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Research Article

Effects of Network Externalities and Recycling Channels on Closed-loop Supply Chain Operation Incorporating Consumers' Dual Preferences

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Consumers' awareness of environmental protection is gradually strengthened, and they often consider environmental protection characteristics of products during purchasing products. Based on the characteristic of consumers' dual preferences for product quality and environmental friendliness, this paper constructs a novel closed-loop supply chain with network externalities for new products and remanufactured products under different waste product recycling channel strategies. It discusses the influence of product recycling channel strategies and network externalities on the optimal decision of the company. By establishing the Stackelberg game model, the analysis found that the retail price of new products and remanufactured products is not affected by network externalities and recycling channel strategies, even if the wholesale price of remanufactured products increases with the influence of network externalities. The changing trend of market demand for new products and remanufactured products is different between products with network externalities and without network externalities. When the cost of remanufactured products is within certain range, the existence of network externalities makes the market demand for new products and remanufactured products increase simultaneously. Our study also found that even if products have positive network externalities that increases consumer utility, when the cost of remanufactured products is outside certain range, the company's profit will be higher when products have no network externalities than that when products have network externalities.

1. Introduction

Today, lots of companies try to develop remanufactured and refurbished products to support the sustainable development of the world environment [1]. For example, in 2017, Apple highly announced that all his future products would be made from recyclable materials. And in 2021, Apple announced that Apple's aim is that all his products and packaging will use only recycled and renewable materials in future. Enterprises such as Kodak, Hewlett-Packard, and Xerox have already participated in the recycling and remanufacturing of products [2]. The government and companies are constantly promoting environmental protection and green products [3, 4]. More and more consumers are also aware of the seriousness of environmental problems [5–7]. Consumers with high environmental protection awareness would pay attention to the effects of products on environment and pay higher price for green products [8, 9]. Due to the difference of consumers' income levels and environmental awareness, a company's quality-differentiated product strategy can meet the needs of more consumers, thereby increasing the market share and profits [10–12]. Remanufactured products have more price advantages, which can help companies expand rapidly in the market. New products can improve consumers' perceptions of remanufactured products through publicity and marketing strategies to promote sales. For example, Apple is actively exploring the market for remanufactured mobile phones and notebooks to gain greater market share and profits. And Apple sells both new products and remanufactured products on his official channel simultaneously, thus satisfying consumers with different needs. Xerox has saved 45%–60% of their manufacturing costs and gained hundreds of millions of dollars by implementing waste product recycling and selling a wider variety of products in the market [13]. More and more consumers consider both quality and environmental friendliness attributes of products when purchasing products. And they are also affected by product network externalities during the purchase process.

The rapid development of e-commerce and the rise of community-based network marketing platforms and forums make product network externalities more and more prominent. Network externalities are typical characteristics of network economy different from traditional economic theory, which are essentially a kind of demand-side scale economy in the process of network scale increase. In 1985, Katz and Shapiro showed that a product with network externalities is one whose utility increases with the increase of the number of consumers who purchase it [14]. It means that consumers' purchase decisions are not only affected by the value of products themselves but also affected by the user size of products and the influence of network value constituted by the strength of network externalities [15]. And high-tech products tend to have strong network externalities [16]. For example, the establishment of an Apple online community has allowed Apple's fans to share their experiences and solutions to problems encountered. Therefore, as the number of consumers who buy Apple's products and join the community increases, the higher utility they receive. Similarly, Samsung, Huawei, and Xiaomi et al., also establish online community to expand network externalities of products.

Based on these observations, we are interested in the following questions: (1) How do different recycling channels affect company's decision making on new products and remanufactured products? (2) Considering the rapid development of network economy, should company continue to amplify network externalities of its products? and (3) When consumers consider both product quality and environmental friendliness, how do product network externalities affect company's decision making?

We attempt to address these issues in this paper. We construct a novel closed-loop supply chain (CLSC) for new products and remanufactured products under different recycling channel strategies, in which new products and remanufactured products both have network externalities or not. Consumers have dual preferences for product quality attributes and environmental friendliness attributes. Then, we examine the influence of recycling channel strategies and product network externalities on the balanced decision-making of brand owners and third-party manufacturers in the supply chain.

The findings reveal that (1) the retail price of new products and remanufactured products is not affected by network externalities and recycling channel strategies. (2) The changing trend of market demand for new products and remanufactured products is different when products have network externalities and no network externalities. (3) Even if products have positive network externalities that increases consumer utility, when the cost of remanufactured products is outside certain range, the company's profit will be higher when products have no network externalities than that when products have network externalities.

This study has three main contributions. First, building on successful practical applications and prior research, we summarize two CLSC models that consider both product quality and environmental friendliness attributes, i.e., the direct and indirect recycling models. Second, network externalities are not only common features of products in network economy but also one of the motivations of consumers to buy products. This study contributes to the literature by understanding whether company should unconditionally expand network externalities of its products. Third, while many CLSC applications have been studied, we contribute the research by not only considering consumers' purchasing decisions but also discussing the impact of consumers' purchase process on the company's sales decisions of new products and remanufactured products.

The remainder of this paper is organized as follows. The relevant literature is discussed in Section 2. Our model and its result analysis are presented in Sections 3 and 4, respectively. We further extend our model by considering consumers' channel preference in Section 5. Numerical example simulation is discussed in Section 6. Our conclusions and directions for future research are presented in Section 7.

2. Literature Review

Our research is mainly related to three streams of literature: consumers' dual preferences, recycling channels of waste products, and network externalities. To highlight our contributions, we review the representative literature in these three streams.

Collection, recycling, and remanufacturing procedures are important components of CLSC [17]. Channel management, especially the selection of recycling channels for waste products, is an important issue in reverse supply chain management [18–20]. Currently, numerous scholars have studied the recycling channels of waste products. Guide and Van Wassenhove [21] put forward the concept of product recycling management, qualitatively analyzed how to manage the uncertainty of recycled product quality, and made an economic value analysis. Savaskan et al. [18]; and Savaskan and Van Wassenhove [22] further studied the relationship between the manufacturer's optimal recycling channel selection and the recycling cost function based on three waste product recycling channel modes. Zheng et al. [23] analyzed the two-level supply chain system with complex relationships such as product remanufacturing, channel competition, and channel intrusion. And they further studied the manufacturer's channel intrusion decision-making strategy under the two modes of manufacturer recycling and retailer recycling, and analyzed its impact on retailers, supply chain system, and consumer surplus [24]. Li et al. [25] discussed the recovery pricing decision of dual channel reverse supply chain under the dual influence of recyclers' loss aversion and consumers' bargaining power. When the ratio of the recycling price to recycling competition coefficient is large, the mixed recycling channel composed of manufacturers, retailers, and third-party recyclers can better maintain the coordination and stability of the supply chain [20]. In recent years, Internet platforms have played important roles in recycling waste products and materials [26]. Internet technology makes online recycling more and more popular, and more and more companies are gradually adopting offline and online recycling channel modes [27, 28].

With the development of social networking platforms, more and more manufacturers have direct online channels to sell their products to consumers [29]. Network externalities are typical features in traditional economic theory of network economy and are essentially economies of scale in the process of expanding a user base [30]. The research on network externalities is gradually deepening. The earliest research on network externalities can be traced back to Katz and Shapiro [14]. They believe that the utility of consumers depends on the number of other users in the same network and will increase with the increase of the total demand for the product. Social networks are universal and the influences of network externalities in the product sales stage have become increasingly prominent [31]. Yi et al. [32] analyzed retailers' bounded rationality of market preferences from the perspective of an evolutionary game in the context of network externalities. Yi and Yang [33] further combined the evolutionary stability strategies and found that the strengths of network externalities directly affected manufacturers' wholesale price decisions. Yi and Yang [15] discussed product quality levels provided by manufacturers and showed that the sizes of network externalities were positively correlated with the levels specified by manufacturers. Zhou et al. [34] studied their own network externalities and interactive network externalities, and then found that the effects of network externalities on product price, demand, and profit occurred only when two types of network externalities exist simultaneously. Guo et al. [35] studied the influences of network externalities on the investment strategies of suppliers and the technology choices of competitive manufacturers, and then found that network externalities increase the manufacturers' equilibrium outputs without affecting the suppliers' wholesale price.

In CLSC, companies collect used products from consumers and utilize the residual value of products through the remanufacturing process [36]. And used products are taken as inputs, restored to as-new condition, and then resold [19, 37, 38]. Some studies [36, 39–41] assume that consumers are qualitative when building theoretical models, that is, consumers have the same cognition of new products and remanufactured products, and there is no preference. However, despite the economic and environmental merits of remanufacturing, there are still difficulties in the resale and remarketing of remanufactured products. Customers have a different perception of remanufactured products and show different levels of willingness to pay [17]. Some studies [42–44] consider the heterogeneity of consumers, but they only think that consumers have different willingness to pay for the two types of products and generally assume that the willingness to pay for recycled products is low [45], without considering consumers' environmental awareness concurrently. With the improvement of environmental protection awareness, many studies of consumer behavior have shown that consumers do consider the low-carbon attributes of products [9]. They will pay attention to the impact of products on environment when purchasing products and are willing to pay a certain fee for environmental protection products, or even pay higher price for environmental friendliness products [8], that is, they have a certain preference for environmental protection attributes of products. Orsdemir et al. [46, 47] studied the quality competition problems of new products and remanufactured products fabricated by manufacturers from the perspective of manufacturers, consumers, and environment, showing that manufacturers can rely on quality and quantity as an effective strategy for competing with remanufacturers. Wang and Hou [48] found that consumers' green preferences were obviously related to the green levels of products, and the adjustment speed of a green supply chain with bounded rationality directly affected market equilibrium. Product performance and environmental friendliness are important factors in CLSC. Chen et al. [49] studied the influence of consumer preferences on supply chain decision making by examining the difference between product performance and environmental quality payment, and then found that consumers' acceptance of product performance was positively correlated with the sales and profits of remanufactured products. Li and Li [50] examined consumers' dual preferences for environmental friendliness and product performance, constructed a CLSC model of different sales channels, and found that consumers' preferences for environmental friendliness were affected by performance recognition.

To sum up, explore the dual preferences of consumers considering both product performance and environmental friendliness, which is closer to the market environment of today's society, especially in the field of CLSC. At the same time, it cannot be ignored that consumers will be affected by product network externalities during the purchase process. Many studies in the existing research have attempted to solve the optimal selection problem in the context of brand owners and third-party manufacturers in CLSC. However, there is a lack of research on recycling channels selected by CLSC members and the impact of network externalities on different waste recycling channels by considering consumers' dual preferences. These studies can provide more realistic and targeted suggestions for the decision making of CLSC, and help company find appropriate operation strategies in the market. Based on the dual preferences of consumers, this paper successively constructs two CLSC models for direct and indirect recycling strategy and solves and analyzes the impact of network externalities on price and profit. Then, we compare the results in different models with numerical analysis and get management enlightenment for CLSC Table 1.

Table 1: The summary of literature and factors addressed by our study.

Literature	Waste products recovery	Competition for new product and remanufactured product	Network externalities	Preference for product quality	Preference for product environmental friendliness
Huang et al. [19]	√				
Orsdemir et al. [48]		√			
Chen et al. [50]	√	√		√	√
Liu et al. [51]	√			√	
Yi et al. [32]			√		
Wang and Hazen [53]				√	√
Li and Zuo [53]	√	√			
Zhu et al. [54]				√	√
Sun and Xiao [9]					√
Li and Li [50]		√		√	√
Suvrat [26]	√				
Cao et al. [44]	√			√	
Wang and Hou [48]					√
Guo et al. [35]			√		
Lin [55]		√		√	√
Chen et al. [20]	√				
Our study	√	√	√	√	√

3. Problems and Modeling

3.1. Problem Description. We considered a CLSC composed of a single brand owner and a single third-party manufacturer (such as Apple and Foxconn). The former does not undertake production but outsources the manufacturing of new products and remanufactured products to the latter and pays the corresponding wholesale price. The brand owner sells new products and remanufactured products simultaneously in the market. A price difference exists between both products sold by the brand owner. For example, Apple outsources the production of new products and remanufactured products to Foxconn, and sales new and remanufactured iPhone, iPad, and MacBook on his office website platform simultaneously. Consumers have dual preference characteristics, i.e., they consider both quality attributes and environmental friendliness attributes during the purchase process. And consumers are also affected by network externalities of products. Network externalities have a positive impact on consumer utility, that is, consumer utility increases with the increase of the number of consumers who buy products.

In recycling stage of waste products, there are two recycling channel strategies: direct and indirect recycling, whose specific structures are shown in Figures 1 and 2, respectively. In the direct recycling channel strategy, the third-party manufacturer directly recycles waste products for remanufacturing from consumers; while in the indirect recycling channel strategy, the brand owner pays unit recycling price to recycle waste products from consumers and resells them to the third-party manufacturer. For example, Apple recycles his waste products directly through "Apple Trade In" plan on his website platform and resells them to Foxconn for remanufacturing. The third-party manufacturer and the brand owner pursue profit maximization by deciding on their wholesale and retail price, respectively.

3.2. Assumptions. This paper made the following assumptions:

(1) Consumers purchase up to one new product or remanufactured product. The brand owner and the third-party manufacturer are risk-neutral, and the information shared between them is completely symmetrical.

(2) Consumers' quality perceptions of new products are higher than those of remanufactured products. In terms of product quality attributes, ν is used to represent quality perception of new products, where $\nu \in [0,1]$, and δ is the degree of consumers' acceptance of remanufactured products, where $\delta \in [0,1]$. Then, quality perception of remanufactured products is $\delta\nu$.

(3) New products and remanufactured products have different environmental friendliness attributes during production and use. Conrad [56] pointed out that different products have different environmental friendliness attributes. Bansal and Gangopadhyay [57] assumed an initial environmental pollution D of a product and the adoption of clean technology to reduce the pollution to $D-e$. Following these two studies, we assumed the initial environmental impact of a unit product produced by the third-party manufacturer to be 1. After material and technical processing, a new product and a remanufactured product would have environmental friendliness of e_n and e_r, respectively, where $e_n, e_r \in [0,1]$ and $e_n < e_r$.

(4) Consumers have no difference in their perceptions of the strengths of network externalities for new products and remanufactured products. A consumer's purchase behavior would be affected by the behaviors of other consumers. And network externalities would be positively correlated with consumer utility, i.e., the more consumers purchase this

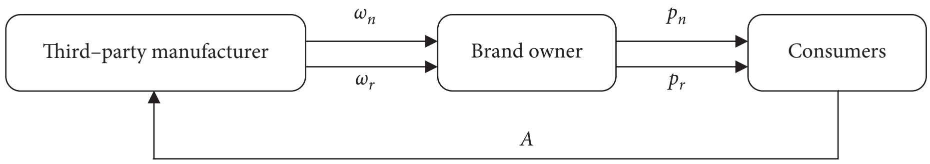


Figure 1: Direct recycling channel strategy of third-party manufacturers.

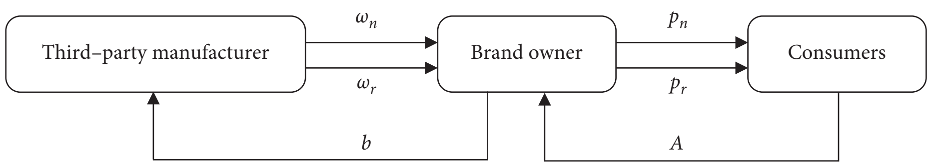


Figure 2: Indirect recycling channel strategy of third-party manufacturers.

product, the higher their utility. Consumer utility is related to the number of other consumers q_k ($k \in (n, r)$) who also purchase this product. Consumers have no difference in their perceptions of the strengths of network externalities for products with the same brand, i.e., the network effect intensity γ, where $\gamma \in [0, 1]$. The increase of consumer utility by network externalities can be expressed as $\gamma v q_n$ for new products and $\gamma\delta\, v q_r$ for remanufactured products [34, 58, 59]. The descriptions of other parameters are shown in Table 2.

3.3. Demand Function. According to the above hypothesis and research methods of Lin [55], the utility function expressions of purchasing new products and remanufactured products can be obtained as $U_n = v + \gamma v q_n^{ij} + \theta e_n - p_n^{ij}$ and $U_r = \delta v + \gamma\delta v q_r^{ij} + \theta e_r - p_r^{ij}$, respectively, where $p_n^{ij} < v$ and $p_r^{ij} < \delta v$. $\gamma v q_n^{ij}$ represent the change in consumer utility caused by product network externalities, and γ ($\gamma \in [0, 1]$) represents the degree of the sensitivity of product network externalities. Higher values of γ indicate higher sensitivity of consumer utility to changes with network externalities. When $0 < \gamma \leq 1$, the products have network externalities, while $\gamma = 0$ denotes that products have no network externalities. θe_n denotes the change in consumer utility caused by environmental friendliness of products, and θ ($\theta \in (0, 1)$) represents the degree of environmental friendliness sensitivity. Higher values of θ indicate higher sensitivity of consumer utility to changes with environmental friendliness. According to the utility function, the utility of consumers is related to quality of products, the number of purchasers, environmental friendliness, and the retail price. θe_n and p_n^{ij} indicate that consumers have dual preferences for products. When consumers buy products, they consider the price and environmental friendliness of products simultaneously. $\gamma v q_n^{ij}$ denotes the influences of product network externalities on purchasing in a social networking environment. This article only considers products' own network externalities and does not consider cross-network externalities.

According to the principle of the maximization of consumer utility [49, 50], when $U_n > U_r$ and $U_n > 0$, consumers would buy new products; and when $U_r > U_n$ and $U_r > 0$, consumers would buy remanufactured products. We get that consumers who are indifferent between new products and remanufactured products are located at $p_n^{ij} - p_r^{ij} - (v + \gamma v q_n^{ij}) + \delta(v + \gamma v q_r^{ij})/e_n - e_r$ and consumers who are indifferent between new products and nothing to buy are located at $p_n^{ij} - v - \gamma v q_n^{ij}/e_n$ (proof is shown in Appendix A). So, the demand functions of new products and remanufactured products with or without network externalities are derived as follows:

When $0 < \gamma \leq 1$ and network externalities are present, the respective demand functions of new products and remanufactured products are

$$q_n^{Ij} = \frac{\gamma\delta v^2 + \delta v e_n + \gamma\delta v e_n - v e_r - \gamma\delta v p_n^{Ij} + e_r p_n^{Ij} - e_n p_r^{Ij}}{-\gamma^2\delta v^2 + \gamma\delta v e_n + e_n^2 + \gamma v e_r - e_n e_r}. \tag{1}$$

$$q_r^{Ij} = \frac{\gamma\delta v^2 + v e_n - \delta v e_n + e_n^2 + \gamma v e_r - e_n e_r - e_n p_n^{Ij} - \gamma v p_r^{Ij} + e_n p_r^{Ij}}{-\gamma^2\delta v^2 + \gamma\delta v e_n + e_n^2 + \gamma v e_r - e_n e_r}. \tag{2}$$

When $\gamma = 0$ and no network externalities are present, the respective demand functions of new products and remanufactured products are

$$q_n^{Nj} = \frac{\delta v e_n - v e_r + e_r p_n^{Nj} - e_n p_r^{Nj}}{e_n (e_n - e_r)}. \tag{3}$$

$$q_r^{Nj} = \frac{v - \delta v + e_n - e_r - p_n^{Nj} + p_r^{Nj}}{e_n - e_r}. \tag{4}$$

4. Model Establishment and Solution

This section is based on different waste recycling channel strategies shown in Figures 1 and 2. Considering network externalities of products, four strategy combinations are

Table 2: Descriptions of parameters.

Parameter symbol	Symbol description
$i \in \{I, N\}$	I has network externalities; N does not have network externalities.
$j \in \{T, O\}$	T third-party manufacturer recycling; O brand recycling
p_r^{ij}	The retail price of remanufactured products under the influence of type i and strategy j
p_n^{ij}	The retail price of new products under the influence of type i and strategy j
ω_r^{ij}	The wholesale price of remanufactured products under the influence of type i and strategy j
ω_n^{ij}	The wholesale price of new products under the influence of type i and strategy j
q_r^{ij}	Demand for remanufactured products under the influence of type i and strategy j
q_n^{ij}	Demand for new products under the influence of type i and strategy j
π_r^{ij}	Profit of the third-party manufacturer under the influence of type i and strategy j
π_n^{ij}	Profit of the brand owner under the influence of type i and strategy j
c_n	Unit production cost of new products
c_r	Unit production cost of remanufactured products
A	Unit price for recycling waste products from consumers
B	Unit price for recycling waste products from brand owners

established in this section, namely, the combination of direct recycling strategy with network externalities, the combination of direct recycling strategy without network externalities, the combination of indirect recycling strategy with network externalities, and the combination of indirect recycling strategy without network externalities. The optimal decisions of the brand owner and the third-party manufacturer in a Stackelberg game are discussed here. First, the third-party manufacturer determines the wholesale price of new products and remanufactured products while the brand owner sets the retail price accordingly. Different waste recycling channel strategies introduce products with or without network externalities.

4.1. Direct Recycling Channel Strategy of the Third-Party Manufacturer. The third-party manufacturer directly recycles waste products for remanufacturing by paying unit recycling price A to consumers and sells new products and remanufactured products to the brand owner. Then, the brand owner sells both products to consumers. So, the respective profit functions of the brand owner and the third-party manufacturer are

$$\pi_n^{iT} = \left(p_n^{iT} - \omega_{\mathrm{n}}^{iT}\right)q_n^{iT} + \left(p_r^{iT} - \omega_r^{iT}\right)q_r^{iT}. \tag{5}$$

$$\pi_r^{iT} = \left(\omega_{\mathrm{n}}^{iT} - c_n\right)q_n^{iT} + \left(\omega_r^{iT} - c_r - A\right)q_r^{iT}. \tag{6}$$

4.1.1. Direct Recycling Channel Strategy with Network Externalities. Derived from equations (1) and (2), the respective demand functions $q_n^{IT} = \gamma\delta v^2 + \delta v e_n + \gamma\delta v e_n - v e_r - \gamma\delta v p_n^{IT} + e_r p_n^{IT} - e_n p_r^{IT} / - \gamma^2\delta v^2 + \gamma\delta v e_n + e_n^2 + \gamma v e_r - e_n e_r$ and $q_r^{IT} = \gamma\delta v^2 + v e_n - \delta v e_n + e_n^2 + \gamma v e_r - e_n e_r - e_n p_n^{IT} - \gamma v p_r^{IT} + e_n\ p_r^{IT} / -\gamma^2\delta v^2 + \gamma\delta v e_n + e_n^2 + \gamma v e_r - e_n e_r$ are brought into the objective function and the equilibrium solution is obtained by reverse recursion. Calculations show that when $\gamma^2\delta v^2 - \gamma\delta v e_n - e_n^2 - \gamma v e_r + e_n e_r > 0$, the definite negative condition of the second-order function Hessian matrix is established and the maximum values of the brand owner and the third-party manufacturer are obtained. The optimal decision is as follows; detailed calculation is shown in Appendix B.1:

$$\omega_{\mathrm{n}}^{IT} = \frac{1}{2}\left(v + c_n + e_n\right). \tag{7}$$

$$\omega_r^{IT} = \frac{1}{2}\left(A + \delta v + c_r + e_r\right). \tag{8}$$

$$p_n^{IT} = \frac{1}{4}\left(3v + c_n + 3e_n\right). \tag{9}$$

$$p_r^{IT} = \frac{1}{4}\left(A + 3\delta v + c_r + 3e_r\right). \tag{10}$$

4.1.2. Direct Recycling Channel Strategy without Network Externalities. Derived from equations (3) and (4), $q_n^{NT} = \delta v e_n - v e_r + e_r p_n^{NT} - e_n p_r^{NT} / e_n\left(e_n - e_r\right)$ and $q_r^{NT} = v - \delta v + e_n - e_r - p_n^{NT} + p_r^{NT} / e_n - e_r$ are brought into the objective function and the optimal decisions of the brand owner, and the third-party manufacturer are also obtained by reverse recursion; detailed calculation is shown in Appendix B.2:

$$\omega_n^{NT} = \frac{1}{2}\left(v + c_n + e_n\right), \tag{11}$$

$$\omega_r^{NT} = \frac{1}{2}\left(A + \delta v + c_r + e_r\right), \tag{12}$$

$$p_n^{NT} = \frac{1}{4}\left(3v + c_n + 3e_n\right), \tag{13}$$

$$p_r^{NT} = \frac{1}{4}\left(A + 3\delta v + c_r + 3e_r\right). \tag{14}$$

4.2. Indirect Recycling Channel Strategy of the Third-Party Manufacturer. The brand owner pays unit recycling price A to recycle waste products from consumers and resells them to the third-party manufacturer at price b. And the third-party manufacturer produces and sells new products and remanufactured products to the brand owner. Then, the brand owner sells both products to consumers in the market.

So, the respective profit functions of the brand owner and the third-party manufacturer are

$$\pi_n^{iO} = \left(p_n^{iO} - \omega_{\mathrm{n}}^{iO}\right)q_n^{iO} + \left(p_r^{iO} - \omega_r^{iO} - A + b\right)q_r^{iO}. \quad (15)$$

$$\pi_r^{iO} = \left(\omega_{\mathrm{n}}^{iO} - c_n\right)q_n^{iO} + \left(\omega_r^{iO} - c_r - b\right)q_r^{iO}. \quad (16)$$

4.2.1. Indirect Recycling Channel Strategy with Network Externalities. Derived from equations (1) and (2), the demand functions $q_n^{IO} = \gamma\delta v^2 + \delta v e_n + \gamma\delta v e_n - v e_r - \gamma\delta v p_n^{IO} + e_r p_n^{IO} - e_n p_r^{IO} / - \gamma^2\delta v^2 + \gamma\delta v e_n + e_n^2 + \gamma v e_r - e_n e_r$ and $q_r^{IO} = \gamma\delta v^2 + v e_n - \delta v e_n + e_n^2 + \gamma v e_r - e_n e_r - e_n p_n^{IO} - \gamma v p_r^{IO} + e_n p_r^{IO} / -\gamma^2\delta v^2 + \gamma\delta v e_n + e_n^2 + \gamma v e_r - e_n e_r$ are brought into the objective function and the equilibrium solution is obtained by reverse recursion. When $\gamma^2\delta v^2 - \gamma\delta v e_n - e_n^2 - \gamma v e_r + e_n e_r > 0$, the definite negative condition of the Hessian matrix is established, so the optimal decisions of the brand owner and the third-party manufacturer are obtained; detailed calculation is shown in Appendix B.3:

$$\omega_n^{IO} = \frac{1}{2}\left(v + c_n + e_n\right), \quad (17)$$

$$\omega_r^{IO} = \frac{1}{2}\left(-A + 2b + \delta v + c_r + e_r\right), \quad (18)$$

$$p_n^{IO} = \frac{1}{4}\left(3v + c_n + 3e_n\right), \quad (19)$$

$$p_r^{IO} = \frac{1}{4}\left(A + 3\delta v + c_r + 3e_r\right). \quad (20)$$

4.2.2. Indirect Recycling Channel Strategy without Network Externalities. Derived from equations (3) and (4), the respective demand functions $q_n^{NO} = \delta v e_n - v e_r + e_r p_n^{NO} - e_n p_r^{NO} / e_n (e_n - e_r)$ and $q_r^{NO} = v - \delta v + e_n - e_r - p_n^{NO} + p_r^{NO} / e_n - e_r$ are brought into the objective function and the equilibrium solution is obtained by reverse recursion. The optimal decisions of the brand owner and the third-party manufacturer are obtained; detailed calculation is shown in Appendix B.4:

$$\omega_n^{NO} = \frac{1}{2}\left(v + c_n + e_n\right), \quad (21)$$

$$\omega_r^{NO} = \frac{1}{2}\left(-A + 2b + \delta v + c_r + e_r\right), \quad (22)$$

$$p_n^{NO} = \frac{1}{4}\left(3v + c_n + 3e_n\right), \quad (23)$$

$$p_r^{NO} = \frac{1}{4}\left(A + 3\delta v + c_r + 3e_r\right). \quad (24)$$

4.3. Comparative Analysis of Supply Chain Operation under Different Recycling Channel Strategies

Proposition 1. *The wholesale price of new products is not affected by product network externalities and waste product recycling channel strategies $\omega_n^{IT} = \omega_n^{IO} = \omega_n^{NT} = \omega_n^{NO}$. When remanufactured products have network externalities, their wholesale price is greater than that without network externalities: $\omega_r^{IT} = \omega_r^{IO} > \omega_r^{NT} = \omega_r^{NO}$.*

The wholesale price of new products is related to different recycling channel strategies and network externalities as follows: $\omega_n^{IT} = \omega_n^{IO} = \omega_n^{NT} = \omega_n^{NO} = 1/2\left(v + c_n + e_n\right)$. The relationships between the wholesale price of remanufactured products in different strategy combinations are $\omega_r^{IT} = \omega_r^{IO} = 1/2\left(-A + 2b + \delta v + c_r + e_r\right)$ and $\omega_r^{NT} = \omega_r^{NO} = 1/2\left(A + \delta v + c_r + e_r\right)$. By comparing $1/2\left(-A + 2b + \delta v + c_r + e_r\right) - 1/2\left(A + \delta v + c_r + e_r\right) = b - A > 0$, we obtain $\omega_r^{IT} = \omega_r^{IO} > \omega_r^{NT} = \omega_r^{NO}$.

Proposition 1 states that the wholesale price of new products is related to product quality level, the production cost of new products, and environmental friendliness, and is not affected by network externalities in the product sales process and recycling channels. First, the third-party manufacturer would not change the wholesale price of new products with the changing utility of consumers. Even network externalities have a positive impact on consumer utility. Next, the third-party manufacturer would not increase the wholesale price to compensate for the recycling cost. Even he must pay high unit recycling price ($b > A$) to the brand owner in the indirect recycling channel strategy. Therefore, in different strategy combinations, the wholesale price of new products remains unchanged.

The wholesale price of remanufactured products varies with different network externalities. Considering remanufactured products with network externalities, the third-party manufacturer decides that the wholesale price is greater than that without network externalities. Network externalities of remanufactured products can increase the utility of consumers, thereby encouraging even more consumers to purchase remanufactured products. With the demand for remanufactured products increasing, the third-party manufacturer would increase the wholesale price in the wholesale market. The calculation shows that in the indirect recycling channel strategy, the third-party manufacturer pays a higher price for recycling waste products. However, when network externalities are determined, the recycling price will not affect the wholesale price of remanufactured products, as $\omega_r^{IT} = \omega_r^{IO}$ and $\omega_r^{NT} = \omega_r^{NO}$. This outcome is different from the usual belief that the wholesale price of remanufactured products is affected by the recycling cost of waste products.

Actually, network externalities of products can encourage more consumers understand and purchase remanufactured products. With the increase of market demand for remanufactured products, manufacturers will increase the wholesale price of remanufactured products. However, product network externalities have no impact on the wholesale price of new products.

Proposition 2. *The equilibrium retail prices of new products and remanufactured products are not affected by product network externalities and waste product recycling channel strategies: $p_n^{ij} = 1/4\left(3v + c_n + 3e_n\right)$ and $p_r^{ij} = 1/4\left(A + 3\delta v + c_r + 3e_r\right)$.*

According to the calculation, the equilibrium prices of both products with different strategy combinations are compared, and the equilibrium prices for new products $p_n^{ij} = 1/4(3\nu + c_n + 3e_n)$ and remanufactured products $p_r^{ij} = 1/4(A + 3\delta\nu + c_r + 3e_r)$ are obtained, where $i \in (I, N)$ and $j \in (T, O)$. When $p_n^{ij} > p_r^{ij}$ is assumed, we obtain $p_n^{ij} - p_r^{ij} = 1/4(-A - 3\delta\nu + 3\nu + c_n - c_r + 3e_n - 3e_r) > 0$.

Proposition 2 states that the brand owner has absolute decision-making power over the retail price and is not affected by network externalities or recycling channel strategies. According to Proposition 1, the third-party manufacturer sets the same wholesale price for new products even different strategy combinations. Therefore, the brand owner will not change the retail price of new products due to changes in network externalities and recycling channel strategies. Even consumer utility increases with the increase of the number of consumers, and the brand owner will not change the retail price to increase his marginal revenues. When considering network externalities of remanufactured products, the third-party manufacturer would increase the wholesale price. The brand owner would change the retail price for remanufactured products even when the marginal revenues become smaller. Because consumers' perceptions of quality of remanufactured products are lower than those of new products. If the latter's wholesale price does not change, the brand owner just increases the retail price of remanufactured products. The rising retail price would weaken consumers' enthusiasm to buy remanufactured products even if they were more environmental friendliness.

In practical terms, when manufacturers increase the wholesale price because of producing cost or labor cost, brand owners will not easily change the retail price even if their marginal revenue decreases, especially when there is a mutual relationship in the market.

Proposition 3. *When network externalities of new products and remanufactured products are determined, changes in recycling channel strategy do not affect their equilibrium demands:* $q_r^{iT} = q_r^{iO}$ *and* $q_n^{iT} = q_n^{iO}$.

When there are network externalities, the market demand of both products with different recycling channel strategies has the following relationship. The equilibrium demand for remanufactured products is

$$\begin{aligned} q_r^{IT} = q_r^{IO} &= \frac{-A\gamma\nu + \gamma\delta\nu^2 - \gamma\nu c_r + Ae_n + \nu e_n - \delta\nu e_n - c_n e_n + c_r e_n + e_n^2 + \gamma\nu e_r - e_n e_r}{-4\left(\gamma^2\delta\nu^2 - \gamma\delta\nu e_n - e_n^2 - \gamma\nu e_r + e_n e_r\right)}, \\ q_r^{NT} = q_r^{NO} &= \frac{A - \delta\nu + \nu - c_n + c_r + e_n - e_r}{4(e_n - e_r)}. \end{aligned} \tag{25}$$

The equilibrium demand for new products is

$$\begin{aligned} q_n^{IT} = q_n^{IO} &= \frac{\gamma\delta\nu^2 - \gamma\delta\nu c_n - Ae_n + \delta\nu e_n + \gamma\delta\nu e_n - c_r e_n - \nu e_r + c_n e_r}{-4\left(\gamma^2\delta\nu^2 - \gamma\delta\nu e_n - e_n^2 - \gamma\nu e_r + e_n e_r\right)}, \\ q_n^{NT} = q_n^{NO} &= \frac{-Ae_n + \delta\nu e_n - c_r e_n - \nu e_r + c_n e_r}{4e_n(e_n - e_r)}. \end{aligned} \tag{26}$$

The above formulas show that when products have network externalities, the direct and indirect recycling channel strategies will not affect the demand for both products. When products have no network externalities, the results are the same as those for products that have network externalities. According to Propositions 1 and 2, we know that different recycling channel strategies only affect the wholesale price of remanufactured products. The retail price of both products for different recycling channel strategies remains unchanged, so the market demand remains unchanged. At the same time, no matter what kind of recycling channel strategy, consumers will resell waste products at the same price A. The marginal income of consumers through recycling waste products will not change. In contrast, for products with or without network externalities, consumers will have different demands for both products. Because network externalities could increase consumer utility and attract more consumers to purchase products.

In reality, consumers are not interested in waste product recycling channels. The recycling of waste products by different companies will not affect consumer demand for products.

Proposition 4. *The Senunz relationships between the equilibrium demands for new products and remanufactured products are as follows:*

For products with network externalities and when $\gamma\nu - 2e_n > 0$*:*

(1) *When* $c_r > \dfrac{A\gamma v - \gamma\delta v c_n - 2Ae_n - ve_n + 2\delta ve_n + \gamma\delta ve_n}{+c_n e_n - e_n^2 - ve_r - \gamma ve_r + c_n e_r + e_n e_r}$ / $-\gamma v + 2e_n$, *then* $q_r^{Ij} < q_n^{Ij}$;

(2) *When* $c_r < \dfrac{A\gamma v - \gamma\delta v c_n - 2Ae_n - ve_n + 2\delta ve_n + \gamma\delta ve_n}{+c_n e_n - e_n^2 - ve_r - \gamma ve_r + c_n e_r + e_n e_r}$ / $-\gamma v + 2e_n$, *then* $q_r^{Ij} > q_n^{Ij}$.

For products without network externalities:

(1) *When* $c_r > 2Ae_n + ve_n - 2\delta ve_n - c_n e_n + e_n^2 + ve_r - c_n e_r - e_n e_r / -2e_n$, *then* $q_r^{Nj} < q_n^{Nj}$;

(2) *When* $c_r < 2Ae_n + ve_n - 2\delta ve_n - c_n e_n + e_n^2 + ve_r - c_n e_r - e_n e_r / -2e_n$, *then* $q_r^{Nj} > q_n^{Nj}$.

The calculations for Proposition 3 show that when both products have network externalities, $q_r^{Ij} - q_n^{Ij} = -A\gamma v + \gamma\delta v c_n - \gamma v c_r + 2Ae_n + ve_n - 2\delta ve_n - \gamma\delta ve_n - c_n e_n + 2c_r e_n + e_n^2 + ve_r + \gamma ve_r - c_n e_r - e_n e_r / -4(\gamma^2\delta v^2 - \gamma\delta ve_n - e_n^2 - \gamma ve_r + e_n e_r)$; when they have no network externalities, $q_r^{Nj} - q_n^{Nj} = 2Ae_n + ve_n - 2\delta ve_n - c_n e_n + 2c_r e_n + e_n^2 + ve_r - c_n e_r - e_n e_r / 4e_n(e_n - e_r)$.

When they have network externalities and meet the conditions $\gamma v - 2e_n > 0$ and $c_r < A\gamma v - \gamma\delta v c_n - 2Ae_n - ve_n + 2\delta ve_n + \gamma\delta ve_n + c_n e_n - e_n^2 - ve_r - \gamma ve_r + c_n e_r + e_n e_r / -\gamma v + 2e_n$, then $q_r^{Ij} - q_n^{Ij} > 0$. Likewise, they meet the conditions $v - 2e_n > 0$ and $c_r > A\gamma v - \gamma\delta v c_n - 2Ae_n - ve_n + 2\delta ve_n + \gamma\delta ve_n + c_n e_n - e_n^2 - ve_r - \gamma ve_r + c_n e_r + e_n e_r / -\gamma v + 2e_n$, and we get $q_r^{Ij} - q_n^{Ij} < 0$.

Similarly, when both products have no network externalities and the condition $c_r < 2Ae_n + ve_n - 2\delta ve_n - c_n e_n + e_n^2 + ve_r - c_n e_r - e_n e_r / -2e_n$ is met, then $q_r^{Nj} - q_n^{Nj} > 0$; otherwise, $q_r^{Nj} - q_n^{Nj} < 0$.

Proposition 4 states that when the brand owner sells both products while the production cost of remanufactured products is sufficiently small, the equilibrium demand for remanufactured products is greater than that for new products. Because the third-party manufacturer will offer a lower wholesale price, the brand owner will sell remanufactured products at a lower retail price. In this case, the price advantage of remanufactured products is more obvious. Consumers will choose to buy remanufactured products by considering product quality, price, and environmental friendliness. So, the equilibrium demand for remanufactured products is greater than that for new products. In contrast, when the production cost of remanufactured products is high, the corresponding wholesale and retail price will increase. So, the price advantage of remanufactured products will weaken. Then, consumers will choose to buy new products at higher price, so the equilibrium demand will be greater than that for remanufactured products.

In practice, product remanufacturing can not only help companies establish a sense of responsibility for green environmental protection but also save the production cost of companies. The advantage of low cost for remanufactured products can help them win more consumers.

Proposition 5. *The relationships between the equilibrium profits for the brand owner and the third-party manufacturer are as follows:* $\pi_r^{ij} = 2\pi_m^{ij}$.

Through calculation, it can be found that:

(1) When the third-party manufacturer directly recycles waste products, the profit of the third-party manufacturer is twice that of the brand owner; and when the third-party manufacturer recycles waste products indirectly, the profit of the third-party manufacturer is still twice than that of the brand owner. In the indirect recycling channel strategy, when the third-party manufacturer recycles waste products from the brand owner, he pays the unit price of waste products higher than the unit price of waste products directly recycled from consumers. So, the third-party manufacturer would pay more recycling costs in the recycling process. While the brand owner can obtain certain recycling income by recycling waste products from consumers and reselling them to the third-party manufacturer. Currently, the profit of the third-party manufacturer is still twice that of the brand owner. So, the brand owner will actively participate in the recycling process of waste products because of the income of recycling channels, and the third-party manufacturer will not hinder the brand owner from entering recycling channels because of the increase of unit recycling price.

(2) When considering network externalities of products, the profit of the third-party manufacturer is twice that of the brand owner, and when network externalities of products are not considered, the profit of the third-party manufacturer is still twice that of the brand owner. When considering network externalities of products, the wholesale price of remanufactured products set by the third-party manufacturer is higher than that without considering network externalities. The brand owner will pay more for remanufactured products to the third-party manufacturer in the wholesale market. At this time, product network externalities can attract more consumers to buy new products and remanufactured products. So, the profit of the brand owner is still just half of the profit of the third-party manufacturer.

5. Extension

In the previous research, we assume that consumers do not consider the recycling of products when purchasing products. And there is no consumer preference for waste product recycling channels. In this section, we will consider that when consumers buy products, they know that new products would be recycled after using. For example, Caterpillar uses a full lifecycle approach to realize consumer value. The product of Caterpillar can be remanufactured at the end of their service life [29]. Therefore, we discuss that when considering the utility of consumers buying new products, the impact of recycling price will be considered. At the same time, some previous studies [29, 60] have pointed out that consumers have different preferences for different channels. Here, we assume that consumers prefer indirect recycling channel more than direct recycling channel. Because when the brand owner can recycle waste products through the official website, consumers' recognition of his enterprise social responsibility will be improved. At the same time, consumers do not have to pay extra costs to download or register third-party enterprise software, but only use the information left when they buy products, to complete the recycling of waste products. We assume that η represents consumers' channel preference for recycling by the brand owner or the third-party manufacturer. Furthermore, it is reasonable to assume $\eta > 1$.

Based on the earlier analysis, we can derive the demand functions in the two models. Among them, when consumers return waste products through indirect recycling, the utility of purchasing new products is $U_n = \nu + \gamma \nu q_n^{ij} + \theta e_n - p_n^{ij} + \eta A$. When consumers return waste products through direct recycling, the utility of purchasing new products is $U_n = \nu + \gamma \nu q_n^{ij} + \theta e_n - p_n^{ij} + A$. Applying the analysis principle in Section 3.3, we can get the demand functions as follows:

In the indirect recycling channel:

$$\begin{aligned} q_n^{IO} &= -\frac{Ak\beta\eta\nu + k\beta\nu^2 + \beta\nu e_n + k\beta\nu e_n - A\eta e_r - \nu e_r - k\beta\nu p_n + e_r p_n - e_n p_r}{k^2\beta\nu^2 - k\beta\nu e_n - e_n^2 - k\nu e_r + e_n e_r}, \\ q_r^{IO} &= -\frac{k\beta\nu^2 + A\eta e_n + \nu e_n - \beta\nu e_n + e_n^2 + k\nu e_r - e_n e_r - e_n p_n - k\nu p_r + e_n p_r}{k^2\beta\nu^2 - k\beta\nu e_n - e_n^2 - k\nu e_r + e_n e_r}. \end{aligned} \tag{27}$$

In the direct recycling channel:

$$\begin{aligned} q_n^{IT} &= -\frac{Ak\beta\nu + k\beta\nu^2 + \beta\nu e_n + k\beta\nu e_n - A e_r - \nu e_r - k\beta\nu p_n + e_r p_n - e_n p_r}{k^2\beta\nu^2 - k\beta\nu e_n - e_n^2 - k\nu e_r + e_n e_r}, \\ q_r^{IT} &= -\frac{k\beta\nu^2 + A e_n + \nu e_n - \beta\nu e_n + e_n^2 + k\nu e_r - e_n e_r - e_n p_n - k\nu p_r + e_n p_r}{k^2\beta\nu^2 - k\beta\nu e_n - e_n^2 - k\nu e_r + e_n e_r}. \end{aligned} \tag{28}$$

By solving the above optimization problem, we can get the optimal equilibrium results of supply chain members, which are presented in Table 3.

Proposition 6. *When consumers have channel preference for waste product recycling, the retail price of new products in the indirect recycling channel is higher than that in the direct recycling channel; the retail price of remanufactured products remains unchanged in different waste product recycling channels.*

The optimal equilibrium results of supply chain members obtained through calculation are shown in Table 3, where

$$\begin{aligned} p_n^{iO} &= \frac{1}{4}\left(3A\eta + 3\nu + c_n + 3e_n\right), \\ p_n^{iT} &= \frac{1}{4}\left(3A + 3\nu + c_n + 3e_n\right), \\ p_r^{iO} &= \frac{1}{4}\left(A + 3\beta\nu + c_r + 3e_r\right), \\ p_r^{iT} &= \frac{1}{4}\left(A + 3\beta\nu + c_r + 3e_r\right). \end{aligned} \tag{29}$$

According to the calculation, we have $p_n^{iO} - p_n^{iT} = 3/4A(-1+\eta) > 0$ and $p_r^{iO} - p_r^{iT} = 0$.

It can be seen from the above calculation that in the waste product recycling process, the brand owner charges different retail prices for new products in different waste product recycling channels, while the retail price is not affected by product network externalities. When the brand owner recycles waste products, the retail price of new products is higher than that of new products when the third-party manufacturer recycles waste products; while the retail price of remanufactured products is not affected by recycling channels. The recycling of waste products by the brand owner can greatly improve his image of high social responsibility and environmental awareness in the hearts of consumers. The brand owner gets profits from waste product recycling channel, but with the increase of consumers' purchase utility, he can still raise the retail price of new products to make greater profits. In particular, the more obvious consumers' preference for recycling channels, the greater the difference in the retail price of new products under different waste product recycling channels. In reality, brand owners should actively participate in the closed-loop supply chain and contribute to the recycling of waste products, product remanufacturing, product environmental protection function design, and other links. Through these measures, brand owners can improve their sense of enterprise social responsibility and product purchase utility in the hearts of consumers, to appropriately increase the retail price of products and obtain greater profits.

Table 3: The optimal equilibrium results of supply chain members.

	Direct recycling by third-party manufacturers	Indirect recycling by brand owners
p_n^{Ij}	$1/4(3A + 3\nu + c_n + 3e_n)$	$1/4(3A\eta + 3\nu + c_n + 3e_n)$
p_r^{Ij}	$1/4(A + 3\beta\nu + c_r + 3e_r)$	$1/4(A + 3\beta\nu + c_r + 3e_r)$
p_n^{Nj}	$1/4(3A + 3\nu + c_n + 3e_n)$	$1/4(3A\eta + 3\nu + c_n + 3e_n)$
p_r^{Nj}	$1/4(A + 3\beta\nu + c_r + 3e_r)$	$1/4(A + 3\beta\nu + c_r + 3e_r)$
q_n^{Ij}	$-Ak\beta\nu + k\beta\nu^2 - k\beta\nu c_n - Ae_n +$ $\beta\nu e_n + k\beta\nu e_n - c_r e_n - Ae_r - \nu e_r +$ $c_n e_r/4(k^2\beta\nu^2 - k\beta\nu e_n - e_n^2 - k\nu e_r + e_n e_r)$	$-Ak\beta\eta\nu + k\beta\nu^2 - k\beta\nu c_n - Ae_n + \beta\nu e_n +$ $k\beta\nu e_n - c_r e_n - A\eta e_r - \nu e_r +$ $c_n e_r/4(k^2\beta\nu^2 - k\beta\nu e_n - e_n^2 - k\nu e_r + e_n e_r)$
q_r^{Ij}	$- - Ak\nu + k\beta\nu^2 - k\nu c_r + 2Ae_n + \nu e_n -$ $\beta\nu e_n - c_n e_n + c_r e_n + e_n^2 + k\nu e_r -$ $e_n e_r/4(k^2\beta\nu^2 - k\beta\nu e_n - e_n^2 - k\nu e_r + e_n e_r)$	$- - Ak\nu + k\beta\nu^2 - k\nu c_r + Ae_n + A\eta e_n + \nu e_n -$ $\beta\nu e_n - c_n e_n + c_r e_n + e_n^2 + k\nu e_r -$ $e_n e_r/4(k^2\beta\nu^2 - k\beta\nu e_n - e_n^2 - k\nu e_r + e_n e_r)$
q_n^{Nj}	$-Ae_n + \beta\nu e_n - c_r e_n - Ae_r - \nu e_r + c_n e_r/4e_n(e_n - e_r)$	$-Ae_n + \beta\nu e_n - c_r e_n - A\eta e_r - \nu e_r + c_n e_r/4e_n(e_n - e_r)$
q_r^{Nj}	$2A + \nu - \beta\nu - c_n + c_r + e_n - e_r/4(e_n - e_r)$	$A + A\eta + \nu - \beta\nu - c_n + c_r + e_n - e_r/4(e_n - e_r)$

6. Numerical Example Simulation

Regarding the complexity of calculations, this section introduces numerical examples to analyze the influence of parameter changes in the modeling of the optimal decision and profit. The parameter assignments are $A = 1/5$, $\delta = 4/5$, $\nu = 1$, $\gamma = 2/5$, $c_n = 2c_r$, and $e_n = e_r/2$. In [29]; He et al. defines the unit cost saving of producing remanufactured products with respect to new product and assume $c_r = 0$ to the simplified model. Here, we assume $c_n = 2c_r$ to analyze the result.

We can obtain the relationships between the equilibrium demands for new products and remanufactured products when products have network externalities or no network externalities as follows:

When $32 - 255e_r + 50e_r^2/5(16 - 95e_r) < c_r < 0.5$, then $q_n^{Ij} < q_n^{Nj}$ and $q_r^{Ij} > q_r^{Nj}$;

When $56 - 195e_r + 25e_r^2/5(24 - 85e_r) < c_r < 32 - 255e_r + 50e_r^2/5(16 - 95e_r)$, then $q_n^{Ij} > q_n^{Nj}$ and $q_r^{Ij} > q_r^{Nj}$;

When $0 < c_r < 56 - 195e_r + 25e_r^2/5(24 - 85e_r)$, then $q_n^{Ij} > q_n^{Nj}$ and $q_r^{Ij} < q_r^{Nj}$.

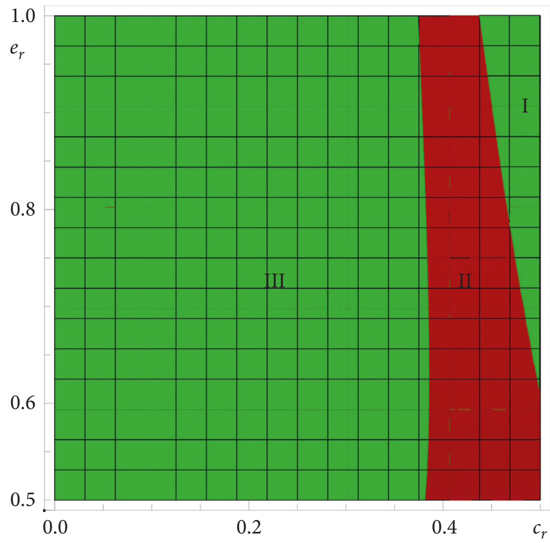
The respective optimal demands for new products and remanufactured products are $q_n^{Ij} = -5(16 - 32c_r - 27e_r + 75c_r e_r)/2(64 - 280e_r + 125e_r^2)$, $q_r^{Ij} = 5(-24 + 40c_r - 60e_r + 50c_r e_r + 25e_r^2)\ /4(64 - 280e_r + 125e_r^2)$, $q_n^{Nj} = -7 + 15c_r/-10e_r$, and $q_r^{Nj} = -4 + 10c_r + 5e_r/20e_r$.

In accordance with Proposition 3, the optimal demands are obtained as follows:

$$q_n^{Ij} - q_n^{Nj} = \frac{4\left(56 - 120c_r - 195e_r + 425c_r e_r + 25e_r^2\right)}{-5e_r\left(64 - 280e_r + 125e_r^2\right)},$$
$$q_r^{Ij} - q_r^{Nj} = \frac{2\left(32 - 80c_r - 255e_r + 475c_r e_r + 50e_r^2\right)}{5e_r\left(64 - 280e_r + 125e_r^2\right)}. \tag{30}$$

Based on the assumptions in previous chapters, $0 < c_r < 0.5$ and $0.5 < e_r < 1$, which can be obtained by calculation. When $0 < c_r < 32 - 255e_r + 50e_r^2/5(16 - 95e_r)$, the demand for new products with network externalities is greater than that without network externalities. When $32 - 255e_r + 50e_r^2/5(16 - 95e_r) < c_r < 0.5$, the converse is true. When $56 - 195e_r + 25e_r^2/5(24 - 85e_r) < c_r < 0.5$, the market demand for remanufactured products with network externalities is greater than that without network externalities. When $0 < c_r < 56 - 195e_r + 25e_r^2/5(24 - 85e_r)$, the converse is true. The details are shown in Figure 3.

Figure 3 shows that in the same interval, the demand trends of new products and remanufactured products are different when products have network externalities or not. It shows that in interval I, $q_n^{Ij} < q_n^{Nj}$ and $q_r^{Ij} > q_r^{Nj}$ are established at the same time while network externalities reduce the demand for new products but increase that for remanufactured products. In interval II, $q_n^{Ij} > q_n^{Nj}$ and $q_r^{Ij} > q_r^{Nj}$ are also established. Network externalities make the market demand for both products increase simultaneously. In interval III, $q_n^{Ij} > q_n^{Nj}$ and $q_r^{Ij} < q_r^{Nj}$ are established at the same time. Network externalities increase the demand for new products but reduce that for remanufactured products. Therefore, when the production cost of remanufactured products is high (in interval I), the brand owner should weaken network externalities of new products and enlarge network externalities of remanufactured products at the same time. Under the condition that the retail price remains unchanged, more consumers buy the product that can increase the income of the brand owner. In interval II, the brand owner can expand network externalities of his products and encourage more consumers to purchase them by establishing network community, implementing network marketing, and gathering users. Thereby, it promotes the simultaneous growth of demands for both products and increases the brand owner's profit. When the production cost of remanufactured products is low (in interval III), the brand owner should expand network externalities of new products and weaken network externalities of remanufactured products. Interestingly, it can be seen from Figure 3 that the equilibrium demand for the product with network externalities is not always greater than that without network externalities. In reality, people usually think that network externalities of products can increase consumer utility and bring higher product demand. The calculation results show that this is not always the case.



I $q_n^{Ij} < q_n^{Nj}$ and $q_r^{Ij} > q_r^{Nj}$

II $q_n^{Ij} > q_n^{Nj}$ and $q_r^{Ij} > q_r^{Nj}$

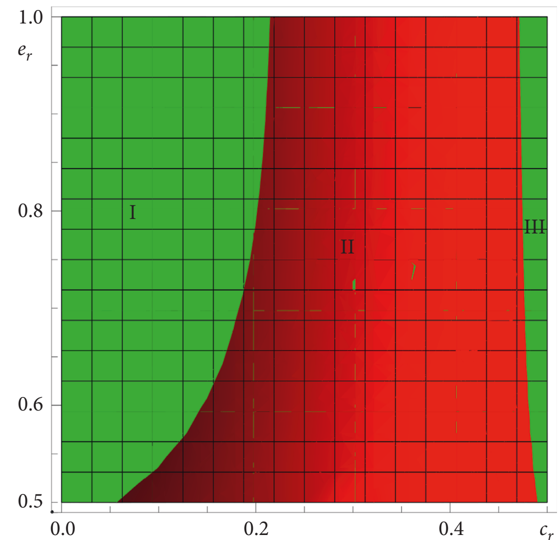
III $q_n^{Ij} > q_n^{Nj}$ and $q_r^{Ij} < q_r^{Nj}$

FIGURE 3: Distributions of demand for new products and remanufactured products.

In the actual operation process, the effect of network externalities on consumers is not invariable. Brand owners should expand or weaken network externalities of products according to different situations, to encourage more consumers to buy products.

We can get the relationships between the profits for new products and remanufactured products when products have network externalities or no network externalities as follows:

When $-192 - 525e_r + 50e_r^2 + \Delta/25 \ (-16 + 49e_r) < c_r < -192 - 525e_r + 50e_r^2 - \Delta/25\,(-16 + 49e_r)$, then



I and III $\pi_n^{Ij} - \pi_n^{Nj} < 0$ and $\pi_n^{Ij} - \pi_n^{Nj} < 0$

II $\pi_n^{Ij} - \pi_n^{Nj} > 0$ and $\pi_n^{Ij} - \pi_n^{Nj} > 0$

FIGURE 4: The profit distributions of the brand owner and the third-party manufacturer.

$\pi_m^{Ij} - \pi_m^{Nj} > 0$ and $\pi_r^{Ij} - \pi_r^{Nj} > 0$, where $\Delta = \sqrt{-5376 + 35360e_r - 76700e_r^2 + 86125e_r^3 - 28125e_r^4}$;

When $0 < c_r < -192 - 525e_r + 50e_r^2 + \Delta/25 \ (-16 + 49e_r)$ or $-192 - 525e_r + 50e_r^2 - \Delta/25 \ (-16 + 49e_r) < c_r < 0.5$; then $\pi_m^{Ij} - \pi_m^{Nj} < 0$ and $\pi_r^{Ij} - \pi_r^{Nj} < 0$, where $\Delta = \sqrt{-5376 + 35360e_r - 76700e_r^2 + 86125e_r^3 - 28125e_r^4}$.

In accordance with results, the profits of both parties can be obtained as follows:

$$\begin{aligned}
\pi_n^{Ij} - \pi_n^{Nj} &= \frac{-528 + 1920c_r - 2000c_r^2 + 1345e_r - 5250c_r e_r + 6125c_r^2 e_r - 525e_r^2 + 500c_r e_r^2 + 125e_r^3}{50e_r\left(64 - 280e_r + 125e_r^2\right)}, \\
\pi_r^{Ij} - \pi_r^{Nj} &= \frac{-528 + 1920c_r - 2000c_r^2 + 1345e_r - 5250c_r e_r + 6125c_r^2 e_r - 525e_r^2 + 500c_r e_r^2 + 125e_r^3}{25e_r\left(64 - 280e_r + 125e_r^2\right)}.
\end{aligned} \tag{31}$$

When we set $\pi_n^{Ij} - \pi_n^{Nj} = 0$ and $\pi_r^{Ij} - \pi_r^{Nj} = 0$, we get the same result as follows:

$$\begin{aligned}
c_{r1} &= -\frac{192 - 525e_r + 50e_r^2 + \Delta}{25\left(-16 + 49e_r\right)}, \\
c_{r2} &= -\frac{192 - 525e_r + 50e_r^2 - \Delta}{25\left(-16 + 49e_r\right)},
\end{aligned} \tag{32}$$

where $\Delta = \sqrt{-5376 + 35360e_r - 76700e_r^2 + 86125e_r^3 - 28125e_r^4}$.

The results for comparing the size relationship of the profits when $0 < c_r < 0.5$ and $0.5 < e_r < 1$ are shown in Figure 4.

Figure 4 shows that in the same interval, the profit trends of the brand owner and the third-party manufacturer are the same when products have network externalities. In intervals I and III, $\pi_n^{Ij} - \pi_n^{Nj} < 0$ and $\pi_r^{Ij} - \pi_r^{Nj} < 0$, network externalities reduce the profits of both parties. In interval II, $\pi_n^{Ij} - \pi_n^{Nj} > 0$, and $\pi_r^{Ij} - \pi_r^{Nj} > 0$, network externalities increase the profits of both parties. When the production cost of remanufactured products changes between intervals I and III, both parties hope that their products will not have network externalities. When the cost is moderate (in interval II), both parties hope for network externalities, which would increase their profits. The brand owner and the third-party manufacturer gain higher profit under the condition of network externalities. According to the above analysis, we know that network externalities do not always lead to an increase in product sales. Therefore, in Figure 4, we also find that under certain conditions, when products have no network externalities, the profits of both parties are greater than that when products have network externalities.

In the actual operation process, brand owners can increase their revenue by expanding network externalities of products through network promotion, but they also need to pay attention to the problem of revenue reduction caused by network externalities.

7. Research Conclusions

This study examined the influence of product network externalities on company decision making in a CLSC model with different waste product recycling channel strategies, incorporating consumers' dual preference factors for product quality and environmental friendliness. The following conclusions are obtained from the analysis and a comparison of the equilibrium results of the wholesale and retail price, product demand, and company profit in different strategy combinations.

(1) The retail price of new products and remanufactured products are not affected by network externalities and recycling channel strategies even if the wholesale price of remanufactured products changes, i.e., the third-party manufacturer increases the wholesale price of remanufactured products with network externalities.

(2) Regardless of whether new products or remanufactured products have network externalities, a value range exists for the production cost of remanufactured products and makes consumers' demand for them greater than that for new products. With the promotion of green environmental protection, green technological innovation will greatly reduce the production cost of remanufactured products, stimulate consumers' enthusiasm for supporting green products, and encourage more consumers to choose environmental friendliness remanufactured products.

(3) Network externalities affect the utility values of products purchased by consumers and affect the demand for new products and remanufactured products. Remanufactured products have an ideal production cost range, that is, the demands of both products with network externalities are greater than those of products without network externalities. It is beneficial to the brand owner who can make better use of the advantages of network interconnections to amplify network externalities of both products, thereby attracting more consumers to purchase them.

(4) Even if it is assumed that network externalities have a positive correlation with consumer utility, there is still a situation that the profits of the brand owner and the third-party manufacturer when products have no network externalities are greater than those when products have network externalities. This conclusion implies that most companies in today's market use social platforms for product network marketing and expanding product network externalities, which may not necessarily lead to increased corporate profit.

This study provides a foundation for further research into CLSC system selection that includes product network externalities and consumers' dual preference characteristics. However, it still has limitations that provide opportunities for future studies.

First, we only considered internal network externalities of products and did not cover cross-network externalities between products when discussing network externalities of products in this study. As there will be cross-network externalities among product sales of the same brand, we will investigate cross-network externalities of new products and remanufactured products and the influence on company decision making in the future study.

Second, this study only considered the dual preferences of consumers for products under the complete information. When the product and environmental protection information are asymmetric, there could be changes in consumers' behaviours. In the next step, we will study how information asymmetry and disclosure of company's new products and remanufactured products affect consumer product selection strategy and company decision making on sales process.

Finally, the process of consumers buying products is often dynamic and multiple in the market. And the recycling and remanufactured product sales issues in the multiperiod dynamic CLSC will be taken into consideration in the future.

Appendix

A. The Proof of θ

When $U_n = U_r$, consumers get the same utility when they buy new products and remanufactured products. $\theta = p_n^{ij} - p_r^{ij} - (v + \gamma v q_n^{ij}) + \delta(v + \gamma v q_r^{ij})/e_n - e_r$ can be obtained by calculation. When $U_n = 0$, consumers buy new products and do not buy products having the same utility. $\theta = p_n^{ij} - v - \gamma v q_n^{ij}/e_n$ can be obtained by calculation. According to the Hoteling model, when $p_n^{ij} - p_r^{ij} - (v + \gamma v q_n^{ij}) + \delta(v + \gamma v q_r^{ij})/e_n - e_r < \theta < 1$, consumers buy remanufactured products; when $p_n^{ij} - v - \gamma v q_n^{ij}/e_n < \theta < p_n^{ij} - p_r^{ij} - (v + \gamma v q_n^{ij}) + \delta(v + \gamma v q_r^{ij})/e_n - e_r$, consumers buy new products; when $\theta < p_n^{ij} - v - \gamma v q_n^{ij}/e_n$, consumers do not buy any products.

B. The Proof of the Optimal Results

B.1 Direct Recycling Strategy of the Third-Party Manufacturer and the Product Has Network Externalities. In this case, the demand function and profit function of new products and remanufactured products can be obtained as follows:

$$q_n^{IT} = \frac{\gamma\delta v^2 + \delta v e_n + \gamma\delta v e_n - v e_r - \gamma\delta v p_n^{IT} + e_r p_n^{IT} - e_n p_r^{IT}}{-\gamma^2\delta v^2 + \gamma\delta v e_n + e_n^2 + \gamma v e_r - e_n e_r}, \tag{A.1}$$

$$q_r^{IT} = \frac{\gamma\delta v^2 + v e_n - \delta v e_n + e_n^2 + \gamma v e_r - e_n e_r - e_n p_n^{IT} - \gamma v p_r^{IT} + e_n p_r^{IT}}{-\gamma^2\delta v^2 + \gamma\delta v e_n + e_n^2 + \gamma v e_r - e_n e_r}, \tag{A.2}$$

$$\begin{aligned} \pi_n^{IT} &= \left(p_n^{IT} - \omega_n^{IT}\right)q_n^{IT} + \left(p_r^{IT} - \omega_r^{IT}\right)q_r^{IT}, \\ \pi_r^{IT} &= \left(\omega_n^{IT} - c_n\right)q_n^{IT} + \left(\omega_r^{IT} - c_r - A\right)q_r^{IT}. \end{aligned} \tag{A.3}$$

We substitute equations (A.1) and (A.2) into the profit function; using backward induction and solving the above model, we get:

$$\begin{aligned}
\omega_{\mathrm{n}}^{IT} &= \frac{1}{2}\left(v + c_n + e_n\right), \\
\omega_r^{IT} &= \frac{1}{2}\left(A + \delta v + c_r + e_r\right), \\
p_n^{IT} &= \frac{1}{4}\left(3v + c_n + 3e_n\right), \\
p_r^{IT} &= \frac{1}{4}\left(A + 3\delta v + c_r + 3e_r\right), \\
q_n^{IT} &= -\frac{\gamma\delta v^2 - \gamma\delta v c_n - A e_n + \delta v e_n + \gamma\delta v e_n - c_r e_n - v e_r + c_n e_r}{4\left(\gamma^2\delta v^2 - \gamma\delta v e_n - e_n^2 - \gamma v e_r + e_n e_r\right)}, \\
q_r^{IT} &= -\frac{-A\gamma v + \gamma\delta v^2 - \gamma v c_r + A e_n + v e_n - \delta v e_n - c_n e_n + c_r e_n + e_n^2 + \gamma v e_r - e_n e_r}{4\left(\gamma^2\delta v^2 - \gamma\delta v e_n - e_n^2 - \gamma v e_r + e_n e_r\right)}, \\
\pi_n^{IT} &= \frac{\left(\begin{array}{c} A^2\gamma v - 2A\gamma\delta v^2 + \gamma\delta v^3 + \gamma\delta^2 v^3 - 2\gamma\delta v^2 c_n + \gamma\delta v c_n^2 + 2A\gamma v c_r \\ -2\gamma\delta v^2 c_r + \gamma v c_r^2 - A^2 e_n - 2Ave_n + 2A\delta v e_n + 2\delta v^2 e_n + 2\gamma\delta v^2 e_n \\ -\delta^2 v^2 e_n + 2Ac_n e_n - 2\delta v c_n e_n - 2\gamma\delta v c_n e_n - 2Ac_r e_n - 2v c_r e_n + 2\delta v c_r e_n \\ +2c_n c_r e_n - c_r^2 e_n - 2Ae_n^2 + 2\delta v e_n^2 + \gamma\delta v e_n^2 - 2c_r e_n^2 - 2A\gamma v e_r - v^2 e_r + \\ 2\gamma\delta v^2 e_r + 2v c_n e_r - c_n^2 e_r - 2\gamma v c_r e_r + 2Ae_n e_r - 2\delta v e_n e_r \\ +2c_r e_n e_r + e_n^2 e_r + \gamma v e_r^2 - e_n e_r^2 \end{array}\right)}{\left(-16\left(\gamma^2\delta v^2 - \gamma\delta v e_n - e_n^2 - \gamma v e_r + e_n e_r\right)\right)},
\end{aligned}$$

$$\pi_r^{IT} = \frac{\begin{pmatrix} A^2\gamma\nu - 2A\gamma\delta\nu^2 + \gamma\delta\nu^3 + \gamma\delta^2\nu^3 - 2\gamma\delta\nu^2 c_n + \gamma\delta\nu c_n^2 \\ +2A\gamma\nu c_r - 2\gamma\delta\nu^2 c_r + \gamma\nu c_r^2 - A^2 e_n - 2A\nu e_n + 2A\delta\nu e_n + 2\delta\nu^2 e_n \\ +2\gamma\delta\nu^2 e_n - \delta^2\nu^2 e_n + 2Ac_n e_n - 2\delta\nu c_n e_n - 2\gamma\delta\nu c_n e_n - 2Ac_r e_n - \\ 2\nu c_r e_n + 2\delta\nu c_r e_n + 2c_n c_r e_n - c_r^2 e_n - 2Ae_n^2 + 2\delta\nu e_n^2 + \gamma\delta\nu e_n^2 - \\ 2c_r e_n^2 - 2A\gamma\nu e_r - \nu^2 e_r + 2\gamma\delta\nu^2 e_r + 2\nu c_n e_r - c_n^2 e_r - 2\gamma\nu c_r e_r + 2Ae_n e_r - \\ 2\delta\nu e_n e_r + 2c_r e_n e_r + e_n^2 e_r + \gamma\nu e_r^2 - e_n e_r^2 \end{pmatrix}}{\left(8\left(\gamma^2\delta\nu^2 - \gamma\delta\nu e_n - e_n^2 - \gamma\nu e_r + e_n e_r\right)\right)}. \tag{A.4}$$

B.2 Direct Recycling Strategy of the Third-Party Manufacturer and the Product Does Not Have Network Externalities. In this case, the demand function and profit function of new products and remanufactured products can be obtained as follows:

$$q_n^{NT} = \frac{\beta\nu e_n - \nu e_r + e_r p_n^{NT} - e_n p_r^{NT}}{e_n(e_n - e_r)}, \tag{A.5}$$

$$q_r^{NT} = \frac{\nu - \beta\nu + e_n - e_r - p_n^{NT} + p_r^{NT}}{e_n - e_r}, \tag{A.6}$$

$$\begin{aligned} \pi_n^{NT} &= \left(p_n^{NT} - \omega_{\mathrm{n}}^{NT}\right)q_n^{NT} + \left(p_r^{NT} - \omega_r^{NT}\right)q_r^{NT}, \\ \pi_r^{NT} &= \left(\omega_{\mathrm{n}}^{NT} - c_n\right)q_n^{NT} + \left(\omega_r^{NT} - c_r - A\right)q_r^{NT}. \end{aligned} \tag{A.7}$$

In the same way, we substitute equations (A.5) and (A.6) into the profit function; using backward induction and solving the above model, we get:

$$\begin{aligned} \omega_n^{NT} &= \frac{1}{2}(\nu + c_n + e_n), \\ \omega_r^{NT} &= \frac{1}{2}(A + \delta\nu + c_r + e_r), \\ p_n^{NT} &= \frac{1}{4}(3\nu + c_n + 3e_n), \\ p_r^{NT} &= \frac{1}{4}(A + 3\delta\nu + c_r + 3e_r), \\ q_n^{NT} &= \frac{-Ae_n + \delta\nu e_n - c_r e_n - \nu e_r + c_n e_r}{4e_n(e_n - e_r)}, \\ q_r^{NT} &= \frac{A + \nu - \delta\nu - c_n + c_r + e_n - e_r}{4(e_n - e_r)}, \\ \pi_n^{NT} &= \frac{\begin{pmatrix} -A^2 e_n - 2A\nu e_n + 2A\delta\nu e_n + 2\delta\nu^2 e_n - \delta^2\nu^2 e_n + 2Ac_n e_n \\ -2\delta\nu c_n e_n - 2Ac_r e_n 2\nu c_r e_n + 2\delta\nu c_r e_n + 2c_n c_r e_n - c_r^2 e_n - 2Ae_n^2 \\ +2\delta\nu e_n^2 - 2c_r e_n^2 - \nu^2 e_r + 2\nu c_n e_r - c_n^2 e_r + 2Ae_n e_r - 2\delta\nu e_n e_r \\ +2c_r e_n e_r + e_n^2 e_r - e_n e_r^2 \end{pmatrix}}{16e_n(e_n - e_r)}, \end{aligned}$$

$$\pi_r^{NT}=\frac{\begin{pmatrix}-A^2e_n-2Ave_n+2A\delta ve_n+2\delta v^2e_n-\delta^2v^2e_n+2Ac_ne_n\\ -2\delta vc_ne_n-2Ac_re_n-2vc_re_n+2\delta vc_re_n+2c_nc_re_n-c_r^2e_n-\\ 2Ae_n^2+2\delta ve_n^2-2c_re_n^2-v^2e_r+2vc_ne_r-c_n^2e_r+2Ae_ne_r-\\ 2\delta ve_ne_r+2c_re_ne_r+e_n^2e_r-e_ne_r^2\end{pmatrix}}{8e_n(e_n-e_r)}. \tag{A.8}$$

B.3 Indirect Recycling Strategy of the Third-Party Manufacturer and the Product Has Network Externalities. In this case, the demand function and profit function of new products and remanufactured products can be obtained as follows:

$$q_n^{IO}=\frac{\gamma\delta v^2+\delta ve_n+\gamma\delta ve_n-ve_r-\gamma\delta vp_n^{IO}+e_rp_n^{IO}-e_np_r^{IO}}{-\gamma^2\delta v^2+\gamma\delta ve_n+e_n^2+\gamma ve_r-e_ne_r}, \tag{A.9}$$

$$q_r^{IO}=\frac{\gamma\delta v^2+ve_n-\delta ve_n+e_n^2+\gamma ve_r-e_ne_r-e_np_n^{IO}-\gamma vp_r^{IO}+e_np_r^{IO}}{-\gamma^2\delta v^2+\gamma\delta ve_n+e_n^2+\gamma ve_r-e_ne_r}, \tag{A.10}$$

$$\begin{aligned}\pi_n^{IO}&=\left(p_n^{IO}-\omega_n^{IO}\right)q_n^{IO}+\left(p_r^{IO}-\omega_r^{IO}-A+b\right)q_r^{IO},\\ \pi_r^{IO}&=\left(\omega_n^{IO}-c_n\right)q_n^{IO}+\left(\omega_r^{IO}-c_r-b\right)q_r^{IO}.\end{aligned} \tag{A.11}$$

In the same way, we substitute equations (A.9) and (A.10) into the profit function; using backward induction and solving the above model, we get:

$$\omega_n^{IO}=\frac{1}{2}(v+c_n+e_n),$$

$$\omega_r^{IO}=\frac{1}{2}(-A+2b+\delta v+c_r+e_r),$$

$$p_n^{IO}=\frac{1}{4}(3v+c_n+3e_n),$$

$$p_r^{IO}=\frac{1}{4}(A+3\delta v+c_r+3e_r),$$

$$q_n^{IO}=-\frac{\gamma\delta v^2-\gamma\delta vc_n-Ae_n+\delta ve_n+\gamma\delta ve_n-c_re_n-ve_r+c_ne_r}{4\left(\gamma^2\delta v^2-\gamma\delta ve_n-e_n^2-\gamma ve_r+e_ne_r\right)},$$

$$q_r^{IO}=-\frac{\begin{matrix}-A\gamma v+\gamma\delta v^2-\gamma vc_r+Ae_n+ve_n-\delta ve_n\\ -c_ne_n+c_re_n+e_n^2+\gamma ve_r-e_ne_r\end{matrix}}{4\left(k^2\delta v^2-\gamma\delta ve_n-e_n^2-\gamma ve_r+e_ne_r\right)},$$

$$\pi_n^{IO} = \frac{\left(\begin{array}{c} A^2\gamma\nu - 2A\gamma\delta\nu^2 + \gamma\delta\nu^3 + \gamma\delta^2\nu^3 - 2\gamma\delta\nu^2 c_n + \gamma\delta\nu c_n^2 + 2A\gamma\nu c_r \\ -2\gamma\delta\nu^2 c_r + \gamma\nu c_r^2 - A^2 e_n - 2A\nu e_n + 2A\delta\nu e_n + 2\delta\nu^2 e_n + 2\gamma\delta\nu^2 e_n \\ -\delta^2\nu^2 e_n + 2Ac_n e_n - 2\delta\nu c_n e_n - 2\gamma\delta\nu c_n e_n - 2Ac_r e_n - 2\nu c_r e_n + 2\delta\nu c_r e_n \\ +2c_n c_r e_n - c_r^2 e_n - 2Ae_n^2 + 2\delta\nu e_n^2 + \gamma\delta\nu e_n^2 - 2c_r e_n^2 - 2A\gamma\nu e_r - \nu^2 e_r + 2\gamma\delta\nu^2 e_r \\ +2\nu c_n e_r - c_n^2 e_r - 2\gamma\nu c_r e_r + 2Ae_n e_r - 2\delta\nu e_n e_r + 2c_r e_n e_r + e_n^2 e_r + \gamma\nu e_r^2 - e_n e_r^2 \end{array}\right)}{\left(-16\left(\gamma^2\delta\nu^2 - \gamma\delta\nu e_n - e_n^2 - \gamma\nu e_r + e_n e_r\right)\right)},$$

$$\pi_r^{IO} = \frac{\left(\begin{array}{c} A^2\gamma\nu - 2A\gamma\delta\nu^2 + \gamma\delta\nu^3 + \gamma\delta^2\nu^3 - 2\gamma\delta\nu^2 c_n + \gamma\delta\nu c_n^2 + 2A\gamma\nu c_r \\ -2\gamma\delta\nu^2 c_r + \gamma\nu c_r^2 - A^2 e_n - 2A\nu e_n + 2A\delta\nu e_n + 2\delta\nu^2 e_n + 2\gamma\delta\nu^2 e_n - \\ \delta^2\nu^2 e_n + 2Ac_n e_n - 2\delta\nu c_n e_n - 2\gamma\delta\nu c_n e_n - 2Ac_r e_n - 2\nu c_r e_n + 2\delta\nu c_r e_n \\ +2c_n c_r e_n - c_r^2 e_n - 2Ae_n^2 + 2\delta\nu e_n^2 + \gamma\delta\nu e_n^2 - 2c_r e_n^2 - 2A\gamma\nu e_r - \nu^2 e_r + 2\gamma\delta\nu^2 e_r \\ +2\nu c_n e_r - c_n^2 e_r - 2\gamma\nu c_r e_r + 2Ae_n e_r - 2\delta\nu e_n e_r + 2c_r e_n e_r + e_n^2 e_r + \gamma\nu e_r^2 - e_n e_r^2 \end{array}\right)}{\left(-8\left(\gamma^2\delta\nu^2 - \gamma\delta\nu e_n - e_n^2 - \gamma\nu e_r + e_n e_r\right)\right)}. \tag{A.12}$$

B.4 Indirect Recycling Strategy of the Third-Party Manufacturer and the Product Does Not Have Network Externalities. In this case, the demand function and profit function of new products and remanufactured products can be obtained as follows:

$$q_n^{NO} = \frac{\delta\nu e_n - \nu e_r + e_r p_n^{NO} - e_n p_r^{NO}}{e_n(e_n - e_r)}, \tag{A.13}$$

$$q_r^{NO} = \frac{\nu - \delta\nu + e_n - e_r - p_n^{NO} + p_r^{NO}}{e_n - e_r}, \tag{A.14}$$

$$\begin{aligned} \pi_n^{NO} &= \left(p_n^{NO} - \omega_n^{NO}\right)q_n^{NO} + \left(p_r^{NO} - \omega_r^{NO} - A + b\right)q_r^{NO}, \\ \pi_r^{NO} &= \left(\omega_n^{NO} - c_n\right)q_n^{NO} + \left(\omega_r^{NO} - c_r - b\right)q_r^{NO}. \end{aligned} \tag{A.15}$$

In the same way, we substitute equations (A.13) and (A.14) into the profit function; using backward induction and solving the above model, we get:

$$\begin{aligned} \omega_n^{NO} &= \frac{1}{2}(\nu + c_n + e_n), \\ \omega_r^{NO} &= \frac{1}{2}(-A + 2b + \delta\nu + c_r + e_r), \\ p_n^{NO} &= \frac{1}{4}(3\nu + c_n + 3e_n), \\ p_r^{NO} &= \frac{1}{4}(A + 3\delta\nu + c_r + 3e_r), \\ q_n^{NO} &= \frac{-Ae_n + \delta\nu e_n - c_r e_n - \nu e_r + c_n e_r}{4e_n(e_n - e_r)}, \\ q_r^{NO} &= \frac{A + \nu - \delta\nu - c_n + c_r + e_n - e_r}{4(e_n - e_r)}, \end{aligned}$$

$$\pi_n^{NO} = \frac{\begin{pmatrix} -A^2e_n - 2Ave_n + 2A\delta ve_n + 2\delta v^2e_n - \delta^2v^2e_n + 2Ac_ne_n - \\ 2\delta vc_ne_n - 2Ac_re_n - 2vc_re_n + 2\delta vc_re_n + 2c_nc_re_n - c_r^2e_n - \\ 2Ae_n^2 + 2\delta ve_n^2 - 2c_re_n^2 - v^2e_r + 2vc_ne_r - c_n^2e_r + 2Ae_ne_r - \\ 2\delta ve_ne_r + 2c_re_ne_r + e_n^2e_r - e_ne_r^2 \end{pmatrix}}{16e_n(e_n - e_r)},$$
$$\pi_r^{NO} = \frac{\begin{pmatrix} -A^2e_n - 2Ave_n + 2A\delta ve_n + 2\delta v^2e_n - \delta^2v^2e_n + 2Ac_ne_n \\ -2\delta vc_ne_n - 2Ac_re_n - 2vc_re_n + 2\delta vc_re_n + 2c_nc_re_n - c_r^2e_n - \\ 2Ae_n^2 + 2\delta ve_n^2 - 2c_re_n^2 - v^2e_r + 2vc_ne_r - c_n^2e_r + 2Ae_ne_r - \\ 2\delta ve_ne_r + 2c_re_ne_r + e_n^2e_r - e_ne_r^2 \end{pmatrix}}{8e_n(e_n - e_r)}. \tag{A.16}$$

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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