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Cooperation vs. Leadership in a Core-Periphery Monetary Union: Inter-country vs. Inter-institutional Policy Coordination

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Abstract: This paper utilises a static representation of a reduced-form Dynamic Stochastic General Equilibrium (DSGE) model to deal with strategic fiscal/monetary policy interactions in a core-periphery monetary union, in which the periphery member’s fiscal authority is always the follower (except in the EMU benchmark where all policymakers move simultaneously). In such a set-up, we examine the policy mix of the alternative institutional arrangements of (i) non-cooperation between the two leading authorities, the lead fiscal authority and the monetary authority; and (ii) in a regime of cooperation between the leaders; and (iii) in a regime of fiscal leadership in which the monetary authority moves between the two fiscal authorities. We explore the welfare implications of these alternative institutional arrangements for the monetary authority, the two fiscal authorities, the social planner, and provide a ranking. Our main results are: (i) the lead fiscal authority’s ability to contribute less counter-cyclically increases with cooperation at the core, while it is unchanged for explicit or implicit cooperation system wide; (ii) monetary leadership provides no advantage over no cooperation in the core; (iii) the ranking for the core member’s fiscal authority is shock independent and favours a cooperative strategic regime, and then fiscal leadership; (iv) the ranking for the other authorities and the social planner is shock dependent, and can either coincide with or be the exact opposite of the core member’s ranking; (v) the ranking for the peripheral fiscal authority always coincides with the social planner’s; and (vi) for common supply shocks, all the policy authorities and the social planner are in favour of the cooperative strategic regime.

Keywords: Currency union; Core-periphery economies; Stabilisation policies; Partial cooperation; Fiscal leadership.

JEL Classification: E61; E63; E65; F45.
1. Introduction

This paper deals with the interaction of fiscal and monetary policies in a monetary union, with a clear reference to the Economic and Monetary Union (EMU) in Europe. The main characteristic of this setup is that monetary policy is centralised, conducted by a single monetary authority, the European Central Bank (ECB), while fiscal policy is decentralised in that there are many fiscal authorities, each responsible for fiscal policy at the country-specific level. However each fiscal authority is subject to the deficit constraint imposed by the European Union (EU)’s Stability and Growth Pact (SGP), refined by the Fiscal Compact¹ (FC) agreement, and to a debt constraint [European Council (2012)]. In this setup, various coordination problems and conflicts of interest arise. Typically, the theoretical literature assumes strategic behaviour on the part of the authorities and investigates the policy interactions in a game-theoretic context; so, through policy games [Hughes Hallett (1986a,b); Beetsma and Bovenberg (1998); Dixit and Lambertini (2001; 2003a,b)].² But, as the Eurozone sovereign debt crisis that followed the Great Recession clearly showed, there is an imperfect institutional structure in EMU for dealing with the various shocks that occurred. This has created renewed interest in the theoretical literature on strategic policy interactions in monetary unions (Foresti, 2018).

Beetsma and Giuliodori (2010) point out that the ‘traditional’ Optimum Currency Area (OCA) theory of Mundell (1961) and McKinnon (1962) largely ignored the implications of monetary unification for fiscal policymaking; while the fiscal authorities, acting as decentralized players, may employ their fiscal policies in a strategic way. This requires a framework to model the strategic interactions of fiscal/monetary policies in a monetary union, mainly in terms of authorities’ objectives, their ability to commit, and the sequencing in the game. According to Fragetta and Kirsanova (2010, p. 856), “there is little doubt that authorities can act strategically”. These authors further point out that an analysis of the potential strategic interactions between fiscal

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¹ Its official name is ‘The Treaty on Stability, Coordination and Governance’ (TSCG).
² There is also a literature based on Dynamic Stochastic General Equilibrium (DSGE) models [see Beetsma and Jensen (2004; 2005); Gali and Monacelli (2008); Ferrero (2009)].
and monetary authorities is crucial, since the assumption of complete cooperation is seldom realistic and different objectives or priorities can lead to conflicts between the authorities.

In this paper, we investigate the policy mix in a monetary union in a strategic context, under alternative institutional arrangements and various shocks in the absence of wage-price flexibility, labour mobility and fiscal transfers. The focus is on macroeconomic stabilisation and the welfare implications of fiscal-monetary policy interactions at both national and union levels. A policy-mix problem appears when policies, in particular the common monetary policy relative to fiscal policy, conflict over how best to resolve the business cycle [Andersen (2008)]. The institutional arrangements will of course determine how monetary and fiscal policies interact. And that will depend on the sequencing of decisions in the game (simultaneous moves vs. policy leadership or cooperation), the degree of cooperation among monetary and fiscal authorities, their objectives, and the constraints imposed by the economy.

Typically, the theoretical literature that studied strategic policy interactions in a monetary union, following the creation of the EMU, embraces the fiscal leadership strategic regime [see Beetsma and Bovenberg (1998); Uhlig (2003); Andersen (2005; 2008); Ferre (2008; 2012)]. The national fiscal authorities move in a non-cooperative simultaneous-move manner at the beginning of the game, while the monetary authority follows. According to Andersen (2008), this works because monetary strategy is made credible and clear, and fiscal decisions precede the monetary decisions. Thus, if the monetary policy objectives are well-known to the fiscal authorities and the policies predictable, then fiscal policymakers can consider the possible monetary threats to their fiscal decisions on fiscal policy. In that way, monetary policy acting as follower can discipline fiscal policies [Libich and Stehlik (2012)]. Put differently, the fiscal authorities have first-mover advantage since once fiscal policy is decided, they cannot react to monetary policy. This rigidity assumes a degree of stickiness or persistence; and applies when debt is a target or constraint since

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3 In complete contrast, the focus of DSGE models is on optimal fiscal/monetary policies in a cooperative setup with well-defined social objectives and transitional dynamics in a monetary union. Recent references include Hjortsoe (2016), Palek and Schwanebeck (2017), Arce et. al. (2016), Jarocinski and Mackowiak (2018). Regime choices however need to be made on the basis of a steady state (equilibrium) representation of the economy, after the transitional dynamics are complete.

4 Hence the literature on surrogate (implicit) coordination and intermediate targets, where outcomes depend strongly on the leadership structure of the game [Hughes Hallett (1992); Hughes Hallett and Weymark (2007)].

5 See Beetsma and Bovenberg (1998). This argument generalizes to any timing scheme [Libich and Stehlik (2012)].
debt is a stock, but the policies a flow. The authority creating the greater persistence necessarily moves first since there will come a point when it cannot react to retaliations from the low persistence player to the *current* decision. In contrast, Kirsanova et. al. (2005) assume a simultaneous-move regime on the argument that, coalitions apart, the fiscal authorities are too many to be considered as leaders in their game with the ECB. But the core may be a coalition, especially if it cooperates with the ECB.

There is little firm empirical evidence on strategic interactions and the sequence of moves between monetary and fiscal authorities. Fragetta and Kirsanova (2010) provide an empirical analysis of policy interactions in the US, UK and Sweden to identify potential leadership regimes. The authors specify a small-scale structural general equilibrium model, like ours, of an open economy and estimate it using Bayesian methods. Their main finding is that fiscal leadership can best describe the actual regime for the UK and Sweden, instead of a simultaneous-move game for the US. Hughes Hallett and Lewis (2015) estimate Taylor rules for anticipated fiscal decisions to condition ECB monetary decisions using real-time data. They find that the fiscal and monetary policies appear to conflict ex-ante, but switch to accommodative (cooperative?) ex-post – which may imply monetary leadership or cooperation to ensure long-run sustainability.

In what follows, we consider an alternative approach with a formalised core-periphery union. This allows us to consider a core leadership game in which there are two leading authorities, the core’s fiscal authority and a single monetary authority, and a follower fiscal authority in the periphery. In this world, we examine the outcomes from cooperation between core and monetary authority. We compare the results in terms of policy mix and overall welfare to four alternative regimes: (i) a simultaneous-move strategic regime that assumes no cooperation with the core; (ii) a simultaneous-move regime where all three policy authorities move together (the benchmark for comparisons); (iii) a cooperative regime at the core, where the core member’s fiscal authority cooperates with the monetary authority explicitly; and (iv) a fiscal leadership regime, in which we assume that the core member’s fiscal authority leads vs. the monetary authority, and both against the periphery – this amounts to implicit cooperation. We further note that monetary leadership delivers the same results as no cooperation among the leaders (regime (i)).

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6 Fiscal leadership, where the monetary authority is the follower, is examined in Chortareas and Mavrodimitrakis (2017). But there is little evidence of core-periphery cooperation in practice. Coalition formation more generally is
The asymmetry in the sequence of moves between the two fiscal authorities rests on the core-periphery assumption, where the lead fiscal authority represents a core member-state. In such a setting, the core member’s fiscal authority retains an informational advantage vs. the periphery which might reflect an unequal distribution of power in the monetary union [Chortareas and Mavrodimitrakis (2016)]. This means that the peripheral member-state chooses its fiscal stance based on a prior observation of the choices made by the core member’s fiscal authority and the monetary authority. There are various ways this assumption can be justified. One explanation might be that countries in the core provide some form of ‘political hegemony’ in the union, in that they, together with the ECB, act pre-emptively to provide economic leadership to the rest of the Eurozone. Another explanation might be that this fiscal sequential asymmetry captures the ability to commit to policy actions. A third is that core economies have an information advantage in terms of knowing what the other policymakers will do and/or what bail out plans may exist. In any event, it seems reasonable for the peripheral member’s fiscal authority to react to the choices made at the core of the monetary union; especially when those member-states are small relative to the core (so they need to adapt or free ride), or have high levels of past debt.

Our point of departure is to examine fiscal vs. monetary leadership as implicit coordination devices in an incomplete economic union like the Eurozone, starting from the ideas in Dixit and Lambertini (2003a,b), Hughes Hallett and Weymark (2007), and the idea that coordination might be counterproductive as a result of de facto institutionalised coalitions [Rogoff (1985)]. Rogoff (1985) shows that the cooperation of two monetary authorities can be counterproductive if there is a third party (the private sector) that does not cooperate. We then investigate whether giving the lead to the core member’s fiscal authority relative to the monetary authority could be welfare-improving. Beetsma and Debrun (2004) argue that coordination in a currency union can be horizontal, across governments, and vertical, between fiscal and monetary authorities. In our core-

7 Canofari et. al. (2015, p. 1621), by contrast, argue that in existing monetary unions (like EMU) policy decisions in the periphery have negligible impacts on the core since “…a leader country can impose its rules on the whole system”. These authors therefore assume no strategic interactions between the core and periphery members.
8 However, we do not deal with country-size asymmetry in this paper, since this has been covered in the literature [see Hughes Hallett and Kavanagh (2001)]. The authors explore how country size affects inflation under different cooperation arrangements. They find that which regime will be preferred overall depends on the particular parameter values in the case at hand; so no general comparisons can be made. Instead, we push the conventional equal size analysis as far as possible to see what rankings we can establish.
periphery monetary union, vertical coordination is explored at the core where it can be either implicit or explicit (fiscal leadership represents implicit coordination). Foresti (2018) stresses that vertical coordination should be of no interest in EMU since the ECB is not allowed to coordinate its policies with the fiscal authorities. However, the author points out that ‘...recent monetary policy arrangements have evidenced that under particular conditions the ECB may overcome this constraint’ [Foresti (2018, p. 235]. Hence, we explore the conditions under which the monetary authority might find it beneficial to cooperate with the core member’s fiscal authority, and vice versa.

Fragetta and Kirsanova (2010) argue that fiscal authorities in European countries might be in different leadership relationships with the ECB, since they are bigger and more independent than fiscal authorities in the US. Knowing that fiscal authorities can affect economic performance we need to explore institutional designs that accommodate that fact. Our paper is a first attempt to provide the necessary theoretical analysis since, in each policy game, the fiscal authorities are set in a different strategic relationship with the monetary authority. The peripheral member’s fiscal authority always plays follower to the monetary authority; while the lead fiscal authority is either (i) in a simultaneous-moves non-cooperative game, or (ii) in a cooperative game, or (iii) has leadership over the monetary authority.

We use a standard two-country monetary-union model based on a static representation of a steady state (equilibrium) DSGE model where the two countries are interconnected through trade, a free flow of investment, and a terms-of-trade effect. Attention is paid on the terms-of-trade effect, since that represents an important transmission channel within a monetary union. In our setting, supply shock asymmetries induce real exchange rate differences in the monetary union which affect aggregate demand and lead to a great source of imbalance. Hence the impact of supply shock asymmetries on the main macroeconomic variables at both the national and union levels is explored. We then consider cyclical stabilisation policies. The common central bank pursues a flexible inflation-targeting policy [Svensson (1997)], while national fiscal authorities care about the output gap and sustainable fiscal balances. These assumptions are common in various papers
in the literature [see Uhlig (2003); Andersen (2008); Chortareas and Mavrodimitrakis (2016; 2017)].

Our main results can be summarised as follows: (i) the fiscal strategic advantage increases with cooperation in the core, while it is unchanged for either explicit or implicit cooperation; (ii) the ranking for the core member’s fiscal authority is shock independent and favours a cooperative strategic regime and failing that fiscal leadership; (iii) the ranking for the other authorities and the social planner is shock dependent, while it can either coincide or be the exact opposite of the core member’s ranking; (iv) the ranking for the peripheral fiscal authority always coincides with the social planner’s; and (v) for common supply shocks, all the authorities and the social planner favour the cooperative strategic regime.

To establish these results, the paper is structured as follows: Section 2 presents the benchmark model; Section 3 solves for the alternative institutional arrangements and provides equilibrium solutions for the main macroeconomic variables at both union and country-specific levels. Section 4 provides a welfare comparison of the alternative regimes with respect to the two fiscal authorities, while Section 5 provides a macroeconomic stabilisation and welfare analysis at the union level. Section 6 concludes.

2. The Model

Following the literature on strategic fiscal/monetary policy interactions based on Dixit and Lambertini (2001; 2003a,b), we assume a standard static reduced-form two-country monetary union model based on an Aggregate Demand (AD) and a Phillips Curve (PC) equation, similar to that employed by Andersen (2005; 2008), Ferre (2008; 2012), and Chortareas and Mavrodimitrakis (2017). This model can be obtained from a reduced-form DSGE model with monopolistic competition and nominal rigidities [see Gali (2015)]. In particular, both equations emerge from a micro-founded model that captures monopolistic competition in product and labour markets, along with sticky wages [see Beetsma and Jensen (2004; 2005); Gali and Monacelli (2008); Ferrero (2009)], after any dynamic adjustments have been completed. The static representation is essential for creating the analytical results which help identify the policy transmission mechanisms and manage the corresponding interactions. This is particularly useful.

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9 These aspects are discussed further in the next section.
10 The reduced-form DSGE model from which our static (equilibrium) version is derived is shown in Appendix A.
in policy games since a relatively simple analytical framework allows us to compare alternative solution concepts, corresponding to alternative institutional arrangements in our set up, *without* having to resort to numerical simulations [Chortareas and Mavrodimitrakis (2016)].

Following Andersen’s (2005) notation and letting $j = l, f$ denote the lead and follower fiscal authorities respectively, economy $j$’s descriptive equations are given by:

$$y_j = -\delta_r (i - \pi_j^e - \bar{r}_j) + \delta_y y_k - \delta_r (\pi_j - \pi_k) + \delta_g g_j + u_j$$  \hspace{1cm} (1)

$$\pi_j = \omega y_j - \varepsilon_j$$  \hspace{1cm} (2)

The first equation represents the AD equation, while the second one is a standard Phillips curve. All variables are defined as log-deviations from long-run equilibrium values, apart from the decimal nominal interest rate, $i$. Thus, $\pi$ represents the inflation rate, while $y$ represents the output gap. The variable $g$ represents fiscal policy, as captured by country $j$’s overall fiscal stance. Variable $\bar{r}_j$ represents the long-run or equilibrium real interest rate which, for simplicity, we set at zero for both countries. Thus, aggregate demand depends negatively on the real interest rate in each country, expressed by the Fisher identity; $u_j$ and $\varepsilon_j$ are independently and identically distributed random demand and supply shocks, with zero means and constant variances. We assume that both are pure and uncorrelated. Thus, the model deals with all kinds of shocks: common, country-specific, symmetric or asymmetric. The index $k$ represents the other country. Thus, the term $\pi_j - \pi_k$ represents the change in the real exchange rate and captures intra-EU competitiveness (the terms-of-trade effect); higher prices for domestic products shift domestic demand to foreign. Finally, $\pi_j^e$ is the private sector’s expectation of country $j$’s inflation rate. The parameters of our model are all positive; $\delta_r$ captures the real interest rate elasticity of aggregate demand arising from country $j$’s share of union-wide investment; and $\delta_g$ the fiscal multiplier (the effectiveness of fiscal policy). Parameters $\delta_r$ and $\delta_y$ therefore represent the direct interconnections between the two countries; specifically, the effect of competitiveness on domestic output, and the relative openness of the economy. Turning to the Phillips curve, equation (2), the parameter $\omega$ is the slope of the curve and shows the degree of nominal price/wage rigidity, with $\omega = 0$ corresponding to complete nominal rigidity [Gali (2015)].
A straightforward manipulation of the descriptive equations (1) – (2) of both member-states gives us country-specific aggregate demand with respect to policy instruments and shocks, as:

\[ y_j = \frac{1}{1-\delta_y} \left[ -\delta_y i + K \delta_y g_j + \Lambda \delta_y g_k + \frac{1-\delta_y}{1+\delta_y+2\delta_y}\delta_t (\varepsilon_j - \varepsilon_k) + Ku_j + \Lambda u_k \right]. \]  

(3)

where \( K = \frac{1+\delta_t \omega}{1+\delta_y+2\delta_t \omega} \) and \( \Lambda = \frac{\delta_y+\delta_t \omega}{1+\delta_y+2\delta_t \omega} \). Equation (3) shows that domestic aggregate demand is directly affected by foreign fiscal policy and demand shocks – so long as there are some inter-connections (\( \Lambda \neq 0 \)). Moreover, domestic aggregate demand is affected directly by supply shock asymmetries only through the terms-of-trade effect \( \delta_t \). Averaging equations (1) and (2) for the two member-states, we get union-wide descriptive equations as follows:

\[ y = \frac{1}{1-\delta_y} (-\delta_y i + \delta_y g + u) \]  

(4)

\[ \pi = \omega y - \varepsilon, \]  

(5)

where \( x = \frac{x_i+x_f}{2} \) represents union-wide average variables.

The authorities’ loss functions are:

\[ L_{Fj} = \frac{1}{2} \left( g_j^2 + a_F y_j^2 \right) \]  

(6)

\[ L_M = \frac{1}{2} (\pi^2 + a_M y^2), \]  

(7)

for the fiscal and monetary authorities, respectively. We assume that preferences are quadratic and that the policy authorities target long-run equilibrium values of the concerned variables. The common central bank cares about union-wide inflation and output gap, while the national fiscal authorities about country-specific output gap and fiscal stance. This specification follows Uhlig (2003), Andersen (2008) and Chortareas and Mavrodimitrakis (2016; 2017) and is meant to capture the actual policymaking concerns in the EMU. The monetary authority pursues a flexible inflation-targeting monetary policy, while the national fiscal authorities are not concerned with inflation stabilisation as such since they have delegated this task to the common monetary authority. Regarding the monetary authority, Andersen (2008, p. 413) argues that ‘…strict inflation targeting may be considered as too rigid an interpretation of the monetary policy being pursued.
in the EMU'.\textsuperscript{11} Regarding the fiscal authorities’ loss function (eq. (6)), the existence of a terms-of-trade effect in aggregate demand works as an implicit concern for inflation stabilisation [Andersen (2005)].\textsuperscript{12}

The time line for decision-making works as follows:

(i) The private sector forms expectations rationally.

(ii) Shocks are realised.

(iii) In the fiscal leadership strategic regime, the lead fiscal authority chooses its fiscal stance; in the other regimes, nobody plays.

(iv) In the fiscal leadership strategic regime, the monetary authority chooses the common nominal interest rate; in the cooperative strategic regime, the lead fiscal authority and the monetary authority cooperate in order to choose the core country’s fiscal stance and the common nominal interest rate; in the non-cooperative regime, the lead fiscal authority and the monetary authority play simultaneously but non-cooperatively; in the benchmark simultaneous-move regime, nobody plays.

(v) In the strategic regimes of fiscal leadership, non-cooperation and cooperation, the follower/peripheral fiscal authority completes the game; in the simultaneous-move regime, all the authorities choose their policy instruments simultaneously but non-cooperatively.

We investigate the case where the monetary authority and lead fiscal authority move simultaneously and non-cooperatively first; this is the non-cooperative strategic regime. Second, we contrast this case with the standard decentralised case where there are no strategic advantages in fiscal policy: the fiscal authorities play simultaneously. That means all three authorities play simultaneously. This setting defines a simultaneous-move strategic regime. Next we examine the case of (partial) co-operation between the lead fiscal authority and common central bank; namely

\textsuperscript{11} The consideration of flexible instead of strict inflation targeting has important implications for the cooperative strategic regime (see Section 3.3 below).

\textsuperscript{12} According to Muscatelli et. al. (2012), it is not unusual for the fiscal authorities not to care about inflation stabilisation in monetary union models, since inflation is usually regarded as an exclusive responsibility of the central bank. Nevertheless, microeconomic foundations for equation (6) are provided by Andersen and Spange (2006). These authors show that this particular loss function can be derived from a representative household’s utility function in the usual way that depends positively on private consumption and the provision of public goods, and negatively on labour supply, where the private consumption bundle is defined over the consumption of domestic and foreign commodities. See also the discussion in Chortareas and Mavrodimitrakis (2017).
the cooperative strategic regime. Finally we analyse fiscal leadership, where the lead fiscal authority leads the game with respect to the common monetary authority (and the latter moves between the two fiscal authorities). This defines the fiscal leadership strategic regime. In all institutional arrangements, the simultaneous-moves regime apart, the peripheral fiscal authority completes the game.

We are particularly interested in the policy mix at the union and country-specific levels, as well as the welfare implications for the policy authorities in the monetary union and for a social planner. We investigate two specific results: (i) Rogoff’s (1985) result that partial cooperation (policy coalitions) can be counterproductive; (ii) Hughes Hallett and Weymark’s (2007) result that fiscal leadership is preferable to simultaneous-moves as it introduces an element of implicit cooperation [see also Dixit and Lambertini (2001; 2003a,b) who do not resolve the point]. The model is solved by backward induction across players, and the final equilibrium rests on sub-game perfection. The above sequence of events, together with the assumptions about the specific structural equations (1) and (2), and the loss functions (equations (6) and (7)), specifically the pure cyclical stabilisation assumption, lead to \( \pi_f^e = \pi^e = 0 \) [see Uhlig (2003); Andersen (2008); among others]. Thus, no time-inconsistency problems emerge.

3. The Policy Mix in the Union under Alternative Institutional Arrangements

This section is dedicated to the analysis of the policy mix at both the union and the country-specific level for the institutional arrangements of non-cooperation and cooperation at the core, as well as fiscal leadership. We begin by solving the follower fiscal authority’s problem, which is the same in each strategic regime. In particular, the authority minimises its loss function (eq. (6)) subject to domestic aggregate demand, equation (3). We get the following fiscal rule:

\[
g_f = - \frac{a_F \delta g K}{1 - \delta_y} y_f = -\phi f y_f, \tag{8}
\]

where \( \phi_f = \frac{a_F \delta g K}{1 - \delta_y} \) is the follower’s unambiguously positive fiscal reaction parameter. Hence, fiscal policy in the periphery is unambiguously countercyclical. It depends on the fiscal multiplier, on the interconnection parameters, and on the degree of nominal rigidity. It also depends positively

\[^{13}\text{See Appendix A.}\]
on the fiscal authorities’ preference for output-gap stabilisation; positively on the trade linkages $\delta_y$; but negatively on the terms-of-trade effect $\delta_r$. The latter makes discretionary fiscal policy less countercyclical as it works as an automatic stabiliser.

Combining the peripheral member’s fiscal rule (eq. (8)) with its domestic aggregate demand (eq. (3), including monetary policy) provides us with the periphery’s fiscal reaction:

$$g_f = \frac{a_F \delta_g K}{a_F \delta_g^2 K^2 + (1-\delta_y)^2} \left[ \delta_r t - \delta_g \Lambda g_l + \frac{1-\delta_y}{1+\delta_y+2\delta_r \omega} \delta_r (\varepsilon_l - \varepsilon_f) - K u_f - \Lambda u_l \right] \tag{9}$$

The peripheral member’s fiscal authority reacts negatively to a core member’s fiscal stance; hence, fiscal expansions at the core of the monetary union will induce fiscal contractions at the periphery, since this would otherwise further increase demand in the periphery through trade and terms-of-trade effects. Equation (9) constrains the leading authorities, so both the monetary authority and the core member’s fiscal authority.

### 3.1 The Non-Cooperative Regime

In the non-cooperative strategic regime, the core member’s fiscal authority and the monetary authority move simultaneously but non-cooperatively at the beginning of the game, and after shocks’ realisation. We begin with the common central bank’s problem. The bank minimises its loss function (equation (7)) subject to the union-wide constraints (4) and (5), and to the periphery’s fiscal reaction, given by equation (9). The resulting monetary rule resolves the standard trade-off between the inflation rate and the output gap at the union level [see Dixit and Lambertini (2001); Uhlig (2003); Andersen (2008); among others]:

$$\gamma = -\frac{\omega \pi}{a_M} \tag{10}$$

Equation (10) shows that the common central bank follows a standard ‘lean against the wind’ monetary policy [Clarida et. al. (1999)]. The monetary authority reacts to a possible rise (fall) in the union-wide inflation rate caused by an average negative (positive) supply shock by reducing (increasing) the union-wide output gap. In order to do that, it raises (decreases) the common nominal interest rate. In addition, its reaction is stronger the larger is the degree of nominal rigidity ($\omega$) and the lower is the weight assigned to output-gap stabilisation ($a_M$). Combining this monetary
rule with the Phillips curve (eq. (5)), we get equilibrium solutions for the union-wide inflation rate and output gap as follows:

\[ y = \frac{\omega}{\omega^2 + a_M} \varepsilon, \quad \text{and:} \]

\[ \pi = -\frac{a_M}{\omega^2 + a_M} \varepsilon \]

(11)

(12)

At equilibrium, both outcomes are standard results, driven by a portion of union-wide supply shocks which are only partially stabilised. Naturally, demand shocks are fully-stabilised at the union level; that is, ‘divine coincidence’ does hold in the absence of supply shocks [Blanchard and Gali (2007)].

The lead fiscal authority meanwhile minimises its own loss function (eq. (6)) subject to its own domestic aggregate demand (eq. (3)) and the follower’s reaction function (eq. (9)). Its fiscal rule is therefore:

\[ g_l = -\phi_f (1 - \chi) y_l = -\phi_l^{nc} y_l, \]

(13)

where ‘nc’ denotes the non-cooperative strategic regime, \( \phi_l^{nc} = \phi_f (1 - \chi) \) is the leader’s fiscal reaction parameter, and \( 0 < \chi = \frac{a_F \delta^2 \Lambda^2}{a_F \delta^2 K^2 + (1 - \delta)^2} < 1 \). The fiscal reaction parameter for the lead fiscal authority is therefore lower than the follower’s reaction parameter in eq. (8), which means that the leader pursues a less countercyclical fiscal policy than the follower. That suggests the following definition:

**Definition:** The absolute difference between the two fiscal reaction parameters, \( |\phi_l - \phi_f| \), defines the leader’s ‘fiscal strategic advantage’, i.e. the lead fiscal authority’s ability to react less countercyclically (than the follower) to changes in domestic aggregate demand.

We can now establish the first result of our paper:

**Result 1:** For the non-cooperative strategic regime, the fiscal strategic advantage depends positively on the interconnections between the core and the periphery in the monetary union. In the special case that the two countries are not connected through trade or competitiveness, this fiscal strategic advantage vanishes.

**Proof:** Using equations (8) and (13), the fiscal strategic advantage is given by:
\[ |\phi_i^{nc} - \phi_f| = |\phi_f \chi| = \frac{\alpha_0^2 \delta_j^2 (1+\delta_\omega)(\delta_y+\delta_\omega)^2}{(1-\delta_y)(1+\delta_y+2\delta_\omega)\alpha_0^2 (1+\delta_\omega)^2+(1-\delta_y)^2(1+\delta_y+2\delta_\omega)^2} \] .

Taking first derivatives of \(|\phi_i^{nc} - \phi_f|\) with respect to \(\delta_y\) and \(\delta_\tau\), and after some tedious algebra, they can be found to be positive: \(\frac{\partial (|\phi_i^{nc} - \phi_f|)}{\partial (\delta_y; \delta_\tau)} > 0\). Moreover, it is straightforward that \(\phi_i^{nc} = \phi_f\) for \(\delta_\tau = \delta_y = 0\).

This result contradicts Chortareas and Mavrodimitrakis (2017), who consider fiscal leadership where the monetary authority plays follower to both fiscal authorities (while the fiscal sequential asymmetry remains; so a three-stage game is assumed) to cause this result to be reversed. In that case, trade and competitiveness reduce any fiscal strategic advantage because the central bank has to become more restrictive in order to safeguard its own objectives. In the case here, we can see that without direct (through interconnections) or indirect (through the sequence of moves) links there is no particular role for the lead fiscal authority to play. In either case, leadership and fiscal advantage work through both direct (economic) and indirect (policy) links between economies.\(^\text{14}\)

Solving the rest of the model, we get the individual country fiscal positions\(^\text{15}\):

\[ g_j = \psi^{nc} \phi_j \left[ -\frac{1}{2} (u_j - u_k) - \delta_\tau (\varepsilon_j - \varepsilon_k) \right] \left[ 1 + \delta_y + 2\delta_\tau \omega + \delta_\phi \phi_k \right] (\omega^2 + \alpha_M) \varepsilon \right], \quad (14) \]

where \(\psi^{nc} = \frac{1}{1+\delta_y+2\delta_\tau \omega+\delta_\phi \phi^{nc}} > 0\) and \(\phi^{nc} = \frac{1}{2} (\phi_i^{nc} + \phi_f)\) is the union-wide (average) fiscal reaction parameter.\(^\text{16}\) Each fiscal authority reacts counter-cyclically to positive (domestic minus foreign) demand or supply shock asymmetries as well as to union-wide supply shocks. Supply shock asymmetries pass through the terms-of-trade effect. Using the country-specific fiscal rules, equation (8) for the periphery fiscal authority and equation (13) for the core, we can compute the country-specific output gaps. They are given by:

\(^{14}\) This result also holds for a regime of monetary leadership, in which the monetary authority leads with respect to the core member’s fiscal authority.

\(^{15}\) Regarding the lead fiscal authority, combine the core member’s fiscal rule (13) with its country-specific aggregate demand (3) in order to get a reaction function. Then use union-wide aggregate demand (4) with the equilibrium outcome for the union-wide output gap (11) to extract a common nominal interest rate as a function of the union-wide fiscal stance. Finally, substitute the common interest rate into the fiscal reaction functions and solve together.

\(^{16}\) The general description for the union-wide fiscal reaction parameter that holds in all the strategic regimes is given by \(\phi = \frac{1}{2} (\phi_i + \phi_k)\). In the rest of the paper, there are upper scripts in all the \(\psi\), \(\phi_i\) and \(\phi\) parameters that denote the alternative institutional arrangements.
\[
y_j = \psi^{nc} \left[ \frac{1}{2} (u_j - u_k) + \delta_\tau (\varepsilon_j - \varepsilon_k) + (1 + \delta_y + 2\delta_\tau \omega + \delta_g \phi_k) \frac{\omega}{\omega^2 + \alpha_M} \varepsilon \right]
\]

(15)

It is straightforward from equation (15) that the country-specific output gaps for both countries are exactly the opposite under shock asymmetries. However, the lead fiscal authority’s output gap is more susceptible to union-wide shocks since \( \psi^{nc} (1 + \delta_y + 2\delta_\tau \omega + \delta_g \phi_f) > 1 \) and \( \psi^{nc} (1 + \delta_y + 2\delta_\tau \omega + \delta_g \phi^{nc}_l) < 1 \). This result widens with the fiscal strategic advantage (i.e. as \( \phi_f \) increases and \( \phi^{nc}_l \) decreases).

The gap between the two fiscal policies is:

\[
g_l - g_f = \psi^{nc} \left\{ - \phi^{nc} \left[ (u_l - u_f) + 2\delta_\tau (\varepsilon_l - \varepsilon_f) \right] + (\phi_f - \phi^{nc}_l) (1 + \delta_y + 2\delta_\tau \omega) \frac{\omega}{\omega^2 + \alpha_M} \varepsilon \right\}
\]

(16)

Equation (16) shows that the fiscal gap increases in absolute terms with the asymmetries in the demand and supply shocks, and with the size of union-wide supply shocks. Under asymmetric shocks, the fiscal gap also widens with the union-wide fiscal reaction parameter, while for union-wide supply shocks it widens with the degree of (fiscal) strategic advantage.\(^\text{17}\) We get exactly the opposite result for the union-wide fiscal stance, pointing to a natural conflict between union and national fiscal policies:

\[
g = \psi^{nc} \left\{ \frac{1}{2} (\phi_f - \phi^{nc}_l) \left[ (u_l - u_f) + \delta_\tau (\varepsilon_l - \varepsilon_f) \right] - \left[ (1 + \delta_y + 2\delta_\tau \omega) \phi^{nc} + \delta_g \phi_f \phi^{nc}_l \right] \frac{\omega}{\omega^2 + \alpha_M} \varepsilon \right\}
\]

(17)

Equation (17) shows that the union-wide fiscal stance depends on demand and supply shock asymmetries so long as there is some fiscal strategic advantage: \( \phi_f \neq \phi^{nc}_l \). Moreover, asymmetric supply shocks are due to the terms-of-trade effect between two member-states. The union-wide fiscal stance however remains countercyclical to union-wide supply shocks (positive for negative union-wide supply shocks). Regarding demand or supply shock asymmetries, when shocks are larger in the core (periphery), union-wide fiscal policy ends up as pro-(counter)-cyclical because the follower puts more effort into output stabilisation than the core economy.

Turning to monetary policy, the common nominal interest rate at equilibrium is given by:

\(^\text{17}\) We get qualitatively the same results for the relative output gap, \( y_l - y_f \), using equation (15).
\[
i = \frac{1}{\delta_r} u + \frac{\psi_{nc}}{\delta_r} \times \left\{ \frac{1}{2} \delta_g (\phi_f - \phi_l^{nc}) \left[ \frac{1}{2} (u_l - u_f) + \delta_t (\epsilon_l - \epsilon_f) \right] - \left[ (1 - \delta_y) (1 + \delta_y + 2e_t \omega) + 2 (1 + \delta_t \omega) \frac{-g}{\delta_g \phi_f} \frac{\phi_l^{nc}}{\omega^{2 + a_M} e} \right] \right\}
\]

Equation (18) shows that the monetary authority reacts explicitly to the union-wide fiscal stance even under asymmetric shocks. Specifically, it is able to counter any fiscal over-expansion (for positive asymmetries) that may emerge from fiscal strategic advantage at the union level, since the equilibrium solutions for both union-wide inflation and output gap, given by equations (11) – (12), are not affected by shock asymmetries.

Hence, the union-wide fiscal stance and the common nominal interest rate act as strategic substitutes in the face of asymmetric shocks. The common central bank meanwhile fully stabilises union-wide demand shocks at the union level. Finally, in the face of union-wide supply shocks, the monetary authority pursues a pro-cyclical policy, acting restrictively to negative supply shocks, which reveals that the two policy instruments are again substitutes in the stabilisation process.

### 3.2 The Simultaneous Move Regime

We can now examine the simultaneous-move regime, where all the authorities play simultaneously (a Nash equilibrium). Since countries are identical, both fiscal authorities follow the fiscal rule described by equation (8). The monetary authority’s problem remains as before, which means that equations (10) – (12) remain the same. This further means that the monetary authority is indifferent between the two policy games, namely the standard simultaneous-move and a non-cooperative game where there is fiscal leadership. It follows that equations (14) – (18) still hold with \( \phi_l^{SM} = \phi_f \frac{a_f \delta_g K}{1 - \delta_y} \) and \( \psi^{SM} = \frac{1}{1 + \delta_y + 2 \delta_t \omega + \delta_g \phi_f} \) where ‘SM’ denotes a standard ‘Simultaneous Move’ game. After setting \( \phi_l = \phi_f \) in equations (16) – (18), we easily see that: (i) the two policy instruments at the union level do not react to shock asymmetries; and (ii) the fiscal gap is not affected by union-wide supply shocks.

### 3.3 Partially Cooperative Regimes: Coalitions in Leadership

This sub-section deals with the case in which the lead fiscal authority cooperates with the common central bank. The follower fiscal authority’s problem remains the same as before, while its fiscal rule, equation (8), is taken into account by the common central bank and the lead fiscal
authority when they jointly decide on their policy instruments. They minimise the following joint loss function:

$$L_{M,F_i} = \beta L_M + (1 - \beta)L_{F_i},$$  \hspace{1cm} (19)$$

subject to the country-specific and union-wide descriptive equations (1) – (2) and (4) – (5) respectively, and the follower fiscal authority’s rule (8), where $\beta \in (0,1)$ is the bargaining power of the common central bank. Solving this variant of our model, we obtain the following fiscal rule for the lead fiscal authority:

$$g_l = -\frac{a_F \delta g}{2(1+\delta_y+2\delta_t \omega)+\delta g \phi_f} y_l = -\phi^c_l y_l,$$  \hspace{1cm} (20)$$

where $\phi^c_l = \frac{a_F \delta g}{2(1+\delta_y+2\delta_t \omega)+\delta g \phi_f} > 0$ is the leader’s fiscal reaction parameter in this regime (the upper script ‘$c$’ denotes the ‘cooperative’ regime). Again it is positive, which means the lead fiscal authority acts counter-cyclically. It is also easy to verify that the lead fiscal authority’s reaction parameter in this cooperative case is lower than the corresponding parameter under non-cooperation, and hence lower than that for the follower fiscal authority. Thus: $\phi^c_l < \phi^{nc}_l < \phi_f$.

Under cooperation, there is a fiscal strategic advantage even when there are no interconnections between the two member-states ($\delta_y = \delta_t = 0$) given by $|\phi^c_l - \phi_f| = \frac{a_F \delta g(1+a_F \delta \gamma)}{2+a_F \delta g}$. The core member’s fiscal authority, by cooperating with the monetary authority, exploits the common nominal interest rate. This provides the strategic advantage.

The lead fiscal authority’s fiscal stance can be computed as
details can be found in the Appendix B.

$$g_l = \psi^c \phi^c_l \left[ -\frac{1}{2} (u_l - u_f) - \delta_t (e_l - e_f) - (1 + \delta_y + 2\delta_t \omega + \delta g \phi_f) \frac{\omega}{\omega^2 + a_M} e \right],$$  \hspace{1cm} (21)$$

where $\psi^c = \frac{1}{1+\delta_y+2\delta_t \omega+\delta g \phi^c+\beta \lambda} > 0$, where $\phi^c = \frac{1}{2} (\phi^c_l + \phi_f)$ is the union-wide average fiscal reaction parameter, and where $\hat{\beta} = \frac{1-\beta}{\beta}$, $\hat{\alpha} = \frac{a_F}{\omega^2 + a_M}$ and $\lambda = \frac{2(1+\delta_y+2\delta_t \omega+\delta g \phi_f)^2}{2(1+\delta_y+2\delta_t \omega+\delta g \phi_f)}$. Equation (21) is symmetric to the corresponding one from the non-cooperative case, (14). A simple comparison reveals that the lead fiscal authority’s reactions to various shocks are milder in the cooperative case.
regime than those in the matching non-cooperative case, which means that cooperation within the coalition is stabilising. Moreover, in the cooperative regime, the follower fiscal authority’s fiscal stance is symmetric to equation (21) for union-wide (supply) shocks, but not symmetric under national asymmetries:

\[
g_f = \psi_f \phi_f \left\{ (1 + 2\mu) \left[ \frac{1}{2} (u_l - u_f) + \delta_r (\epsilon_l - \epsilon_f) \right] - (1 + \delta_y + 2\delta_r \omega + \delta_g \phi^c_l) \frac{\omega}{\omega^2 + a_M} \right\} \tag{22}
\]

where \( \mu = \frac{\beta \lambda}{1 + \delta_y + 2\delta_r \omega + \delta_g \phi^c} \). So, it is straightforward to see that, for asymmetric shocks on either the demand or the supply side, the follower authority’s reactions are stronger than those of the leader – a new source of friction. In fact, \((1 + 2\mu) \phi_f > \phi_f > \phi^c_l\).

It is also straightforward to obtain the equilibrium solutions for both countries’ output gaps, using their corresponding fiscal rules. Qualitatively, the results will be the same. Regarding union-wide supply shocks, results are similar to the corresponding analysis for the previous institutional arrangement, following equation (15), but now the peripheral member’s fiscal authority would be better off since \(\phi^c_l < \phi^c_{nc}\).

The union-wide fiscal stance meanwhile is:

\[
g = \psi_f \psi^c \left\{ \frac{1}{2} \left[ (1 + 2\mu) \phi_f - \phi^c_l \right] \left[ \frac{1}{2} (u_l - u_f) + \delta_r (\epsilon_l - \epsilon_f) \right] - \left[ (1 + \delta_y + 2\delta_r \omega) \phi^c_l + \delta_g \phi^c \phi^c_l \frac{\omega}{\omega^2 + a_M} \right] \right\} \tag{23}
\]

Equation (23) therefore shows that the union-wide fiscal stance ends up reacting pro-cyclically under positive (leader minus follower) shock asymmetries (shocks that are stronger in the core than in the periphery), since the reaction of the follower is stronger than that of the leader. But, at the same time, it is counter-cyclical under union-wide supply shocks.

One implication is that, in contrast to non-cooperative regimes, the degree of flexibility for an inflation-targeting central bank \(a_M\) now affects the parameter \(\psi^c\). That affects all equilibrium solutions, including fiscal reactions to asymmetric shocks. Specifically, \(\frac{\partial \psi^c}{\partial (a_M)} > 0\) which means a more flexible inflation-targeting central bank will strengthen the leader’s reaction to asymmetric shocks.

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\(^{19}\) It is so, since \(\psi^c \phi^c_{nc} - \psi^c \phi^c_l = \psi^c \psi^c \left[ \frac{\beta \lambda}{1 + \delta_y + 2\delta_r \omega + \frac{1}{2} \delta_g \phi^c} \right] > 0.\)
shocks, \( \frac{\partial g_l}{\partial (\alpha_M)} = \frac{\partial \psi^c}{\partial (\alpha_M)} > 0 \). A similar result holds if the monetary authority’s bargaining power, \( \beta \), were to increase.

**Outcomes:** The union-wide output gap and the inflation rate at equilibrium are now given by:

\[
y = \psi^c \left\{ (-\mu) * \left[ \frac{1}{2} (u_l - u_f) + \delta_t (\epsilon_l - \epsilon_f) \right] + (1 + \delta_y + 2\delta_t \omega + \delta_g \phi^c) \frac{\omega}{\omega^2 + \alpha_M} \epsilon \right\} + \left(1 + \delta_y + 2\delta_t \omega + \delta_g \phi^c \right) \alpha_M + \frac{\beta a_M \lambda}{\omega^2 + a_M} \epsilon
\]

(24)

\[
\pi = \psi^c \left\{ (-\omega \mu) * \left[ \frac{1}{2} (u_l - u_f) + \delta_t (\epsilon_l - \epsilon_f) \right] - \left(1 + \delta_y + 2\delta_t \omega + \delta_g \phi^c \right)a_M + \frac{\beta a_M \lambda}{\omega^2 + a_M} \epsilon \right\}
\]

(25)

Equations (24) and (25) underline a major difference with the non-cooperative case and with the rest of the literature. Under partial cooperation, as here, shock asymmetries affect both the union-wide inflation rate and the output gap.\(^{20}\) The crucial parameter is \( \beta \); without partial cooperation, \( \beta = 1 \), then \( \hat{\beta} = \mu = 0 \). So the equilibrium solutions are again given by equations (11) and (12).

We can also compute the common nominal interest rate at equilibrium by substituting for the union-wide output gap (24) and fiscal stance (23) in union-wide aggregate demand (4). We get:

\[
i = \frac{u}{\delta_r} + \psi^c \left\{ \left[ \frac{1}{2} \delta_g (\phi_f - \phi^c_l) + (1 - \delta_y + \delta_g \phi_f) \mu \right] * \left[ \left[ \frac{1}{2} (u_l - u_f) + \delta_t (\epsilon_l - \epsilon_f) \right] - \left(1 - \delta_y \right) (1 + \delta_y + 2\delta_t \omega) + 2(1 + \delta_t \omega) \delta_g \phi^c + \delta_g^2 \phi_f \phi^c \right] \frac{\omega}{\omega^2 + \alpha_M} \epsilon \right\}
\]

(26)

It follows that the common central bank will fully stabilise union-wide and common demand shocks. But it can only partially stabilise asymmetric shocks on the demand side or supply side. It will also react to the fiscal strategic advantage, \( \phi_f - \phi^c_l \), as in the non-cooperative case (see eq. (18)). But it cannot fully counter the fiscal reactions, even if its reaction parameter is now stronger by the factor \( (1 - \delta_y + \delta_g \phi_f) \mu \). It is interesting to note that this parameter increases as the central bank loses bargaining power to the lead fiscal authority since \( \frac{\partial \mu}{\partial \beta} < 0 \). It is also easy to verify that the monetary authority reacts pro-cyclically to union-wide supply shocks, which makes the two policy instruments, the common nominal interest rate and the union-wide fiscal stance, strategic substitutes when stabilising union-wide supply shocks.

\(^{20}\) Chortareas and Mavrodimitrakis (2016) show that demand shock asymmetries pass through at the union level if fiscal policy can directly affect the inflation rate in a fiscal leadership game with sequential fiscal asymmetries.
3.4 The Fiscal Leadership Strategic Regime

In this sub-section, we investigate the case of fiscal leadership, where the lead fiscal authority leads the monetary authority; hence, the latter plays between the two fiscal authorities. Its problem is the same as in the non-cooperative strategic regime, defined by equations (10) – (12). In this game, the monetary authority’s optimal reaction function\(^{21}\) will be:

\[ i = \frac{1}{\delta_t [2(1+\delta_y+2\delta_t\omega)+\delta_g\phi_f]} \left\{ (1 + \delta_y + 2\delta_t\omega + \delta_g\phi_f)^2 g_l + \delta_g\phi_f\delta_t (\varepsilon_l - \varepsilon_f) + 2(1 + \delta_y + 2\delta_t\omega)u + \delta_g\phi_f u_l - 2[(1 - \delta_y)(1 + \delta_y + 2\delta_t\omega) + (1 + \delta_t\omega)\delta_g\phi_f] \right\} \cdot \frac{\omega}{\omega^2 + a_M} \varepsilon. \] (27)

This equation shows that the monetary authority will react counter-cyclically to the lead fiscal authority’s fiscal stance, to union-wide demand shocks, and to supply shock asymmetries, as well as to the follower’s reaction to a demand shock that hits the core member-state \((\phi_f u_l)\). But it reacts pro-cyclically to union-wide supply shocks. Equation (27) therefore works as a constraint on the lead fiscal authority’s decision-making problem.

Meanwhile the fiscal authority in the core member-state minimises its loss function (eq. (6)) by taking into account the peripheral state’s reaction function (eq. (9)) and the central bank’s reaction function (eq. (27)). The fiscal rule for the lead fiscal authority is computed as:

\[ g_l = \frac{a_r \delta_g}{2(1+\delta_y+2\delta_t\omega)+\delta_g\phi_f} y_l = -\phi_l^c y_l, \] (28)

where \( \phi_l^c = \frac{a_r \delta_g}{2(1+\delta_y+2\delta_t\omega)+\delta_g\phi_f} > 0 \). This equals the leader’s fiscal reaction parameter under cooperation: equation (20). Thus, the leader’s fiscal reaction parameter, when the monetary authority plays between the two fiscal authorities, equals that when the lead fiscal authority cooperates with the monetary authority. This means that fiscal leadership (over the monetary authority) enhances the cooperation between the two authorities [supporting Hughes Hallett and Weymark (2007)]. In this case too, there exists a fiscal strategic advantage even in the absence of interconnections, as the lead fiscal authority can exploit its ability to influence the common nominal interest rate by setting its fiscal policy as it would with a degree of implicit coordination.

\(^{21}\) In order to compute the monetary authority’s reaction function, we start with the union-wide aggregate demand equation (4), into which we substitute for the union-wide output gap at equilibrium (eq. (11)) and for the periphery member’s fiscal stance using its reaction function (eq. (9)).
Solving this model again, we find that equations (14) – (18) that describe the non-cooperative case still hold for this game of fiscal leadership for the core fiscal authority’s reaction parameter equal to $\phi^c$ and $\psi^{FL} = \frac{1}{1+\delta_y+2\delta_o+\delta_y \phi^c}$, where ‘FL’ stands for the ‘Fiscal Leadership’ strategic regime.

3.5 Discussion

Following the lead fiscal authority’s reaction parameters in all the alternative strategic regimes and the follower fiscal authority’s reaction parameter, namely equations (13), (20), (28) and equation (8), we can establish a key result regarding the fiscal strategic advantage and explicit/implicit coordination.

**Corollary 1:** As cooperation increases at the core of the monetary union, the fiscal strategic advantage increases since the core member’s fiscal authority follows a less countercyclical fiscal policy. Moreover, there is no distinction between the cooperative and the fiscal leadership strategic regimes; that is between explicit or implicit cooperation at the core.

Thus fiscal strategic advantage depends on direct (through interconnections) and indirect (through the sequence of moves) links between the two member-states. In the non-cooperative strategic regime, the sequential fiscal asymmetry alone cannot provide a fiscal strategic advantage if there are no interconnections. Since the monetary authority moves simultaneously, there is nothing to be exploited by the lead fiscal authority. However, there is a fiscal strategic advantage in both the cooperative and the fiscal leadership strategic regimes, since the lead fiscal authority also exploits the monetary authority even if there are no interconnections between the two member-states. In those cases, the fiscal strategic advantage works through both direct and indirect links.

The existence of a fiscal strategic advantage is indeed quite informative regarding the welfare implications of different institutional arrangements for the core fiscal authority, so on the latter’s incentives to adopt those regimes. However, this does not mean that there is a welfare advantage at all times. Moreover, it is interesting to investigate the welfare implications for the peripheral member’s fiscal authority, the monetary authority, as well as of a social planner. Which regime is preferable from whom; do they coincide or differ to some specific shock configurations; which regime is more possible to prevail as an endogenous outcome? These are all important questions that we answer in the next two sections.
4. National Fiscal Authorities’ Welfare Analysis

This section assesses the welfare implications of the alternative institutional arrangements for the two national fiscal authorities in the union. This assessment is based on a computation of their expected losses. The authorities are the core fiscal authority and the peripheral fiscal authority. We provide a welfare ranking for the two fiscal authorities. This gives us an idea of their incentive to adopt different decision making arrangements in the monetary union. It also provides the first part of a possible answer to a more difficult question: which regime is more likely to endogenously prevail among the leading authorities?22 The evaluation of the authorities’ loss functions is made ex ante, conditional on demand and/or supply shocks. In the next section, we go on to a welfare comparison of the alternative regimes from the monetary authority’s perspective, as well as from a social planner’s.

The expected losses for the two national fiscal authorities and the monetary authority are presented in the Appendix C. We can see there the impact of the terms-of-trade effect under supply shocks. To be specific, it shows the analysis depends on the size of the shocks, since the solutions depend on both asymmetric and aggregate supply shocks. In the special case that there are no terms-of-trade effects, \( \delta_T = 0 \), we could consider union-wide supply shocks. However, in general where \( \delta_T > 0 \), we assume that in both countries supply shocks share a common standard deviation, \( \sigma_{\varepsilon_j} = \sigma_{\varepsilon_k} = \sigma_{\varepsilon} \), but are linked by a correlation coefficient \( \rho_{\varepsilon} \) to indicate the degree of communality. This way we distinguish between idiosyncratic and common shocks. In a similar vein, Ferre (2008) treats an exogenous terms-of-trade effect as a shock in competitiveness which is uncorrelated to supply and demand shocks. That shock then works exactly like an asymmetric demand shock, and allows the author to consider union-wide supply shocks. In our case, the analysis for idiosyncratic supply shocks coincides with that for asymmetric demand shocks.24

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22 The second part is given in the next section; it is essential the monetary authority’s welfare analysis.
23 This assumption is made to get analytical results and follows Beetsma and Jensen (2005). We follow this approach in the rest of the paper. By using this assumption, we examine only common and idiosyncratic shocks. The solutions for less than idiosyncratic asymmetric shocks lie in between. So, this assumption is not as restrictive as it may seem.
24 To summarise: (i) all results regarding macroeconomic stabilisation and welfare comparisons among the alternative strategic regimes for idiosyncratic supply shocks coincide with the corresponding ones for asymmetric demand shocks; and (ii) in the special case that there are no (or negligible) terms-of-trade effects, \( \delta_T = 0 \), all the results for common supply shocks hold for union-wide (aggregate) shocks.
We start our analysis by comparing the expected losses of the national fiscal authorities in each regime to determine whether a fiscal strategic advantage delivers a welfare advantage for the lead fiscal authority. We establish the following result:

**Result 2:** The fiscal strategic advantage delivers a welfare advantage in all the strategic regimes under asymmetric demand shocks, and under idiosyncratic supply shocks, if a terms-of-trade effect exists between the two member-states.

**Proof:** See Appendix D.

The core member’s fiscal authority is always better off than the peripheral fiscal authority under asymmetric demand shocks. The rationale is the following: demand shocks have similar effects on output and inflation, hence no conflict of interest between the monetary and the fiscal authorities exists. However, there is a conflict of interest between the two fiscal authorities, since each authority tries to diminish the demand shock asymmetries. So the lead fiscal authority has an incentive to pass the stabilisation burden onto the follower by pursuing a less countercyclical fiscal policy. Meanwhile the monetary authority does not neutralise the union-wide fiscal stance. This story is similar to Chortareas and Mavrodimitrakis (2017) in the standard fiscal leadership regime where the monetary authority is the last player in the game. But it is more pronounced in our core-periphery setting, since (i) the peripheral fiscal authority is always the follower in all the strategic regimes (hence the monetary authority always leads with respect to the peripheral fiscal authority); and (ii) the core member’s fiscal authority cooperates with the monetary authority in the cooperative strategic regime.

The situation for supply shocks is more complicated. Those shocks have opposite effects on output and inflation, leading to a conflict of interest between the monetary and fiscal authorities. Chortareas and Mavrodimitrakis (2017) show that in the absence of a terms-of-trade effect the follower fiscal authority is always better off than the leader under union-wide shocks. This result rests entirely on the game since both monetary authority and follower fiscal authority move at a later stage than the lead fiscal authority, providing a ‘second-mover’ advantage. In our core-periphery setting, this is not always the case, even in the absence of a terms-of-trade effect, since in all regimes the monetary authority leads the peripheral fiscal authority; so the former may not always be able to counter fiscal policies. The existence of terms-of-trade effects further complicates the analysis, since each fiscal authority’s response will always be in conflict with the
other fiscal authority through shock asymmetries, or with its response to the monetary authority’s response to union-wide supply shocks (if, e.g., both shocks are negative but the one at the core is larger).

A critical observation in the cooperative strategic regime is that the central bank’s bargaining power, $\beta$, plays no role in ranking welfare gains between the two fiscal authorities under union-wide supply shocks. It can be the case that the peripheral state’s fiscal authority is better off than the core fiscal authority even under common supply shocks. This result undermines the case for creating a single economic governance system in the Euro area – especially among the lead policy-makers – because single governance does not guarantee an independent central bank.

We begin our comparisons of the alternative institutional arrangements with the core fiscal authority. We establish the following result:

**Result 3:** The welfare ranking for the lead fiscal authority is shock-independent. In particular, the cooperative strategic regime is preferable to the fiscal leadership one, the fiscal leadership to the non-cooperative one, and the latter to the simultaneous-move strategic regime.

**Proof:** See Appendix D.

Hence the lead fiscal authority’s preferred choice will be independent of the type or size of shocks. More cooperation means a better position for the core member’s fiscal authority. Following **Corollary 2**, as cooperation at the core is enhanced, the lead fiscal authority pursues a less countercyclical fiscal policy. However, there is a preference for explicit over implicit cooperation since the fiscal leadership regime is inferior to the cooperative one; but still superior to the non-cooperative regime. The irrelevance of shocks means that the result is entirely driven by the strategic game. It is also independent on the monetary authority’s bargaining power, $\beta$. Even if this is really large, which means that the monetary authority actually chooses the leader fiscal authority’s fiscal stance, the latter prefers this to the fiscal leadership strategic regime. This is not the case for the follower fiscal authority, though.

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25 See the exact computations for the cooperative strategic regime in Appendix D, Proof of Result 2.
Result 4: The welfare ranking for the follower fiscal authority is shock-dependent. In particular, it coincides with the lead fiscal authority’s ranking under common supply shocks, but it is exactly the opposite under asymmetric demand and/or idiosyncratic supply shocks.

Proof: See Appendix D.

In the face of asymmetric demand shocks, the ranking for the peripheral fiscal authority is the exact opposite to that of the lead fiscal authority since sequential fiscal asymmetries work in favour of the leader under asymmetric demand shocks, implying a natural conflict between core and periphery. In the face of supply shocks with the same standard deviation in each country, the follower authority prefers a simultaneous-move regime under idiosyncratic shocks; but prefers the cooperative regime under common shocks, and then the fiscal leadership. Thus, partial cooperation (either explicit or implicit) enhances the position of the peripheral member-state in the union under common supply shocks.

5. Macroeconomic Stabilisation and Welfare Analysis at the Union Level – The Monetary Authority and the Social Planner

This section is dedicated to the macroeconomic stabilisation and welfare implications of our alternative institutional arrangements at the union level. We are interested in comparing the volatility of the main macroeconomic variables, union-wide inflation, the output gap and fiscal stance, across regimes. The analysis for inflation and the output gap leads to a welfare analysis on the part of the monetary authority, following its loss function given by equation (7), while the consideration of the union-wide fiscal stance enables us to discuss welfare analysis from a social planner’s perspective for the union as a whole.\footnote{The social planner’s loss function is going to be defined further down.} We proceed with the following results.

Proposition 1: Both the inflation rate and the output gap: (i) can be fully stabilised under demand and idiosyncratic supply shocks, while they are equally volatile under union-wide supply shocks for the simultaneous-move, non-cooperation and fiscal leadership strategic regimes; (ii) but they are more volatile in the cooperative regime under asymmetric demand and idiosyncratic supply shocks, but (likely, for inflation) less volatile under common supply shocks.

Proof: See Appendix D.
The cooperative strategic regime delivers quite different results regarding both the inflation rate and the output gap, since shock asymmetries pass through at the union level. Regarding common supply shocks, the output gap is unambiguously less volatile while inflation is more likely to be less volatile too. What are then the implications for the monetary authority? We have the following result:

**Result 5**: The welfare ranking for the monetary authority is shock-dependent: (i) for asymmetric demand and idiosyncratic supply shocks, the monetary authority is indifferent to the institutional arrangements involving simultaneous-moves, non-cooperation, and fiscal leadership, while all are preferable to the cooperative strategic regime; (ii) common demand shocks can be fully stabilised in all strategic regimes; (iii) for common supply shocks, the cooperative regime is more likely to be superior the higher is the degree of nominal rigidity, so long as the monetary authority follows a flexible inflation-targeting approach.

**Proof**: See Appendix D.

For the non-cooperative, simultaneous-moves and fiscal leadership regimes, the monetary authority’s expected losses are exactly the same. In the cooperative regime, the interesting case is common supply shocks. There, the degree of nominal rigidities in product and labour markets, and the degree of flexibility in the central bank’s inflation-targeting framework define the actual outcome. The superiority of the cooperative strategic regime holds when there is a high degree of nominal rigidities but flexible inflation targeting. The special case of a strict inflation targeting central bank makes the non-cooperative strategic regime superior since union-wide inflation cannot be fully stabilised in the cooperative regime: see equation (25). In this case, the common central bank is forced to include the fiscal authorities’ objectives if its bargaining power \( \beta < 1 \).

Both the assumptions of a flexible inflation-targeting central bank and a flatter Phillips curve are

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27 Following the Phillips curve equation (2) and the monetary authority’s loss function (eq. (7)), where the latter provides the monetary rule described by equation (10), the monetary authority is indifferent to any strategic regimes in the monetary union, even when there are asymmetries on the fiscal side, as long as the monetary authority moves independently (i.e., does not cooperate with the lead fiscal authority).

28 Assuming \( \omega < 1.0 \) and low. Most of the theoretical literature proceeds by means of numerical illustrations which assume \( 0.2 < \omega < 0.5 \) [van Aarle et. al. (2002); Andersen (2005; 2008)], although there is a worry that \( \omega > 1 \) may prevail in inflationary periods. More recent OECD experience suggests \( \omega < 1 \) and really low, a possible bi-product of structural reforms and a ‘flattening’ Phillips curve after the Great Recession [Bokan and Hughes Hallett (2008)]. In particular, there is an important part of literature that tries to explain (i) the missing disinflation in the wake of the crisis, and (ii) the excessive disinflation after 2012, especially in the euro area [IMF (2013)]. See Daly and Hobijn (2014), Murphy (2014) and Hughes Hallett (2018).
good approximations of the situation in the EMU during the sovereign debt crisis. We now turn to the union-wide fiscal stance:

**Proposition 2:** The union-wide fiscal stance: (i) for asymmetric demand or idiosyncratic supply shocks, it is less volatile in a simultaneous-move strategic regime than in the non-cooperative one, which in turn is less volatile than in fiscal leadership, which is less volatile than in the cooperative regime, with; (ii) has exactly the opposite ordering under common supply shocks.

**Proof:** See Appendix D.

So, the outcome is shock-dependent. More cooperation at the core produces a more volatile union-wide fiscal stance under asymmetric demand or idiosyncratic supply shocks, but less volatility under common supply shocks. The union-wide fiscal stance is an important, although ‘mechanical’ concept for the EMU. Ways to assess and manage the ‘appropriate’ union-wide fiscal stance are at the centre of the discussion in the EMU. This is defined as the one that balances the objectives of sustainable public finances and macroeconomic stabilisation, implying a trade-off between long-run and short-run objectives for expansionary policies. The current institutional framework in the EMU has no instruments to manage the union-wide fiscal stance, or to assess conflicts between the union and national levels. These comparisons of union-wide fiscal volatility allow us to construct welfare comparisons at the union level on the part of a social planner.

Following Beetsma and Bovenberg (1998) and Andersen (2008), among others, we define the social planner’s loss function in union-wide variables:

$$ L_{EC} = \frac{1}{2} \left( \pi^2 + c_y y^2 + c_g g^2 \right), $$

(29)

where ‘EC’ stands for the ‘European Council’, designated as the social planner for EMU, and where $c_y$ and $c_g$ are the EC’s weights on the union-wide output gap and fiscal stance stabilisation relative to inflation stabilisation. It is easy to show that minimisation of equation (29) brings the same outcomes for union-wide macroeconomic variables as full fiscal-monetary policy coordination between all three policy authorities. This holds in particular for $c_g = 1.0, c_y = a_F +$

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29 There are no debt considerations in our model. We have only assumed that the discretionary use of fiscal policy is costly for the fiscal authorities. For the importance of the concept of euro-area fiscal stance, see ECB Economic Bulletin (2016) and the report of the European Fiscal Board (2017).

30 This loss function is also used in DSGE models that consider optimal fiscal/monetary policies, such as Gali and Monacelli (2008) and Ferrero (2009).
\( a_M \) and the joint objective \( L_M + \frac{1}{2}(L_F_j + L_F_k) \) [see Andersen (2008); Chortareas and Mavrodimitrakis (2017)]. We instead use equation (29) as a welfare criterion to compare the alternative institutional arrangements. We can establish the following corollary, which can be easily obtained starting from Result 4, Proposition 1 and Proposition 2:

**Corollary 2:** The social planner’s welfare ranking is shock-dependent and coincides with the follower fiscal authority’s ranking in all cases.

**Proof:** See Appendix D.

The social planner’s welfare ranking is entirely driven by the volatility of the union-wide fiscal stance therefore, apart from in the cooperative strategic regime.\(^{31}\) This further demonstrates the importance of the union-wide fiscal stance and justifies the attention paid to it by the European Commission. Is there a way to compute the ‘appropriate’ fiscal stance in our model, so to compare with the fiscal stances of the alternative strategic regimes? We assume the ‘appropriate’ union-wide fiscal stance is the outcome of the minimisation of EC’s loss function, equation (29), subject to the union-wide descriptive equations (4) and (5); then we will find that the union-wide fiscal stance should be neutral; i.e., \( g = 0 \).\(^{32}\) Thus, Proposition 2 suffices for an assessment of the alternative strategic regimes in terms of the ‘appropriateness’ of the union-wide fiscal stance.

In summary, our analysis reveals that the least preferable regime for the social planner under asymmetric demand shocks is the cooperative one. In this case, the famous Rogoff (1985) result that partial cooperation can be counterproductive continues to hold. In the special case that supply shocks have the same standard deviation the social planner prefers non-cooperation for idiosyncratic shocks and cooperation for common shocks. In the other special case, with no terms-of-trade effects, \( \delta_r = 0 \), cooperation is always preferable from the social planner’s point of view under union-wide supply shocks which contradicts Rogoff’s result. Thus, the social planner would like to impose a non-cooperative strategic regime under asymmetric demand shocks; but partial cooperation under union-wide supply shocks. Which they choose will depend on which type of shock is larger or more prevalent in particular cases.

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\(^{31}\) See Proof of Corollary 2 in Appendix D.

\(^{32}\) The result will be the same under overall policy coordination, as discussed above.
The fiscal leadership regime always lies in the middle, which means that its outcomes are worse under non-cooperation with asymmetric demand or idiosyncratic supply shocks (at least given equal standard deviations). But fiscal leadership is likely to be better than non-cooperation under common supply shocks (but worse than partial cooperation). These results modify the conclusions in Hughes Hallett and Weymark (2007) and Dixit and Lambertini (2001; 2003a,b). In a core-periphery monetary union where the periphery member’s fiscal authority is always the follower, giving the lead to the core fiscal authority over the monetary authority can be better under common supply shocks, but is welfare-reducing under asymmetric demand shocks. The existence of a terms-of-trade effect complicates the analysis for union-wide supply shocks, reducing the desirability of implicit coordination in the core via fiscal leadership.

So, which strategic regime is more likely to prevail in a core-periphery monetary union? If we assume that the core member’s fiscal authority has the ability to choose the strategic regime at the beginning of the game, before the realisation of the particular shocks, then the most likely answer would be fiscal leadership. The fiscal leadership regime is the second-best choice for the core fiscal authority, with the first best being cooperation. However, a cooperative regime is only preferable for the monetary authority in the special case of common supply shocks. It delivers the worst outcomes for asymmetric demand shocks. This means that the common central bank is highly unlikely to proceed to partial cooperation, which means the core fiscal authority is likely to end up with a worse outcome. Moreover, the bank is indifferent between fiscal leadership and a non-cooperative strategic regime. Thus the core fiscal authority is more likely to try to lead, which makes the monetary authority the follower.

Combining Result 3 with Result 5 and Corollary 2, we get the following corollary:

**Corollary 3:** Under common supply shocks, more cooperation at the core is preferred by all the authorities in the monetary union, as well as by the social planner.

In the special case of common supply shocks, it is more likely that the monetary authority and lead fiscal authority would try to cooperate, a choice which is also unambiguously preferred by the follower (as well as the social planner). And if the monetary authority and the lead fiscal authority do decide to cooperate, the union-wide macroeconomic variables will be less volatile. If there are no terms-of-trade effects, $\delta_r = 0$, this result also holds unambiguously for union-wide supply shocks. That not only shows that partial cooperation may not always be counter-productive,
as suggested by Rogoff (1985), but also that the Rogoff position may be preferred by third parties who do not wish or are unable to cooperate with the policy making leadership. It would be safe to conclude that a cooperative strategic regime will be the dominant outcome under common supply shocks if the authorities are left to themselves to choose the actual regime at the start of the game, prior to policy actions.

6. Conclusion

We have compared and contrasted the performance of alternative institutional arrangements in a core-periphery monetary union from the point of view of the policy authorities and the social planner. We used a static two-country monetary union model based on a reduced-form structural DSGE model. In particular, we explored possibilities of (explicit or implicit) cooperation between the monetary authority and core fiscal authority when they both lead the game with respect to other members of the union. A regime of fiscal leadership at the core provides a form of implicit cooperation, where the monetary authority moves between the two fiscal authorities. This regime is compared to two regimes of non-cooperation vs. explicit cooperation at the core. We find a fiscal strategic advantage as we move towards more cooperation at the core, meaning that the core fiscal authority follows a less countercyclical policy, while it is equally countercyclical under implicit or explicit coordination. This means that the fiscal leadership strategic regime enhances cooperation between the monetary authority and core fiscal authority as previously argued in Hughes Hallett and Weymark (2007). We also find that a fiscal strategic advantage delivers a welfare advantage only for (asymmetric) demand and idiosyncratic supply shocks, but not for common supply (or demand) shocks, similar to Chortareas and Mavrodimitrakis (2017). On the contrary, in our model, the fiscal strategic advantage is the outcome of the existence of direct links (interconnections) between the two member-states in the monetary union, apart from the cooperative strategic regime.

Our main results can be summarised as follows: (i) the lead fiscal authority’s welfare ranking of the alternative regimes is shock-independent; (ii) the lead fiscal authority prefers a cooperative regime to fiscal leadership, and the latter to a non-cooperative regime, and a non-cooperative regime to a simultaneous-move (Nash) game between all three policy authorities; (iii) the welfare rankings for the common central bank, the follower fiscal authority, and the social planner are shock-dependent; (iv) the welfare ranking of the follower coincides with the social planner’s in every case.
In more detail: under asymmetric demand shocks the monetary authority is indifferent between the simultaneous-moves regime, the non-cooperative regime and fiscal leadership, all three being superior to a cooperative strategic regime. The follower prefers the simultaneous-move regime to non-cooperation, the latter to fiscal leadership, and fiscal leadership to a cooperative regime. This is the reverse ranking order to the lead fiscal authority, making hard to get agreement on which regime should prevail.

Turning now to supply shocks, the existence of a terms-of-trade effect between member states complicates our story, as shock asymmetries pass through the country-specific equilibrium solutions in conjunction with union-wide shocks. To simplify the analysis, we have assumed that shocks in both countries share the same standard deviation. That way we can consider idiosyncratic and common supply shocks. The results for idiosyncratic shocks are exactly the same as for asymmetric demand shocks. But with common shocks, the common central bank would prefer a cooperative strategic regime to the other three regimes so long as there are significant nominal rigidities in the product and/or labour markets. Meanwhile the follower fiscal authority prefers cooperation to fiscal leadership, the latter to a non-cooperative regime, and non-cooperation to simultaneous-moves between all authorities. The follower fiscal authority coincides with the lead fiscal authority in this case. Finally, if there are no (or insignificant) terms-of-trade effects between Union members, these results hold unambiguously for union-wide shocks and for common shocks.

Thus the welfare ranking for the fiscal authorities can either coincide (for common supply shocks) or be the reverse (asymmetric demand and idiosyncratic supply shocks). Moreover, the social planner’s welfare ranking coincides with that for the follower. For common supply shocks, all authorities agree on the cooperative strategic regime; hence this is likely to be the prevailing outcome. However, for asymmetric demand or idiosyncratic supply shocks, fiscal leadership is likely to prevail, being the second-best choice for the core fiscal authority while the monetary authority is indifferent between this regime and non-cooperative or simultaneous-move games. If the regime is chosen by the social planner instead, he/she “protects” the periphery member-state in the monetary union, even if their preferences seem to be different; and he/she will go for the simultaneous-move (Nash) game between all authorities.

Finally, the famous Rogoff (1985) result that partial cooperation can be counter-productive still holds under either (asymmetric) demand or idiosyncratic supply shocks (for shocks of equal
size), but breaks down under common supply shocks. Also fiscal leadership will be welfare inferior to the non-cooperative regime with asymmetric demand or idiosyncratic supply shocks, restricting the Hughes Hallett and Weymark (2007) and Dixit and Lambertini (2001; 2003a,b) results in those cases. These results are of prime importance for the Eurozone since: (i) the role of asymmetric demand shocks has been stressed, leading to the possibility of inferior outcomes (volatile fiscal outcomes) for the periphery and the union as a whole under fiscal leadership; (ii) the role of the terms-of-trade effect is crucial to determine if more cooperation at the core is welfare-enhancing under supply shocks; and (iii) it might be thought that the periphery was better served being a member of the union since the periphery’s welfare ranking coincides with the social planner’s ranking. The first point also mitigates against enlargement since new entrants may hesitate to join or existing members to stay if they face the prospect of being at the periphery in a union dominated by others. These results are also a warning that the Eurozone could be worse-off with a unified fiscal policy under the control of a European Ministry of Finance if it led to fiscal leadership or cooperation among the core policymakers when asymmetric demand or supply shocks prevail (which is likely most of the time).

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Appendix A

The reduced-form DSGE model for a two-country monetary union can be summarized as follows:

\[ y_{j,t} = E_t y_{j,t+1} - \delta_r (i - E_t \pi_{j,t+1} - \bar{r}) + \delta_y g_{j,t} + \delta_y y_{k,t} - \delta_r (\pi_{j,t} - \pi_{k,t}) + v_{j,t}, \]  
(A.1)

\[ \pi_{j,t} = \gamma E_t \pi_{j,t+1} + \omega y_{j,t} - e_{j,t}. \]  
(A.2)

It is common in the literature to assume that demand and supply shocks follow an AR(1) process, in that:

\[ v_{j,t} = \theta v_{j,t-1} + u_{j,t}, \]  
(A.3)

\[ e_{j,t} = \zeta v_{j,t-1} + \epsilon_{j,t}, \]  
(A.4)

where \( \theta, \zeta \in [0,1) \) and define shocks’ persistence [see Gali (2015)]. Both \( u_{j,t} \) and \( \epsilon_{j,t} \) are i.i.d. random variables with zero means and constant variances. The micro-foundations for this kind of model are well-known and can be found in Beetsma and Jensen (2005) and Gali and Monacelli (2008). Macroeconomic variables at the union level are defined as \( x_t = nx_{j,t} + (1-n)x_{k,t} \), where \( n \in (0,1) \).

In order to obtain equations (1) and (2) in the paper, we need some simplified assumptions. We assume that both shocks are purely transitory, so \( \theta = \zeta = 0 \), which essentially make the shocks as pure random, and that the equilibrium real interest rate is \( \bar{r}_j = 0 \) for both countries. We further assume that the private sector in each country forms expectations about future inflation rationally, that these expectations are formed prior to the shocks’ realization, and that all the authorities, namely the monetary authority and the two fiscal authorities, care about cyclical macroeconomic stabilisation. This latter assumption means that the authorities care about minimising deviations of macroeconomic variables from steady-state (long-run) equilibrium (see equations (6) and (7)). Those assumptions guarantee that policies are time-consistent; hence expected inflation should be equal to zero in both countries, i.e. \( \pi_j^e = 0 \). We follow Uhlig (2003) by using this result at the beginning, rather than deriving it as the last step of the calculation.

Thus, our model can be described by equations (1) and (2), so a static AD/AS two-country monetary union model, in which we got rid of the time sub-scripts. We further impose country-size symmetry, so that \( n = 0.5 \).
Appendix B

The minimisation of equation (19) provides us with another first order condition, instead of equation (20), with respect to the common nominal interest rate, which can be written as:

\[ \omega \pi + a_M y = -\frac{1-\beta}{\beta} \cdot a_F \cdot \frac{2(1+\delta_y+2\delta_\tau \omega+\delta_g \phi_f)}{2(1+\delta_y+2\delta_\tau \omega)+\delta_g \phi_f} y_t. \]  

(B.1)

Equation (B.1) can be easily converted to link the two fiscal stances. We first substitute for the union-wide inflation rate through the union-wide Phillips curve, equation (5), and then incorporate the two country-specific fiscal rules, equations (8) and (20). We end up with:

\[ \left[ \frac{1}{2}(\omega^2 + a_M) \delta_g + \frac{1-\beta}{\beta} \cdot 2(1+\delta_y+2\delta_\tau \omega+\delta_g \phi_f) \phi_f \right] \phi_f g_t = -\delta_g \phi_f \phi_f \omega \epsilon - \frac{1}{2}(\omega^2 + a_M) \delta_g \phi_f \phi_f g_f \]  

(B.2)

Equation (B.2) shows that the core member’s fiscal authority reacts negatively to a ‘foreign’ fiscal expansion from outside the coalition (a follower at the periphery of the monetary union, for example), acting as a strategic substitute. Equations (20) and (B.2) are then solved together to provide us the equilibrium solutions for the two fiscal stances. For equation (21), we substitute for the core member-state’s aggregate demand equation (3), and we substitute for the common nominal interest rate through the union-wide aggregate demand equation (4), incorporating also the two country-specific fiscal rules (equations (8) and (20)) to substitute for the union-wide output gap.

Appendix C

The analysis assumes that demand and supply shocks are uncorrelated. We compute expected losses for the two fiscal authorities, the monetary authority and the social planner.

C.1 The National Fiscal Authorities’ Expected Losses

The expected loss for country j’s fiscal authority for all the alternative strategic regimes is given by:

\[ E \left( L_{F_j} \right) = \frac{1}{2}(a_F + \phi_j^2)\psi^2 \left\{ \frac{1}{4} Var(u_j - u_k) + Var \left[ \delta_\tau (\epsilon_j - \epsilon_k) + (1 + \delta_y + 2\delta_\tau \omega + \delta_g \phi_k) \frac{\omega}{\omega^2 + a_M} \epsilon \right] \right\}, \]  

(C.1)
where ‘E’ is the expectations operator and \( \psi, \phi_j \) and \( \phi_k \) are all regime-dependent. For the non-cooperative strategic regime (Section 3.1) in particular, we insert the equilibrium solutions for the country-specific fiscal stances (eq. (14)), and fiscal rules (equations (8) and (13)) to the fiscal authorities’ loss functions (eq. (6)). The regimes of simultaneous-move and fiscal leadership are exactly the same. For the former we use \( \phi_l^{SM} = \phi_f = \frac{a_F \delta_g \phi_f}{1 - \delta_y} \) and \( \psi^{SM} = \frac{1}{1 + \delta_y + 2 \delta_{\tau} \omega + \delta_g \phi_f} \), and for the latter equation (28) as the leader’s fiscal rule. We have seen that the cooperative regime delivers both a fiscal rule (eq. (20)) and a fiscal stance (eq. (21)) for the lead fiscal authority similar to those in the non-cooperative regime (equations (13) and (14), respectively). Thus, the lead fiscal authority’s expected loss is given by the same expression as equation (C.1):

\[
E\left(L_{F_1}\right)^c = \frac{1}{2} \left[a_F + (\phi_f^c)^2 \right] \left(\psi^c\right)^2 \left\{ \frac{1}{4} \text{Var}(u_l - u_f) + \text{Var} \left[ \delta_{\tau} (\varepsilon_l - \varepsilon_f) + (1 + \delta_y + 2 \delta_{\tau} \omega + \delta_g \phi_f)^{-1} \varepsilon \right] \right\}, \tag{C.2}
\]

where the upper script in the \( E\left(L_{F_1}\right)^c \) expression (here ‘c’) denotes the strategic regime (here the cooperative one). However, to compute the follower fiscal authority’s expected loss we use the follower’s fiscal stance (eq. (22)), since it differs to the leader’s regarding shock asymmetries. We then get:

\[
E\left(L_{F_2}\right)^c = \frac{1}{2} \left[a_F + \phi_f^2 \right] \left(\psi^c\right)^2 \left\{ \left(\frac{1}{2} + \mu\right)^2 \text{Var}(u_l - u_f) + \text{Var} \left[ (1 + 2 \mu) \delta_{\tau} (\varepsilon_l - \varepsilon_f) + (1 + \delta_y + 2 \delta_{\tau} \omega + \delta_g \phi_f^c)^{-1} \varepsilon \right] \right\}, \tag{C.3}
\]

C.2 The Monetary Authority’s Expected Loss

In order to compute the monetary authority’s expected loss, we plug the equilibrium solutions for the union-wide inflation rate and the output gap in the authority’s loss function (eq. (7)), and then take expectations. The equilibrium solutions for the strategic regimes of simultaneous move, non-cooperation and fiscal leadership, are given by equations (11) and (12). For the cooperative strategic regime, we use equations (24) and (25). We find:

\[
E(L_M)^{nc} = E(L_M)^{SM} = E(L_M)^{FL} = \frac{1}{2} * \frac{a_M}{\omega^2 + a_M} \text{Var}(\varepsilon), \tag{C.4}
\]

\(~35~\)
\[ E(L_M)^c = \frac{1}{2} \langle \psi^c \rangle^2 \{ \mu^2(\omega^2 + a_M)Var \left[ \frac{1}{2}(u_l - u_f) \right] + \omega^2 Var[-(\mu \delta\tau + B)\varepsilon_l + (\mu \delta\tau - B)\varepsilon_f] + a_M Var[(-\mu \delta\tau + \Gamma)\varepsilon_l + (\mu \delta\tau + \Gamma)\varepsilon_f] \} \]. \tag{C.5}

where \[ B = \frac{1}{2} \left[ (1 + \delta_y + 2\delta_\tau \omega + \delta_g \phi^c) a_M + \beta a_F \lambda \right] \frac{1}{\omega^2 + a_M} \] and \[ \Gamma = \frac{1}{2} \left( 1 + \delta_y + 2\delta_\tau \omega + \delta_g \phi^c \right) \frac{\omega}{\omega^2 + a_M} \].

**Appendix D**

We provide proofs of the results in the main text; specifically, in Section 4 and Section 5.

**Proof of Result 2**: Following Appendix C.1, we compare and contrast the expected losses of the two fiscal authorities for each and every strategic regime. We use equations (C.1) for both the non-cooperative and the fiscal leadership strategic regimes for both fiscal authorities, while for the cooperative one we use equations (C.2) and (C.3). Demand and supply shocks are assumed to be uncorrelated. Let us start with demand shocks. We have:

\[ E(L_{F_l})^{nc} - E(L_{F_f})^{nc} = -\frac{1}{4} \phi^{nc} \langle \psi^{nc} \rangle^2 Var(u_l - u_f)(\phi_f - \phi_l^{nc}) < 0, \quad \text{and:} \]

\[ E(L_{F_l})^{c} - E(L_{F_f})^{c} = -\frac{1}{4} \langle \psi^c \rangle^2 Var(u_l - u_f)[(\phi_f - \phi_l^c)\phi^c + \mu(a_F + \phi_f^2)(1 + 2\mu)] < 0, \]

where the sub-scripts in the expressions for the expected losses define demand/supply shocks and upper scripts the strategic regimes. The expression for the fiscal leadership strategic regime is equivalent to the non-cooperative one, since they both share the same expression for the authorities’ expected loss (equation (C.1)) and \( \phi_f - \phi_l^{FL} > 0 \). For supply shocks, we assume \( \sigma_{\varepsilon_j} = \sigma_{\varepsilon_k} = \sigma_{\varepsilon} \). However, the general case is far more complicated as it depends on the size of the shocks. The general solutions are:

\[ E(L_{F_l})^{nc} - E(L_{F_f})^{nc} = -\langle \psi^{nc} \rangle^2 (\phi_f - \phi_l^{nc})\sigma_{\varepsilon}^2 \left\{ 2\phi^{nc} \delta^2(1 - \rho_{\varepsilon}) + \frac{1}{2K} \left( \frac{\omega}{\omega^2 + a_M} \right)^2 (1 + \rho_{\varepsilon}) \left[ (\delta_y + \delta_\tau \omega)^2 - (1 + \delta_\tau \omega)(1 - 2\delta_y) - \frac{1}{2} \delta_g [(1 - \delta_y)\phi_f - (1 + \delta_y + 2\delta_\tau \omega)\phi_l^{nc}] \right] \} \], and:
\[
E(L_{F_1})^c - E(L_{F_f})^c = (\psi^c)^2\sigma^2_\epsilon \left\{ \delta^2_\epsilon (1 - \rho_\epsilon)[a_F + (\phi^c_F)^2 - (a_F + \phi^2_F)(1 + 2\mu)^2] + \frac{1}{2}(1 + \rho_\epsilon) \left( \frac{\omega}{\omega^2 + \alpha M} \right)^2 \left[ (a_F + (\phi^c_F)^2)(1 + \delta_y + 2\delta_t\omega + \delta_g\phi_f)^2 - (a_F + \phi^2_F)(1 + \delta_y + 2\delta_t\omega + \delta_g\phi_f)^2 \right] \right\}
\]

For idiosyncratic shocks (\(\rho_\epsilon = -1.0\)), we get:

\[
E(L_{F_1})^{nc} - E(L_{F_f})^{nc} = -4\phi^{nc}(\psi^{nc})^2\delta^2_\epsilon \sigma^2_\epsilon (\phi_f - \phi^{nc}_f) < 0, \text{ and:}
\]

\[
E(L_{F_1})^c - E(L_{F_f})^c = -2(\psi^c)^2\delta^2_\epsilon \sigma^2_\epsilon [(\phi_f - \phi^c_f)\phi^c + \mu(a_F + \phi^2_F)(1 + 2\mu)] < 0,
\]

where in the special case that \(\delta_\epsilon = 0\), then \(E(L_{F_1})_\epsilon = E(L_{F_f})_\epsilon\) for both the strategic regimes.

Naturally, the analysis for the fiscal leadership strategic regime coincides with the one for the non-cooperative strategic regime. The result for common shocks (\(\rho_\epsilon = 1.0\)) is still uncertain.

The term in brackets in the expression that refer to union-wide supply shocks in the cooperative strategic regime is not affected by the central bank’s bargaining power, \(\beta\). Thus, it plays no role in ranking welfare gains between the two fiscal authorities under union-wide supply shocks.

**Proof of Result 3:** Following Appendix C.1, we use equations (C.1) and (C.2). We get:

\[
E(L_{F_1})^{nc} - E(L_{F_f})^{SM} = \frac{1}{2} * \frac{A}{1+\delta_t\omega} * \left\{ \frac{1}{4} Var(u_i - u_f) + Var[\delta_\tau (e_j - e_k)] + (1 + \delta_y + 2\delta_t\omega + \delta_g\phi_f) \frac{\omega}{\omega^2 + \alpha M} \epsilon \right\} < 0,
\]

\[
E(L_{F_1})^{nc} - E(L_{F_f})^{FL} = \frac{1}{2} * \left\{ \frac{1}{4} Var(u_i - u_f) + Var[\delta_\tau (e_i - e_f)] + (1 + \delta_y + 2\delta_t\omega + \delta_g\phi_f) \frac{\omega}{\omega^2 + \alpha M} \epsilon \right\} * (\psi^{nc})^2 \psi^{FL}[2(1 + \delta_y + 2\delta_t\omega) + \delta_g\phi_f] > 0, \text{ and:}
\]

\[
E(L_{F_1})^c - E(L_{F_f})^{FL} = -\frac{1}{2} * \left\{ \frac{1}{4} Var(u_i - u_f) + Var[\delta_\tau (e_i - e_f)] + (1 + \delta_y + 2\delta_t\omega + \delta_g\phi_f) \frac{\omega}{\omega^2 + \alpha M} \epsilon \right\} * [a_F + (\phi^c_F)^2] * \beta \lambda \lambda^c \psi^{FL} (\psi^c + \psi^{FL}) < 0,
\]

where \(A = \frac{1}{2}(1 - \delta_y)(1 + \delta_y + 2\delta_t\omega)[2(1 + \delta_y + 2\delta_t\omega) + \delta_g\phi_f + \delta_g\phi^{nc}_f]\phi_f - (1 + \delta_t\omega)(1 + \delta_y + 2\delta_t\omega + \frac{1}{2}\delta_g\phi_f)[2(1 + \delta_y + 2\delta_t\omega + \frac{1}{2}\delta_g\phi_f)\phi^{nc}_f + \delta_g\phi_f\phi^{nc}_f] < 0\). Thus:
\[ E(L_{F_l})^c < E(L_{F_l})^{FL} < E(L_{F_l})^{nc} < E(L_{F_l})^{SM}, \] following the transitivity condition, while this welfare ranking is independent on shocks.

**Proof of Result 4:** Following Appendix C.1, we use equations (C.1) and (C.3). Let us start with asymmetric demand shocks and/or idiosyncratic \((\rho_{\epsilon} = -1.0)\) supply shocks. For demand shocks, we get:

\[
E \left( L_{F_l} \right)^{nc}_u - E \left( L_{F_l} \right)^{SM}_u = \frac{1}{16} \text{Var}(u_l - u_f)(a_F + \phi^2_f)(\psi^{nc} \psi^{SM})^2 \delta_g(\phi_f - \phi^{nc}_l)[2(1 + \delta_y + 2\delta_{\tau}\omega) + \delta_g \phi_f + \delta_g \phi^{nc}] > 0,
\]

\[
E \left( L_{F_l} \right)^{nc}_u - E \left( L_{F_l} \right)^{FL}_u = - \frac{1}{16} * (a_F + \phi^2_f) \text{Var}(u_l - u_f) * \delta_g(\phi^{nc}_l - \phi^{nc}_f) * \psi^{nc} \psi^{FL}(\psi^{nc} + \psi^{FL}) < 0,
\]

and:

\[
E \left( L_{F_l} \right)^{c}_u - E \left( L_{F_l} \right)^{FL}_u = \frac{1}{8} * (a_F + \phi^2_f) \text{Var}(u_l - u_f) * (1 + \delta_y + 2\delta_{\tau}\omega + \delta_g \phi^{nc}_f) \mu \psi^{nc} \psi^{FL}[(\psi^{FL} + (1 + 2\mu)\psi^c] > 0.
\]

Thus, the welfare ranking of the alternative institutional arrangements for the peripheral state’s fiscal authority under asymmetric demand shocks follows: \(E \left( L_{F_l} \right)^{SM}_u < E \left( L_{F_l} \right)^{nc}_u < E \left( L_{F_l} \right)^{FL}_u < E \left( L_{F_l} \right)^{c}_u\), following the transitivity condition; i.e., exactly the opposite from the lead fiscal authority’s ranking, as this is demonstrated in Result 3.

For supply shocks, \(\sigma_{\epsilon_j} = \sigma_{\epsilon_k} = \sigma_{\epsilon} \) is assumed. It can be easily verified that the previous ranking holds for idiosyncratic \((\rho_{\epsilon} = -1.0)\) supply shocks, too. For common \((\rho_{\epsilon} = 1.0)\) supply shocks, we get:

\[
E \left( L_{F_l} \right)^{nc}_e - E \left( L_{F_l} \right)^{SM}_e = - \frac{1}{4} (a_F + \phi^2_f) \left( \frac{\omega}{\omega^2 + a_M} \right)^2 \sigma_{\epsilon}^2 \psi^{nc} \psi^{SM} \delta_g(\phi_f - \phi_l)(1 + \delta_y + 2\delta_{\tau}\omega + \delta_g \phi_f)[(1 + \delta_y + 2\delta_{\tau}\omega + \delta_g \phi^{nc}_l)\psi^{nc} + (1 + \delta_y + 2\delta_{\tau}\omega + \delta_g \phi_f)\psi^{SM}] < 0,
\]

\[
E \left( L_{F_l} \right)^{nc}_e - E \left( L_{F_l} \right)^{c}_e = \frac{1}{2} (a_F + \phi^2_f) \psi^{nc} \psi^{c} \left( \frac{\omega^2 \sigma_{\epsilon}^2}{(\omega^2 + a_M)^2} \right) \left[ \psi^{nc} (1 + \delta_y + 2\delta_{\tau}\omega + \delta_g \phi^{nc}_l) + \psi^{c} (1 + \delta_y + 2\delta_{\tau}\omega + \delta_g \phi^{nc}_f) \right] \left[ (1 + \delta_y + 2\delta_{\tau}\omega + \delta_g \phi^{nc}_l) \beta \delta \lambda + \frac{1}{2} (1 + \delta_y + 2\delta_{\tau}\omega + \delta_g \phi_f) \delta_g(\phi^{nc}_l - \phi^{nc}_f) \right] > 0.
\]
\[
E \left( L_{f_f} \right)_\varepsilon^{nc} - E \left( L_{f_f} \right)_\varepsilon^{FL} = \frac{1}{4} \left( \frac{\omega}{\omega^2 + a_M} \right)^2 \sigma_\varepsilon^2 (a_F + \phi_f^2) \delta_g (\phi_f^{nc} - \phi_f^{c}) \psi^{nc} \psi^{FL} \left( \frac{a_F}{\phi_f^c} - 1 \right) \left[ (1 + \delta_y + 2 \delta_t \omega + \delta_g \phi_f^c) \right] \geq 0, \text{ and:}
\]
\[
E \left( L_{f_f} \right)_\varepsilon^{c} - E \left( L_{f_f} \right)_\varepsilon^{FL} = -\frac{1}{2} \left( a_F + \phi_f^2 \right) \left( 1 + \delta_y + 2 \delta_t \omega + \delta_g \phi_f^c \right) \left( \frac{\omega}{\omega^2 + a_M} \right)^2 \sigma_\varepsilon^2 \beta \lambda \psi^c \psi^{FL} \left( \psi^c + \psi^{FL} \right) < 0.
\]

The sign of \( E \left( L_{f_f} \right)_\varepsilon^{nc} - E \left( L_{f_f} \right)_\varepsilon^{FL} \) is ambiguous. However, it is more likely that \( E \left( L_{f_f} \right)_\varepsilon^{nc} - E \left( L_{f_f} \right)_\varepsilon^{FL} > 0 \) holds. It certainly holds if \( a_F > \left[ 1 - 2 \delta_g (1 + \delta_y + 2 \delta_t \omega) \right] \frac{1 - \delta_y}{\delta_g} \), which is true for fiscal multipliers large enough to make \( 1 - 2 \delta_g (1 + \delta_y + 2 \delta_t \omega) \) negative, with \( 1 - \delta_y > 0 \) (even a value of \( \delta_g = 0.5 \) suffices). In that case, the welfare ranking of the alternative institutional arrangements for the peripheral state’s fiscal authority under common supply shocks follows:\n
\[
E \left( L_{f_f} \right)_\varepsilon^{c} < E \left( L_{f_f} \right)_\varepsilon^{FL} < E \left( L_{f_f} \right)_\varepsilon^{nc} < E \left( L_{f_f} \right)_\varepsilon^{SM}; \text{ i.e., exactly the same with the lead fiscal authority’s ranking, as this is demonstrated in Result 3.}
\]

**Proof of Proposition 1:** We use equations (11) and (12) for the non-cooperative, the simultaneous move and the fiscal leadership strategic regimes, and equations (24) and (25) for the cooperative one. For asymmetric demand shocks, it is straightforward to verify that \( 0 = Var(\pi)^{SM}_u = Var(\pi)^{nc}_u = Var(\pi)^{FL}_u < Var(\pi)^c_u \) and \( 0 = Var(y)^{SM}_u = Var(y)^{nc}_u = Var(y)^{FL}_u < Var(y)^c_u \). The sub-scripts denote demand/supply shocks, while the super-scripts the usual strategic regimes.

For supply shocks, it is straightforward to verify that \( Var(\pi)^{SM} = Var(\pi)^{nc} = Var(\pi)^{FL} = \left( \frac{a_M}{\omega^2 + a_M} \right)^2 Var(\varepsilon) \) and \( Var(y)^{SM} = Var(y)^{nc} = Var(y)^{FL} = \left( \frac{\omega}{\omega^2 + a_M} \right)^2 Var(\varepsilon) \). However, in order to compare with the cooperative strategic regime, we need to assume \( \sigma_{\varepsilon_j} = \sigma_{\varepsilon_k} = \sigma_{\varepsilon} \). For idiosyncratic supply shocks (\( \rho_\varepsilon = -1.0 \)), it can be easily verified that the previous analysis for asymmetric demand shocks hold. We then compare for common shocks (\( \rho_\varepsilon = 1.0 \)). We get:

\[
Var(y)^{nc}_\varepsilon - Var(y)^c_\varepsilon = \frac{\beta \lambda \psi^c \omega^2 \sigma_\varepsilon^2}{(\omega^2 + a_M)^2} \left[ 1 + \psi^c \left( 1 + \delta_y + 2 \delta_t \omega + \delta_g \phi^c \right) \right] > 0, \text{ and:}
\]
\[
Var(\pi)^{nc}_\varepsilon - Var(\pi)^c_\varepsilon = \frac{\psi^c \sigma_\varepsilon^2}{(\omega^2 + a_M)^2} \left\{ a_M + \omega \psi^c \left[ \beta \lambda \psi^c \left( 1 + \delta_y + 2 \delta_t \omega + \delta_g \phi^c \right) \right] \right\} \left[ a_M (1 - \omega)(1 + \delta_y + 2 \delta_t \omega + \delta_g \phi^c) + \frac{a_M(1 - \omega) - \omega^3}{\omega^2 + a_M} \beta \lambda \phi^c \right].
\]

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where the latter is likely to be positive for low enough values of $\omega$.

**Proof of Result 5:** Following Appendix C.2, the first part is straightforward from equation (C.4). For demand shocks (either symmetric or asymmetric), it would be equal to zero. For the cooperative strategic regime, equation (C.5) shows that the expected loss is unambiguously positive for asymmetric demand shocks since both the inflation rate and the output gap are affected by demand shock asymmetries in equilibrium. For supply shocks, we assume $\sigma_{\varepsilon j} = \sigma_{\varepsilon k} = \sigma_{\varepsilon}$. The previous analysis also works for idiosyncratic supply shocks ($\rho_{\varepsilon} = -1.0$).

For common supply shocks ($\rho_{\varepsilon} = 1.0$), the result is not straightforward, so we need to compute $E(L_M)^{nc} - E(L_M)^{c}$. After some tedious algebra, we get:

$$E(L_M)^{nc} - E(L_M)^{c} = \frac{1}{2}(\psi^c)^2\frac{\sigma_{\varepsilon}^2}{(\omega^2 + a_M)^3}\left\{2(1 - \omega^2)a_M(\omega^2 + a_M)\beta a_F \lambda (1 + \delta_y + 2\delta_t \omega + \delta_g \phi^c) + [a_M - \omega^2(\omega^2 + a_M)](\beta a_F \lambda)^2 + [1 - \omega^2(1 + \delta_y + 2\delta_t \omega + \delta_g \phi^c)]a_M(\omega^2 + a_M)(1 + \delta_y + 2\delta_t \omega + \delta_g \phi^c)\right\}.$$

For low enough values of the slope of the Phillips curve, $\omega$, then $E(L_M)^{nc} > E(L_M)^{c}$ for $a_M > 0$. In the special case of strict inflation targeting, $a_M = 0$, then $E(L_M)^{nc} - E(L_M)^{c} = -\frac{1}{2}\omega^4(\beta a_F \lambda)^2(\psi^c)^2\frac{\sigma_{\varepsilon}^2}{(\omega^2 + a_M)^3} < 0$.

**Proof of Proposition 2:** We use equation (17) for the non-cooperative, the simultaneous move and the fiscal leadership strategic regimes, and equation (23) for the cooperative one. For asymmetric demand shocks, we get:

$$Var(g)^{nc}_u > Var(g)^{SM}_u = 0,$$

$$Var(g)^{FL}_u - Var(g)^{nc}_u = \frac{1}{4}Var(u_l - u_f)(1 + \delta_y + 2\delta_t \omega + \delta_g \phi_f)\delta_g \phi_f (\phi^nc - \phi^c)\psi^{nc}\psi^{FL}[\psi^{FL}(\phi_f - \phi^c) + \psi^{nc}(\phi_f - \phi^nc)] > 0,$$

and:

$$Var(g)^{FL}_u - Var(g)^{c}_u = -\frac{1}{2}Var(u_l - u_f)[(1 + \delta_y + 2\delta_t \omega)\phi^c + \delta_g \phi_f (\frac{\phi^nc + \phi^c}{2})]*\mu\psi^c\psi^{FL}[\psi^{FL}(\phi_f - \phi^c) + \psi^c[(1 + 2\mu)\phi_f - \phi^c]] < 0.$$

That means: $Var(g)^{SM}_u < Var(g)^{nc}_u < Var(g)^{FL}_u < Var(g)^{c}_u$, following also the transitivity condition.
For supply shocks, \( \sigma_{\varepsilon_j} = \sigma_{\varepsilon_k} = \sigma_{\varepsilon} \) is assumed. The previous analysis exactly holds for idiosyncratic supply shocks (\( \rho_{\varepsilon} = -1.0 \)). For common supply shocks (\( \rho_{\varepsilon} = 1.0 \)), we get:

\[
\text{Var}(g)_{\varepsilon}^{nc} - \text{Var}(g)_{\varepsilon}^{SM} = -\frac{1}{4} \chi \left( \frac{\phi_f \psi^SM \sigma_{\varepsilon}}{\omega^2 + \alpha_M} \right)^2 \left( 1 + \delta_y + 2\delta_t \omega + \delta_g \phi_f \right) \left[ (1 + \delta_y + 2\delta_t \omega) (4 - \chi) + \delta_g \phi_f (4 - 3\chi) \right] < 0 \text{ since } \chi \in (0, 1).
\]

\[
\text{Var}(g)_{\varepsilon}^{FL} - \text{Var}(g)_{\varepsilon}^{nc} = -\left( 1 + \delta_y + 2\delta_t \omega + \delta_g \phi_f \right)^2 \left( \frac{\omega}{\omega^2 + \alpha_M} \right)^2 \sigma_{\varepsilon}^2 \left( \phi_{\varepsilon}^{nc} - \phi_{\varepsilon}^c \right) \psi_{nc} \psi_{FL}^* \left[ \psi_{FL}^* \left( 1 + \delta_y + 2\delta_t \omega \right) \phi^c + \delta_g \phi_f \phi_{\varepsilon}^c \right] < 0, \text{ and:}
\]

\[
\text{Var}(g)_{\varepsilon}^{FL} - \text{Var}(g)_{\varepsilon}^c = 2 \left( 1 + \delta_y + 2\delta_t \omega \right) \phi^c + \delta_g \phi_f \phi_{\varepsilon}^c \right)^2 \left( \frac{\omega}{\omega^2 + \alpha_M} \right)^2 \sigma_{\varepsilon}^2 \beta \alpha \lambda^c \psi_{FL}^c (\psi_{FL}^* + \psi_c) > 0.
\]

In this case: \( \text{Var}(g)_{\varepsilon} < \text{Var}(g)_{\varepsilon}^{FL} < \text{Var}(g)_{\varepsilon}^{nc} < \text{Var}(g)_{\varepsilon}^{SM} \), following the transitivity condition; i.e., exactly the opposite of the previous ordering.

**Proof of Corollary 2:** Let us start with asymmetric demand and idiosyncratic supply shocks (\( \rho_{\varepsilon} = -1.0 \)). For supply shocks, we again assume \( \sigma_{\varepsilon_j} = \sigma_{\varepsilon_k} = \sigma_{\varepsilon} \). Following Result 4, we need:

\( E(L_{EC})_u^{SM} < E(L_{EC})_u^{nc} < E(L_{EC})_u^{FL} < E(L_{EC})_u^c \). Following Proposition 1, we have:

\[
0 = \text{Var}(y, \pi)_u^{SM} = \text{Var}(y, \pi)_u^{nc} = \text{Var}(y, \pi)_u^{FL} < \text{Var}(y, \pi)_u^c,
\]

for either the output gap or the inflation rate. Following Proposition 2, we have:

\[
\text{Var}(g)_u^{SM} < \text{Var}(g)_u^{nc} < \text{Var}(g)_u^{FL} < \text{Var}(g)_u^c.
\]

Following equation (29), EC’s loss function is additive in the variances of inflation, output gap and fiscal stance. Hence: \( E(L_{EC})_u^{SM} < E(L_{EC})_u^{nc} < E(L_{EC})_u^{FL} < E(L_{EC})_u^c \).

Let us consider common supply shocks (\( \rho_{\varepsilon} = 1.0 \)). Following Result 4, we need: \( E(L_{EC})_{\varepsilon}^c < E(L_{EC})_{\varepsilon}^{FL} < E(L_{EC})_{\varepsilon}^{nc} < E(L_{EC})_{\varepsilon}^{SM} \). Following Proposition 1, we have:

\[
\text{Var}(y, \pi)_{\varepsilon}^{SM} = \text{Var}(y, \pi)_{\varepsilon}^{nc} = \text{Var}(y, \pi)_{\varepsilon}^{FL} > \text{Var}(y, \pi)_{\varepsilon}^c,
\]

for either the output gap or inflation. Following Proposition 2, we have:

\[
\text{Var}(g)_{\varepsilon}^c < \text{Var}(g)_{\varepsilon}^{FL} < \text{Var}(g)_{\varepsilon}^{nc} < \text{Var}(g)_{\varepsilon}^{SM}.
\]

Following equation (29), EC’s loss function is additive in the variances of inflation, output gap and fiscal stance. Hence: \( E(L_{EC})_{\varepsilon}^c < E(L_{EC})_{\varepsilon}^{FL} < E(L_{EC})_{\varepsilon}^{nc} < E(L_{EC})_{\varepsilon}^{SM} \).

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