Dynamic efficiency and arbitrage potential in Bitcoin: a long-memory approach

Dynamic Efficiency and Arbitrage Potential in Bitcoin: A Long-memory Approach

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Abstract

Employing a long-memory approach, we provide a study of the evolution of informational efficiency in five major Bitcoin markets and its influence on cross-market arbitrage. While all the markets are close to full informational efficiency over the whole sample period, the degree of market efficiency varies across markets and over time. The cross-market discrepancy in market efficiency gradually vanishes, suggesting the segmented markets are developing to a consensus where all markets are equally efficient. Through a fractionally cointegrated vector autoregressive (FCVAR) model we show that when the efficiency in Bitcoin/USD and Bitcoin/AUD markets improves the cross-market arbitrage potential narrows, whereas it widens when the efficiency in Bitcoin/CAD, Bitcoin/EUR, and Bitcoin/GBP markets improves. A battery of robustness checks reassure our main findings.

Keywords: Bitcoin, Market Efficiency, Cryptocurrency, Long Memory, FCVAR
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1. Introduction

The Efficient Market Hypothesis (EMH) has been extensively discussed in the finance literature for decades since the seminal work by Fama (1970). While traditional tests of market efficiency mainly treat the issue as a static question, arriving at either an acceptance or rejection of the EMH, recent literature has quantified the extent to which the market is efficient and shown that it can vary over time (See, e.g., Fernandez, 2010, Tabak and Cajueiro, 2007, Wang and Liu, 2010, Wang et al., 2009). Rösch et al. (2017) state that such time-varying property of market efficiency can be governed by the financial frictions that vary over time. However, despite the financial frictions in the Bitcoin market that can vary over time, the attention paid to the time-varying property of its market efficiency is still surprisingly scant. The first part of our paper, therefore, addresses this issue by studying the time-varying property of the Bitcoin market.

In parallel, due to the segmented nature of its trading venues, Bitcoin often exhibits remarkable cross-market arbitrage opportunities. For example, Makarov and Schoar (2020) find strong evidence that the Bitcoin market possesses a high degree of segmentation and cross-market arbitrage potential. They ascribe this arbitrage potential to capital controls along with insufficient regulatory supervision. Theoretically, Perlin et al. (2014) provide a microstructure model in which such type of cross-market arbitrage to occur when the speed of convergence towards the contemporaneous equilibrium price varies across markets. While there has been a growing body of literature on Bitcoin market efficiency and arbitrage,[1] few have looked into the relationship between the efficiency of segmented markets and the cross-market arbitrage in Bitcoin. The second part of our paper fills this gap by investigating the association between the dynamic market efficiency of segmented markets and the cross-market arbitrage potential.

We categorise global Bitcoin trading into five segmented markets by the base currency against which Bitcoin is traded, namely, Australia dollar (AUD), Canadian dollar (CAD), Euro (EUR), British pound (GBP), and US dollar (USD).[2] Using daily data from 1st

[1]See, e.g. Bariviera (2017), Hattori and Ishida (2020), Kroeger and Sarkar (2017), Nadarajah and Chu (2017), Urquhart (2016), Wei (2018), Zargar and Kumar (2019).

[2]Our categorisation ensures a sample that is liquid and spans a long enough period that allows for a meaningful dynamic efficiency analysis by rolling-window estimations. This categorisation is also justified by the fact that most Bitcoin traders use only one fiat currency, usually their home currency,

January 2013 to 7th January 2020 for the five Bitcoin markets, our study is conducted in three steps.

First, we quantify the degree of market efficiency measuring the long-memory in the price series. Traditionally, testing Bitcoin market efficiency is treated as a polar question and conclusions are made mainly based on (1) whether the price process is a random walk (Aggarwal, 2019, Nadarajah and Chu, 2017, Urquhart, 2016) and (2) whether the returns are predictable (Shynkevich, 2020, Urquhart and McGroarty, 2016). Translated into econometric terms, all above treatments attempt to answer the question that if the price series has an integration order (d) of one (i.e. I(1) series). In reality, however, the integration order (d) of the price series does not have to be restricted to integer values such as 1 or 0. Instead, d might be a fractional value, implying a long memory process in the prices, i.e. the reflection of the available information can be slowly-decayed over a long time period. Given that the long-dependence is known to be characterised in the Bitcoin market (Cheah et al., 2018, Takaishi and Adachi, 2020, Tiwari et al., 2018), a serious bias and informational loss regarding the interpretation of the market efficiency degree could occur unless we relax the premise of d of the price series to a fraction and capture its long-memory feature.

Therefore, our paper allows for potential existence of fractional values of d to capture the long-memory in the price series through which the degree of efficiency in the market can be well gauged.[3] Moreover, since our approach does not rely on any pricing model, it avoids the famous joint hypothesis problem (Fama, 1991).[4] Employing the Feasible Exact Local Whittle (FELW) estimator of Shimotsu (2010) for the fractional integration order (d) calculation, we show that all the five Bitcoin markets possess a high degree of informational efficiency over the full sample period.[5] Across a variety of estimation bandwidths employed, our estimates of d values of Bitcoin prices remain generally close

in the trading (Makarov and Schoar, 2020).

[3]We measure the closeness to an efficient market through the absolute difference between d and 1. This measure is denoted as D, whose value decreases as the degree of market efficiency increases and is 0 if the market is fully efficient. Detailed discussions regarding D are in Section 4.

[4]The joint hypothesis problem states that any attempts of testing whether information is properly translated into an equilibrium price defined by a pricing model jointly test both EHM and that pricing model (Fama, 1991).

[5]See a detailed discussion of the employed method for the calculation of d in Section 3.

to unity, ranging from 0.841 for AUD (bandwidth = 0.4) to 1.099 for CAD (bandwidth = 0.6). The main findings further remain robust when replacing the currently-employed d estimator with alternatives.

Second, in the spirit of Bariviera (2017) and Rösch et al. (2017), our paper acknowledges that the efficiency degree of the Bitcoin market can fluctuate over time and investigates the dynamic evolution of market efficiency. To this end, we recursively estimate d on each market through a rolling window approach. It can be conducted by first recursively estimating the fractional integration order (d) over a specified window size, and then an indicator of market efficiency degree (denoted as D in the paper) that is defined as the absolute difference between d and one. In doing so, we enable the degree of Bitcoin market efficiency to evolve over time in a manner consistent with the Adaptive Market Hypothesis (Lo, 2004), while a cross-market comparison of the efficiency is also possible.

A graphical analysis demonstrates that the degree of efficiency of each Bitcoin market varies over time but generally remains highly close to efficient throughout the sample period. However, in the period of 2016 to 2017 (the Bitcoin price boom), most markets experienced a reduction in the degree of efficiency with the d value significantly less than one but different from zero. More importantly, we observe that the discrepancy in the degree of efficiency between markets shrinks gradually over time except for the period of 2016 to 2017 that coincides with the Bitcoin boom. In the light of Makarov and Schoar (2020), a possible explanation for the observed narrowing gap of the efficiency degree across markets is that as the Bitcoin market matures, the cross-market information exchanges are enhanced overtime and less arbitrage profits are available.

Third, we further construct the arbitrage index following Makarov and Schoar (2020) to capture the cross-market arbitrage potential and then model its nexuses with the efficiency degree (D) of the five individual markets through a fractionally cointegrated vector autoregressive (FCVAR) model recently proposed by Johansen and Nielsen (2012). Rather than the conventional I(1)/I(0) framework, the FCVAR model relaxes the strict assumption of integer integration and cointegration orders, and allows that both the orders to be fractional, through which the potential existence of the long-memory feature in the variable system can be well identified. Therefore, the FCVAR model offers a novel and robust way to uncover how the arbitrage level can be truly impacted by the degree of informational efficiency in individual Bitcoin markets in both the short- and long-run terms.[6]

Our empirical results illustrate that the cross-market arbitrage potential is distinctively higher in the first half of our sample period compared to the second half, echoing our previous findings that the discrepancy in efficiency across markets narrows along with time. Moreover, in the light of the long-run relationship identified by the FCVAR estimation, we find that the degree of Bitcoin market efficiency in the US and Australia exerts positive impacts on the cross-market arbitrage potential, indicating that an increase in efficiency degrees in these markets leads to an increase in the arbitrage opportunities. On the contrary, the efficiency degree of Bitcoin market in Canada, Europe and UK demonstrates negative effects on the arbitrage opportunities, i.e. the less efficient in these markets, the more cross-market arbitrage opportunities can be expected.

We arrange the rest of this paper as follows. Section 2 reviews the extant key literature with a discussion of our contribution. Section 3 discusses the two main econometric models used in this study, i.e. the fractional integrated model and the fractional cointegrated vector autoregressive model. Section 4 describes the data employed. Section 5 documents our empirical findings. Section 6 presents robustness checks. Finally, Section 7 concludes the paper.

2. Literature Review

With promising application potentials since its proposal by Nakamoto (2008), the literature on Bitcoin which has exploded with papers documenting the hedging and diversification benefits (Borri, 2019, Corbet et al., 2018a, Urquhart and Zhang, 2018), the existence of bubbles (Cheah and Fry, 2015, Corbet et al., 2018b), investor attention (Shen et al., 2019, Urquhart, 2018), the trading potential (Bouri et al., 2019b, Corbet et al., 2019a, Hudson and Urquhart, 2019, Kajtazi and Moro, 2019, Platanakis and Urquhart, 2020), the volatility dynamics (Bouri et al., 2019a, Katsiampa, 2017, 2018, Katsiampa et al., 2019, Shen et al., 2020), and the multifractality in Bitcoin markets (Kristjanpoller and Bouri, 2019, Kristjanpoller et al., 2020, Mensi et al., 2019a, Takaishi and Adachi, 2020).[7]

[6] Detailed discussions of FCVAR model can be seen in the methodology section.

[7] See Corbet et al. (2019b) for a recent review of the empirical literature on cryptocurrencies.

Urquhart (2016) is amongst the first studies on Bitcoin market efficiency. He employs five different independence tests and rejects the EMH over the first subsample period but find evidence of a possible move towards efficiency in the more recent subsample period. This evolutionary view of the Bitcoin market efficiency is also share by Sensoy (2019) who find that the Bitcoin market exhibits some degree of weak form efficiency at intraday level since 2016. Nadarajah and Chu (2017) employ an odd integer power transformation of Bitcoin returns and show that returns are weakly efficient. On the contrary, Zargar and Kumar (2019) examine the intraday efficiency of Bitcoin and find evidence of the Bitcoin inefficiency at higher frequency levels. Simiarly, Aslan and Sensoy (2020) provide evidence of Bitcoin return predictability at the intraday level. Akyildirim et al. (2020) find cryptocurrency return predictability using at daily and minutely levels using common machine learning techniques, including the support vector machines, logistic regressions, neural network, and random forests. Moreover, Bouri et al. (2019b) study the predictability of trading volume to cryptocurrency returns via through a copula-quantile causality approach. Thus, there is an inconclusive debate on the degree of informational efficiency of Bitcoin markets, and a consensus has yet been reached so far.

At the same time, the existence of long-memory in financial asset price series has been widely embraced such as in the stock market, commodity market, and real estate market (See, e.g., Canarella et al., 2019, Kristoufek and Vosvrda, 2014, Mensi et al., 2019b). Instead of taking the issue of market efficiency as an 'all-or-nothing' question and assuming that the market can only be either completely efficient or inefficient, this strand of literature considers the possibility that a market could possess a certain degree of efficiency (i.e. quasi-efficient), which can be captured by measuring the long-memory in the price series. (See, e.g., Cuñado et al., 2005, Liow, 2009, Ngene et al., 2015). Recently, though scant, the existence of long-memory is also found in the Bitcoin price series, indicating the rejection of the EMH in the Bitcoin market while it still possesses certain efficiency degree (See, e.g., Cheah et al., 2018, Tiwari et al., 2018).

With regard to estimators of the long memory, although Hurst exponent computed based on R/S and DFA methods is popular for the measurement of long-memory in the extant literature (See, e.g. Bariviera, 2017, Takaishi and Adachi, 2020, in the Bitcoin market), it has been long documented that the effectiveness of both the methods tends to be prone to be affected by the nature of the data and initial parameter settings (Hauser and Reschenhofer, 1995, Kantelhardt et al., 2001, Lo, 1991). Instead, a recently-developed FELW estimator (Shimotsu, 2010) is known to improve the weaknesses encountered by the traditional methods and enhance the accuracy of the long-memory estimates against alternative parametric and semi-parametric ones (See, recent applications Berger et al., 2009, Dolatabadi et al., 2018, Kumar and Okimoto, 2007).

Furthermore, scholars have been interested in the cross-market arbitrage and market efficiency. For example, Gromb and Vayanos (2002) propose a model that explains the relationship between cross-market arbitrage and allocative market efficiency. On the other hand, Perlin et al. (2014) provide a model showing that the cross-market arbitrage behaviour can be affected by three factors, one of which is the discrepancy of informational efficiency across individual markets.

Our paper contributes to the extant literature in a fourfold manner. First, instead of taking the issue of market informational efficiency as a static problem as conventional in the literature, we view market efficiency in an evolving manner to shed light on Bitcoin market efficiency. Second, we employ the recent developed long-memory estimator, Feasible Exact Local Whittle estimator (Shimotsu, 2010), that mitigates the weaknesses of the traditional methods, like the Hurst exponent. Third, we investigate the dynamic evolution of this market efficiency degree over time via a rolling window approach and add to the literature that argues the market efficiency should not be taken as a static concept (Rösch et al., 2017, Takaishi and Adachi, 2020). Fourth, we examine the effect of individual market efficiency degree on cross-market arbitrage potential via a long memory cointegration framework. To the best of our knowledge, our paper is the first to shed light upon how segmented Bitcoin market efficiency may contribute to the arbitrage potential across multiple markets.

3. Methodology

For a given asset market, to uncover the extent of its deviation from an efficient market, we relax the traditional I(1)/I(0) assumption of the integration order (d) of market price series by considering the possibility that d can be a fractional value. By doing this, the degree of market efficiency can be truly quantified, rather than the strict premise of integer d values, which imply that the market should either satisfy the efficient market hypothesis (EMH), i.e. an I(1) price series, or completely inefficient, i.e. an I(0) price series. Identifying Considering A cointegration analysis is then conducted

This section first follows Hamilton (1994) to conduct a thorough discussion of various types of 'memory' of a given time series by identifying its potentially-existing fractional integration order, through which the degree of market informational efficiency can be well interpreted. We then employ a fractionally cointegrated vector autoregressive (FCVAR) model proposed by Johansen (2008) and Johansen and Nielsen (2012) to uncover both the short-run error corrections and the long-run relationship(s) among target variables, while allowing for the existence of the fractional cointegration order by considering the long-memory in the model system.

3.1. Fractional integration, long memory and market efficiency

(i) Theoretical discussion

By convention, an integrated process (y_t) of order d can be expressed as follows given $t = 1, \ldots, T$.

$$(1-L)^d y_t = \psi(L)\varepsilon_t = \sum_{j=0}^{\infty} \psi(L^j)\varepsilon_{t-j} \tag{1}$$

where $(1-L)^d$ is the difference operator of order d; $\psi(L^j)$ is the coefficient of error term (ε) at time period $t-j$ with $\sum_{j=0}^{\infty} |\psi(L^j)| < \infty$ to meet the covariance-stationarity requirement of $(1-L)^d y_t$ and $\psi(L^0) = 1$, $j = 0, 1, 2, \ldots$; error term (ε) is a white noise process, viz. $\varepsilon_t \sim iid(0, \sigma^2)$.

Specifically, in a conventional integer case when $d = 0$, Equation (1) can be transformed as:

$$y_t = \psi(L)\varepsilon_t = \sum_{j=0}^{\infty} \psi(L^j)\varepsilon_{t-j} \tag{2}$$

where y_t is defined as a covariance stationary series with zero mean and constant variance, indicating that the impact of a unit shock to the past error term (ε_{t-j}) on the current value of y_t, i.e. the impulse response coefficient of y_t, can be either zero such as a white noise process (viz. $y_t = \varepsilon_t$) or decay geometrically such as a stationary ARMA process (viz. $y_t = \sum_{i=0}^{\infty} \rho^j \varepsilon_{t-j}$). Thus, a market can be defined as the one with no efficiency in the condition when its price series has a 0 d value featured by short-memory.

When $d = 1$, y_t in Equation (1) can be then defined as a unit root process where contains one character root in the character function of y_t that lies on the unit circle.

In this case, y_t is no longer covariance-stationary and its statistical property can be intuitively observed through its infinite moving average (MA(∞)) representation:

$$y_t = \varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \varepsilon_{t-3} + \dots \tag{3}$$

where the impulse response coefficient of y (i.e. $\psi(L^j) = 1, j = 0, 1, 2, \dots$) constantly equals to one over time, indicating that past information of the unit root process y can permanently affect its current value and never dies away. Thus, a series with order $d = 1$ is known to process permanent memory, which also characters the price series in a market that satisfies the weak-form EMH, i.e. an efficient market.

However, the conventional discussion is still limited to a strict assumption of an integer integration order, it nevertheless fails to consider the possibility that d can be a fraction, which depicts long-memory featured shocks implied in such a fractionally integrated series. Thus, we further extend the conventional discussion to the field of fraction. In the light of Hamilton (1994), when d is a fraction and $0 < d < 1$, by multiplying the inverse of $(1 - L)^d$ on both sides of Equation (1), a fractionally integrated series y_t can be then formulated as:

$$y_t = (1 - L)^{-d}\psi(L)\varepsilon_t = \sum_{j=0}^{\infty} \gamma_j L^j \varepsilon_{t-j} \tag{4}$$

where

$$\gamma_j = \frac{(d + j - 1)(d + j - 2)\cdots(d + 2)(d + 1)(d)}{j!}, \tag{5}$$

γ_j is the impulse-response coefficient of y_t at time period $t - j$, viz. the coefficient of error term at the time period $t - j$, and $\gamma_0 \equiv 1$; $\gamma_j \cong (j + 1)^{d-1}$ given that $d < 1$ as a fraction and j is large. Therefore, the integrated series y_t defined in Equation (4) can be further expressed through the following infinite moving average (MA(∞)) process.

$$y_t = (1 - L)^{-d}\varepsilon_t = \gamma_0\varepsilon_t + \gamma_1\varepsilon_{t-1} + \gamma_2\varepsilon_{t-2} + \gamma_3\varepsilon_{t-3} + \cdots \tag{6}$$

In the light of (4), (5), and (6), impacts of a shock to the error term at time $t - j$ on (y_t) are highly-persistent and slowly converged over time (implied long-memory), in contrast to no or geometrically-decayed impact in a I(0) series (implied short-memory) and permanently-persistent impact in a I(1) series (implied permanent-memory) as previously discussed. According to the statistical property of a MA(∞) process, a series can be stationary only when $d < 1/2$ instead of $d = 0$ as conventionally defined.

Overall, it is evident that the impulse-response factor of a given series can actually reflect its information transmission pattern over time. The factor in past time period $t-j$ can represent the extent of how much the information about the series y in time $t-j$ can be transmitted to its current value (y_t), shedding lights on the degree of informational efficiency of a target market. For a given price series (y_t), its statistical properties with different d values and the associated status of market efficiency are presented in Table 1.

[Table 1 about here.]

Specifically, regarding the price series of the market under consideration, when its integration order $d = 1$, the market is efficient, indicating that past (price-related) information can be entirely transmitted to the current value of the price series featured by perfect autocorrelation. When $d = 0$, the market has no efficiency indicating that past information has no or very short reflection on current dynamics of the price series featured by no or weak autocorrelation. When $0 < d < 1$, although the market is not as efficient as the case when $d = 1$ because that the information transmission over time is not perfect, the transmission is highly persistent and slowly converged over past periods, indicating a greater degree of market efficiency than the case when $d = 0$. With regard to 'abnormal' conditions, when $d > 1$, the price series is known as an 'explosive' series, indicating an inefficient market as there is an increasingly excessive transmission of past information on the current price value with the increase of time lags. This is inconsistent with the spirit of EMH regarding the true reflection of the past information. When $d < 0$, the market is also inefficient due to the incomplete information transmission and the existence of autocorrelation in the price return series.

In summary, by identifying the fractional integration order of the price series, we extend the conventional $I(1)/I(0)$ framework and capture the potential 'long-memory' pattern of the series, through which various degrees of market efficiency are truly uncovered. While a market is 'completely' efficient only when the integration order (d) of its price series equals to 1, we propose a novel way to evaluate the extent of market efficiency by measuring the absolute difference between d and 1. The gap between d and 1 is negatively correlated with the degree of market efficiency. That is, a market is more efficient when d value of its price series is closer to 1, and vice versa. Thus, our strategy can not only identify whether the market is efficient or not, but also compare the efficiency degree among multiple target markets.

(ii) d estimates

Regarding the estimation of d, although conventional methods such as R/S and DFA estimators are widely employed, the validity of these methods is nevertheless questioned by existing literature so that the reliability of conclusions drawn by using these estimators could be seriously weakened. Specifically, it is documented that the Hurst index derived via the R/S method would give rise to biased and inconsistent d estimates (Lo, 1991) and its obtained results are largely affected by the sample size and initial parameter settings (Hauser and Reschenhofer, 1995). DFA method is known to underestimate d when the memory degree of the target series is unknown (Kantelhardt et al., 2001).

While the semi-parametric Exact Local Whittle (ELW) estimator (Shimotsu et al., 2005) could overcome the encountered weakness of the conventional methods (Kumar and Okimoto, 2007), its performance tends to be unstable and largely relies on the stationarity of the target series (Berger et al., 2009). A serious bias of the ELW estimator would occur if the target series is non-stationary, especially when its fractional integration order is greater than 0.75 (Velasco, 1999). As a further improvement, a recently developed semi-parametric estimator, i.e. Feasible Exact Local Whittle (FELW) estimator (Shimotsu, 2010), is documented to accommodate an unknown mean and time trend and ensure the estimation accuracy irrespective of the stationarity of the target series (See, e.g. Berger et al., 2009, Dolatabadi et al., 2018, Kumar and Okimoto, 2007). However, surprisingly scant application has emerged so far in the field of the Bitcoin market. Hence, we will employ the FELW estimator to estimate the d value of individual Bitocin price series in our main empirical discussion; to reassure our main results and make them comparable with existing literature, the ELW estimator will be also employed as a robustness check.

3.2. Fractionally cointegrated VAR model

How to uncover the actual linkage(s) between target variables in both short- and long-run terms considering the presence of long-memory in the variables? To answer this question, we follow Johansen and Nielsen (2012) and employ a fractionally cointegrated vector autoregressive (FCVAR) model. As an extension of the conventional vector error correction model (VECM) initially proposed by Johansen (1995), the FCVAR model offers an effective way to extend the I(1)/I(0) framework where most conventional analyses rest on. Through this, it can well capture the potential long-memory feature by identifying potential fractional integration and cointegration orders in the variable system, while investigating the short-run error corrections and the long-run equilibrium relation(s) among target variables. The FCVAR model specification is formulated as follows:

$$\Delta^d(Y_t - \rho) = \alpha\beta' L_d(Y_t - \rho) + \sum_{i=1}^{p} \Gamma_i \Delta^d L_d^i (Y_t - \rho) + \varepsilon_t \tag{7}$$

where Δ^d and L_d are fractional difference and lag operators, $\Delta^d = 1 - L_d = (1 - L)^d$, d can be either a fraction or integer value; Y_t is a K-dimensional column vector including K numbers of variables that are integrated with order d and cointegrated to order 0 to ensure that the obtained long-run relationship(s) are established on a stationary environment with no memory; the long-run relation(s) among variables in Y_t are defined by $\beta' L_d(Y_t - \rho)$ where β is a $K \times r$ matrix and describes the specified values of target variables in each cointegration relationship given that r indicates the rank of in Y_t, i.e. numbers of cointegration relationship; α is also a $K \times r$ matrix that identifies the speed of error correction towards the long-run equilibrium for each target variable in Y_t; Γ_i represents the short-run dynamics of variables in Y_t at time $t - i$, $i = 1, \ldots, p$; ε_t is a white noise process ($\varepsilon_t \sim iid(0, \Omega)$); ρ is a drift term that shifts the vector Y_t by a constant value ρ to relax the strict assumption regarding zero value of observations before the start of the finite empirical dataset (Johansen and Nielsen, 2016). Moreover, the FCVAR model is estimated by using the Maximum Likelihood estimator, which is examined to be normally distributed and provide unbiased estimation results (Johansen and Nielsen, 2012).

Regarding the endogeneity issue, the FCVAR model can ameliorate the issue regarding simultaneity and omitted explanatory variables. Specifically, due to the fact that the FCVAR model is built based on a VAR framework, the potential simultaneity issue, i.e. reverse causality, can be well captured. Moreover, given that our employed FCVAR model specification (7) incorporates a constant term (ρ) in the long-run relationship(s), it can well absorb the omitted explanatory power of the long-run determination of the specific variable, which is used to identify and normalize the long-run relationship(s) among all incorporated variables. Thus, through the above mechanisms, the FCVAR model is able to alleviate the endogeneity problem, enhancing the accuracy of our empirical results.

4. Data

Bitcoins are traded in a number of exchanges in many countries with different currencies. In most cases, the base currency to which Bitcoin is traded against depends on the country of the exchange's operation. Even in the less common cases where a large exchange operates across multiple countries, the order books for different base currencies are normally held separately and investors from different countries can only choose the local currency as the base currency (Makarov and Schoar, 2020). Therefore, we classify different Bitcoin markets and thereby capture the trading behaviour of geographically different investors by the base currency to which Bitcoin is traded against. Specifically, in the light of Cheah et al. (2018), Gillaizeau et al. (2019), we use the closing Bitcoin price data from the five developed markets, specifically the US, Europe, UK, Australia, and Canada, which represent the global Bitcoin market by covering more than 80% trading volumes of the worldwide Bitcoin transactions (Gillaizeau et al., 2019).

[Table 2 about here.]

We retrieve daily closing prices from Bitcoin Charts (`https://bitcoincharts.com/`) over the period of 1st January 2013 to 7th January 2020. For each base currency, we collect data from the most actively traded exchange that is selected by jointly considering data availability and trading volume. All the data we have from Bitcoin Charts use UTC timestamp in the trading record, therefore our closing prices are synchronised. Table 2 reports the details of the exchanges and sample periods employed in our study. In order to eliminate the foreign exchange risks, we convert all prices into US dollars using daily FX rates retrieved from Yahoo Finance (UTC time).

[Table 3 about here.]

[Figure 1 about here.]

Intuitively, the moving tendency of the five Bitcoin closing price series are reported in Figure 1 where we can obverse a co-movement pattern of the market price series under consideration. The figure also demonstrates visual evidence of the high fluctuation of the five price series, which is also consistent with the high standard deviations that are even greater than the mean values of the series as reported in Table 3, while overall they all depict an upward tendency over time, especially since the mid of 2017. Moreover, descriptive statistics of the five price series are reported in Table 3, and we observe that the closing price in Australia is ranked as the highest (4031.958) with the highest standard deviation (5213.299) while that in the UK is the lowest (2225.768) among the five markets. We can clearly see that each price series has a large standard deviation relative to its mean value, while for each series different locations (percentiles) of its price distribution depict large differences with the mean value.

5. Empirical analysis

5.1. Degree of Bitcoin market efficiency and its evolutionary dynamics

Our empirical analysis starts with estimating the fractional integration order ('d' values) of each target Bitcoin price series, from which we construct our measure for the degree of market efficiency. The Feasible Exact Local Whittle estimator (FELW) (Shimotsu, 2010) is employed for the 'd' estimation, which is conducted in two manners. Specifically, over the whole data sample, we first estimate the 'd' value for each series by using the FELW estimator with different bandwidths ranging from 0.4 to 0.8. The results are presented in Table 4. Across different bandwidths employed, our estimates of the fractional integration order of Bitcoin prices remain generally close to unity over the full sample period, ranging from 0.841 for AUD (bandwidth = 0.4) to 1.099 for CAD (bandwidth = 0.6). This result suggests that the five Bitcoin markets in our sample period is at a high level of informational inefficiency in general.

Next, to study the evolutionary nature of informational efficiency of each target market over time, we measure the degree of market efficiency using a self-calculated index D, which is built through the absolute difference between 1 and the fractional integration order:

$$D_t = |1 - d_t| \tag{8}$$

where d_t is the estimated fractional integration order at t, using the same FELW estimator on a rolling basis. Following Bariviera (2017), we set the window size as 1-year.[8] In the light of our discussions in Section 3, the index D calculated by the distance between d values and 1 actually represents the degree of market efficiency in an inverse way, i.e., the greater the D, the greater the gap (in absolute terms), the more inefficient the market, the less the market efficiency degree will be. Thus, D can be also regarded as a proxy of the market inefficiency degree.

[8] In Section 6 we re-estimate the fractional integration orders using different window sizes as robustness check. Our conclusion qualitatively remains unchanged.

The results are shown in Figure 2. Panel (a) plots the estimated fractional integration order (d) where we observe its time-varying behaviour in all markets. Specifically, the estimated 'd' value bounces around 0.6 at the beginning of the sample period to around 0.8 in the later phase. Moreover, it appears that the discrepancy in terms of 'd' across markets also exhibit time-varying patterns. The largest disagreement across the markets is observed mainly in the first half of our sample period, when the CAD/BTC market being the most deviating market. However, this discrepancy shrinks dramatically along with time in the second half of the sample period and is virtually disappeared after July 2017. This vanishing discrepancy in the 'd' value across markets implies that the Bitcoin markets trading with different fiat currencies are developing to a consensus where they are equally efficient.

Similar pattern can be observed in Panel (b) Figure 2, which depicts the degree of market efficiency (D) that is defined as the absolute difference between 1 and 'd'. Over the full sample period, D values fluctuates between 0 to 0.6, where virtually half of the times being close to 0. The discrepancy across markets is enlarged over the period from Jan 2016 to July 2017, after which it converges to an agreement. Rösch et al. (2017) find significant comovement in efficiency across individual stocks. Our results show that different Bitcoin markets are evolving towards a consensus in which there exists comovement in efficiency across markets.

[Table 4 about here.]

[Figure 2 about here.]

Urquhart (2016) find evidence that Bitcoin market is inefficient but might be in the process of evolving towards a higher degree of market efficiency. Takaishi and Adachi (2020) study the association between Bitcoin market liquidity and market efficiency. Similar to Urquhart (2016), they find that the Bitcoin market is evolving towards a higher degree of efficiency and this evolution echos the development of market liquidity. Bouri et al. (2019a) document structure changes in the dynamics of Bitcoin and find the lack of mean reversion in some sample periods. Our results are consistent with this stream of literature in that we find multiple Bitcoin markets exhibit higher degree of efficiency in the second half of our sample. Moreover, we show that different markets are developing to a consensus at which all markets are equally efficient.

5.2. Cross-market arbitrage potential and degree of market efficiency

The EMH lies on foundation that price has fully incorporated public information(see, e.g., Fama, 1970) whilst inefficiency can originate from inner- and cross- market wide development (see, e.g., Kühl, 2010). Theoretically, arbitrage opportunities should not be available for assets cross-listing on multi-markets, given the condition that EMH holds. Considering fiat currency cannot flow seamlessly across regions, Bitcoin cross-market wide information fails to be promptly reflected in price fluctuations of individual markets, which is indeed a crucial portion of information for Bitcoin price fluctuation. This type of friction prevents Bitcoin markets from forming consensus and thus cross-market price discrepancy will not fad and is bound to have an intrinsic linkage with failure in market efficiency. This intuition coincides with evidences from existing financial markets. For example, Suarez (2005) provokes that cross-market arbitrage opportunities for American Depository Receipt implies inefficiency. However, to the best of our knowledge, transmission mechanism directly linking cross-market price deviation and individual market efficiency remains unclear and this omission impedes us from drawing a full picture for Bitcoin price formation. To tackle this gap, in this subsection, we quantify across-market price deviation via an arbitrage index and further systemically examine its interactive roles against five developed Bitcoin markets' efficiency levels.

Analysing high-frequency data from a wealth of exchanges, Makarov and Schoar (2020) conclude that the cross-exchange correlation of returns on crypto markets are low at 5-minute level and there exist cross-exchange arbitrage opportunities. Further examinations in their study show that this opportunity is greater across regions than within regions. Following the same approach, we first study the arbitrage opportunity across markets. Table 5 reports the correlation coefficients for daily returns. While there appears a strong correlation among AUD/BTC, EUR/BTC, and USD/BTC markets, most pair-wise correlations are fairly low, resulting an average cross-market correlation coefficient of 0.40. This implies, even at daily frequency, the return correlation is low across markets clarified by base currency and the arbitrage opportunity reported in Makarov and Schoar (2020) might exist across these markets.

[Table 5 about here.]

To further examine the arbitrage across the five markets, we construct the Makarov and Schoar (2020) arbitrage index (AINX) using our daily data. In particular, AINX is computed as the ratio of the maximum price and the minimum price across markets on each day.[9] This ratio is never below unity and a greater value implies larger arbitrage opportunity. Figure 3 plots the arbitrage index over time. As shown by the figure, the year of 2016 serves as a clear watershed prior to which both the mean and variance of the arbitrage index are significantly greater than that in the period after 2016. Figure 4 depicts the frequency of each country being the maximum (Panel A) and minimum (Panel B) prices across markets.

[Figure 3 about here.]

[Figure 4 about here.]

While our research focus lies in how the cross-market arbitrage opportunities can be determined by the degree of individual market efficiency, their relationship can be bidirectional, giving rise to a potential endogeneity concern of simultaneity. Specifically, the cross-market arbitrage potential we measured can be closely related with the proportion of active arbitrageurs in each market and their inter-market migration behaviour. A market that has more arbitrage activities digests new information faster, thus has higher degree of market efficiency, than a market that is less actively arbitraged. Moreover, a sudden change in the arbitrage potentials may cause inter-market migration of arbitrageurs which might then lead to changes of market efficiency. In contrary, changes in the degree of market efficiency can in turn cause the migration behaviour of arbitrageurs that may generate price discrepancy across markets.[10]

Thus, our empirical analysis is greatly motivated to employ the FCVAR model to uncover the impact of individual market efficiency degree on cross-market arbitrage potentials. As explained in Methodology Section, the aforementioned simultaneity issue can be well ameliorated by the FCVAR model, while it can also alleviate the endogeneity issue of potential unobserved determinants of the arbitrage opportunities. The FCVAR model

[9] Due to the lack of intraday foreign exchange data, we use daily last prices instead of intraday prices as in Makarov and Schoar (2020).

[10] More generally, such mechanism does not necessarily require physical migration of arbitrageurs. For example, if the arbitrageurs are active only when the market is relatively inefficient and become inactive when the market is relatively efficient, then the migration of the behaviour of arbitrage will occur without the arbitrageurs needing to move across markets.

builds a long-memory cointegration framework, though which both the long-memory featured short-run error corrections and long-run relationships of our target variables can be well estimated.

Prior to estimating the FCVAR model, its model specification should be first determined, involving the lag-order selection and model rank tests (Jones et al., 2014). Regarding the lag order selection with the result shown in Table 6, while $p = 7$ appears to be selected due to the minimum AIC value, the residual is significantly auto-correlated given the 0 P-value of the Ljung-Box Q test, i.e. $PmvQ = 0$. Although the FCVAR estimation built by $p = 5$ and $p = 6$ possesses the second- and third-minimized AIC values, respectively, both of their model residuals tend to be serially correlated.[11] Thus, following a three-pronged strategy, the optimal lag order can be eventually selected as $p = 10$ due to its significant lag coefficient ($P\text{-}value = 0.000$), relatively minimized value of the information criteria (AIC value), and no auto-correlation in the model residual ($P\text{-}value$ of the Ljung–Box Q test is 1.000). Moreover, the model rank test is based on a series of Log-Likelihood ratio tests with null hypothesis of rank=k, $k = 1, \ldots, K$ against the same alternative hypothesis regarding the full rank, rank=K. The model rank is chosen as $Rank = 1$ at 5% significance level with the results shown in Table 7. Thus, the FCVAR model specification can be determined as the one with 10 lags of the short-run dynamics and 1 model rank.

Next, we conduct the FCVAR estimation to examine the relationship between the arbitrage index ($ANIX$) and the degree of informational efficiency for the five bitcoin markets (D) in the long-run. The results are presented in Equation (9). The estimated value of the fractional cointegration order ($\hat{d}$) is 0.868, demonstrating the existence of long-memory in the variable system involving the arbitrage index and efficiency degree. It indicates that while there exists a long-run relationship in the system, the equilibrium errors induced by occurred shocks to the target variables exhibit slow reversion to zero, i.e. error corrections towards the equilibrium are slow and thus deviations from equilibrium

[11] Although when $p = 6$ the model residual of the overall FCVAR model considering all six target variables just reject the null hypothesis of autocorrelation at 5% significance level, there still exists significant autocorrelation in the model residual of the six individual equations in the FCVAR model system. Results of the autocorrelation test for the residual in each individual equation are available from the authors upon request.

are highly persistent over time featured by long-memory. In economic terms, this means that the effect of a shock to the arbitrage potential and/or the market efficiency degree does not transmit instantly but rather is highly persistent, which might be caused by the delay in the arbitrage migration. The P value of the model Ljung-Box Q test is large and approaching to 1, indicating no auto-correlation in the residual of Equation (9).[12]

More importantly, the long-run relationship is identified by the arbitrage index and presented in Equation (10). In the long-run system equilibrium, the degree of market inefficiency of the US (USD) and Australian (AUD) markets exert a negative effect on the arbitrage potential with a coefficient of -3.505 and -0.337, respectively, whereas the inefficiency degree of the Canadian (CAD), European (EUR), and the UK (GBP) markets exhibit a positive relationship with a coefficient of 0.091, 2.831, and 0.842, respectively.[13]

[Table 6 about here.]

[Table 7 about here.]

Estimated FCVAR model (1Y rolling-FELW estimator):

$$\Delta^{\hat{d}} \left(\begin{bmatrix} AINX \\ USD \\ AUD \\ CAD \\ EUR \\ GBP \end{bmatrix} - \begin{bmatrix} 0.151 \\ 0.128 \\ 0.045 \\ 0.050 \\ 0.144 \\ 0.191 \end{bmatrix} \right) = L_{\hat{d}} \begin{bmatrix} -0.176 \\ -0.009 \\ -0.003 \\ -0.001 \\ 0.015 \\ 0.027 \end{bmatrix} \nu_t + \sum_{i=1}^{10} \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (Y_t - \hat{\rho}) + \hat{\varepsilon}_t \tag{9}$$

$$\hat{d} = \underset{(0.016)}{0.868},\ Q_\varepsilon(12) = \underset{(1.000)}{154.881},\ LogL = 29992.233$$

[12]The estimates of $\{\hat{\Gamma}_i\}_{i=1}^{10}$ are suppressed as we are only concerned with the long-run relationship. Complete estimation results including coefficient estimates of $\{\hat{\Gamma}_i\}_{i=1}^{10}$ are available from the authors upon request.

[13]Based on the calculation of the index D as previously discussed, USD, AUD, CAD, EUR, and GBP represent the degree of market efficiency of the five markets in an inverse way. That is, they respectively behave a negative relationship with the efficiency degree in their markets. Concurrently, they can be also regarded as proxies of the market inefficiency degree, i.e. the greater the D, the greater the market inefficiency, the less the market efficiency degree will be.

The Equilibrium Relationships (long-run):

$$AINX_t = -1.042 - 3.505 \times USD_t - 0.337 \times AUD_t + 0.091 \times CAD_t + 2.831 \times EUR_t + 0.842 \times GBP_t + \nu_t \tag{10}$$

Our obtained long-run cointegrating relationship built by β coefficients shown in Equation (10) suggests that the degree of Bitcoin market efficiency in the US and Australia exerts positive impacts on the cross-market arbitrage potential, indicating that an increase in efficiency degrees in these markets leads to an increase in the arbitrage opportunities. The efficiency degree of Bitcoin market in Canada, Europe and UK demonstrates negative effects on the arbitrage opportunities, i.e. the more inefficient in these markets, the greater their D values (i.e. CAD, EUR, GBP), the more cross-market arbitrage opportunities can be expected.

Taking the arbitrage index as a measure of market segmentation, our empirical results shed light on the effect of individual market efficiency on the overall market segmentation. A decrease in efficiency of a market may lead to the emigration of domestic liquidity trading due to a rising cost of transaction.[14] If the US and Australian trading are a mass of the overall Bitcoin trading, a decrease in their market efficiency might cause liquidity trading to overflow to other smaller markets, thus increases the degree of market segmentation. On the contrary, a decrease in the Canadian, European, and UK markets might cause the movement of liquidity trading towards the large markets like US, therefore reduces the degree of market segmentation.

Moreover, α coefficients shown in Equation (9) define the error correction speed of each incorporated variable that pushes the variable system back to the equilibrium defined as $\nu_t = 0$. In response to deviations from equilibrium, i.e. a fractionally lagged increase in ν_t, a correcting movement in $AINX$, USD, AUD, CAD, EUR, and GBP is -0.176, -0.009, -0.003, -0.001, -0.015, and 0.027, respectively. The magnitudes imply that $AINX$ moves the variable system towards equilibrium quicker than the efficiency degree of individual markets does. A possible explanation for our empirical findings is that, in the long-run, the overall market price discovery is dominated by the US and Australian markets (particularly the US), so that when the market efficiency improves in these two markets,

[14] Similar to the migration of arbitrageurs, this cross-market movement of liquidity traders need not be physical. A reduction of liquidity trading in a market with a simultaneous intensification in another market would be effectively counted as a migration of liquidity trading across the two markets.

the global price discrepancy narrows. On the contrary, for Bitcoin priced in CAD, EUR and GBP, their inefficiency levels exhibit a positive bond with arbitrage opportunities. This suggests that when market efficiency condition in any of these markets worsens, the rest markets will not follow the pattern and as a result, this kind of anti-herding behaviour broaden cross-market wide price deviation.

6. Robustness Check

In this section, we revisit our main findings and conduct a series of robustness checks in two dimensions, i.e. the estimation of market efficiency degree and its relationship with the arbitrage index, respectively.

6.1. Degree of market efficiency estimation: The change of window size

Our robustness check starts by re-estimating the dynamic 'd' and degree of market efficiency using the FELW estimator on a 6-month rolling basis. Figure 5 confirms the pattern found in our main analysis where 1-year rolling window is employed. In detail, the overall level of 'd' bounces around unity where the largest discrepancy across markets is observed during the first half of the whole sample period. Moving into the second half of the sample period, gaps among five markets' integration orders shrink to an agreement as observed in the main analysis. Since our measure of market efficiency degree is computed as the absolute difference between the 'd' value and 1, i.e. the index D, the corresponding pattern of market efficiency degree (D) shown in Panel (b) of Figure 5 resembles the pattern in the order of integration. To conclude, our results are not sensitive when changing the window size.

[Figure 5 about here.]

6.2. Degree of market efficiency estimation: The change of d estimator

To further examine the robustness of our main results with regard to estimation of integration order (d) and corresponding calculation of market efficiency degrees, we employ two alternative semi-parametric estimators, namely the Exact Local Whittle estimator ($\hat{d}_{ELW}$) and the Feasible Exact Local Whittle estimator with demeaned and detrended data, i.e. $\hat{d}_{FELW_D}$. We first estimate the 'd' values for the five Bitcoin price series over the full sample period. As shown in Table 8, the $\hat{d}_{ELW}$ and $\hat{d}_{FELW_D}$ estimates are generally consistent with their $\hat{d}_{FELW}$ counterparts in the main analysis, ranging from 0.801 for AUD ($\hat{d}_{ELW}$, bandwidth 0.4) and 1.103 for CAD ($\hat{d}_{ELW}$, bandwidth 0.6). This further supports our argument that the markets are generally at a high level of information efficiency over the full sample period. However, it is worth noting that none of the markets are fully efficient with the price series as an I(1) process.

Next, through the ELW and FELW$_D$ estimators, we recursively estimate the d values and accordingly calculate the market efficiency degree over time using both 1-year and 6-month rolling windows, respectively. Corresponding results are plotted in Figures 6 to 9, where similar patterns of the 'd' values and degree of efficiency to that in our main analysis can be observed, further confirming that our results are not sensitive to the integration order estimator employed.

[Table 8 about here.]

[Figure 6 about here.]

[Figure 7 about here.]

[Figure 8 about here.]

[Figure 9 about here.]

6.3. *The Relationship between Arbitrage Index and Market Efficiency Degree: Changing the Estimator*

As another robustness check, we perform again the FCVAR model with the market efficiency degree estimated through the FELW$_D$ estimator using a 1-year rolling window. Specifically, regarding the lag order selection, we follow the same three pronged strategy used in the main estimation and select the optimal highest lag order as $p = 10$ given the result of lag order selection presented in Table 9.[15] Moreover, the results shown in Table 10 indicate 1 rank in the model. Therefore, the FCVAR model specification can be determined with 10 lag orders and 1 model rank, and corresponding estimation results are saved in Equation (11).

[15] Although the value of the information criteria is minimized when $p = 7$, we do not select it as its associated model residuals are significantly auto-correlated. The optimal lag order is eventually selected as $p = 10$ based on the same strategy applied in the main estimation.

Intuitively, the estimated value of the system order is 0.807, further indicating the existence of long-memory in the variable system involving the efficiency degree of the five Bitcoin markets and the arbitrage index. The residual of Equation (11) is checked to have no auto-correlation indicated by the rejection of the null hypothesis of the Ljung-Box Q test. The obtained one cointegrationship relationship is identified by the arbitrage index and shown in Equation (12). In the long-run system equilibrium, we find a negative effect of market inefficiency degree of the US (USD) and Australian (AUD) markets on the arbitrage potential with a coefficient of -2.520 and -0.320, respectively, whereas the inefficiency degree of the Canadian (CAD), European (EUR), and the UK (GBP) markets exhibit a positive relationship with a coefficient of 0.059, 1.852, and 0.851, respectively. Overall, the results are highly consistent with that of the main estimation, reassuring the robustness of our main conclusions.

[Table 9 about here.]

Estimated FCVAR model (1Y rolling-FELW$_D$ estimator):

$$\Delta^{\hat{d}}\left(\begin{bmatrix} AINX \\ USD \\ AUD \\ CAD \\ EUR \\ GBP \end{bmatrix} - \begin{bmatrix} 1.141 \\ 0.128 \\ 0.040 \\ 0.043 \\ 0.144 \\ 0.192 \end{bmatrix}\right) = L_{\hat{d}} \begin{bmatrix} -0.245 \\ -0.018 \\ -0.011 \\ -0.011 \\ 0.013 \\ 0.031 \end{bmatrix} \nu_t + \sum_{i=1}^{10} \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^{i} (Y_t - \hat{\rho}) + \hat{\varepsilon}_t \tag{11}$$

$\hat{d} = \underset{(0.025)}{0.807}$, $Q_\varepsilon(12) = \underset{(1.000)}{144.959}$, $LogL = 29793.286$

The Equilibrium Relationships (long-run):

$$AINX_t = 1.044 - 2.520 \times USD_t - 0.320 \times AUD_t + 0.059 \times CAD_t + 1.852 \times EUR_t + 0.851 \times GBP_t + \nu_t \tag{12}$$

6.4. Inclusion of JPY/BTC transactions

How sensitive are our main findings to the inclusion of the Japanese Yen/Bitcoin (i.e. JPY/BTC) transactions? It is known that Japan is one of the countries with the most liquid exchanges where Bitcoin is heavily traded against the local currency (e.g.

the Japanese Yen) (Makarov and Schoar, 2020).[16] In this section, we further investigate the robustness of our main findings after considering the role of JPY/BTC transactions. Given that the Bitcoin trading was primarily concentrated on the exchange of MtGox before it folded due to bankruptcy in early 2014, thus, there was a data break at that time. Accordingly, we trim the dataset that starts from 1st June 2014 when considering the Bitcoin market in Japan. Following extant literature (See, e.g., Kliber et al., 2019) and the strategy employed in our main analysis, the Bitcoin price series in Japan is constructed by collecting data from the most actively traded exchanges involving Anxbtc (2014/06/01-2015/06/24) and Bitflyer (2015/06/25-2020/01/07).

[Figure 10 about here.]

[Table 10 about here.]

Estimated FCVAR model (1Y rolling-FELW estimator including Japan):

$$\Delta^{\hat{d}}\left(\begin{bmatrix} AINX \\ USD \\ AUD \\ CAD \\ EUR \\ GBP \\ JPY \end{bmatrix} - \begin{bmatrix} 1.150 \\ 0.143 \\ 0.080 \\ 0.191 \\ 0.104 \\ 0.252 \\ 0.103 \end{bmatrix}\right) = L_{\hat{d}} \begin{bmatrix} -0.363 \\ 0.022 \\ 0.031 \\ 0.004 \\ 0.031 \\ 0.018 \\ -0.015 \end{bmatrix} \nu_t + \sum_{i=1}^{10} \hat{\Gamma}_i \Delta^{\hat{d}} L^i_{\hat{d}}(Y_t - \hat{\rho}) + \hat{\varepsilon}_t \tag{13}$$

$$\hat{d} = \underset{(0.012)}{0.898},\ Q_\varepsilon(12) = \underset{(1.000)}{191.600},\ LogL = 27925.846$$

The Equilibrium Relationships (long-run):

$$\begin{aligned} AINX_t = 0.883 - 0.063 \times USD_t - 0.030 \times AUD_t + 0.128 \times CAD_t + 0.391 \times EUR_t \\ +0.558 \times GBP_t - 0.709 \times JPY_t + \nu_t \end{aligned} \tag{14}$$

Based on the FELW estimator with a 1-year rolling window setting, the fractional integration order (d) and the corresponding efficiency degree (D) of individually-segmented

[16] We thank the referee for pointing this out and suggesting the inclusion of Japan as a robustness check.

Bitcoin markets including Japan are presented in Panels (a) and (b) in Figure 10. It is clear that the moving tendency of both d and D of the five markets (i.e. AUD, CAD, EUR, GBP, and USD) mimics that of our main findings (shown in Figure2), and the corresponding pattern in Japan (i.e. JPY) broadly exhibits a co-movement with that in other major Bitcoin markets except for Canada. Moreover, we investigate the relationship between the arbitrage index ($ANIX$) and the efficiency degree of the six individual Bitcoin markets (D) through the FCVAR estimation. The FCVAR model specification is determined with 10 short-run terms and 1 model rank according to results of the lag-order selection and the rank test shown in Tables 11 and 12, respectively. The FCVAR estimation with 1 identified long-run relationship is reported in Equations (13) and (14). Overall, the influences of the efficiency degree (D) of the five individual Bitcoin markets are highly consistent with the counterparts in our main estimation (shown in Equation (10)). We also find that the impact direction of D of Japanese (JPY) Bitcoin market is the same as that of the US (USD) and Canadian (CAD) markets, showing to be negative on the arbitrage index ($ANIX$).

7. Conclusion

With an increasing popularity of Bitcoin since Nakamoto (2008), the informational efficiency of its market has aroused a great interest amongst both regulators and academics. Focusing on five segmented Bitcoin markets, in this paper, we capture the degree of informational efficiency through the fractional integration order (d) of the price series and study its dynamic evolution over time. Furthermore, by employing a fractionally cointegrated vector autoregressive (FCVAR) model, we investigate the impact of individual market efficiency upon overall cross-market arbitrage potential.

Our paper contributes to the existing literature in that we estimate the integration order (d) of the Bitcoin price series and allow for its value to be fractional, so that we refuse to take the EMH as a yes or no question and evaluate the dynamic of Bitcoin market efficiency. Based on this, more importantly, we estimate the extent to which individual market efficiency affects overall cross-market arbitrage potential, linking informational efficiency to market segmentation of Bitcoin.

Specifically, the d value of Bitcoin price series is estimated by a robust semi-parametric estimator, the Feasible Exact Local Whittle estimator (Shimotsu, 2010), and the efficiency degree of each of our five target Bitcoin markets can be then computed based on a self-proposed index D which is the absolute difference between 1 and d. Over the full sample period, we observe that the integration orders of the Bitcoin markets are fairly close to one (D being close to zero), suggesting high degree of efficiency in general. However, recursively estimating the d value shows that the degree of efficiency of each Bitcoin market varies over time. While it generally remains highly close to efficient throughout the sample period, the market efficiency deteriorates significantly over the period of 2016 to 2017, which coincides with the Bitcoin boom. More importantly, we observe that the gap in the degree of efficiency between markets narrows gradually over time, suggesting that all segmented markets are evolving to a consensus in terms of informational efficiency.

Then we move a step further to investigate if the cross-market arbitrage potentials is determined by the efficiency degree of individual markets in the context of Bitcoin. If so, how to identify the determination in both the short- and long-terms while allowing for the potential long-memory feature in the system? We answer these questions through a recently-proposed Fractionally Cointegrated Vector Autoregressive (FCVAR) model (Johansen and Nielsen, 2012). Our results show that the degree of Bitcoin market efficiency in the US and Australia impose positive impacts on the cross-market arbitrage potential, i.e. an increase in efficiency degrees in these markets leads to an increase in the arbitrage opportunities, whereas the efficiency degree of Bitcoin market in Canada, Europe and UK demonstrates negative effects on the arbitrage opportunities, i.e. the less efficient in these markets, the more cross-market arbitrage opportunities can be expected.

Our results are consistent to a series of robustness checks and possess insightful implications that should be of interest to policymakers and market investors. From the perspective of investors, clear comprehension regarding different impacts of the efficiency degree of individually-segmented Bitcoin markets on the cross-market arbitrage potential provides investors with useful information for designing rational investment strategies and maximizing their profits. Moreover, the Bitcoin market inefficiency manifests the existence of irrational and speculative investment behaviours, indicating the importance of pricing the actual value of Bitcoin prudently rather than entering such a risky market arbitrarily. From the policymakers' standpoint, as capital controls and the devoid of regulatory oversight on exchanges of cryptocurrencies would induce market segmentation (Makarov and Schoar, 2020), potentially leading to cross-border speculative behaviours, policymakers should control such speculative arbitrage and ensure financial stability by increasing openness of the economy and strengthening the supervision on crypto exchanges.

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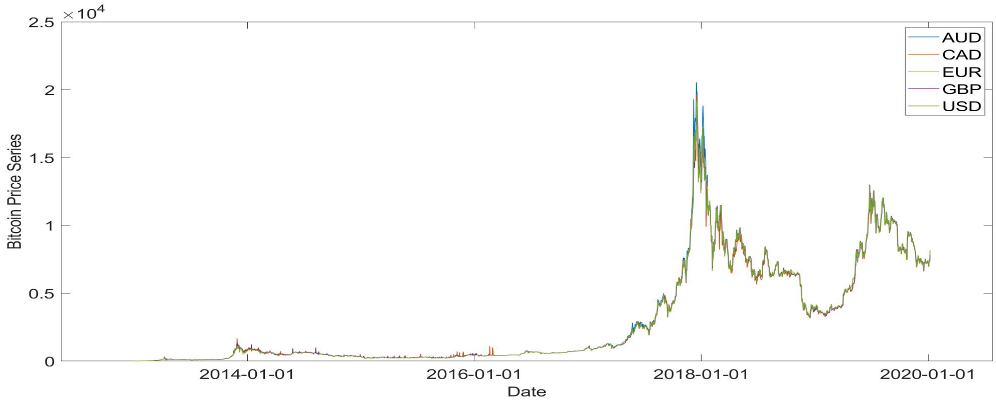
Figures

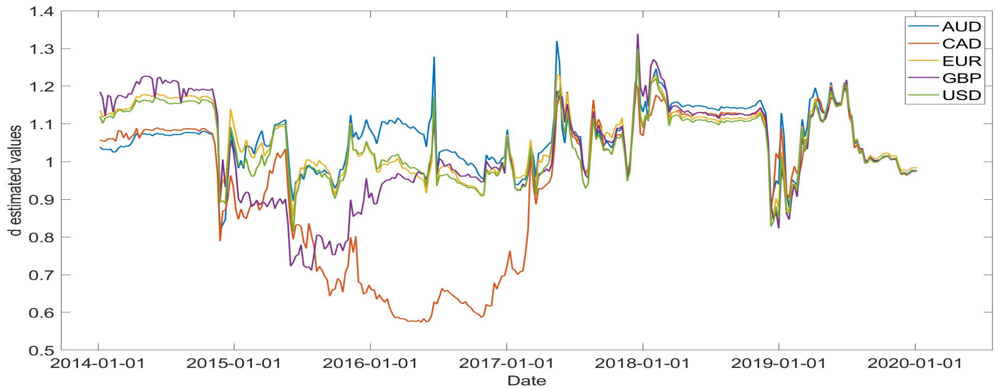
Figure 1: Movements of Bitcoin Price Series



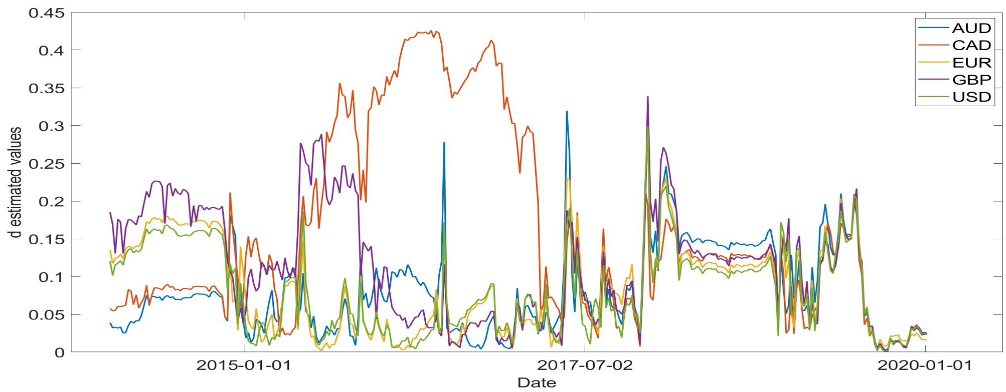
Note: This figure depicts the moving tendency of the Bitcoin closing price series in the five markets, viz. US (USD), Europe (EUR), UK (GBP), Australia (AUD), and Canada (CAD).

Figure 2: 1-Year Rolling-window FELW Estimates of d and Market efficiency Degree (D)

(a) d estimates

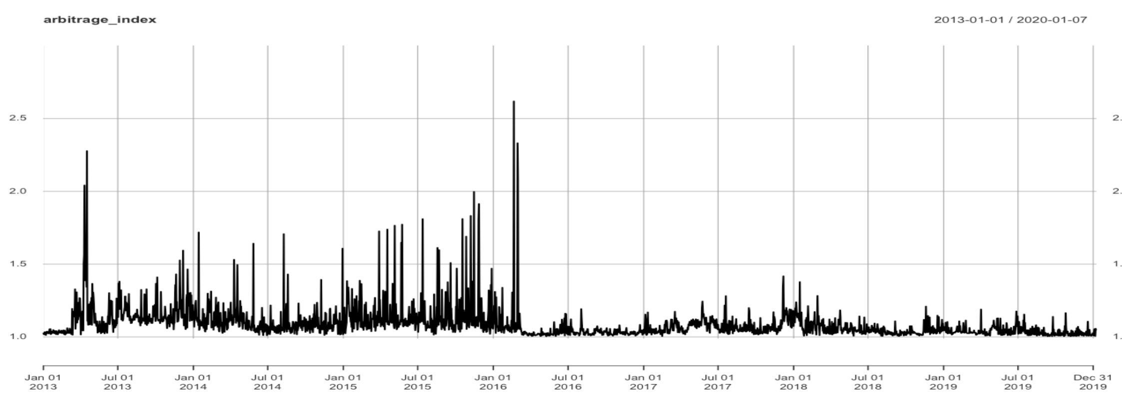


(b) Degree of Informational Efficiency (D)



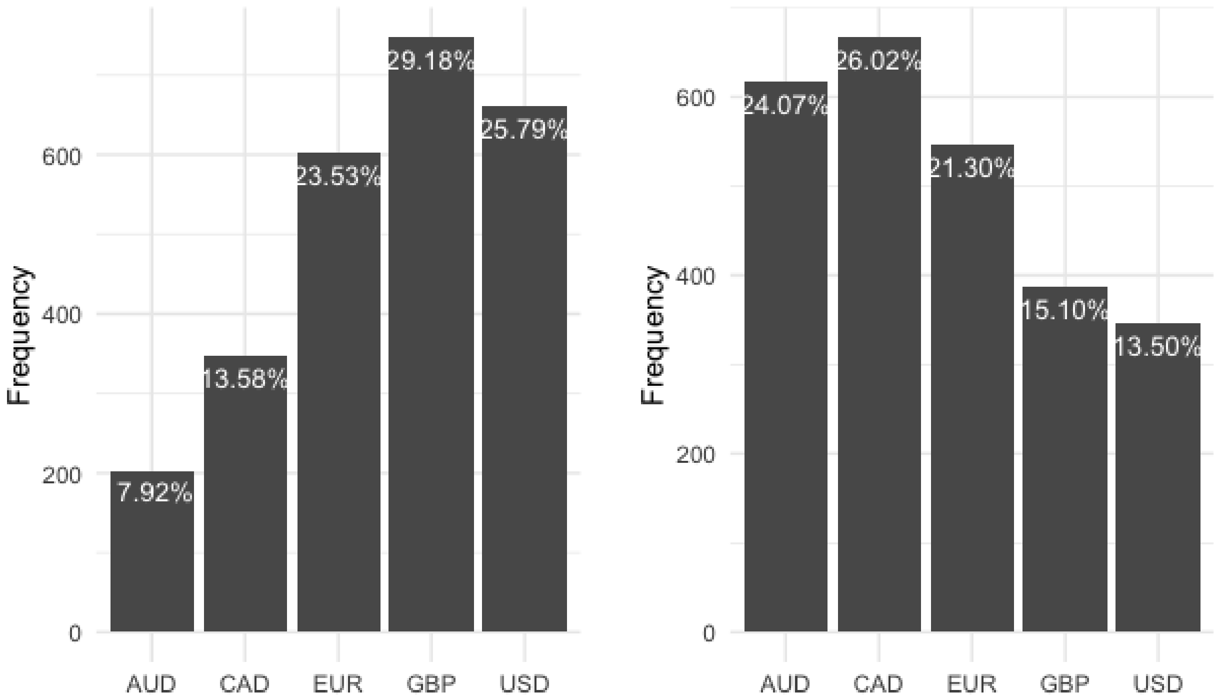
Note: The sub-figures (a) and (b) respectively report dynamics of d of the five Bitcoin price series and efficiency degree of (D) the five Bitcoin markets estimated by using the 1-year rolling-window FELW estimator. The index D is obtained by computing the absolute difference between d and 1, and it represents the market efficiency degree in an inverse way. AUD, CAD, EUR, GBP, and USD represent Bitcoin price series in the segmented markets in Australia, Canada, Europe, the UK and the US, respectively.

Figure 3: Arbitrage Index



Note: This figure depicts the Arbitrage Index that is constructed following Makarov and Schoar (2020). In particular, the index is computed as the ratio of the maximum price and the minimum price across markets on each day.

Figure 4: Frequency of Entering the Arbitrage Index Calculation

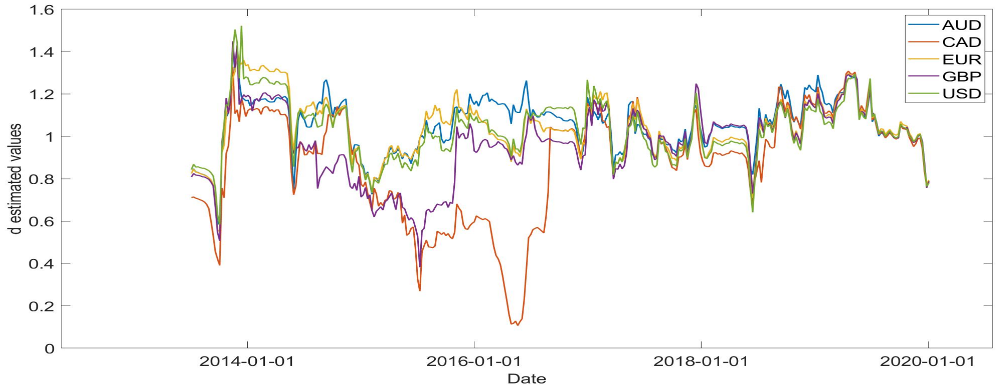


Panel A: Frequency of being Maximum   Panel B: Frequency of being Minimum

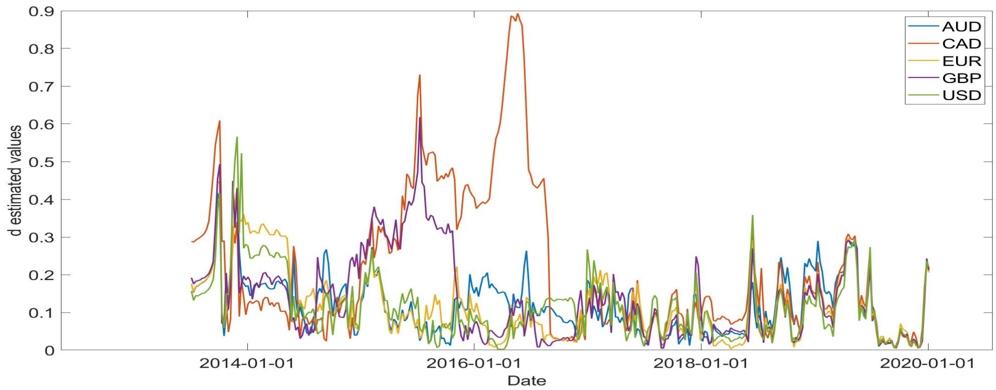
Note: This figure depicts the frequency of each market enters the calculation of the Arbitrage Index, which is constructed following Makarov and Schoar (2020). Panel A shows the frequency of each country having the maximum price across markets. Panel B shows the frequency of each country having the minimum price across markets.

Figure 5: 6-Month Rolling-window FELW Estimates of d and Efficiency Degree (D)

(a) d estimates



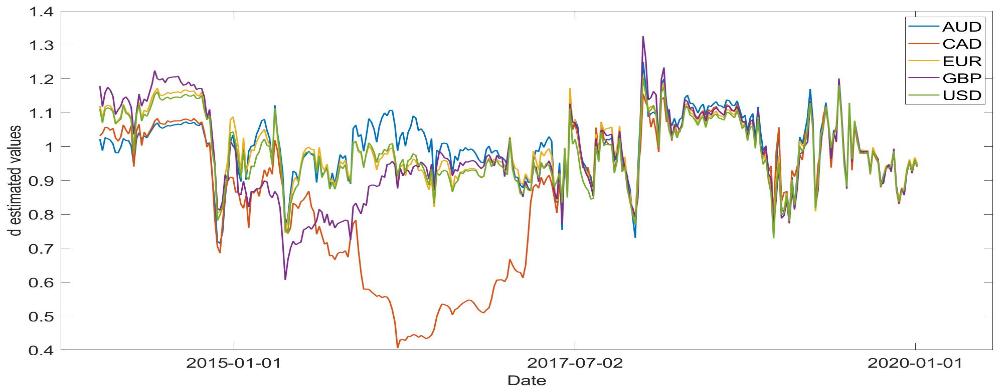
(b) Degree of Informational Efficiency (D)



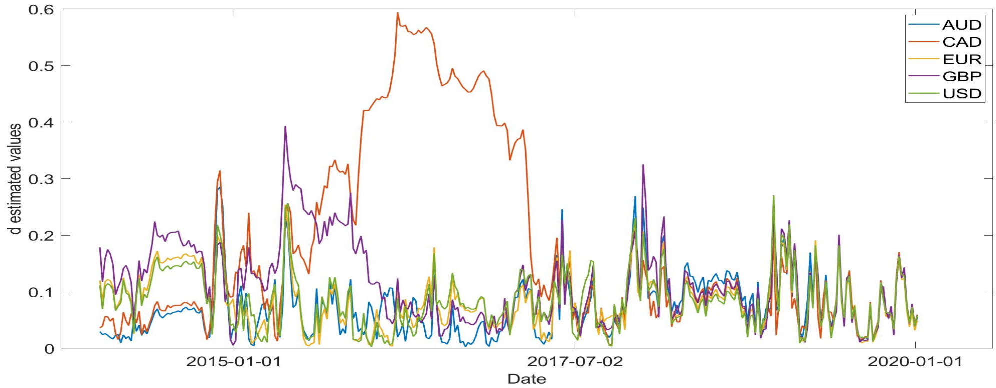
Note: The sub-figures (a) and (b) respectively report dynamics of d of the five Bitcoin price series and efficiency degree of (D) the five Bitcoin markets estimated by using the 6-month rolling-window FELW estimator. The index (D) is obtained by computing the absolute difference between d and 1, and it represents the market efficiency degree in an inverse way. AUD, CAD, EUR, GBP, and USD represent Bitcoin price series in the segmented markets in Australia, Canada, Europe, the UK and the US, respectively.

Figure 6: 1-Year Rolling-window ELW Estimates of d and Market Efficiency Degree (D)

(a) d estimates



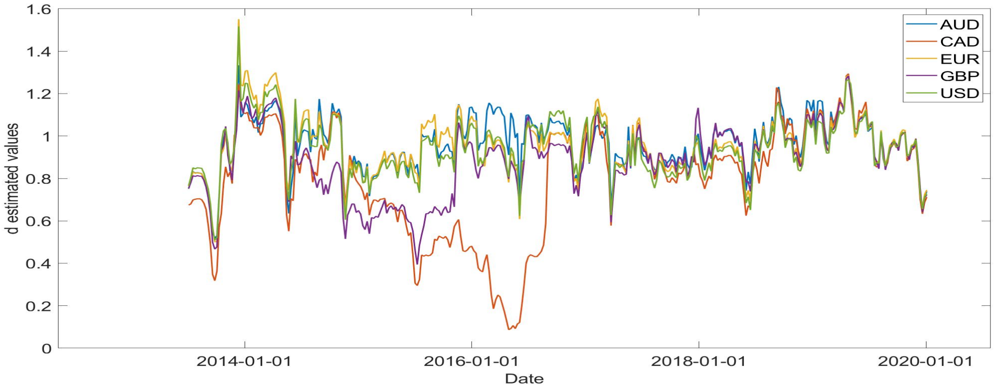
(b) Degree of Informational Efficiency (D)



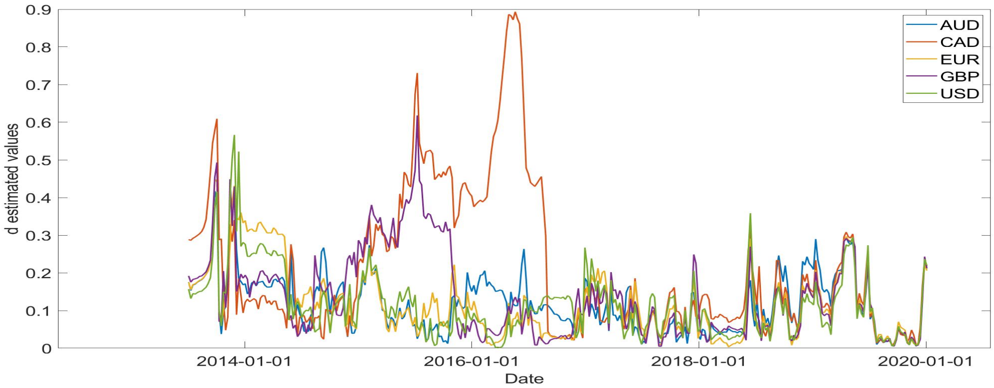
Note: The sub-figures (a) and (b) respectively report dynamics of d of the five Bitcoin price series and efficiency degree of (D) the five Bitcoin markets estimated by using the 1-year rolling-window ELW estimator. The index D is obtained by computing the absolute difference between d and 1, and it represents the market efficiency degree in an inverse way. AUD, CAD, EUR, GBP, and USD represent Bitcoin price series in the segmented markets in Australia, Canada, Europe, the UK and the US, respectively.

Figure 7: 6-month Rolling-window ELW Estimates of d and Market Efficiency Degree (D)

(a) d estimates



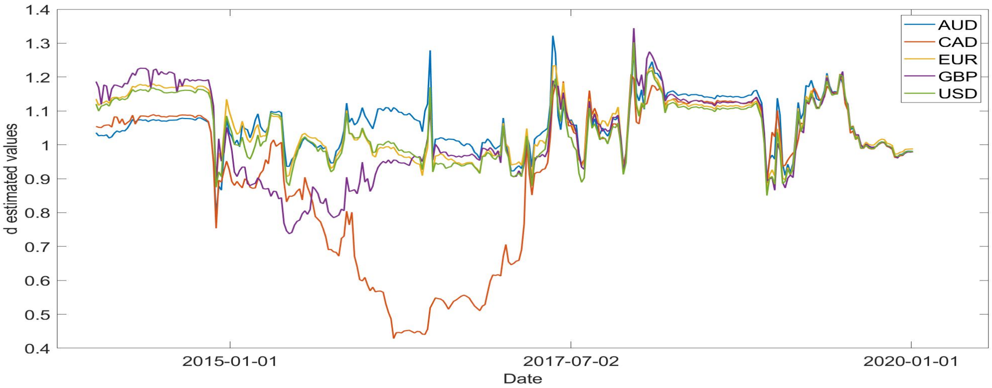
(b) Degree of Informational Efficiency (D)



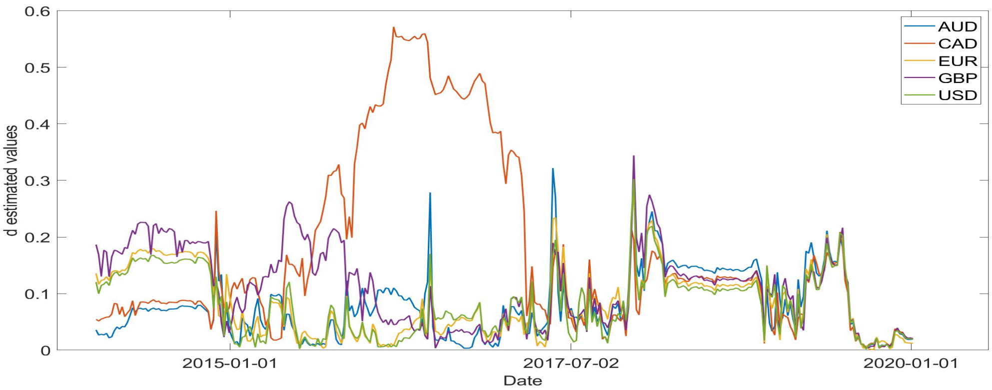
Note: The sub-figures (a) and (b) respectively report dynamics of d of the five Bitcoin price series and efficiency degree of (D) the five Bitcoin markets estimated by using the 6-month rolling-window ELW estimator. The index D is obtained by computing the absolute difference between d and 1, and it represents the market efficiency degree in an inverse way. AUD, CAD, EUR, GBP, and USD represent Bitcoin price series in the segmented markets in Australia, Canada, Europe, the UK and the US, respectively.

Figure 8: 1-Year Rolling-window FELWD Estimates of d and Efficiency Degree (D)

(a) d estimates



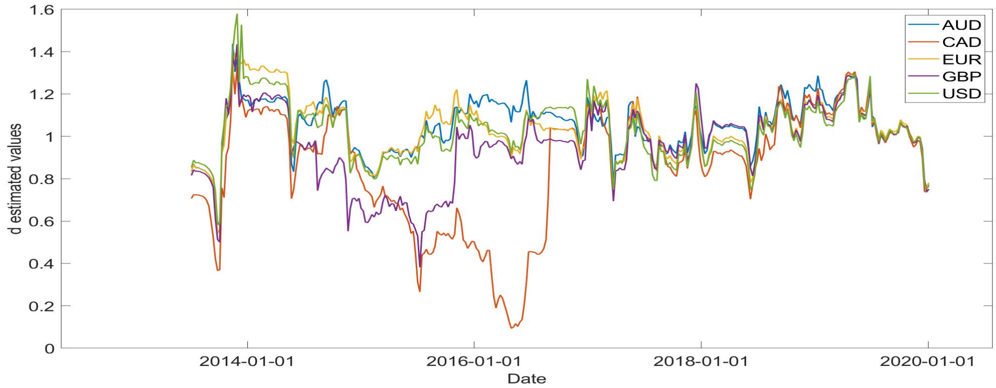
(b) Degree of Informational Efficiency (D)



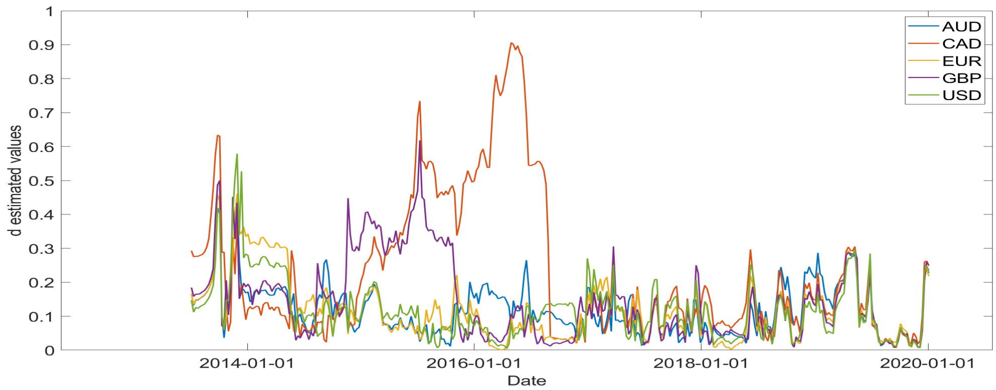
Note: The sub-figures (a) and (b) respectively report dynamics of d of the five Bitcoin price series and efficiency degree of (D) the five Bitcoin markets estimated by using the 1-year rolling-window FELW$_D$ estimator. The index D is obtained by computing the absolute difference between d and 1, and it represents the market efficiency degree in an inverse way. AUD, CAD, EUR, GBP, and USD represent Bitcoin price series in the segmented markets in Australia, Canada, Europe, the UK and the US, respectively.

Figure 9: 6-month Rolling-window FELW$_D$ Estimates of d and Efficiency Degree (D)

(a) d estimates



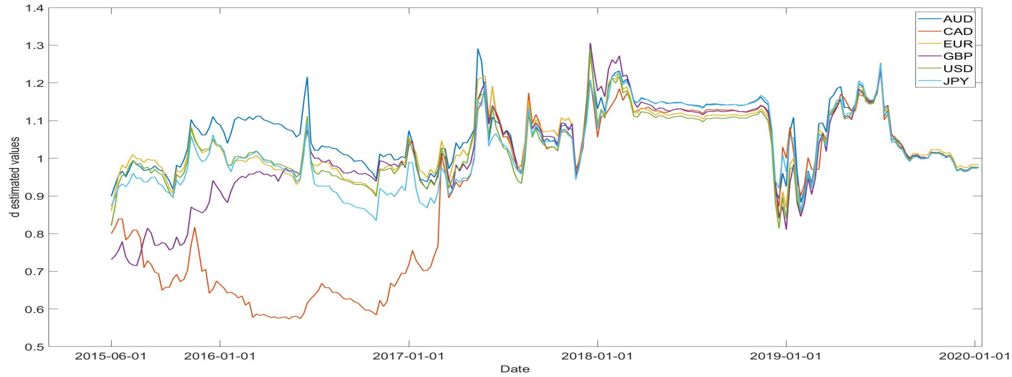
(b) Degree of Informational Efficiency (D)



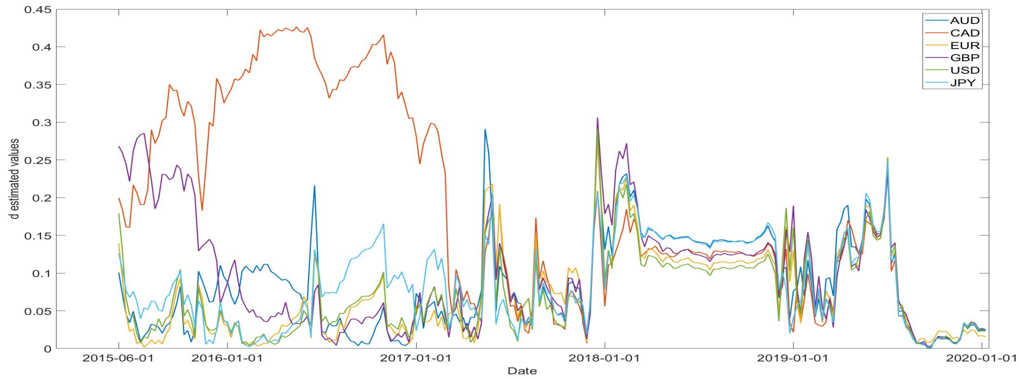
Note: The sub-figures (a) and (b) respectively report dynamics of d of the five Bitcoin price series and efficiency degree (D) of the five Bitcoin markets estimated by using the 6-month rolling-window FELW$_D$ estimator. The index D is obtained by computing the absolute difference between d and 1, and it represents the market efficiency degree in an inverse way. AUD, CAD, EUR, GBP, and USD represent Bitcoin price series in the segmented markets in Australia, Canada, Europe, the UK and the US, respectively.

Figure 10: 1-Year Rolling-window FELW Estimates of d and Market Efficiency Degree (D) (Including Japan)

(a) d estimates



(b) Degree of Informational Efficiency (D)



Note: The sub-figures (a) and (b) respectively report dynamics of d of the five Bitcoin price series and efficiency degree of (D) the five Bitcoin markets estimated by using the 1-year rolling-window ELW estimator. The index D is obtained by computing the absolute difference between d and 1, and it represents the market efficiency degree in an inverse way. AUD, CAD, EUR, GBP, USD, and JPY represent Bitcoin price series in the segmented markets in Australia, Canada, Europe, the UK, the US, and Japan, respectively.

Tables

Table 1: Memory properties of a given price series (y_t) with different d values

d Value	Persistence of shocks	Market Efficiency	Information transmission	The close degree to an efficient market
$d > 1$	Expansionary memory, explosive over time	Inefficiency	Excessive transmission	-
$d = 1$	Permanent memory	Efficiency	Complete transmission	Efficient market
$0.5 <= d < 1$	Long memory	Inefficiency	Partial transmission	High degree
$0 < d < 0.5$	Long memory	Inefficiency	Partial transmission	Lower degree
$d = 0$	Short memory	Inefficiency	None	Zero degree
$d < 0$	Long memory	Inefficiency	Reverse transmission	-
d Value	Stationarity	Mean	Variance	Return series
$d > 1$	Non-stationary	No mean reversion	Infinite	Positively autocorrelated
$d = 1$	Non-stationary , unit root process	No mean reversion	Infinite	No autocorrelation , white noise
$0.5 <= d < 1$	Non-stationary	Reversion	Infinite	Negatively autocorrelated
$0 < d < 0.5$	Stationary	Reversion	Finite	Negatively autocorrelated
$d = 0$	Stationary	Reversion	Finite	Negatively autocorrelated
$d < 0$	Stationary	Reversion	Finite	Negatively autocorrelated

Note: This table reports statistical properties of a given price series (Y_t) with various integration orders (d) and its corresponding implications on the market efficiency.

Table 2: Data Source

Currency	Exchange	Source	Sample Period
AUD	Mtgox	Bitcoincharts	2013/01/01 - 2014/01/03
	Btcmarkts	Bitcoincharts	2014/01/14 - 2014/05/19
	Anxbtc	Bitcoincharts	2014/05/20 - 2015/01/10
	Btcmarkts	Bitcoincharts	2015/01/11 - 2020/01/07
CAD	Mtgox	Bitcoincharts	2013/01/01 - 2013/03/12
	Localbitcoins	Bitcoincharts	2013/03/13 - 2014/05/18
	Anxbtc	Bitcoincharts	2014/05/19 - 2015/01/10
	Localbitcoins	Bitcoincharts	2015/01/11 - 2016/03/08
	Kraken	Bitcoincharts	2016/03/09 - 2020/01/07
EUR	Mtgox	Bitcoincharts	2013/01/01 - 2013/09/11
	Kraken	Bitcoincharts	2013/09/12 - 2020/01/07
GBP	Mtgox	Bitcoincharts	2013/01/01 - 2013/03/10
	Localbitcoins	Bitcoincharts	2013/03/11 - 2016/01/10
	Coinfloor	Bitcoincharts	2016/01/11 - 2020/01/07
USD	Bitfinex	Bitcoincharts & Bitfinex	2013/01/01 - 2020/01/07

Note: This table reports the base currencies, exchanges, data source, and sample period of the data employed in this study.

Table 3: Descriptive Statistics of Bitcoin Closing Price Series

	N	Mean	Standard Deviation	Min	Max	P25	P50	P75
USD	2563	2971.280	3803.005	13.484	20544.250	331.939	676.578	5709.526
EUR	2563	2917.005	3672.968	13.292	19694.404	337.781	677.913	5690.767
GBP	2563	2928.568	3712.275	13.293	18999.604	328.191	663.351	5663.575
AUD	2563	2938.645	3710.532	13.465	18719.725	346.055	677.856	5681.999
CAD	2563	2937.662	3719.482	13.300	19187.000	327.675	663.000	5746.000

Note: This table reports the descriptive statistics of the Bitcoin price series for each of the five markets. We report number of observations, mean, standard deviation, minimum value, maximum value, and the 25th, 50th, and 75th percentiles.

Table 4: Estimation of Fractional Integration Order (d) of Bitcoin Price Series

	$m = T^{0.4}$	$m = T^{0.5}$	$m = T^{0.6}$	$m = T^{0.7}$	$m = T^{0.8}$
AUD	0.841	0.954	1.012	1.069	1.008
	(0.104)	(0.070)	(0.047)	(0.032)	(0.022)
CAD	0.885	0.993	1.099	1.056	0.955
	(0.104)	(0.070)	(0.047)	(0.032)	(0.022)
EUR	0.891	0.965	1.053	1.050	1.022
	(0.104)	(0.070)	(0.047)	(0.032)	(0.022)
GBP	0.898	0.965	1.052	1.056	1.010
	(0.104)	(0.070)	(0.047)	(0.032)	(0.022)
USD	0.892	0.964	1.014	1.043	1.004
	(0.104)	(0.070)	(0.047)	(0.032)	(0.022)

Note: This table reports univariate d estimates for the five Bitcoin price series using the FELW estimator. AUD, CAD, EUR, GBP, and USD represent the price series in segmented Bitcoin markets in Australia, Canada, Europe, the UK and the US, respectively. Stand errors of the estimates are in parentheses, and are calculated as $(4m)^{-1/2}$, $m = T^B$, where T is the number of observations, T=2470, and B represents estimation bandwidths ranging from 0.4 to 0.8.

Table 5: Correlation of Daily Returns

	AUD	CAD	EUR	GBP	USD
AUD	1	0.38	0.89	0.05	0.86
CAD		1	0.40	0.05	0.37
EUR			1	0.03	0.91
GBP				1	0.02
USD					1

Note: This table reports pair-wise correlation coefficients of Bitcoin daily returns amongst the five markets clarified by the base currencies against which Bitcoin is traded.

Table 6: **Lag-order Selection - FCVAR (1Y rolling-FELW estimator)**

p	K	$\hat{d}$	*LogL*	*LR*	*P-value*	*AIC*	*PmvQ*
12	6	0.921	30088.92	62.91	0.004	-59227.84	1.000
11	6	0.838	30057.46	65.12	0.002	-59236.93	1.000
10	6	0.909	30024.90	83.44	0.000	-59243.80	1.000
9	6	0.842	29983.18	76.59	0.000	-59232.36	1.000
8	6	0.773	29944.88	-180.17	1.000	-59227.77	1.000
7	6	0.045	30034.97	258.11	0.000	-59479.94*	0.000
6	6	0.392	29905.91	102.51	0.000	-59293.82	0.150
5	6	0.539	29854.66	96.86	0.000	-59263.31	0.030
4	6	0.677	29806.23	116.03	0.000	-59238.46	0.000
3	6	0.840	29748.21	117.18	0.000	-59194.42	0.000
2	6	0.772	29689.62	152.89	0.000	-59149.24	0.000
1	6	0.691	29613.18	1454.72	0.000	-59068.35	0.000
0	6	0.599	28885.82	0.00	0.000	-57685.63	0.000

Note: This table presents results of the lag-order selection for the FCVAR model system involving the arbitrage index ($AINX$) and the efficiency degree (D) of the five individual Bitcoin markets. D is constructed based on the 1-year rolling FELW estimator. p indicates the number of lags; K indicates the total number of variables in the system; $\hat{d}$ is the estimated d value of the system; $LogL$ and LR are values of log-likelihood and LR test statistic in each lag-order selection, respectively; *P-value* is the p-value of the coefficient of the lag order p; AIC reports the value of Akaike information criterion (AIC); $PmvQ$ is the p-value of the white noise test for the model residual. Number of observations (T) in sample is 2200; order for the white noise test is 12.

Table 7: **Rank Tests - FCVAR (1Y rolling-FELW estimator)**

Rank	$\hat{d}$	*LogL*	*LRstatistic*	*P-value*
0	0.860	29965.401	119.001	0.000
1	0.868	29992.233	65.337	0.089
2	0.891	30006.891	36.021	0.452
3	0.901	30015.800	18.204	0.695
4	0.904	30022.125	5.553	0.919
5	0.908	30024.436	0.931	0.918
6	0.909	30024.902	-	-

Note: This table presents rank test results for the FCVAR model system involving the arbitrage index ($AINX$) and the efficiency degree (D) of the five individual Bitcoin markets. D is constructed based on the 1-year rolling FELW estimator. *Rank* is the number of ranks being tested; $\hat{d}$ is the estimated d value of the system; $LogL$ and LR are values of log-likelihood and LR test statistic in each test, respectively; *P-value* is the p-value of each test. Number of observations (T) in sample is 2200; order of lags is 10.

Table 8: Estimation of Fractional Integration Order (d) of Bitcoin Price Series (Robustness check)

Bandwidth	$m = T^{0.4}$		$m = T^{0.5}$		$m = T^{0.6}$		$m = T^{0.7}$		$m = T^{0.8}$	
	$\hat{d}_{ELW}$	$\hat{d}_{FELW_D}$	$\hat{d}_{ELW}$	$\hat{d}_{FELW_D}$	$\hat{d}_{ELW}$	$\hat{d}_{FELW_D}$	$\hat{d}_{ELW}$	$\hat{d}_{FELW_D}$	$\hat{d}_{ELW}$	$\hat{d}_{FELW_D}$
AUD	0.801	0.820	0.948	0.952	1.013	1.012	1.063	1.069	0.979	1.008
	(0.104)	(0.104)	(0.070)	(0.070)	(0.047)	(0.047)	(0.032)	(0.032)	(0.022)	(0.022)
CAD	0.849	0.870	0.989	0.992	1.103	1.100	1.050	1.056	0.932	0.955
	(0.104)	(0.104)	(0.070)	(0.070)	(0.047)	(0.047)	(0.032)	(0.032)	(0.022)	(0.022)
EUR	0.855	0.877	0.959	0.963	1.055	1.053	1.045	1.050	0.994	1.022
	(0.104)	(0.104)	(0.070)	(0.070)	(0.047)	(0.047)	(0.032)	(0.032)	(0.022)	(0.022)
GBP	0.858	0.884	0.954	0.962	1.051	1.052	1.049	1.056	0.980	1.010
	(0.104)	(0.104)	(0.070)	(0.070)	(0.047)	(0.047)	(0.032)	(0.032)	(0.022)	(0.022)
USD	0.855	0.878	0.957	0.962	1.014	1.013	1.037	1.043	0.976	1.004
	(0.104)	(0.104)	(0.070)	(0.070)	(0.047)	(0.047)	(0.032)	(0.032)	(0.022)	(0.022)

Note: This table reports univariate d estimates for the five Bitcoin price series using the ELW estimator and the FELW estimator with demeaned and detrended data (i.e. FELW$_D$). AUD, CAD, EUR, GBP, and USD represent the price series in segmented Bitcoin markets in Australia, Canada, Europe, the UK and the US, respectively. Stand errors of the estimates are in parentheses, and are calculated as $(4m)^{-1/2}$, $m = T^B$ where T is the number of observations, $T = 2470$, and B represents the value of estimation bandwidth ranging from 0.4 to 0.8.

Table 9: Lag-order Selection - FCVAR (1Y rolling-FELW$_D$ estimator)

p	K	$\hat{d}$	*LogL*	*LR*	*P-value*	*AIC*	*PmvQ*
12	6	0.934	29875.04	53.91	0.028	-58800.09	1.000
11	6	0.867	29848.09	60.78	0.006	-58818.18	1.000
10	6	0.846	29817.70	72.69	0.000	-58829.40	1.000
9	6	0.853	29781.35	58.72	0.010	-58828.71	1.000
8	6	0.746	29751.99	-192.78	1.000	-58841.99	1.000
7	6	0.052	29848.38	279.27	0.000	-59106.77*	0.000
6	6	0.285	29708.75	111.26	0.000	-58899.50	0.010
5	6	0.557	29653.12	106.70	0.000	-58860.24	0.170
4	6	0.570	29599.77	109.97	0.000	-58825.54	0.000
3	6	0.819	29544.79	118.33	0.000	-58787.57	0.000
2	6	0.759	29485.62	140.71	0.000	-58741.24	0.000
1	6	0.687	29415.27	1653.35	0.000	-58672.54	0.000
0	6	0.603	28588.59	0.00	0.000	-57091.18	0.000

Note: This table presents results of the lag-order selection for the FCVAR model system involving the arbitrage index ($AINX$) and the efficiency degree (D) of the five individual Bitcoin markets. D is constructed based on the 1-year rolling FELW estimator with demeaned and detrended data (i.e. FELW$_D$). p indicates the number of lags; K indicates the total number of variables in the system; $\hat{d}$ is the estimated d value of the system; *LogL* and *LR* are values of log-likelihood and LR test statistic in each lag-order selection, respectively; *P-value* is the p-value of the coefficient of the lag order p; *AIC* reports the value of Akaike information criterion (AIC); *PmvQ* is the p-value of the white noise test for the model residual. Number of observations (T) in sample is 2200; order for the white noise test is 12.

Table 10: Rank Tests - FCVAR (1Y rolling-FELW$_D$ estimator)

Rank	$\hat{d}$	*LogL*	*LRstatistic*	*P-value*
0	0.748	29711.60	80.787	0.044
1	0.739	29730.34	43.302	0.449
2	0.744	29739.54	24.910	0.671
3	0.728	29745.81	12.371	0.765
4	0.740	29750.68	2.631	0.965
5	0.748	29751.91	0.162	0.968
6	0.746	29751.99	-	-

Note: This table presents rank test results for the FCVAR model system involving the arbitrage index ($AINX$) and the efficiency degree (D) of the five individual Bitcoin markets. D is constructed based on the 1-year rolling FELW estimator with demeaned and detrended data (i.e. FELW$_D$). *Rank* is the number of ranks being tested; $\hat{d}$ is the estimated d value of the system; *LogL* and *LR* are values of log-likelihood and LR test statistic in each test, respectively; *P-value* is the p-value of each test. Number of observations (T) in sample is 2200; order of lags is 10.

Table 11: Lag-order Selection - FCVAR (1Y rolling-FELW estimator including Japan)

p	K	$\hat{d}$	*LogL*	*LR*	*P-value*	*AIC*	*PmvQ*
12	7	0.990	28053.36	90.61	0.000	-54816.71	1.000
11	7	0.998	28008.05	82.58	0.002	-54824.10	1.000
10	7	0.919	27966.76	143.84	0.000	-54839.52	1.000
9	7	0.832	27894.84	113.52	0.000	-54793.68	1.000
8	7	0.911	27838.08	-190.10	1.000	-54778.17	1.000
7	7	0.024	27933.13	397.42	0.000	-55066.27*	0.000
6	7	0.784	27734.42	83.29	0.002	-54766.85	0.300
5	7	0.661	27692.78	125.41	0.000	-54781.56	0.000
4	7	0.840	27630.07	117.96	0.000	-54754.15	0.000
3	7	0.799	27571.09	143.37	0.000	-54734.19	0.000
2	7	0.740	27499.41	213.40	0.000	-54688.81	0.000
1	7	0.663	27392.71	1628.89	0.000	-54573.41	0.000
0	7	0.575	26578.26	0.00	0.000	-53042.52	0.000

Note: This table presents results of the lag-order selection for the FCVAR model system involving the arbitrage index ($AINX$) and the efficiency degree (D) of the six individual Bitcoin markets including Japan. D is constructed based on the 1-year rolling FELW estimator (i.e. FELW). p indicates the number of lags; K indicates the total number of variables in the system; $\hat{d}$ is the estimated d value of the system; *LogL* and *LR* are values of log-likelihood and LR test statistic in each lag-order selection, respectively; *P-value* is the p-value of the coefficient of the lag order p; *AIC* reports the value of Akaike information criterion (AIC); *PmvQ* is the p-value of the white noise test for the model residual. Number of observations (T) in sample is 2047; order for the white noise test is 12.

Table 12: Rank Tests - FCVAR (1Y rolling-FELW estimator including Japan)

Rank	$\hat{d}$	*LogL*	*LRstatistic*	*P-value*
0	0.885	27899.312	134.899	0.006
1	0.898	27925.846	81.831	0.272
2	0.902	27939.700	54.124	0.475
3	0.907	27951.619	30.287	0.770
4	0.914	27958.525	16.474	0.817
5	0.920	27964.798	3.928	0.985
6	0.918	27966.411	0.703	0.955
7	0.919	27966.762	-	-

Note: This table presents rank test results for the FCVAR model system involving the arbitrage index ($AINX$) and the efficiency degree (D) of the six individual Bitcoin markets including Japan. D is constructed based on the 1-year rolling FELW estimator (i.e. FELW). *Rank* is the number of ranks being tested; $\hat{d}$ is the estimated d value of the system; *LogL* and *LR* are values of log-likelihood and LR test statistic in each test, respectively; *P-value* is the p-value of each test. Number of observations (T) in sample is 2047; order of lags is 10.