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Research Article

Cooperative Advertising Strategy Selection Problem for considering Pricing and Advertising Decisions in a Two-Period Online Supply Chain

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This article studies the cooperative advertising problem of a two-period online supply chain consisting of a manufacturer and an online retail platform. The manufacturer provides national advertising in the first period to build the brand image and increase the awareness of the product. And the online retail platform provides platform advertising for selling the product to consumers on its platform during two periods. The manufacturer and the online retail platform may choose different cooperative advertising strategies for national advertising and platform advertising, which are one-way subsidy strategy, two-way subsidy strategy, and revenue-share strategy. We formulate a Stackelberg game model to study the cooperative advertising problem by taking price and advertising effect into account and analyze how the profit is influenced in different cooperative advertising strategies. We find that under the revenue-share strategy, the manufacturer provides a higher subsidy rate for the online retail platform advertising than that in other cooperative advertising strategies. Interestingly, there are conditions where, while just the manufacturer contributes a percentage of the platform advertising and the online retail platform has no effort on the national advertising, the total profit would be better than that in revenue-share strategy even in revenue-share strategy, the cooperative relationship is closer between the manufacture and the online retail platform.

1. Introduction

Cooperative advertising is a powerful tool commonly used in marketing channels where one party in the channel agrees to subsidize the advertising expenditure of the other [1]. Nagler [2] found that in 2000, the total expenditures on cooperative advertising in the USA were estimated at $15 billion, and about $50 billion was spent on cooperative advertising in 2009 [3]. The practice of cooperative advertising has seen a significant increase in recent years [1, 4]. In a typical arrangement, manufacturers agree to share part of the local advertising cost for retailers, which is called a one-way subsidy. While manufacturers are willing to bear part of the local advertising cost for retailers, retailers are also willing to bear part of the cost of manufacturers' national advertising, which is called a two-way subsidy [5]. In this case, manufacturers and retailers bear part of each other's advertising expenses, commonly known as the subsidy rate.

In the era of online retail, the cooperative advertising relationship is reflected in the cooperation process between national advertising and platform advertising. Generally, platform advertising (such as "Tianhe Plan" of Alibaba and "Donglian Plan" of JD.com) is managed by online retail platforms because online retail platforms usually better understand the effectiveness of platform advertising media, platform preferences, and the interests of demographic consumers. As a powerful tool commonly used in marketing channels, cooperative advertising not only enables manufacturers and retailers to share part of the advertising costs for each other but also enables them to benefit from this alliance by increasing sales [6]. For example, in China, "The Media Resource Replacement Program" is currently a

collaborative marketing strategy widely adopted by online retail platforms and manufacturers. In this plan, manufacturers would reflect the brand visual elements of online retail platforms (platform name, logo, etc.) in their national advertising, and online retailer platforms provide a corresponding proportion of platform advertising resource replacement in return [7]. Manufacturers intend to build a brand image by national advertising whereas online retail platforms aim at increasing sales by platform advertising [8, 9]. Advertising cooperation between manufacturers and online retail platforms can effectively improve the profit of the participating parties [10]. For example, since 2015, Tmall (B2C platform of Alibaba) has established advertising cooperation with more than 200 brands, such as Maserati, Beats, and Swarovski, and helps brands build the double harvest of brand and sales on "Super Brand Day." And Amazon announced a new advertising program for its retail business to compete with Google and Facebook in 2018. Apart from cooperative advertising, manufacturers and online retail platforms also have other methods to strengthen the cooperation relationship, such as revenue-share strategy [11]. For example, the Apple App Store and Google Play use uniform revenue-share strategies to engage with all developers whose apps they sell; through Amazon Marketplace, the seller pays Amazon a certain percentage of the retail price and a fixed fee [12]. Under the revenue-share strategy, manufacturers could be motive to spend more on adverting and reduce the wholesale price because they could get a portion of online retail platforms' sales profit, and online retail platforms may benefit from the advertising and the lower wholesale price even if they must share their revenue with manufacturers in the retail market.

Based on these observations, we are interested in the following questions. (1) What is the impact of cooperative advertising on firms' advertising decisions and price decisions during a two-period supply chain? (2) How do the profits for the manufacturer and the online retail platform change in different cooperative advertising strategies? (3) Whether and when can the firms benefit from cooperative advertising?

We attempt to address these issues in this paper; we develop a two-period online supply chain system consisting of a manufacturer and an online retail platform. Different from previous models, this paper considers that the impact of national advertising, platform advertising, and product price on demand is changing over time, and this naturally divides the selling season into two periods. The manufacturer sells his product through the online retail platform and chooses different levels of national advertising. The online retail platform sells the product to consumers and chooses different levels of platform advertising. The manufacturer and the online retail platform may choose different cooperative advertising strategies for national advertising and platform advertising, which are one-way subsidy strategy, two-way subsidy strategy, and revenue-share strategy. We formulate a Stackelberg game model to study the cooperative advertising problem by taking price and advertising effect into account and analyze how the profit is influenced in different cooperative advertising strategies.

The findings reveal the impacts of the cooperation strategies on cooperative advertising. First, under the revenue-share strategy, the manufacturer provides a higher subsidy rate for platform advertising than that in other strategies. Second, in the revenue-share strategy, there is a special region. With the increase of the revenue-share rate, the online retail platform obtains the manufacturer's advertising subsidy support by transferring more revenue to the manufacturer. Thrid, by identifying the value of the cooperative advertising, the manufacturer and the online retail platform just taking a two-way subsidy strategy is the best choice for the online supply chain system.

This paper contributes to the literature in the following aspects. First, it considers the cooperative advertising problem in a multi-period supply chain, while most articles consider the cooperative advertising problem in a single-period supply chain [10, 13, 14]. Second, this paper considers the impact of price and advertising on market demand simultaneously and considers the different effects of national advertising and platform advertising on consumers in different periods. Third, different from Zhang et al. [15] who consider the cooperative advertising problem under one- and two-way subsidy strategies, this paper adds a revenue-share strategy to enrich the cooperative advertising choice for enterprises.

We organize the remainder of this paper as follows. We discuss the relevant literature in Section 2. The model and the analysis of results are presented in Sections 3 and 4. We provide a numerical analysis in Section 5. In Section 6, we conclude the paper and point out further research for future work. The derivation of equilibrium and the proof of propositions are provided in the Appendix.

2. Literature Review

Cooperative advertising was first used in the United States in the 1990s. As cooperative advertising programs are widespread in practice, many practitioners and researchers pay more attention to the effect of cooperative advertising on demand and profit [16]. Chen [17] studied the profit model of manufacturers and retailers from the perspective of traditional retail channels and online direct sales channels. On the basis of Chen [17], Chen et al. [13] studied the problem of cooperative advertising under the simultaneous price and advertising competition between manufacturers' online channels and traditional retail channels. Chen et al. [13] pointed out that the increase of price discounts will reduce the enthusiasm of retailers' advertising investment. From the perspective of the free-rider effect, Tao and Li [14] considered the pricing strategies of members in the O2O channel supply chain and the impact of the free-rider effect on channel profits and analyzed the optimal pricing strategy under centralized and decentralized. Different from Tao and Li [14], Wang and Shu [18] considered the cooperative advertising strategy of the O2O supply chain, studied three game theory models, and deduced the optimal decision of advertising effort and participation rate among supply chain members. On this basis, Shu and Wang [7] further studied the impact of channel power structure on the cooperative

advertising decision-making of O2O supply chain channel members. They found that the manufacturer's national advertising investment and the overall profit of the supply chain in the centralized decision-making model were the largest.

In the following papers, some researchers have extended the cooperative advertising model and explored the cooperative advertising strategy in the O2O supply chain composed of sellers and online platform agents under the "Internet plus" environment [19]. Li et al. [19] established a mathematical model of cooperative advertising in O2O supply chain composed of sellers and network platform agents, proposed three cooperative advertising modes, and deduced the optimal decision-making of advertising level and participation rate among supply chain members by using quantitative modeling method for cooperative advertising in the supply chain. On the basis of Li et al. [19], Li et al. [20] conducted further extended research. They constructed three different game decision models for a dual-channel manufacturer supply chain system carrying out online direct sales business and analyzed and compared the optimal decisions of dual-channel commodity pricing and advertising investment in different situations. When the manufacturer carries out an online direct sales channel through the third-party online retail platform, the manufacturer should actively choose to carry out a cooperative advertising strategy with the online retail platform and traditional retailers at the same time [10]. In the cooperative advertising practice between online retail platforms and manufacturers, the platform cross-sales effect is the key factor affecting the formulation and implementation of a cooperative advertising plan. When the platform cross-sales effect is large, the profit of online retail platform is greater than that of manufacturers [21].

Taking into account the impact of advertising on the reference price, Zhang et al. [15] propose a dynamic cooperative advertising model for a manufacturer-retailer supply chain. Assuming that consumers' goodwill and product reference price are affected by advertising, they use differential dynamics equation to model and consider how the reference price effect affects the decision-making of all channel members under the advertising cooperation strategy of one- and two-way subsidies. Different from them, our paper extends the understanding of the research by considering the case where there is a two-period operation in the online supply chain system, and the manufacturer and the online retail platform could use a two-way subsidy strategy or revenue-share strategy to enhance the cooperative degree based on the result in Qi and Xie [22]. The demand function is affected by price and advertising simultaneously, and the national advertising and the platform advertising have different effects on consumers during the two periods.

Table 1 reveals that some publications considered a two-period supply chain, while others considered one or two aspects of cooperative advertising. The studies rarely considered different cooperative advertising strategies, the effects of advertising, and price simultaneously, so we will address this gap in our study.

3. Model Framework

In this section, we consider an online supply chain consisting of a single manufacturer and a single online retail platform in a two-period setting, and the online retail platform sells the product to consumers by its retail platform. Both the manufacturer and the online retail platform would provide the advertising for the product. The manufacturer provides national advertising in the first period to build the brand image and increase the awareness of the product, and the online retail platform provides platform advertising to promote consumers to buy the product on its platform during two periods [23]. For example, Apple would make the national advertising on Google and other browsers to build Apple's image. At the same time, Amazon Marketplace would provide the platform advertising for Apple to attract consumers to buy these products on his online platform. The manufacturer and the online retail platform choose different strategies for the cooperation between national advertising and platform advertising, which are one-way subsidy strategy, two-way subsidy strategy, and revenue-share strategy. In this paper, the manufacturer is the leader, and the online retail platform is the follower. The manufacturer decides the national advertising level A and the subsidy rate ϕ_1 and ϕ_2 for platform advertising [24]. And the online retail platform decides the platform advertising level e_i and the retail price p_i during two periods ($i \in (1, 2)$). Especially, the wholesale price w is an exogenous variable and $w < p_i$. The specific structure is shown in Figure 1.

By researching the prevalent assumption in the literature, we can assume the consumer demand as the following form:

$$D(p_i, e_i, A) = g(p_i) \times h(e_i, A). \tag{1}$$

The demand function is similar to Xie and Wei [25], SeyedEsfahani et al. [26], Aust and Buscher [27], and He et al. [28], where $g(p_i)$ reflects the impact of the retail price on the demand and $h(e_i, A)$ reflects the impact of the advertising expenditures on the demand. The demand expression for the first period is as follows:

$$D_1 = (1 - \delta p_1)(a_1 + \beta A + \lambda e_1), \tag{2}$$

where δ, β, and λ are positive constants and β and λ reflect the efficacy of each type of advertising in generating sales. $a_1 > 0$ is the potential demand for the first period. Because $1 - \delta p_1 > 0$, we get $0 < p_1 < 1/\delta$ and $w < p_1 < 1/\delta$.

Generally, manufacturers usually do national advertising when the product first appears in the market. Because manufacturers in the market are normally good at R&D (research and development) and producing but not marketing. We assume that the manufacturer only does the national advertising in the first period.

Therefore, the demand function in the second period would be as follows:

$$D_2 = (1 - \delta p_2)(a_2 + \theta \lambda e_2), \tag{3}$$

where $a_2 > 0$ is the potential demand for the second period and $0 < \theta \leq 1$ means the platform advertising effect would

Table 1: The summary of literature and factors addressed by our study.

Reference	Online cooperative advertising	Two period supply chain	One-way Subsidy Contract	Two-way Subsidy Contract	Revenue-Share Contract	Advertising and Price
Zhang et al [15]			✓	✓		✓
He et al [23]		✓	✓	✓	✓	
Chen [17]	✓		✓		✓	✓
Chen et al [13]	✓		✓		✓	✓
Tao and Li [14]			✓			
Li et al [19]	✓		✓	✓		
Yan and he [24]		✓	✓	✓	✓	✓
Shu and Wang [7]	✓		✓	✓		
Shu and Wang [21]	✓			✓	✓	
Gao et al [10]	✓		✓			
Li et al [20]	✓		✓			✓
Our study	✓	✓	✓	✓	✓	✓

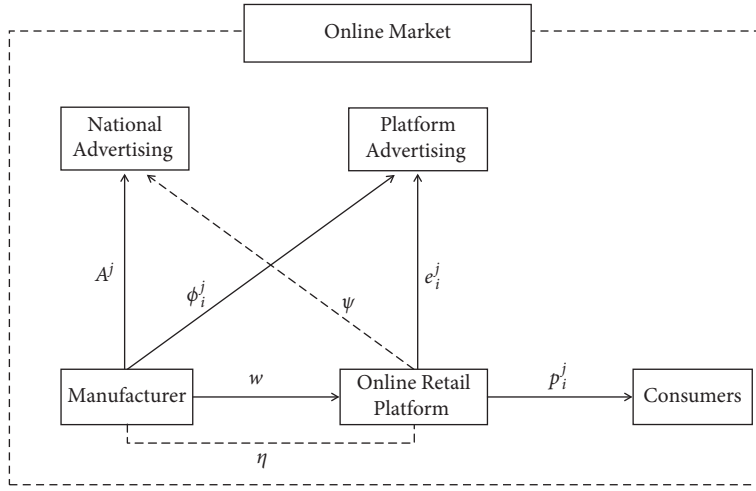

Figure 1: Manufacture and online retail platform cooperative structure.

weaken in the second period. For the same reason, we can get $0 < p_2 < 1/\delta$ and $w < p_2 < 1/\delta$.

And the cost functions for the national advertising and the platform advertising with respect to the advertising level are as follows:

$$C(A) = \frac{1}{2}A^2,$$
$$C(e_1) = \frac{1}{2}e_1^2, \quad (4)$$
$$C(e_2) = \frac{1}{2}e_2^2.$$

The descriptions of the other parameters are shown in Table 2.

3.1. One-Way Subsidy Strategy. In this situation, only the manufacturer shares part of the platform advertising cost for the online retail platform. And the game sequence is as follows: first, the manufacturer determines the subsidy rate for the platform advertising. Second, the manufacturer determines the national advertising level. Third, the online retail platform determines the retail price and the platform advertising level for the first period. Finally, the online retail platform determines the above two variables for the second period. The profit functions for the manufacturer and the online retail platform are given by

$$\pi_M^o = w \times D_1 + w \times D_2 - \frac{1}{2}A^{o2} - \frac{1}{2}\phi_1^o e_1^{o2} - \frac{1}{2}\phi_2^o e_2^{o2},$$
$$\pi_R^o = (p_1^o - w)D_1 + (p_2^o - w)D_2 - \frac{1}{2}(1 - \phi_1^o)e_1^{o2} \quad (5)$$
$$- \frac{1}{2}(1 - \phi_2^o)e_2^{o2},$$

where ϕ_i^o is the manufacturer's subsidy rate for the platform advertising and $i \in (1, 2)$.

Table 2: Descriptions of parameters.

Parameter symbol	Symbol description
$i \in \{1,2\}$	The first period and the second period
$j \in \{o,t,r\}$	o – one-way subsidy strategy, t – two-way subsidy strategy, and r – revenue-share strategy
p_i^j	Retail prices of the products under the strategy j in the i period
e_i^j	Platform advertising level of the product under the strategy j in the i period
ϕ_i^j	The subsidy rate of the platform advertising under the strategy j in the i period
A^j	National advertising level of the product under the strategy j
ψ	The subsidy rate of the national advertising by the online retail platform
η	The revenue-share rate
π_R^j	Profit of the online retail platform under the strategy j
π_M^j	Profit of the manufacturer under the strategy j

Using backward induction, we obtain the Stackelberg equilibrium results.

Proposition 1. *The optimal advertising level, subsidy rate, and retail price are as follows:*

$$\begin{aligned} A^o &= \frac{1}{2}w\beta(1-w\delta), \\ \phi_i^o &= \frac{(5w\delta-1)}{(1+3w\delta)}, \\ e_1^o &= \frac{(1-w\delta)(1+3w\delta)\lambda}{8\delta}, \\ e_2^o &= \frac{(1-w\delta)(1+3w\delta)\theta\lambda}{8\delta}, \\ p_i^o &= \frac{(1+w\delta)}{2\delta}, \end{aligned} \tag{6}$$

where $i \in \{1,2\}$. The profits for the manufacturer and the online retail platform are as follows:

$$\begin{aligned} \pi_M^o &= \frac{(-1+w\delta)\left(-16w^2\beta^2\delta^2+16w^3\beta^2\delta^2-\lambda^2-5w\delta\lambda^2-3w^2\delta^2\lambda^2+9w^3\delta^3\lambda^2-\theta^2\lambda^2-5w\delta\theta^2\lambda^2-3w^2\delta^2\theta^2\lambda^2+9w^3\delta^3\theta^2\lambda^2-64w\delta^2a_1-64w\delta^2a_2\right)}{128\delta^2}, \\ \pi_R^o &= \frac{(-1+w\delta)^2\left(-8w\beta^2\delta+8w^2\beta^2\delta^2-\lambda^2-2w\delta\lambda^2+3w^2\delta^2\lambda^2-\theta^2\lambda^2-2w\delta\theta^2\lambda^2+3w^2\delta^2\theta^2\lambda^2-16\delta a_1-16\delta a_2\right)}{-64\delta^2}. \end{aligned} \tag{7}$$

Proposition 1 shows that as the wholesale price w varies, the optimal subsidy rate ϕ_i^o falls into two different regimes. When the wholesale price is high ($1/5\delta \le w < 1/\delta$), the manufacturer chooses to invest optimal platform advertising level, and when the wholesale price is low ($0 < w < 1/5\delta$), the manufacturer chooses to invest nothing in the platform advertising.

In fact, if the wholesale price is lower in the wholesale market, manufacturers would not get enough money to invest in the cooperative advertising strategy. The profit of manufacturers in the wholesale market will affect their willingness and decision on sharing the advertising cost of retailers.

Then, differentiating A^o and e_i^o with respect to w, we have Proposition 2.

Proposition 2. *Studying the sensitivity analysis with respect to w.*

(a) $\partial A^o/\partial w > 0$ and $\partial e_i^o/\partial w > 0$, when $0 < w < 1/3\delta$

(b) $\partial A^o/\partial w > 0$ and $\partial e_i^o/\partial w < 0$, when $1/3\delta < w < 1/2\delta$

(c) $\partial A^o/\partial w < 0$ and $\partial e_i^o/\partial w < 0$, when $1/2\delta < w < 1/\delta$

In this research, we consider the national and platform advertising levels are affected by the wholesale price in the market and get the different changing trends for A^o and e_i^o

presented in Figure 2 and express them mathematically as follows.

From Figure 2, we know that when the wholesale price is very low in region I ($0 < w < 1/3\delta$), with the wholesale price increasing, the manufacturer and the online retail platform could pay more effort on the advertising. The manufacturer could get more profits in the wholesale market with the wholesale price increasing, and the online retail platform hopes to incentive more consumers to buy the product with the platform advertising. When the wholesale price is moderate in region II ($1/3\delta < w < 1/2\delta$), with the wholesale price increasing, the manufacturer could pay more effort on the national advertising, but the online retail platform reduces the effort on the platform advertising. The online retail platform could pay more money in the wholesale market with the increasing wholesale price. When the wholesale price is very high in region III ($1/2\delta < w < 1/\delta$), with the wholesale price increasing, the manufacturer and the online retail platform could pay less effort on the advertising. The manufacturer could get a lot of profits in the wholesale market and have no incentive to make great efforts on the national advertising.

In reality, when the wholesale price of products in the wholesale market is higher, manufacturers have no motivation to do advertising for products. Higher wholesale price may be due to the higher value of the product itself or no alternative product in the market. So manufacturers do not need advertising to increase the market demand for products.

In this section, we characterize that just the manufacturer contributes some efforts for platform advertising. Next section, we characterize the two cases: "two-way subsidy strategy" and "revenue-share strategy" for cooperation advertising between manufacturer and the online retail platform.

3.2. Two-Way Subsidy Strategy. In this situation, both the manufacturer and the online retail platform share the other's adverting costs. And the game sequence is as follows: first, the manufacturer and the online retail platform both determine the subsidy rate for each other simultaneously. Second, the manufacturer determines the national advertising level. Third, the online retail platform determines the retail price and the platform advertising level for the first period. Finally, the online retail platform determines the two variables for the second period. Without loss of generality, we omit the same demand functions to avoid redundancy and just define the profit functions for the manufacturer and the online retail platform. We denote ψ for the online retail platform's subsidy rate and $0 < \psi < 1$. We can express the profit functions as follows:

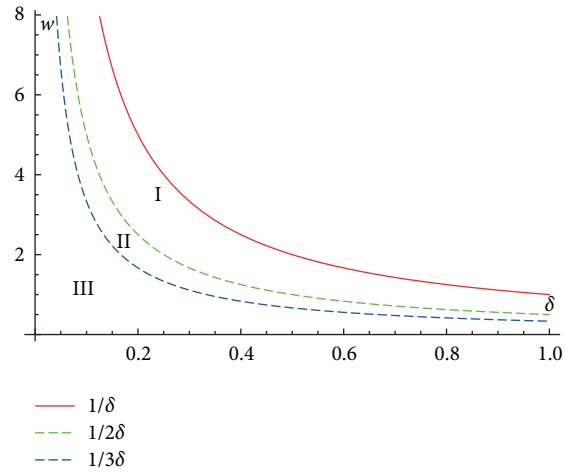

FIGURE 2: The changing trend for A^o and e_i^o with respect to w.

$$\pi_M^t = w \times D_1 + w \times D_2 - \frac{1}{2}(1-\psi)A^{t2} - \frac{1}{2}\phi_1^t e_1^{t2} - \frac{1}{2}\phi_2^t e_2^{t2},$$

$$\pi_R^t = \left(p_1^t - w\right)D_1 + \left(p_2^t - w\right)D_2 - \frac{1}{2}\psi A^{t2} - \frac{1}{2}\left(1-\phi_1^t\right)e_1^{t2} - \frac{1}{2}\left(1-\phi_2^t\right)e_2^{t2}. \tag{8}$$

Solve this game by using backward induction to get the following results and use superscripts t to denote a two-way subsidy strategy. For example, π_M^t represents the profit of the manufacturer under the two-way subsidy strategy.

Proposition 3. *The optimal advertising level, subsidy rate, and retail price are as follows:*

$$A^t = \frac{\beta(1-w\delta)}{4\delta},$$

$$\psi = 1 - 2w\delta,$$

$$\phi_i^t = \frac{(5w\delta - 1)}{(1+3w\delta)},$$

$$e_1^t = \frac{(1-w\delta)(1+3w\delta)\lambda}{8\delta}, \tag{9}$$

$$e_2^t = \frac{(1-w\delta)(1+3w\delta)\theta\lambda}{8\delta},$$

$$p_i^t = \frac{(1+w\delta)}{2\delta},$$

where $i \in \{1, 2\}$. *Furthermore, to ensure* $0 < \psi < 1$, *the condition* $w\delta < 1/2$ *must be satisfied. The optimal profits for the manufacturer and the online retail platform are given as follows:*

$$\pi_M^t = \frac{(-1+w\delta)\left(-8w\beta^2 + 8w^2\beta^2\delta^2 - \lambda^2 - 5w\delta\lambda^2 - 3w^2\delta^2\lambda^2 + 9w^3\delta^3\lambda^2 - \theta^2\lambda^2 - 5w\delta\theta^2\lambda^2 - 3w^2\delta^2\theta^2\lambda^2 + 9w^3\delta^3\theta^2\lambda^2 - 64w\delta^2(a_1+a_2)\right)}{128\delta^2},$$

$$\pi_R^t = \frac{(-1+w\delta)^2\left(2\beta^2 + \lambda^2 + 2w\delta\lambda^2 - 3w^2\delta^2\lambda^2 + \theta^2\lambda^2 + 2w\delta\theta^2\lambda^2 - 3w^2\delta^2\theta^2\lambda^2 + 16\delta a_1 + 16\delta a_1\right)}{64\delta^2}. \tag{10}$$

Proposition 3 shows that with the wholesale price increasing, the online retail platform could reduce the subsidy rate for the national advertising ($\partial\psi/\partial w = -2\delta < 0$). If the increase in the wholesale price leads to a decrease in the product's marginal profit, the online retail platform allowance policy for the manufacturer will reduce.

Therefore, in practical terms, manufacturers can appropriately lower wholesale price; retailers would have more incentive and money to invest in cooperative advertising.

3.3. Revenue-Share Strategy. In this situation, there is a revenue-share strategy between the manufacturer and the online retail platform. The online retail platform would share the revenue with the manufacturer in the retail market, and the revenue-share rate is an exogenous variable related to the power of the manufacturer and the online retail platform. We denote η for the revenue-share rate and $0 < \eta < 1$. When $1/2 < \eta < 1$, the online retail platform has the strong power in the market; when $0 < \eta < 1/2$, the manufacturer has the strong power. And the game sequence is as follows: first, the manufacturer determines the subsidy rate. Second, the manufacturer determines the national advertising level. Finally, the online retail platform determines the retail price and the platform advertising level for the first period and the two variables for the second period. For the same reason in the two-way subsidy strategy section, we just express the profit functions here. We can define the profit functions as follows:

$$\begin{aligned}\pi_M^r &= (1-\eta)\left(p_1^r D_1 + p_2^r D_2\right) + w \times D_1 + w \times D_2 - \frac{1}{2}A^{r2} \\ &\quad - \frac{1}{2}\phi_1^r e_1^{r2} - \frac{1}{2}\phi_2^r e_2^{r2}, \\ \pi_R^r &= \eta\left(p_1^r D_1 + p_2^r D_2\right) - w\left(D_1 + D_2\right) - \frac{1}{2}\left(1-\phi_1^r\right)e_1^{r2} \\ &\quad - \frac{1}{2}\left(1-\phi_2^r\right)e_2^{r2}.\end{aligned} \tag{11}$$

We also solve this game using backward induction and use superscripts r to denote the results in the revenue-share strategy.

Proposition 4. *The optimal advertising level, subsidy rate, and retail price are as follows:*

$$\begin{aligned}A^r &= -\frac{\beta(w\delta-\eta)\left(w\delta+\eta+w\delta\eta-\eta^2\right)}{4\delta\eta^2}, \\ \phi_i^r &= \frac{2w\delta+2\eta+3w\eta-3\eta^2}{2w\delta+2\eta+w\delta\eta-\eta^2}, \\ e_1^r &= -\frac{(w\delta-\eta)\left(2w\delta+2\eta+w\delta\eta-\eta^2\right)\lambda}{8\delta\eta^2}, \\ e_2^r &= -\frac{(w\delta-\eta)\left(2w\delta+2\eta+w\delta\eta-\eta^2\right)\theta\lambda}{8\delta\eta^2}, \\ p_i^r &= \frac{w\delta+\eta}{2\delta\eta}.\end{aligned} \tag{12}$$

And the optimal profits of the manufacturer and the online retail platform are given as follows:

$$\pi_M^r = \frac{1}{128\delta^2\eta^4}(w\delta-\eta)\begin{pmatrix} 4w^3\beta^2\delta^3 + 4w^2\beta^2\delta^2\eta + 8w^3\beta^2\delta^3\eta - 4w\beta^2\delta\eta^2 - 8w^2\beta^2\delta^2\eta^2 + 4w^3\beta^2\delta^2\eta^2 - \\ 4\beta^2\eta^3 - 8w\beta^2\delta\eta^3 - 12w^2\beta^2\delta^2\eta^3 + 8\beta^2\eta^4 + 12w\beta^2\delta\eta^4 - 4\beta^2\eta^5 + 4w^3\beta^3\lambda^2 + 4w^2\delta^2\eta\lambda^2 + 4w^2\delta^3\eta\lambda^2 - \\ 4w\delta\eta^2\lambda^2 - 4w^2\delta^2\eta^2\lambda^2 + w^3\delta^3\eta^2\lambda^2 - 4\eta^3\lambda^2 - 4w\delta\eta^3\lambda^2 - 3w^2\delta^2\eta^3\lambda^2 + 4\eta^4\lambda^2 + 3w\delta\eta^4\lambda^2 - \eta^5\lambda^2 + \\ 4w^3\delta^3\theta^2\lambda^2 - 4w^2\delta^2\eta\theta^2\lambda^2 + 4w^3\delta^3\eta\theta^2\lambda^2 - 4w\delta\eta^2\theta^2\lambda^2 - 4w^2\delta^2\eta^2\theta^2\lambda^2 + w^3\delta^3\eta^2\theta^2\lambda^2 - 4\eta^3\theta^2\lambda^2 - \\ 4w\delta\eta^3\theta^2\lambda^2 - 3w^2\delta^2\eta^3\theta^2\lambda^2 + 4\eta^4\theta^2\lambda^2 + 3w\delta\eta^4\theta^2\lambda^2 - \eta^5\theta^2\lambda^2 - 32w\delta^2\eta^2\alpha_1 - 32\delta\eta^3\alpha_1 - 32w\delta^2\eta^3\alpha_1 + \\ 32\delta\eta^4\alpha_1 - 32w\delta^2\eta^2\alpha_2 - 32\delta\eta^3\alpha_2 - 32w\delta^2\eta^3\alpha_2 + 32\delta\eta^4\alpha_2 \end{pmatrix},$$

$$\pi_R^r = -\frac{1}{64\delta^2\eta^3}(w\delta-\eta)^2\left(4w^2\beta^2\delta^2 + 4w^2\beta^2\delta^2\eta - 4\beta^2\eta^2 - 8w\beta^2\delta\eta^2 + 4\beta^2\eta^3 + 2w^2\delta^2\lambda^2 + w^2\delta^2\eta\lambda^2 - 2\eta^2\lambda^2 - 2w\delta\eta^2\lambda^2 + \eta^3\lambda^2 + 2w^2\delta^2\theta^2\lambda^2 + w^2\delta^2\eta\theta^2\lambda^2 - 2\eta^2\theta^2\lambda^2 - 2w\delta\eta^2\theta^2\lambda^2 + \eta^3\theta^2\lambda^2 - 16\delta\eta^2\alpha_1 - 16\delta\eta^2\alpha_2\right). \tag{13}$$

Proposition 4 shows that, under the revenue-share strategy, the revenue-share rate will affect the retail price of the product ($\partial p_i^r/\partial\eta = -w/2\eta^2 < 0$). With the increase of the revenue-share rate, the retail price will decrease; otherwise, the retail price increases.

As a matter of fact, after establishing a revenue-share strategy between manufacturers and retailers, if manufacturers have strong power and obtain more revenue from retailers, retailers will transfer the loss to consumers and increase the retail price to make up for the loss revenue.

4. Comparison and Managerial Implications

By comparing the equilibrium results among the three strategies, summarize some important management insight in the following propositions.

Furthermore, to ensure the online retail platform would share national adverting costs for the manufacturer in a two-way subsidy strategy, the condition $w < 1/2\delta$ must be satisfied.

Proposition 5. *The manufacturer provides (a) the same subsidy rate in one-way subsidy strategy and two-way subsidy strategy ($\phi_i^o = \phi_i^t = (5w\delta - 1)/(1 + 3w\delta)$)and (b) the higher subsidy rate for the platform advertising in revenue-share strategy ($\phi_i^r > \phi_i^t$).*

First, part (a) of Proposition 5 states that even though the online retail platform contributes the subsidy rate on the manufacturer's national advertising, the manufacturer also provides the same subsidy rate in one- and two-way subsidy strategies. One would consider that in a two-way subsidy strategy, the online retail platform provides money for the manufacturer's national advertising, the manufacturer would transfer the save cost ($1/2\psi A^2$) to the platform advertising by charging the higher subsidy rate (ϕ) in order to incentive the online retail platform to improve the platform advertising level for two periods. However, actually, the manufacturer pays more money to improve its own advertising level even if the national advertising just acts on the demand in the first period.

According to Proposition 5(b), the manufacturer provides a higher subsidy rate in the revenue-share strategy. In this strategy, there is a revenue-share relationship between the manufacturer and the online retail platform. The manufacturer maximizes his revenues from selling his product to the online retail platform and sharing part income from the online retail platform. So the manufacturer hopes to attract more consumers and increase the demand by his more effort on the platform advertising. When the manufacturer agrees to pay more on the platform advertising, the online retail platform would have the motive power to make the platform advertising and maximize its profit by selling more products.

Actually, compared with the strategy that retailers bear part of the national advertising investment cost, manufacturers prefer to improve the investment rate of retailers' advertising in revenue-share strategy. Retailers can sign an appropriate revenue-share strategy with manufacturers to promote manufacturers' investment.

Proposition 6. *The manufacturer's national advertising level has the following relationship among the three strategies:*

(a) $A^t > A^o$, $A^r > A^o$

(b) $A^r \geq A^t$, *when* $\eta^{3/2}(3\sqrt{\eta} + 4\sqrt{-4+5\eta})/2(1+\eta)\delta \leq w \leq 1/2\delta$; $A^r < A^t$, *others*

Proposition 6(a) states that in a two-way subsidy strategy, the online retail platform would provide a subsidy rate for the national advertising; the manufacturer will heighten the national advertising level to increase the demand in the first period to maximize his profit. In revenue-share strategy, the manufacturer would get part sales income from the online retail platform, and he has the power and the fund to heighten the national advertising level. So, when the cooperation relationship gets closer in a two-way subsidy strategy and revenue-share strategy, the manufacturer would be willing to increase the national advertising to attract potential customers.

In Proposition 6(b), it is very interesting that when $\eta^{3/2}(3\sqrt{\eta} + 4\sqrt{-4+5\eta})/2(1+\eta)\delta \leq w \leq 1/2\delta$, the

manufacturer would set higher-level national advertising in revenue-share strategy than that in a two-way subsidy strategy. The reason is that the manufacturer would get more profits from the wholesale market when the wholesale price is high. The manufacturer has the power to pay more effort on the national advertising even if the online retail platform does not share any portion of the national advertising cost in revenue-share strategy. When the wholesale price is not high enough $(0 < w < \eta^{3/2}(3\sqrt{\eta} + 4\sqrt{-4+5\eta})/2(1+\eta)\delta)$, the manufacturer hopes to increase his profit by increasing demand. The online retail platform would be willing to pay money on the national advertising when the wholesale price is low, so the manufacturer would make high-level national advertising to attract potential consumers in a two-way subsidy strategy. However, in a revenue-share strategy, the manufacturer's income consists of both wholesale market income and retail market income. Even if the wholesale price is low, the manufacturer could get part sales income from the online retail platform. So the manufacturer would not pay attention to the demand increasing by enhancing the national adverting level.

In reality, both cost-sharing and revenue-sharing can encourage manufacturers to increase their investment in national advertising, so as to better build a brand image to attract more potential consumers.

Proposition 7. *The online retail platform's platform advertising level has the following relationship among the three strategies:*

(a) $e^o = e^t$

(b) $e^o = e^t \geq e^r$, *when* $\sqrt{1-\eta}/\delta\sqrt{(2+\eta-3\eta^2)/\eta^2} \leq w \leq 1/2\delta$; $e^o = e^t < e^r$, *others*

Proposition 7(a) states that even if, in a two-way subsidy strategy, the online retail platform contributes the subsidy rate (ψ) on the national advertising, the platform advertising level is the same as that in one-way subsidy strategy. Because the demand increases with respect to the efficacy of the national advertising and $A^t > A^o$, when the online retail platform observes that the manufacturer improves the national advertising level ($A^t \geq A^o$) in a two-way subsidy strategy, it would set the same platform advertising level to get more demand and profits.

One would consider that in revenue-share strategy, the online retail platform shares the part income with the manufacturer and it would pay less money on the platform advertising. However, the game equilibrium shows that under the condition $0 < w < \sqrt{1-\eta}/\delta\sqrt{(2+\eta-3\eta^2)/\eta^2}$, the online retail platform would increase the platform advertising expenditure (Proposition 7(b)). Consistent with earlier studies, the manufacturer sets a higher subsidy rate for the platform advertising in revenue-share strategy and the same subsidy rate in one- and two-way subsidy strategies (Proposition 5). So we know that when the wholesale price is not high and the manufacturer shares more expenditure for the platform advertising, the online retail platform would set a higher platform advertising level to stimulate consumer's buying behavior to get more profits. With the wholesale price increasing, when w is more than the threshold $(\sqrt{1-\eta}/\delta\sqrt{(2+\eta-3\eta^2)/\eta^2} \leq w \leq 1/2\delta)$, the online retail platform would minus the expenditure in the platform advertising because it must pay more money in the wholesale market and loss part income with the revenue-share profit.

In reality, retailers do not arbitrarily change their advertising investment level by sharing advertising costs. So, in a two-way subsidy strategy, retailers' advertising level is the same as that in the one-way subsidy strategy. However, in the revenue-share strategy, the change of retailers' income will affect their investment level in advertising. Especially, with the wholesale price of products changing, retailers' investment in product advertising will also change.

Proposition 8. *The online retail platform's retail price has the relationship $p^o = p^t < p^r$ in the three cases.*

This proposition states that in one- and two-way subsidy strategies, the online retail platform would charge the same retail price for the product, but in the revenue-share strategy, the online retail platform charges the larger retail price. It is interesting that with one- and two-way subsidy strategies, the manufacturer and the online retail platform have the same value decision variable for ϕ, e, and p (Proposition 5, 7, and 8). Just the manufacturer's national advertising level is higher in a two-way subsidy strategy than that in one-way subsidy strategy. The reason is that the national advertising is just to influence potential consumers to consider this brand and to help develop brand knowledge and preference in the first period. if the manufacturer enhances the national advertising level in a two-way subsidy strategy (Proposition 6) and the online retail platform shares a portion of the national advertising, the online retail platform still sets the same platform advertising level and charges the same retail price compared with a two-way subsidy strategy in order to maximize its profit.

In revenue-share strategy, the manufacturer indirectly gets a portion of selling product income from the online retail platform. The online retail platform enhances the retail price to increase the profit even if the manufacturer sets a large subsidy rate (Proposition 5(b)) to minus his share in revenue-share strategy. Especially, from Propositions 7(b) and 8, we find that the online retail platform would charge a higher retail price in revenue-share strategy even if it pays less money for the platform advertising than that in other strategies.

Actually, when retailers and manufacturers reach a revenue-share strategy, retailers often transfer their revenue loss by increasing the retail price in the retail market. This will partly weaken consumers' willingness to buy products.

Proposition 9. *Studying the sensitivity analysis with respect to η in revenue-share strategy.*

(a) $\partial\phi_i^r/\partial\eta > 0$, *when* $0 < \eta < (-1+\sqrt{2})w\delta$; $\partial\phi_i^r/\partial\eta < 0$, *when* $(-1+\sqrt{2})w\delta < \eta < 1$

(b) $\partial e_i^r/\partial\eta < 0$, *when* $0 < \eta < 3^{1/3}w^2\delta^2 + \Delta^{2/3}/3^{2/3}\Delta^{1/3}$; $\partial e_i^r/\partial\eta < 0$, *when* $3^{1/3}w^2\delta^2 + \Delta^{2/3}/3^{2/3}\Delta^{1/3} < \eta < 1$

where $\Delta = (-18w^2\delta^2 + \sqrt{3}\sqrt{-w^4\delta^4(-108 + w^2\delta^2)})$.

In this research, we consider the revenue-share rate to be an exogenous variable and affected by the power of the online retail platform and the manufacturer in the market. When $0 < \eta < (-1 + \sqrt{2})w\delta < 1/2$, the manufacturer has a strong power and gets more profits in revenue-share strategy. The manufacturer could pay more effort on the platform advertising to incentivize consumers to buy the product as the revenue-share rate increases. When $(-1 + \sqrt{2})w\delta < \eta < 1$, the online retail platform has much power in the bargaining. Because the manufacturer does not get enough profit in the revenue-share strategy, he would decrease the subsidy rate of the platform advertising with the revenue-share rate increasing. Next, we analyze the platform advertising level changing trend by the revenue-share rate. From Proposition 9(b), we know that when $0 < \eta < 3^{1/3}w^2\delta^2 + \Delta^{2/3}/3^{2/3}\Delta^{1/3}$, the online retail platform would increase the subsidy rate of the platform advertising with the revenue-share rate increasing. The online retail platform wants to enhance consumers' demand to minus the loss profit in revenue-share strategy by the platform advertising's effect. When $3^{1/3}w^2\delta^2 + \Delta^{2/3}/3^{2/3}\Delta^{1/3} < \eta < 1$, the online retail platform is satisfied with the profit from the sales market, and with the revenue-share rate increasing, it would cut down the platform advertising level.

And, in conclusion, we get the different changing trends for ϕ_i^r and e_i^r presented in Figure 3 and express them mathematically as follows.

First, in region I, we get that ϕ_i^r and e_i^r have the same changing trend. With η increasing, the manufacturer decreases the subsidy rate for the platform advertising and the online retail platform reduces the platform advertising level simultaneously. It is very interesting that in region I, even if the online retail platform has a strong bargaining power ($\eta \longrightarrow 1$), it still decreases the investment in the platform advertising. Maybe the online retail platform considers that it has got enough profit by the higher retail price (Proposition 5(b)) and the manufacturer's higher subsidy rate (Proposition 8) comparing other cases, and it could minus the lost share by reducing the platform advertising level.

Second, in region II, we get that with η increasing, the online retail platform would pay more effort on the platform advertising even if the manufacturer's subsidy rate is decreasing. Because with η increasing, the online retail platform gets more profits in revenue-share strategy and has the capital to pay more effort on the platform advertising even if the manufacturer reduces the subsidy rate for the platform advertising.

Finally, in region III, we get with η increasing, the manufacturer raises the subsidy rate for the platform advertising and the online retail platform enhances the platform advertising level simultaneously. In region III, we consider that the manufacturer has a strong power in the bargaining. When the manufacturer has more sharing in a revenue-share strategy, he would be willing to undertake a high subsidy rate for the platform advertising. When the online retail platform observes this situation, it could enhance the platform advertising level to incentive more consumers to buy the product.

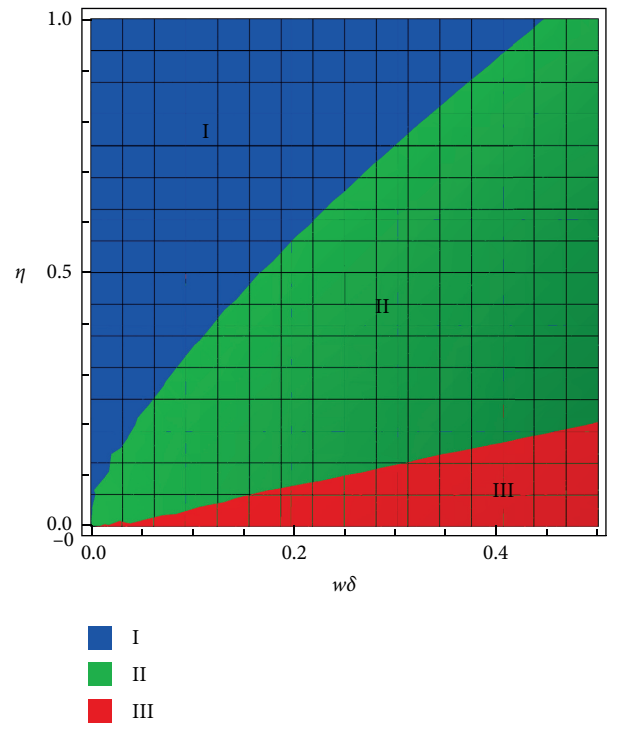

Figure 3: The distributions of ϕ_i^r and e_i^r.

In the actual operation process, in revenue-share strategy, retailers can obtain manufacturers' support for their advertising by transferring more revenue to manufacturers. Even if retailers increase the advertising investment, manufacturers would increase the share rate for the advertising.

5. Numerical Analysis

In this section, we would give the values for the different parameters to compare these strategies and the optimal profits. In the numerical analysis, we set the following parameters for the manufacturer and the online retail platform: $a_1 = 1$, $a_2 = 1$, $\beta = 1$, $\lambda = 1$, $\theta = 1/2$, and $\delta = 1$ and set $\eta = 1/2$ to let the manufacturer and the online retail platform have the same power in the bargaining game. Because of the condition $1 - 2w\delta > 0$, we get $0 < w < 1/2$. We would define S^o (S^t or S^r) represents the total profit of the supply chain with the manufacturer and the online retail platform under situation one-way subsidy strategy (two-way subsidy strategy or revenue-share strategy) and get the total profit under three strategies as follows:

$$S^o = \frac{(-1+w)\left(-271 - 355w + 99w^2 + 15w^3\right)}{512},$$

$$S^t = \frac{(-1+w)(7+w)\left(-41 - 38w + 15w^2\right)}{512}, \tag{14}$$

$$S^r = \frac{(-1+2w)\left(-1147 - 2462w + 828w^2 + 984w^3\right)}{2048}.$$

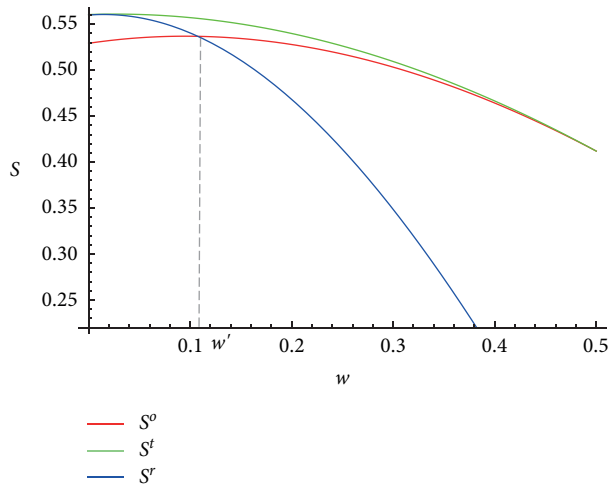

Figure 4: The value of the total profit.

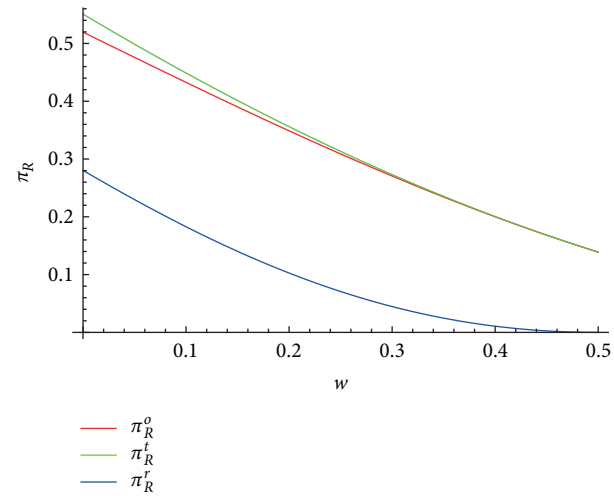

Figure 5: The value of the online retail platform's profit.

And the total profit is presented in Figure 4 and is expressed mathematically as follows.

From Figure 4, we find that in a two-way subsidy strategy, the total profit of the supply chain is always better than that in other cooperation strategies ($S^t > S^r$ and $S^t > S^o$). In a two-way subsidy strategy, the manufacturer and the online retail platform bear each other's advertising investment costs and share the advertising investment risks. A two-way subsidy strategy encourages them to pursue more corporate profits by appropriately increasing advertising investment and brings higher total profits of the supply chain. At the same time, we also find that when the wholesale price of the product takes a special value, the total profit of the supply chain in a one-way subsidy strategy is the same as that in the revenue-share strategy. Set this specific value to $w\prime$ here. When $w < w\prime$, the closer cooperation between the manufacturer and the online retail platform in revenue-share strategy could make the total profit better than that in one-way subsidy strategy ($S^r > S^o$). Because when the wholesale price of the product is low, the online retail platform has greater power and ability to invest in advertising to promote the product. The total profit of the whole supply chain will increase due to the improvement of the product popularity and competitiveness. When $w > w\prime$, the closer cooperation between the manufacturer and the online retail platform in revenue-share strategy could make the total profit worse than that in one-way subsidy strategy ($S^r < S^o$). Because when the wholesale price of the product is high, the online retail platform needs to invest more purchase costs in the wholesale market. At the same time, the online retail platform also needs to share its sales revenue with the manufacturer in revenue-share strategy. It would have no power and ability to invest in advertising, which will affect the market sales of the product and reduce revenue.

In practical terms, starting from the overall interests of the supply chain, supply chain participants should actively explore reasonable cooperative alliances to increase the total profit of the supply chain.

We compare the profit of the online retail platform under three strategies and get Figure 5 as follows.

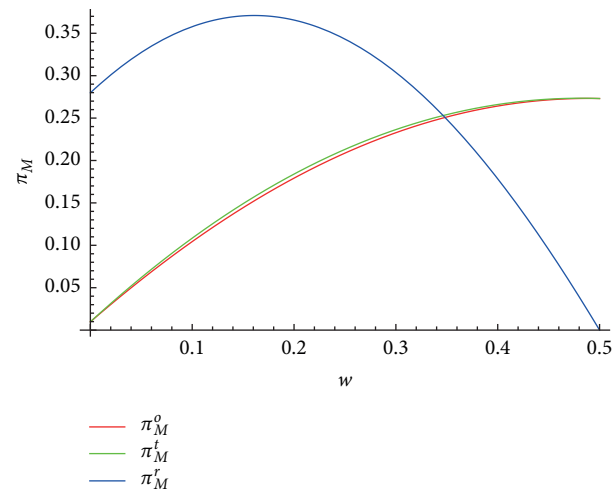

Figure 6: The value of the manufacturer's profit.

From Figure 5, we find that the profit of the online retail platform under a two-way subsidy strategy is always better than that in one-way subsidy strategy and revenue-share strategy even under two-way subsidy strategy, the online retail platform must provide some money for the manufacturer's national advertising. In particular, in the revenue-share strategy, the online retail platform needs to transfer part of the product sales revenue to the manufacturer in exchange for cooperation so that his revenue is far less than the profit in other cooperation strategies. In fact, retailers should actively strengthen the cooperation between retailers and manufacturers and improve their profit by signing a two-way subsidy strategy.

Furthermore, we compare the manufacturer's profit under three strategies and also observe the interesting conclusion (see Figure 6).

Because, under the revenue-share strategy, the manufacturer would get a portion of the online retail platform's selling product profit, we consider that the profit of the manufacturer under the revenue-share strategy is better than that in other strategies. Figure 6 shows that when the

wholesale price is high, the manufacturer's profit is worse than that in other strategies even if he would get the income from the wholesale market and the retail market. In revenue-share strategy, the online retail platform needs to share the product sales revenue of its own retail market with the manufacturer. With the increase of product wholesale price, the online retail platform will also increase product retail price to strive for more profits. Too high product retail prices will hurt consumers' enthusiasm to buy the product. When the manufacturer's revenue in the wholesale market decreases, his share of sales revenue will also decrease. Therefore, when the wholesale price of the product is too high, the manufacturer's profit may be lower than that in the other two strategies. Actually, manufacturers can choose the cooperation strategy with retailers according to the changes of product wholesale price, rather than invariable.

6. Conclusion

As an important consideration when consumers decide whether to buy a product or not, the retail price and the advertising can both influence consumers' willingness [15]. This paper investigates the cooperative advertising problem by taking price and advertising effect into account in the two-period online supply chain. Utilizing the Stackelberg game, the manufacturer's and the online retail platform's optimal level on national and platform advertising are calculated in three different cooperation adverting strategies. As Zhang et al. [15], in a two-way subsidy strategy, both the manufacturer and the online retail platform share the other's adverting costs. Especially, in a revenue-share strategy, the manufacturer both share the online retail platform's adverting costs and the retailing income.

The main results of this paper we found as follows: first, under revenue-share cooperative advertising strategy, the manufacturer provides a higher subsidy rate for the online retail platform advertising than that in other cooperative advertising strategies. However, the online retailer platform does not blindly invest more platform advertising costs in revenue-share strategy than that in other strategies because of the high platform advertising subsidy rate. The level of advertising is affected by the wholesale price of products. Second, in the revenue-share strategy, there is a special region. With the increase of the revenue-share rate, the online retail platform obtains the manufacturer's advertising subsidy support by transferring more revenue to the manufacturer. When the online retail platform increases its investment on platform advertising, the manufacturer will also increase his subsidy rate for platform advertising. Thrid, with identifying the value of the cooperative advertising, it is the best choice for the online supply chain system that the manufacturer and the online retail platform just take two-way subsidy strategy. Interestingly, there are conditions where, while just the manufacturer contributes a percentage of the online retail platform's platform advertising in a one-way subsidy strategy, the total profit of the online supply chain would be better than that in revenue-share strategy even if the cooperation relationship is closer.

In future research, some valuable extensions of this paper could include the following. First, the characteristics of the network market as bilateral network externalities can be considered. It may be more interesting if the impact of the definition on advertising can be positive or negative. Second, it may be interesting that assume a dynamic model for a two-period supply chain, in which the impact ability of cooperative advertising in the latter stage is based on the impact of the previous stage. Finally, there are multiple manufacturers and multiple online retail platforms, and there is competition between them. The level of competition has an impact on the best subsidy rate of all manufacturers and the best advertising level of all online retail platforms [29].

Appendix

Proposition 10. *Using backward induction and solving the first-order condition of π_R, we get*

$$p_2^o = \frac{1 + w\delta}{2\delta}, \qquad e_2^o = -\frac{(-1 + w\delta)^2 \theta\lambda}{4\delta\left(-1 + \phi_2^o\right)}. \tag{A.1}$$

Substituting p_2 and e_2 into π_r and solving the first-order condition of π_R, we have

$$p_1^o = \frac{1 + w\delta}{2\delta}, \qquad e_1^o = -\frac{(-1 + w\delta)^2 \lambda}{4\delta\left(-1 + \phi_1^o\right)}. \tag{A.2}$$

Substituting p_i and e_i into π_M and solving the first-order condition of π_M with respect to A yield, we get

$$A^o = \frac{1}{2} w\beta(1 - w\delta). \tag{A.3}$$

Using the above results and solving the first-order condition of π_M with respect to ϕ_1 and ϕ_2 yield, we have

$$\phi_i^o = \frac{(5w\delta - 1)}{(1 + 3w\delta)}. \tag{A.4}$$

Substituting (A.4) into e_i^o, π_M^o, and π_R^o, we get the optimal solutions shown in Proposition 10.

Proposition 11. *It can be obtained by solving the first-order condition of A^o and e_i^o as follows:*

$$\frac{\partial A^o}{\partial w} = -\frac{1}{2}\beta(-1 + 2w\delta) = 0, \tag{A.5}$$

$$\frac{\partial e_i^o}{\partial w} = -\frac{1}{4}(-1 + 3w\delta)\lambda = 0. \tag{A.6}$$

Solving equations (A.5) and (A.6), we get the result as shown in Proposition 11.

Proposition 12. *Using backward induction and solving the first-order condition of π_R, we get*

$$p_2^t = \frac{1+w\delta}{2\delta},$$
$$e_2^t = -\frac{(-1+w\delta)^2\theta\lambda}{4\delta\left(-1+\phi_2^o\right)}. \tag{A.7}$$

Substituting p_2 and e_2 into π_r and solving the first-order condition of π_R, we have

$$p_1^t = \frac{1+w\delta}{2\delta},$$
$$e_1^t = -\frac{(-1+w\delta)^2\lambda}{4\delta\left(-1+\phi_1^o\right)}. \tag{A.8}$$

Substituting p_i and e_i into π_M and solving the first-order condition of π_M with respect to A yield, we get

$$A^t = \frac{w\beta(-1+w\delta)}{2(-1+\psi)}. \tag{A.9}$$

Using the above results and solving the first-order condition of π_M with respect to ϕ_1 and ϕ_2 yield, we have

$$\phi_i^t = \frac{(5w\delta-1)}{(1+3w\delta)}. \tag{A.10}$$

Using the above results and solving the first-order condition of π_R with respect to ψ, we get

$$\psi = 1-2w\delta. \tag{A.11}$$

Substituting equations (A.10) and (A.11) into e_i^o, A^t, π_M^o, and π_R^o, we get the optimal solutions shown in Proposition 11.

Proposition 13. *The proof is similar to Proposition 10 and thus is omitted.*

Proposition 14. *To prove this, we need to compare the subsidy rate for the platform advertising under different strategies:*

$$\phi_i^o = \phi_i^t = \frac{(5w\delta-1)}{(1+3w\delta)},$$
$$\phi_i^r - \phi_i^t = \frac{4(-1+\eta)\left(-w\delta+w^2\delta^2-\eta-w\delta\eta\right)}{(1+3w\delta)\left(2w\delta+2\eta+w\delta\eta-\eta^2\right)} > 0. \tag{A.12}$$

Proposition 15. *To prove this we need to compare the national advertising level under different strategies:*

$$A^o = \frac{1}{2}w\beta(1-w\delta),$$
$$A^t = \frac{w\beta(-1+w\delta)}{2(-1+\psi)},$$
$$A^r = -\frac{\beta(w\delta-\eta)\left(w\delta+\eta+w\delta\eta-\eta^2\right)}{4\delta\eta^2}, \tag{A.13}$$
$$A^r - A^o = \frac{\beta(-1+\eta)\left(w^2\delta^2+2w^2\delta^2\eta-\eta^2\right)}{4\delta\eta^2} > 0,$$
$$A^t - A^o = \frac{\beta(-1+w\delta)(-1+2w\delta)}{4\delta} > 0.$$

When $A^r = A^t$ satisfies, we get $w = \eta^{3/2}(3\sqrt{\eta} - \sqrt{-4+5\eta})/2(1+\eta)\delta$ or $w = \eta^{3/2}(3\sqrt{\eta} + \sqrt{-4+5\eta})/2(1+\eta)\delta$. Because we have the condition as $0 < w < 1/2\delta$, we can obtain the result in Proposition 15.

Proposition 16. *To prove this, we need to compare the platform advertising level under different strategies:*

$$e_i^o = -\frac{(-1+w\delta)(1+3w\delta)\lambda}{8\delta},$$
$$e_i^t = -\frac{(-1+w\delta)(1+3w\delta)\lambda}{8\delta}, \tag{A.14}$$
$$e_i^r = -\frac{(w\delta-\eta)\left(2w\delta+2\eta+w\delta\eta-\eta^2\right)\lambda}{8\delta\eta^2}.$$

We can obtain that $e_i^o = e_i^t$. When $e_i^r = e_i^t$ satisfies, we get $w = -\sqrt{1-\eta}/\delta\sqrt{(2+\eta-3\eta^2/\eta^2)}$ or $w = \sqrt{1-\eta}/\delta\sqrt{(2+\eta-3\eta^2/\eta^2)}$. Because we have the condition as $w < 1/2\delta$, we can obtain the result in Proposition 16.

Proposition 17. *To prove this, we need to compare the retail price under different strategies:*

$$p_i^o = \frac{(1+w\delta)}{2\delta},$$
$$p_i^t = \frac{(1+w\delta)}{2\delta}, \tag{A.15}$$
$$p_i^r = \frac{w\delta+\eta}{2\delta\eta}.$$

We can obtain that $p_i^o = p_i^t$ and $p_i^r - p_i^t = -w(-1+\eta)/2\eta > 0$. We can obtain the result in Proposition 17.

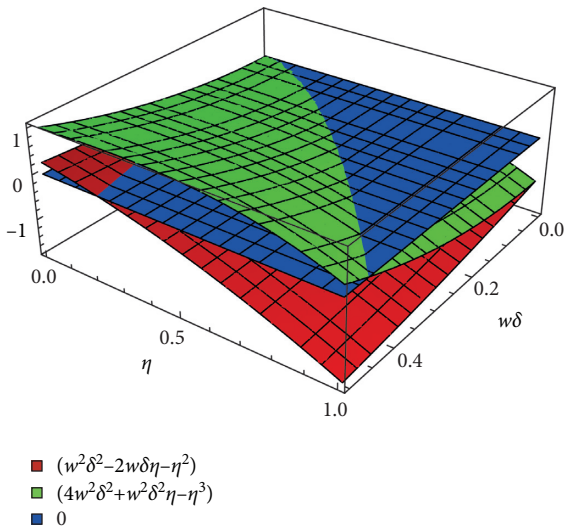

Figure 7: Comparing relationship.

Proposition 18. *It can be obtained by solving the first-order condition of* $\partial\phi_i^r/\partial\eta$ *and* $\partial e_i^r/\partial\eta$ *as follows:*

$$\frac{\partial\phi_i^r}{\partial\eta}=\frac{4\left(w^2\delta^2-2w\delta\eta-\eta^2\right)}{\left(2w\delta+2\eta+w\delta\eta-\eta^2\right)^2},$$
$$\frac{\partial e_i^r}{\partial\eta}=\frac{\left(4w^2\delta^2+w^2\delta^2\eta-\eta^3\right)\lambda}{8\delta\eta^3}. \tag{A.16}$$

We can obtain that if $(w^2\delta^2-2w\delta\eta-\eta^2)>0$, then $\partial\phi_i^r/\partial\eta>0$, and if $(4w^2\delta^2+w^2\delta^2\eta-\eta^3)>0$, then $\partial e_i^r/\partial\eta>0$. To compare $(w^2\delta^2-2w\delta\eta-\eta^2)$, $(4w^2\delta^2+w^2\delta^2\eta-\eta^3)$, and 0, we get Figure 7.

Through Figure 7, we can obtain the result in Proposition 18.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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