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Testing Stability in Functional Event Observations with an Application to IPO Performance

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Abstract

Many sequentially observed functional data objects are available only at the times of certain events. For example, the trajectory of stock prices of companies after their initial public offering (IPO) can be observed when the offering occurs, and the resulting data may be affected by changing circumstances. It is of interest to investigate whether the mean behaviour of such functions is stable over time, and if not, to estimate the times at which apparent changes occur. Since the frequency of events may fluctuates over time, we propose a change point analysis that has two steps. In the first step, we segment the series into segments in which the frequency of events is approximately homogeneous using a new binary segmentation procedure for event frequencies. After adjusting the observed curves in each segment based on the frequency of events, we proceed in the second step by developing a method to test for and estimate change points in the mean of the observed functional data objects. We establish the consistency and asymptotic distribution of the change point detector and estimator in both steps, and study their performance using Monte Carlo simulations. An application to IPO performance data illustrates the proposed methods.

Keywords: Functional data analysis, Change point analysis, IPO

1 Introduction

This work is motivated by the problem of performing event studies and change point analysis on initial public offering (IPO) data. The specific data that we consider was downloaded from the Wind Economic Database, and contains the daily stock prices of 1,297 companies whose IPO occurred on the Shanghai Stock Exchange and Shenzhen Stock Exchange during the period from December 1, 2015 to September 30, 2020, which consists of $N = 1,181$ trading days[1]. Denote by n_t the number of IPOs on day t, $P_{t,m}(0)$ the IPO issuance price of stock m on day t, and $P_{t,m}(u)$ the price of stock m traded at time u after its IPO. We consider the 60-day *cumulative abnormal return curves* (CARCs) defined by

$$\text{CARC}_{t,m}(u) = \frac{P_{t,m}(u) - P_{t,m}(0)}{P_{t,m}(0)} - \frac{P_t^{(mkt)}(u) - P_t^{(mkt)}(0)}{P_t^{(mkt)}(0)}, \quad 0 \le u \le 60, 1 \le m \le n_t, 1 \le t \le 1181.$$

[1]We consider the IPOs issued between December 1, 2015 to September 30, 2020. In order to fully observe the 60-day *cumulative abnormal return curves*, we include the price data up to December 31, 2020.

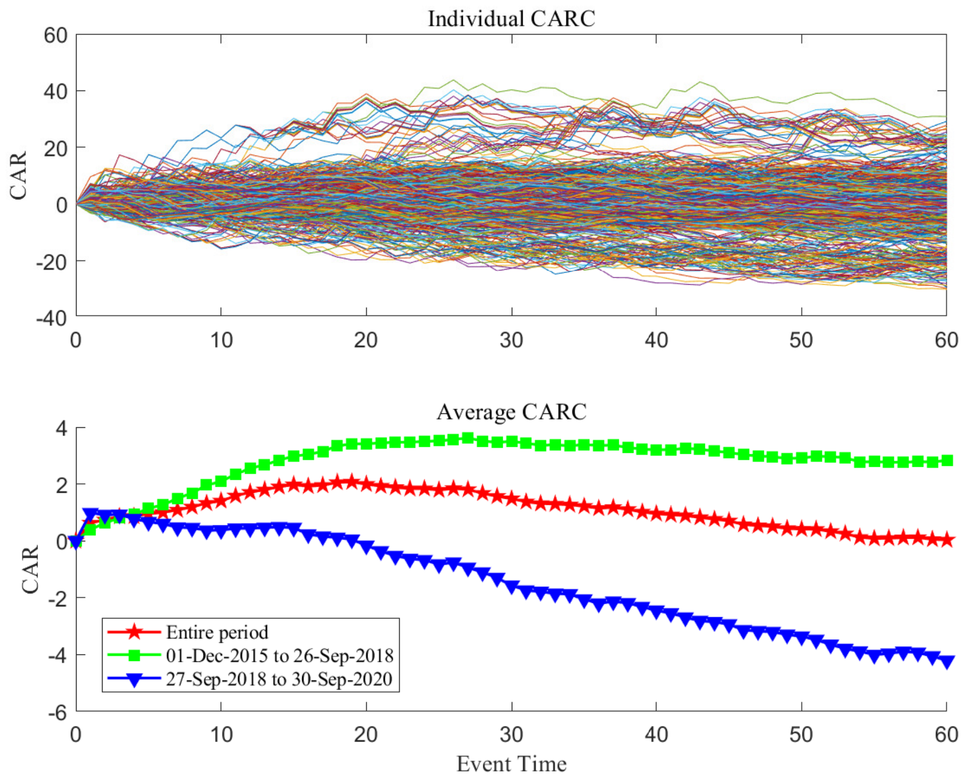

Figure 1: Top panel: Cumulative abnormal return curves of 1,297 IPOs between December 2015 and September 2020 in China. Bottom panel: Estimates of the mean CARC curves from the entire sample, as well as based on a segmentation before and after September 26, 2018.

The second term depending on $P_t^{(mkt)}(u)$ is determined based on an overall market index, and is used to estimate the excess return of stock m over the market; the details of this are provided in Section 6. The CARCs can be used to compare and measure the performance of IPOs in their first 60 days, and, after interpolating the daily resolution data, are naturally viewed as functional data objects, as can be seen in Figure 1.

Since the CARC curves viewed over time can be used to understand the effects of changing governmental policies and economic conditions on IPOs, performing change point analysis on such curves is of interest, and is the main objective of this article. A challenge encountered though in analyzing this data is that IPO events occur irregularly over time, and the CARCs

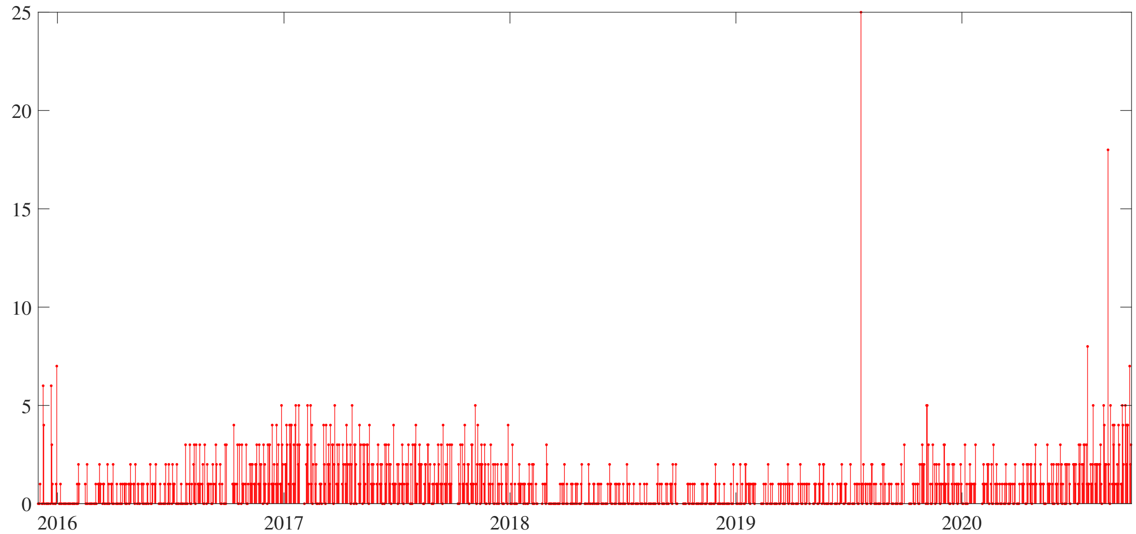

Figure 2: Number and frequency of IPOs on the Shanghai Stock Exchange and Shenzhen Stock Exchanges one each day during the period from December 1, 2015 to September 30, 2020.

are only observable when the IPOs occur. Although there are typically one or two IPOs on the stock exchanges considered each day, there could be many more, or none. The number and frequency of IPOs in the data set is illustrated in Figure 2. Additionally, clustering of IPO events and the overlapping time periods on which the CARCs are computed suggest that the data will exhibit serial dependence, which must be taken into account in any analysis of this data.

To generally frame this problem, consider a specific type of event, such as an IPO, that can happen for different firms at different times over a sample period of N days. To facilitate computation, the event time u is rescaled to the unit interval $[0, 1]$. Let n_t denote the number of events on day t. If no event occurs on day t, so that n_t is zero, we do not have any observation on that day, but if at-least-one event occurs on day t, we assume that we observe functional data objects, which we term as "functional event observations". The main example of interest are CARCs. We suppose that functional event observations are square integrable curves defined on the unit interval $[0, 1]$ of the form $X_{t,1}(u), X_{t,2}(u), \ldots, X_{t,n_t}(u)$,

$u \in [0,1]$, $1 \le t \le N$, that satisfy the basic model

$$X_{t,m}(u) = \mu_t(u) + y_t(u) + \epsilon_{t,m}(u), \;\; 0 \le u \le 1, \; 1 \le m \le n_t, \; 1 \le t \le N, \tag{1.1}$$

where $\mu_t(u)$ is the unknown mean curve[2] of $X_{t,m}(u)$, $y_t(u)$ represents a random effect[3] on the date t, and $\epsilon_{t,m}(u)$ is associated with the idiosyncratic effect of a specific firm. Model (1.1) appears to model the CARCs well due to the scaling and recentering by an overall market index applied to compute each CARC trajectory. We assume that

Assumption 1.1. *(i) $Ey_t(u) = 0$ and $E\epsilon_{t,m}(u) = 0$, $u \in [0,1]$, $1 \le m \le K$, (ii) $E\|y_t\|^\kappa < \infty$ and $E\|\epsilon_{t,\ell}\|^\kappa < \infty$ with some $\kappa > 4$, (iii) $0 \le n_t \le K$ with some $K > 0$ for all t.*

Assumption 1.1(i) is needed to identify $\mu_t(u)$ as the mean curve of $X_{t,m}(u)$, whereas Assumption 1.1(ii) defines a moment condition of $y_t(u)$ and $\epsilon_{t,m}(u)$, and Assumption 1.1(iii) posits that the number of events on a given day is bounded.

Under the null hypothesis the mean of the observed curves is stable:

$$H_0: \;\; \mu_1(u) = \cdots = \mu_N(u), \tag{1.2}$$

whereas under the alternative hypothesis there is a change in the global mean curve at an unknown date k^*:

$$H_A: \;\; \mu_1(u) = \cdots \mu_{k^*}(u) \ne \mu_{k^*+1}(u) = \cdots = \mu_N(u). \tag{1.3}$$

[2]Under H_0, the unknown mean curve $\mu_t(u)$ on different dates becomes a common one $\mu(u)$ for all t.

[3]It is also possible to suppress $y_t(u)$ from (1.1) in place of other conditions on the serial dependence in the data. In the literature, it is usually assumed that $\{y_t(u), 1 \le t \le N\}$ and $\{\varepsilon_{t,\ell}, 1 \le \ell \le n_t, 1 \le t \le N\}$ are independent or at least uncorrelated. Due to Assumption 3.1, we do not need conditions between $y_t(u)$ and $\epsilon_{t,m}(u)$. Thus, we choose to include $y_t(u)$ to emphasize that time dependence is allowed between functional event observations at different dates.

The key challenge in conducting a hypothesis test of H_0 versus H_A is a fluctuating number of events n_t. To demonstrate this challenge, let us consider the daily averages defined by

$$Y_t(u) = \begin{cases} 0, & \text{if } n_t = 0, \\ \dfrac{1}{n_t} \displaystyle\sum_{m=1}^{n_t} X_{t,m}(u), & \text{if } n_t > 0. \end{cases}$$

In our empirical application, $Y_t(u)$ is obtained by averaging over companies for which the IPO occurred on day t if there is at-least-one IPO; $Y_t(u)$ is a zero function if there is no IPO on day t. Under the assumption that n_t and $\{X_{t,m}(u),\ u \in [0,1],\ 1 \leq m \leq K\}$ are independent,

$$EY_t(u) = (1 - P(\{n_t = 0\}))\, \mu(u), \tag{1.4}$$

where $\mu(u)$ denotes the common mean under H_0. In particular, the expected value of the daily averages of event observations might change even if the mean curves in model (1.1) are stable, but the probability of the event $\{n_t = 0\}$ changes.

As a result of this, our proposed stability test has two steps: In the first step, we segment the event frequencies during the sample period into segments for which $P(\{n_t = 0\})$ is approximately stable using a new binary segmentation procedure for event frequencies. After doing so, the daily averages of $Y_t(u)$ are adjusted based on the estimated event frequencies, and a change point analysis is carried out for the adjusted daily average curves. In Sections 2 and 3, we develop test statistics that are used in each step, and establish their asymptotic properties in order to conduct change point analyses of the event frequency and mean curves, respectively. Details and consistency results on estimating the covariance kernels that are needed to implement these tests are given in Section 4. In Section 5 we present the results of Monte Carlo simulations that aimed to investigate the finite sample properties of our methods, which shows that for the examples considered this two-step testing and estimation procedure works well. Section 6 illustrates the application of these methods to the IPO data set introduced above, which supports the existence of a single change in the mean CARC curves in September of 2018. An online supplement contains the detailed proofs of all technical results, as well as additional simulation results, and practical guidance on the implementation of our methods.

This work is most closely related to the literature on event studies and change point analysis with functional data. Event studies, which refers to empirical analyses examining the impact of significant events on the value of a security, are widely considered in the fields of economics, finance, marketing, and political science. Important references, which serve as entry points to the area, include Dolley (1933), Brown and Weinstein (1985), MacKinlay (1997), Campbell et al. (1997), Kothari and Warner (2007), and Linton (2019). Textbook length treatments of functional data analysis can be found in Eubank and Hsing (2008), Ramsay and Silverman (2005), and Horváth and Kokoszka (2012). The literature on change point analysis of functional data has grown considerably in the last 10 years, with notable references including Bucchia and Wendler (2017), Chiou et al. (2019), Aston and Kirch (2012b), Aston and Kirch (2012a), Berkes et al. (2009), Rice and Zhang (2022), Sharipov et al. (2016), and Aue et al. (2018), who consider change point testing and multiple change point estimation for the mean of functional data, and Sharipov and Wendler (2019), Stoehr et al. (2019), and Horváth, Rice, et al. (2022), who consider change point analysis of covariance operators of functional data. We note that all previous literature on the change point analysis of functional data concerns functional observations observed at a regular frequency. In this paper, we contribute to the literature by developing a new method to test for the mean stability of functional data observed irregularly and subject to changes on observation frequency.

Briefly we highlight some notations. There are N total trading dates, and on each date there are n_t events (n_t can be zero or a positive number). In the sample period, there could be R change points in frequency of events, resulting in $R+1$ segments. For the index notation, we generally use t ($1 \leq t \leq N$) to index which date out of N total dates, m ($1 \leq m \leq n_t$) to index which event out of n_t events on a date, j ($1 \leq j \leq R$) to index which change point in frequency, and i, ℓ ($1 \leq i, \ell \leq R+1$) to index which homogenous segment.

2 Test for frequency change

In this section, we develop methods to segment the frequency at which at-least-one event occurs into approximately homogenous segments. Our method is motivated by the change point testing procedure in Horváth and Serbinowska (1995). Since we are interested in conducting change point analysis on the probability that at-least-one event occurs each day, we introduce the Bernoulli variables

$$\xi_t = \begin{cases} 1, & \text{if } \; n_t > 0, \\ \\ 0, & \text{if } \; n_t = 0. \end{cases}$$

We wish to test

$$H_{0,P}: \quad P\{\xi_1 = 1\} = P\{\xi_2 = 1\} = \ldots = P\{\xi_N = 1\},$$

against the alternative

$$H_{A,P}: \text{ there are } 1 < t_1 < t_2 < \ldots t_R < N \text{ such that } P\{\xi_1 = 1\} = P\{\xi_2 = 1\} = \ldots = P\{\xi_{t_1} = 1\}$$
$$\neq P\{\xi_{t_1+1} = 1\} = P\{\xi_{t_1+2} = 1\} = \ldots = P\{\xi_{t_2} = 1\} \neq P\{\xi_{t_2+1} = 1\} = \ldots P\{\xi_{t_R} = 1\}$$
$$\neq P\{\xi_{t_R+1} = 1\} = P\{\xi_{t_R+2} = 1\} = \ldots = P\{\xi_N = 1\},$$

i.e. under the alternative the frequency of having at least one event in a day changes at the unknown times $1 < t_1 < \ldots < t_R < N$. Horváth and Serbinowska (1995) derived several test statistics to test $H_{0,P}$ versus $H_{A,P}$, assuming that the Bernoulli variables $\xi_1, \xi_2, \ldots, \xi_N$ are independent. Let

$$\Lambda_k = \frac{\hat{p}_N^{N\hat{p}_N}(1-\hat{p}_N)^{N(1-\hat{p}_N)}}{\hat{p}_k^{k\hat{p}_k}(1-\hat{p}_k)^{k(1-\hat{p}_k)}\bar{p}_{N-k}^{(N-k)\bar{p}_{N-k}}(1-\bar{p}_{N-k})^{(N-k)(1-\bar{p}_{N-k})}}, \tag{2.1}$$

with

$$\hat{p}_k = \frac{1}{k}\sum_{t=1}^{k}\xi_t, \qquad \bar{p}_{N-k} = \frac{1}{N-k}\sum_{t=k+1}^{N}\xi_t,$$

and

$$\Delta_k = \frac{N(k\hat{p}_k - k\hat{p}_N)^2}{\hat{p}_N(1-\hat{p}_N)k(N-k)}. \tag{2.2}$$

One may see that if we compare the probability of successes in the subsamples $\{\xi_1, \xi_2, \ldots, \xi_k\}$ and $\{\xi_{k+1}, \xi_{k+2}, \ldots, \xi_N\}$, then Λ_k in (2.1) is a likelihood ratio, and (2.2) is the classic χ^2 test statistic. Since it is unknown where to divide the sample, it is suggested to consider the maximally selected statistics

$$\bar{\Lambda}_N = \max_{1\le k<N}(-2\log\Lambda_k) \quad \text{and} \quad \bar{\Delta}_N = \max_{1\le k<N}\Delta_k.$$

Simulation studies in Horváth and Serbinowska (1995) showed that $\bar{\Lambda}_N$ and $\bar{\Delta}_N$ converge to their limits rather slowly, so they also studied their weighted versions

$$\tilde{\Lambda}_N = \max_{1\le k<N} \frac{k(N-k)}{N^2}(-2\log \Lambda_k) \quad \text{and} \quad \tilde{\Delta}_N = \max_{1\le k<N} \frac{k(N-k)}{N^2}\Delta_k.$$

The variable $\bar{\Delta}_N^{1/2}$ is the maximum of a self normalized CUSUM process. Horváth and Serbinowska (1995) derived the limit distributions of $\bar{\Lambda}_N, \bar{\Delta}_N, \tilde{\Lambda}_N$ and $\tilde{\Delta}_N$ in case of independent and identically distributed Bernoulli random variables. Since independence is too strong of an assumption to apply to the data of interest in the present study, we need to modify their results to allow for serial dependence. We assume that under the null hypothesis the sequence $\{\xi_t, -\infty < t < \infty\}$ is a weakly dependent Bernoulli shift:

Assumption 2.1. $\xi_t = g(\eta_t, \eta_{t-1}, \ldots)$ *with some non-random function* g *defined on* $\mathcal{S}^\infty$, $\{\eta_t, -\infty < t < \infty\}$ *are independent, identically distributed random variables with values in a measurable space* $\mathcal{S}$,

$$(E|\xi_t - \xi_{t,s}|^{\kappa_1})^{1/\kappa_1} \le cs^{-\kappa_2} \ \ \textit{with some} \ \ c > 0, \kappa_1 > 2 \ \ \textit{and} \ \ \kappa_2 > 1,$$

where $\xi_{t,s} = g(\eta_t, \eta_{t-1}, \ldots, \eta_{t-s+1}, \boldsymbol{\eta}^*_{t,s})$, *and* $\boldsymbol{\eta}^*_{t,s} = (\eta^*_{t,s,t-s},\ \eta^*_{t,s,t-s-1}, \ldots)$, $\{\eta^*_{t,s,s'}, -\infty < t, s, s' < \infty\}$ *are independent, identically distributed as* η_0, *also independent of* $\{\eta_t, -\infty < t < \infty\}$.

Assumption 2.1 holds for a large class of time series, including AR, MA, ARMA and all linear processes as well as several nonlinear time series sequences. The idea of decomposability of Bernoulli sequences goes back at least to Ibragimov (1962). One of the advantages of such a condition in this context, in which we are concerned with integer valued sequences, over other potential weak dependence conditions, such as mixing, is that no continuity properties are required of the distribution of the variables.

Since the observations are not independent, $\hat{p}_N(1-\hat{p}_N)$ does not estimate the long run variance of the sum of the indicators. Assumption 2.1 implies that the infinite sum

$$\lim_{N\to\infty} \operatorname{var}\left(N^{-1/2}\sum_{t=1}^{N}\xi_t\right) = \sigma^2, \tag{2.3}$$

defining the long run variance, which is absolutely convergent. We require

Assumption 2.2. $\sigma^2 > 0$.

There are several methods to estimate σ^2. The most popular one is the kernel-lag window long run variance or spectral density estimators, $\hat{\sigma}_N^2$. Its consistency,

$$\hat{\sigma}_N \overset{P}{\to} \sigma, \tag{2.4}$$

was studied by several authors. Hörmann and Kokoszka (2010) and Liu and Wu (2010) proved (2.4) under Assumption 2.1 and minor requirements on the window (smoothing parameter) and the kernel. To obtain limit results for $\bar{\Lambda}_N$ and $\bar{\Delta}_N$, we need an upper bound for the rate of convergence in (2.4). Namely,

$$|\hat{\sigma}_N - \sigma| = o_P((\log\log N)^{-1/2}), \tag{2.5}$$

which follows from Liu and Wu (2010) and Xiao and Wu (2012). For further results we refer to Chapter 16.3 of Horváth and Kokoszka (2012). Let

$$\hat{\tau}_N^2 = \frac{\hat{p}_N(1-\hat{p}_N)}{\hat{\sigma}_N^2}. \tag{2.6}$$

Theorem 2.1. *If $H_{0,P}$, Assumptions 2.1, 2.2 and* (2.4) *are satisfied, then*

$$\lim_{N\to\infty} P\{\hat{\tau}_N^2 \tilde{\Lambda}_N \le x\} = P\left\{\sup_{0\le u\le 1} B^2(u) \le x\right\} \tag{2.7}$$

and

$$\lim_{N\to\infty} P\{\hat{\tau}_N^2 \tilde{\Delta}_N \le x\} = P\left\{\sup_{0\le u\le 1} B^2(u) \le x\right\}, \tag{2.8}$$

where $\{B(u), 0 \le u \le 1\}$ denotes a Brownian bridge.

In addition, if (2.5) *also holds, then*

$$\lim_{N\to\infty} P\left\{a(\log N)\hat{\tau}_N \bar{\Lambda}_N^{1/2} \le x + b(\log N)\right\} = \exp(-2e^{-x}) \tag{2.9}$$

and

$$\lim_{N\to\infty} P\left\{a(\log N)\hat{\tau}_N \bar{\Delta}_N^{1/2} \le x + b(\log N)\right\} = \exp(-2e^{-x}) \tag{2.10}$$

where $a(x) = (2\log x)^{1/2}$ and $b(x) = 2\log t + (1/2)\log\log x - (1/2)\log \pi$.

If $H_{A,P}$ holds, we are interested in the estimation of R, the number of change points and $t_1, t_2, \ldots, t_R$, the times of the changes. The method of Serbinowska (1996) can be adapted to our case, but, since we also wish to estimate the locations of the change points, we employ binary segmentation, as initially put forward in Vostrikova (1981). We assume that

Assumption 2.3. $t_j = \lfloor N\theta_j \rfloor, \quad 1 \le j \le R$ *with* $0 < \theta_1 < \theta_2 < \ldots < \theta_R < 1$.

Binary segmentation is generally applied as follows: first we test if the null hypothesis holds for the observations $\{\xi_1, \xi_2, \ldots, \xi_N\}$. If the null hypothesis is rejected, we find the point, $\hat{t}_1$, the argument where the change point test statistic attains its largest value. The observations are then segmented into two subsets, $\left\{\xi_1, \xi_2, \ldots, \xi_{\hat{t}_1}\right\}$ and $\left\{\xi_{\hat{t}_1+1}, \xi_{\hat{t}_1+2}, \ldots, \xi_N\right\}$ based on the point $\hat{t}_1$. Now the procedure is repeated on each subset. We continue this procedure until the null hypothesis cannot be rejected for any subsets resulting in the estimators $\hat{t}_1, \hat{t}_2, \ldots, \hat{t}_{\hat{R}}$. Let

$$\frac{\hat{t}_j}{N} = \hat{\theta}_{j,N} = \hat{\theta}_j, \quad 1 \le j \le \hat{R}.$$

If we use the binary segmentation method where each test is performed with significance level α, standard arguments give that for all $\delta > 0$

$$\liminf_{N\to\infty} P\{\hat{R} = R, |\hat{\theta}_j - \theta_j| \le \delta\} \ge 1 - (R+1)\alpha. \tag{2.11}$$

In order to get a consistent estimators of the number and locations of the change points we must then take a suitable sequence $\alpha = \alpha_N \to 0$, which equivalently defines an asymptotically increasing threshold for the change point statistics used in the binary segmentation procedure. In practice though the user must decide on a value for α_N, and we have found in practice that a fixed α is reasonable in this application, especially in that it tends to lead to estimating a larger number of change points.

We also note that Pan and Chen (2006) and Ciuperca (2011) use penalized likelihood and least squares to estimate the number of changes and the locations of changes. Their estimates $\tilde{R}$ and $\tilde{\theta}_j$, $1 \le j \le \tilde{R}$, satisfy that $\lim_{N\to\infty} P\left\{R = \tilde{R}, |\tilde{\theta}_i - \theta_i| \le \delta\right\} = 1$ for all $\delta > 0$.

We conclude this section by presenting how to practically use the proposed statistics of Section 2. In order to compare the change point detectors with their critical values, one can further calculate

$$\bar{\Lambda}_N^* = a(\log N)\hat{\tau}_N \bar{\Lambda}_N^{1/2} - b(\log N), \qquad \bar{\Delta}_N^* = a(\log N)\hat{\tau}_N \bar{\Delta}_N^{1/2} - b(\log N),$$

$$\tilde{\Lambda}_N^* = \hat{\tau}_N^2 \tilde{\Lambda}_N, \quad \text{and} \quad \tilde{\Delta}_N^* = \hat{\tau}_N^2 \tilde{\Delta}_N.$$

Based on the simulation in Appendix C of the online supplement, the critical values of $\bar{\Lambda}_N^*$ and $\bar{\Delta}_N^*$ are 2.944 at 10% level, 3.663 at 5% level, and 5.293 at 1% level, and the critical values of $\tilde{\Lambda}_N^*$ and $\tilde{\Delta}_N^*$ are 1.486 at 10% level, 1.829 at 5% level, and 2.632 at 1% level.

3 Test for mean curve change

In this section we proceed to develop change point tests and estimators for the mean curve of event observations, after adjusting for changes in the event frequency as described in Section 2. First, we relax Assumption 2.1 to allow for sequences that are only piecewise stationary over the observation period $[1, 2, \ldots, N]$, with the stationary subsegments coinciding with the subsegments on which the event frequency is homogeneous. Let $\|\cdot\|$ denote the L^2 norm of functions defined on $[0, 1]$; in case the input is a vector valued function, $\|\cdot\|$ is the standard L^2 norm on the product space.

Assumption 3.1. *We assume that* $\mathbf{z}_t = (n_t, y_t(u), \epsilon_{t,1}(u), \ldots, \epsilon_{t,K}(u)) = \mathbf{g}_i(\eta_t, \eta_{t-1}, \ldots)$, $t_{i-1} < t \le t_i, 1 \le i \le R+1$, *where* $\mathbf{g}_i, 1 \le i \le R+1$ *are non-random measurable functions defined on* $\mathcal{S}^\infty$, *with* $\mathcal{S}$ *a measurable space, and* $\{\eta_t, -\infty < t < \infty\}$ *are independent,* *identically distributed random variables with values in $\mathcal{S}$. Additionally, we assume that*

$$(E\|\mathbf{z}_t - \mathbf{z}_{t,s}\|^{\kappa_1})^{1/\kappa_1} \leq cs^{-\kappa_2} \textit{ with some } c > 0, \kappa_1 > 4 \textit{ and } \kappa_2 > 1,$$

where $\mathbf{z}_{t,s}(u) = (n_{t,s}, y_{t,s}(u), \epsilon_{t,1,s}(u), \epsilon_{t,2,s}(u), \ldots, \epsilon_{t,K,s}(u)) = \mathbf{g}_i(\eta_t, \eta_{t-1}, \ldots, \eta_{t-s+1}, \boldsymbol{\eta}^*_{t,s})$, $1 \leq i \leq R+1$, *and* $\boldsymbol{\eta}^*_{t,s} = (\eta^*_{t,s,t-s}, \eta^*_{t,s,t-s-1}, \ldots)$, $\{\eta^*_{t,s,s'}, -\infty < t, s, s' < \infty\}$ *are independent, and identically distributed as* η_0, *and independent of* $\{\eta_t, -\infty < t < \infty\}$.

Assumption 3.1 imposes that the processes in the model (1.1) are weakly dependent Bernoulli shifts on the intervals $(t_{i-1}, t_i], 1 \leq i \leq R+1$. Hörmann and Kokoszka (2010) and Aue et al. (2014) show that Assumption 3.1 holds for a large class of time series under suitable regularity conditions, including functional AR, MA, ARMA processes, general functional linear processes, and several nonlinear time series sequences, including volatility processes. Further discussion is provided in Horváth and Kokoszka (2012). Dalla et al. (2015) and Xu (2015) note that in several applications the errors are heteroscedastic, which should be taken into account. Bardsley et al. (2017) and Górecki et al. (2018) suggest several test to detect change in the mean when the error is not stationary. They study models that satisfy Assumption (3.1). Busetti and Taylor (2004), Cavaliere et al. (2011), Cavaliere and Taylor (2008), Hanson (2002) and Harvey et al. (2006) introduce change point tests when the nonstationarity of the data is allowed.

According to Assumption (3.1) we do not require that the curves defining the event observations form a stationary sequence, rather we only require stationarity on the subintervals $(t_{i-1}, t_i], 1 \leq i \leq R+1$. This means that even if the functional mean remains the same between frequency changes, the volatility might change. However, such a change in the volatility will not affect the detection of the stability of the mean. We require that the expected value of the errors conditionally on the number of events that occurred on that day is 0:

Assumption 3.2. *$E(y_t|n_t = m') = 0, E(\epsilon_{t,m}|n_t = m') = 0$, for $1 \leq m, m' \leq K$, $1 \leq t \leq N$,*

and on every interval of stationarity, the frequency of at-least-one event is positive,

Assumption 3.3. *$q_{0,i} = P\{n_{t_i} > 0\} > 0$ for all $1 \leq i \leq R+1$.*

Let $\hat{t}_0 = 0, \hat{t}_{R+1} = N$ and $\hat{t}_1 < \hat{t}_2 < \ldots < \hat{t}_R$ be the change points in the mean of n_t found by the binary segmentation introduced in Section 2. In order to estimate $q_{0,i}$ consistently and to discuss asymptotic consistency of the change point procedures that follow, we assume that we may consistently estimate the change points in the event frequencies with a mild rate.

Assumption 3.4. *For each $1 \leq j \leq R$, $\hat{t}_j = t_j + o_P(N)$.*

First we define the average of the days between $(\hat{t}_{i-1}, \hat{t}_i]$ when events happen,

$$\tilde{q}_i = \frac{1}{\hat{t}_i - \hat{t}_{i-1}} \sum_{t=\hat{t}_{i-1}+1}^{\hat{t}_i} \mathbb{1}\{n_t > 0\}, \tag{3.1}$$

under Assumption 3.4 $\tilde{q}_i \xrightarrow{P} q_i$. This along with the consistency of binary segmentation is discussed more in Section A of the online supplement.

Due to (1.4) we modify the definition of the daily averages:

$$\hat{Z}_t(u) = \frac{1}{\tilde{q}_i} Y_t(u), \quad \text{if } \hat{t}_{i-1} < t \leq \hat{t}_i, 1 \leq i \leq R+1.$$

Our testing procedure is based on the L^2 norm of the CUSUM process of the $\hat{Z}_t$'s,

$$\hat{Z}_N^*(v,u) = N^{-1/2}\left(\sum_{t=1}^{\lfloor Nv \rfloor} \hat{Z}_t(u) - v\sum_{t=1}^{N} \hat{Z}_t(u)\right), \quad 0 \le u, v \le 1. \tag{3.2}$$

We show that $\hat{Z}_N^*(v,u)$ is asymptotically Gaussian. The limiting process is denoted by $\Gamma^0(v,u)$, and

$$\Gamma^0(v,u) = \Gamma(v,u) - v\Gamma(1,u),$$

where $\Gamma(v,u)$ is a Gaussian process with $E\Gamma(v,u) = 0$ and $E\Gamma(v,u)\Gamma(v',u') = C(v,v',u,u')$. To define the covariance kernel $C(v,v',u,u')$, we need some further notation. Due to the changes in the frequency of events, we write the model of (1.1) as

$$X_{t,i,m}(u) = \mu_t(u) + y_{t,i}(u) + \epsilon_{t,i,m}(u), \;\; 0 \le u \le 1, \; 1 \le m \le n_{t,i}, \; t_{i-1} < t \le t_i, \; 1 \le i \le R+1. \tag{3.3}$$

The model in (3.3) reflects that on the days $t_{i-1} < t \le t_i$ the second order properties of $(y_{t,i}(u), \epsilon_{t,i,m}(u))$ are the same, although they could be different if we consider the model of (1.1) on a different sub-interval $t_\ell < t \le t_{\ell+1}, i \ne \ell$. The functions $y_t(u)$, $\epsilon_{t,m}(u)$ of (1.1) are stationary only on subintervals. We define $n_{t,i}^{(e)}$, $y_{t,i}^{(e)}(u)$, $\epsilon_{t,i,m}^{(e)}(u)$ as the extension of $n_{t,i}$, $y_{t,i}(u)$, $\epsilon_{t,i,m}(u)$ defined only on $(t_{i-1}, t_i]$ to $(-\infty, \infty)$, and (3.3) can be generalized as

$$X_{t,i,m}^{(e)}(u) = \mu_t(u) + y_{t,i}^{(e)}(u) + \epsilon_{t,i,m}^{(e)}(u), \;\; 0 \le u \le 1, \; 1 \le m \le n_{t,i}^{(e)}, \; -\infty < t < \infty,$$

where $\mathbf{z}_{t,i}^{(e)} = \mathbf{z}_{t,i}^{(e)}(u) = (n_{t,i}^{(e)}, y_{t,i}^{(e)}(u), \epsilon_{t,i,1}^{(e)}(u), \epsilon_{t,i,2}^{(e)}(u), \ldots, \epsilon_{t,i,K}^{(e)}(u)) = \mathbf{g}_i(\eta_t, \eta_{t-1},, \ldots)$, and $\mathbf{g}_i, 1 \le i \le R+1$ are non-random measurable functions defined on $\mathcal{S}^\infty$, with $\mathcal{S}$ a measurable space, and $\{\eta_t, -\infty < t < \infty\}$ are independent, identically distributed random variables with values in $\mathcal{S}$. Then, we can have

$$Y_{t,i}^{(e)}(u) = \left(\frac{1}{n_{t,i}^{(e)}} \sum_{m=1}^{n_{t,i}^{(e)}} \{ y_{t,i}^{(e)}(u) + \epsilon_{t,i,m}^{(e)}(u) \} \right) \mathbb{1}\{n_{t,i}^{(e)} > 0\},\ 1 \le i \le R+1, -\infty < t < \infty.$$

Now for $0 \le u, u' \le 1, \theta_{i-1} < v \le \theta_i, v \le v'$, we have

$$\begin{aligned} C(v, v', u, u') = \sum_{\ell=1}^{i-1} \frac{\theta_\ell - \theta_{\ell-1}}{q_{0,\ell}^2} \Big\{ & C_{1,\ell}(u, u') - (\theta_\ell - \theta_{\ell-1})\mu(u')C_{3,\ell}(u) - (\theta_\ell - \theta_{\ell-1})\mu(u)C_{3,\ell}(u') \\ & + (\theta_\ell - \theta_{\ell-1})^2 \mu(u)\mu(u')C_{2,\ell} \Big\} + \frac{v - \theta_{i-1}}{q_{0,i}^2} \Big\{ C_{1,i}(u, u') - \mu(u)(\min(v', \theta_i) - \theta_{i-1})C_{3,i}(u') \\ & - \mu(u')(\min(\theta_i, v') - \theta_{i-1})C_{3,i}(u) + \mu(u)\mu(u')(\theta_i - \theta_{i-1})(\min(v', \theta_i) - \theta_{i-1})C_{2,i} \Big\}. \end{aligned} \tag{3.4}$$

Clearly, $E\Gamma_0(v, u) = 0$ and $E\Gamma_0(v, u)\Gamma_0(v', u') = D(v, v', u, u')$ with

$$D(v, v', u, u') = C(v, v', u, u') - v'C(v, 1, u, u') - vC(1, v', u, u') + vv'C(1, 1, u, u').$$

We note that the formulas for $C_{1,i}$, $C_{2,i}$ and $C_{3,i}$ based on the processes defined on the sub-interval $(t_{i-1}, t_i]$ are provided by (A.23), (A.24) and (A.25) in the online supplement. Using the extension notations, we can write the functions $C_{1,i}, C_{2,i}$ and $C_{3,i}$ as

$$C_{1,i}(u, u') = \sum_{t=-\infty}^{\infty} \mathrm{cov}(Y_{0,i}^{(e)}(u), Y_{t,i}^{(e)}(u')), \tag{3.5}$$

$$C_{2,i} = \sum_{t=-\infty}^{\infty} \mathrm{cov}(\mathbb{1}\{n_{0,i}^{(e)} > 0\}, \mathbb{1}\{n_{t,i}^{(e)} > 0\}), \tag{3.6}$$

and

$$C_{3,i}(u) = \sum_{t=-\infty}^{\infty} \mathrm{cov}(Y_{0,i}^{(e)}(u), \mathbb{1}\{n_{t,i}^{(e)} > 0\}). \tag{3.7}$$

Theorem 3.1. *If H_0 and Assumptions 2.3–3.4 are satisfied, then for each N we can define a Gaussian process $\Gamma_N^0(v,u)$ such that*

$$\int_0^1 \int_0^1 \left(\hat{Z}_N^*(v,u) - \Gamma_N^0(v,u)\right)^2 dvdu \overset{P}{\to} 0$$

and

$$\left\{\Gamma_N^0(v,u), 0 \le v, u \le 1\right\} \overset{\mathcal{D}}{=} \left\{\Gamma^0(v,u), 0 \le v, u \le 1\right\}.$$

Corollary 3.1. *If the assumptions of Theorem 3.1 are satisfied, then*

$$T_N = \int_0^1 \int_0^1 \left(\hat{Z}_N^*(v,u)\right)^2 dvdu \overset{\mathcal{D}}{\to} \sum_{g=1}^{\infty} \lambda_g \mathcal{N}_g^2,$$

where $\mathcal{N}_1, \mathcal{N}_2, \dots$ are independent standard normal random variables and $\lambda_1 \ge \lambda_2 \ge \dots$ are the eigenvalues of $D(v,v',u,u')$.

By definition, $\lambda_1 \ge \lambda_2 \ge \dots$ are determined by the integral equation

$$\lambda_g \phi_g(v',u') = \int_0^1 \int_0^1 D(v,v',u,u')\phi_g(v,u)dvdu, \quad g \ge 1,$$

where $\phi_1, \phi_2, \dots$ are orthonormal elements of $L^2([0,1]\times[0,1])$. Since D is unknown, we need to estimate it from the sample. If the estimator $\hat{D}_N$ satisfies

$$\int_0^1 \cdots \int_0^1 \left(\hat{D}_N(v, v', u, u') - D(v, v', u, u')\right)^2 dvdudv'du' \xrightarrow{P} 0, \tag{3.8}$$

then we have automatically (cf. Horváth and Kokoszka, 2012, p.31) that for every g

$$\hat{\lambda}_{g,N} \xrightarrow{P} \lambda_g, \tag{3.9}$$

where the empirical eigenvalues $\hat{\lambda}_{1,N} \geq \lambda_{2,N}, \geq \ldots, \hat{\lambda}_{N,N}$ satisfy

$$\hat{\lambda}_{g,N}\hat{\phi}_{g,N}(v', u') = \int_0^1 \int_0^1 \hat{D}_N(v, v', u, u')\hat{\phi}_{g,N}(v, u)dvdu, \quad 1 \leq g \leq N,$$

and $\hat{\phi}_{1,N}, \hat{\phi}_{2,N} \ldots, \hat{\phi}_{N,N}$ are orthonormal bivariate functions.

These results suggest a simple approach to a consistent test of H_0 in (1.2) versus H_A (1.3): we reject H_0 at level α if T_N is larger than the $1 - \alpha$ quantile of the random variable

$$\sum_{g=1}^{M} \hat{\lambda}_{g,N} N_g^2,$$

where M is a large user-selected integer. The distribution above can be approximated easily, conditionally on the sample, using Monte Carlo simulations. We select $M = 15$, which generally gives us satisfactory approximation of the limit distribution in Corollary 3.1.[4]

Lastly, an estimator of the time of mean curve change is defined by

$$\hat{v}_N = \arg\max_v \int_0^1 \left(\hat{Z}^*(v, u)\right)^2 du. \tag{3.10}$$

[4]We also provide a discussion of this choice in the Appendix D of the online supplement.

Under H_A, there is a change in the global mean curve at an unknown date $k^* = \lfloor N\omega \rfloor, 0 < \omega < 1$. Specifically, $E\hat{Z}_t(u) = a_1(u), 1 \le t \le k^*$ is changed to $E\hat{Z}_t(u) = a_2(u), k^*+1 \le t \le N$. This introduces a drift in the CUSUM process of the form

$$\bar{a}_{\lfloor Nv \rfloor}(u) = \begin{cases} \lfloor Nv \rfloor a_1(u) - vk^* a_1(u) + (N - k^*) a_2(u), & \text{if } 0 < v \le \omega, \\ \\ \lfloor N\omega \rfloor a_1(u) + (\lfloor Nv \rfloor - \lfloor N\omega \rfloor) a_2(u) & \\ \qquad\qquad -vk^* a_1(u) + (N - k^*) a_2(u), & \text{if } \omega < v \le 1. \end{cases} \tag{3.11}$$

In this case,

$$\sup_{0<v<1} \left| \int_0^1 \left(\hat{Z}_N^*(v,u) \right)^2 du - \frac{1}{N} \int_0^1 \bar{a}^2_{\lfloor Nv \rfloor}(u) du \right| = O_P(1). \tag{3.12}$$

If

$$N \int_0^1 (a_1(u) - a_2(u))^2 \, du \to \infty,$$

then this drift is the dominating term. The function $\int_0^1 \bar{a}^2_{\lfloor Nv \rfloor}(u) du$ reaches its largest value at ω, and we get that $\hat{v}_N/N \to \omega$. Additionally, (3.12) can imply the consistency of our test of mean curve change. We also provide more details on the consistency of our test in Section A of the online supplement.

4 Estimation of the long run covariance function

To implement the above test, we require an estimator for D which satisfies (3.8). As a simplified case, we discuss the estimation of D when the event observations are modeled as volatility processes that are serially uncorrelated in Section B of the online supplement. In a more general case, we relax the uncorrelated assumptions and illustrate the consistency of kernel estimators. The times of changes, $t_1, t_2, \dots, t_R$ are estimated by $\hat{t}_1, \hat{t}_2, \dots, \hat{t}_{\hat{R}}$ using the binary segmentation method. Recall that the estimates satisfy (2.11).

When the processes are stationary on the intervals $(t_{i-1}, t_i], 1 \le i \le R+1$, we use the observations on these time intervals to estimate $C_{1,i}, C_{2,i}$ and $C_{3,i}$. Let $\mathcal{K}$ be the kernel satisfying

Assumption 4.1. *(i)* $\mathcal{K}(0) = 1$, *(ii)* $\mathcal{K}(x) = 0$ *if* $|x| > c_*$ *with some* $c_* > 0$, *(iii)* $\mathcal{K}$ *is differentiable on* $[-c_*, c_*]$.

We let $h = h_N$ be a window or bandwidth parameter satisfying

Assumption 4.2. $h_N \to \infty$ *and* $h_N/N \to 0$.

We estimate the correlations of lag $-(\hat{t}_i - \hat{t}_{i-1} + 1) < d < \hat{t}_i - \hat{t}_{i-1} + 1$ on the interval $(\hat{t}_{i-1}, \hat{t}_i]$ with

$$\hat{\gamma}_{i,d}^{(1)}(u,u') = \frac{1}{\hat{t}_i - \hat{t}_{i-1} - |d|} \sum_{t=t_{i-1}+1}^{t_i} (Y_t(u) - Y_i^*(u))(Y_{t+d}(u') - Y_i^*(u')),$$
$$\hat{\gamma}_{i,d}^{(2)} = \frac{1}{\hat{t}_i - \hat{t}_{i-1} - |d|} \sum_{t=t_{i-1}+1}^{t_i} (\mathbb{1}\{n_t > 0\} - \tilde{q}_i)(\mathbb{1}\{n_{t+d} > 0\} - \tilde{q}_i),$$
$$\hat{\gamma}_{i,d}^{(3)}(u) = \frac{1}{\hat{t}_i - \hat{t}_{i-1} - |d|} \sum_{t=t_{i-1}+1}^{t_i} (Y_t(u) - Y_i^*(u))(\mathbb{1}\{n_{t+d} > 0\} - \tilde{q}_i),$$

and

$$Y_i^*(u) = \frac{1}{\hat{t}_i - \hat{t}_{i-1} + 1} \sum_{t=\hat{t}_{i-1}}^{\hat{t}_i} Y_t(u), \quad 0 \le u, u' \le 1, 1 \le i \le R+1. \tag{4.1}$$

where $\tilde{q}_i$ is defined in (3.1). Using these empirical correlations, we define the long run covariance (LRC) estimates

$$\hat{C}_{1,i}^{(LRC)}(u,u') = \sum_{d=-(t_i - t_{i-1})}^{t_i - t_{i-1}} \mathcal{K}\left(\frac{d}{h}\right) \hat{\gamma}_{i,d}^{(1)}(u,u'),$$

$$\hat{C}_{2,i}^{(LRC)} = \sum_{d=-(t_i-t_{i-1})}^{t_i-t_{i-1}} \mathcal{K}\left(\frac{d}{h}\right) \hat{\gamma}_{i,d}^{(2)},$$

and

$$\hat{C}_{3,i}^{(LRC)}(u) = \sum_{d=-(t_i-t_{i-1})}^{t_i-t_{i-1}} \mathcal{K}\left(\frac{d}{h}\right) \hat{\gamma}_{i,d}^{(3)}(u).$$

Applying (2.11), Assumptions 3.1, 4.1 and 4.2 one can show that

$$\int_0^1 \int_0^1 \left(\hat{C}_{1,i}^{(LRC)}(u,u') - C_{1,i}(u,u')\right)^2 du du' \overset{P}{\to} 0, \tag{4.2}$$

$$\hat{C}_{2,i}^{(LRC)} \overset{P}{\to} C_{2,1} \tag{4.3}$$

and

$$\int_0^1 \left(\hat{C}_{3,i}^{(LRC)}(u) - C_{3,i}(u)\right)^2 du \overset{P}{\to} 0. \tag{4.4}$$

The plug in estimator for $C(v,v',u,u')$ is

$$\begin{aligned}
&\hat{C}_N^{(LRC)}(v,v',u,u') \\
&= \sum_{\ell=1}^{i-1} \frac{\hat{\theta}_\ell - \hat{\theta}_{\ell-1}}{\tilde{q}_\ell^2} \Bigg\{ \hat{C}_{1,i}^{(LRC)}(u,u') - (\hat{\theta}_\ell - \hat{\theta}_{\ell-1})\hat{\mu}_\ell(u')\hat{C}_{3,\ell}^{(LRC)}(u) - (\hat{\theta}_\ell - \hat{\theta}_{\ell-1})\hat{\mu}_\ell(u)\hat{C}_{3,\ell}^{(LRC)}(u') \\
&\quad + (\hat{\theta}_\ell - \hat{\theta}_{\ell-1})\hat{\mu}_\ell(u)\hat{\mu}_\ell(u')\hat{C}_{2,\ell}^{(LRC)} \Bigg\} + \frac{v - \hat{\theta}_{i-1}}{\tilde{q}_i^2} \Bigg\{ \hat{C}_{1,i}^{(LRC)}(u,u') - \hat{\mu}_i(u)(\min(v',\hat{\theta}_i) - \hat{\theta}_{i-1})\hat{C}_{3,i}^{(LRC)}(u') \\
&\quad - \hat{\mu}_i(u')(\min(\hat{\theta}_i, v') - \hat{\theta}_{i-1})\hat{C}_{3,i}^{(LRC)}(u) + \hat{\mu}_i(u)\hat{\mu}_i(u')(\min(v',\hat{\theta}_i) - \hat{\theta}_{i-1})\hat{C}_{2,i}^{(LRC)} \Bigg\}.
\end{aligned}$$

The results in (2.11) and (4.2)–(4.4) imply that

$$\int_0^1 \cdots \int_0^1 \left(\hat{C}_N^{(LRC)}(v,v',u,u') - C(v,v',u,u')\right)^2 dv dv' du du' \overset{P}{\to} 0$$

and therefore

$$\int_0^1 \cdots \int_0^1 \left(\hat{D}_N^{(LRC)}(v, v', u, u') - D(v, v', u, u') \right)^2 dv dv' du du' \overset{P}{\to} 0,$$

where $\hat{D}_N^{(LRC)}(v, v', u, u') = \hat{C}_N^{(LRC)}(v, v', u, u') - v'\hat{C}_N^{(LRC)}(1, v, u, u') - v\hat{C}_N^{(LRC)}(1, v', u, u') + vv'\hat{C}_N^{(LRC)}(1, 1, u, u')$. Hence (3.8) holds when kernel estimators are used for the long run variances.

We note that the results in this section follow from Theorem 2.3 of Berkes et al. (2016) applied on each stationary sub-interval separately. A similar argument is also used in Horváth, Kokoszka, et al. (2022). We refer the reader to that literature for more technical details on the estimation of the long run covariance function.

5 Monte Carlo simulations

In this section we present the results of Monte Carlo simulations that aimed to study finite sample performance of the developed stability test. The stability test consists of the frequency change test in Section 2 as well as the mean curve change test in Section 3. We performed some simulations to evaluate the performance of the event frequency change point analyses by itself, which showed generally satisfactory performance for both the detectors $\tilde{\Lambda}_N$ and $\tilde{\Delta}_N$. We have relegated these results to Section C of the online supplement, and below we consider perform the frequency change tests based on $\tilde{\Delta}_N$.

We instead use this space then to focus on the analysis of simulation results for the "overall" stability test.

Empirical size under H_0

For the purpose of simplicity, we set only one frequency change ($R = 1$) in the middle of sample period. The detailed steps of our Data Generating Processes (DGP) under the null hypothesis is as follow:

1. Set the time of frequency change at $t_1 = \lfloor N/2 \rfloor$, resulting in two homogeneous segments $(1, \lfloor N/2 \rfloor]$ and $(\lfloor N/2 \rfloor, N]$.

2. Generate the Bernoulli variable ξ_t with success rate 0.3 for $1 \leq t \leq \lfloor N/2 \rfloor$ and with a different success rate 0.7 for $\lfloor N/2 \rfloor < t \leq N$.

3. If $\xi_t = 1$, generate the number of events at day t by $n_t \sim \min(\text{Pois}(1) + 1, 15)$, where Pois(1) denotes the Poisson distribution with parameter equal to 1, which enables n_t strictly larger than zero and capped at 15. If $\xi_t = 0$, then $n_t = 0$.

4. Set $\mu(u)$, the common mean under H_0, to one of the three choices: *i*) a constant function $\mu(u) = 1$, *ii*) a sine function $\mu(u) = \sin(2\pi u)$, or *iii*) a trend function $\mu(u) = u$.

5. Generate $y_t(u)$ for either the IID case or the functional autoregressive (AR) case

   - IID case: $y_t(u) = \sum_{r=1}^{5} a_{t,r}\psi_r(u)$, where $a_{t,r} \sim i.i.d.\ \mathcal{N}(0, 0.2^2)$ and $\psi_r(u)$ are Fourier basis functions.

   - AR case: $y_t(u) = \sum_{r=1}^{5} b_{t,r}\psi_r(u)$, where $b_{t,r}$ follows autoregressive process $b_{t,r} = \rho b_{t-1,r} + z_{t,r}$, $1 \leq r \leq 5$. We set $\rho = 0.3, 0.5$, or 0.7, and $z_{t,r} \sim i.i.d.\ \mathcal{N}(0, \sigma_r^2)$, where $(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5) = (0.1, 0.05, 0.05, 0.025, 0.025)$.

6. Generate $\epsilon_{t,m}(u) = \sum_{r=1}^{5} b_{t,r,m}\psi_r(u)$, where $b_{t,r,m} \sim i.i.d.\ \mathcal{N}(0, 0.1^2)$, $\psi_r(u)$ are Fourier basis functions, and $1 \leq m \leq n_t$.

7. If $n_t > 0$, generate $X_{t,m}(u) = \mu(u) + y_t(u) + \epsilon_{t,m}(u)$.

We calculate the test statistic T_N by the steps detailed in Section D.1 of the online supplement. To approximate its limit distribution, we use the estimated eigenvalues based on the estimator covariance kernel of Section B for the IID case and use a kernel estimator for the AR case. For the kernel estimator, we use the Bartlett (BT) kernel $\mathcal{K}(x) = (1 - |x|)\mathbb{1}\{|x| < 1\}$ with the bandwidth $h = \lfloor 4(N/100)^{2/9} \rfloor$. We also tried the flat-top kernel with bandwidth $h = \log(N)$, and similar level of empirical sizes and power are observed. Section D in the online supplement provides the detailed guidance on the implementing the two estimators, which may be useful for researchers who want to apply it without studying the underlying theory.

We repeat the Monte Carlo simulation 1,000 times and report the empirical sizes at the significance levels of 1%, 5%, and 10% for the IID case in Table 1 and the AR case in Table 2. For the IID case, it can be observed that the empirical sizes under three different $\mu(u)$ are generally close to their theoretical levels, though there are some slight deviations. For the AR case, the empirical sizes are slightly under-sized if $\rho = 0.3$ and 0.5, but closer to theoretical levels if $\rho = 0.7$. Comparing with $\mu(u) = sin(u)$, the empirical sizes are generally higher if $\mu(u) = u$ and lower when $\mu(u) = 1$. Overall, the test has reasonably good empirical sizes close to the nominal sizes as suggested by the theory developed in Section 3.

Table 1: Empirical size of the IID case

	$\mu(u) = 1$			$\mu(u) = \sin(2\pi u)$			$\mu(u) = u$		
	1%	5%	10%	1%	5%	10%	1%	5%	10%
$N = 100$	0.3%	5.1%	9.4%	0.7%	4.5%	10.7%	1.6%	7.1%	13.9%
$N = 200$	0.2%	2.7%	6.3%	0.5%	3.6%	9.0%	0.7%	4.4%	10.2%
$N = 300$	0.8%	3.5%	7.5%	0.4%	4.3%	8.4%	0.6%	5.0%	11.0%

Table 2: Empirical size of the AR case

	$\rho = 0.3$			$\rho = 0.5$			$\rho = 0.7$		
	$\mu(u) = 1$								
	1%	5%	10%	1%	5%	10%	1%	5%	10%
$N = 100$	0.0%	1.6%	5.3%	0.1%	2.6%	6.5%	0.4%	2.7%	7.1%
$N = 200$	0.3%	1.9%	5.7%	0.2%	2.6%	6.7%	0.5%	2.7%	7.5%
$N = 300$	0.2%	1.3%	4.8%	0.1%	2.6%	5.5%	0.4%	2.6%	7.4%
	$\mu(u) = \sin(2\pi u)$								
	1%	5%	10%	1%	5%	10%	1%	5%	10%
$N = 100$	0.2%	2.9%	6.9%	0.0%	1.9%	6.4%	0.4%	4.9%	9.9%
$N = 200$	0.1%	2.3%	6.0%	0.1%	2.3%	7.2%	0.3%	4.2%	11.2%
$N = 300$	0.1%	2.7%	6.7%	0.2%	2.3%	7.1%	0.7%	4.9%	12.2%
	$\mu(u) = u$								
	1%	5%	10%	1%	5%	10%	1%	5%	10%
$N = 100$	0.1%	2.9%	7.9%	0.1%	3.1%	8.9%	0.9%	6.3%	13.1%
$N = 200$	0.3%	3.0%	7.3%	0.2%	1.9%	6.4%	1.0%	6.2%	12.7%
$N = 300$	0.4%	3.6%	7.5%	0.3%	2.5%	7.7%	0.8%	6.1%	12.5%

Empirical power under H_A

We now turn to the analysis of the empirical power. For the purpose of demonstration, we choose the random effect and error distributions to follow the AR case above, with the change in the mean curve in following four scenarios:

- Scenario 1: $\mu_t(u) = 0.5$ if $1 \leq t \leq k_\mu^*$, $\mu_t(u) = 1$ if $k_\mu^* < t \leq N$;
- Scenario 2: $\mu_t(u) = 0.5$ if $1 \leq t \leq k_\mu^*$, $\mu_t(u) = \sin(2\pi u)$ if $k_\mu^* < t \leq N$;
- Scenario 3: $\mu_t(u) = 0.5$ if $1 \leq t \leq k_\mu^*$, $\mu_t(u) = u$ if $k_\mu^* < t \leq N$;
- Scenario 4: $\mu_t(u) = \sin(2\pi u)$ if $1 \leq t \leq k_\mu^*$, $\mu_t(u) = u$ if $k_\mu^* < t \leq N$,

where k_μ^* is set to be at $\lfloor 0.25N \rfloor$, $\lfloor 0.5N \rfloor$, or $\lfloor 0.75N \rfloor$.

We investigate the empirical power by using the same procedure we used under the null. We generate the data under the alternative, calculate the test statistics, and approximate the limit distribution of the statistics. The Monte Carlo simulation is also repeated 1,000 times and the empirical power is shown in Table 3. There are some distinctions between the four scenarios. Scenarios 2 and 4 have very high power (close to 100%) for all k_μ^* and ρ, even when N is as small as 100. This is mainly because those two scenarios involve a sine function, which is a more 'obvious' change. Scenario 1 is a parallel shift from 0.5 to 1, while Scenario 3 is a change from a flat curve to a slope. For Scenario 1 and 3, the empirical power is generally high when N is more than 150. Comparing different k_μ^*, the empirical power is highest when the mean curve change occurs in the middle $k_\mu^* = \lfloor 0.5N \rfloor$, followed by the late change $k_\mu^* = \lfloor 0.75N \rfloor$, while the early change $k_\mu^* = \lfloor 0.75N \rfloor$ has lowest power. The higher power under $k_\mu^* = \lfloor 0.5N \rfloor$ can be intuitively explained by the fact that the time of frequency change t_1 and the time of mean curve change coincide at $\lfloor 0.5N \rfloor$. Lastly, the empirical power deteriorate slightly with a higher ρ.

6 An application to IPO performance

There is a vast amount of literature on studying IPO performance, which is a classic example in event studies. One central topic in the literature is IPO underpricing which is an observation that in a typical IPO the stock price rises above the initial offer price after one trading day. Four theories to explain IPO underpricing have been proposed, including information asymmetry (Baron and Holmstrom, 1980; Aggarwal et al., 2002; Hanley, 2008), the legal system (Tinic, 1988; Hughes and Thakor, 1992; Lowry and Shu, 2002), ownership separation (Booth and Chua, 1996; Brennan and Franks, 1997; Stoughton and Zechner, 1998), and behavioral finance (Derrien, 2005; Ljungqvist et al., 2006; Cornelli et al., 2006). Among

Table 3: Empirical power of the AR case

	$k_\mu^* = \lfloor 0.5N \rfloor$			$k_\mu^* = \lfloor 0.25N \rfloor$			$k_\mu^* = \lfloor 0.75N \rfloor$		
ρ	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
Scenario 1: $\mu_t(u) = 0.5$ if $1 \le t \le k_\mu^*$, $\mu_t(u) = 1$ if $k_\mu^* < t \le N$.									
$N = 100$	99.3%	99.0%	96.9%	56.5%	55.7%	52.6%	89.0%	85.9%	82.4%
$N = 150$	100.0%	99.8%	99.7%	80.6%	81.3%	75.2%	98.6%	97.7%	94.6%
$N = 200$	100.0%	100.0%	100.0%	92.8%	93.4%	89.4%	100.0%	99.3%	98.8%
Scenario 2: $\mu_t(u) = 0.5$ if $1 \le t \le k_\mu^*$, $\mu_t(u) = \sin(2\pi u)$ if $k_\mu^* < t \le N$.									
$N = 100$	100.0%	100.0%	100.0%	98.9%	98.7%	98.7%	100.0%	100.0%	100.0%
$N = 150$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
$N = 200$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Scenario 3: $\mu_t(u) = 0.5$ if $1 \le t \le k_\mu^*$, $\mu_t(u) = u$ if $k_\mu^* < t \le N$.									
$N = 100$	95.6%	94.7%	91.7%	56.4%	55.6%	55.4%	69.2%	65.2%	65.3%
$N = 150$	99.7%	99.8%	99.2%	85.0%	82.7%	78.4%	90.0%	88.8%	85.4%
$N = 200$	100.0%	100.0%	100.0%	95.4%	94.6%	93.2%	98.9%	98.2%	97.2%
Scenario 4: $\mu_t(u) = \sin(2\pi u)$ if $1 \le t \le k_\mu^*$, $\mu_t(u) = u$ if $k_\mu^* < t \le N$.									
$N = 100$	100.0%	100.0%	100.0%	99.6%	99.6%	98.9%	99.9%	99.7%	99.9%
$N = 150$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
$N = 200$	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

them, the explanatory theories based on information asymmetry, legal system and ownership separation all assume that investors are rational people, and the price on the secondary market can reflect the stock value. The relatively high closing price is caused by the low IPO issuance price, while the explanatory theory based on behavioral finance considers the secondary market. There are irrational traders, herding effect and excessive pursuit of new shares, leading to high closing prices.

However, less attention has been paid to investigate whether the IPO performance is stable over time. Due to evolving circumstances, such as changes in the economic environments and interventions by regulators, the average of IPO cumulative (abnormal) returns might have changed during the sample period. To this end, we apply the developed stability test on the cumulative (abnormal) returns of IPOs in mainland China. Our data is downloaded from

Wind Economic Database, and the sample period is from December 1, 2015 to September 30, 2020, excluding non-trading days.[5] The total number of trading days is 1,181. Recall the notation that there are n_t number of IPO issued on the trading day t. There are 1,297 IPOs on the Shanghai Stock Exchange and Shenzhen Stock Exchanges during our sample period. We consider the cumulative (abnormal) return within three months (60 trading days).

Below, we first apply the stability test developed above to the raw data of cumulative returns of IPO stocks. We note here that the changes in the mean trajectory of the cumulative returns could simply be due to changes in the market returns. Subsequently then, we consider a similar analysis based on the cumulative abnormal returns of IPO stocks, in which the market return[6] is approximately removed, which is more inline with classic event studies. The instability based on the cumulative abnormal returns will reveal the change in the "net" IPO performance.

Exploratory analysis based on cumulative return

The cumulative return curves (CRC) of the stock m with its IPO on date t is defined as

$$\mathrm{CRC}_{t,m}(u) = \frac{P_{t,m}(u) - P_{t,m}(0)}{P_{t,m}(0)}, \qquad 0 \le u \le 60, 1 \le m \le n_t, 1 \le t \le 1181 \tag{6.1}$$

where $P_{t,m}(0)$ is the IPO issuance price, and $P_{t,m}(u)$ is the price of the stock traded at time u after its IPO.[7] To facilitate computation, we have rescaled u to the unit interval $[0, 1]$. As discussed in the introduction, the number and frequency of IPOs fluctuate throughout the

[5] The sample period is chosen to avoid a prolonged period of IPO suspension in China. The China Securities Regulatory Commission (CSRC) suspended IPOs between July 4, 2015 and November 31, 2015 in order to slow a devastating stock market crash in 2015. Such suspension was lifted and the resumption of IPO was allowed in December 1, 2015.

[6] We used a value-weighted portfolio of all stocks in China's A-share market.

[7] This definition allows us to use both high frequency intraday data as well as daily data. Due to the computational cost, we choose to use daily data in this study.

sampling period.

In the first step, we are interested in whether the frequency of IPO issuances has changed in the sample period. To this end, we employ the developed frequency change test (specifically based on $\tilde{\Delta}_N$) in Section 2 to segment the days according to the change of frequency. Because there could be multiple changes in the frequency, we further use binary segmentation to obtain the remaining changes. Table 4 shows the results of the segmentation test. We find five changes in the frequency at 5% significance level, resulting in six segments over our sample period. The frequency jumped from 23.0% to 51.4% on March 3, 2016 and further increased to 84% on August 5, 2016. Afterwards, the frequency reduced to 40.7% on December 6, 2017, and then moved to 70.6% on October 18, 2019, followed by another increase to reach its highest value of 94.5% on June 18, 2020.

In the second step, we focus on investigating whether the mean curve of IPO cumulative returns is stable over the sample period. To answer this question, we use the proposed mean curve change test procedure in Section 3. If the null hypothesis of stability is rejected, we find the time of mean curve change by the estimator $\hat{v}_N$ in (3.10). Again, it is likely that the mean curve is subject to multiple changes. Thus, we use binary segmentation to find the rest of the changes over the sample period. Our stability test procedure reveals three changes in the mean curves on the following dates: February 22, 2017 (p-value: 0.000), May 28, 2018 (p-value: 0.027), and January 15, 2019 (p-value: 0.026). Figure 3 shows how the mean curves evolved over different periods. It is clear that the mean curve of IPO cumulative return was at its highest level before February 22, 2017. Since then the mean curve declined twice on May 28, 2018 and on January 15, 2019. Interestingly, the mean curve bounced back to a slightly higher level between January 16, 2019 and September 30, 2020.

Table 4: Results of frequency change test

	Start	End	Total IPOs	Total Days	Total Days with at least one IPO	Total Days with no IPO	Frequency $\tilde{q}_\ell$
Seg. 1	1-Dec-15	2-Mar-16	37	61	14	47	23.0%
Seg. 2	3-Mar-16	4-Aug-16	70	107	55	52	51.4%
Seg. 3	5-Aug-16	5-Dec-17	569	325	273	52	84.0%
Seg. 4	6-Dec-17	17-Oct-19	254	452	184	268	40.7%
Seg. 5	18-Oct-19	17-Jun-20	180	163	115	48	70.6%
Seg. 6	18-Jun-20	30-Sep-20	187	73	69	4	94.5%

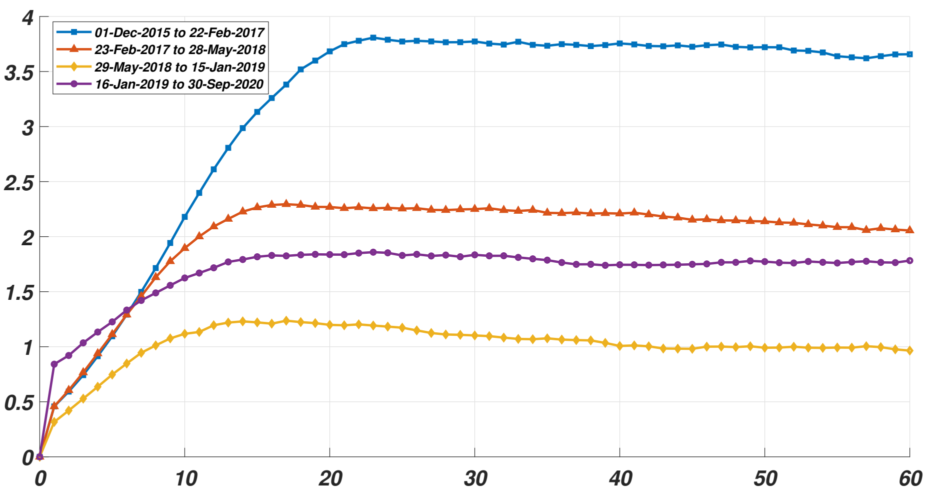

Figure 3: Mean curve of IPO cumulative returns

Formal analysis based on abnormal cumulative return

Following the typical approach of event studies, we now consider the cumulative abnormal returns. To obtain abnormal returns, it is typical to specify an asset pricing model for the stock returns, such as the one-factor capital asset pricing model (CAPM) and the three-factor model (Fama and French, 1993), and then estimate the factor loadings based on the historical data before the events. However, in the study of IPOs, one does not have any historical data to fit to the asset pricing model. Linton (2019) explains that it is common to just use the market return as a proxy for the normal return of the IPOs. Following this common practice, we define the cumulative abnormal return curve (CARC) of the stock m

with its IPO on date t as

$$\mathrm{CARC}_{t,m}(u) = \frac{P_{t,m}(u) - P_{t,m}(0)}{P_{t,m}(0)} - \frac{P_t^{(mkt)}(u) - P_t^{(mkt)}(0)}{P_t^{(mkt)}(0)}, \tag{6.2}$$

where $P_t^{(mkt)}(0)$ is the opening price of market index on date t, and $P_t^{(mkt)}(u)$ is the market index at time u after the opening on day t. Intuitively, the CARC of IPO is the cumulative return curve of the stock minus the cumulative market return curve.

We repeat the same two-step procedure, the frequency change test and the mean curve change test, on the CARC of IPOs. The first step of the frequency change test has the same results as the cumulative return because the frequency of IPO issuance is the same in both cases. In the second step of the mean curve test, we only find one change in the CARC on September 26, 2018 (p-value: 0.000). Binary segmentation does not suggest further changes. The identified change could be related to announcement made by the China Securities Regularly Commission (CSRC) on September 30, 2018; CSRC unveiled to significantly reduce the member size of its issuance examination committee from 66 to 35, in particular removing most part-time committee members. The lower panel of Figure 1 displays the mean curves of cumulative abnormal return before and after the change. Before the change, the CARC are positive with a upward trend before 20 days and then level off. However, after the change, the CARC has a very different pattern after the change, showing a clearly downside trend. This finding could be insightful for policy regulators and investors.

Supplemental material

Online Supplement: In Section A, we provide the detailed proofs of all technical results. In Section B, we discuss the estimation of the long run variance function with uncorrelated errors. In Section C, we present the simulation results of the frequency change test. In Section D, we outline practical guidance on the implementation of our stability test. (PDF)

Computer Code: MATLAB code to perform the test described in the article. (zip file)

IPO data set: Data set used in the illustration of methods in Section 6. (.xlsx file)

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Conflicts of interest

The authors report there are no competing interests to declare.

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