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Article

Accepted Version

The Paradoxical Prices of Options*

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Abstract

The synchronised relationship between financial and fundamental prices has been topical for years now. It seems that option pricing theory (OPT) has not be used to disentangle that relationship between two prices during merger and acquisition (M&A) activities. This article uses Put-Call parity theorem to explore the divergence of financial and fundamental prices in any firm during acquisition process. The results illustrate that price differentials are persistent; moreover, the differentials are caused by the exponential factor. Despite the fact that some principles are drawn from the REIT literature, the results have wider implications for industries with similar traits to REITs.

Keywords: NAV, share price, Put-Call parity, REIT

JEL: G13

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*The idea that share prices and NAVs trade at different levels, especially in the REIT industry, was initiated and stimulated by Simon Martin. We are grateful for valuable comments from George Alexandridis, Donald DePamphilis, Pablo Fernandez, Simon Martin, Simon Stevenson, Cambridge-Reading PhD Research Day participants, especially Eva Steiner. All the remaining errors are our own. Corresponding author: **T** +27(0)11 717-7668, **F** +27(0)86 765-8628 and **E** tumellano.sebehela@wits.ac.za.

1 Background

Marcato et al. (2019) explored exchange options in the REIT industry. In that study, some exchange options occurred between REITs and real estate operating companies (REOCs), some between REOCs and REOCs, some between REITs/REOCs and other type of firms. In terms of funding, some transactions were funded through cash only, stocks and cash (either debt or equity, or both) and some via stocks only. In presenting the results, Marcato et al. (2019) grouped the results based on the following parameters; (i) conflict of interest, (ii) market growth, (iii) funding (i.e. type of funding) and (v) specialisation. The results illustrated that the four parameters explain exchange options occurrence and persistent strategic investment decisions. However, Marcarto et al. (2019) never explained volatilities behaviour in those exchange options and the volatility patterns for determined window periods.

Marcato et al. (2018) investigated the latter statement. Their study used the Black-Scholes (1973) model (from here B-S) for only cash-finaced options. The deals used are classified as being small to medium transactions. The salient results of Marcato et al. (2018) illustrate that initial put value is in-the-money and synergies are 30% on average. In terms of converge to their long-term average volatilities, spot volatilities took some time to converge. These converge challenges are common on actual M&A deals, but what about on actual exchange options? The slower converges were evident both for underlying stock and converge towards the closing prices of target firms. Furthermore, Marcato et al. (2018) illustrated evident risks and inherent excess returns in those exchange options. One of the main question is that can those listed latter factors explain price differentials. For this article, the price differentials are between fundamental (i.e. net asset value per share, form here NAV) and financial (i.e. share) prices.

According to Coskun et al. (2020), the issue of NAV versus share price diversion has been a long-standing point of discussion including within the Turkish REIT industry. In Coskun et al. (2020), parameters that led the divergence of financial and fundamental prices include leverage, financial performance, liquidity, size, market sentiments and appraisal biasness of appraisers. They argue specifically that levered NAVs traded at lower levels than share price while fair value accounting leads to NAVs trading at premiums. The issue of appraisal biasness is well documented in developed countries. Capozza and Israelsen (2007) stated the reason why values of real assets are uncertain is due to acceptable margin error valuation, which leads to subjective NAV values. Moreover, it can be inferred from Capozza and Israelsen

(2007), assessors make conservative assumptions when doing valuations and that leads to diversion.

The widely used closed-form derivatives formula that illustrates relationship between fundamental and financial price is the Put-Call parity. Put-Call parity was pioneered by Hans Stoll on 1969. At the heart of Put-Call parity is depicting the relationship between put and call options in a closed-form solution. The parity relationship is explored in depth in the modelling section of this article. The study that is close to this article is Klemkosky and Resnick (1980), where Put-Call parity includes dividends. Another article that comes close to this study is Hsieh et al. (2008). Although, they looked at futures and index options while this study uses Put-Call approach on compound options. In Hsieh et al. (2008), implied volatilities for both futures and options indices were noticeable and statistically significant. Moreover, the results show that when there is Put-Call parity violation, there is information content is evident in the model. Overall, despite of the brilliant illustrations of Hsieh et al. (2008), it does not answer what causes violations between fundamental and financial option prices.

Firstly, the objective of this article is to illustrate the price differentials when there are two underlying assets. In particular, it focuses when one price is a fundamental one and other is a financial one price). Is price differentials sustatanable? This article differs from the two mentioned studies because this article takes into account risk-free interest rate, dividends, the lagging effect which is common in most industries and the effect of tau (τ). Finally, this article focuses on pricing, hedging and calibration of options when mentioned parameters are taken into account. In principle, this article addresses more issues relating the Put-Call parity theorem. The resulst of this article illustrates price differentails are common during corporate activities. Furthermore, due to the price differentials, one needs to use appropriate formuals for pricing options during bear and bull market conditions.

The balance of the article is as follows. Section 2 reviews prior literature and develops the hypothesis. Section 3 is on modelling. Section 4 is on the investigation of exponetial factor and section 5 concludes the article.

2 Literature Review

2.1 Price Relationship in Exchange Options

Chen and Suchanecki (2011) created a Parisian exchange options based occupation time derivatives and Parisian options. The authors opined that those type of options are not just valuable for hedging but for merger arbitrage. That is, other than the two firms merging, the actual merging time is taken into account-this is more like path dependent exchange options. Chen and Suchanecki (2011) state that in case of merger, prices of bidding and taken firms should be underpinned by what happens in those firms respectively. For their modelling framework, they start from the usual exchange option formula. Then, proceed to the barrier exchange option. The barrier element is because each option (i.e. put and call) have different probabilities of being in-the-money. By extension, the prices of acquiring and target firms have different probabilities of being in-the-money. On the other hand, Ross (1976) opined that option defined by a range of random returns. Similarly, in an exchange option, predators and preys are equitable compensated. Interesting, that moneyness leading to the entire exchange option being down-and-in the exchange call/put option. Moreover, the price of the latter option can be written differently, the difference between ordinary exchange option and subsequent down-and-in exchange call option. The inverse is true about up-and-out exchange call option. Chen and Suchanecki (2011) also express the latter option as Radon-Nikodym derivative. The point here is that prices of the main exchange option are sensitive to prices of sub-options of the main option.

There were also special cases of standard Parisian exchange options. In addition, cases of cumulative Parisian exchange options where their values are underpinned by standard Parisian exchange options. By extension, “main” prices are influenced by standard sub-prices. The numerical results illustrate that a decrease in barrier levels, standard Parisian options are more valuable than barrier options and cumulative Parisian options are the less valuable. That can be attributed to the fact that barrier exchange options had less barriers than other options. The major finding by Chen and Suchanecki (2011) is that more parameters are included in option pricing, the more variables influence the prices of different option legs. Moreover, the impact of those parameters of different price legs is not always the same. Hence, difference in prices of an exchange option.

Ma et al. (2020) and Johnson and Shanno (1987) show how changes in variance of options lead to different prices in an exchange options. In Johnson and Shanno (1987) the variation in ex post variance while in Ma et al. (2020) variation is due to clustered jump dynamics. Johnson and Shanno (1987) assumed that both the return and stock price follow a stochastic process. Moreover, the hedging strategy and the portfolio of assets have random parameters. Call prices are generated using Monte Carlo simulations. The results of Johnson and Shanno (1987) show that the value of out-of-money options increase with an increase in correlation effect. Thus, due to increase in correlation including in prices, the variation increases lead to increase value of options. The increase in implied volatility had the same effect.

Ma et al. (2020) incorporated Hawkes process in an exchange option in order to account jumps one an asset and across assets. They argue that there is no perfect hedging given clustering and each asset by implication its price has its own Greeks (Delta, Theta, Vega and Gamma). Fundamentally, the Greeks of a target firm tend to be higher than the Greeks of an acquiring firm. The numerical results show that prices of both assets increase dramatically because of the jumps, which leads to increase volatility. And that leads to increased option prices. It seems the prices of both assets do not increase at the same amount of change. The empirical results based on S&P500 index reveal the same findings as the numerical results.

2.2 Premium versus Discounts of Exchange Options

2.2.1 Differentials in Option Prices

The price differentials in multi options can be explained by numerous things including option trading at premium or discount to other prices of other options. Longstaff (1992) investigated a rare situation indeed, where there is a possibility of negative prices. In the context B-S model, negative is only possible when prices do not follow a log normal distribution-where an investor is paid to invest in an option. Another point put forward by Longstaff (1992) in relation to negative prices is the presence of friction. In industries like REITs, friction is evident; therefore, there should be a possibility of negative option prices. He uses a simple long call option formula to compute callable and non-callable bonds. Preliminary estimates reveal that over 340 calls, 61.5% have negative values. Furthermore, there is much correlation between callable bonds and bond prices. Overall, the results show that the Treasury-bond market is imperfect; hence, characterised by frictions. Longstaff (1992) went further and investigated the call policy of the Treasury. Based on the policy the results reveal that negative prices are due to Treasury calling bonds when it is advantageous to the bondholder. He states that other potential explanations of negative prices, by extension price differentials include Treasury activities (i.e. liquidity, different tax treatments, premium amortization and tax-timing options), and market frictions and arbitrage (i.e. taxes, short-selling costs and actual arbitrage strategies). The issue of (il)liquidity is covered again in Brenner et al. (2001).

According to Brenner et al. (2001), illiquidity is one such character that explains price differentials. They argue that common determinants of illiquidity are bid-offer spreads and transactions costs-which are well documented in microstructure literature. For their study, they compare currencies issued over-the-counter (OTC) by Bank of Israel (BI) and options traded on Tel-Aviv Stock Exchange (TASE) during the period of April 1994 to June 1997. For modelling, they used implied standard deviation (ISD) weighted by *vega* and forward currency option formula. The preliminary results show that BI options traded at lower levels than the TASE options. In addition, ATM options showed less difference because of the tradability. They went further and tested illiquidity discount based on two methods-currency options techniques. Method 1 illustrated that TASE options traded about 21% on average higher than BI options. Method 2 is on validation of illiquidity. Those results show that illiquidity is statistically significant. Finally, transactions costs on BI options contributed to illiquidity. Deuskar et al. (2011) investigate the price differentials by exploring liquidity in OTC options markets.

Deuskar et al. (2011) explored liquidity effect in OTC options markets, specifically focusing on whether prices trade at discounts or premiums in the market. According to the authors, the prominent factors that lead to price differentials in OTC markets are interest rate caps and floor markets. They used data from Westdeutsche Landsbank Girozentrale (WestLB) and Fixed Income Group during the period of January 1999 to May 2001. They present prices of caps and floors at OTM, ATM and ITM. The used Black-GBM and GARCH models to estimated implied volatilities. The initial result reveal that bid-offer spreads ranges from 8% to 9%. The results broadly reveal that the higher the liquidity, the less the differentials. By extension, it implies lower premiums and/or discounts. Moreover, the results reveal that premiums and discounts are affected by moneyness of options-ATM options have less price differentials and/or discounts, ITM options hardly have differentials and OTM options have significant differentials.

2.2.2 Jumps of Prices

Deng (2015) priced an American put option on a zero-coupon bond in a jump-extended Cox-Ingersoll-Ross (1985) model (from here CIR). Recall that CIR model is a short term interest model with jumps. He argues that short term-and-long term interest rates can be modelled in pure diffusion models; although the latter assumption contradicts empirical findings of some studies. Moreover, he states that jump-diffusion interest rates models do not give closed-form solutions. The reason why his study focuses on American style options is because according to him most traded interest rate products in the bond markets give option holders the right to chose the dates of exercisability. He illustrates the latter point using CIR model. Secondly, Deng (2015) derives a characteristic function using a Fourier transform in order to recover the corresponding cumulative probability. In principle, he prices a Bermudan option because the American option in his study has different exercisable dates that the holders chose from.

He used (i) affine jump-diffusion CIR model (from here CIR_EJ) that allows random jumps in order to capture term structure of short term rate and defaultable-free, zero-coupon bond and (ii) the discounted zero-coupon bond with maturity T with pre-specified date T in future without intermediate payments. His extension on the CIR model allows jumps to move randomly. Then, he prices the American put option on zero-coupon bond using two Geske and Johnson (1984) approach. Deng (2015) states as number of dates in Bermudan option increases, so its value approaches a true American option value. Similarly, when nodes in a binomial tree increase, the option values approaches B-S values. Secondly, he inverted a formula for pricing a bond of American nature with two dates (simply a Bermudan option with two exercisable dates).

The main finding of Deng (2015) was that Poisson process does not increase the value of the American option. Furthermore, numerical results show that the ‘higher the initial interest rate leads to lower price of the zero-coupon bond’. Secondly, the jump intensity has a significant impact on the option values. According to Deng (2015), the latter statement illustrates that jump intensity is central to the term structure of the interest rates. In order to validate and test reliability of his results, Deng (2015) used least-square Monte Carlo (MC) simulation with at least 100,000. Surprisingly, the results of least-square MC were unsatisfactory. Possible reasons for that might be the quality of data or some jumps were inaccurately captured in the models.

Zhang and Schmidt (2016) approximate a small-time probability density functions in a case of a general jump diffusion process. They argue that general jump diffusion models that combine stochastic volatility and local volatility are common in practice. Due to the demand of those models in practice, Zhang and Schmidt (2016) state that finding a joint probability distribution for of the asset price (or return) and its volatility and its volatility at a certain point is a challenge. The theorems that can address the latter statement according to Zhang and

Schmidt (2016) is the Fokker-Planck equation (FPE) because it is either a partial differential equation (PDE) when there are no jumps or partial integral-differential equation (PIDE) when jumps are added to models. One advantage of FPE according to authors is the fact that it can be discretized numerically and its approximations tend to be accurate due to an iteration process.

For modelling, Zhang and Schmidt (2016) introduce the fast Fourier transform (FFT). Other techniques used include Itô-Taylor expansion. In order to maintain simplicity in Itô-Taylor expansion, Zhang and Schmidt (2016) dealt first with stochastic process without any jumps. For lemma 1, which is on real functions and matrices, they used Euler-Maruyama approximation as it provides reasonable drifts with constant diffusion functions over short times. Other technique that improves accuracy and speedy in (re)calibration of options is by using calculated options as a numéraire. Thereafter, they built characteristic functions that included Milstein approximation in which the investigated only one double stochastic integral for each pair of jump. When they added jumps, they focused on compound poison process. The latter process is similar to Cox-and-Hawks process. Those jump processes were applied on Heston and Bates models. Then Zhang and Schmidt (2016) applied that compound Poisson process in a general stochastic volatility model. For the probability density functions, they used inverse Fourier transform (IFT) because IFT compute algorithms efficiently. The numerical results illustrate that the model of Zhang and Schmidt (2016) performed better than Bates and Heston models. Furthermore, under the Heston model, their computation has shown to be more efficient and more accurate on large databases compared with other closed-form solutions. Finally, in the context of put-call parity, what drives disequilibrium?

2.3 Violation of Put-Call parity theorem

Sternberg (1994) argued that before the 1980s, it was a held view that stock markets exhibit semi-strong efficient market hypothesis. However, since the dawn of index futures and related products, people started to hold the notion that there are price violations in derivatives products. Sternberg (1994) used the Put-Call parity to explore price violations. He opined that with that Put-Call approach one does not need to deal with data problems and non-synchronization in trading environments. Given the information asymmetric, and the relationship between fundamental and financial options, this study anchors its illustration based on synchronization principle-that principle will be demonstrated later in this study. The Put-Call approach in Sternberg (1994) is used in its original form.

The preliminary results show that early exercise of options led to arbitrage opportunities. He went further and analysed arbitrage values for May-June 1983. The results illustrate that absolute deviations ranged between $50 and $60 for most of the sample. The mean value for the sample was -$30. Normally, negative means and skewness implies lower arbitrage opportunities. On the tax-modified boundary conditions, the arbitrage for early exercise is more than the arbitrage of late exercisability. The futures had specific months when they are closed-March, June, September and December. The latter pattern closing periods are similar to South African closing months. Overall, the results show market inefficiencies under rigorous conditions. In addition, it is very hard for one to eliminate mispricing opportunities in the derivatives markets.

Wagner et al. (1996) explore intra-day Put Call parity violations. In case of Wagner et al. (1996), even synchronisation does not lead to harmonised Put-Call formula. The latter principle will be explored further in this article. The explanatory variables that Put-Call divergence in Wagner et al. (1996) are (i) dividends, (ii) interest rate, (iii) trading volume, (iv) intraday volatility, (v) intraday price trend and (vi) time to expiration. In the context of Put-Call parity, violations are due to overpriced calls occurring frequently within shorter periods-Wagner et al. (1996). In Wagner et al. (1996) dividends are over 365-this is largely due to the fact that dividends are company policy; therefore, there is no formula not determining dividends with exception in the REIT industry. Wagner et al. (1996) argue that because of timing risk, risks are inherent in the Put-Call parity theorem. They also created numerous pairs of puts and calls for synchronisation purposes. The results of Wagner et al. (1996) show that out of 437,649 of puts-calls pairs tested, 93% of pairs trade in one minute or less. Both the lower and upper bounds of violations were evident and violations were 100%. Broadly, (under)over pricing led to the violations in the Put-Call theorem.

Wagner et al. (1996), De Roon and Veld (1996) explore Put-Call violations based on early exercise. De Roon and Veld (1996) argue that index options on S&P 100 are not corrected for dividends; therefore, a violation on the Put-Call parity of S&P 100 is given. De Roon and Veld (1996) assumed that on the performance index, dividends are reinvested in the index-this makes it similar to nondividend paying stock. The performance index used in De Roon and Veld (1996) has been used before. The Put-Call parity used in De Roon and Veld (1996) is similar to one in Wagner et al. (1996) except the fact that in the latter studies the parameters are discrete in nature while the former parameters are continuous in nature. The results of De Roon and Veld (1996) illustrate that the difference between upper and lower bounds is notable. And the Put-Call parity violation is mainly driven by early exercise premium. Interestingly, they found a negative relationship was found between violation and time to expiration, although the coefficient is not statistically significant. Then, how are those violations modelled in the presence of fundamental and financial option prices?

3 Modelling

In modelling option, one of the central rules is that input parameters should at least mimic the model. In order to abide with that rule, this article first synchronise the input parameters; thereafter, NAV and share price are derived from the Put-Call formula. Note that the resulting NAV and share price are of Put-Call parity nature.

3.1 Synchronization of Financial and Fundamental Prices

In order for share price and NAV to be comparable, share price and NAV should be illustrated by one numéraire using Brownian motion (BM). Prior studies illustrated that prices mean revert after some time and in complete markets expected returns of stock prices are zero. A similar pattern on REIT share prices was observed by Clayton and MacKinnon (2003) when they explored REIT returns taking into account other capital market instruments. What was observed by Clayton and MacKinnon (2003) is synonymous with BM in the context of mean-reversion. Some studies have shown that listed real estate investments are increasing becoming integrated into capital markets. As REITs are tradable securities, they should follow a GBM. In order to standardize share price and NAV using same numéraire it is assumed that:

- Interpretation: no drift, no trend.
- For i.i.d. if $\int NAV\mu_t(NAV)\partial NAV = m = 0$.
- Markov process:$\int_{\mathcal{R}} NAV_1\rho(t, s, NAV_0, \partial NAV_1) = 0$.
- Brownian motion is a martingale.
- Can have very complicated dependence on the past.

It also assumed that the final process is a natural filtration of some past process. In order for a future stochastic process to be naturally filtrated by some prior process, the following should hold:

- Encoding of knowledge.
- $\mathcal{F}_t$ is a σ-algebra; describes knowledge of agent at time t.
- Collection $(\mathcal{F}_t)_{t\in I}$ is called filtration if for all $s \ll t$ holds $\mathcal{F}_s \subset \mathcal{F}_t$.
- A stochastic process is called adapted if NAV_t is $\mathcal{F}_t$ measurable.
- Natural filtration of a process $(NAV_t)_{t\gg 0}$: $\mathcal{F}_t$ is the σ-algebra generated by $(NAV_s)_{s\ll t}$.
- Any process is adapted with respect to (w.r.t.) its own filtration.
- A stochastic process $(Y_t)_t$ is adapted w.r.t. the natural filtration of process $(NAV_t)_t$ if Y_t is a function only of $(NAV_s)_{s\ll t}$.

Any stochastic process is called a martingale if for all $s \ll t$ holds and its expectation is represented as follows; $\mathrm{E}\left[X_t - X_s \,\middle|\, (X_t)_{t\ll s}\right] = 0$, where X is share price of a stock that process follows BM. The conditional expectation is also a martingale process in the sense that increases and decreases of share price from their relative level overtime are equal such that its expectation over time is zero. That is, share price and NAV are similar because they follow the BM. In practice, exercise price (X) is deteremined through valuations; hence, X can be replaced by NAV. Sometimes X is illustaretd as K. Fisher (1978) argued that in pricing exchange options, exercise price can be illustrated by fundamental price such NAV. Taking into that the future process can be filtrated w.r.t. its past process, f_s, then it can be proven that the excepted value of a random process, NAV_t at future time t, given that its present time s, taking the whole history of the BM process is $\mathrm{E}[NAV_t \,|\, f_{s\ \ s<t}]$, where E represents the expectation. Filtration insures that the flow of information is continuous. Then NAV_t is decomposed into known value of NAV_s and the random increment of $[NAV_t - NAV_s]$:

$$\mathrm{E}\left[NAV_t \,\middle|\, \mathcal{F}_{s\ \ s<t}\right] = \mathrm{E}\left[NAV_t \,\middle|\, \mathcal{F}_s\right] \tag{1a}$$

$$= \mathrm{E}[NAV_s + NAV_t - NAV_s | \mathcal{F}_s] \tag{1b}$$

$$= \mathrm{E}\left[NAV_s \,\middle|\, \mathcal{F}_s\right] + \mathrm{E}\left[NAV_t - NAV_s \,\middle|\, \mathcal{F}_s\right] \tag{1c}$$

$$= NAV_s + \mathrm{E}\left[NAV_t - NAV_s \,\middle|\, \mathcal{F}_s\right] \tag{1d}$$

$$= NAV_s + 0 \tag{1e}$$

$$= NAV_s \tag{1d}$$

The NAV_s is due to the history of time s and it is not a random variable but the known value, so $\mathrm{E}\left[NAV_s \;\middle|\mathcal{F}_s\right] = NAV_s$ which proves that the process could be either a sub or super-martingale. Then $NAV_t - NAV_s$ equation is an increment value from s to t, normally treated as a single random and it is independent of the value of the BM at time s or earlier and the unconditional expectation of $\mathrm{E}\left[NAV_t - NAV_s \;\middle|\mathcal{F}_s\right] = \mathrm{E}[NAV_t - NAV_s] = 0$ meaning that on average, number of increases or decreases in share price over time are equal, a phenomenon synonymous with martingale process. Thereafter, share price and NAV are derived based on the Put-Call parity. The latter principle entices similarity of share price and NAV further.

3.2 Disetangling Prices

One technique that illustrates that there is no arbitrage opportunity between options parameters is Put-Call parity theorem. Alexander (2008) illustrated that Put-Call parity formula is written as follows when there are no arbitrage opportunities:

$$f^{call}\left(S,t \;\middle|K,T\right) - f^{put}\left(S,t\middle|K,T\right) = S(t)e^{-q(T-t)} - Ke^{-r(T-t)} \tag{2a}$$

where f^{call} is call option, f^{put} is put option, S is stock price, t is current time, K is exercise price, T is time to maturity of option contract, $(T - t)$ is time to expiration of the contract, r is risk-free interest rate, q is dividend yield, and $(S,t \;|K,T)$ is interpreted as changes in option (either call or put) price is due to change in S and t, and those changes are conditional upon K and T. The latter part of the statement illustrates filtration process. Although, dividend component in Put-Call parity is important, dividend uncertainty does not have major impact on Put-Call parity. Taylor (1990: 207) stated that “dividend uncertainty may be only one or two cents, which is relatively low”. What drives an option contract in the context of time is t in relation to T of an option contract. That is, tau drives an option price together with S and there is some history embedded in S. Moreover, $(S,t \;|K,T)$ illustrates that option variables are interrelated. Now, when one looks at $(S,t \;|K,T)$, one sees that K and T are fixed during life of an option, and S and t change during life of an option. According to Alexander (2008) the reason why LHS is equal to RHS of Put-Call parity is that eq. (2a) shows that a long call and short put options with same strike and maturity (M). Furthermore, Alexander (2008) stated that if M>0, put is worth nothing if it is exercised now, but call is worth:

$$f^{call}\left(S,t \,\middle|\, K,T\right) = S(t)e^{-q(T-t)} - Ke^{-r(T-t)} \tag{2b}$$

And if M<0, call is worth nothing if exercised now, but put is worth:

$$f^{put}\left(S,t \,\middle|\, K,T\right) = Ke^{-r(T-t)} - S(t)e^{-q(T-t)} \tag{2c}$$

Thereafter, LHS of eq. (2c) is simplified:

$$f^{put}\left(S,t \,\middle|\, K,T\right) = -\left[S(t)e^{-q(T-t)} - Ke^{-r(T-t)}\right] \tag{2d}$$

$$f^{put}\left(S,t \,\middle|\, K,T\right) = -f^{call}(S,t \mid K,T) \tag{2e}$$

When one explores eqs 2 (d and e), it can be seen that $f^{call}(S,t \mid K,T)$ and $f^{put}(S,t \mid K,T)$ are the same value except that they have opposite signs. It follows from eqs 2 (d and e) that LHS (i.e. put) and RHS (i.e. call) have the same risk as two different products of the same value should have the same amount of risk. In addition, eqs 2 (d and e) illustrate that there are no arbitrage opportunities between of $f^{call}(S,t \mid K,T)$ and $f^{put}(S,t \mid K,T)$. According to Alexander (2008) $f^{call}(S,t \mid K,T)$ and $f^{put}(S,t \mid K,T)$ are independent of volatility, implying that $f^{call}(S,t \mid K,T)$ and $f^{put}(S,t \mid K,T)$ have the same vega (v). While it might be true that $f^{call}(S,t \mid K,T)$ and $f^{put}(S,t \mid K,T)$ have the same vega, the impact of volatility on call and put options in Put-Call parity has opposite effects on call and put options, i.e. if a long call increases due to the volatility then a long put option decreases at the same time. The same concept should hold on betas of call and put options. In addition, it can be inferred from Alexander (2008) that long call and put options in Put-Call parity have same risks amounts of different signs. Following Alexander (2008), the negative risks of long put options and positive risks of long call options cancel each other in Put-Call parity, thereby causing the net effect of risks in Put-Call parity to be zero. Similar effects should apply to volatilities as well when they are used as risks measures. Merton (1973) stated that options unlike warrants are side contracts and when there are equal number of sellers and buyers, aggregated sum of options would be zero. While for warrants, aggregated sum when there are equal sellers and buyers is positive because warrants are part of capital structures of firms. The other possibility in eqs 2 (d and e) is that long call and put options decrease by the same rate of *r*; therefore, long call and put options are independent of *r* as they have the same rho (ρ). Then, *K* in Put-Call parity is replaced by NAV and eq. (2a) becomes:

$$f^{call}\left(S,t\ \middle|K,T\right) - f^{put}\left(S,t\middle| K,T\right) = S(t)e^{-q(T-t)} - NAVe^{-r(T-t)} \tag{3}$$

In order to illustrate share price and NAV diversion using Put-Call parity, this article proposes that eq. (3) should be re-written such that firstly, share price is the subject of the formula; thereafter, NAV as the subject of the formula. Although, NAV and share price of a firm are non-synchronous with each other that should lead to errors in the application of Put-Call parity given that share prices converge to NAVs in a long-run. In order for Put-Call parity to give fair values, call, put and underlying asset should be traded simultaneously. Thus, eq. (3) is re-written such that share price is the subject of the formula. Appendix A illustrates that when a share price is the subject of Put-Call parity, the resulting formula is:

$$S(t)_{PCP} = NAVe^{q-r} \tag{4a}$$

Note that in eq. (4a) NAV is the actual NAV of Put-Call parity. Eq. (4a) illustrates that share price is equal to NAV times the exponential factor that incorporates the difference of dividend yields and risk-free interest rate. That is, the exponential factor from Put-Call parity causes divergence of share price and NAV of a firm. Moreover, NAV trades above share price of a REIT firm, phenomenon that occurs rarely in the REIT industry. For a failed merger, there would not be expoenetial factor as the option price would be equivalent the price of underlying asst. “A call with a zero exrcise price is equivalent to the primitive asset on which it is written”, Ross (1976: 79). Note that although eq. (4a) is inconsistent with prior studies on the REIT industry, it cannot be ruled out. There is one special circumstance where firms declare dividend yields higher than risk-free interest rate is during bear market conditions such as in the United States in 1960s during the Great Depression. This is partly to induce investors to invest in companies during bearish conditions. In addition, financial markets should be in bearish state for a long period. Share price in eq. (4a) can never be negative because share prices are always positive. Therefore, $S(t)_{PCP}$ will decline at a decreasing rate of $(q - r)$ to some minimum level. Now, eq. (4a) is re-written such that NAV is the subject of the formula. Appendix B illustrates that a NAV as a subject of Put-Call parity is given by the following formula:

$$NAV_{PCP} = S(t)e^{r-q} \tag{4b}$$

Note that in eq. (4b) $S(t)$ is the actual $S(t)$ of Put-Call parity. In eq. (4b), NAV of Put-Call parity is equal to share price times the exponential factor that incorporates the difference of the risk-free interest rate and dividend yields. Hence, the NAV is more than share price by the exponential factor amount. The difference of NAV and share price is consistent with occurrences in some industries such as REIT. The exponential factor in eqs 4 (a and b) contributes to the divergence of share price and NAV of a firm. From eqs 4 (a and b), one can see that the exponential factor has smoothing effect in the sense that it can be hedged until share price and NAV are equal in trading environment. Just like eq. (4a), eq. (4b) illustrates there should be arbitrage opportunities in industries because of the divergence of share price and NAV. However, arbitrage opportunities are most likely to be short lived given that share price converges to its NAV in a long-run. One of the ways of minimising arbitrage opportunities is by expressing options as ratio of price of an underlying asset. In addition, a technique in the latter statement leads to options returns expressed in relation to returns of an underlying asset.

The relationship between risk-free interest rate and dividend yields in eqs 4 (a and b) illustrates that in the former formula risk-free interest rate is less that dividend yields while the latter formula risk-free interest rate is more than dividend yields. The special relationship in both formulas can be attributed to non-linearity of those two variables. Normally, when a share price of firm increases, management declare higher dividend yields. When interest rates are high, consumers tend to consume less of goods and services provided by firms and that makes share prices of those firms less desirable at the point in time. Intuitively, dividends declared by firms should be less than risk-free interest rate. The linkage of dividend yields and risk-free interest rate in eq. (4b) seem to represent bullish market conditions. As share price of a firm changes, options parameters changes. Glascock and Hung (2010: 128) stated that “according to volatility feedback theory, the news will decrease future volatilities of winners, and therefore decrease their required rate of returns, causing an increase in immediate stock prices". The information asymmetry in industries is most likely to contribute to Put-Call parity violations. Interestingly, eq. (4b) just like eq. (2a) illustrates that when NAV is viewed as an option, NAV of Put-Call parity is driven by S and t, a phenomenon synonymous with Put-Call parity. Therefore, it is recommended that eq. (4b) be used to calculate share price and NAV of a REIT firm during bull market conditions. For calculation of share price of a firm during bull market conditions, it is recommended that eq. (4b) is re-written such that share price is the subject of the formula as opposed using eq. (4a) for reasons stated earlier. When share price is the subject of the formula, eq. (4b) changes to:

$$S(t)_{PCP} = \frac{NAV_{actual}}{e^{\lambda}} \tag{5a}$$

or

$$S(t)_{PCP} = NAV_{actual}e^{-\lambda} \tag{5b}$$

where λ is the difference between risk-free interest rate and dividend yield. The reason why lambda is the difference between risk-free interest rate and dividend yields is because eqs 5 (a and b) are locked, otherwise eqs 5 (a and b) will revert back to bear market conditions where the dividend declared are higher than risk-free interest rate. Arbitrageurs can written option strategies and earn riskless profits. Note that the exponential lambda in eqs 5 (a and b) can be written as $e^{\lambda} = \frac{NAV_{actual}}{S(t)_{PCP}}$. This is similar to eqs (3) and (4) in Kamara and Miller (1995). Those formulas in Kamara and Miller (1995) synthetic riskless lending[†] and borrowing[‡] rates, respectively. In the context of REITs, one can end up with negative R_L and R_B because the NAV can be negative. Similar, the exponential factor (i.e. arbitrage) can be negative. That would be typically in a deflationary environment, where the cost of living is high indeed and it costly to own assets. Although, that scenario is very rare but it is not impossible to occur. It in the context of the real estate industry including REIT, it is highly likely to have negative arbitrage. According to Kamara and Miller (1995), what leads to negative arbitrage is the delaying in execution time-similar to lagging effect in the real estate industry. To one's knowledge, no study has explained negative arbitrage so far. The exponential factor in eqs 4 (a and b) supports the latter statement. It is recommended that one should use eq. (4a) when

[†] $R_L \equiv \frac{X}{(S_a - D + t_s) + (P_a + t_p) - (C_b - t_c)} - 1$, where S_a and S_b are bid and ask prices of one S&P share, S_T price of S&P share at date T, D the present value of dividends, t_i transaction costs of buying or selling assets, λ_s percent of proceeds available to S&P short-sellers and X is exercise price.

[‡] $R_B \equiv \frac{X}{(\lambda_s(S_b - D) - t_s) + (P_a + t_p) - (C_b - t_c)} - 1$.

calculating NAV and share price where investors are induced to participate in mergers in bearish market conditions. When investors are not induced to participate in mergers, eq. (4b) should be used to calculate NAV and share price. It seems that the latter statement refers to the bullish market conditions. Although, in eqs 4 (a and b) there is no time factor, it is fair to assume that there is the time factor in both formulas given that share prices and NAVs are different at different points in time. In this study, costs are not taken into account, but when costs are minimal, NAV and share price of a firm are most likely to be equal.

The fundamental exponential factor conjecture drawing from Ross (1976) is that it (exponential factor) is that its presence dependent of underlying assets. This is because it can be inferred from theorem 1 from Ross (1976) that option returns are inherent from underlying assets. Moreover, symmetry environment option returns are most likely to be zero. This article follows from Marcato et al. (2018 and 2019)-each REIT is portfolio of a number of properties which form one fund in the form a REIT firm. On a single option portfolio being shown a number of single simple options¸ Ross (1976) stated that they can be 'written with no loss of efficiency'. The latter resonates with REITs given that their uniqueness and the fact hostile merger are rare indeed. The salient taking would be that the exponential factor is not due to REIT market ineffincies but the REIT structure. Similarly, the production theorem from Ross (1976), that each contributes positive returns seem to be probale. Moreover, Marcato et al (2018) illustrated that REIT mergers generate alpha.

4 Investigation of Exponential Factor

In order to exploit exponential factor, the following procedures are carried out; minimization of exponential factor, integral transforms and second order partial differential equations (PDEs). Those mentioned techniques will give a rounded picture of exponential factor.

4.1 Exponential Factor

Numerical techniques are used to explore the exponential factor that causes the diversion of share price and NAV of a firm as illustrated by eqs 4 (a and b). The idea behind exploring eqs 4 (a and b) is to collapse those two equations to the level where the exponential factor is the subject of the formula. Wherever it is possible to eliminate the exponential factor is eliminated it will be done so. Appendix C illustrates that without the exponential factor, eqs 4 (a and b) yield:

$$f^{call}(S,t\,|\,K,T) = f^{put}(S,t\,|\,K,T) \tag{6}$$

Eq. (6) illustrates that without the exponential factor the value of the buyer (i.e. call option) is equal to the value of the seller (i.e. put option). In order to verify whether eq. (6) is maximum (minimum) value, options boundaries are explored.

4.2 Options Boundaries

Option boundaries are normally used to show maximum and/or minimum values of options. Other ways of illustrating options boundaries includes taking derivatives of options prices in relation to their spot prices. Table 4.1 illustrates option boundaries:

Table 4.1: Option Boundaries

Boundaries	Long Call Options	Long Put Options
Upper	$C \ll S_0$	$P \ll K$
Lower	$C < S_0 - Ke^{-rT}$	$P < Ke^{-rT} - S_0$

The changes in spot prices of options values can lead to arbitrage opportunities. Therefore, it is proposed that options boundaries must be illustrated using first partial derivatives of options boundaries w.r.t. spot prices. Table 4.2 illustrates derivatives of options boundaries:

Table 4.2: Options Boundaries Derivatives

Boundaries	Long Call Options	Long Put Options
Upper	$\frac{\partial C}{\partial S_0} \ll 1$	$\frac{\partial P}{\partial S_0} \ll 0$
Lower	$\frac{\partial C}{\partial S_0} < 1$	$\frac{\partial C}{\partial S_0} < -1$

Bakshi et al. (1997) stated when options are divided by spot prices and resulting ratios are around one, those options are regarded as being at-the-money position. Ratios higher than one are regarded as being in-the-money position and ratios less than one are regarded as being at out-of-the-money position. That is, upper and lower boundaries of call options exist when calls are at least at-the-money. At same time, when one calculates Put-Call parity parameters using options which are in-the-money it leads to mispricing. The latter challenge is also illustrated eralier in Easton (1994). The mispricing can be solved by buying calls which are in-the-money as they are generally undervalued and selling puts which are out-of-the-money as they are generally overvalued. The presence of mispricing and deep out-of-the-money especially for long calls signals the presence of skewness with higher kurtosis. Similar findings are in Bakshi et al. (1997). However, arbitrage opportunities when options are out-the-money are very difficult to fully exploit. The first partial derivatives illustrate that long puts have upper and lower boundaries when puts are in out-of-the-money. Options that are in out-the-money are hardly traded in stock markets. The contradiction positions of calls and puts should lead to mispricing as options and their underlying assets are not traded simulatenously. On the other hand, those contradictions lead to options boundaries. Given that exploitations have been of derivatives nature, this article proposes exploring the integral transforms of the same formulas. The reason why eqs (4b) and 5 (a and/or b) are explored is because those are two formulas that seem to consistent with some studies in most industries.

4.3 Integral Transforms

Fourier and Laplace transforms are used to transform eqs (4b) and 5 (a and/or b) respectively. First, eq. (4b) is transformed using Fourier transform because the subject of eq. 5 (a and/or b) is NAV. And, NAVs can be negative and Fourier transform integrates values between positive and negative infinity. Suppose function (*f*) is continuous and piecewise smooth, and $f: \mathbb{R} \longrightarrow \mathbb{R}$ is integrable over the real line $\mathbb{R}$, then Fourier transform $\mathcal{F}[f(x)] \coloneqq \hat{f}: \mathbb{R} \longrightarrow \mathbb{R}$ of f is defined by:

$$\hat{f}(x) \coloneqq \mathcal{F}[f(s)](x) \equiv \int_{-\infty}^{+\infty} e^{isx} f(s) d(s) \tag{7}$$

It is assumed that the initial condition for Fourier transform can be illustrated as follows; $A(S) + B(S) = \hat{f}(s)$, $\mathrm{A(s)} - \mathrm{B(s)} = 0$ and so $A(S) = B(S)\frac{1}{2}\hat{f}(s)$ and $\frac{1}{2}$ is an integer.

Let $y' = r + q$ and $y' = r - q$, and and $NAV_{PCP} = S(t)e^{r-q}$ from eq. (4b). Firstly, eq. (4b) is converted into continuous and piecewise formula, a requirement for Fourier transform. When $e^{(r-q)}$ is discretised, it becomes $(1 + r - q)$ which gives rise to arbitrage opportunities. Therefore, an arbitrage opportunity is written off by taking a short position in the same formula; $(1 - 1 + r - q)$ ignoring any related costs. The function becomes $(r - q)$ and NAV_{PCP} becomes $NAV_{PCP} = S(t)(r - q)$. Then $NAV_{PCP} = S(t)(r - q)$ is convert into continuous form; $NAV_{PCP} = \ln(1 + NAV_{PCP})$ or $e^{NAV_{PCP}} - 1$. Thereafter, arbitrage is written off by taking a long position of the same amount as a short position $NAV_{PCP} = e^{NAV_{PCP}} - 1 + 1$ or $NAV_{PCP} = e^{NAV_{PCP}}$. Appendix D shows that Fourier term for NAV is given by the following formula:

$$= \sqrt{\pi}e^{-{[S(t)(r-q)]^1}/_2} \tag{8}$$

Eq. (8) shows that when dividends are less than risk-free interest rate, eq. (8) increases with time. Moreover, given that $\sqrt{\pi}$ is positive; therefore, eq. (8) has a minimum function. $[S(t)(r - q)]^2$ portion increases with eq. (8) exponentially over time. The increasing factor of eq. (8) with time is similar to a logarithm function as long as $(r - q)^2$ decreases with time. However, *ceteris paribus*, if $(r - q)^2$ increases with time, eq. (8) decreases with time. That is, eq. (8) is dual-direction in the sense that it has ability to increase and/or decrease with time depending what happens to other variables. The latter statement shows that eq. (8) is entropic. Moreover, values can converge either from top or bottom. It seems that NAV based strategy works in bull and bear market conditions. Thereafter, Laplace transform of share price of eq. 5 (a and/or b) is exemplified. Laplace transform is used in eq. 5 (a and/or b) because share prices can never be negative. By definition, Laplace transform is $\bar{X}(s) = \mathcal{L}[X(t)](s) \equiv \int_0^{+\infty} e^{-st}X(t)dt$ given a function, $X(t)$ which is defined for $t \geq 0$. Function $(\bar{X})s$ is defined for all values of s such that the integral converges and the function is derived w.r.t. any variable of s. Appendix E illustrates that Laplace transform of share price is:

$$= e^{-NAV\lambda} \tag{9}$$

Although eq. (8) includes more variables, eq. (9) is simpler as it is based on the change in NAV because lambdas tend to constant for a specific period. The phenomenon on lambdas is inferred from Anand and Weaver (2006). NAVs tend to be stable in the long run and that leads to consistency is an implementable strategy. Given that Laplace transform integrates values between zero and positive infinity and prices do not increase forever. That is, a long term average is most likely to be between zero and positive infinity.

4.4 Second Order PDEs

Options have non-linear payoffs; this implies that they pay differently at different points in time. In addition, payoff structures differ from option to the next one. Second order PDEs are some of the techniques used to illustrate options payoff structures. Second order PDE is represented by the following formula:

$$f\left(u_x, u_y, u, x, y\right) = Au_{xx} + Bu_{xy} + Cu_{yy} \tag{10}$$

where $f\left(u_x, u_y, u, x, y\right)$ is a function that has parameters; u, x and y, and one wants second derivatives of the same function w.r.t. x and y. A, B and C are constants of the second derivatives of the function. Functions can be described as elliptic when $B^2 < AC$, as hyperbolic when $B^2 > AC$ and as parabolic when $B^2 = AC$. Payoff structures that are illustrated are of eqs (A8) and (B4). The reason why equations in the latter statement are used is because they have all variables that make a function elliptic, hyperbolic or parabolic. *S(t)* is the function of eq. (A8) and its parameter is q. The three parts on the RHS of eq. (A8) relate to A, B and C. Therefore, A=2, B=-2 and C=1. That is, $B^2 = 4$ and AC=2. This illustrates that eq. (A8) is a hyperbolic function. In a hyperbola, values start at a specific starting point and as time changes, value increases. Similar principle would imply that options value increase with time; moreover, one has numerous points where a desired value is located. Thus, one can design option strategy as per desired option value taking into account time and other options parameters. It seems that eq. (A8) has a minimum point (usually a starting point) and the same equation has ability to increase options values. NAV is a function of eq. (B4) and its parameters are risk-free interest rate and tau. The three parts on the RHS of eq. (B4) relate to A, B and C. Therefore, A=1, B=2 and C=-2. That is, $B^2 = 4$ and AC=-2. This illustrates that eq. (B4) is a hyperbolic function. The views that apply to eq. (A8) apply equally to eq. (B4) as well because they are both hyperbolic functions.

5 Conclusion

Firstly, this article illustrates that there are price differentials in options during corporate activities. Those price differentials are due to the presence of financial and fundamental prices. In principle, financial and fundamental prices hardly trade at the same level(s). Secondly, there are appropriate formulas for pricing options during bear and bull market conditions. Thirdly, negative option premium are possible during corporate activities. Fourth, options strategies are complex as shown by PDEs and hyperbola. Finally, although the article focuses on the REIT industry, emanating strategies can be replicated in other industries.

The implications of this article are as follows. Firstly, price differentials create arbitrage opportunities. Moreover, those opportunities can be customised to suit short-term and/or long-term period. Secondly, option formulas need to be improved in order for them to be suitable for pricing in certain situations. Thirdly, unforeseen products like negative option premiums do emerge sometimes. Fourth, strategists need to improve their technical knowledge for appropriately pricing of capital market products. Finally, some strategies are not industry and/or product specific.

Appendix

Appendix A: Put-Call Share Price

If Alexander (2008) arguments on call and put options when M>0 and M<0 are taken into account then Put-Call parity can be re-written as:

$$S(t)e^{-q(T-t)} = NAVe^{-r(T-t)} + \underbrace{\left\{f^{call}\left(S,t \,\middle|\, K,T\right) - f^{put}(S,t \,|K,T)\right\}}_{LHS\ of\ eq.(12)} \tag{A1}$$

Thereafter, f^{call} and f^{put} are replaced by their formulas and eq. (A1) becomes:

$$S(t)e^{-q(T-t)} = NAVe^{-r(T-t)} + \underbrace{\left\{S(t)e^{-q(T-t)} - Ke^{-r(T-t)}\right\}}_{f^{call}} - \underbrace{\left\{Ke^{-r(T-t)} - S(t)e^{-q(T-t)}\right\}}_{f^{put}} \tag{A2}$$

Removing brackets in eq. (A2) yields:

$$S(t)e^{-q(T-t)} = NAVe^{-r(T-t)} + \underbrace{S(t)e^{-q(T-t)} - Ke^{-r(T-t)}}_{f^{call}} - \underbrace{Ke^{-r(T-t)} + S(t)e^{-q(T-t)}}_{f^{put}} \tag{A3}$$

Putting together like terms in eq. (A3) yields:

$$S(t)e^{-q(T-t)} = NAVe^{-r(T-t)} + 2S(t)e^{-q(T-t)} - 2Ke^{-q(T-t)} \tag{A4}$$

Taking out the common factor on both sides of eq. (A4) leads to:

$$S(t)e^{-q}e^{(T-t)} = e^{(T-t)}[NAVe^{-r} + 2S(t)e^{-q} - 2Ke^{-r}] \tag{A5}$$

Thereafter, one divides throughout eq. (A5) by common factor, $e^{(T-t)}$:

$$\frac{S(t)e^{-q}e^{(T-t)}}{e^{(T-t)}} = \frac{e^{(T-t)}[NAVe^{-r}+2S(t)e^{-q}-2Ke^{-r}]}{e^{(T-t)}} \tag{A6}$$

After dividing throughout by the common factor, eq. (A6) becomes:

$$S(t)e^{-q} = NAVe^{-r} - 2Ke^{-r} + 2S(t)e^{-q} \tag{A7}$$

The exercise price, *K* in eq. (A7) is replaced by NAV because it can be inferred from Fisher (1978) that *K* can be replaced by fundamental price when one prices exchange options when underlying assets are real ones.

$$S(t)e^{-q} = NAVe^{-r} - 2NAVe^{-r} + 2S(t)e^{-q} \tag{A8}$$

Factorising eq. (A8) yields:

$$2S(t)e^{-q} - S(t)e^{-q} = 2NAVe^{-r} - NAVe^{-r} \tag{A9}$$

Eq. (A9) is simplified further:

$$S(t)e^{-q} = NAVe^{-r} \tag{A10}$$

Then, share price in eq. (A10) is written as the subject of the formula:

$$S(t) = \frac{NAVe^{-r}}{e^{-q}} \tag{A11}$$

Exponential terms in eq. (A11) are put together:

$$S(t)_{PCP} = NAVe^{q-r} \tag{A12}$$

Appendix B: Put-Call NAV

When arguments by Alexander (2008) on the f^{call} and the f^{put} when M>0 and M<0 are taken into account, Put-Call parity can be re-written as:

$$NAVe^{-r(T-t)} = S(t)e^{-q(T-t)} + f^{put}(S,t \,|\, K,T) - f^{call}(S,t \,|\, K,T) \quad \text{(B1)}$$

Thereafter, f^{call} and f^{put} are replaced by their formulas and eq. (B1) becomes:

$$NAVe^{-r(T-t)} = S(t)e^{-q(T-t)} + \underbrace{\{Ke^{-r(T-t)} - S(t)e^{-q(T-t)}\}}_{f^{put}} - \underbrace{\{S(t)e^{-q(T-t)} - Ke^{-r(T-T}\}}_{f^{call}} \quad \text{(B2)}$$

Removing brackets in eq. (B2) yields:

$$NAVe^{-r(T-t)} = S(t)e^{-q(T-t)} + \underbrace{Ke^{-r(T-t)} - S(t)e^{-q(T-t)}}_{f^{put}} - \underbrace{S(t)e^{-q(T-t)} + Ke^{-r(T-t)}}_{f^{call}} \quad \text{(B3)}$$

When like terms are put together, eq. (B3) becomes:

$$NAVe^{-r(T-t)} = S(t)e^{-q(T-t)} + 2Ke^{-r(T-t)} - 2S(t)e^{-q(T-t)} \quad \text{(B4)}$$

The K in eq. (B4) is replaced by NAV for the same reason as in appendix A, and like times on the RHS of eq. (B4) are put together:

$$NAVe^{-r(T-t)} = 2NAVe^{-r(T-t)} - S(t)e^{-q(T-t)} \quad \text{(B5)}$$

Factorising eq. (B5) leads to:

$$NAVe^{-r(T-t)} = S(t)e^{-q(T-t)}) \quad \text{(B6)}$$

Both sides of eq. (B6) are divided by common factor $(i.e.\, e^{(T-t)})$ in order to simplify further:

$$\frac{NAVe^{-r}e^{(T-t)}}{e^{(T-t)}} = \frac{S(t)e^{-q}e^{(T-t)}}{e^{(T-t)}} \quad \text{(B7)}$$

The resulting formula after dividing eq. (B7) by common factor is:

$$NAVe^{-r} = S(t)e^{-q} \tag{B8}$$

NAV in eq. (B8) is written as the subject of the formula:

$$NAV = \frac{S(t)e^{-q}}{e^{-r}} \tag{B9}$$

Exponential terms in eq. (B9) are put together:

$$NAV_{PCP} = S(t)e^{r-q} \tag{B10}$$

Appendix C: Disentangling Exponential Factor

When the exponential factor is the subject of the formula, eq. (13) changes to:

$$e^{q-r} = \frac{S(t)_{PCP}}{NAV_{actual}} \tag{C1}$$

Note that eq. (C1) includes $S(t) = NAVe^{q-r} + [f^{call}(S,t|K,T) - f^{put}(S,t|K,T)]e^{+q(T-t)}$. The $S(t)_{PCP}$ in eq. (C1) is replaced by the full *S(t)* term:

$$e^{q-r} = \frac{\overbrace{NAVe^{q-r}+[f^{call}(S,t|K,T)-f^{put}(S,t|K,T)]e^{q(T-t)}}^{S(t)_{PCP}}}{NAV} \tag{C2}$$

The RHS of eq. (C2) is decomposed into appropriate ratios:

$$e^{q-r} = e^{q-r} + \frac{[f^{call}(S,t|K,T)-f^{put}(S,t|K,T)]e^{q(T-t)}}{NAV} \tag{C3}$$

Thereafter, one factorises eq. (C3):

$$0 = \frac{[f^{call}(S,t|K,T)-f^{put}(S,t|K,T)]e^{q\tau}}{NAV} \tag{C4}$$

In eq. (C4), one multiplies throughout by the common denominator, NAV:

$$0 = [f^{call}(S,t|K,T) - f^{put}(S,t|K,T)]e^{q\tau} \tag{C5}$$

The put and call options are separated from one another in eq. (C5):

$$0 = f^{call}(S,t|K,T)e^{q\tau} - f^{put}(S,t|K,T)e^{q\tau} \tag{C6}$$

A call option is equated to a put option in eq. (C6):

$$f^{call}(S,t|K,T)e^{q\tau} = f^{put}(S,t|K,T)e^{q\tau} \tag{C7}$$

Eq. (C7) illustrates that when dividend yields are discounted on continuous basis over time, the value of call and put options are equal in Put-Call parity. Taylor (1990) stated that the inclusion of dividend yields in Put-Call parity minimises mispricing opportunities. Eq. (C7) can still be simplified further by diving throughout by the common factor, $e^{q\tau}$ on LHS and RHS of the formula.

$$f^{call}(S,t\,|\,K,T) = f^{put}(S,t\,|\,K,T) \tag{C8}$$

Then, when the exponential factor is the subject of the formula, eq. (14) changes to:

$$e^{r-q} = \frac{NAV_{PCP}}{S(t)_{actual}} \tag{C9}$$

Note that eq. (C9) includes $NAV = S(t)e^{r-q} + [f^{put}(S,t|K,T) - f^{call}(S,t|K,T)]e^{r(T-t)}$. NAV_{PCP} in eq. (C9) is replaced by full NAV term:

$$e^{r-q} = \frac{\overbrace{S(t)e^{r-q} + [f^{put}(S,t|K,T) - f^{call}(S,t|K,T)]e^{r\tau}}^{NAV_{PCP}}}{S(t)} \tag{C10}$$

The RHS of eq. (C10) is decomposed into appropriate ratios:

$$e^{r-q} = e^{r-q} + \frac{[f^{put}(S,t|K,T) - f^{call}(S,t|K,T)]e^{r\tau}}{S(t)} \tag{C11}$$

Thereafter, eq. (C11) is factorised:

$$0 = \frac{[f^{put}(S,t|K,T) - f^{call}(S,t|K,T)]e^{r\tau}}{S(t)} \tag{C12}$$

In eq. (C12), one multiplies throughout by the common denominator, *S(t)*:

$$0 = f^{put}(S,t|K,T)e^{r\tau} - f^{call}(S,t|K,T)e^{r\tau} \tag{C13}$$

Put and call options are separated from one another in eq. (C13):

$$f^{put}(S,t|K,T)e^{r\tau} = f^{call}(S,t|K,T)e^{r\tau} \tag{C14}$$

Eq. (C14) is similar to eq. (C7) except that instead of dividend yields there is risk-free interest rate. When firms pay out dividends, they usually take into account the level of risk-free interest rate. Thus, dividends declared are based on the level of risk-free interest rate and normally dividends declared are lower than risk-free interest rate. Similarly, risk-free interest rate should have the same effect as dividends in Put-Call parity. Bakshi et al. (1997) stated that proper modelling of interest rates in improves the quality of options models. Eq. (C14) can be simplified further by dividing throughout by the common factor; $e^{r\tau}$:

$$f^{call}(S,t \,|\, K,T) = f^{put}(S,t \,|\, K,T) \tag{C15}$$

Appendix D: NAV Transformation

When $NAV_{PCP} = e^{S(t)(r-q)}$ is transformed into algebraic, it yields:

$$\hat{f}(x) \coloneqq \mathcal{F}[f(NAV_{PCP})](x) \coloneqq \int_{-\infty}^{+\infty} e^{isx} f(NAV_{PCP}) d(NAV_{PCP}) \quad \text{(D1)}$$

Then likes terms are put together:

$$\mathcal{F}[e^{NAV_{PCP}}](s) = \int_{-\infty}^{+\infty} e^{-isNAV_{PCP}} f(NAV_{PCP})\, dNAV_{PCP} \quad \text{(D2)}$$

Eq. (D2) can be simplified further:

$$= \int_{-\infty}^{+\infty} e^{NAV_{PCP}} e^{-isNAV_{PCP}} d(NAV_{PCP}) \quad \text{(D3)}$$

NAV term in eq. (D3) is squared:

$$= \int_{-\infty}^{+\infty} e^{-is(NAV_{PCP})^2} d(NAV_{PCP}) \quad \text{(D4)}$$

Positive and negative infinities are separated:

$$= \int^{+\infty} e^{-is(NAV_{PCP})^2} d(NAV_{PCP}) - \int_{-\infty} e^{-is(NAV_{PCP})^2} d(NAV_{PCP}) \quad \text{(D5)}$$

Note that the second part of eq. (D5) approaches zero as infinity becomes a huge number. Thus, the second part of eq. (D5) disappears:

$$= \sqrt{\pi} e^{-{(NAV_{PCP})^2}/{4}} \quad \text{(D6)}$$

NAV term is replaced by appropriate parameters in eq. (D6):

$$= \sqrt{\pi} e^{-{[S(t)(r-q)]^2}/{4}} \quad \text{(D7)}$$

Eq. (D7) is simplified further:

$$= \sqrt{\pi} e^{-{[S(t)(r-q)]^1}/{2}} \quad \text{(D8)}$$

Appendix E: Share Price Transformation

Just like in Fourier transform, in order for Laplace transform to exist, eq. (15) should be converted into continuous and piecewise formula. First, e^{λ} is discretised into $(\lambda + 1)$. Thereafter, arbitrage opportunities are written off by taking a short position ignoring any related costs, $(\lambda + 1 - 1) = \lambda$.

Then eq. (15) collapse to $S(t)_{PCP} = NAV\lambda$. The continuous form of $S(t)_{PCP}$ is $S(t)_{PCP} = e^{NAV\lambda} + 1$. Finally, the share price is $S(t)_{PCP} = e^{NAV\lambda}$ after taking a short position and ignoring any costs related to $S(t)_{PCP} = e^{NAV\lambda} + 1 - 1$. $S(t)_{PCP}$ is put into Laplace formula:

$$\mathcal{L}[S(t)_{PCP}](s) = \int_0^{+\infty} e^{-sS(t)_{PCP}} d(S(t)_{PCP}) \tag{E1}$$

Eq. (E1) can be simplified further:

$$= \left[-\frac{1}{S(t)_{PCP}} e^{-sS(t)}\right]_0^{\infty} \tag{E2}$$

One takes the limit of eq. (E2) between zero and positive infinity:

$$= \lim_{T\to\infty} \left|-\frac{1}{S(t)_{PCP}} e^{-sS(t)_{PCP}}\right|_0^{T} \tag{E3}$$

Positive infinity and zero terms are separated from each other in eq. (E3):

$$= \lim_{T\to\infty} \left|-\frac{1}{S(t)_{PCP}} e^{-TS(t)_{PCP}} + \frac{1}{S(t)_{PCP}}\right|_0^{T} \tag{E4}$$

The first term in brackets in eq. (E4) approaches zero:

$$= (0 + \frac{1}{S(t)_{PCP}}) \tag{E5}$$

The zero term in eq. (E5) is left out:

$$= \frac{1}{S(t)_{PCP}} \tag{E6}$$

$S(t)_{PCP}$ reverts back to appropriate term:

$$= \frac{1}{e^{NAV\lambda}} \tag{E7}$$

Eq. (E7) can be re-written as:

$$= e^{-NAV\lambda} \tag{E8}$$

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