this indirect cell, mid and  
385 high latitude warm air moves downward in a large band between 240 and 285 K. As shown later,  
386 this warm air is found in both hemispheres, at temperatures substantially larger than the zonal  
387 mean.

388 Figure 11 can help link these results with the geometric space. For MJ, the reference state, in  
389 the lower levels, the tropical stratosphere is colder than the mid-latitudes. In the upper levels the  
390 warm hemisphere is broadly isothermal, whereas the cold hemisphere shows strong temperature  
391 gradients in the mid-latitudes. In ND, the cold pole is warmer than in MJ, and stronger temperature  
392 gradients are found in the warm hemisphere.

393 Although there is no unequivocal correspondence between the thermodynamic space and the  
394 zonally averaged temperature, in MJ the geometric interpretation is perhaps straightforward: The

395 indirect cell in the lower stratosphere is attributable to overshooting convection or to the lower  
396 branch of the Brewer-Dobson circulation. It is, to a great extent, symmetric between the two  
397 hemispheres. Figure 11 shows that the equatorial air is on average colder than polar and midlat-  
398 itude air. As seen in Figure 10, the air moves upward through the cold tropical tropopause in a  
399 confined region of the  $T$ - $z$  space, and moves downward in a thermodynamically broader region,  
400 corresponding mainly to the mid-latitudes.

401 The direct circulation starts being detectable at about 50 hPa, which is the level where Figure 11  
402 shows the inversion of the equator-to-pole temperature gradient in the cold hemisphere. At these  
403 levels, equatorial air is at intermediate temperatures, between a warm pole, which is inactive,  
404 and a cold pole, which is active and corresponds to the descending branch of the direct cell.  
405 Comparing Figs. 10 and 11 it can be concluded that in ND the indirect circulation in the mid  
406 stratosphere associated with wave-forcing involves temperatures that are typical or higher than the  
407 zonal mean temperature of the warm hemisphere. In fact, the portion of the  $T$ - $z$  space with highest  
408 temperatures is a combination of extra-tropical air in the winter hemisphere at temperatures much  
409 higher than the climatological zonal mean, and air in the warm hemisphere, as shown by Figure 12  
410 and 13.

411 Figure 12 shows mass densities of four regions of the atmosphere defined in table 1. Equatorial  
412 air is colder than the rest of the atmosphere in the lower stratosphere but at higher levels the  
413 lowest temperatures are found at the cold pole. The tropics and warm extratropics span a narrow  
414 temperature range at constant pressure, while the cold pole and surf zone span a much wider range.  
415 In presence of waves (ND), a part of the mass of the cold pole and of the surf zone is shifted to  
416 very high temperatures that are found typically in the warm hemisphere.

417 This is further investigated in the analysis of the 10 hPa layer presented in Figure 13. Figure 13a  
418 shows the mass density in ND in temperature space for two regions, the warm extratropics and the

419 combination of cold pole and surf zone (the cold extratropics) defined as in table 5. This panel  
420 demonstrates that the wide temperature range spanned by air in the cold extratropics, at constant  
421 pressure, is indeed compatible with a small portion (note the logarithmic scale) of high temperature  
422 air in the winter extratropics. The mass-weighted mean vertical velocity, shown in Figure 13b (note  
423 that negative corresponds to downward), indicates downward motion at any temperature in the  
424 cold extratropics. After a threshold temperature, downwelling becomes stronger as temperature  
425 increases. Downward motion in the warm extratropics is weaker and upwelling is found around  
426 225 K. These panels of Figure 13 are useful to interpret the mass flux ( $\Omega$ ) shown in Figure 13c,  
427 which is equal to  $-\partial\Psi/\partial T$ .

428 The extratropics in both hemispheres have predominantly downward motion for temperatures  
429 higher than 230 K, which corresponds to the descending branch of the indirect circulation (see  
430 Figure 10). It can be concluded that the downwelling branch of the thermally indirect cell in  
431 the upper stratosphere in ND is associated with: 1) a small fraction of air in the cold hemisphere,  
432 “shifted” to very high temperatures, that have a large downward velocity and 2) larger air masses in  
433 the warm hemisphere, whose temperature is slightly larger than the zonal mean temperature in the  
434 region, with a relatively weak downward motion. The coexistence of these two distinct branches  
435 is consistent with the presence of the poleward heat flux at the tropopause found in Fig. 8.

436 Figure 14 synthesises the above findings on the effect of large scale Rossby waves in the  $T$ - $z$   
437 space by showing the anomalous (ND minus MJ) stream function. For the upper stratosphere, this  
438 can be interpreted as the thermodynamic stream function forced by Rossby wave breaking in the  
439 stratosphere. We can see that the mechanical forcing of the breaking Rossby waves maintains an  
440 anomalous indirect circulation with relatively mild air rising in the tropics and warm air sinking  
441 in extratropical regions. In Lagrangian terms, it shifts extra-tropical masses in the  $T$ - $z$  space  
442 from cold to warm regions (Figs. 9 and 12), it intensifies the upward equatorial branch of the

443 overturning circulation, and induces a thermodynamically indirect motion that converts kinetic  
444 energy to potential energy (Fig. 14).

## 445 **5. Concluding remarks**

446 The thermodynamic properties of the overturning circulation in the stratosphere have been anal-  
447 ysed. Stream functions in a temperature-log(pressure) thermodynamic space have been computed  
448 with realistic data from an atmospheric reanalysis. The stream function  $\Psi$  detects the thermo-  
449 dynamically active component of the circulation (for instance, air rising and sinking at the same  
450 temperature and pressure would result in a net zero mass flux in this coordinates system). It is  
451 clear that the coordinate system potentially loses geometric information although some obvious  
452 geometric information is maintained: pressure reduces with altitude, and temperature in the trop-  
453 ics has a small range which is easily located in temperature-log(pressure) space. An important  
454 advantage of this approach is the possibility to quantify whether the circulation converts thermal  
455 and potential energy to kinetic energy, or the other way around. This advantage has proved to be  
456 particularly appropriate for the stratosphere system considering that the mechanisms discussed in  
457 section 1 and 2 imply the existence of both kinds of circulation.

458 The analysis clearly shows the coexistence of two cells, a direct cell such that:

- 459 – it is confined to the mid and upper stratosphere (above 50 hPa)
- 460 – the associated circulation peaks around the solstices (Fig. 5)
- 461 – the associated circulation scales weakly and inversely with the magnitude of wave injection  
462 (i.e. it is typically weaker when waves are stronger, Fig. 14 )
- 463 – it produces work at the rate of about 7 TW in the annual mean
- 464 – it has a maximum at about 10 hPa and 225 K in the annual mean (Fig. 3)
- 465 – it has similar values at different pressure levels



466 – by virtue of the sign in the  $z$ - $T$  space it implies conversion of potential energy to kinetic  
467 energy (Eq. 14)

468 and an indirect cell such that:

- 469 – it spans the whole vertical structure of the stratosphere
- 470 – the associated circulation peaks at different levels in different periods of the years, but is  
471 typically strongest in late winter and summer (Fig. 6)
- 472 – the associated circulation scales with the magnitude of wave injection (Fig. 14)
- 473 – it requires work to be maintained at the rate of about 137 TW in the annual mean (7 TW  
474 above 50 hPa)
- 475 – it is maximum in the proximity of the tropopause and 200 K in the annual mean
- 476 – decreases monotonically with height, spanning three order of magnitudes between 225 and 1  
477 hPa
- 478 – by virtue of the sign in the  $z$ - $T$  space it implies conversion of kinetic energy to potential  
479 energy (Eq. 14)
- 480 – involves also motion that happens at temperatures that are very different from the zonal mean  
481 (Figs. 9-11)

482 The thermodynamic description of motion in the stratosphere yields an estimate of tropical up-  
483 welling that ranges from about  $8.1 \times 10^9$  Kg/s in ND to  $5.1 \times 10^9$  Kg/s in MJ, which is broadly  
484 consistent with previous estimates (e.g. Seviour et al. 2012, Fig. 6).

485 The diagnostics presented in Section 4 offer a thermodynamic picture of the qualitative dif-  
486 ference between the lower and upper stratosphere. The lower stratosphere is always dominated  
487 by a shallow indirect motion with weak seasonal variations. In the upper stratosphere, say above  
488 50 hPa, the direct and indirect components are both present with varying relative and total strength:

489 the underlying circulation in the upper stratosphere is thermodynamically direct, while an indirect  
490 anomaly is imposed by mechanical wave forcing; when the wave forcing is strong, the indirect  
491 component dominates the circulation in parts of thermodynamic space, in particular at anoma-  
492 lously high temperatures in  $T-z$  space (which, geometrically, are found in both the warm and the  
493 cold hemisphere).

494 The downward mass flux for the direct branch happens geometrically in the cold winter hemi-  
495 sphere at cold temperatures, whereas downward motion in the indirect branch is found in both  
496 hemispheres at warm temperatures. Both the direct and the indirect component imply cross-  
497 isentropic flow, as they must to close the circulation thermodynamically.

498 The thermodynamic analysis presented in this study is focused on overturning motion in the  
499 stratosphere only but it has been applied also in the troposphere, where it reveals the thermally  
500 driven, direct Hadley cell (not shown). It should be noted anyway that below 500 hPa the correction  
501 introduced by Eq. 22 becomes relatively large even if the presence of orography is taken into  
502 account.

503 The indirect cell between the tropopause and the 50 hPa layer indicates conversion from kinetic  
504 energy to potential energy, and requires a net flux of eddy kinetic energy to be maintained. A  
505 reasonable explanation is that the kinetic energy can be supplied for the lowermost part by break-  
506 ing small scale Rossby waves (Plumb 2002) and by convective overshooting (Fueglistaler et al.  
507 2009). Overshooting convection (i.e. essentially by the inertia of upwelling air) and gravity waves  
508 generated by deep convection (Yu et al. 2019) may therefore be important for a part of the annual  
509 mean indirect flow in the lower stratosphere. This view is supported by observations that link cold  
510 anomalies at the tropical tropopause to deep convection (Gettelman et al. 2002; Schoeberl et al.  
511 2019; Johnston et al. 2018). On the other hand, even models that have no convection up to the cold  
512 point manage to describe the thermal structure of this region well (Gettelman and Birner 2007).

513 The absence of a thermally driven cell in the lower stratosphere is consistent with findings of  
514 Semeniuk and Shepherd (2001). The lack of a semiannual cycle and the peak of the indirect cell  
515 intensity in a solstice season (Figure 5) indicate that it is unlikely that a direct component here is  
516 masked by a stronger indirect one.

517 Being an inertial mechanism with a source in the troposphere, convective overshooting becomes  
518 less relevant at higher altitudes, in the middle stratosphere; similarly small-scale (synoptic) Rossby  
519 waves are filtered out by the lower layers above the tropopause. Here, large-scale Rossby waves  
520 have a major role, as the indirect component in the mid-stratosphere is absent when forcing from  
521 these waves is absent. Breaking Rossby waves act as an “extratropical wave pump” (Holton et al.  
522 1995), inducing poleward motion to balance the horizontal shear of zonal momentum flux. How-  
523 ever, proportionality between the magnitude of the indirect cell and tropospheric wave propagation  
524 is not straightforward: seasonal variations in the vertical distribution of zonal mean zonal wind  
525 (not shown) suggest that, by Charney–Drazin filtering, the indirect cell in the uppermost part of  
526 the stratosphere is well controlled by individual NH and SH heat flux in solstice seasons. In inter-  
527 mediate seasons and far from the upper stratosphere (approximately below 5 hPa), it is controlled  
528 by the total heat flux (i.e. NH plus SH). The direct component is dominant around the solstices,  
529 and it shows a semiannual cycle at all levels.

530 Overall, the perspective summarised by Plumb (2002) can be recovered in the  $T$ - $z$  description  
531 of the circulation. However, the present analysis emphasises the energetics of the circulation and  
532 highlights the fact that the basic part of the circulation is thermally forced, and that, when they  
533 are strong, breaking Rossby waves add a thermodynamically independent mechanically driven  
534 component.

535 Comparison with other and larger datasets may help discern robust features and highlight dif-  
536 ferences between reanalyses that have been pointed out by Abalos et al. (2015). Similarly, the

537 application of similar diagnostics in a multi-model framework can be insightful for a process-wise  
 538 interpretation of biases. Applying this thermodynamic description to idealised model experiments  
 539 can be illustrative, as it can show, for instance, the circulation responding to diabatic forcing only  
 540 or to wave forcing with constant, prescribed heating.

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## 544 APPENDIX

545 The mass of a portion of atmosphere at a certain pressure level, longitude and latitude has been  
 546 computed as

$$M_{lm} = \left| \frac{\Delta p \Delta A_{lm}}{g} \right| = \left| \frac{a^2 \Delta p \Delta \phi_l \Delta \theta_m \cos \theta_m}{g} \right| \quad (\text{A1})$$

547 Here  $a$  is the Earth's radius,  $\theta$  is latitude,  $\phi$  is longitude,  $\Delta A_{lm}$  is the area of the grid cell. The mass  
 548 density in  $T$ - $z$  space is defined as

$$\Phi(T, z) = \sum_{l,m} \frac{M_{lm} \delta_{T_{lm}, T}}{\Delta z \Delta T}. \quad (\text{A2})$$

549 Logarithm of mass density  $\Phi$ , displayed in Figure 12, is  $\log \left( \frac{\Phi}{\langle \Phi \rangle_{\text{top}}} \right)$  where  $\langle \Phi \rangle_{\text{top}}$  is the mass  
 550 density averaged over temperature at 1 hPa. The mass weighted average of the absolute value of  
 551 latitude of air masses in  $T$ - $z$  space have been computed as

$$\Theta(T, z) = \frac{\sum_{l,m} |\theta_{lm}| M_{lm} \delta_{T_{lm}, T}}{\sum_{l,m} M_{lm} \delta_{T_{lm}, T}}. \quad (\text{A3})$$

552  $\Omega, \Psi, \Theta$  are computed as functions of time and then time-averaged.



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TABLE 1. Definition of 4 areas used in the analysis

	MJ	ND
Warm Extratropics	90°N-30°N	90°S-30°S
Tropics	30°S-30°N	
Surf Zone	30°S-60°S	30°N-60°N
Cold Pole	60°S-90°S	60°N-90°N

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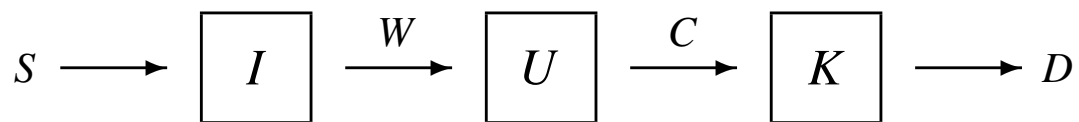
684 **Fig. 10.** Stream function  $\Psi$  (contours, drawn at  $\pm 0.2, 0.5, 1, 2, 5, 10, 20, 50, 10^8 \text{ Kg s}^{-1}$ ), solid  
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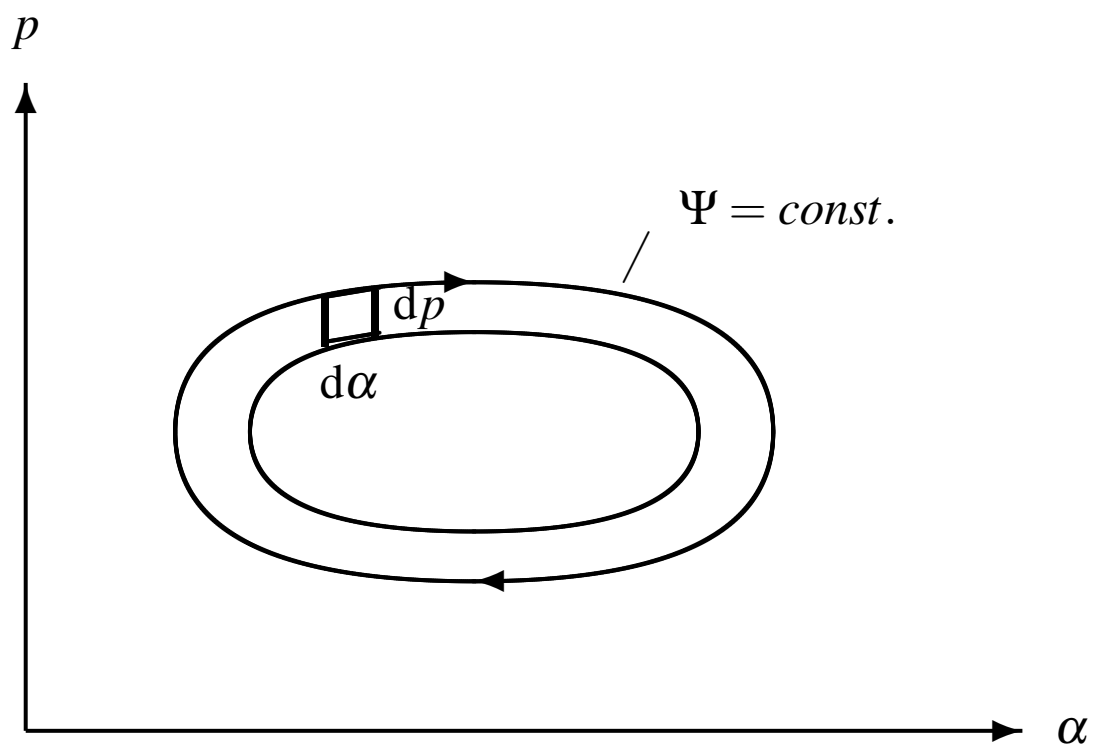
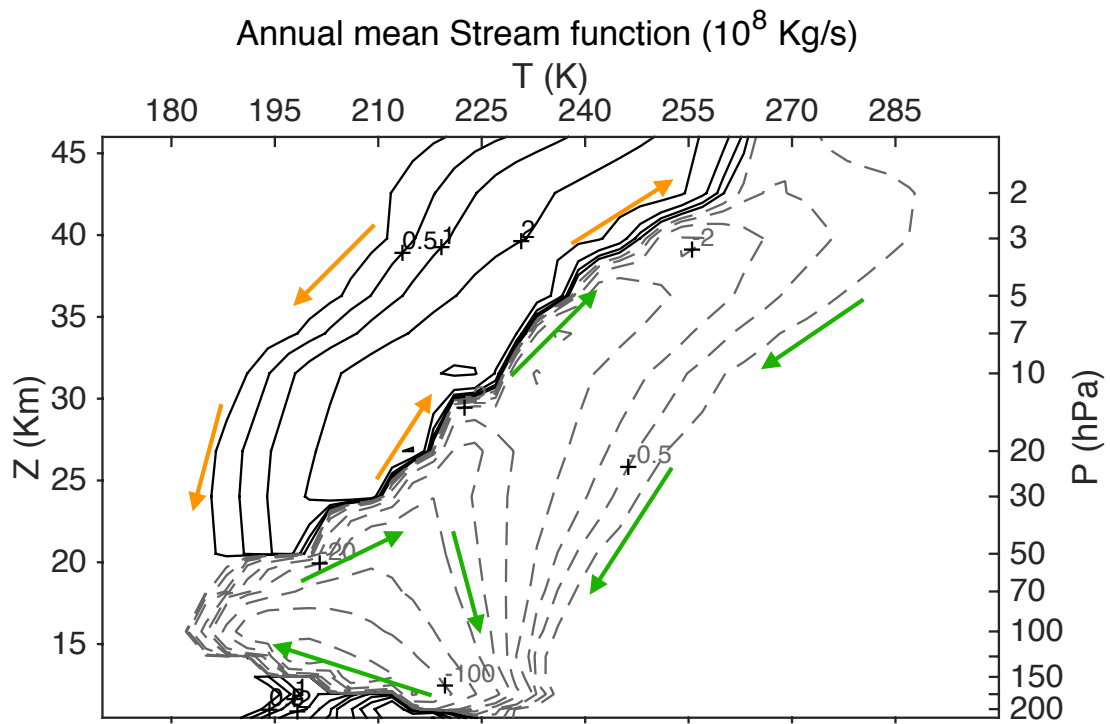
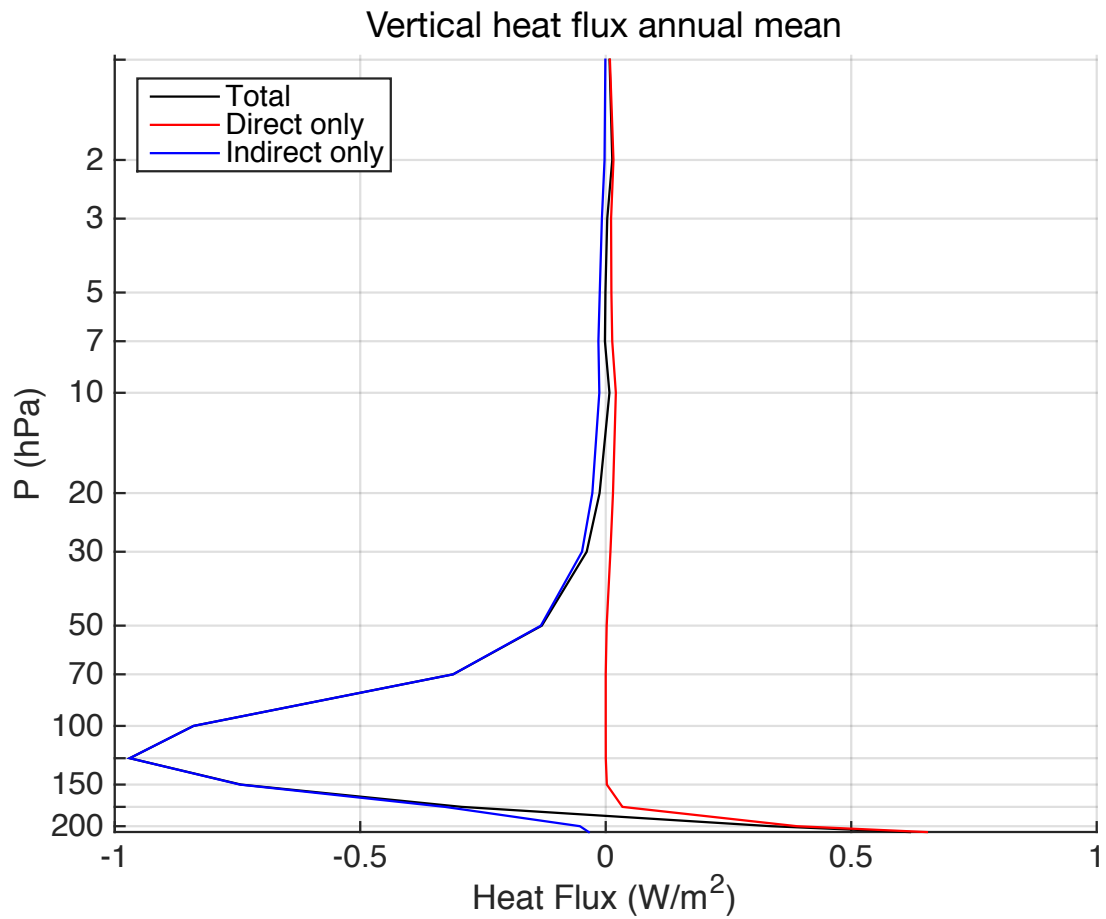


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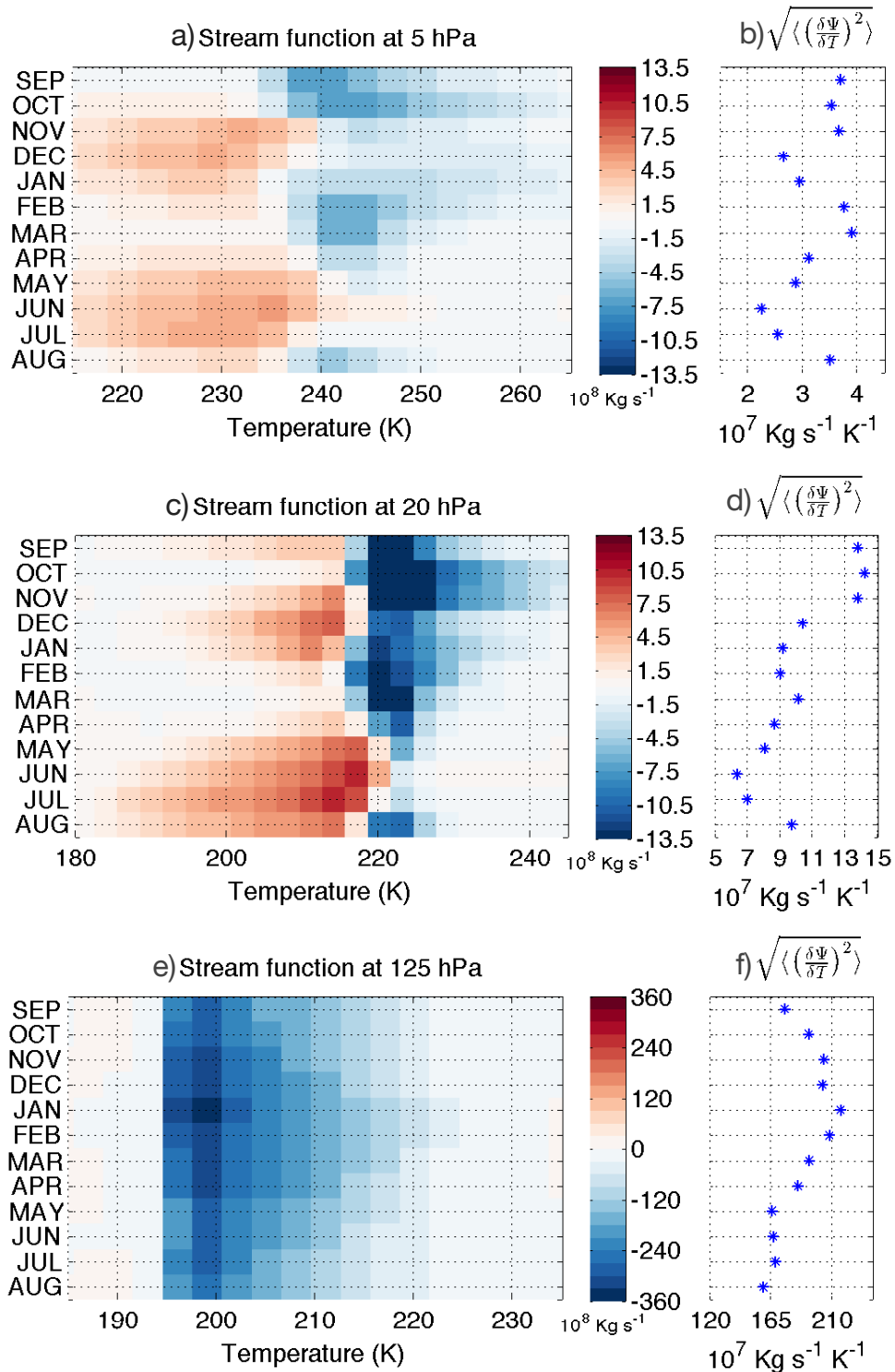




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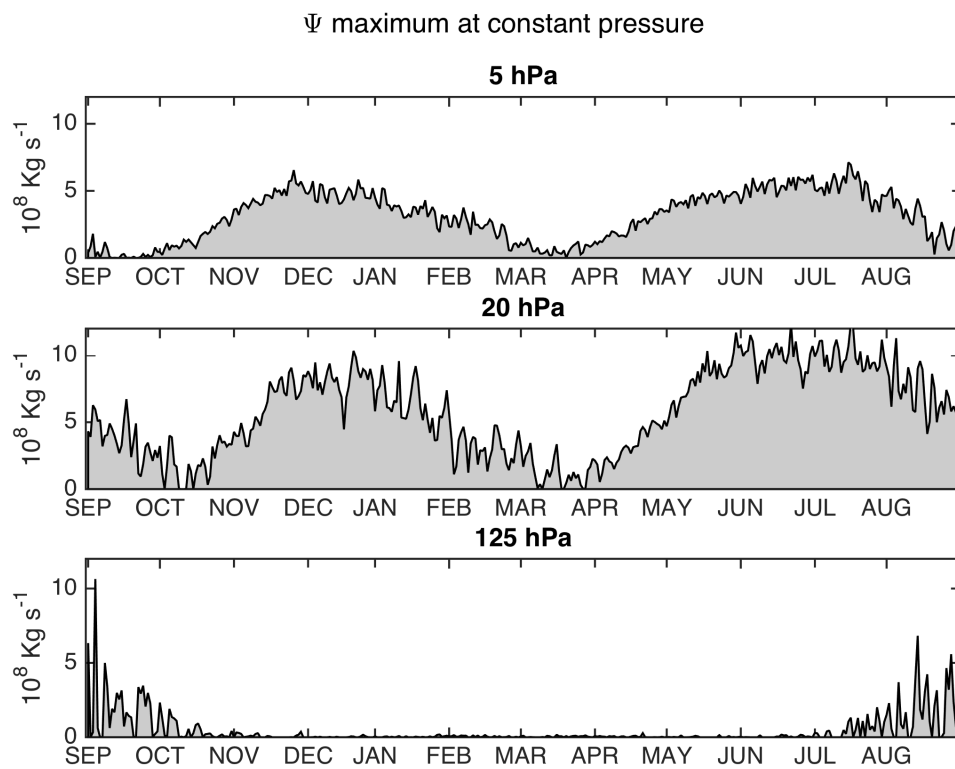


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$\Psi$  minimum at constant pressure

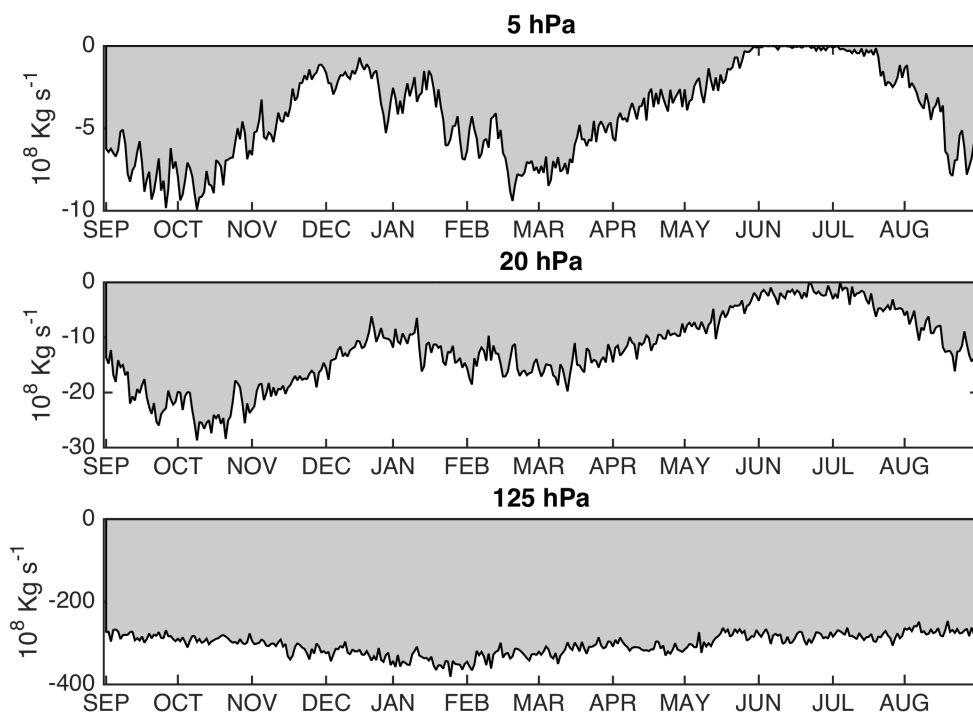
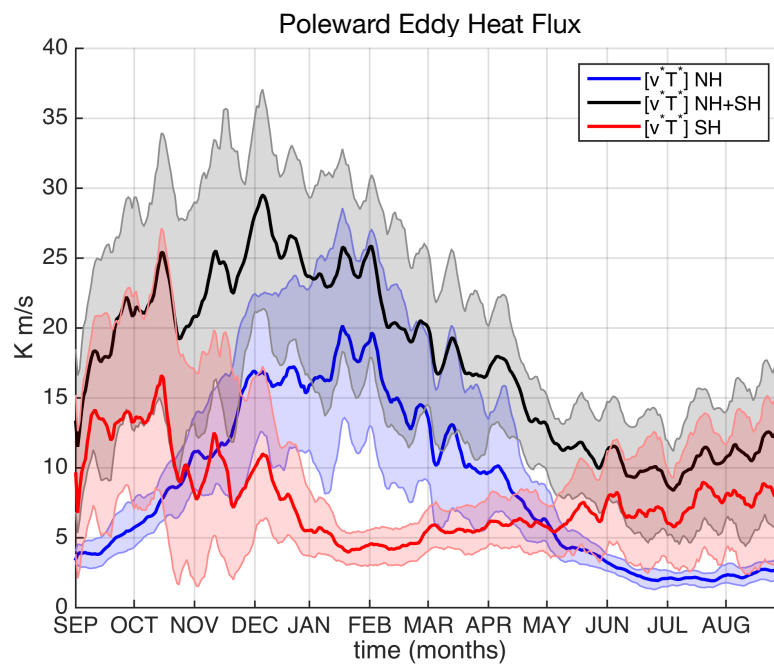
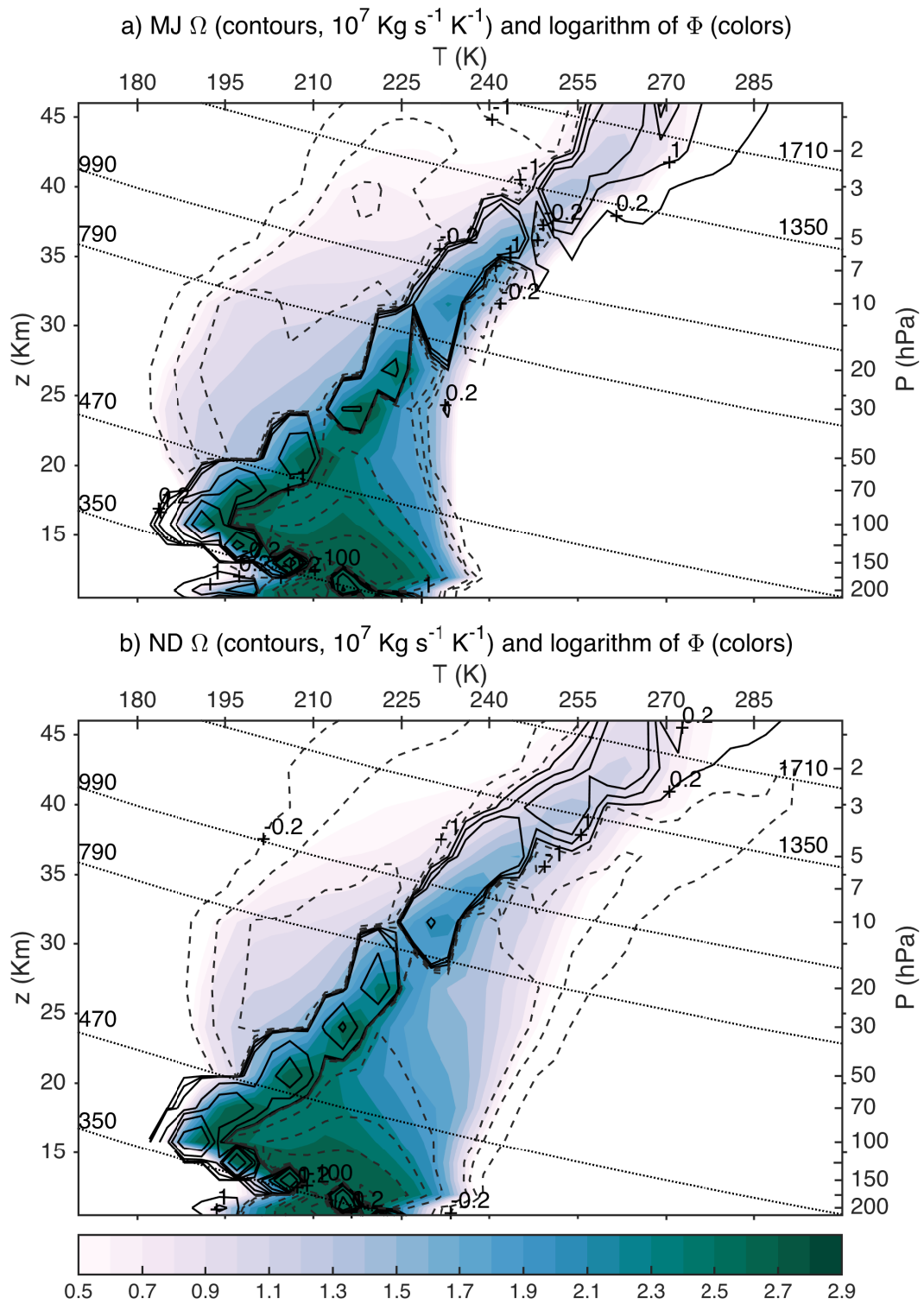


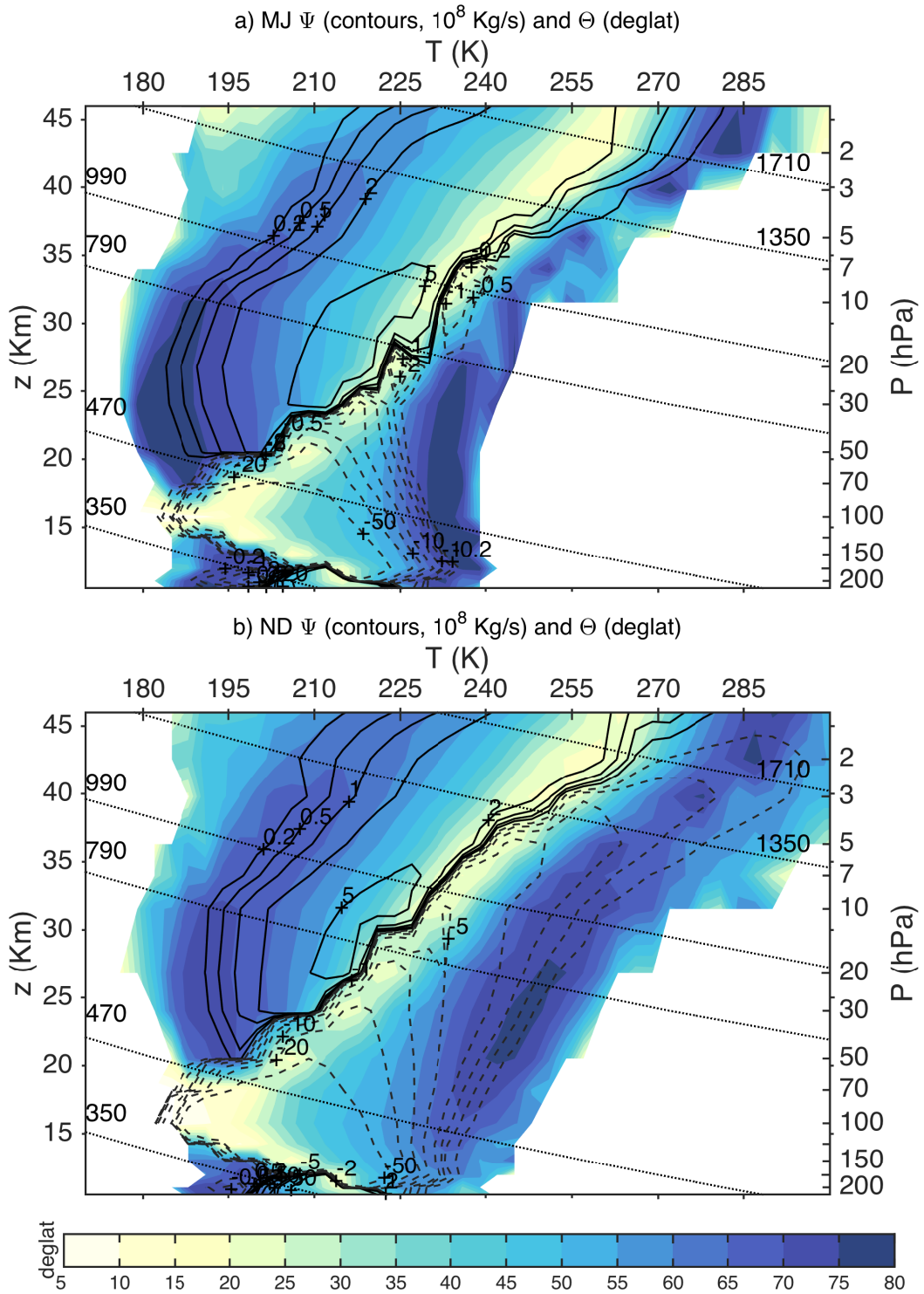
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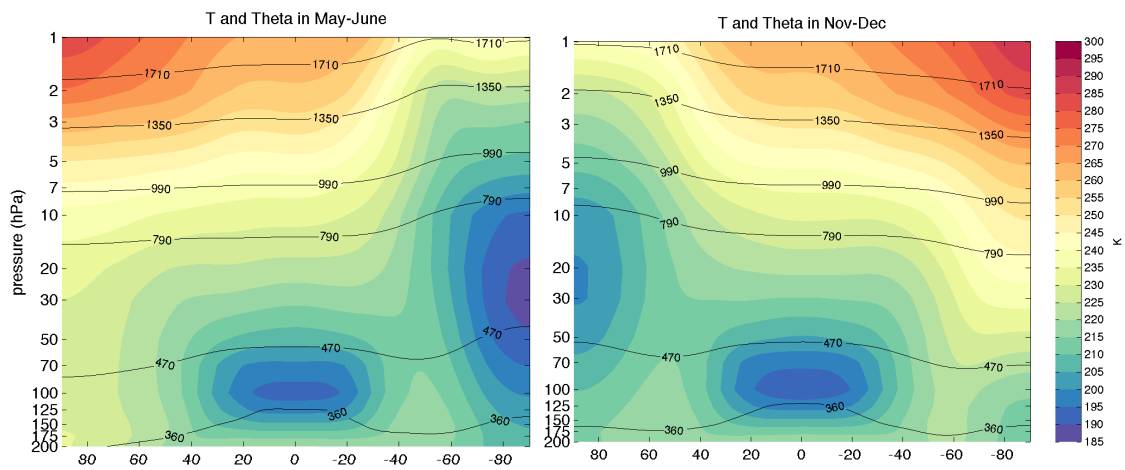


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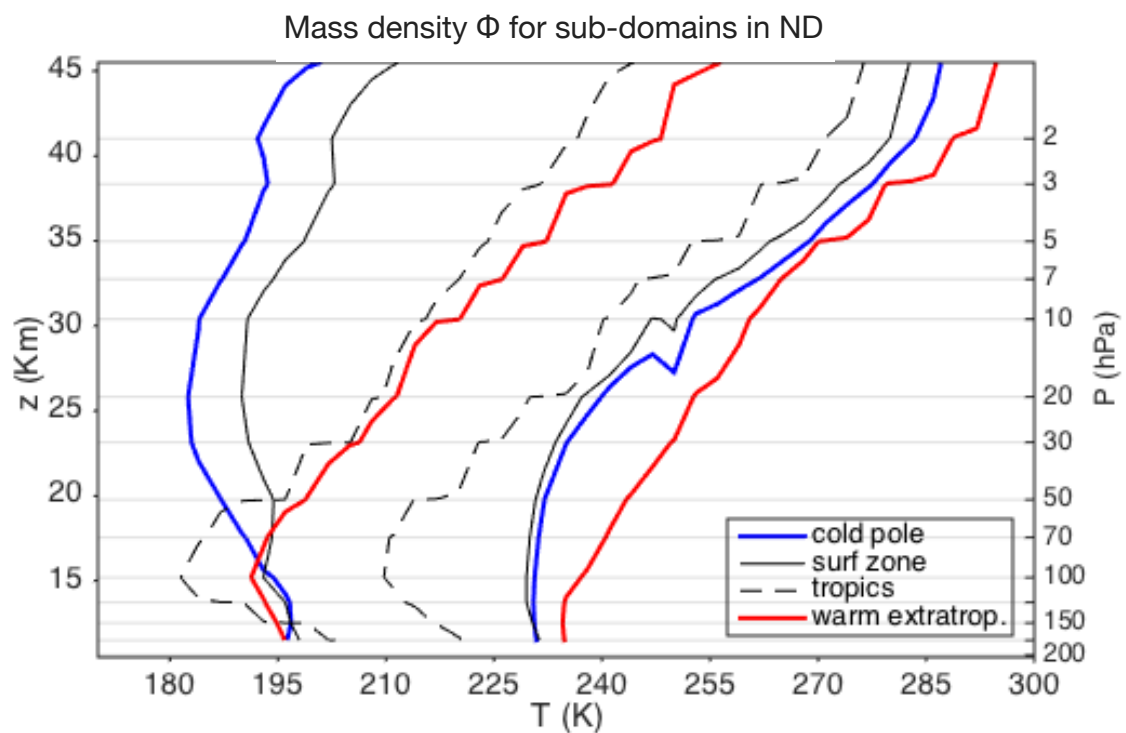
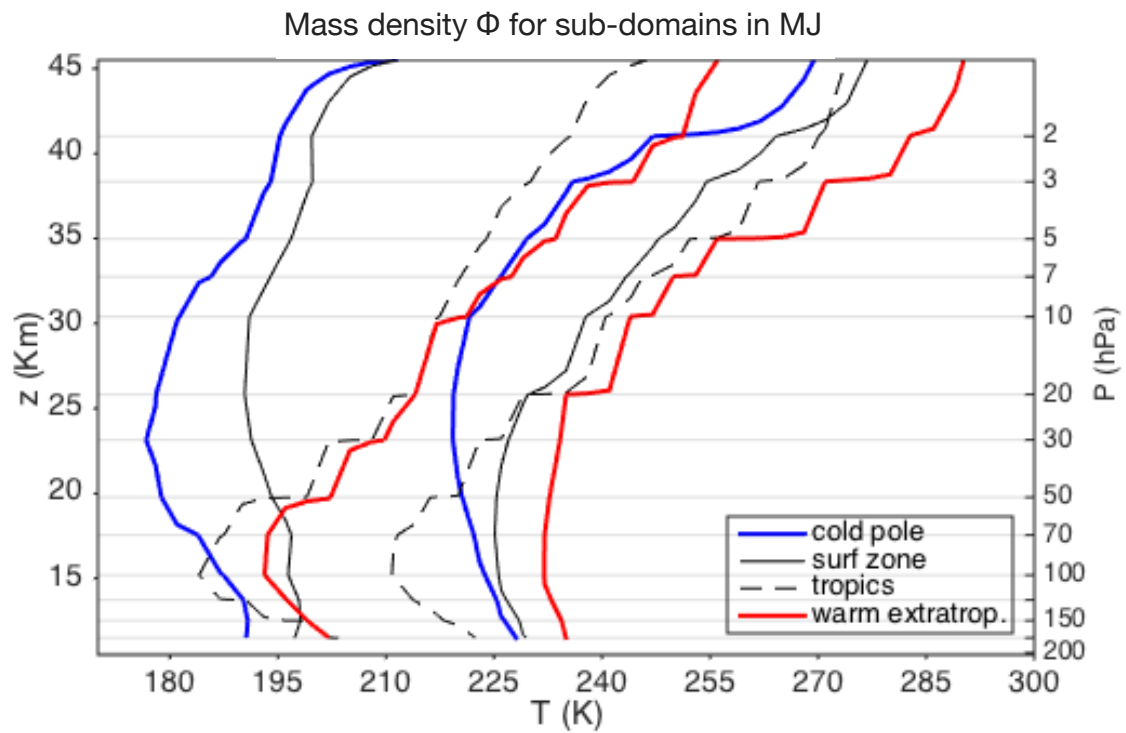


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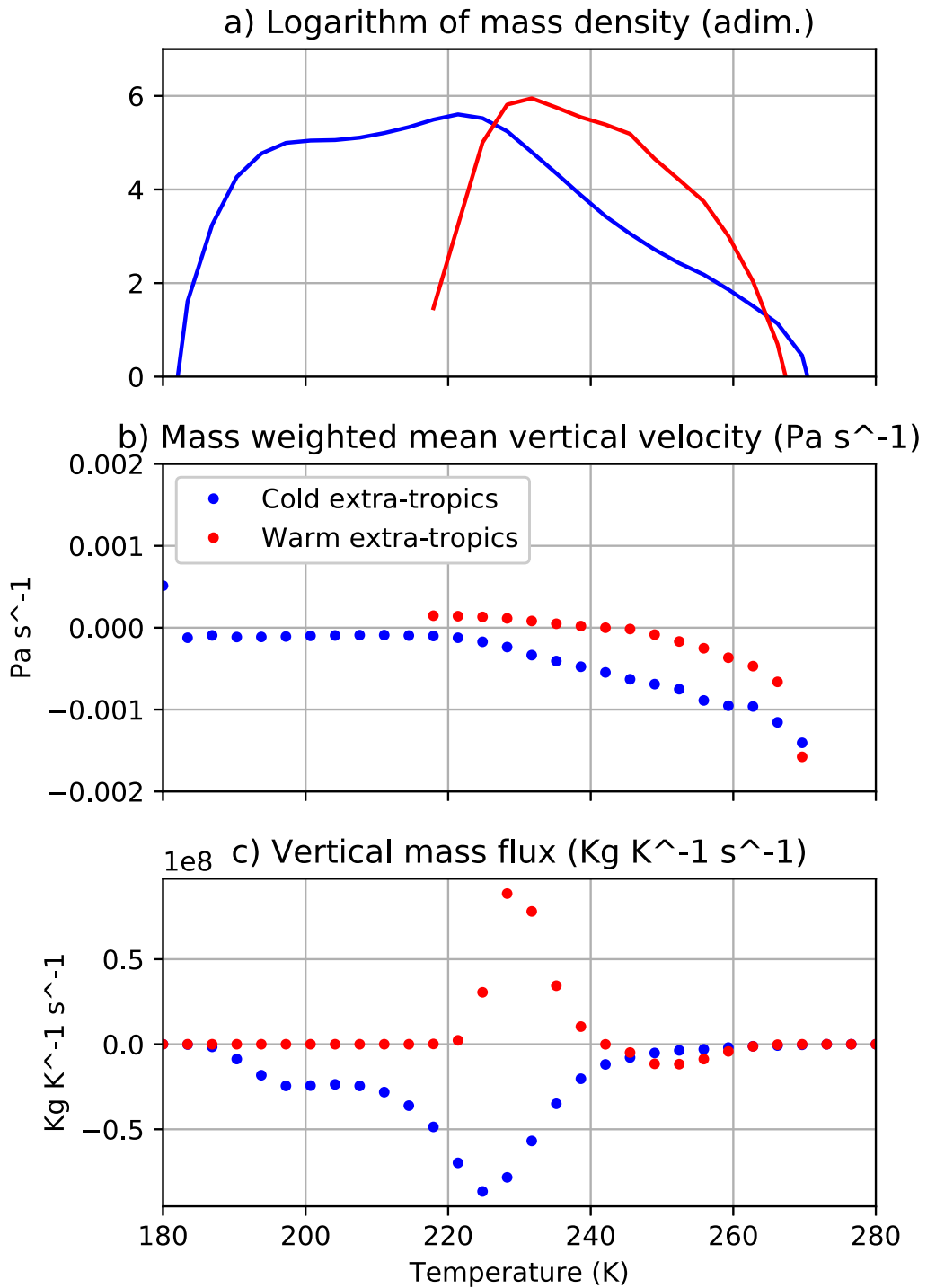




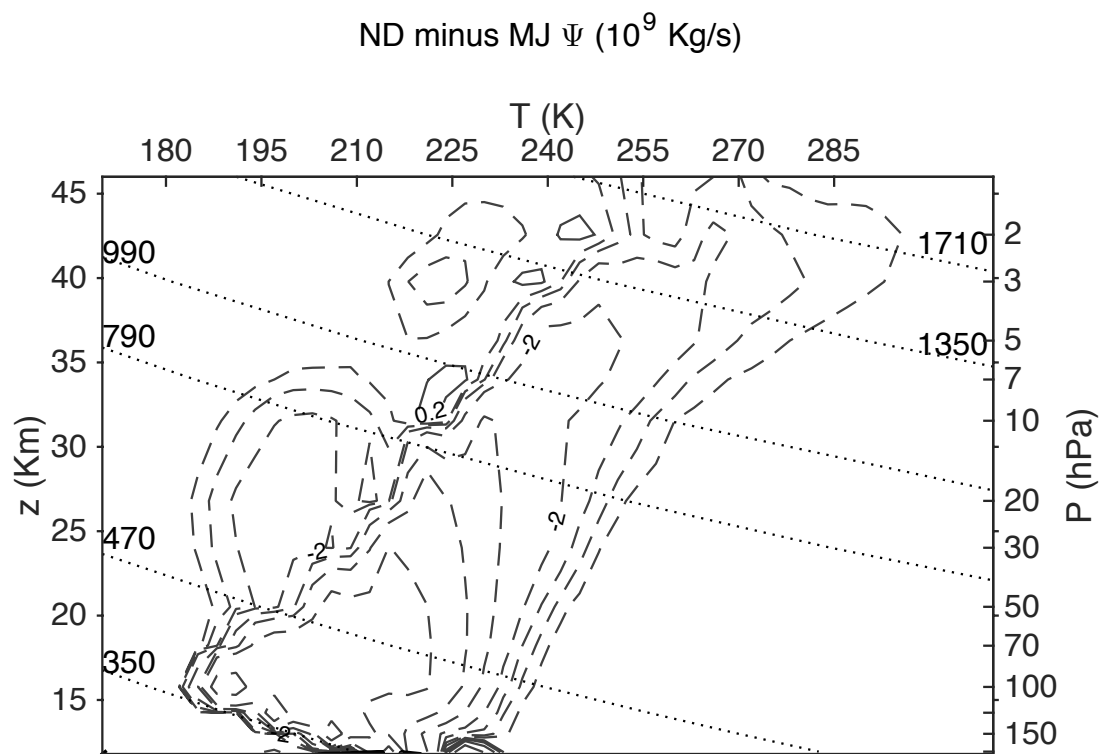
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