Policy conflict, coordination, and leadership in a monetary union under imperfect instrument substitutability

Policy Conflict, Coordination, and Leadership in a Monetary Union under Imperfect Instrument Substitutability

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Abstract

This paper investigates the implications of strategic fiscal-monetary policy interactions on the policy mix and coordination in a monetary union under imperfect policy instrument substitutability. We develop a model that incorporates the key features of the New-Keynesian framework augmented by a cost channel of monetary policy. Both policy instruments can directly affect inflation, hence having supply-side effects, too. We consider alternative strategic and fiscal regimes. We show that relative policy effectiveness and the cost-channel effect together define policy-mix outcomes, policies' cyclicality, and coordination problems. The cost channel limits union-wide demand shocks' stabilization, the monetary authority can no longer manage the cycle, and cooperation and commitment irrelevance do not hold anymore. The lead authority reacts to the follower authority's reaction parameter, hence to the follower's preference parameter, while it might choose not to trade-off its objectives. In the leadership strategic regimes for demand-side policy instruments, the leader reacts positively/negatively to the follower's preference parameter, if its instrument is more/less effective in stabilizing inflation (relative to aggregate demand) than the follower's policy instrument.

Keywords: *Monetary union; Fiscal/monetary policies; Cost channel of monetary policy; Strategic interactions; Coordination*.

JEL Classification: *E52; E61; E62; E63; F45*.

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1. Introduction

Twenty years after the official launch of the Economic and Monetary Union (EMU) in Europe, the "*…greatest monetary reform since Bretton Woods*" (Buti, 2003, p. 24), the interactions between fiscal and monetary policy remain bewildering. The recent travails of the Eurozone, including the sovereign debt crisis and the attempts to respond to the COVID-19 pandemic, reveal that the institutional structure of policymaking in a monetary union has been proved more challenging than initially thought. Monetary policy is conducted by the independent supranational European Central Bank (ECB), while fiscal policy remains decentralized at the national level, although constrained by the Stability and Growth Pact (SGP) and the Treaty on Stability, Coordination and Governance (or Fiscal Compact). In the absence of adjustment mechanisms (e.g., labor mobility) or cross-country risk-sharing schemes (e.g., cross-border asset holdings), fiscal policy is the only available arm of stabilization policy at the national level (Beetsma and Debrun, 2004). Non-coordinated national fiscal policies can generate inefficiencies, however, because of the externalities to other member-states. This motivates concerns about policy coordination. Beetsma and Giuliodori (2010) stress the need to develop a formal framework to model the strategic interactions in monetary unions, for the analysis of the policy mix and coordination. In a recent literature survey on the matter, Foresti (2018) presents an up-to-date and versatile theoretical framework to highlight key aspects of the literature, regarding the impact of multiplicative (parameter) uncertainty, authorities' preferences, the role of commitment to policy rules, and coordination, on the policy mix.

We develop a model that captures strategic fiscal-monetary policy interactions in a monetary union in the spirit of Dixit and Lambertini (2001, 2003a), to characterize the policy-mix outcome when the policy instruments (interest rate and fiscal stance) are imperfect substitutes in stabilizing the economy. In a New-Keynesian framework, this assumes that (at least) one policy instrument can directly affect inflation, independently of its effect through the output gap. Thus, we allow both policy instruments to have short-run supply-side effects, along with their usual demand-side ones. Fiscal policy can have either positive or negative direct effects on inflation, as various fiscal instruments can have (positive/negative) short-run effects on the supply side of the economy (see, e.g., Andersen, 2005, 2008; Debrun, 2000), whereas monetary policy can have a direct positive effect on inflation, mainly through the cost channel (Ravenna and Walsh, 2006). A key contribution of this paper is the introduction of a cost channel of monetary policy, which reflects the existence of a financial sector in the economy. Considering both policy instruments' direct effects on inflation, we provide a general framework that uncovers the importance of policy

instruments' relative effectiveness for the policy-mix outcome and coordination problems in a monetary union under alternative institutional arrangements; i.e., strategic and fiscal regimes.[1]

The typical New-Keynesian model does not explicitly incorporate investment and its response to the interest rate. The working capital channel (e.g., Christiano, 1991; Christiano and Eichenbaum, 1992), however, assumes that the labor input must be financed by loans, implying that monetary policy affects the economy by changing variable production costs. In addition, the presence of the cost channel of monetary policy provides an alternative interpretation of the price puzzle; i.e., the observation that a contractionary monetary policy shock is followed by a rise in the price level (Walsh, 2017). Barth and Ramey (2002) argue that an increase in interest rates raises the cost of holding inventories, acting as a positive cost-push shock. Rabanal (2003) considers whether the cost-channel effect dominates the output effect on inflation in the US and the Euro area, finding that the (conventional) demand-side effects of monetary policy prevail.[2]

The literature on macroeconomic policies in monetary unions typically focuses on two types of policy interactions: (i) the links between deficits, debts, inflation, and interest rates via the (dynamic) government budget constraints; and (ii) the links between fiscal and monetary policies in a macroeconomic stabilization perspective (Plasmans et. al., 2006). Our work builds on the second strand of the literature.[3] The relevant early literature was based on the traditional Barro-Gordon (1983) set-up, extended to include fiscal policy (Alesina and Tabellini, 1987), where the effects of monetary/fiscal policies are often set to work on the supply side of the economy, assuming that the '*law of one price*' (hereafter, LOOP) holds and the monetary authority cares about country-specific data (see, e.g., Beetsma and Bovenberg, 1998; Dixit and Lambertini, 2001, 2003a,b; and Foresti, 2018, for a unified framework). The most recent literature is based on the New-Keynesian framework, which focuses on the demand side of the economy, while the supply is often held fixed (see, e.g., Uhlig, 2003; Cavallari and Di Gioacchino, 2005; Bofinger and Mayer, 2007; Ferre, 2008, 2012; Oros and Zimmer, 2015; Chortareas and Mavrodimitrakis, 2017; Hughes Hallett and Mavrodimitrakis, 2019; among others). This literature assumes a conventional Phillips curve, where inflation is only linked to the output gap, along with expected inflation. Thus, the two policy instruments emerge as perfect substitutes in the stabilization process. An expansionary fiscal/monetary policy raises aggregate demand, hence increasing inflation. Moreover, the LOOP does not hold, hence inflation varies

[1] Beetsma and Debrun (2004) distinguish between 'horizontal' (across governments) and 'vertical' (between the monetary and the fiscal authorities) coordination problems.

[2] For further empirical evidence on the cost channel see Gaiotti and Secchi (2006), Chowdhury et. al. (2006), and Henzel et. al. (2009).

[3] We acknowledge that issues of debt accumulation and stabilization are important, since they have implications for the strategic fiscal-monetary policy interactions in monetary unions. An online appendix to this paper examines the model's implications when we introduce debt considerations. We thank an anonymous referee for insisting on examining those implications. An adequate treatment of these issues, however, would require a dynamic policy game (see, e.g., Beetsma and Boveneberg (1999, 2005)).

across countries. Building on the New-Keynesian framework, our motivation is to unify the literature by allowing the two policy instruments to have short-run supply-side effects, too.

Dixit and Lambertini (2001, 2003a) show that in a strategic game of pure macroeconomic stabilization[4] under non-conflicting objectives, there is '*symbiosis*' of monetary and fiscal policies, in that the actual targets can be obtained irrespective of the ordering of moves, of fiscal authorities' cooperation, or of identical preference priorities. Under conflicting objectives, the simultaneous-move strategic regime is inferior to any leadership regime. Kempf and von Thadden (2013) build on Dixit and Lambertini (2001, 2003a) and Chari and Kehoe (2008) to provide the general (sufficient) conditions for the irrelevance of the monetary authority's commitment capacity and the sequencing of moves, as well as for fiscal policies' coordination, in monetary unions under both private and fiscal spillovers; i.e., the '*cooperation and commitment irrelevance*' proposition.[5] These are: (i) the direct spill-over effects must have no strategic significance; and (ii) the number of instruments must match the number of squared gaps in all the authorities' payoff functions. Furthermore, under non-conflicting objectives, the bliss points can be achieved (*symbiosis* is feasible). In the absence of those conditions, however, both cooperation and commitment patterns (the sequence of moves) matter, while the difference between the non-cooperative and the cooperative outcomes depends on the number of countries in the monetary union. The authors show that the monetary union can benefit from fiscal authorities' cooperation under fiscal leadership.

The symbiosis result holds only under specific modeling assumptions (see, e.g., Della Posta and De Bonis, 2009; Di Bartolomeo and Giuli, 2011; Oros and Zimmer, 2015, 2020). Otherwise, both coordination and timing issues emerge, creating a policy-mix bias (Andersen, 2008; Foresti, 2018).[6] In order to explore this bias, our paper builds on a static representation of a reduced-form New-Keynesian model, following mainly Andersen (2005, 2008), in which the LOOP does not hold. We consider non-conflicting objectives of the authorities involved, but strategically significant (both direct and indirect) spill-over effects together with a shortage of policy instruments. So, by assumption, we depart from the before-mentioned conditions for commitment and coordination irrelevance. We show, instead, that the condition for commitment and coordination irrelevance at the union level is perfect instrument substitutability, since the system of equations is fully controllable by the central bank, meaning that equilibrium outcomes are independent on fiscal policies; and we show the connection with the symbiosis result. Under imperfect instrument

[4] Pure macroeconomic stabilization means that the policymakers care about inflation and output-gap stabilization, while non-conflicting objectives imply that they agree on the ideal targets of those macroeconomic variables (Uhlig, 2003). If the targets are equal to the long-run equilibrium values, we refer to pure cyclical macroeconomic stabilization (Dixit and Lambertini, 2003a).

[5] The private spillovers refer to the (wage) decisions by (multiple) private agents (non-coordinated wage setters) within countries (Chari and Kehoe, 2008). In the Barro-Gordon (1983) framework, instead, a representative private sector exists.

[6] E.g., fiscal authorities' cooperation has been found beneficial in various cases (see, e.g., Uhlig, 2003; Ferre, 2008; Beetsma and Bovenberg, 2005). Kempf (2020) considers a fiscal federalism and shows that adding another non-cooperative player in the policy game is not always preferable to fiscal authorities' cooperation.

substitutability, instead, both the strategic and the fiscal regimes (that is, alternative institutional arrangements) matter for pure cyclical macroeconomic stabilization at the union level. Then, we show how alternative institutional arrangements interact with the relative policy effectiveness and the cost-channel effect, and the implications for the policy-mix outcome and coordination problems in a monetary union.

Andersen (2008) examines the policy mix and coordination problems in a monetary union using a New-Keynesian model with strategically significant spill-over effects when fiscal policy can directly affect inflation and the country-specific fiscal policy instrument is costly for the national fiscal authorities. The author finds that under fiscal leadership and in the face of aggregate shocks the fiscal authorities underestimate the monetary reaction, resulting in a more countercyclical fiscal policy, whereas in the case of idiosyncratic shocks, the monetary response is over-estimated and fiscal policy is insufficiently countercyclical. Moreover, flexible inflation targeting can be welfare-improving under aggregate shocks. The present paper differs in four main ways: (i) we introduce a cost channel of monetary policy so that the common nominal interest rate can directly affect country-specific inflation in a positive way; (ii) we explore the strategic regimes of simultaneous move and monetary leadership, as well as the fiscal-monetary (overall) policy coordination regime, together with the fiscal leadership one; (iii) we allow for non-conventional effects of both policy instruments on inflation, which could transform them into supply-side policy instruments; and (iv) we deal with financial frictions in the monetary union, through supply shock asymmetries. We simplify by assuming pure demand/supply shocks and a two-country monetary union.

Our main results can be summarized as follows: (i) in a New-Keynesian framework, coordination and commitment irrelevance at the union level requires perfect instrument substitutability; (ii) for perfect instrument substitutability and common shocks, the bliss points are achievable at both the union and country level only under demand shocks; and (iii) a cost channel of monetary policy limits union-wide demand shocks' stabilization. Moreover, under imperfect instrument substitutability, (iv) the deficit-bias result does not hold and fiscal policy becomes active; (vi) the lead authority reacts to the follower authority's reaction parameter, while it might choose not to trade-off its objectives; and (vi) in the fiscal leadership strategic regime, the monetary authority affects the horizontal coordination problem. Focusing on demand-side policy instruments, we show that the policy-mix outcome depends entirely on relative policy effectiveness. In particular: (i) the leader's reaction to the follower's preference parameter; (ii) the monetary authority's reaction in the monetary leadership regime compared to the simultaneous-move one, and between the two fiscal regimes; (iii) the cyclicality of the union-wide fiscal stance in both the fiscal leadership regime for fiscal authorities' cooperation and the overall policy coordination regime; (iv) the way monetary authority affects the horizontal coordination problem in the fiscal leadership strategic regime; (v) the relevance of the specific fiscal regime for the monetary authority in the monetary leadership strategic regime.

The literature on the implications of relative policy effectiveness stemming from imperfect instrument substitutability for the policy-mix outcome in a monetary union is scant. Both Dixit and Lambertini (2001, 2003a) and Andersen (2008) assume imperfect instrument substitutability in a supply/demand-side model, respectively, but none of them considers a cost channel of monetary policy; so, the implications of relative policy effectiveness. Surpassing, we contribute to the literature in two aspects. First, we provide a generic framework to study short-run stabilization discretionary (time-consistent) fiscal/monetary policies in a monetary union in a strategic context and in a regime of policy conflict, based on a static representation of a reduced-form New-Keynesian model. The model is rich enough to cover aspects of the literature as special cases, including models with or without interconnections, strict vs. flexible inflation targets, alternative strategic regimes, fiscal policies' coordination, cost channels, and policies' supply-side effects. Second, we define the relative effectiveness of monetary/fiscal policies through a comparison of the marginal rates of transformation on inflation and output gap. We then show how relative policy effectiveness interacts with alternative institutional arrangements in a monetary union, and the implications for the policy mix.[7] To the best of our knowledge, this is the first paper that does so.

The next section presents the baseline model. Section 3 describes the general solution at the union level and defines relative policy effectiveness. Section 4 analyzes the policy mix for all the alternative strategic and fiscal regimes, while Section 5 explores union-wide and country-specific equilibrium solutions. Finally, Section 6 concludes.

2. The Model

We consider a monetary union consisting of two identical countries interconnected via traditional trade links and monetary policy. We model the monetary union as a closed area and assume that both countries have no interconnections with countries outside the union. The model is a static representation of a reduced-form New-Keynesian model based on an Aggregate Demand (AD) equation and a Phillips Curve (PC) equation, which constitutes a first-order approximation to a dynamic stochastic general equilibrium (DSGE) model with monopolistic competition and nominal rigidities (see, e.g., Gali, 2015). Both equations can emerge from a micro-founded model that captures monopolistic competition in product (and/or labor) markets, along with sticky prices (see, e.g., Beetsma and Jensen, 2005; Gali and Monacelli, 2008).[8] The static representation provides analytical results, which make the policy transmission mechanisms tractable

[7] The importance of the policy mix is recently stressed in Bartsch et. al. (2020), where the authors discuss capabilities and boundaries between fiscal/monetary interventions in face of policy responses to the COVID-19 crisis, and the institutional framework needed to support them.

[8] Kempf (2020) discusses how a fully-fledged dynamic model can be reduced to a static linear-quadratic model like the one employed here, while the general method to express a fully developed micro-founded macro model into a linear-quadratic framework is developed in Benigno and Woodford (2012).

and the study of the corresponding interactions manageable. This proves particularly useful in policy games, where a relatively simple analytical framework is required to allow comparisons of different solution concepts without resorting to numerical simulations. The baseline model is mainly based on Andersen (2005, 2008), and is extended to include a cost channel of monetary policy, while it follows the same notation to make the comparison of the results more transparent.

For each country j, the non-policy block of equations is given by:

$$y_j = -\delta_r\left(i - \pi_j^e - \bar{r}_j\right) - \delta_\tau\left(\pi_j - \pi_k\right) + \delta_y y_k + \delta_g g_j + u_j \quad (1)$$

$$\pi_j = \pi_j^e + \omega_y y_j + \omega_g g_j + \lambda_j i_{l_j}, \quad (2)$$

where the index k represents the other country. All variables represent log-deviations from long-run equilibrium values, apart from the decimal nominal interest rates, i and i_{l_j}. The former is the common nominal interest rate set by the central bank, i.e., the monetary policy instrument. Thus, $i - \pi_j^e$ is the real policy rate that affects aggregate demand. The variable i_{l_j} is the loan (lending) rate at which firms in each country should borrow from commercial banks in order to pay for wages in advance (see, e.g., Ravenna and Walsh, 2006). The variables π, y, and g represent respectively inflation, the output gap, and fiscal policy as captured by the overall fiscal stance. We assume that both economies have balanced budgets prior to the realization of shocks. The variable $\bar{r}_j$ represents the long-run equilibrium real interest rate, which for simplicity is set equal to zero for both countries, while u_j is an independently and identically distributed (i.i.d.; random) demand shock, with zero mean and known constant variance. The inflation differential $\pi_j - \pi_k$ represents the change in the real exchange rate and captures changes in intra-union competitiveness (the terms-of-trade effect); higher prices for domestic products shift domestic demand to foreign products. Finally, π_j^e denotes the private sector's (rational) expectation on country j's future inflation.

All the parameters in the AD equation (1) are positive. The parameter δ_r captures the interest sensitivity of aggregate demand, while δ_g captures the demand effect of fiscal policy. The parameters δ_τ and δ_y capture the interconnections between the two countries; that is, the effect of competitiveness on domestic output (a cost spill-over effect) and the relative openness of the economy (a demand spill-over effect), respectively (Ferre, 2008). They both represent strategically significant direct spill-over effects (Kempf and von Thadden, 2013), potentially leading to insufficient stabilization (Andersen, 2005). Higher domestic activity leads to higher prices and thus makes it possible for foreign partners to increase their market share, while a domestic fiscal expansion benefits trading partners by an increase in demand for foreign products. Moreover, the inflation differential works as an (automatic) stabilization mechanism, compensating for the lack of an independent monetary policy. If, e.g., a country experiences a negative

supply shock that increases inflation, a real appreciation would decrease exports to the other country, reducing output demand. This means that the country loses in competitiveness vis-a-vis the other country; and the resulting reduction in demand eventually decreases inflation.[9]

The existence of country-specific Phillips curves, equation (2), implies that the LOOP does not hold, assuming also the production of non-tradable goods, hence allowing for asymmetric supply shocks (as we will see) and real (ex-post) interest rate differentials across the union (Bofinger and Mayer, 2007). The parameter $\omega_y > 0$ is the slope of the PC equation and captures nominal (price/wage) rigidities in the economy (see, e.g., Clarida et. al., 1999; Walsh, 2017). This provides a rationale for the monetary authority to influence output, as this passes through to inflation. Following, e.g., the Calvo (1983) model of staggered price adjustment, where a fraction of firms are assumed to adjust prices each period, the PC is a relation between inflation, expected inflation, and firms' real marginal costs; where the latter's impact on inflation mainly depends on the degree of price stickiness (the fraction of firms that do not adjust their prices each period), here given by $\lambda_j = \frac{\partial \pi_j}{\partial i_{l_j}} > 0$. Thus, by including both the country-specific fiscal stance and the lending rate in the specified PC equation (2), we assume that they both directly affect real marginal costs.

Fiscal policy has a positive effect on output and through this a positive effect on inflation. Following Andersen (2005), however, it may also have (temporarily) separate effects on wage (price) inflation depending on the instrument used. For example, public expansions financed by value-added and excise taxes add (temporarily) to the inflationary pressure in the economy. A negative effect can emerge, however, via public investment or a production subsidy that raises private productivity, increasing the supply of goods (Dixit and Lambertini, 2003a). Another channel can exist through deep-habit formation, when private agents form habits from the consumption of individual goods (Ravn et. al., 2006).[10] In an economy with imperfectly competitive product markets, deep-habit formation creates a time-varying mark-up, which negatively depends on government spending through price-elasticity and intertemporal effects.[11] We thus allow the parameter that captures fiscal policy's direct effect on inflation, ω_g, to be of either sign.

Following Ravenna and Walsh (2006), we assume that monopolistically competitive firms must borrow from a financial intermediary to pay for wages in advance. Thus, prices set by firms directly depend on the cost of borrowing (the loan rate); in particular, under a high (low) loan rate, prices will be also high (low).[12] Next we define the link between the country-specific loan rate and the common nominal interest

[9] This procedure represents the adjustment of the real exchange rate through inflation differentials.
[10] We thank an anonymous referee for pointing this channel to us.
[11] Gali and Monacelli (2008) provide micro-foundations for a direct negative effect in a monetary union see also Palek and Schwanebeck (2017) and Vieira et. al. (2018). De Grauwe and Foresti (2020) also assume a negative direct effect.
[12] See an online Appendix to this paper for a simple exposition.

rate. This requires an explicit characterization of financial intermediaries; in particular, of bank behavior. In general, the country-specific loan rate can be expressed as:

$$i_{l_j} = z_j\left(i + v_j \bar{l}_j\right), \tag{3}$$

where $z_j, v_j > 0$ and $\bar{l}_j$ is a shock to the demand for loans. Various structural characteristics can affect the parameters z_j and v_j; e.g., operational costs, real/nominal rigidities, risky loans, to name just a few.[13] Combining equations (2) and (3), we get the following country-specific PC relation:

$$\pi_j = \pi_j^e + \omega_y y_j + \omega_g g_j + \omega_i i - \varepsilon_j, \tag{4}$$

where $\omega_i = \frac{\partial \pi_j}{\partial i} = \frac{\partial \pi_j}{\partial i_{l_j}} * \frac{\partial i_{l_j}}{\partial i} = \lambda_j * z_j = \lambda z > 0$ for symmetric countries, i.e., for $\lambda_j = \lambda_k = \lambda$ and $z_j = z_k = z$, and $\varepsilon_j = -\lambda_j z_j v_j \bar{l}_j = -\lambda z v \bar{l}_j$ for $v_j = v_k = v$, too. We consider identical countries with no structural heterogeneities. We assume that ε_j constitutes an (i.i.d.) supply shock with zero mean and constant known variance, similar to demand shock u_j. We further assume that both shocks are pure and uncorrelated. So, a positive shock in the country-specific demand for loans can be captured by a negative supply shock that increases inflation.

The country-specific PC relation described by eq. (4) shows that the common nominal interest rate directly affects inflation, following the cost channel of monetary policy effect of Ravenna and Walsh (2006) (see, also, Walsh, 2017). The cost channel creates a meaningful policy trade-off for the central bank without the need for an exogenous cost-push shock. Thus, both inflation and the output gap fluctuate in response to supply/demand disturbances under the optimal policy.

We can compute the descriptive non-policy block of equations at the union level by averaging the country-specific equations (1) and (4) to obtain:

$$y = \frac{1}{1-\delta_y}\left[-\delta_r(i - \pi^e) + \delta_g g + u\right] \tag{5}$$

$$\pi = \pi^e + \omega_y y + \omega_g g + \omega_i i - \varepsilon, \tag{6}$$

where for every variable x, it follows that $x = \frac{1}{2}\left(x_j + x_k\right)$.[14] The trade effect, δ_y, works as a multiplier, since domestic aggregate demand is affected by foreign aggregate demand, which is affected by domestic aggregate demand; so, increases in either domestic or foreign aggregate demand initiate consequent

[13] In an online Appendix to this paper we provide a derivation of equation (3) from a simple model of banks' behavior and an analysis on the determination of the relevant parameters. Our purpose is to provide a justification of the cost channel of monetary policy and we do not deal with strategic interactions between policy authorities and the banking sector. For such a paper in a closed economy, see Ismihan and Ozkan (2012).

[14] In this paper, we consider a two-country model of a monetary union under country-size symmetry. All the results, however, hold qualitatively in a multi-country setting with country-size asymmetry. Proofs are available by the authors, upon request.

increases in domestic aggregate demand, where their overall impact at the union level is captured by $\frac{1}{1-\delta_y} > 0$; hence must be $\delta_y \in (0,1)$ (see, also, Landmann, 2018). The terms-of-trade effect cancels out at the union level, since it affects both countries in a perfectly asymmetric way.

All authorities have complete control over their policy instrument and preferences over some variables that can be approximated by a quadratic loss function. The policy instruments for the national fiscal authorities and the monetary authority are g_j, g_k and i, respectively. Policymakers minimize a typical quadratic loss function subject to the economy's constraints. The authorities' loss functions are given by:

$$L_M = \frac{1}{2}(\pi^2 + a_M y^2) \tag{7}$$

$$L_{F_j} = \frac{1}{2}\left(g_j^2 + a_F y_j^2\right), \tag{8}$$

where '*M*' stands for the 'Monetary' authority and '*F*' for the national 'Fiscal' authorities. All the authorities seek to minimize deviations of their concerned variables from long-run equilibrium, which means that they agree on the steady state of overall optimal policy (Uhlig, 2003); hence the non-conflicting objectives. The national fiscal authorities share identical preferences and they are concerned with the output gap and the deviation from the balanced budget, where the parameter $a_F > 0$ is the weight that the authorities place on output-gap stabilization relative to fiscal-stance stabilization. The common central bank is concerned with the average output gap and inflation in the union, where the parameter $a_M > 0$ defines the weight it puts on output-gap stabilization relative to inflation stabilization. The larger this weight, the more flexible is the inflation-targeting framework that the common central bank follows (Svensson, 1997); hence the case of $a_M = 0$ corresponds to strict inflation targeting.

The specification of the loss functions given by equations (7) and (8) follows Uhlig (2003) and Andersen (2008), and represents a realistic mapping of the actual policymaking concerns in the EMU. Following Uhlig (2003), the inclusion of each country's fiscal stance in the fiscal authorities' loss functions reflects the desire of governments to both stabilize their economy and run a fiscally balanced budget. In this sense, the parameter a_F can be thought of as the weight the fiscal authority puts on a stable economy relative to a balanced budget; the lower this weight, the less the fiscal policy's flexibility to stabilize (country-specific) shocks. An $a_F = 0$ assumes a passive country-specific fiscal policy, while the symmetry assumption on positive/negative fiscal stance in the loss function would assume an unpleasant debt arithmetic in the case of a permanently positive fiscal stance; i.e., a structural deficit (Bofinger and Mayer, 2007). Since the fiscal stance is the fiscal authorities' policy instrument, it simultaneously defines a target and an instrument; hence a policy conflict (Kempf and von Thadden, 2013). Fiscal policymakers are assumed not to be directly concerned with (country-specific) inflation stabilization, since the task of

controlling (union-wide) inflation is delegated to the common central bank. Including a terms-of-trade effect in the aggregate demand equation, however, creates an implicit preference for (country-specific) inflation stabilization for the national fiscal authorities (see, e.g., Andersen, 2005, 2008).

To investigate the macroeconomic policy mix in the monetary union we have just described, we analyze the standard one-shot policy games of simultaneous move, fiscal leadership, and monetary leadership. In all scenarios, the time context begins with the private sector forming expectations about future inflation rationally and not strategically (Uhlig, 2003); then, demand and supply shocks are realized; finally, the authorities choose their control instrument in order to achieve their goals according to the particular institutional arrangement (strategic regime), hence acting in discretion. The strategic regime of simultaneous move demands all the authorities to act independently and simultaneously, where the equilibrium is described by a Cournot-Nash equilibrium. In the leadership regimes, the authority having the lead makes its move prior to the follower authority, while it considers the way the latter will react to its choice of the policy instrument. To solve these Stackelberg games we use backward induction and equilibrium rests on sub-game perfection. The fiscal leadership regime requires the two fiscal authorities to lead the game with the common central bank, while in the monetary leadership regime the monetary authority has the lead and the national fiscal authorities follow.[15] The above time context guarantees that policies are time-consistent; hence $\pi_j^e = \pi^e = 0$ (see, e.g., Uhlig, 2003; Andersen, 2008; among others).[16] In any case, we assume that the national fiscal authorities move simultaneously. The model also assumes that there is no uncertainty about structural parameters between the two fiscal authorities and between them and the monetary authority.

In each strategic regime, we also consider the case of fiscal authorities' cooperation. Following the rest of the literature, the national fiscal authorities minimize a joint loss function according to a straightforward utilitarian criterion that corresponds to simply averaging the two loss functions given by equation (8) (see, e.g., Debrun, 2000; Dixit and Lambertini, 2001, 2003a; Cavallari and Di Gioacchino, 2005; Ferre, 2008, 2012; Andersen, 2005, 2008; among others). This joint loss function is given by:

$$L_F = \frac{1}{2}\left(L_{F_j} + L_{F_k}\right) = \frac{1}{4}\left[g_j^2 + g_k^2 + a_F\left(y_j^2 + y_k^2\right)\right] \qquad (9)$$

We also consider a strategic regime of fiscal-monetary (overall) policy coordination, in which all the authorities choose their policy instruments to achieve their joint objectives. This loss function is defined by

[15] An online Appendix to this paper further discusses those regimes in real policy-making scenarios. In general, different regimes may emerge under different circumstances, while the empirical evidence on the actual sequencing of moves in the EMU is scant.
[16] Time-consistent policies emerge when: (i) the private sector forms expectations rationally; (ii) the expectations are formed prior to the realization of the shocks; and (iii) the policymakers target long-run equilibrium values. However, an online appendix to this paper shows that expected inflation is not equal to zero if we consider debt-constrained fiscal authorities, instead.

the sum of the monetary authority's loss function with the average of the two fiscal authorities', given by equations (7) and (9), respectively, as below:

$$L_{OC} = L_M + L_F = \frac{1}{2}\left[\pi^2 + \frac{1}{2}\left(g_j^2 + g_k^2\right) + a_M y^2 + \frac{1}{2}a_F\left(y_j^2 + y_k^2\right)\right], \quad (10)$$

where '*OC*' stands for 'Overall Coordination'. Naturally, this joint loss function includes both union-wide and country-specific variables, while all spill-over effects are fully internalized.[17]

We conclude this section by computing the reduced-form country-specific aggregate demand equation with respect to the policy instruments and shocks.[18] In a world of strategic policy interactions and perfect information, this relation would be known to all the authorities in the policy game.[19] For country j:

$$y_j = Z_\pi \pi_j^e + Z_\pi^* \pi_k^e - Z_i i + Z_g g_j + Z_g^* g_k + Z_\varepsilon\left(\varepsilon_j - \varepsilon_k\right) + Z_u u_j + Z_u^* u_k, \quad (11)$$

where $Z_\pi = \frac{\partial y_j}{\partial \pi_j^e} = \frac{(1+\delta_\tau\omega_y)\delta_r - (1-\delta_y)\delta_\tau}{(1-\delta_y)(1+\delta_y+2\delta_\tau\omega_y)}$, $Z_\pi^* = \frac{\partial y_j}{\partial \pi_k^e} = \frac{(\delta_y+\delta_\tau\omega_y)\delta_r - (1-\delta_y)\delta_\tau}{(1-\delta_y)(1+\delta_y+2\delta_\tau\omega_y)}$, $Z_i = \left|\frac{\partial y_j}{\partial i}\right| = \frac{\delta_r}{1-\delta_y}$, $Z_g = \frac{\partial y_j}{\partial g_j} = \frac{\delta_g - \delta_\tau\omega_g + \delta_\tau(\omega_g\delta_y + \omega_y\delta_g)}{(1-\delta_y)(1+\delta_y+2\delta_\tau\omega_y)}$, $Z_g^* = \frac{\partial y_j}{\partial g_k} = \frac{\delta_\tau(\omega_g+\omega_y\delta_g) + \delta_y(\delta_g - \delta_\tau\omega_g)}{(1-\delta_y)(1+\delta_y+2\delta_\tau\omega_y)}$, $Z_\varepsilon = \left|\frac{\partial y_j}{\partial(\varepsilon_j - \varepsilon_k)}\right| = \frac{\delta_\tau}{1+\delta_y+2\delta_\tau\omega_y}$, $Z_u = \frac{\partial y_j}{\partial u_j} = \frac{1+\delta_\tau\omega_y}{(1-\delta_y)(1+\delta_y+2\delta_\tau\omega_y)}$, and $Z_u^* = \frac{\partial y_j}{\partial u_k} = \frac{\delta_y+\delta_\tau\omega_y}{(1-\delta_y)(1+\delta_y+2\delta_\tau\omega_y)}$. Equation (11) defines a target variable, namely country-specific output demand, with respect to the policy instruments and exogenous shocks. It represents the constraint faced by country j's fiscal authority when solving its own minimization problem, given both the country k's fiscal authority and monetary authority's decisions. All the 'Z' parameters are country-specific output-demand elasticities (multipliers) relative to the three policy instruments (Z_i, Z_g, Z_g^*), to domestic and foreign demand shocks (Z_u, Z_u^*), and to supply shock asymmetries (Z_ε), where the latter is defined by $\varepsilon_j - \varepsilon_k$.[20] The corresponding ones that refer to the policy instruments define policy effectiveness, as long as they are different to zero.

The importance of the interconnections for those elasticities is profound. First, domestic output demand is directly affected by foreign demand shocks, by supply shock asymmetries, and by foreign fiscal

[17] Foresti (2018) stresses that vertical coordination should be of no interest in the EMU since the ECB is not allowed to coordinate its policies with the fiscal authorities. The author further points out that *'Although recent monetary policy arrangements have evidenced that under particular conditions the ECB may overcome this constraint, it does not imply that the ECB will coordinate with national governments in the policy mix'* (Foresti, 2018, p. 235). Hughes Hallett and Mavrodimitrakis (2019) explore the conditions under which the monetary authority might find it beneficial to cooperate (only) with the core member's fiscal authority in a core-periphery monetary union.

[18] We solve together the two aggregate demand equations (eq. (1)) for both countries. We then subtract the two PC equations (eq. (4)) to create $\pi_j - \pi_k$, and we incorporate the latter to both the aggregate demand equations, which we solve together.

[19] Real-world examples of information sharing can refer to the Economic and Financial Affairs Council (ECOFIN) meetings, and the fact that ECB's president may attend those meetings; and vice versa the president of ECOFIN is also present at the meetings of the governing council of the ECB (Hughes Hallett, 2005).This situation is described in the literature as narrow (or informal positive) coordination (see, e.g., Ferre, 2008; Onorante, 2004).

[20] Both the elasticities of the nominal interest rate, Z_i, and the supply shock asymmetries, Z_ε, are in absolute terms. The ones with respect to expected domestic and foreign inflation are also shown, but recall that expected inflation is equal to zero in our setting.

policy only through the interconnections. In the opposite case of $\delta_y = \delta_\tau = 0$, domestic aggregate demand is only affected by domestic demand shocks. Supply shock asymmetries are present only because of the terms-of-trade effect. Second, both domestic and foreign fiscal policy affect domestic aggregate demand. Regarding domestic fiscal policy, there is a direct positive effect, but also an indirect one through the terms of trade. The latter is positive through the output gap and ambiguous from the direct effect of fiscal policy on inflation. Moreover, the ambiguous overall impact on inflation creates a trade effect. The foreign fiscal policy affects domestic output through both the trade and the terms-of-trade effects, depending positively on the overall effect of fiscal policy on aggregate demand and inflation. Third, all the above elasticities are independent of the cost-channel effect, as the latter is assumed to be the same for the two countries. This means that the cost channel does not affect the terms-of-trade effect and aggregate demand.[21] For $\delta_y \in (0,1)$, all the 'Z' parameters are positive, apart from domestic and foreign fiscal policy's impact on domestic aggregate demand; i.e., Z_g and Z_g^*, respectively. In an attempt to capture standard reasoning in fiscal policymaking, Andersen (2005, 2008) assumes that the overall impact of country-specific fiscal policy on both country-specific inflation and the output gap is positive; i.e., $\frac{\partial \pi_j}{\partial g_j} = \omega_g + \omega_y \delta_g > 0$ and $\frac{\partial y_j}{\partial g_j} = \delta_g - \delta_\tau \omega_g > 0$. These assumptions make fiscal policy a demand-side policy instrument; and both the elasticities of domestic and foreign fiscal policy on domestic aggregate demand are now unambiguously positive.[22]

3. The General Solution at the Union Level and Relative Policy Effectiveness

The monetary authority and the national fiscal authorities have two targets but only one instrument: (i) the monetary authority controls the common nominal interest rate, i, and minimizes its loss function (eq. (7)) subject to the union-wide descriptive equations (equations (5) and (6)); and (ii) each national fiscal authority controls its fiscal stance, g_j, and minimizes its loss function (eq. (8)) subject to the country-specific aggregate demand equation (eq. (11)). Each authority manipulates the instrument under its control by equating the marginal rate of transformation with the marginal rate of substitution between the two target variables. The former is defined as the ratio of the first derivatives of the policy instruments on the target variables, while the latter as the ratio of the marginal losses, which is based on the authorities' preference

[21] Supply shock asymmetries could also represent country-specific lending rates, reflecting asymmetries to the demand for loans. Specifically, $\varepsilon_j = -\lambda z v \bar{l}_j = -\omega_i v \bar{l}_j$ and $\varepsilon_j - \varepsilon_k = -\omega_i v(\bar{l}_j - \bar{l}_k)$. See an online Appendix to this paper for further discussion.

[22] Both derivatives do not consider the interconnections. Extreme positive (negative) values of fiscal policy's direct effect on inflation, ω_g, would be required for a domestic (foreign) fiscal contraction to have expansionary effects; specifically in the former case, the country-specific fiscal stance must become a strong supply-side policy instrument, with $\omega_g > \frac{\delta_g}{1-\delta_y}\left(\frac{1}{\delta_\tau} + \omega_y\right)$. We can safely assume that both Z_g and Z_g^* are positive.

parameters.[23] Each authority's optimization program ends up with a corresponding policy rule that combines the concerned macroeconomic variables. Within each strategic regime, a cooperative fiscal regime is also considered, in which the national fiscal authorities choose their policy instruments by minimizing the joint loss function given by equation (9). Finally, in the overall policy coordination regime, all the authorities choose their policy instruments by minimizing equation (10).

The country-specific fiscal rule for the national fiscal authorities under decentralization is given by:

$$g_j = -a_F \frac{dy_j}{dg_j} y_j = -\phi_{g_j} y_j, \tag{12}$$

where $\phi_{g_j} = a_F \frac{dy_j}{dg_j}$ is the country-specific fiscal reaction parameter. The symmetry assumption for the two countries ensures that $\frac{dy_j}{dg_j} = \frac{dy_k}{dg_k}$, which suggests that the two fiscal rules are symmetric, too; hence $\phi_{g_j} = \phi_{g_k}$. The first order condition for the centralized fiscal regime is:

$$g_j + a_F \left(\frac{dy_j}{dg_j} y_j + \frac{dy_k}{dg_j} y_k \right) = 0 \tag{13}$$

Equation (12) clearly demonstrates that the national fiscal authorities change their fiscal stances only in response to a change in domestic output gap, whereas in the centralized fiscal regime, following equation (13), they also react to changes in foreign output gap. In both the simultaneous move and the monetary leadership strategic regimes, the national fiscal authorities consider the common nominal interest rate as given. This means that $\frac{dy_j}{dg_j} = \frac{\partial y_j}{\partial g_j}$ and $\frac{dy_k}{dg_j} = \frac{\partial y_k}{\partial g_j}$, both of which can be directly derived from the country-specific aggregate demand equation (11). On the contrary, in the fiscal leadership regime the national fiscal authorities consider the monetary authority's reaction function; hence $\frac{dy_j}{dg_j} = \frac{\partial y_j}{\partial g_j} + \frac{\partial y_j}{\partial i} * \frac{\partial i}{\partial g_j}$.

At the union level, the monetary/fiscal rules emerge as:

MR: $y = -\phi_\pi \pi$ (14)

FR: $g = -\phi_g y$, (15)

where '*MR*' stands for 'Monetary Rule' and '*FR*' for 'Fiscal Rule'. The two rules show how both monetary and union-wide fiscal policy react to changes in the authorities' concerned macroeconomic variables. The parameters ϕ_π and ϕ_g correspond to the monetary and the (union-wide) fiscal reaction parameters, respectively, and are functions of the model's structural ($\delta_r, \delta_\tau, \delta_y, \delta_g, \omega_y, \omega_i, \omega_g$) and preference ($a_M, a_F$) parameters, while they can be of either sign; in particular, a positive sign defines a (subjective) trade-off

[23] For each authority, the elasticity of substitution between the target variables is equal (in absolute terms) to the preference parameter (see, e.g., Acocella et. al., 2013).

between the authorities' target variables. Different combinations of strategic and fiscal regimes would generally produce differing reaction parameters.[24]

The monetary reaction parameter is given by $\phi_\pi = \frac{1}{a_M} * \frac{\frac{d\pi}{di}}{\frac{dy}{di}}$, where $\frac{d\pi}{di} = \frac{\partial \pi}{\partial y} * \frac{dy}{di} + \frac{\partial \pi}{\partial g} * \frac{\partial g}{\partial i} + \frac{\partial \pi}{\partial i}$. The first impact is the standard monetary policy one, the second is the one through fiscal policy under monetary leadership, so it disappears under either simultaneous move or fiscal leadership, since $\frac{\partial g}{\partial i} = 0$, and the third effect is the cost channel. The union-wide fiscal reaction parameter, instead, explicitly depends on the fiscal regime. For the decentralized fiscal regime, it is straightforward that $\phi_g = \phi_{g_j} = a_F \frac{dy_j}{dg_j}$, following equation (12), while for the centralized case, we get $\phi_g = a_F \left(\frac{dy_j}{dg_j} + \frac{dy_j}{dg_k} \right)$ by averaging equation (13); hence in the latter case the spill-over (trade) effect is also taken into account. A comparison yields:

$$\phi_{g_{nc}} - \phi_{g_c} = -a_F \frac{dy_j}{dg_k} \tag{16}$$

This means that, if the impact of foreign fiscal policy on domestic aggregate demand is positive (negative), then union-wide fiscal policy is more (less) reactionary in the cooperative fiscal regime. In the two leadership regimes, the follower authority's reaction parameter will be equal to the corresponding one in the simultaneous-move strategic regime, while the lead authority's, as we show, may or may not depend on the follower's reaction parameter.

At the union level, the AD equation (5) and the PC equation (6), together with the monetary rule (eq. (14)) and the fiscal rule (eq. (15)) create a $4 * 4$ system of (log)-linear equations, with unknowns being inflation, the output gap, the fiscal stance, and the common nominal interest rate. The two former variables represent the target variables, while the two latter the policy instruments. The country-specific fiscal stance, however, is simultaneously a target and an instrument, following the fiscal authorities' loss functions (eq. (8)). Introducing quadratic instrument costs makes the number of instruments lower than the number of target variables. Solving all four equations simultaneously yields the following equilibrium solutions:

$$\pi = \frac{1}{\Omega} (\omega_i u - \delta_r \varepsilon) \tag{17}$$

$$y = -\frac{\phi_\pi}{\Omega} (\omega_i u - \delta_r \varepsilon) \tag{18}$$

$$g = \frac{\phi_g \phi_\pi}{\Omega} (\omega_i u - \delta_r \varepsilon) \tag{19}$$

$$i = \frac{[1+(\omega_y - \omega_g \phi_g)\phi_\pi] u - (1 - \delta_y + \delta_g \phi_g)\phi_\pi \varepsilon}{\Omega}, \tag{20}$$

[24] It turns out that equations (14) and (15) also hold for the overall policy coordination regime.

where:

$$\Omega = \delta_r \left\{1 - \left[\left(\frac{\partial \pi}{\partial y} + \frac{\partial \pi}{\partial i} * \frac{\partial i}{\partial y}\right) + \left(\frac{\partial \pi}{\partial g} + \frac{\partial \pi}{\partial i} * \frac{\partial i}{\partial g}\right)\frac{\partial g}{\partial y}\right]\frac{\partial y}{\partial \pi}\right\} =$$

$$= \delta_r + \left[\omega_y \delta_r - \left(1 - \delta_y\right)\omega_i\right]\phi_\pi - \left(\delta_g \omega_i + \delta_r \omega_g\right)\phi_g \phi_\pi \qquad (21)$$

We call Ω the '*reference parameter*'; it 'refers' to a specific institutional arrangement, capturing differences on equilibrium solutions of the union-wide macroeconomic variables across strategic and fiscal regimes.

Following the union-wide equilibrium solutions, namely equations (17) – (20), we can make two initial observations. Firstly, the cost channel of monetary policy makes the full stabilization of union-wide demand shocks at the union level unattainable. In the opposite case that a cost channel of monetary policy does not exist, namely $\omega_i = 0$, the union-wide demand shocks are fully stabilized at the union level. Countering demand shocks pushes both the output gap and inflation in the same direction, since there is no trade-off between those two; thus, the monetary authority succeeds in fully stabilizing demand shocks and fiscal policy at the union level is passive ($g = 0$). This is the *'divine coincidence'* property of the standard closed-economy New-Keynesian model (Blanchard and Gali, 2007), which illustrates the optimality of the strict inflation-targeting monetary policy framework (see, also, Clarida et. al, 1999). That is, keeping inflation stable is a way of keeping output at its potential level. By contrast, the cost channel of monetary policy creates a trade-off between inflation and the output gap, even in the absence of supply shocks. Then, the union-wide fiscal stance is not passive, and reacts as it would react to (negative) supply shocks.[25]

Secondly, all the union-wide macroeconomic variables are affected by union-wide (demand/supply) shocks and not by shock asymmetries; hence idiosyncratic shocks are fully stabilized at the union level. Following the equilibrium solutions for the two policy instruments, equations (19) and (20), neither of them depends on shock asymmetries. In the case of idiosyncratic shocks, regardless of the ordering of moves, the two national fiscal authorities respond in exactly the opposite way, since the two countries are identical; hence, their responses cancel out at the union level. The common central bank does not react in the simultaneous move and the fiscal leadership strategic regimes, since there is no average shock to the monetary union, whereas in the case of monetary leadership it anticipates that the reactions of the fiscal authorities will be offset.

The aggregate descriptive equations (5) and (6) can be expressed in reduced form in matrix terms as:

[25] Andersen (2008) considers shocks that are not pure, since they can simultaneously affect demand and supply in various ways; but pure demand shocks emerge as a special case. Demand shocks are partially stabilized in models that assume interest-rate smoothing on the part of the monetary authority, following Woodford (2003), since the monetary reaction to shocks is now milder (see, e.g., Beetsma et. al., 2001; Oros and Zimmer, 2015).

$$\begin{matrix}\pi \\ y\end{matrix} = \begin{matrix}-\frac{\omega_y\delta_r}{1-\delta_y}+\omega_i & \frac{\omega_y\delta_g}{1-\delta_y}+\omega_g \\ -\frac{\delta_r}{1-\delta_y} & \frac{\delta_g}{1-\delta_y}\end{matrix} * \begin{matrix}i \\ g\end{matrix} + \begin{matrix}\frac{\omega_y}{1-\delta_y}u & -\varepsilon \\ \frac{1}{1-\delta_y}u & 0\end{matrix}, \tag{22}$$

where we can call $J = \begin{bmatrix}-\frac{\omega_y\delta_r}{1-\delta_y}+\omega_i & \frac{\omega_y\delta_g}{1-\delta_y}+\omega_g \\ -\frac{\delta_r}{1-\delta_y} & \frac{\delta_g}{1-\delta_y}\end{bmatrix}$ the *Jacobian* matrix (see Acocella et. al., 2013). It is a matrix of multipliers, in which each element indicates the effect on a target variable of changes in the corresponding policy instrument; e.g., $\frac{d\pi}{dg} = \frac{\partial\pi}{\partial y} * \frac{\partial y}{\partial g} + \frac{\partial\pi}{\partial g} = \frac{\omega_y\delta_g}{1-\delta_y} + \omega_g$. The former effect is the one through aggregate demand and the latter is the direct effect. Since the trade effect works as a multiplier, it enhances the demand-side effects of the policy instruments. If no element of the matrix J is equal to zero, this assumes policy effectiveness of the specific policy instrument on the corresponding target variable. We further define a marginal rate of transformation between union-wide inflation and output gap with respect to the two policy instruments using the above reduced-form system described by equation (22). The two marginal rates of transformation are given by:

$$MRT^i_{y\pi} = -\frac{d\pi}{dy} = \frac{\frac{\partial\pi}{\partial i}}{\frac{\partial y}{\partial i}} = \frac{\omega_y\left(-\frac{\delta_r}{1-\delta_y}\right)+\omega_i}{-\frac{\delta_r}{1-\delta_y}} = \omega_y - \frac{(1-\delta_y)\omega_i}{\delta_r} \tag{23}$$

$$MRT^g_{y\pi} = -\frac{d\pi}{dy} = \frac{\frac{\partial\pi}{\partial g}}{\frac{\partial y}{\partial g}} = \frac{\omega_y\left(\frac{\delta_g}{1-\delta_y}\right)+\omega_g}{\frac{\delta_g}{1-\delta_y}} = \omega_y + \frac{(1-\delta_y)\omega_g}{\delta_g}, \tag{24}$$

where $MRT^i_{y\pi}$ and $MRT^g_{y\pi}$ define the marginal rates of transformation between union-wide inflation and output gap for monetary and union-wide fiscal policy, respectively. The latter refers to the union-wide fiscal stance as the outcome of the game between the two national fiscal authorities, described by the specific fiscal regime.[26] Each marginal rate of transformation measures the impact on inflation of a marginal change in the output gap induced by the corresponding authority that controls the specific policy instrument; i.e., the gradient of the inflation to output-gap transformation as a result of changes in each policy instrument.

The rank of matrix J is of particular importance. In the special case of $rank[J] = 1$, the two policy instruments are linearly dependent, so they are perfect substitutes in the stabilization process, and they share the same marginal rate of transformation. If we further assume that the two policy instruments cannot directly affect inflation, i.e. $\omega_g = \omega_i = 0$, then $MRT^i_{y\pi} = MRT^g_{y\pi} = \omega_y$ and the PC equation (6) describes a transformation curve (linking the one target variable to the other). In the general case, instead, of

[26] Since the national fiscal authorities do not care about country-specific inflation, following their loss function (eq. (8)), they do not face the trade-off described by equation (24), not even at the union level.

$rank[J] = 2$, the marginal rates of transformation differ, and there is imperfect instrument substitutability. We can state the following definition.

Definition 1: *Perfect instrument substitutability requires* $rank[J] = 1$*; which holds for* $MRT^{i}_{y\pi} = MRT^{g}_{y\pi}$. *Imperfect instrument substitutability requires* $rank[J] = 2$*, hence* $MRT^{i}_{y\pi} \neq MRT^{g}_{y\pi}$.

The impact of fiscal/monetary policies on country-specific inflation is ambiguous, following $\frac{\partial \pi_j}{\partial g_j} = \omega_g + \omega_y \delta_g$ and $\frac{\partial \pi_j}{\partial i} = \omega_i - \delta_r \omega_y$ for fiscal and monetary policy, respectively. A positive sign for the former derivative and a negative sign for the latter indicate that the policy instruments are demand-side and both the marginal rates of transformation are positive (so, the two macroeconomic variables move together); and vice versa for supply-side policy instruments.[27] When the marginal rates of transformation differ in sign, the two policy instruments are complements; then, a possible equality in absolute terms implies perfect complementarity. The relation between the two marginal rates of transformation assumes the degree of substitutability between the policy instruments, and defines relative policy effectiveness; i.e., how much more effective is one policy instrument in stabilizing the one target variable relative to the other than the other policy instrument. It turns out that relative policy effectiveness combined with the cost channel of monetary policy and leadership regimes define policies' cyclicality and coordination problems, hence the policy-mix outcome in the monetary union. We can state the following definition.

Definition 2: *Relative policy effectiveness is defined as the effectiveness of one policy instrument in stabilizing one target variable relative to the other (target variable) compared to the other policy instrument. If* $MRT^{i}_{y\pi} > MRT^{g}_{y\pi}$*, then monetary policy is more effective in stabilizing inflation (relative to aggregate demand; or the output gap) than (union-wide) fiscal policy; and vice versa for* $MRT^{i}_{y\pi} < MRT^{g}_{y\pi}$. *If* $MRT^{i}_{y\pi} = MRT^{g}_{y\pi}$*, then the two policy instruments are equally effective.*

Comparing the two marginal rates of transformation, namely equations (23) and (24), produces the following lemma.

Lemma 1: *For* $MRT^{i}_{y\pi} \gtreqless MRT^{g}_{y\pi}$ *it holds that* $\delta_g \omega_i + \delta_r \omega_g \lesseqgtr 0$.

Proof: Following equations (23) and (24), we get:

[27] The derivatives exclude the interconnections. The overall impact of fiscal policy on aggregate demand, $\frac{\partial y_j}{\partial g_j}$, should be positive for demand-side fiscal policy. The assumption that $\frac{\partial \pi_j}{\partial i} < 0$ means that monetary policy has the usual overall effect upon inflation. It further implies that a price puzzle does not exist, being consistent with empirical evidence (see, e.g., Rabanal, 2003). Note that the marginal rates of transformation are expressed in terms of union-wide variables, but the same reasoning holds for $\delta_y \in (0,1)$.

$$MRT^{i}_{y\pi} \gtreqless MRT^{g}_{y\pi} \Rightarrow \omega_y - \frac{(1-\delta_y)\omega_i}{\delta_r} \gtreqless \omega_y + \frac{(1-\delta_y)\omega_g}{\delta_g} \Rightarrow -\frac{\omega_i}{\delta_r} \gtreqless \frac{\omega_g}{\delta_g} \Rightarrow \frac{\omega_g}{\delta_g} + \frac{\omega_i}{\delta_r} \lesseqgtr 0 \Rightarrow \frac{\delta_g\omega_i+\delta_r\omega_g}{\delta_g\delta_r} \lesseqgtr 0 \Rightarrow \delta_g\omega_i + \delta_r\omega_g \lesseqgtr 0,$$

where the conditions for ω_g and the links to demand/supply-side policy instruments can be found in the Appendix A.1. ■

The intuition behind **Lemma 1** is as follows: If fiscal policy directly affects inflation positively, then a change in fiscal policy has an additional effect on inflation. If, moreover, there is a cost channel of monetary policy, then the impact of the nominal interest rate on inflation is moderated, since the cost channel works in the opposite direction; e.g., a higher interest rate reduces inflation through its impact on aggregate demand, but increases inflation through the cost channel. This renders fiscal policy more effective in stabilizing inflation (relative to output gap) than monetary policy. Allowing, instead, fiscal policy to have a negative direct effect on inflation allows for the two policy instruments to have the same qualitative impact on both macroeconomic variables, namely inflation and the output gap. In particular, an expansionary (contractionary) fiscal/monetary policy directly increases (decreases) the output gap and directly decreases (increases) inflation. Then, the exact values of the structural parameters matter for defining relative policy effectiveness, while there is a special case where the two policy instruments can become perfect substitutes; so, they are linearly dependent. Moreover, the interconnections play no role in defining relative policy effectiveness; although the trade effect enhances the demand-side effects of the policy instruments. Naturally, a strong cost-channel effect makes monetary policy less effective than fiscal policy in tackling inflation relative to aggregate demand.

4. The Policy Mix under Alternative Institutional Arrangements

What determines policies' (counter)-cyclicality? Is it possible for policies to be pro-cyclical, and, if so, what induces this pro-cyclical behavior? What is the role of policies' relative effectiveness and of the authorities' preferences for policies' cyclicality and coordination? To answer those questions, we explore the policy mix in the monetary union for different combinations of the strategic and the fiscal regimes. The analysis asks for a comparison of the fiscal/monetary reaction parameters, presented in Table 1.[28]

[Insert Table 1]

[28] Details on the construction of Table 1 can be found in an online Appendix to this paper.

4.1 The Simultaneous-Move Strategic Regime

The monetary reaction parameter now includes an additional negative effect, given by $-\frac{1-\delta_y}{\delta_r}\omega_i$. This presents the relative effect of monetary policy to stabilize inflation relative to output gap, where the negative sign reveals the effect's opposite direction; i.e., an increase in the monetary policy instrument directly increases inflation but decreases the output gap. The cost channel makes the monetary policy less reactionary, since it reduces its effectiveness on union-wide inflation; but this effect decreases (in absolute terms) with the effect of the interest rate on the union-wide aggregate demand. Hence both the trade effect, δ_y, and the interest sensitivity of aggregate demand, δ_r, now affect the monetary reaction parameter positively, lessening the cost-channel effect. If the effect through the output gap prevails, then the monetary reaction parameter is positive and represents the standard trade-off between inflation and output gap. The cost-channel effect needs to be large enough for the monetary reaction parameter to become negative; so, the common nominal interest rate would become a supply-side policy instrument at the union level[29], and the monetary authority would choose not to trade-off its objectives. In the case of a positive union-wide supply shock that decreases inflation at the union level, e.g., the monetary authority increases the nominal interest rate to increase inflation, since the negative impact through the output gap is dominated. Naturally, the monetary reaction parameter is exactly the same for both fiscal regimes and with the corresponding one for the fiscal leadership regime, while it differs from the one for the overall policy coordination regime only in the weight that the authorities who set policy place on output-gap stabilization relative to inflation, namely $a_F + a_M$, rendering monetary policy less reactionary.

The union-wide fiscal stance ends up countercyclical in both fiscal regimes. In the decentralized fiscal regime, the national fiscal authorities respond only to changes in domestic output gap, following equation (12); hence the fiscal reaction parameter depends positively on the overall impact of domestic fiscal policy on aggregate demand, Z_g, which is substantially affected by the trade externalities (see eq. (11)). Each fiscal authority tries to exploit the terms-of-trade effect to gain in competitiveness vis-a-vis the other country. This channel works through the output gap, but also through fiscal policy's direct impact upon inflation (see Andersen, 2005, 2008). In the special case that this latter channel does not exist, i.e. for $\omega_g = 0$, the terms of trade affects the fiscal reaction parameter negatively, since it works as an automatic stabilizer (see, e.g., Ferre, 2012; Landmann, 2018; Hughes Hallett and Mavrodimitrakis, 2019).

In the centralized fiscal regime, following equation (13), the national fiscal authorities also react to changes in foreign output gap in a countercyclical manner; hence the externalities are internalized (see, e.g.,

[29] The monetary policy's impact on aggregate demand is multiplied through the trade effect; this does not happen with the cost-channel effect, since the terms-of-trade effect vanishes.

Uhlig, 2003; Andersen, 2005). At the union level, the fiscal reaction parameter equals the union-wide fiscal multiplier, $\frac{\delta_g}{1-\delta_y}$, multiplied by the weight, a_F. The fiscal authorities' responses to output changes cancel out, so their joint reaction has the same result with the effect of fiscal policy upon the union-wide output gap. Comparing the two fiscal reaction parameters, following equation (16), (union-wide) fiscal policy is more countercyclical under fiscal cooperation, since $\phi_{g_c}^{SM} - \phi_{g_{nc}}^{SM} = a_F Z_g^* > 0$. By cooperating with each other, the national fiscal authorities succeed in strengthening their strategic position relative to the common central bank (see, e.g., Beetsma and Bovenberg, 1998). In the decentralized fiscal regime, instead, their attempt to exploit one another is completely inefficient, weakening their strategic position. There is a free-rider problem, since the fiscal policy instrument is costly for the national fiscal authorities (Uhlig, 2003).

4.2 The Leadership Strategic Regimes and Coordination Problems

In the leadership strategic regimes, relative policy effectiveness and the cost-channel effect together define policies' cyclicality and coordination problems. We have already discussed that the overall effects of both domestic and foreign fiscal policies on domestic aggregate demand are both positive, following the country-specific aggregate demand equation (11). A strong cost-channel effect, however, may turn the monetary policy instrument into a supply-side one at the union level. In this case, the monetary reaction parameter is negative and monetary policy is less effective in stabilizing inflation (relative to aggregate demand) than fiscal policy, even if fiscal policy directly affects inflation negatively. We can establish the following result and propositions.

Result 1: *Under imperfect instrument substitutability, the lead authority (i) reacts to the follower authority's reaction parameter, hence to the follower's preference parameter; and (ii) might choose not to trade-off its objectives (for specific parameterizations).*

Proof: See Appendix A.2. ■

Result 1 states that the two policy instruments must be imperfect substitutes at the union level for the lead authority to take advantage of the follower; so, becoming less reactionary to induce a moderate reaction. If the anticipated reaction is strong enough, then the lead authority might choose not to trade-off its objectives. We proceed with the two leadership regimes.

Proposition 1: *Consider the fiscal leadership regime and allow the monetary policy instrument to be demand-sided at the union level (for a small cost-channel effect). If monetary policy is less effective in stabilizing inflation (relative to aggregate demand) than (union-wide) fiscal policy, then:*

(i) the fiscal reaction parameter depends positively on the monetary authority's preference on output-gap stabilization (relative to inflation); and vice versa;

(ii) the monetary authority reduces (exacerbates) the horizontal coordination problem by pursuing a more (less) flexible inflation-targeting framework of monetary policy; and vice versa.

If the cost-channel effect is strong enough to turn the monetary policy instrument into a supply-side one at the union level, monetary policy becomes less effective in stabilizing inflation (relative to aggregate demand) than fiscal policy. Then:

(i) the fiscal reaction parameter depends negatively on the monetary authority's preference on output-gap stabilization (relative to inflation);

(ii) the monetary authority exacerbates (reduces) the horizontal coordination problem by pursuing a more (less) flexible inflation-targeting framework of monetary policy.

Proof: See Appendix A.2. ■

In the fiscal leadership strategic regime, the national fiscal authorities take into account the monetary authority's reaction function. The parameter V_i defines the reaction of the common nominal interest rate to a possible change in the average fiscal stance, namely $V_i = \frac{\partial i}{\partial g}$; hence, it defines the slope of the reaction function. This can be of either sign; it can become negative if the one policy instrument is demand-sided and the other one supply-sided at the union level. The fiscal reaction parameter for the decentralized fiscal regime, given by $Z_g - \frac{1}{2} Z_i V_i$, is of ambiguous sign, which means that fiscal policy can be either countercyclical (positive sign) or pro-cyclical (negative sign). Same-side (demand/supply) policy instruments induce a substitutability effect, hence reducing the fiscal reaction parameter in comparison to the one in the simultaneous-move regime; i.e., $\phi_{g_{nc}}^{SM} - \phi_{g_{nc}}^{FL} = \frac{1}{2} Z_i V_i > 0$. The national fiscal authorities anticipate the monetary reaction, hence becoming less countercyclical. If the fiscal authorities anticipate that the impact of fiscal policy on country-specific output gap will be larger than the one of the monetary policy's response, then the fiscal reaction parameter will be positive and fiscal policy will be unambiguously countercyclical (Andersen, 2008); however, a pro-cyclical behavior can be the outcome of a strong monetary response.[30] If, instead, the cost-channel effect is strong enough, and the fiscal policy instrument is demand-sided, then the two policy instruments might become complements; hence fiscal

[30] Empirical evidence exists for pro-cyclical fiscal policies within the EMU (see, e.g., Fatas and Mihov, 2010; Candelon et al., 2010; and recently Gootjes and de Haan, 2020). Possible explanations are discussed in Bartsch et. al. (2020). The lack of counter-cyclicality may have rendered the euro-zone more vulnerable in the run-up to the Great Recession (Benetrix and Lane, 2013).

policy might become more countercyclical. Under perfect instrument substitutability, the fiscal reaction parameter is unambiguously positive and independent on the monetary reaction parameter.

The monetary authority's preference parameter affects the monetary reaction parameter, which affects the monetary response. In the fiscal leadership regime, this would be taken into consideration from the national fiscal authorities, making them less/more reactionary. **Proposition 1(i)** clearly shows that the way the inflation-targeting framework that the monetary authority follows affects fiscal policy's cyclicality depends on relative policy effectiveness and the cost-channel effect. A small cost-channel effect when the fiscal policy's instrument directly affects inflation positively generates a milder monetary response for a more flexible inflation-targeting monetary policy framework, inducing a larger fiscal reaction parameter to induce a stronger monetary response.

In the cooperative fiscal regime, under perfect instrument substitutability, the union-wide fiscal stance is passive. The monetary authority's response exactly offsets the impact of the union-wide fiscal stance on the union-wide output gap, which equals the union-wide fiscal multiplier, $\frac{\delta_g}{1-\delta_y}$ (see, e.g., Ferre, 2008). Like before, fiscal policy in the simultaneous-move strategic regime is more countercyclical than in the fiscal leadership regime, since $\phi_{g_c}^{FL} = \phi_{g_c}^{SM} - a_F Z_i V_i \Rightarrow \phi_{g_c}^{FL} < \phi_{g_c}^{SM}$ for $V_i > 0$. Comparing the two (union-wide) fiscal reaction parameters for the two fiscal regimes, following equation (16), we get:

$$\phi_{g_{nc}}^{FL} - \phi_{g_c}^{FL} = -a_F \left(Z_g^* - \frac{1}{2} Z_i V_i \right) \tag{25}$$

Their difference depends on the effect of foreign fiscal policy on domestic aggregate demand minus the monetary response. If the former (latter) effect prevails, then the union-wide fiscal stance for the centralized fiscal regime ends up more (less) countercyclical; or less (more) pro-cyclical. In Andersen (2008), equation (25) demonstrates the horizontal coordination problem. Each fiscal authority only perceives a fraction of its fiscal decision on the common monetary policy, here half since we use a two-country model, while in the cooperative case the authorities consider the aggregate nature of the shock and the implied monetary response. Thus, the decentralized case delivers an inefficiency in fiscal policymaking.

The common central bank can reduce the horizontal coordination problem, since in the fiscal leadership strategic regime, in contrast to the simultaneous-move one, the monetary authority's preference parameter, a_M, affects the fiscal reaction parameter under imperfect instrument substitutability (**Result 1**). **Corollary 1** directly emerges from **Proposition 1(ii)**.

Corollary 1: *In the fiscal leadership strategic regime, the monetary authority affects the horizontal coordination problem as long as the two policy instruments are imperfect substitutes in the stabilization process; i.e.,* $MRT_{y\pi}^{i} \neq MRT_{y\pi}^{g}$.

Proposition 1(ii) is consistent with a generalization of the corresponding result by Andersen (2008) when there is a cost channel of monetary policy, too. A more flexible inflation-targeting monetary policy framework means a less steep trade-off between inflation and output gap, hence a milder monetary response to a fiscal expansion. In Andersen (2008) this reduces the difference between the cooperative and the non-cooperative policy, since there is no cost channel of monetary policy and the author assumes a positive direct effect of fiscal policy on inflation. We, instead, show that this depends on the cost-channel effect. This holds if monetary policy is less effective in stabilizing inflation (relative to aggregate demand) than fiscal policy when the cost channel is not too large. But a large cost-channel effect overturns this result; hence a less flexible inflation-targeting monetary policy framework is needed.[31]

Proposition 2: *In the monetary leadership regime, the relative effectiveness of fiscal-monetary policies fully determines the monetary policy's reaction to fiscal authorities' preferences, its intensity compared to that in the simultaneous-move regime and between fiscal regimes, and the relevance of the specific fiscal regime for the monetary authority. Specifically, if monetary policy is less effective in stabilizing inflation (relative to aggregate demand) than fiscal policy, then:*

- *(i) the monetary reaction parameter is negatively related to the fiscal authorities' preference on output-gap stabilization (relative to the fiscal stance); and vice versa;*
- *(ii) monetary policy is less reactionary compared to the simultaneous-move strategic regime, and for the centralized fiscal regime, compared to the decentralized one; and vice versa;*
- *(iii) an increase in the relative preference for the fiscal authorities makes the specific fiscal regime more relevant for the monetary authority; and vice versa.*

Proof: See Appendix A.2. ■

The monetary reaction parameter is the same across strategic and fiscal regimes for perfect instrument substitutability. Furthermore, if there is no cost-channel effect (so, no direct effect of fiscal policy on inflation, too), then the standard monetary policy trade-off emerges (see, e.g., Clarida et. al., 1999). The cost-channel effect makes monetary policy less reactionary, reducing the monetary authority's subjective trade-off. In the monetary leadership regime, if monetary policy is less effective in stabilizing inflation (relative to aggregate demand) than fiscal policy, then it becomes even less reactionary. When, e.g., the cost-channel effect is large or fiscal policy directly affects inflation positively, the negative effect is reinforced from the fiscal reaction. Monetary policy may even become expansionary in response to an inflation increase, since this triggers a countercyclical (contractionary) reaction from the national fiscal

[31] An online Appendix to this paper shows the exact value of the monetary authority's preference parameter needed to eliminate the horizontal coordination problem.

authorities, which is enhanced by the positive direct effect of fiscal policy on inflation; more so in the centralized fiscal regime, since the union-wide fiscal policy is more countercyclical. Moreover, the fiscal authorities' preference parameter matters. If the fiscal authorities care more about aggregate demand, then the monetary authority can be less reactionary since the fiscal authorities are more effective on inflation; and this requires them to induce more effort to stabilize aggregate demand. Thus, the fiscal regimes matter for the monetary authority, while the stronger is the fiscal authorities' preference for output-gap stabilization (relative to the monetary authority's) the larger is the monetary reaction parameter's difference between the two fiscal regimes. A more policy-concerned fiscal authority makes the fiscal regimes less relevant for the monetary authority. Combining **Propositions 1** and **2**, we can produce **Corollary 2**.

Corollary 2: *In the leadership strategic regimes, for demand-side policy instruments, the way the lead authority reacts to the follower's preference parameter depends entirely on relative policy effectiveness. The leader reacts positively, if its instrument of control is more effective in stabilizing inflation (relative to aggregate demand) than the follower's policy instrument; and vice versa.*

The intuition behind **Corollary 2** is the following: Recall that the preference parameters, a_M and a_F, denote the weight put on output-gap stabilization. Then, if the leader's policy instrument is less effective in aggregate demand, this makes the follower's policy instrument more effective in aggregate demand. Then, the increase in the latter's preference parameter for aggregate demand decreases its response to the leader's. The leader takes this into account and becomes more reactionary to increase the follower's reaction.

4.3 Explicit vs. Implicit Policy Coordination

The strategic regimes of overall policy coordination and fiscal leadership for centralized fiscal policies deliver similar fiscal reaction parameters, following Table 1. We can establish the following proposition.

Proposition 3: *In both the fiscal leadership strategic regime under fiscal authorities' cooperation and the overall policy coordination strategic regime:*

(i) *if monetary policy is less effective in stabilizing inflation (relative to aggregate demand) than fiscal policy (for a low cost-channel effect), then the union-wide fiscal stance is pro-cyclical; and vice versa;*

(ii) *if the cost-channel effect is strong enough to turn the monetary policy instrument into a supply-side one at the union level, then monetary policy becomes less effective in stabilizing inflation (relative to aggregate demand) than fiscal policy, and the union-wide fiscal stance is countercyclical;*

(iii) if monetary policy is equally effective in stabilizing inflation (relative to aggregate demand) to fiscal policy, then the union-wide fiscal stance is passive.

Proof: See Appendix A.2. ■

In the overall policy coordination regime, the union-wide fiscal reaction parameter is positively related to the monetary reaction parameter ($\frac{\partial \phi_g}{\partial \phi_\pi} > 0$) when monetary policy is less effective in stabilizing inflation (relative to aggregate demand) than fiscal policy ($MRT^i_{y\pi} < MRT^g_{y\pi}$). This means that the union-wide fiscal policy supplements monetary policy in the stabilization process. Moreover, the higher the authorities' joint weight on output-gap stabilization, namely $a_F + a_M$, and/or the cost-channel effect, ω_i, the stronger is the pro-cyclicality of fiscal policy. In the special case that both policies are equally effective in stabilizing inflation (relative to aggregate demand), namely $MRT^i_{y\pi} = MRT^g_{y\pi}$, which renders the two policy instruments perfect substitutes in the stabilization process, then the union-wide fiscal stance is passive and monetary policy takes all the burden of stabilizing the cycle. This is so, since fiscal policy is costly. Finally, none of the reaction parameters depends on the terms-of-trade effect, δ_τ, since the latter is not being exploited by the fiscal authorities when they cooperate.

5. Union-wide and Country-specific Equilibrium Solutions

The equilibrium solutions for the union-wide macroeconomic variables (equations (17)-(20)) also require the computation of the reference parameter, Ω, following equation (21). This is also presented in Table 1. What is essentially provided is a map of all possible equilibrium solutions for all possible combinations of strategic and fiscal regimes under various shocks. Under perfect instrument substitutability, the reference parameter is unambiguously positive and the same for all the strategic and fiscal regimes. In the case of imperfect instrument substitutability, however, the reference parameter differs and can even become negative, apart from the strategic regimes of overall policy coordination and of fiscal leadership under fiscal authorities' cooperation where it is unambiguously positive.

Consider the standard case of a positive reference parameter (see, e.g., Ferre, 2008, 2012; among others). Under imperfect instrument substitutability, a positive reference parameter can unambiguously emerge, e.g., for the simultaneous-move strategic regime if: (i) there is a demand-side monetary policy instrument (small cost-channel effect) and monetary policy is more effective in stabilizing inflation (relative to aggregate demand) than fiscal policy (there should be a strong enough negative direct effect of fiscal policy on inflation); or (ii) there is a supply-side monetary policy instrument (large cost-channel effect).

Union-wide inflation is positively related to demand shocks and negatively related to supply shocks, for all strategic and fiscal regimes. Thus, a positive (negative) country-specific demand (supply) shock that has union-wide consequences leaves inflation higher than before, hence being partially stabilized. The union-wide output gap depends on the monetary authority's optimal choice. If there is a subjective trade-off, then it is negatively related to demand shocks and positively related to supply shocks. This works when the monetary policy instrument is demand-sided at the union level (for a small cost-channel effect). Lastly, a similar analysis holds for the union-wide fiscal stance. If fiscal policy is countercyclical, then with a demand-side monetary policy instrument the union-wide fiscal stance follows inflation. In the leadership regimes, we have seen that the lead authorities' might choose not to trade-off their objectives (**Result 1**). This will make the output gap follow inflation in the monetary leadership strategic regime, and the union-wide fiscal stance to follow the output gap in the fiscal leadership regime; hence a pro-cyclical union-wide fiscal stance. A negative reference parameter, however, delivers the opposite result. In general, this can emerge for positive direct effects of the two policy instruments on inflation.

Result 2: *Under perfect instrument substitutability, neither the strategic nor the fiscal regimes matter for pure cyclical macroeconomic stabilization at the union level; i.e., there is controllability by the monetary authority, being indifferent to alternative strategic and fiscal regimes. Under imperfect instrument substitutability, however, the system is no longer controllable by the monetary authority, and alternative strategic and fiscal regimes become relevant.*

Proof: See Appendix A.2. ■

Result 2 modifies a result by Kempf and von Thadden (2013), since we show that when the national fiscal authorities care about their fiscal stance and not about inflation, coordination and commitment irrelevance holds if the two policy instruments are perfect substitutes in the stabilization process. Utilizing a standard New-Keynesian framework, in which policies are assumed to work on the demand side, the irrelevance result holds even under strategically significant (direct/indirect) spill-over effects. In this setting, the central bank can always achieve its (optimal) trade-off, although not achieving its bliss points. This means that the system in terms of inflation and output gap at the union level is controllable by the common central bank. In this case, the two targets are dependent, since $rank[J] = 1$. Thus, in the absence of supply shocks, the monetary authority ends up with one instrument and one target. The bliss points are only achieved when no policy instrument directly affects inflation in the absence of supply shocks.

In the case of perfect instrument substitutability, the (union-wide) fiscal reaction parameter, ϕ_g, does not affect the reference parameter, Ω (eq. (21)), hence neither the equilibrium inflation nor the output gap, given by equations (17) and (18), respectively. It affects, however, both the union-wide fiscal stance and

the common nominal interest rate, following equations (19) and (20), respectively.[32] This represents the '*deficit-bias result*' which holds for all the strategic regimes (see, e.g., Beetsma and Bovenberg, 1998; Uhlig, 2003; Andersen, 2008; among others). Naturally, the fiscal reaction parameter is a function of the fiscal authorities' preference parameter, a_F. Thus, the fact that the fiscal authorities' preference parameter cannot affect the equilibrium union-wide output gap and inflation corresponds to (endogenous, union-wide) fiscal policy neutrality (see Acocella et. al., 2013). The national fiscal authorities determine the distribution of the union-wide output gap between them; hence the country-specific and union-wide fiscal stance. This is not the case when the policy instruments are imperfect substitutes.[33]

Finally, we discuss the country-specific equilibrium solutions (expressed in relative terms). A straightforward manipulation of the country-specific descriptive equations (1) and (4), or the reduced-form aggregate demand equation (11), gives the following equation that holds for all strategic and fiscal regimes:

$$y_j - y_k = \left(Z_g - Z_g^*\right)\left(g_j - g_k\right) + 2Z_\varepsilon\left(\varepsilon_j - \varepsilon_k\right) + \left(Z_u - Z_u^*\right)\left(u_j - u_k\right), \tag{26}$$

where $Z_g - Z_g^* = \frac{\delta_g - 2\delta_\tau\omega_g}{1+\delta_y+2\delta_\tau\omega_y}$ and $Z_u - Z_u^* = \frac{1}{1+\delta_y+2\delta_\tau\omega_y} > 0$. Equation (26) simply demonstrates that country-specific equilibrium solutions for output gap differ if there are shock asymmetries and/or differences in the fiscal stances. Supply shock asymmetries create a wedge in the relative output gap, since they positively affect country-specific aggregate demand through the terms of trade, making domestic economy more competitive. We combine equation (26) with the country-specific fiscal rules for each strategic and fiscal regime to obtain the equilibrium solutions. These will differ across fiscal regimes.

For the decentralized fiscal regime for all the alternative strategic regimes, combining the country-specific fiscal rule equation (12), we get:

$$y_j - y_k = \frac{2Z_\varepsilon}{1+\phi_g\left(Z_g - Z_g^*\right)}\left(\varepsilon_j - \varepsilon_k\right) + \frac{Z_u - Z_u^*}{1+\phi_g\left(Z_g - Z_g^*\right)}\left(u_j - u_k\right) \tag{27}$$

Equation (27) shows that the two country-specific output gaps differ if there are asymmetric demand or supply shocks. For the centralized fiscal regime,[34] we obtain:

$$y_j - y_k = \frac{2Z_\varepsilon}{1+a_F\left(Z_g - Z_g^*\right)^2}\left(\varepsilon_j - \varepsilon_k\right) + \frac{Z_u - Z_u^*}{1+a_F\left(Z_g - Z_g^*\right)^2}\left(u_j - u_k\right), \tag{28}$$

[32] This can be easily verified by combining the aggregate PC equation (6) with the monetary rule (eq. (14)), for $\delta_g\omega_i + \delta_r\omega_g = 0$.
[33] In Hughes Hallett and Mavrodimitrakis (2019), this is not the case when there is a lead fiscal authority that cooperates with the monetary authority, and another fiscal authority follows.
[34] We use the country-specific fiscal rules given by equation (13). In particular, for the fiscal leadership strategic regime, the fiscal rule is given by $g_j = -a_F\left(Z_g y_j + Z_g^* y_k - Z_i V_i y\right)$; see also equation (D.2) in an online Appendix to this paper. The country-specific equilibrium solution for the overall policy coordination regime can be found in an online Appendix to this paper.

where the sign of the denominator is unambiguous. Thus, the two country-specific output gaps differ if there are shock asymmetries in all strategic and fiscal regimes. We can establish the following result, excluding the overall policy coordination regime.

Result 3: *For common shocks, the country-specific macroeconomic variables are equal to the union-wide ones for all strategic and fiscal regimes. Under perfect instrument substitutability, the values for inflation and output gap are the same across all strategic and fiscal regimes. For demand shocks, these are also equal to their target (long-run equilibrium) values if the two policy instruments cannot directly affect inflation.*

Proof: See Appendix A.2. ■

Result 3 is a generalization of standard results in the literature for imperfect instrument substitutability (see, e.g., Dixit and Lambertini, 2001, 2003a; Foresti, 2018). The existence of a cost channel of monetary policy makes the equilibrium solutions under demand shocks differ across strategic and fiscal regimes. The country-specific equilibrium solutions, described by equations (27) – (28), further demonstrate that even in the case of perfect instrument substitutability, monetary policy affects the country-specific output demand, since the latter is affected by the monetary preference parameter, a_M; in particular, $y_j = y_j\left(y\left(\phi_\pi(a_M)\right)\right)$. However, this does not work in the case of idiosyncratic shocks, since $y = 0$.

6. Conclusion

We consider the monetary-fiscal policy mix, policies' cyclicality, and coordination in a monetary union under imperfect instrument substitutability and in a strategic context. We use a static representation of a two-country monetary union model based on the New-Keynesian framework to study the strategic interactions of the monetary authority and the national fiscal authorities under alternative strategic regimes and forms of fiscal authorities' cooperation. The model allows for direct effects of fiscal/monetary policies on inflation and significant spill-over effects between the two national fiscal policies in a regime of policy conflict. We compare our results with the standard case of perfect instrument substitutability. At the union level, the presence of a cost channel of monetary policy limits union-wide demand shocks' stabilization, while the deficit-bias result does not hold, and fiscal policy becomes active. There is no system controllability on the part of the monetary authority, and no coordination and commitment irrelevance; hence alternative strategic and fiscal regimes produce differing policy-mix outcomes. The lead authority reacts to the follower authority's reaction parameter, hence to the follower's preference parameter, while it

might choose not to trade-off its objectives. In the fiscal leadership strategic regime, the monetary authority affects the horizontal coordination problem.

Our results are tied through the policies' relative effectiveness in stabilizing inflation (relative to aggregate demand), expressed by the marginal rates of transformation of the two policy instruments, and by demand/supply-side policy instruments, especially regarding monetary policy. If both policy instruments affect inflation positively, then fiscal policy becomes more effective in stabilizing inflation than monetary policy, while if fiscal policy's effect is negative, then monetary policy becomes more effective, unless a strong cost-channel effect is present. Focusing on demand-side policy instruments, we show that relative policy effectiveness entirely defines: (i) the lead authority's reaction to the follower authority's preference parameter; (ii) the monetary policy's reaction compared to the simultaneous-move strategic regime and between the two fiscal regimes, in the monetary leadership strategic regime; (iii) the cyclicality of the union-wide fiscal stance in both the fiscal leadership strategic regime under fiscal authorities' cooperation and the overall policy coordination strategic regime; (iv) how the monetary authority affects the horizontal coordination problem in the fiscal leadership strategic regime; (vi) the relevance of the specific fiscal regime for the monetary authority in the monetary leadership strategic regime.

The model characterizes the policy-mix outcome and coordination problems in a monetary union under alternative strategic and fiscal regimes, relative policy effectiveness, and demand/supply-side policy instruments. We do not take a stand on which combination of assumptions better describes the real policy-making situations (e.g., in the EMU) because there is no well-documented empirical evidence on these issues, while different scenarios can emerge under different circumstances. What we show is how these characteristics interact, hence affecting the policy-mix outcome. Focusing, e.g., in the case of fiscal leadership with decentralized fiscal policies, demand-side policy instruments, and a direct positive effect of fiscal policy on inflation, we find that the fiscal reaction parameter is positively related to the monetary authority's preference for output-gap stabilization (relative to inflation), fiscal policies might be pro-cyclical, and the monetary authority reduces (exacerbates) the horizontal coordination problem by pursuing a more (less) flexible inflation-targeting framework of monetary policy.

Conventional wisdom holds that the union monetary authority, regardless of the strategic regime, can always manage the cycle at the union level according to its preferences and irrespective of fiscal authorities' cooperation. Our findings show that a cost channel of monetary policy, reflecting the presence of the financial sector, makes the common central bank's preference for the actual fiscal regime not a trivial one, even if there are no financial frictions and heterogeneities. For a strong cost-channel effect that turns the nominal interest rate into a supply-side policy instrument, it is more likely that monetary policy becomes less effective in stabilizing inflation (relative to aggregate demand) than fiscal policy, even if there is a

direct negative effect of fiscal policy on inflation. We find that the monetary authority chooses not to trade-off its objectives in all strategic regimes, the union-wide fiscal stance is countercyclical for both the fiscal leadership regime under fiscal centralization and the overall policy coordination regime, the fiscal reaction parameter depends negatively on the monetary authority's preference on output-gap stabilization (relative to inflation), and the monetary authority exacerbates (reduces) the horizontal coordination problem by pursuing a more (less) flexible inflation-targeting framework of monetary policy in the fiscal leadership strategic regime. In the monetary leadership strategic regime, the monetary reaction parameter is negatively related to the fiscal authorities' preference on output-gap stabilization (relative to the fiscal stance), monetary policy is less reactionary compared to the simultaneous-move regime, and less reactionary in the centralized fiscal regime compared to the decentralized one, and an increase in the relative preference for the fiscal authorities makes the specific fiscal regime more relevant for the monetary authority.

Finally, one can identify two main issues that need further consideration and constitute directions for future research. First, the impact of the financial sector has been only considered implicitly through the cost channel of monetary policy. Future research can explicitly consider strategic interactions between fiscal-monetary authorities and the financial sector. Second, in an online Appendix to this paper, we consider debt accumulation showing that debt stabilization under imperfect instrument substitutability results in a policy-mix bias at the union level, even in the absence of shocks. Debt accumulation in a monetary union and the corresponding policy responses would be key issues in a dynamic model, along with possible solutions to the corresponding policy-mix bias.

Table 1

Reaction & Reference Parameters	Overall Policy Coordination Regime	Simultaneous-Move Regime		Fiscal Leadership Regime		Monetary Leadership Regime	
		No Cooperation	Cooperation	No Cooperation	Cooperation	No Cooperation	Cooperation
$\boldsymbol{\phi_\pi}$	$\frac{\omega_y\delta_r-(1-\delta_y)\omega_i}{(a_F+a_M)\delta_r}$	$\frac{\omega_y\delta_r-(1-\delta_y)\omega_i}{a_M\delta_r}$				$\phi_\pi^{SM}-\frac{(\delta_g\omega_i+\delta_r\omega_g)\phi_g^{SM}}{a_M\delta_r}$	
$\boldsymbol{\phi_g}$	$-\frac{\delta_g\omega_i+\delta_r\omega_g}{\delta_r\phi_\pi^{OC}}$	a_FZ_g	$a_F(Z_g+Z_g^*)$ $=\frac{a_F\delta_g}{1-\delta_y}$	$a_F\left(Z_g-\frac{1}{2}Z_iV_i\right)$	$a_F(Z_g+Z_g^*-Z_iV_i)$ $=-\frac{a_F(\delta_g\omega_i+\delta_r\omega_g)\phi_\pi^{SM}}{\delta_r[1+a_M(\phi_\pi^{SM})^2]}$	a_FZ_g	$a_F(Z_g+Z_g^*)$ $=\frac{a_F\delta_g}{1-\delta_y}$
$\boldsymbol{\Omega}$	$\delta_r[1+(a_F+a_M)(\phi_\pi^{OC})^2]$ $+\frac{(\delta_g\omega_i+\delta_r\omega_g)^2}{\delta_r}$	$\delta_r[1+a_M(\phi_\pi^{SM})^2]$ $-a_FZ_g(\delta_g\omega_i+\delta_r\omega_g)$ $*\phi_\pi^{SM}$	$\delta_r[1+a_M(\phi_\pi^{SM})^2]$ $-\frac{a_F\delta_g}{1-\delta_y}(\delta_g\omega_i$ $+\delta_r\omega_g)\phi_\pi^{SM}$	$\delta_r[1+a_M(\phi_\pi^{SM})^2]$ $-(\delta_g\omega_i+\delta_r\omega_g)$ $*\phi_\pi^{SM}a_F\left(Z_g-\frac{1}{2}Z_iV_i\right)$	$\delta_r[1+a_M(\phi_\pi^{SM})^2]$ $+\frac{a_F(\delta_g\omega_i+\delta_r\omega_g)^2(\phi_\pi^{SM})^2}{\delta_r[1+a_M(\phi_\pi^{SM})^2]}$	$\Omega^{SM}-(\delta_g\omega_i+\delta_r\omega_g)\phi_g^{SM}\phi_\pi^{ML}$	

'*OC*' stands for Overall (policy) Coordination, '*SM*' for Simultaneous Move and '*ML*' for Monetary Leadership; $V_i=\frac{\delta_g(1+\omega_y\phi_\pi^{SM})+(1-\delta_y)\omega_g\phi_\pi^{SM}}{\delta_r\left[1+a_M(\phi_\pi^{SM})^2\right]}$.

Appendix A

A.1: Structural Parameters' Restrictions regarding Relative Policy Effectiveness and Demand/Supply-Side Policy Instruments

The case of $\delta_g\omega_i + \delta_r\omega_g \leq 0$ holds for $\omega_g < 0$ and $|\omega_g| \geq \frac{\delta_g\omega_i}{\delta_r}$. In the case of a demand-side fiscal policy instrument, i.e., when $\frac{\partial \pi_j}{\partial g_j} = \omega_g + \omega_y\delta_g > 0$, then this holds for $|\omega_g| < \omega_y\delta_g$. Thus, must be $\frac{\delta_g\omega_i}{\delta_r} \leq |\omega_g| < \omega_y\delta_g$ when monetary policy instrument is also demand-sided, since $\frac{\partial \pi_j}{\partial i} = \omega_i - \delta_r\omega_y < 0 \Rightarrow \frac{\omega_i}{\delta_r} < \omega_y$. In the opposite case, that both policy instruments are supply-side, it suffices that $|\omega_g| \geq \frac{\delta_g\omega_i}{\delta_r}$, since the latter is also greater than $\omega_y\delta_g$ by assumption. Exactly the opposite holds for $\delta_g\omega_i + \delta_r\omega_g > 0$ when $\omega_g < 0$. Finally, when $\omega_g > 0$, then $\delta_g\omega_i + \delta_r\omega_g > 0$, and for demand-side fiscal policy instrument must be $\frac{\partial y_j}{\partial g_j} = \delta_g - \delta_\tau\omega_g > 0 \Rightarrow \omega_g < \frac{\delta_g}{\delta_\tau}$, which we can safely assume that it holds.

A.2: Proofs for Results and Propositions

Proof of Result 1: For the monetary leadership strategic regime, following Table 1, $\phi_\pi^{ML} = \phi_\pi^{SM} - \frac{(\delta_g\omega_i+\delta_r\omega_g)\phi_g^{SM}}{a_M\delta_r}$. For $MRT_{y\pi}^i \neq MRT_{y\pi}^g$, which following **Lemma 1** means that $\delta_g\omega_i + \delta_r\omega_g \neq 0$, then $\phi_\pi^{ML} = \phi_\pi^{ML}\left(\phi_g^{SM}(a_F)\right)$. For $MRT_{y\pi}^i < MRT_{y\pi}^g$, which means that $\delta_g\omega_i + \delta_r\omega_g > 0$ (**Lemma 1**), then ϕ_π^{ML} can become negative, even if $\phi_\pi^{SM} > 0$ (for a small cost-channel effect). That is, the monetary authority chooses not to trade-off inflation with output gap. For the fiscal leadership regime, $\phi_{g_{nc}}^{FL} = a_F\left(Z_g - \frac{1}{2}Z_iV_i\right)$. For $MRT_{y\pi}^i \neq MRT_{y\pi}^g$, which following **Lemma 1** means that $\delta_g\omega_i + \delta_r\omega_g \neq 0$, then $\phi_{g_{nc}}^{FL} = \phi_{g_{nc}}^{FL}\left(V_i\left(a_M, \phi_\pi^{SM}(a_M)\right)\right)$. Moreover, $\phi_{g_{nc}}^{FL}$ can become negative, since $V_i = \frac{\partial i}{\partial g} > 0$ for demand/supply-side policy instruments at the union level, which means that the fiscal authorities act pro-cyclically. In the opposite case where $\delta_g\omega_i + \delta_r\omega_g = 0$, then $V_i = \frac{\delta_g}{\delta_r}$ and $\phi_{g_{nc}}^{FL} = \frac{1}{2}a_F\left(Z_g - Z_g^*\right) = \frac{1}{2} * \frac{a_F(\delta_g - 2\delta_\tau\omega_g)}{1+\delta_y+2\delta_\tau\omega_y} > 0$ for either $\omega_g = -\frac{\delta_g\omega_i}{\delta_r}$ or $\omega_g = 0$; so independent on a_M. In order to compute V_i, one needs to substitute for $\omega_g = -\frac{\delta_g\omega_i}{\delta_r}$.

Proof of Proposition 1: Following **Result**'s **1** proof, (i) $\frac{\partial \phi_{g_{nc}}^{FL}}{\partial a_M} = \frac{1}{2} * \frac{(\delta_g \omega_i + \delta_r \omega_g)\phi_\pi^{SM}}{a_M \delta_r \left[1 + a_M (\phi_\pi^{SM})^2\right]^2}$, which leads to $sign\left\{\frac{\partial \phi_{g_{nc}}^{FL}}{\partial a_M}\right\} = sign\{(\delta_g \omega_i + \delta_r \omega_g)\phi_\pi^{SM}\}$, and (ii) $\frac{\partial (\phi_{g_{nc}}^{FL} - \phi_{g_c}^{FL})}{\partial a_M} = -\frac{a_F(\delta_g \omega_i + \delta_r \omega_g)\phi_\pi^{SM}}{a_M \delta_r \left[1 + a_M (\phi_\pi^{SM})^2\right]^2}$, which means that $sign\left\{\frac{\partial (\phi_{g_{nc}}^{FL} - \phi_{g_c}^{FL})}{\partial a_M}\right\} = -sign\{(\delta_g \omega_i + \delta_r \omega_g)\phi_\pi^{SM}\}$. It is straightforward that for $MRT_{y\pi}^{i} = MRT_{y\pi}^{g}$, which means that $\delta_g \omega_i + \delta_r \omega_g = 0$ (**Lemma 1**), then $\frac{\partial \phi_{g_{nc}}^{FL}}{\partial a_M} = 0$ and $\frac{\partial (\phi_{g_{nc}}^{FL} - \phi_{g_c}^{FL})}{\partial a_M} = 0$. Then, for $\phi_\pi^{SM} > 0$, which holds for $\omega_i < \frac{\omega_y \delta_r}{1 - \delta_y}$, for $MRT_{y\pi}^{i} < MRT_{y\pi}^{g}$, which means that $\delta_g \omega_i + \delta_r \omega_g > 0$ (**Lemma 1**), then $\frac{\partial \phi_{g_{nc}}^{FL}}{\partial a_M} > 0$ and $\frac{\partial (\phi_{g_{nc}}^{FL} - \phi_{g_c}^{FL})}{\partial a_M} < 0$; and vice versa. For $\phi_\pi^{SM} < 0$, which holds for $\omega_i > \frac{\omega_y \delta_r}{1 - \delta_y}$, so a strong cost-channel effect, it is more plausible to be $\delta_g \omega_i + \delta_r \omega_g > 0$, even if $\omega_g < 0$, since must be $\frac{\omega_y}{1 - \delta_y} > \frac{|\omega_g|}{\delta_g}$; so $MRT_{y\pi}^{i} < MRT_{y\pi}^{g}$. Then $\frac{\partial \phi_{g_{nc}}^{FL}}{\partial a_M} < 0$ and $\frac{\partial (\phi_{g_{nc}}^{FL} - \phi_{g_c}^{FL})}{\partial a_M} > 0$.

Proof of Proposition 2: Following **Result**'s **1** proof and Table 1, (i) $\frac{\partial \phi_\pi^{ML}}{\partial a_F} = -\frac{(\delta_g \omega_i + \delta_r \omega_g)\phi_g^{SM}}{a_F a_M \delta_r}$, which leads to $sign\left\{\frac{\partial \phi_\pi^{ML}}{\partial a_F}\right\} = -sign\{\delta_g \omega_i + \delta_r \omega_g\}$, (ii) $\phi_\pi^{ML} - \phi_\pi^{SM} = -\frac{(\delta_g \omega_i + \delta_r \omega_g)\phi_g^{SM}}{a_M \delta_r}$ and $\phi_{\pi_{nc}}^{ML} - \phi_{\pi_c}^{ML} = \frac{(\delta_g \omega_i + \delta_r \omega_g)(\phi_{g_c}^{SM} - \phi_{g_{nc}}^{SM})}{a_M \delta_r} = \frac{(\delta_g \omega_i + \delta_r \omega_g) a_F Z_g^*}{a_M \delta_r}$, which both lead to $sign\{\phi_\pi^{ML} - \phi_\pi^{SM}\} = -sign\{\phi_{\pi_{nc}}^{ML} - \phi_{\pi_c}^{ML}\} = -sign\{\delta_g \omega_i + \delta_r \omega_g\}$, and (iii) $\phi_{\pi_{nc}}^{ML} - \phi_{\pi_c}^{ML} = \frac{(\delta_g \omega_i + \delta_r \omega_g)(\phi_{g_c}^{SM} - \phi_{g_{nc}}^{SM})}{a_M \delta_r} = \frac{(\delta_g \omega_i + \delta_r \omega_g) a_F Z_g^*}{a_M \delta_r}$, which means that $sign\left\{\frac{\partial (\phi_{\pi_{nc}}^{ML} - \phi_{\pi_c}^{ML})}{\partial \left(\frac{a_F}{a_M}\right)}\right\} = sign\{\delta_g \omega_i + \delta_r \omega_g\}$; since $\phi_g^{SM} > 0$ for both fiscal regimes and $Z_g^* > 0$ for plausible values of the direct effect of fiscal policy on inflation. It is straightforward that for $MRT_{y\pi}^{i} = MRT_{y\pi}^{g}$, which means that $\delta_g \omega_i + \delta_r \omega_g = 0$ (**Lemma 1**), (i) $\frac{\partial \phi_\pi^{ML}}{\partial a_F} = 0$, (ii) $\phi_\pi^{ML} = \phi_\pi^{SM}$ and $\phi_{\pi_{nc}}^{ML} = \phi_{\pi_c}^{ML}$ $(= \phi_\pi^{SM})$, and (iii) $\frac{\partial (\phi_{\pi_{nc}}^{ML} - \phi_{\pi_c}^{ML})}{\partial \left(\frac{a_F}{a_M}\right)} = 0$. For $MRT_{y\pi}^{i} < MRT_{y\pi}^{g}$, which means that $\delta_g \omega_i + \delta_r \omega_g > 0$ (**Lemma 1**), then (i) $\frac{\partial \phi_\pi^{ML}}{\partial a_F} < 0$, (ii) $\phi_\pi^{ML} < \phi_\pi^{SM}$ and $\phi_{\pi_{nc}}^{ML} > \phi_{\pi_c}^{ML}$, and (iii) $\frac{\partial (\phi_{\pi_{nc}}^{ML} - \phi_{\pi_c}^{ML})}{\partial \left(\frac{a_F}{a_M}\right)} > 0$; and vice versa.

Proof of Proposition 3: Following Table 1, the union-wide fiscal reaction parameter for the strategic regime of fiscal leadership under fiscal authorities' cooperation is given by $\phi_{g_c}^{FL} = -\frac{a_F(\delta_g \omega_i + \delta_r \omega_g)\phi_\pi^{SM}}{\delta_r \left[1 + a_M (\phi_\pi^{SM})^2\right]}$, while the corresponding one for the overall policy coordination regime by $\phi_g^{OC} = -\frac{\delta_g \omega_i + \delta_r \omega_g}{\delta_r \phi_\pi^{OC}}$. Then,

$sign\{\phi_{g_c}^{FL}\} = -sign\{(\delta_g\omega_i + \delta_r\omega_g)\phi_\pi^{SM}\}$ and $sign\{\phi_g^{OC}\} = -sign\left\{\frac{\delta_g\omega_i+\delta_r\omega_g}{\phi_\pi^{OC}}\right\}$. It is straightforward that for $MRT_{y\pi}^{i} = MRT_{y\pi}^{g}$, which means that $\delta_g\omega_i + \delta_r\omega_g = 0$ (**Lemma 1**), then $\phi_{g_c}^{FL} = \phi_g^{OC} = 0$, so a passive union-wide fiscal stance. For $MRT_{y\pi}^{i} < MRT_{y\pi}^{g}$, which means that $\delta_g\omega_i + \delta_r\omega_g > 0$ (**Lemma 1**), and $\phi_\pi^{SM} > 0$ and $\phi_\pi^{OC} > 0$, which holds for $\omega_i < \frac{\omega_y\delta_r}{1-\delta_y}$, then $\phi_{g_c}^{FL} < 0$ and $\phi_g^{OC} < 0$, so a pro-cyclical union-wide fiscal stance; and vice versa. For $\phi_\pi^{SM} < 0$ and $\phi_\pi^{OC} < 0$, instead, which holds for $\omega_i > \frac{\omega_y\delta_r}{1-\delta_y}$, it is more plausible to be $\delta_g\omega_i + \delta_r\omega_g > 0$, following **Proposition**'s **1** Proof. Then, $\phi_{g_c}^{FL} > 0$ and $\phi_g^{OC} > 0$, so a countercyclical union-wide fiscal stance.

Proof of Result 2: Let us consider the case of perfect instrument substitutability. It is straightforward from Table 1 that for $MRT_{y\pi}^{i} = MRT_{y\pi}^{g}$, which means that $\delta_g\omega_i + \delta_r\omega_g = 0$ (**Lemma 1**), then $\phi_\pi = \frac{\omega_y}{a_M} - \frac{(1-\delta_y)\omega_i}{a_M\delta_r}$ and $\Omega = \delta_r + \frac{[\omega_y\delta_r-(1-\delta_y)\omega_i]^2}{a_M\delta_r}$ for all strategic and fiscal regimes. Thus, following equations (17) and (18) that show the equilibrium solutions for the inflation rate and the output gap at the union level, respectively, then $\pi = \frac{a_M\delta_r}{a_M\delta_r^2+[\omega_y\delta_r-(1-\delta_y)\omega_i]^2}(\omega_i u - \delta_r\varepsilon)$ and $y = -\frac{a_M\delta_r[\omega_y\delta_r-(1-\delta_y)\omega_i]}{a_M\delta_r^2+[\omega_y\delta_r-(1-\delta_y)\omega_i]^2}(\omega_i u - \delta_r\varepsilon)$. This is the case for $\omega_g = -\frac{\delta_g\omega_i}{\delta_r}$. Naturally, this result holds for $\omega_g = \omega_i = 0$, too, by setting $\omega_i = 0$ to the previous solutions. For imperfect instrument substitutability, namely $MRT_{y\pi}^{i} \neq MRT_{y\pi}^{g}$, which following **Lemma 1** means that $\delta_g\omega_i + \delta_r\omega_g \neq 0$, then the reference parameter, Ω, differs across strategic and fiscal regimes. This means that equilibrium solutions for both inflation and the output gap at the union level would differ. We can further compute the monetary authority's expected loss by combining its loss function (eq. (7)) with the monetary rule (eq. (14)) and the union-wide equilibrium solution for inflation (eq. (17)), and then taking expectations. We get:

$$E(L_M) = \frac{1}{2} * \frac{1+a_M\phi_\pi^2}{\Omega^2} * \left[\omega_i^2 Var(u) + \delta_r^2 Var(\varepsilon)\right]$$

For the general case of imperfect instrument substitutability, it is straightforward from Table 1 that both the monetary reaction parameter, ϕ_π, and the reference parameter, Ω, are susceptible to the specific strategic and fiscal regime. Under perfect instrument substitutability, however, the monetary authority's expected loss is identical for all strategic and fiscal regimes.

Proof of Result 3: It is straightforward from the country-specific equilibrium solutions equations (27) and (28) that for common shocks, namely $u_j = u_k = u$ and $\varepsilon_j = \varepsilon_k = \varepsilon$, then $y_j = y_k = y$. Then, following equation (26), must be $g_j = g_k = g$. Finally, following equation (4), then $\pi_j = \pi_k = \pi$. For perfect instrument substitutability, since the output gap and inflation at the union level are the same across strategic

and fiscal regimes, following the proof of **Result 2**, then this holds for both country-specific inflation and output gap. Furthermore, since when there is no cost channel of monetary policy and no supply shocks both the union-wide inflation and output gap are equal to their long-run equilibrium values, then this holds for the country-specific variables, too.

Supplementary Material

Supplementary material associated with this article can be found in the online version.

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