

Thomas Ströhlein's Endgame Tables: a 50th Anniversary

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Accepted Version

The article

Haworth, G. (2020) Thomas Ströhlein's Endgame Tables: a 50th Anniversary. *ICGA Journal*, 42 (2-3). pp. 165-170. ISSN 1389-6911 doi: <https://doi.org/10.3233/ICG-200151> Available at <http://centaur.reading.ac.uk/90000/>

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Published version at: <https://content.iospress.com/articles/icga-journal/icg200151>

To link to this article DOI: <http://dx.doi.org/10.3233/ICG-200151>

Publisher: The International Computer Games Association

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CEN: Thomas Ströhlein's Endgame Tables, a 50th Anniversary

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We should not let February 2020 recede too far into the distance without celebrating the 50th anniversary of Thomas Ströhlein's (1970) Ph.D. thesis, *Research on Combinatorial Games*, see Fig. 1. Previously, Bellman (1965) had indicated that Dynamic Programming could be applied to endgames. Ingo Althöfer (2019) relates that the topic was proposed by F. L. Bauer after the backwards analysis of games and puzzles had been mentioned to him in the Netherlands by two Dutch colleagues, Max Euwe and Wim van der Poel (van den Herik, 2020).

The thesis considered perfect-information, win-loss games using the concepts and results of graph theory and boolean matrices. The properties of winning and optimal strategies were then described. After defining Graph Kernels, Ströhlein brought chess into scope and described the first realisation of a retrograde algorithm to create endgame tables. In the last of nine chapters, results including correct maximal depth figures² were presented for the five pawnless endgames KRk, KQk, KRkb, KRkn and KQkr. While Bellman's Dynamic Programming (1965) noted that optimal endgame play could be defined, Ströhlein's computational graph-theory discovered, illustrated and analysed it.³



Fig. 1. Richard Bellman, Thomas Ströhlein (CPW, 2020b) and the title page of *Research on Combinatorial Games*.

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² TS' figures in today's depth-to-conversion notation 'DTC': KRk (maxDTC = 16 winner's moves), KQk (10 moves), KRkb (18m), KRkn (27m) and KQkr (31m). Computers can quickly prove that Kk, KBk and KNk feature no wins.

³ Bellman (1965) notes that two positions may be regarded as equivalent, $P_1 \sim P_2$ in the sense that one can move from either to the other. However, the equivalence classes so defined do not quite correspond to all the positions of an endgame force. There is, for example, no KNNk position that precedes or succeeds 8/8/8/8/1NN5/2K5/k7 b.

The actual computations were carried out in the period 1967-9 (Schmidt and Ströhlein, 1989, 1993). These were the first years of a West German National Research Programme. They were also the last years of the AEG-Telefunken TR4 computer at the ‘LRZ’ Leibniz Rechenzentrum, see Fig. 2 (Bauer, 2007; Bitsavers, 2007; CPW, 2020a/b; Sapper, 2020). Lest we forget, this computer’s 0.25MB of core memory, 50 MB of disc, and speeds of 4.5/30 μ s for fixed-point add/multiply represented leading edge performance in Europe when it was installed for \$2.5m in 1964.⁴



Fig. 2. 1964: The LRZ, Richard-Wagner-Strasse 18, and the AEG TR4 (Bauer, 2007)

Table 1. The thesis’ table on p62 – plus column six.

#	Endgame	md = maxDTC	~ comp. time t	t/md secs.	p > no. pos.	t/(md*p) msec.
1	KRk	16	9m	33.8	65,536	516
2	KQk	10	6.5m	39	65,536	595
3	KRkb	18	6h 30m	1,296	4,194,304	309
4	KRkn	27	14h 16m	1,902	4,194,304	454
5	KQkr	31	29h 9m	3,384	4,194,304	808

Ströhlein’s computer model of chess simplified the code for practical reasons. The king was not mated but captured after being surrounded like Richard III. This capture could also notionally be done by the opposing king but that would have been captured first! ‘Capture depths’ on some print-outs were therefore one more than *dtc* depths in today’s ‘DTC’ Depth to Conversion metric. With no pawns and with castling considered unavailable as now, the squares a1, a8, h8 and h1 were rightly considered equivalent. However, for simplicity of programming and of reading the output, the sides a1-a8 and a1-h1 were not, so Ströhlein’s raw count of maxDTC positions is only slightly less than double the number of distinct positions. White is the stronger side and the focus is on wins for White, mainly White to move. The table on p62 of the thesis gives the correct maxDTC figures as in Table 1 here. Clearly, the step up from 3-man to 4-man endgames was a major one and a considerable feat worth pondering.

⁴ Similarly, Cambridge University’s ~\$5m 1965 TITAN ATLAS by 1968 sported 0.75MB core-store, 40MB disc and fixed-point add/multiply times of 1.6/5.0 μ s. \$2.5m in 1964 \approx \$21m in 2020, ‘top ten’ petaflop money.

In addition to consulting the relevant KQk and KRk positions, the number of positions involved exceeded the number of bits in memory so data had to be managed to and from disc. Thomas credits his computer scientist wife, Ingeborg, with the finer points of the computing including the fast bit manipulation in machine code. Some remarkably long computer runs were involved: the TR4 was notably more reliable than its successor, the TR440 (Bauer, 2007, p102). KQkr later became the icon of non-trivial endgames thanks in part to Thompson (Kopec, 1990) and Jansen (1992).

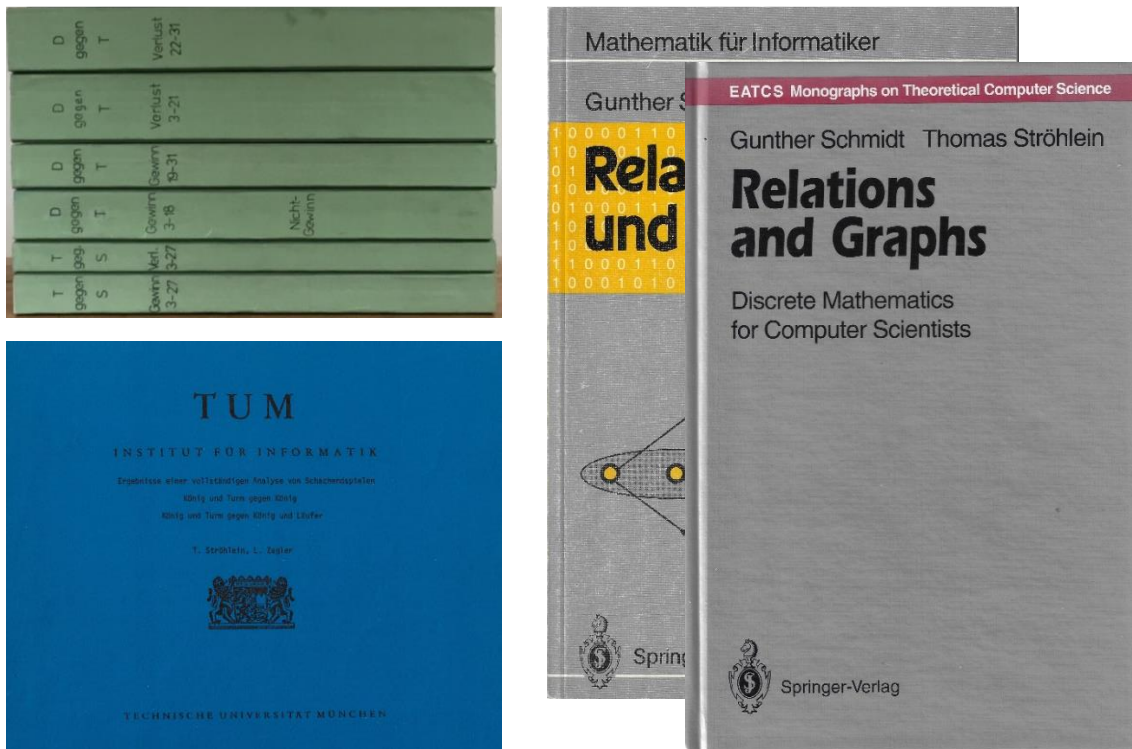


Fig. 3. The KRkn and KQkr results, ‘TUM-INFO’ (1978) and *Relations and Graphs* (1989, 1993).

Consideration of the computer results continued after 1970 in association with Gunther Schmidt, acknowledged in the thesis. The outputs for KRk and KRkb were photocopied and bound, see Fig. 3, and further analysed in Ströhlein and Zagler (1978) which included, see Figs. 4 and 5:

- 1) pp 003-088: all KRk positions with an optimal move; ‘!’ indicates uniqueness,
- 2) pp 089-100: a list of KRk positions with $d_{tc} \geq 4$ and a unique optimal winning line,
- 3) pp 101: a list of the maxDTC positions, i.e., with $d_{tc} = 16$ moves,⁵
- 4) pp 105-202: a lexicographic list of all winning KRkb positions with $d_{tc} \geq 4$.

A winning move is given: ‘*’ \equiv ‘only winning move’ and ‘!’ \equiv ‘uniquely optimal’.

Other work on relations, graphs and games (Schmidt, G. and Ströhlein, T., 1985; Ströhlein, T., 1976; Ströhlein, T. and Zagler, L., 1977) contributed to their definitive books on the subject (Schmidt and Ströhlein, 1989, 1993). These include the broader applications of games, one of which – program verification – is also relevant in the world of models and games.

⁵ 229 positions with $d_{tc} = 31$ ply: 121 distinct, being 108 pairs ‘mirrored’ in a1-h8 plus 13 exclusively on a1-h8.

Thomas Ströhlein's thesis and subsequent work has been an inspiration to later workers. We have since enjoyed Ken Thompson's sub-6-man 'EGT' endgame tables on CD (Tamplin, J.T. and Haworth, G.M., 2001) and the Nalimov (2000) 6-man EGTs online (Bleicher, 2010). We now benefit from sub-8-man results (de Man et al, 2018; Lomonosov, 2012) and look forward to 8-man EGTs. However, this pioneering work and thesis is where it all started and they deserve to be better known. Thomas himself celebrated his '50th' with his family and longtime friend and colleague Gunther Schmidt on Feb. 23rd, 2020, see Fig. 6.

Thomas Ströhlein (2020) has made a generous contribution of original and immaculately conserved material to the author's EGT archive: this will afford further study. I also thank him for reviewing this note. I welcome any offers of help for my halting and inadequate attempts to do justice to his thesis in translation.

The e-version of this note (Haworth, 2020) provides supporting files including an archive on the TR4, the thesis, some extracts from references cited in the thesis, various pgn and data files, and the 40th anniversary celebration of LRZ in 2007.

```
a) p03 2 T STROEHLIN A LRZ MUENCHEN 30.11.07 5
      wT1 wT2 wT3 wT4 wT5 wT6 wT7 wT8 wT9 wT10 wT11 wT12 wT13 wT14 wT15 wT16 wT17 -T18 WK1
      **** **** **** **** **** **** **** **** **** **** **** **** **** **** **** **** SKA1
      **** **** 1TA2 1TA2 1TA2 1TA2 1TA2 1TA2 1TA2 1KA2 1KA2 1KA2 1KA2 1KA2 1KA2 1KA2 1KA2 SKA2
      **** 1TA3 **** 1TA3 1TA3 1TA3 1TA3 1TA3 1TA3 9TB2 9TB1 1TA3 9TB1 9TB1 9TB1 9TB1 9TB1 SKA3
      **** 1TA4 1TA4 **** 1TA4 1TA4 1TA4 1TA4 1TA4 8KA2 8KA2 8KA2 1TA4 10TB1 9TB2 9KA2 9KA2 SKA4
      **** 1TA5 1TA5 1TA5 **** 1TA5 1TA5 1TA5 1TA5 9TB2 9TB1 8KB2 9TB1 1TA5 9TB1 9TB1 9TB1 SKA5
      **** 1TA6 1TA6 1TA6 1TA6 **** 1TA6 1TA6 1TA6 8KA2 8KA2 8KA2 9TB3 1TA6 9TB3 9TB3 SKA6
      **** 1TA7 1TA7 1TA7 1TA7 **** 1TA7 9TB2 9TB1 8KB2 8KA2 9TB1 9TB1 1TA7 9TB1 SKA7
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b) p89 Stellungen mit eindeutiger Hauptvariante
      ZUEGEZAHL 4
      A4A5A2;TB5 A4A6A2;TB6 A4A7A2;TB7 A4A8A2;TB8 A4R1A7;KB5 A4R2A7;KB5 A4R3A7;KB5 A4B4A7;KB5 A4B5A7;KB5 A4B6A7;KB5 A4B7A7;KB5 A4C1B7;KB5 A4C2B7;KB5 A4C3B7;KB5 A4C4B2;KB4 A4C4B7;KB5 A4C5B2;KB4 A4C5B7;KB5 A4C6B2;KB4 A4C7B2;KB4 A4C8B2;KB4
      A4E4A2;TB4 A4E5A2;TB5 A4E6A2;TB6 A4E7A2;TB7 A4E7B;KB5 A4E8A2;TB8 A4F4A2;TB4 A4F5A2;TB5 A4F6A2;TB6 A4F7A2;TB7 A4G7A8;KB5 A4F8A2;TB8 A4G4A2;TB4 A4G5A2;TB5 A4G6A2;TB6 A4G7A2;TB7 A4G7A8;KB5 A4G8A2;TB8 A4H4A2;TB4 A4H5A2;TB5
```

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c) p101 36 T STROEHLIN A LRZ MUENCHEN 12.12.07 4
      MAXIMALZUGER 17
      A1B2C3 A1B2D4 A1B2D5 A1B2E4 A1B2E5 A1B3C4 A1B3D4 A1B3D5 A1B3E4 A1B3E5 A1B4C5 A1B5C4 A1B6C5 A1B6D4 A1B6D5
      A1B6E4 A1B6E5 A1B7C6 A1B7D5 A1B7E4 A1B7E5 A1B8D5 A1B8E4 A1B8E5 A1C2D3 A1C2D4 A1C2D5 A1C2E4 A1C2E5 A1C3D4
      A1C3D5 A1C3D6 A1C3F4 A1C3E5 A1C3E6 A1C3F5 A1C3F6 A1C4D5 A1C6D3 A1C6D4 A1C6D5 A1C6E4 A1C6E5 A1C6F4
      A1C6F5 A1C7D3 A1C7D4 A1C7D5 A1C7D6 A1C7E4 A1C7E5 A1C7F4 A1C7F5 A1C7F6 A1C8D3 A1C8D4 A1C8D5 A1C8E4 A1C8E5 A1C8F4
      A1C8E5 A1C8E6 A1D2E3 A1D3E4 A1D4E5 A1D6E4 A1D6E5 A1D6F4 A1D6F5 A1D7E6 A1E2D3 A1F2D4 A1F2D5 A1F2E3 A1F2E4
      A1F2E5 A1F3C4 A1F3D4 A1F3D5 A1F3D6 A1F3E4 A1F3F5 A1F3E6 A1F4D5 A1F4D6 A1F4E5 A1F4E6 A1F6C5 A1F6D4 A1F6D5
      A1F6E3 A1F6E4 A1F6E5 A1F7C6 A1F7D4 A1F7D5 A1F7D6 A1F7E3 A1F7E4 A1F7E5 A1F7E6 A1F8D5 A1F8E3 A1F8E4 A1F8E5
      A1F8E6 A1F8E7 A1G2D5 A1G2E4 A1G2E5 A1G2F3 A1G3C4 A1G3D4 A1G3D5 A1G3D6 A1G3E4 A1G3E5 A1G3E6 A1G3F4 A1G3F5
      A1G3F6 A1G4F5 A1G6C5 A1G6D4 A1G6D5 A1G6E4 A1G6E5 A1G6F4 A1G6F5 A1G6F6 A1G7D4 A1G7D5 A1G7D6 A1G7E4 A1G7E5
      A1G7E6 A1G7F4 A1G7F5 A1G7F6 A1G8D5 A1G8E4 A1G8F5 A1G8F6 A1G8F7 A1G8F8 A1G8F9 A1G8F10 A1G8F11 A1G8F12
      A1H3E4 A1H3E5 A1H3F5 A1H3G4 A1H4C5 A1H4D5 A1H4E5 A1H4E6 A1H4E7 A1H4E8 A1H4E9 A1H4E10 A1H4E11 A1H4E12
      A1H7D6 A1H7D5 A1H7E4 A1H7E5 A2B3C4 A2B3D4 A2B3E5 A2B4C5 A2B4D4 A2B4D5 A2B4E4 A2B4E5 A2B4E6 A2B4E7
      A2F2E3 A2F3C4 A2F3D4 A2F3E5 A2F3E6 A2F3E7 A2F4E5 A2F4E6 A2F4E7 A2F4E8 A2F4E9 A2F4E10 A2F4E11 A2F4E12
      A2G3F4 A2G4F5 A2G6F5 A2G7F6 B1B6C5 B1C2D3 B1C2D4 B1C2E4 B1C6D3 B1C6D4 B1E6D5 B1E6E5 B1E6E6 B1E6E7
      B1F6E5 B1D6F5 B1E7E6 B1F2E4 B1F2E3 B1F2E4 B1F3D4 B1F3D5 B1F3E4 B1F3E5 B1F6D4 B1F6D5 B1F6E3 B1F6E4 B1F6E5
      B1F7E6 B1D3F4 B1G6F5 B1G7E6
```

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d) p107 wK wT aK aL aL aL aL aL aL aL aL aL aL aL
      a3 d6 a1 b2: 4Kb31 a7: 10Tc61 b2: 5Kb3* b8: 5Tb61 c1: 4Kb3* c3: 4Kb3* c7: 9Tc61 #3: 10Tc61 #5: 10Tc61 #2: 10Tc61
      b1 a5: 6Td51 f4: 10Tc61 g3: 10Tc61 g5: 10Tc6* g7: 10Tc6* h2: 10Tc61 h4: 10Tc6* h8: 10Tc6
      d7 a1 b2: 4Kb31 a5: 5Ka41 b2: 5Kb3* b6: 6Tb7* b8: 6Tb11 c1: 4Kb3* c3: 4Kb3* c5: 4Kb3* e3: 10Tc71 #2: 10Tc7* #4: 11Td4*
      b1 a5: 5Ka41 b2: 5Kb3* b6: 6Tb7* b8: 6Tb11 c1: 4Kb3* c3: 4Kb3* c5: 4Kb3* e3: 10Tc71 #2: 10Tc7* #4: 11Td4*
      f6: 10Tc7* g5: 10Tc71
```

Fig. 4. Extracts from pages of Ströhlein and Zagler (1978) including the exemplar positions of Fig. 5:
 (a) p03, the first results, wtm KRk positions, wK on a1, the bK (later captured) on a1...a7, the R on a1...b8;
 (b) p89, wtm 'positions with a clear main variant': first move and others uniquely optimal, correct depths;
 (c) p101, the correct list of KRk max-depth positions, the last move capturing the Black king as suggested by the '17';
 (d) p107, KRkb wtm wins with $d_{tc} \geq 4$

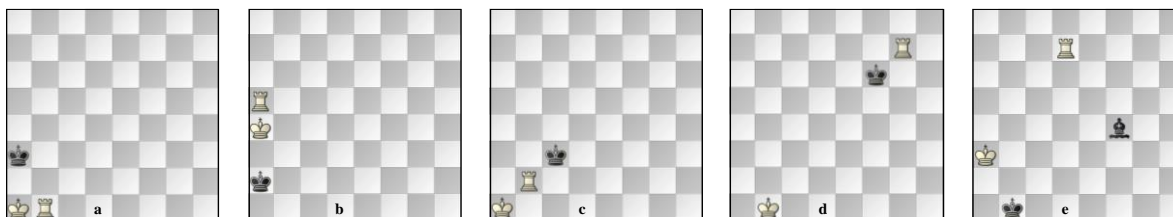


Fig. 5. White to move positions taken from the extracts of Fig. 4, annotated wKwR/bK(bB), T ≡ Turm ≡ Rook:

- (a) p03 row 3 col. 9, a1b1/a3, ‘9TB2!’ ≡ **1.Rb2** is uniquely optimal and *dtc* = 8 white moves;
- (b) p89 r1 c1, a4a5/a2, ‘TB5’ ≡ uniquely optimal **1.Rb5!**, Ka1° 2.Kb3! Kb1° 3.Rc5! Ka1° 4.Rc1#!, *dtc* indeed is 4m;
- (c) p89 last position, d4h2/c1, ‘KD3’ ≡ uniquely optimal **1.Kd3!**, Kb1! 2.Kc3! Ka1! 3.Kb3! Kb1° 4.Rh1#!, *dtc* = 4m;
- (d) p101 last position, b1g7/f7, ‘17’ (counting the capture of the king), i.e., *dtc* = maxDTC = 16m;
- (e) p107 r5 last position, a3d7/b1f4, ‘11Td4*’ ≡ ‘*dtc* = 11m, **1.Rd4!!** is the only winning move’.

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Fig. 6. From left to right: Thomas Ströhlein, his wife Ingeborg and longtime friend and colleague Gunther Schmidt at the family '50th' celebration on 23rd February, 2020 of his 1970 doctorate.