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A B S T R A C T

The Frankel–Wei regression framework measures the relationship between one currency and another currency solely by using foreign exchange rates as regression variables, where investors choose a *common numéraire* for the foreign exchange rates. When the common numéraire is a single currency, the foreign exchange rates are bilateral exchange rates. Option *implied volatilities* are easily estimated from listed option prices for bilateral exchange rates. In this paper, we use the implied volatilities to estimate *implied betas* for the Frankel–Wei regression framework. We show that the average beta and the average implied beta are both *exactly* 0.5, for each currency in a system of currencies.

1. Introduction

The Frankel–Wei regression framework measures the comovements between two currencies. Both the dependent, and the independent, regression variables are log returns of foreign exchange rates relative to a *common numéraire*. When the common numéraire is a single currency, the foreign exchange rates are bilateral exchange rates. For example, if investors were interested in estimating the movements of the Eurozone euro against the movements of the British pound, the US dollar could be used as the common numéraire. Two popular single-currency numéraires are the Swiss franc (Frankel and Wei, 1994; Bénassy-Quéré, 1999; Frankel, 2019), and the US dollar (Bénassy-Quéré, 1999).

In this paper, we use option *implied volatilities* of bilateral exchange rates to calculate *implied betas* for the Frankel–Wei regression framework. The implied betas allow investors to measure the ex-ante betas that are priced in the market. Similar research has mainly focused on estimating *implied correlations* in the exchange rate market, rather than on estimating implied betas (see Beer and Fink, 2019; Campa and Chang, 1998; Walter and Lopez, 2000). We show that the average beta and the average implied beta are both *exactly* 0.5, for each currency in a system of currencies.

2. Material and methods

2.1. Exchange rates

The ith/jth bilateral exchange rate is the price at which an amount of the ith currency can be exchanged for an amount of the jth currency, where there are N_C distinct currencies, with $i, j = 1, \ldots, N_C$. The focus of this paper is on bilateral exchange rates, which will be simply referred to as exchange rates. In log terms at time t, let $s_{i/j,t}$ represent the ith/jth exchange rate for the ith currency in terms of the jth (numéraire) currency. Assuming that there are no arbitrage opportunities (see Chacholiades, 1971), exchange rates are related by:

$$s_{i/j,t} = s_{i/k,t} - s_{j/k,t}, \tag{1}$$

where $i, j, k = 1, \ldots, N_C$; $t = 0, \ldots, T$; $s_{i/j,t}$ is the ith/jth exchange rate; $s_{i/k,t}$ is the ith/kth exchange rate; $s_{j/k,t}$ is the jth/kth exchange rate; and $s_{i/i,t} = 0$.

In matrix notion, the log returns of the exchange rates in (1) can be written as:

$$\Delta \mathbf{s}_{i/j} = \Delta \mathbf{s}_{i/k} - \Delta \mathbf{s}_{j/k} \tag{2}$$

where $i, j, k = 1, \ldots, N_C$; $\Delta \mathbf{s}_{i/j}$ is a $T \times 1$ vector of the ith/jth exchange rate returns; $\Delta \mathbf{s}_{i/k}$ is a $T \times 1$ vector of the ith/kth exchange rate returns; and $\Delta \mathbf{s}_{j/k}$ is a $T \times 1$ vector of the jth/kth exchange rate returns.

The variance of the exchange rate returns in (2) is given by:

$$\sigma^2_{i/j} = \sigma^2_{i/k} + \sigma^2_{j/k} - 2\sigma_{i/k,j/k} \tag{3}$$

where $i, j, k = 1, \ldots, N_C$; $\sigma^2_{i/j}$ is the variance of the ith/jth exchange rate returns; $\sigma^2_{i/k}$ is the variance of the ith/kth exchange rate returns; $\sigma^2_{j/k}$ is the variance of the jth/kth exchange rate returns; and $\sigma_{i/k,j/k}$ is the covariance between the ith/kth exchange rate returns and the jth/kth exchange rate returns. Rearranging (3) for the covariance term produces:

$$\sigma_{i/k,j/k} = \frac{1}{2}\left(\sigma^2_{i/k} + \sigma^2_{j/k} - \sigma^2_{i/j}\right) \tag{4}$$

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where $i, j, k = 1, \ldots, N_C$; $\sigma_{i/k,j/k}$ is the covariance between the ith/kth exchange rate returns and the jth/kth exchange rate returns; and the variance terms are from (3).

2.2. Implied volatilities

Let the *implied volatility* be an expectation of the volatility to give:

$$\tilde{\sigma}_{i/j} = E(\sigma_{i/j}) \tag{5}$$

where $i, j = 1, \ldots, N_C$ with $j \neq i$; and $\tilde{\sigma}_{i/j}$ is the implied volatility of the ith/jth exchange rate returns. The implied volatility in (5) can be substituted into (4) to create the *implied covariance* by:

$$\tilde{\sigma}_{i/k,j/k} = \frac{1}{2}\left(\tilde{\sigma}^2_{i/k} + \tilde{\sigma}^2_{j/k} - \tilde{\sigma}^2_{i/j}\right) \tag{6}$$

where $i, j, k = 1, \ldots, N_C$; $\tilde{\sigma}_{i/k,j/k}$ is the implied covariance between the ith/kth exchange rate returns and the jth/kth exchange rate returns; $\tilde{\sigma}^2_{i/k}$ is the implied variance of the ith/kth exchange rate returns; $\tilde{\sigma}^2_{j/k}$ is the implied variance of the jth/kth exchange rate returns; and $\tilde{\sigma}^2_{i/j}$ is the implied variance of the ith/jth exchange rate returns.

2.3. Frankel-Wei regressions

The Frankel–Wei regression framework models the relationship between comovements in two currencies and assumes that both currencies have a *common numéraire* (Frankel and Wei, 1994). The focus of this paper is on using single currencies as the common numéraire, where the regression variables are the log returns of bilateral exchange rates.

The form of the Frankel–Wei regression model for a dependent ith currency being determined by an independent jth currency with a common-numéraire kth currency can be written as:

$$\Delta \boldsymbol{s}_{i/k} = \alpha_{i,j/k} + \Delta \boldsymbol{s}_{j/k}\beta_{i,j/k} + \boldsymbol{u}_{i,j/k} \tag{7}$$

where $i, j, k = 1, \ldots, N_c$ with $i \neq j$ and $i, j \neq k$; $\Delta \boldsymbol{s}_{i/k}$ is a $T \times 1$ vector of log returns of the ith/kth exchange rate returns; $\alpha_{i,j/k}$ is an intercept term; $\Delta \boldsymbol{s}_{j/k}$ is a $T \times 1$ vector of the jth/kth exchange rate returns; $\beta_{i,j/k}$ is a regression coefficient (beta); $\boldsymbol{u}_{i,j/k}$ is a $T \times 1$ vector of disturbance terms; and the single-currency common numéraire is the kth currency.

The estimate of beta in (7) is given by:

$$\mathrm{E}(\beta_{i,j/k}) = \frac{\sigma_{i/k,j/k}}{\sigma^2_{j/k}} = \frac{1}{2}\left(1 + \frac{\sigma^2_{i/k} - \sigma^2_{i/j}}{\sigma^2_{j/k}}\right) \tag{8}$$

where $i, j, k = 1, \ldots, N_c$ with $i \neq j$ and $i, j \neq k$; $\sigma_{i/k,j/k} = \frac{1}{2}(\sigma^2_{i/k} + \sigma^2_{j/k} - \sigma^2_{i/j})$ is the covariance between the ith/kth exchange rate returns and the jth/kth exchange rate returns from (4); $\sigma^2_{j/k}$ is the variance of the jth/kth exchange rate returns; $\sigma^2_{i/k}$ is the variance of the ith/kth exchange rate returns; and $\sigma^2_{i/j}$ is the variance of the ith/jth exchange rate returns.

The *implied beta* for the Frankel–Wei regression model for a dependent ith currency being determined by an independent jth currency with a common-numéraire kth currency can be estimated by substituting $\tilde{\sigma}_{i/k,j/k}$ for $\sigma_{i/k,j/k}$ and $\tilde{\sigma}^2_{j/k}$ for $\sigma^2_{j/k}$ in (8) to give:

$$\mathrm{E}(\tilde{\beta}_{i,j/k}) = \frac{\tilde{\sigma}_{i/k,j/k}}{\tilde{\sigma}^2_{j/k}} = \frac{1}{2}\left(1 + \frac{\tilde{\sigma}^2_{i/k} - \tilde{\sigma}^2_{i/j}}{\tilde{\sigma}^2_{j/k}}\right) \tag{9}$$

where $i, j, k = 1, \ldots, N_c$ with $i \neq j$ and $i, j \neq k$; $\tilde{\sigma}_{i/k,j/k} = \frac{1}{2}(\tilde{\sigma}^2_{i/k} + \tilde{\sigma}^2_{j/k} - \tilde{\sigma}^2_{i/j})$ is the implied covariance between the ith/kth exchange rate returns and the jth/kth exchange rate returns from (6); $\tilde{\sigma}^2_{j/k}$ is the implied variance of the jth/kth exchange rate returns; $\tilde{\sigma}^2_{i/k}$ is the implied variance of the ith/kth exchange rate returns; and $\tilde{\sigma}^2_{i/j}$ is the implied variance of the ith/jth exchange rate returns.

2.4. Deterministic relationship

Interestingly, there is a deterministic relationship for each dependent ith currency between the estimate of beta $\mathrm{E}(\beta_{i,j/k})$ in (8) and the estimate of beta $\mathrm{E}(\beta_{i,k/j})$ when the independent jth currency is swapped with the common-numéraire kth currency:

$$\mathrm{E}(\beta_{i,j/k}) + \mathrm{E}(\beta_{i,k/j}) = 1 + \frac{\sigma^2_{i/k} - \sigma^2_{i/j}}{2\sigma^2_{j/k}} + \frac{\sigma^2_{i/j} - \sigma^2_{i/k}}{2\sigma^2_{k/j}} = 1 \tag{10}$$

where $i, j, k = 1, \ldots, N_c$ with $i \neq j$ and $i, j \neq k$; and the variance of the jth/kth exchange rate returns is equal to the variance of the kth/jth exchange rate returns, with $\sigma^2_{j/k} = \sigma^2_{k/j}$. In this situation, the average of the two betas in (10) is exactly 0.5.

Similarly, there is a deterministic relationship for each dependent ith currency between the implied beta $\mathrm{E}(\tilde{\beta}_{i,j/k})$ in (9) and the implied beta $\mathrm{E}(\tilde{\beta}_{i,k/j})$ when the independent jth currency is swapped with the common-numéraire kth currency:

$$\mathrm{E}(\tilde{\beta}_{i,j/k}) + \mathrm{E}(\tilde{\beta}_{i,k/j}) = 1 + \frac{\tilde{\sigma}^2_{i/k} - \tilde{\sigma}^2_{i/j}}{2\tilde{\sigma}^2_{j/k}} + \frac{\tilde{\sigma}^2_{i/j} - \tilde{\sigma}^2_{i/k}}{2\tilde{\sigma}^2_{k/j}} = 1 \tag{11}$$

where $i, j, k = 1, \ldots, N_c$ with $i \neq j$ and $i, j \neq k$; and the implied variance of the jth/kth exchange rate returns is equal to the implied variance of the kth/jth exchange rate returns, with $\tilde{\sigma}^2_{j/k} = \tilde{\sigma}^2_{k/j}$. In this situation, the average of the two implied betas in (11) is exactly 0.5.

More generally, the *average beta* for each dependent ith currency across all possible independent jth currencies and all possible common-numéraire kth currencies is:

$$\begin{aligned}\overline{\beta}_{i,./.} &= \frac{1}{(N_c-1)(N_c-2)}\sum_{j=1}^{N_c} I(j \neq i)\sum_{k=1}^{N_c} I(k \neq i,j)\mathrm{E}(\beta_{i,j/k}) \\ &= \frac{1}{2} + \frac{1}{(N_c-1)(N_c-2)}\sum_{j=1}^{N_c} I(j \neq i)\sum_{k=1}^{N_c} I(k \neq i,j)\left(\frac{\sigma^2_{i/k} - \sigma^2_{i/j}}{2\sigma^2_{j/k}}\right) \\ &= \frac{1}{2}\end{aligned} \tag{12}$$

and *average implied beta* for each dependent ith currency across all possible independent jth currencies and all possible common-numéraire kth currencies is:

$$\begin{aligned}\overline{\tilde{\beta}}_{i,./.} &= \frac{1}{(N_c-1)(N_c-2)}\sum_{j=1}^{N_c} I(j \neq i)\sum_{k=1}^{N_c} I(k \neq i,j)\mathrm{E}(\tilde{\beta}_{i,j/k}) \\ &= \frac{1}{2} + \frac{1}{(N_c-1)(N_c-2)}\sum_{j=1}^{N_c} I(j \neq i)\sum_{k=1}^{N_c} I(k \neq i,j)\left(\frac{\tilde{\sigma}^2_{i/k} - \tilde{\sigma}^2_{i/j}}{2\tilde{\sigma}^2_{j/k}}\right) \\ &= \frac{1}{2}\end{aligned} \tag{13}$$

where $i, j, k = 1, \ldots, N_c$ with $i \neq j$ and $i, j \neq k$; $\overline{\beta}_{i,./.}$ is the average beta for the dependent ith currency that is averaged over all possible independent jth currencies and all possible common-numéraire kth currencies; $\overline{\tilde{\beta}}_{i,./.}$ is the average implied beta for the dependent ith currency that is averaged over all possible independent jth currencies and all possible common-numéraire kth currencies; $I(j \neq i)$ is an indicator function that is one when $j \neq i$ and zero otherwise; $I(k \neq i, j)$ is an indicator function that is one when $k \neq i, j$ and zero otherwise; and all other variance terms are from (8) and (10).

Finally, for a system of N_c currencies, the overall average beta and the overall average implied beta are:

$$\overline{\beta}_{.,./.} = \frac{1}{N_c}\sum_{i=1}^{N_c}\overline{\beta}_{i,./.} = \frac{1}{2} \quad \text{and} \quad \overline{\tilde{\beta}}_{.,./.} = \frac{1}{N_c}\sum_{i=1}^{N_c}\overline{\tilde{\beta}}_{i,./.} = \frac{1}{2} \tag{14}$$

where $\overline{\beta}_{.,./.}$ is the average beta over all dependent ith currencies, all possible independent jth currencies, and all possible common-numéraire kth currencies with $i \neq j$ and $i, j \neq k$; $\overline{\beta}_{i,./.}$ is the average beta for the dependent ith currency from (12); $\overline{\tilde{\beta}}_{.,./.}$ is the average implied beta over all dependent ith currencies, all possible independent jth currencies and all possible common-numéraire kth currencies with $i \neq j$ and $i, j \neq k$; and $\overline{\tilde{\beta}}_{i,./.}$ is the average implied beta for the dependent ith currency from (13). These results provide a clear yardstick for both the observed betas, and the observed implied betas, of the Frankel–Wei regression framework.

2.5. Discussion

In this section, we investigate the underlying reason for the deterministic relationship that results in the average beta and the average implied beta being exactly 0.5. In the equity market, for example, a market beta appears in the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965). The market beta for an individual equity represents a measure of its exposure to market systematic risk. In addition, the average beta across all equities is expected to be approximately one (see Elton et al., 2003).

We begin by writing the two regression equations associated with the deterministic relationship of $\mathrm{E}(\beta_{i,j/k}) + \mathrm{E}(\beta_{i,k/j}) = 1$ in (9). The form of the first regression model for the dependent ith currency being determined by the independent jth currency with the common-numéraire kth currency can be written as:

$$\Delta \boldsymbol{s}_{i/k} = \alpha_{i,j/k} + \Delta \boldsymbol{s}_{j/k} \beta_{i,j/k} + \boldsymbol{u}_{i,j/k} \tag{15}$$

and the form of the second regression model for the dependent ith currency being determined by the independent kth currency with the common-numéraire jth currency can be written as:

$$\Delta \boldsymbol{s}_{i/j} = \alpha_{i,k/j} + \Delta \boldsymbol{s}_{k/j} \beta_{i,k/j} + \boldsymbol{u}_{i,k/j} \tag{16}$$

where $i, j, k = 1, \ldots, N_c$ with $i \neq j$ and $i, j \neq k$; $\Delta \boldsymbol{s}_{i/k}$, $\Delta \boldsymbol{s}_{i/k}$, $\Delta \boldsymbol{s}_{i/k}$, $\Delta \boldsymbol{s}_{i/k}$ are all $T \times 1$ vectors of log returns of the associated exchange rate returns; $\alpha_{i,j/k}$ and $\alpha_{i,j/k}$ are intercept terms; $\beta_{i,j/k}$ and $\beta_{i,k/j}$ are beta terms; and $\boldsymbol{u}_{i,j/k}$ and $\boldsymbol{u}_{i,k/j}$ are $T \times 1$ vectors of disturbance terms.

The no arbitrage relationship for $\Delta \boldsymbol{s}_{i/j}$ in (2) can be substituted into (16) to give:

$$\begin{aligned} \Delta \boldsymbol{s}_{i/k} - \Delta \boldsymbol{s}_{j/k} &= \alpha_{i,k/j} + \Delta \boldsymbol{s}_{k/j} \beta_{i,k/j} + \boldsymbol{u}_{i,k/j} \\ &= \alpha_{i,k/j} - \Delta \boldsymbol{s}_{j/k} \beta_{i,k/j} + \boldsymbol{u}_{i,k/j} \end{aligned} \tag{17}$$

where $i, j, k = 1, \ldots, N_c$ with $i \neq j$ and $i, j \neq k$; and $\Delta \boldsymbol{s}_{k/j} = -\Delta \boldsymbol{s}_{j/k}$. Adding $\Delta \boldsymbol{s}_{j/k}$ to both sides of (17) produces:

$$\Delta \boldsymbol{s}_{i/k} = \alpha_{i,k/j} + \Delta \boldsymbol{s}_{j/k}(1 - \beta_{i,k/j}) + \boldsymbol{u}_{i,k/j} \tag{18}$$

where $i, j, k = 1, \ldots, N_c$ with $i \neq j$ and $i, j \neq k$. The two regression equations for $\Delta \boldsymbol{s}_{i/k}$ in (15) and (18) can be compared to uncover the deterministic relationship:

$$\beta_{i,k/j} + \beta_{i,j/k} = 1 \tag{19}$$

Thus, the deterministic relationship is driven by the no arbitrage relationship in (2). Single-currency numéraires restrict the movements of bilateral exchange rates, which consequently restrict the betas in the Frankel–Wei regression framework.

In summary, the choice of common numéraire matters in the Frankel–Wei regression framework when the r-squared of the regression model is less than 100% (Frankel and Wei, 2007). It has been argued that a common numéraire currency should be one that is independent of the other currencies in the regression model (Bénassy-Quéré et al., 2006). Consequently, researchers typically expect that the choice of common numéraire will have minimal impact on the regression coefficients in the Frankel–Wei regression framework.

Table 1
Average implied volatilities.

	USD	EUR	JPY	AUD	CHF	GBP	CAD
USD		9.756	10.250	11.837	9.764	10.057	9.158
EUR	9.756		11.953	10.392	6.537	9.143	9.503
JPY	10.250	11.953		14.382	10.704	12.819	12.734
AUD	11.837	10.392	14.382		11.612	10.993	9.083
CHF	9.764	6.537	10.704	11.612		10.008	10.237
GBP	10.057	9.143	12.819	10.993	10.008		10.015
CAD	9.158	9.503	12.734	9.083	10.237	10.015	
AVG	**10.137**	**9.547**	**12.141**	**11.383**	**9.810**	**10.506**	**10.122**

Notes: Table 1 reports the average implied volatilities over time for all currency pair combinations of the seven currencies, and the average (AVG) grouped by each currency.

However, the deterministic dependencies for both the betas in (9) and the implied betas in (11) provide strong evidence that using single currencies as the common numéraire have a significant impact on the regression coefficients in the Frankel–Wei regression framework. Single-currency numéraires restrict the movements of bilateral exchange rates, which is driven by the standard no arbitrage relationship in (2). Consequently, the betas in the Frankel–Wei regression framework are also restricted.

Recent research has shown that the estimator of beta in (8) is biased when single currencies are used as the common numéraire, when compared to a multicurrency basket as the common numéraire (see Kunkler, 2021). Using a large multicurrency numéraire increases the number of currencies in the no arbitrage relationship (Kunkler, 2022).

3. Empirical analysis

3.1. Data

We consider a set of seven ($N_C = 7$) developed market currencies, namely, the US dollar (USD), the Eurozone euro (EUR), the Japanese yen (JPY), the Swiss franc (CHF), the British pound (GBP), the Canadian dollar (CAD) and the Australian dollar (AUD). The end-of-month data sample consists of one-year at-the-money implied volatilities for all pair combinations of the seven currencies, and is sourced from Bloomberg from 1st July 2008 to 31st December 2021.

3.2. Implied volatilities

Fig. 1 displays, and Table 1 reports, the average implied volatilities over time for all currency pair combinations of the seven currencies. Table 1 also reports the average implied volatilities grouped by each currency. The Japanese yen (JPY) has highest average implied volatility of 12.141. In addition, the Japanese yen (JPY) and the Australian dollar (AUD) have the highest individual implied volatility of 14.382. In contrast, the Eurozone euro (EUR) has lowest average implied volatility of 9.547. Furthermore, the Eurozone euro (EUR) and the Swiss franc (CHF) have the lowest individual implied volatility of 6.537.

3.3. Implied betas

Fig. 2 displays, and Table 2 reports, the average implied betas in (9) for the seven currencies averaged across all possible single-currency numéraires (k), where $k = 1, \ldots, N_C$ and $k \neq i, j$. Table 1 also reports the average implied betas grouped by each dependent currency (i) and each independent currency (j). The

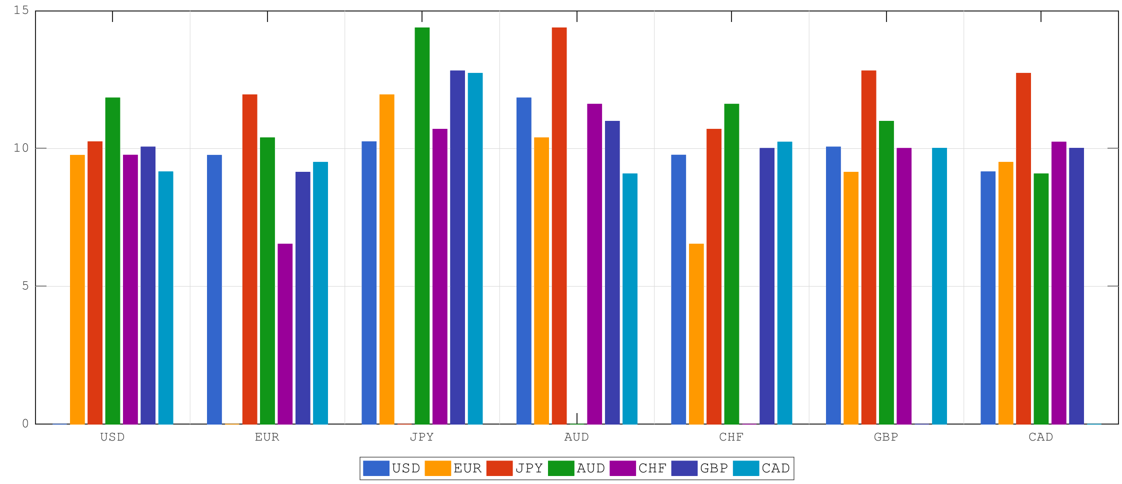

Fig. 1. Average implied volatilities. *Notes:* Fig. 1 displays the average implied volatilities over time for all currency pair combinations of the seven currencies.

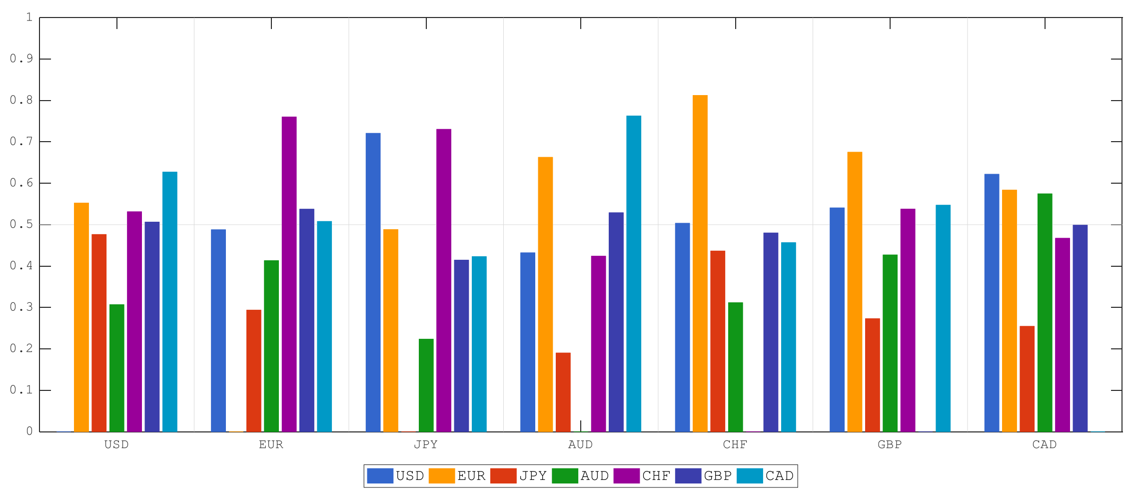

Fig. 2. Average implied betas. *Notes:* Fig. 2 displays the average implied betas for all currency pair combinations of the seven currencies.

Table 2
Average implied betas.

i	$\tilde{\tilde{\beta}}_{i,\mathrm{USD}/.}$	$\tilde{\tilde{\beta}}_{i,\mathrm{EUR}/.}$	$\tilde{\tilde{\beta}}_{i,\mathrm{JPY}/.}$	$\tilde{\tilde{\beta}}_{i,\mathrm{AUD}/.}$	$\tilde{\tilde{\beta}}_{i,\mathrm{CHF}/.}$	$\tilde{\tilde{\beta}}_{i,\mathrm{GBP}/.}$	$\tilde{\tilde{\beta}}_{i,\mathrm{CAD}/.}$	AVG
USD		0.552	0.476	0.307	0.531	0.506	0.627	**0.500**
EUR	0.488		0.294	0.413	0.760	0.537	0.508	**0.500**
JPY	0.721	0.488		0.224	0.730	0.414	0.423	**0.500**
AUD	0.432	0.663	0.190		0.424	0.529	0.763	**0.500**
CHF	0.503	0.812	0.436	0.311		0.480	0.457	**0.500**
GBP	0.541	0.675	0.273	0.427	0.538		0.547	**0.500**
CAD	0.622	0.584	0.254	0.574	0.467	0.499		**0.500**
AVG	**0.551**	**0.629**	**0.321**	**0.376**	**0.575**	**0.494**	**0.554**	**0.500**

Notes: Table 2 reports the average implied betas for all currency combinations of the seven currencies.

Eurozone euro (EUR) has highest average implied beta of 0.830 against the Swiss franc (CHF). In contrast, the Japanese yen (JPY) has lowest average implied beta of 0.196 against the Australian dollar (AUD). In should be noted that for each dependent currency (i) the average implied beta is 0.500, where the average is across possible independent currencies (j) and all possible single-currency numéraires (k) from (13). Furthermore, the overall average implied beta is also 0.500, where the average is across all dependent currencies (i), all possible independent currencies (j), and all possible single-currency numéraires (k) from (14).

Fig. 3 displays the observed monthly implied beta for the European euro (EUR) against the Swiss franc (CHF) over time that is averaged across all possible single-currency numéraires. The European sovereign debt crisis started in 2010, which saw the implied beta fall below 0.4 during 2011 from a previous high of above 0.9 in early 2010. The implied beta jumped when the Swiss National Bank pegged the Swiss franc (CHF) to the European euro (EUR) in September of 2011, which resulted in a jump of the implied beta to above 0.7. Subsequently, the implied beta continued to increase back above 0.9 until the Swiss National Bank unpegged the Swiss franc (CHF) from the European euro (EUR) in January of 2015, which resulted in a large decrease in the implied beta below 0.4.

4. Conclusion

In this paper, we provided a method to estimate the implied betas for the Frankel–Wei regression framework, which allows investors to measure the ex-ante betas that are priced in the foreign exchange options market.

Data availability

Data will be made available on request.

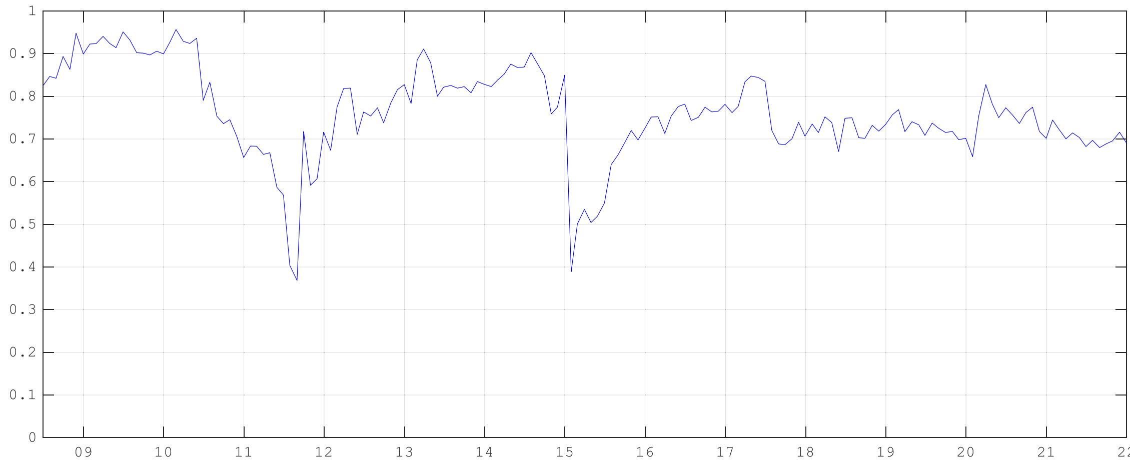

Fig. 3. Implied betas for the European euro against the Swiss franc. *Notes:* Fig. 3 displays the observed implied betas for the European euro (EUR) against the Swiss franc (CHF).

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