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Testing Factor Models in the Cross-Section*

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Abstract

The standard full-sample time-series asset pricing test suffers from poor statistical properties, look-ahead bias, constant-beta assumptions, and rejects models when average factor returns deviate from risk premia. We therefore confront prominent equity pricing models with the classical Fama & MacBeth (1973) cross-sectional test. For all models, we uncover three main findings: (i) the intercept coefficients are economically large and highly statistically significant; (ii) cross-sectional factor risk premium estimates are generally far below the average factor excess returns; and (iii) they are usually not statistically significant. Overall, *all* new factor models are inconsistent with no-arbitrage pricing and cannot accurately explain the cross-section of stock returns.

JEL classification: G12, G11, G17

Keywords: Factor models, cross-sectional tests, no-arbitrage pricing, beta estimation

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I Introduction

In recent years, several new equity factor models have been proposed (e.g., Fama & French, 2015, 2018; Hou et al., 2015, 2021; Stambaugh & Yuan, 2017; Daniel et al., 2020a). These are shown to price the cross-section of stock returns far better than the classical Fama & French (1993) 3-factor model. Nevertheless, the authors to all these papers also have to concede that their models fail the Gibbons et al. (1989) time-series GRS test of zero average pricing errors and are ultimately rejected.

The time-series test, though, makes a number of implicit assumptions in its standard setup applied in these studies (e.g., investors already know all moments of returns, betas are constant, and factor risk premia equate the time-series averages) and has very poor finite-sample properties (Bekaert & De Santis, 2021). In this paper, we therefore employ the cross-sectional asset pricing test to analyze the fit of the new factor models for a large set of test assets. Importantly, our most general setup relaxes all implicit assumptions of the time-series test while also using real-time, time-varying betas. Our main contribution is a thorough and comprehensive analysis of the new asset pricing models with the cross-sectional test.

In the main part of the paper, we test the Fama & French (2015) 5-factor model. Since it is unclear how to best perform the cross-sectional test (portfolios vs. stocks, estimation method for the cross-sectional regression, how to estimate beta, ...), we conduct a very thorough analysis using a variety of estimators and approaches that have been proposed in the literature.

Our first result in a nutshell: The Fama & French (2015) 5-factor model clearly fails the cross-sectional test. Independent of the combination of specifications we choose, we uncover three main findings: (i) the estimated intercept is economically large, annualized generally between 4% and 10%, and statistically clearly different from zero; (ii) the estimated factor risk premia are far smaller than the average excess returns of the factors; while (iii) they are often not even significantly different from zero. Our findings thus show that neither using an alternative test with different statistical properties, nor accounting for look-ahead bias, time-varying betas, or potential differences between factor risk premia and their average returns reverse the model's rejection by the time-series GRS test. Furthermore, not only is the market beta close to useless in determining cross-sectional differences in returns, also the SMB, HML, RMW, and CMA betas are of very limited value.

The logical next question relates to the performance of the main competitor models of the Fama & French (2015) 5-factor model. Perhaps these models perform better in the cross-sectional test? The brief answer is no. We find that our previous results extend well beyond the Fama & French (2015) 5-factor model. The Stambaugh & Yuan (2017) 4-factor model, the Fama & French (2018) 6-factor model, the Daniel et al. (2020a) 3-factor model, the modified Fama & French (2015) 5-factor model of Daniel et al. (2020b), the Hou et al. (2021) 5-factor model, and a model based on principal components analysis (PCA) perform little better. For all models, we obtain the same set of three stylized findings. None of the models appears to be consistent with no-arbitrage pricing and the arbitrage pricing theory (APT).

To be fair, each model appears to have one or two "reasonable" factor(s) that partially explain differences in asset returns. The Stambaugh & Yuan (2017) 4-factor model performs comparably best in our cross-sectional test. The management factor and to some extent also the performance factor often yield significantly positive risk premia. Further useful factors include the cash-based operating profitability factor in the Fama & French (2018) 6-factor model, the post-earnings-announcement-drift factor in the Daniel et al. (2020a) 3-factor model, the expected growth factor in the Hou et al. (2021) 5-factor model, and the second and third principal component in the PCA model. However, even for these factors the risk premium point estimates are generally clearly smaller than the corresponding average factor excess returns.

While the literature focuses mainly on the (full-sample) time-series test, the use of variations of the cross-sectional test by Chordia et al. (2017), Jegadeesh et al. (2019), and Raponi et al. (2020) constitutes notable exceptions. Chordia et al. (2017) examine whether the Fama & French (2015) 5-factor model factor loadings or the underlying characteristics explain more of the cross-sectional variation in returns. They find that characteristics have substantially higher explanatory power than factor loadings. Jegadeesh et al. (2019) reach the same conclusion using an in-sample instrumental variable approach. Raponi et al. (2020) develop a methodology for testing factor models with the number of stocks exceeding the number of time periods. In the empirical part of their paper, using contemporaneously estimated (ex-post) betas, they find that the risk premia for their method are more strongly significant than under the standard approach. Nevertheless, they also find that characteristics outperform factor loadings in explaining the cross-section of stock returns. It is unclear how the results of the latter two papers generalize to an adaptive-expectations setup when avoiding a look-ahead bias. More broadly, our contribution relative to these three papers lies in answering a more basic question: Do the new factor models pass the cross-sectional test? That is, do the factor models qualify as APT models? We thus complement the previous studies by performing direct cross-sectional tests of a multitude of factor models (not only that of Fama & French, 2015, on which the mentioned papers' main focus lies). Importantly, this paper provides a thorough and robust test of the factor models, using a variety of different methods, all in one place.

Several recent papers analyze the performance of the Fama & French (2015) factors vis-à-vis other alternatives (e.g., Barillas & Shanken, 2018; Kozak et al., 2018; Chib et al., 2020; Daniel et al., 2020b; Harvey & Liu, 2021). In this paper, we account for these developments and also consider several other empirical factor models and improvements of the original

Fama & French (2015) factors. For none of these do our conclusions differ markedly.[1]

Over the last couple of years, several modifications of the classical Fama & MacBeth (1973) approach have been proposed in the literature (e.g., Burnside, 2011; Kleibergen & Zhan, 2020; Bryzgalova et al., 2021; Giglio & Xiu, 2021; Liao & Liu, 2022). Kroencke & Thimme (2021) analyze the size and power properties of various methodologies with empirically relevant sample sizes. They show that the standard approach with Kan et al. (2013) standard errors performs well in all situations. In this paper, we therefore follow their advice and use the classical approach with Kan et al. (2013) standard errors. We also study several adjustments to tackle a potential weak-factor problem (Bryzgalova, 2015).

A further contribution of this study is an analysis of the impact of different beta estimation techniques on the empirical results. Given the apparent errors-in-variables (EIV) problem, beta estimation deserves additional consideration. However, little is known about the performance of different possible estimators. Surprisingly little research has been devoted to "improving" individual beta estimates. As Levi & Welch (2017) note, the vast majority of academic studies still broadly uses the 60-month monthly window approach of Fama & MacBeth (1973). However, there are a number of alternative approaches that may provide better beta estimates (e.g., Hollstein & Prokopczuk, 2016; Levi & Welch, 2017; Hollstein et al., 2019; Welch, 2019; Hollstein, 2020).

The remainder of this paper is organized as follows. In Section II, we introduce the data as well as the theoretical background and the methodology of the empirical analysis. In addition, this section presents the summary statistics. We present our main empirical results for the Fama & French (2015) 5-factor model in Section III. Section IV examines alternative factor models. We use Section V to conclude.

[1] Different from Barillas & Shanken (2018) and Chib et al. (2020), our focus is not primarily on the relative comparison between different asset pricing models. Instead, we subject different models to an absolute test to examine whether, and to what extent, they are consistent with no-arbitrage pricing.

II Data and Methodology

A Data

We base our analysis on three different sets of portfolio returns as well as individual stocks. The first set consists of portfolios retrieved from Kenneth French's website.[2] We use the 25 portfolios sorted on each size and book-to-market, size and investment, and size and operating profitability. This results in a total of 75 portfolios. These portfolios constitute the home playing ground for the Fama & French (2015) factors and are also those that are most widely studied in the previous literature.

Our second set of portfolios consists of 100 double-sorted portfolios based on historical Fama & French (2015) 5-factor betas. That is, for market betas, value betas, profitability betas, and investment betas we conduct 5×5 double sorts with size betas. The portfolios are rebalanced each month based on individual stock betas based on the data described in the next paragraph and estimated using the past year of daily returns. As the final set of portfolios, we use 116 anomaly long–short portfolios of Chen & Zimmermann (2020).[3]

We obtain daily data on stock returns, prices, and shares outstanding from the Center for Research in Security Prices (CRSP). We use all stocks traded on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the National Association of Securities Dealers Automated Quotations (NASDAQ) that are classified as ordinary common shares (CRSP share codes 10 or 11). We exclude closed-end funds and REITs (SIC codes 6720–6730 and 6798). Furthermore, following Amihud (2002) and Zhang (2006), we exclude highly illiquid stocks. We expunge firm–month observations with prices below 3 dollars (e.g.,

[2]The website can be reached at: `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

[3]The data can be downloaded from: `https://www.openassetpricing.com/data/`. Of the total of 171 anomalies, we use those 116 that are available for our full sample period. To limit ourselves to these is necessary because some approaches require a complete dataset without missing values. The results for all other tests are qualitatively similar when using all 171 anomaly portfolios.

Zhang, 2006). We adjust for delisting returns following Shumway (1997) and Shumway & Warther (1999).

We start our sample period in January 1963 (when the investment and profitability factors become available) and end it in December 2018. Data on the risk-free (1-month Treasury Bill) rate as well the Fama & French (2015, 2018) factors come from Kenneth French's data library. The Stambaugh & Yuan (2017) factors come from Robert Stambaugh's webpage. Data on the Daniel et al. (2020a,b) factors come from Kent Daniel's website and those on the Hou et al. (2021) factors come from the *global-q* webpage.[4,5]

B The General Setup and the APT

Ross (1976) makes two main assumptions to derive the APT: (i) No-arbitrage and (ii) that there is a latent factor structure that describes the covariance of asset returns.[6] It then follows that

$$\mathbb{E}(r_j - r_f) = a_j + \lambda' \beta_j, \tag{1}$$

where r_j and r_f denote the return of one asset j and the risk-free rate, respectively. a_j is the intercept term. λ and β_j are $k \times 1$ vectors of factor risk premia and factor betas of asset j toward the k factors of a model, respectively.

The APT no-arbitrage condition then states that

$$a_1^2 + ... + a_N^2 < \delta < \infty, \tag{2}$$

[4]The corresponding websites are, in order: `http://finance.wharton.upenn.edu/~stambaug/`, `http://www.kentdaniel.net/data.php`, and `http://global-q.org/index.html`.

[5]For part of the factors, the daily or monthly (or both) returns are not available. In these cases, we use data from CRSP and Compustat and follow the methodologies described in the respective studies to obtain the missing time series.

[6]Further technical requirements include that (iii) the idiosyncratic part of the asset returns is mean zero and independently and identically distributed (in particular, it is uncorrelated to the factors), (iv) the number of factors is much smaller than that of assets, (v) capital markets are perfect, and (vi) expectations are homogeneous.

for some unknown constant δ. Intuitively, the upper bound on asset-specific returns and hence Sharpe ratios makes the implicit assumption that very attractive investment opportunities in terms of their risk–return profile will be used by investors, which pushes up their prices and makes them less attractive (Cochrane & Saa-Requejo, 2000).

The empirical factor models deviate slightly from the APT by assuming that $a_j = 0$ for all j. Therefore, for most of this study, we test the specification

$$\mathbb{E}(r_j - r_f) = a + \lambda' \beta_j, \tag{3}$$

with a common intercept a. It is included to allow all test asset returns to be mispriced by a common amount (Parker & Julliard, 2005). That is, the common intercept captures the amount by which the zero-beta rate of the factor model exceeds the risk-free rate: the return that could be realized on average while hedging out all factor risk. Fama (1976) shows that the estimated intercept coefficient corresponds with a portfolio of the left-hand-side assets, whose weights sum to one. The portfolio has zero betas toward all factors. Clearly, under the null hypothesis of a certain factor model a high zero-beta rate in excess of the risk-free rate constitutes a "good deal". Investors would be willing to make use of such good deals. Higher demand for stocks that contribute positively to the a and lower demand for those contributing negatively should make the prices adjust. Hence, the APT predicts that such "good deals" do not persist in the market. Thus, if the model is a good approximation to the true return generating process, the a should be (close to) zero.

C The Cross-Sectional Asset Pricing Test

The asset pricing hypothesis underlying the cross-sectional regression test is that of Equation (3). Empirical tests of this equation are typically performed with the two-pass procedure of Fama & MacBeth (1973). The first pass involves the estimation of betas for all test assets.

The second pass includes a cross-sectional regression of stocks' excess returns each period on these beta estimates.

1 The First Pass: Beta Estimation

A very important ingredient for the two-pass cross-sectional regression procedure is an estimate for the conditional beta ($\hat{\beta}_{j,t}$). Previous studies in part use the full sample period to estimate the conditional betas (e.g., Hu et al., 2013; Kan et al., 2013; Adrian et al., 2014; Buraschi et al., 2014). For initial benchmark results, we also follow this procedure. However, it creates two problems. First, the literature provides compelling evidence of time-variation in betas (e.g., Lettau & Ludvigson, 2001; Avramov & Chordia, 2006). Thus, one may reject a factor model simply because of the approximation of time-varying betas with a full-sample estimate (Hansen & Richard, 1987; Jagannathan & Wang, 1996). Second, by estimating betas over the full sample period, one relies on information investors would not have been able to use in real time. Hence, any full-sample test essentially uses a joint hypothesis of the asset pricing model and the assumption that investors know all (constant) moments of asset returns at all times.

For our main specification, we thus estimate betas using a historical window. We use the simple historical estimator. That is, we perform time-series regressions of an asset's excess return on a constant and the different factors of a model

$$r_{j,\tau} - r_{f,\tau} = \alpha_{j,t} + \beta'_{j,t} F_\tau + \epsilon_{j,\tau}, \tag{4}$$

where $r_{j,\tau}$ denotes the return of asset j observed at time τ. $r_{f,\tau}$ is the corresponding risk-free interest rate over the same interval. Depending on the data frequency, τ can measure time steps in days, months, or quarters (in tables, we indicate the data frequency used in the first pass by "D", "M", and "Q", respectively; the superscript denotes the historical window in months, which is left blank for full-sample and 60-month estimators). $\beta_{j,t}$ is a $k \times 1$ vector of conditional betas for asset j at time t. F_τ is a $k \times 1$ vector of factor returns observed at time τ. For example, in the case of the Fama & French (2015) 5-factor model, F_τ consists of the returns of the market factor (MKT), the size factor (SMB), the value factor (HML), the profitability factor (RMW), and the investment factor (CMA). We use data from time $t-k$ to t, observed at discrete intervals τ, with k being the length, in months, of the estimation window (indicated in the superscript in the tables).[7]

2 The Second Pass: Cross-Sectional Regression

In a second pass, we run a cross-sectional regression at each time period of excess returns on a constant and the estimated betas ($\hat{\beta}$)

$$r_{j,t} - r_{f,t} = a_t + \lambda_t' \hat{\beta}_{j,t} + \nu_{j,t}. \tag{5}$$

These regressions are typically performed at the monthly frequency. That is, $r_{j,t} - r_{f,t}$ is the stock excess return during month t and $\hat{\beta}_{j,t}$ is an estimate for beta obtained at the end of the previous month. $\nu_{j,t}$ is the regression residual. In the empirical part of this paper, we follow this approach.

The statistical tests are based on the time-series averages $\bar{\hat{a}}$ and $\bar{\hat{\lambda}}$ of the estimated intercept and slope coefficients. In tables, we denote $\bar{\hat{a}}$ as *Intercept* and the elements of $\bar{\hat{\lambda}}$ by their factor acronyms. We adjust the standard errors for errors-in-variables, model misspecification, as well as heteroskedasticity and autocorrelation following Kan et al. (2013) (with 5 Newey & West, 1987 lags). We choose this approach based on the results of Kroencke & Thimme (2021), who analyze the size and power properties of different estimation and

[7] To estimate Equation (4), we require at least half the returns (as well as the most recent one) in the historical window to be non-missing. Stocks and portfolios, for which this is not the case, obtain missing beta estimates and are not considered for the second pass at that time.

standard error approaches for the cross-sectional test. They show that the standard Fama & MacBeth (1973) approach with Kan et al. (2013) standard errors has very good properties for empirically relevant sample sizes.

D The Cross-Sectional vs. the Time-Series Test

For traded factors, both the time-series test and the cross-sectional test can be performed. The latter has a number of advantages. First, in the standard setup, the time-series test suffers from a look-ahead bias caused by the use of full-sample betas. The assumption underlying the use of just one ex-post time-series regression is that investors already know all moments of returns (and hence the beta factors). This assumption is clearly difficult to uphold in practice (Barahona et al., 2021). Boguth et al. (2011) show that the use of information that is not available in real time can bias the time-series regression results. As shown in Section II.C, the cross-sectional test easily incorporates a relaxation of this assumption. It can be cast in an adaptive-expectations setting where all beta estimates are obtained based on real-time information.

Second, the standard full-sample time-series test ignores time-variation in betas. That betas are changing over time is a well-established empirical fact (e.g., Fama & French, 1997; Lettau & Ludvigson, 2001; Avramov & Chordia, 2006; Daniel & Moskowitz, 2016). Indeed, Morana (2007) and Becker et al. (2021) document that betas clearly have long-memory properties in the time series, rendering a constant approximation over the entire sample period particularly inaccurate. The cross-sectional test can easily account for changing betas.

Third, observed alphas in the time-series test can be caused by differences between factor risk premia and their time-series means. As shown by Cochrane (2005), time-series alphas are functions of the differences between the factors' unknown risk premia and their time-series averages. Time-series means may differ from the true factor expected returns due to measurement errors (Roll, 1977; Shanken, 1987; Jagannathan et al., 2010). The cross-sectional test avoids the assumption that the time-series average returns coincide with the factor risk premia, which is imposed by the time-series test. Instead, it empirically estimates the factor premia from cross-sectional information.

Finally, Bekaert & De Santis (2021) show with simulations that the time-series Gibbons et al. (1989) GRS test tends to overreject the null hypothesis in finite samples.

Some, although not all, of these issues can also be addressed with a suitably adjusted time-series test. We choose to perform a cross-sectional test because it easily accounts for all these issues.

E Variations of the Test Design

The main plague of the two-pass cross-sectional test is the EIV problem introduced by using estimated betas as independent variables. The first consequence of this problem is that one has to adjust the standard errors of the risk-premium coefficients in the second pass (which we do by using the procedure of Kan et al., 2013). The second consequence is the attenuation bias that arises in the risk premium estimates if the independent variables are measured with error. To alleviate the EIV problem, the literature has developed several adjustments of the test design. We consider these below.

1 Test Portfolios

The first such adjustment is the grouping of stocks into portfolios. Building portfolios drastically reduces the attenuation bias on the coefficients. Given that the portfolios are well diversified, a large part of the residual variation is eliminated and the portfolio betas suffer from meaningfully lower estimation errors (Black et al., 1972; Blume & Friend, 1973; Fama & MacBeth, 1973). Therefore, an analysis at the portfolio level will provide the starting point of our empirical analysis. Several subsequent studies, however, came to criticize this scheme based on the dependence of the results on the way the portfolios are formed (Lo & MacKinlay, 1990; Lewellen et al., 2010) and loss of power (Grauer & Janmaat, 2010; Ang et al., 2020).

We address the first point by considering different sets of portfolios for the empirical tests, thereby examining the sensitivity of the results to the portfolio formation scheme. To alleviate the loss of power and address the weak-factor problem, Grauer & Janmaat (2010) suggest repackaging the portfolio data to increase the spread in betas. For their approach, one has to first estimate the (market) betas of different portfolios. Subsequently, the portfolio returns of the half of the highest-market-beta portfolios are subtracted from those of half of the lowest-market-beta portfolios (the lowest minus the highest, the second lowest minus the second highest, ...). These repackaged portfolios replace the original half of portfolios with the lowest betas.

2 Estimating Beta

There are several alternatives for beta estimation. The beta estimator affects the results through two channels: measurement errors and conditionality. Start with the former. Lower measurement errors in beta reduce the EIV problem and the attenuation bias in the risk premium estimates. Based on this insight, one might tend to increase the beta estimation window as much as possible to obtain a more precise historical estimate. The problem with this approach is that one loses conditionality. Given that betas vary over time, a more timely sample likely provides a more precise estimate of the current conditional beta.

In our benchmark analysis, we rely on 60-month windows, as in Fama & MacBeth (1973), with either monthly or daily data (denoted by M and D in the tables), but also on a shorter 12-month window of daily data (D^{12mon}). For robustness, we also consider 6-month and 36-month windows of daily data (D^{6mon} and D^{36mon}, respectively) and a 60-month window of (monthly-overlapping) quarterly return data (Q).

Additionally, the ordinary least squares (OLS) estimator of Equation (4) might not be the best choice when aiming at a conditional estimate with as little measurement error as possible. We alternatively consider substitutes with exponentially decaying weights and several shrinkage estimators. Finally, small and illiquid stocks might be traded only infrequently (and not synchronously with other stocks). An infrequent-trading adjustment may help reduce their measurement errors. We therefore also consider two such adjustments. We describe all alternative estimators in detail in Appendix A.

The vast majority of previous studies use the simple historical estimator of Equation (4) when estimating betas for multifactor models (or market betas) (e.g., Chang et al., 2013; Kan et al., 2013; Buraschi et al., 2014; Savor & Wilson, 2014; Hollstein & Prokopczuk, 2018), while a few use a Dimson (1979) infrequent-trading adjustment (e.g., Asness et al., 2013; Cremers et al., 2015).

3 Choice of Estimation Method

A further alternative concerns the estimator chosen to estimate Equation (5). The standard approach is to use OLS. A viable alternative is the use of generalized least squares (GLS). For the GLS approach, the inverse of the stock return covariance matrix serves as a weighting matrix. Thus, stocks with more volatile returns are weighted less strongly than those that are less volatile.[8] An alternative weighted least squares (WLS) weighting scheme uses a diagonal weighting matrix containing the stocks' current market capitalizations.

For individual stocks, i.e., in the case where the number of stocks, N, is large and the beta-estimation horizon, T, is fixed, there is a further adjustment method. It directly tackles the attenuation bias in the estimated factor risk premium coefficients, which is induced by

[8]Note that with the GLS approach, the cross-sectional test does not entirely rely on real-time information any longer. This is because the estimation of the covariance matrix requires large amounts of data, which is why we use the full sample to do so.

measurement errors in the beta estimates. Based on Shanken (1992), Chordia et al. (2017) and Raponi et al. (2020) use the following adjustment to the OLS estimator of Equation (5)

$$\begin{bmatrix} \hat{a}_t \\ \hat{\lambda}_t \end{bmatrix} = \left(\hat{X}_t' \hat{X}_t - \sum_{j=1}^{N_t} M' \hat{\Sigma}_{j,t} M \right)^{-1} \hat{X}_t' R_t, \tag{6}$$

where $\hat{X}_t = \begin{bmatrix} \mathbf{1}_N \ \hat{\beta}_t' \end{bmatrix}$, with $\hat{\beta}_t$ being an $k \times N$ matrix of estimated betas for the N assets. $\mathbf{1}_N$ indicates a vector of N ones. $M = \begin{bmatrix} \mathbf{0}_k \ I_k \end{bmatrix}$, with I_k being a $k \times k$ diagonal matrix of ones. $\hat{\Sigma}_{j,t}$ is the White (1980) covariance matrix of the time-series beta estimates for asset j from the first pass. R_t is a $N \times 1$ vector of stock excess returns observed in month t. The term $\sum_{j=1}^{N_t} M' \hat{\Sigma}_{j,t} M$ provides the EIV-bias correction.[9] We refer to this estimation method as OLS^{BC}.

F Summary Statistics

Before starting the main empirical analysis, it is instructive to first have a look at the summary statistics presented in Table 1. Panel A of Table 1 starts with a description of the factor returns. We find that all the Fama & French (2015) factors yield positive average returns. The market factor (MKT) has the highest average excess return of 6.15% per year with a standard deviation of 15.21%. For SMB, both the average return and the standard deviation are substantially smaller with 2.83% and 10.46% per annum. The average returns of HML (3.90%), RMW (3.09%), and CMA (3.40%) all exceed 3% per year while their standard deviations are even slightly smaller, all being below 10%.

The factor returns are non-normal with in part substantial excess kurtosis (in particular for RMW). The factor returns all have some very mild positive first-order monthly

[9] Based on the advice of Chordia et al. (2017), we use Fama & MacBeth (1973) standard errors when applying this bias-correction. The authors find in simulations that these are well behaved. Raponi et al. (2020) provide asymptotic standard errors for the case of betas estimated ex-post. These formulas, though, cannot be applied in our ex-ante setting.

autocorrelation. The correlations are mostly low, but reach 0.696 between HML and CMA.

In Panels B and C of Table 1, we present the summary statistics of the beta estimates resulting from the OLS estimator of Equation (4). Panel B reports the summary statistics for 12-month daily and 60-month monthly estimators for our (first) Fama–French portfolio sample and Panel C those for the same estimators and our individual stock sample. The value-weighted average betas are, as they should be, close to 1 for the market and close to 0 for all other factors.[10] The average standard deviations of the factor betas are substantially higher than those of the market betas. For portfolios, the highest cross-sectional standard deviations occur for the SMB betas. For individual stocks, the most volatile factor sensitivities are those toward RMW.

The cross-sectional standard deviations are substantially higher for the 12-month daily estimator than they are for the 60-month monthly estimator. It thus seems that the choice of sampling frequency and estimation window might have important consequences for the resulting beta estimates and their properties. Whether the increased standard deviation and dispersion of the 12-month daily estimator reflects its economic content or rather increased noise, though, remains to be seen. All factor betas display excess kurtosis. Interestingly, the kurtosis is much larger for the 12-month daily estimator for individual stocks than for the 60-month monthly estimator.

Finally, the correlations between the beta estimates of different factors are generally low. In particular, that between HML and CMA betas is substantially smaller in magnitude than that between the two factor returns. Thus, the second-stage regressions of Equation (5) are not likely to suffer from any issues due to multicollinearity. For individual stocks, however, we detect substantial correlations exceeding 0.3 between MKT and SMB and between HML and RMW. This indicates that, to some extent, the factor sensitivities carry

[10] The figures are not exactly 1 and 0, because we exclude part of the stocks from the CRSP universe; see Section II.A.

similar economic content.

III Main Empirical Results

In this section, we present the main empirical results of our cross-sectional test of the Fama & French (2015) 5-factor model. We begin with an analysis of the Fama–French portfolios, providing the results for different test specifications. Afterwards, we examine the model with different sets of portfolios and analyze alternative beta estimators. Finally, we repeat the analysis on the individual-stock level.

A Analysis With the Fama–French Portfolios

The first main goal of this study is to test the Fama & French (2015) 5-factor model with the cross-sectional test. We start the analysis by examining the model's ability to explain the returns of its base portfolios: the 75 portfolios consisting of different sets of 25 that are jointly sorted on size and each book-to-market, profitability, and investment.

The main results are in Table 2. For comparison, Panel A presents the time-series-test results. That is, we test the statistical significance of the factor returns both with monthly and daily data. We find that all average factor returns except for that of SMB are strongly statistically significant. These average returns are important because they provide a natural benchmark for the size of the cross-sectional risk premia which, under the assumptions of the time-series test, should be of a similar magnitude.

Furthermore, we present the results of the standard full-sample time-series GRS test of Gibbons et al. (1989). Its hypothesis is that the alphas of all 75 portfolios are jointly zero. Consistent with Fama & French (2015), this test strongly rejects the model with a test statistic of 2.48 with monthly data and a p-value below 0.001. The average absolute alpha of the portfolios is 0.95% per month, and 31% of the portfolios have individually statistically significant alphas. When using daily return data, the numbers are of similar magnitudes. Thus, the time-series test clearly rejects the Fama & French (2015) 5-factor model, even for its base portfolios.

It would be premature, though, to dismiss the model right away based on this result. As discussed in Section II.D, there are various possible reasons why the time-series GRS test may reject a well-specified model: (i) its statistical properties of the GRS test are not optimal, (ii) the basic time-series GRS test does not allow for time-variation in factor betas, (iii) it uses information investors could not have had in real time, which may lead to a look-ahead bias, and (iv) the time-series alphas can be non-zero simply because the historical average factor return differs from the unobservable factor risk premium. In order to account for these issues, we next perform the cross-sectional test.

As a further benchmark, we perform the cross-sectional test with full-sample beta estimates, either based on monthly or on daily data. Note that this approach still suffers from time-invariant betas and a look-ahead bias. However, it allows the factor risk premia to differ from the time-series average returns. The results are in the first two columns of Panel B of Table 2. We detect some modest support for all factors except for the market. With full-sample betas based on monthly data, the estimated HML and RMW risk premia are significantly positive at the 5% level. Those for SMB and CMA are at least significantly positive with respect to the 10% level. However, for all factors except for SMB, the risk premium point estimates are somewhat smaller than the average realized factor excess return. With daily return data, the support for the model is a bit more modest. Only the HML factor yields a significant risk premium.

The common theme across both specifications is that the intercept term is very high, with 7.67% for monthly betas and 9.37% with daily data. Thus, even for its base portfolios (but with potentially look-ahead-biased constant beta estimates), the Fama & French (2015) 5-factor model creates a huge common mispricing component and, thus, fails the main part of the cross-sectional test. One might argue that the zero-beta rate could exceed the risk-free rate, implying that the intercept does not necessarily have to be zero. However, even in such a case one would expect the intercept to be only mildly positive and surely not exceeding 7.67% on an annualized basis.

However, as mentioned above, both the look-ahead bias and the lack of accounting for potential time-variation in betas may have a strong impact on these full-sample results. Thus, next we consider rolling-window beta estimates. With the classical 60-month monthly estimator ("M"), we obtain an intercept of 10.2% per year, which is highly statistically significant (t-stat > 4). With 60-month daily betas ("D"), for which we have roughly 21 times as many data points to obtain more precise portfolio beta estimates, the results are overall similar. Finally, for beta estimates based on a shorter historical window of 12 months ("D^{12mon}"), which likely yields more precise estimates in the presence of time-varying betas, the same holds true. The intercept estimates are 10.43% and 9.66%, respectively. However, the risk premium estimates are overall of similar magnitudes to those for the full-sample estimates. The HML and RMW risk premia are significantly positive for all beta estimation specifications. The CMA risk premium is for two and the SMB premium for one.[11]

Given that the results for all specifications are qualitatively similar, we focus the remainder of the paper on the approach with rolling historical betas. It provides a "real-time" test of the factor model avoiding a look-ahead bias, allows for time-varying betas, while permitting the factor risk premia to differ from the average excess returns. The results of Table 2 suggest that the use of shorter beta estimation windows do not adversely affect the outcome of the cross-sectional test.

Another commonality across all specifications examined so far is that the estimated market risk premium is negative, between −1.21% and −3.96%, and not statistically significant.

[11] In Table A1 of the Online Appendix, we further present the results for 6-month and 36-month daily as well as 60-month quarterly estimation windows. The results are qualitatively similar to those of Table 2.

This finding echoes, in a multifactor setup, the finding of previous studies that the market factor performs very poorly when it comes to explaining cross-sectional differences in stock returns (see, among many others, Fama & French, 1992; Frazzini & Pedersen, 2014). On the other hand, it is well known that the market factor is important for explaining time-series variation in, and hence the level of, returns (Fama & French, 1993).

We can therefore conclude thus far that the Fama & French (2015) 5-factor model, although it is ultimately rejected, explains its base portfolios reasonably well. The intercept is way too large and the market factor is clearly not helpful for explaining cross-sectional differences in returns. However, the remaining factor betas are clearly valuable for explaining the cross-section of the Fama–French portfolio returns.

B Alternative Estimation Schemes

It is possible that the reasons for the ultimate model rejection are of a technical nature. Improved estimation schemes may mend the remaining shortcomings. Thus, we next examine the impact of a changed test design. First, Grauer & Janmaat (2010) argue that portfolios do not provide sufficient variation in population betas, making the cross-sectional test less powerful to reject the null hypothesis of zero factor risk premia. To account for this possibility, we follow the authors' approach and repackage the portfolios to increase the spread (see Section II.E for further details).

We present these results in Table 3 (column heading "OLS-GJ"). The main finding is that the repackaging does not improve but does rather impair the model's performance. The intercept remains large and statistically significant. In most cases, the risk premium estimates are even smaller for the repackaged data than for the original data and the market risk premium turns out significantly negative in two out of three cases.

Next, we turn to using an alternative GLS estimator of Equation (5). Relying less strongly on the most noisy portfolios may improve the efficiency of the risk premium estimates. Presenting these results in the middle columns of Table 3, though, we find that there is only a small improvement in using GLS instead of OLS. In fact, the intercept is still large, between 8.64% and 9.69% per year, and the risk premium estimates are generally smaller and less statistically significant. Thus, using the GLS instead of the OLS estimator does not salvage the ultimate rejection of the Fama & French (2015) model.

Finally, we also consider the case without intercept. This specification directly imposes the theoretical restriction of the factor models and is recommended, e.g., by Cochrane (2005). For perfectly specified models, it should not make a difference whether the model is estimated with or without intercept, as it is equal to zero anyway. In this case, the risk premium estimates would simply be more efficient. However, the previous results indicate that the Fama & French (2015) 5-factor model is severely misspecified, leading to a large positive common intercept coefficient. It is thus interesting to see what happens when constraining the common intercept to zero.

We present the results in the final three columns of Table 3. We find that indeed, the risk premium estimates are much larger without common intercept. In most cases, the point estimates are close to and statistically indistinguishable from the average factor excess returns (see Table 2). Even the market factor yields a significant positive risk premium estimate. However, these significant positive risk premium estimates need not to imply a correct model specification. Instead, the coefficient that can signal model misspecification most clearly is simply forced to zero.

Nevertheless, it is still interesting to examine the pricing errors of the portfolios as can be obtained from average regression residual components. In Table 3, we present the average absolute pricing errors in the line denoted as "$Intercept$". We find that these are still around 1% per annum. A χ^2-test strongly rejects the hypothesis that all average pricing errors are jointly zero with p-values smaller than 0.001. Even individually, between 23% and 31% of the average pricing errors are statistically significant. Thus, also when forcing the common mispricing to zero there is some support for the Fama & French (2015) 5-factor model when tested with its base portfolios. However, this support is not exactly overwhelming.

C Alternative Test Portfolios

Another possible reason for why performance of the Fama & French (2015) 5-factor model may not be outstanding is that its factors could be weak (Bryzgalova, 2015). That is, for factors that have small covariances with test asset returns, inference about risk premia can be problematic. The OLS-GJ repackaging may not be sufficient since it focuses mainly on the market factor. To examine whether the weak-factor problem affects the model performance, we next examine a set of factor-beta-sorted portfolios. By definition, these have large cross-sectional dispersion in (ex-ante) factor betas.

We present the results in Panel A of Table 4. Somewhat surprisingly, we find that the Fama & French (2015) 5-factor model performs even worse for the factor-beta-sorted portfolios. The intercept coefficients are somewhat smaller than for the Fama–French base portfolios examined so far, ranging between 4.64% and 6.81%. However, the risk premium estimates are generally also smaller and less statistically significant. The reason for this is likely that the factor betas may pick up even more unpriced factor risk in the factor-beta-sorted portfolios (Daniel et al., 2020b). Overall, we can conclude that it is unlikely that the weak-factor problem causes any of the failures of the Fama & French (2015) 5-factor model in the cross-sectional asset pricing test.

Having so far examined the Fama & French (2015) 5-factor model within its home territory of the factor base portfolios, we next examine its performance in a less friendly environment. That is, we test the model for the set of anomaly long–short portfolios of Chen & Zimmermann (2020). We present these results in Panel B of Table 4. For this set of portfolios, the failure of the Fama & French (2015) 5-factor model becomes much more apparent. As is natural when moving from long-only portfolios examined thus far to the long–short portfolios considered now, the intercept estimates are somewhat smaller. However, they are still economically large in all cases and clearly statistically significant. Even more importantly, barely any of the risk premium estimates are significantly positive. Even when the common intercept coefficient is forced to be zero, the average absolute pricing error is 3.52% per annum, with 50% of the individual pricing errors being significantly different from zero. Only 2 out of the 5 factors yield significant risk premium coefficients even without common intercept. Thus, based on the anomaly long–short portfolios, the case against the Fama & French (2015) 5-factor model becomes particularly strong.

Consistent with our results, Liao & Liu (2022) develop a Fama & MacBeth (1973) regression approach that treats EIV biases as return innovations and show, based on their slightly different methodology, that the Fama & French (2015) factors cannot explain more than 50% in a set of anomaly returns. Liao & Liu (2022), however, do not consider alternative factor models.

Next, we examine the relation between the intercept and slope t-statistics and the number of anomalies considered. We start with a setup with (almost) no model misspecification: including only the factor long and short portfolios as well as the market portfolio. Then, we gradually expand the set of anomalies by randomly adding the long–short portfolios of Chen & Zimmermann (2020) one by one without replacement. We create 1,000 samples/paths for the anomaly inclusion.

We present the results in Figure 1. To establish the validity of the test procedure, we first look at the benchmark case without any additional anomaly variables. Indeed, in this setup the Fama & French (2015) 5-factor model passes the cross-sectional test: The intercept is statistically insignificant while the extra-market factors on average generate statistically significant risk premium estimates. Adding one anomaly long–short return changes little in the model performance. However, already when 2 to 3 anomalies are contained in the test set, the intercept parameters typically turn to be significantly positive and the slope parameters on average turn statistically insignificant. The slope t-statistics converge to low levels below one once 20–30 anomalies are included. The intercept t-statistics continue to increase when adding more anomalies. Thus, model misspecification plays an important role for the poor performance of the Fama & French (2015) 5-factor model.

D Beta Estimators

For the final part of our portfolio analysis, we turn to alternative beta estimators. The penultimate remaining possibility to salvage the Fama & French (2015) 5-factor model is that the factor betas are indeed time-varying and that this time-variation is not accurately reflected by the simple historical estimator. Thus, we now consider several alternative beta estimators that may yield more accurate forecasts. In particular, we use several shrinkage estimators (HIST^V, HIST^K, and LW), exponential weighting approaches (EWMA and EWMA^{ex}), two estimators correcting for asynchronous trading (Dim and SW), and a beta forecast combination approach (COMB). We describe the details of these alternatives in Appendix A. The results are presented in Table 5. For all estimators, we rely on 12-month daily estimation windows and use the anomaly long–short portfolios.[12]

We find that the choice of estimator has an effect on the risk premium point estimates. In particular, applying shrinkage to the beta estimates tends to increase them. For example, with the LW estimator, the risk premium estimates on HML (1.03% vs. 0.58%), RMW (3.95% vs. 2.41%), and CMA (1.05% vs. 0.52%) are substantially higher. However, this effect is largely mechanical and the standard errors rise along with the risk premium esti-

[12]We also repeat the analysis for individual stocks with qualitatively similar results; see Tables A2 and A3 of the Online Appendix.

mates.[13] Hence, there is no appreciable increase in statistical significance. The EWMA estimators, as well as those that correct for infrequent trading, do not improve things markedly either. Most importantly, for all beta estimators, the estimated intercept coefficients are between 3.36% and 3.90% per annum and highly statistically significant. Thus, independently of how the betas are estimated, the cross-sectional test clearly rejects the Fama & French (2015) 5-factor model.

E Analysis on the Stock Level

Based on the findings of Ang et al. (2020) that the portfolio approach is less powerful than an analysis at the stock level, we lastly test the Fama & French (2015) 5-factor model for individual stocks. We present these results in Table 6.[14]

Starting with the standard 60-month monthly OLS approach, we find that the results are not materially different for the model than with the portfolio approach. For individual stocks, the point estimates of both the MKT and SMB premia are negative (although the estimates are not statistically significant). The estimated RMW and CMA risk premia are also smaller than for the portfolio approach. Only the HML premium is statistically significant at 5%. The point estimate, though, is still less than half the average factor excess return (1.83% vs. 3.90%). Most importantly, the intercept estimate of 8.53% per annum is similar in magnitude to that of the portfolio approach.

Thus, with monthly data and individual stocks, we still clearly reject the Fama & French (2015) 5-factor model. The same is true for 60-month and 12-month daily beta estimates.

[13]Intuitively, think of a 1-factor model. The return–beta line is found to be too flat. Shrinkage pushes the most extreme estimates (both the largest and the smallest) more toward the center. With the more concentrated data, the slope of the return–beta line will increase. A similar mechanism applies with a multifactor model.

[14]Note that for this part, we have to skip the model misspecification part in the Kan et al. (2013) standard errors. For the large cross-sections of individual stocks, the approach requires the inversion of near-singular matrices, which occasionally leads to implausibly large standard error estimates. Additionally accounting for this part only makes the failure of factor models in the cross-sectional test stronger.

The intercept estimates are economically large, with t-statistics exceeding 4. Overall, the model appears to work best with 12-month daily beta estimates. In this case, both the HML and RMW risk premium estimates are statistically significant at the 5% level (albeit with magnitudes still less than half that of the average factor excess returns).[15]

It is possible that the Fama & French (2015) 5-factor model works well for big stocks, but fails mainly among small stocks. Larger companies are typically traded more frequently and have more liquid stock markets. Thus, it might be easier to precisely estimate the factor sensitivities for these than it is for smaller stocks. To account for this possibility, we estimate Equation (5) with WLS instead of OLS, weighting the stocks each month by their lagged market capitalizations.

Presenting these results in Table 6, we find that the results change only little when moving from an OLS to a WLS estimation. The only major difference is that the SMB risk premium estimates turn from negative to positive. When using daily data, these are marginally positively significant (at the 10% level). However, the increase in the SMB premium when estimated with WLS comes at the cost of a reduced and insignificant HML premium. The intercept term, for the 12-month daily approach, still amounts to 5.99% per annum and is highly statistically significant.

Finally, we turn to the bias-corrected OLS estimation approach. The bias-correction is designed to mend the attenuation bias that results from the use of estimated betas as exogenous regressors in Equation (5). In the presence of measurement errors in the betas, the OLS slope estimates are biased downward. Such downward-bias would be consistent with our observation that the estimated factor risk premia are far lower than the corresponding average factor excess returns.

We present the results for the bias-corrected OLS approach in the final columns of Table

[15]In Table A4 of the Online Appendix, we repeat the analysis with alternative daily estimation windows as well as with an estimator based on quarterly data. For all specifications, the results are qualitatively similar to those presented in Table 6.

6. Consistent with the notion that the OLS risk-premium estimates are affected by an attenuation bias, we find that the bias-corrected OLS risk-premium estimates are generally larger than with the plain OLS approach. However, the magnitude of the increase is far too low to salvage the Fama & French (2015) 5-factor model. For example, for the 12-month daily beta estimates, the bias-corrected risk premia on HML and RMW are still more than one third lower than the corresponding average factor excess returns. More importantly, none of the other factor risk premia comes even close to being significant and the intercept estimates remain at similar magnitudes for the bias-corrected to those for the plain OLS approach.

To some extent, our conclusions differ from those drawn in Raponi et al. (2020). The authors use contemporaneous (ex-post) betas estimated over several time windows and find that the risk premium estimates for all factors are significant, at least for part of the sample period. There are two important reasons for these differences. First, we use ex-ante rather than ex-post beta estimates. Thus, while their test is in-sample, we perform a test of the model that relies on information available in real time only. Second, and more importantly, Raponi et al. (2020) count both positive and negative significance. The sign-changing behavior observed in their paper is consistent with insignificant unconditional factor risk premia over the full sample that we observe. Since Fama & French (2015) construct their factors with unconditionally positive excess returns, it is difficult to argue that a significant negative risk premium estimate provides support for the model. Chordia et al. (2017) perform a regression similar to Equation (5) for individual stocks. However, in addition to the factor betas they also include several characteristics. The inclusion of these characteristics seems to affect the risk premium estimates. For example, they also obtain a statistically significant CMA premium. However, consistent with ours, their overall conclusion is that stock characteristics explain more of the variation in expected returns than the factor sensitivities do. Ultimately, this type of test is just another way to reject the model.

F Interpretation

Having documented the failure of the Fama & French (2015) 5-factor model in the cross-sectional test, we next turn to the following questions: What are potential causes of this result? How can we interpret it? Can the model still be used for asset pricing purposes?

Start with the first question: Why does the model fail the cross-sectional test? The most obvious answer would be that the model is only an approximation of reality and simply not perfect. Indeed, Fama & French (2015) also show (and we confirm it in Table 2) that their model is rejected by the time-series GRS test. Our results show that this result is not caused by the statistical properties of the time-series test. Neither does it seem to be due to look-ahead bias, failure to account for time-variation in factor betas, or deviations of factor risk premia from their time-series means. The model is clearly misspecified and assets are mispriced relative to the model by a large common amount. Thus, the Fama & French (2015) 5-factor model is clearly inconsistent with the APT.

Related to the second question, Welch (2008) delivers an interpretation for the generally low estimates of the factor risk premia. The cross-sectional risk premium estimates are upward-biased if the historical factor performance exceeds a factor's true risk premium. Similarly, the risk premium estimates are downward-biased if the factors perform worse than expected. Thus, the low risk premium estimates we obtain would be consistent with a worse-than-expected performance of the factor portfolios. We are skeptical about this explanation, though, mainly because the factors were chosen based on their past performance. It is thus more likely that the historical factor performance was better than that it was worse than expected.

Hence, two important questions remain. First, when aiming to control for the Fama & French (2015) factors in a cross-sectional test, which econometric design should one use? It appears to make sense to use individual stocks and daily data along with the attenuation-bias-correction. Shrinkage or an exponential weighting scheme in the beta estimation also appear to strengthen the risk premia. The bigger question, though, is: Should we use the model at all? The answer, in part, depends on whether there are better alternatives.

IV Alternative Factor Models

In this section, we turn toward alternative factor models. First, we examine the Stambaugh & Yuan (2017) 4-factor model. Afterwards, we also confront the Fama & French (2018) 6-factor model, the Daniel et al. (2020a) 3-factor model, the modified Fama & French (2015) 5-factor model of Daniel et al. (2020b), and the Hou et al. (2021) 5-factor model with the cross-sectional test. To keep the presentation manageable, for each model, we limit ourselves to the anomaly portfolios and individual stocks. We first present the time-series test results and then focus on the cross-sectional test generally based on a 12-month daily horizon for beta estimation and present the results only for three of the alternative beta estimation methods: (i) a shrinkage (LW), (ii) an exponential weighting (EWMA^{ex}), and (iii) an infrequent-trading approach (Dim).

A The Stambaugh & Yuan (2017) 4-Factor Model

The results for the Stambaugh & Yuan (2017) 4-factor model are in Table 7.[16] While the average factor excess returns are large and strongly statistically significant, the model is clearly rejected by the time-series GRS test. More than 40% of the anomaly long–short returns cannot be explained by the model. The results for the cross-sectional test are overall similar to those of the Fama & French (2015) 5-factor model: (i) the intercept is economically large and statistically highly significant, (ii) the factor risk premia are often not statistically

[16]The factor model contains a market factor (MKT), a size factor (SMB^{SY}), a management factor ($MGMT$), and a performance factor ($PERF$). The $MGMT$ and $PERF$ factors result from sorts on the average stock ranks to various anomalies. The MKT and $MGMT$ factors have a correlation of -0.524.

significant, and (iii) they are usually clearly smaller than the average factor returns. We reach this conclusion independently of whether we use portfolios or stocks as test assets, for all estimation methods, and for all beta estimation methods. The model's $MGMT$ and $PERF$ factors, though, obtain some support. The risk premium estimates are statistically significant for many specifications. In addition, for $PERF$ the point estimates are quite close to the average factor returns, at least for the anomaly portfolio set. For pricing individual stocks, the $PERF$ factor works clearly less well.

B The Fama & French (2018) 6-Factor Model

We show the results for the Fama & French (2018) 6-factor model in Table 8.[17] The results are overall similar to those for the Fama & French (2015) 5-factor model. The intercept is economically large and highly statistically significant and most factor risk premia are indistinguishable from zero. The RMW^{Cash} factor seems to be more strongly priced than the normal RMW factor and the WML factor is helpful in explaining the anomaly portfolio returns (although not those of individual stocks). However, this is by far not enough to salvage the model.

C The Daniel et al. (2020a) 3-Factor Model

The results for the Daniel et al. (2020a) 3-factor model can be found in Table 9.[18] The results are qualitatively similar to those for the other models. The intercept coefficient is large and statistically significant. While the FIN and $PEAD$ factors obtain some support,

[17]The factor model contains a market factor (MKT), the size (SMB) and investment (CMA) factors, as well as a monthly value factor (HML^M), a cash operating profitability factor (RMW^{Cash}), and a momentum factor (WML). The monthly value factor uses the most recent market capitalization for the book-to-market ratio (Asness & Frazzini, 2013). The RMW^{Cash} uses a cash-based definition of operating profitability. The correlation between HML^M and WML is -0.647.

[18]The factor model contains a market factor (MKT), a financing factor (FIN) and a post-earnings-announcement-drift factor ($PEAD$). The financing factor is based on two variables capturing equity issues. The correlation between MKT and FIN is -0.50.

it is not entirely consistent across all specifications.

D The Modified Fama & French (2015) 5-Factor Model of Daniel et al. (2020b)

Next, we present the results for the Fama & French (2015) 5-factor model modified to hedge out unpriced factor risk by Daniel et al. (2020b) in Table 10. The factors are the same as for the Fama & French (2015) 5-factor model, only they are optimally combined with so-called hedge portfolios that are long low-factor-beta portfolios and short high-factor-beta portfolios. The results are very similar to those for the main Fama & French (2015) 5-factor model. The intercept estimates are strongly statistically significant and the risk premium estimates are almost all close to, and not significantly different from, zero. Thus, hedging out unpriced factor risk also does not salvage the Fama & French (2015) 5-factor model.

E The Hou et al. (2021) 5-Factor Model

We present the results for the Hou et al. (2021) 5-factor model in Table 11.[19] Overall, the results are also similar to those for the other models. The intercept coefficients are economically large and statistically significant and the estimated factor risk premia are generally substantially smaller than the average factor returns. The expected growth factor is the only one that yields a quite consistently significant risk premium estimate.

F Principal Components Factor Model

Finally, we analyze a PCA factor model. We construct the PCA factors based on the anomaly return dataset following Haddad et al. (2020). That is, we use the Chen & Zimmer-

[19]The factor model contains a market factor (MKT), a size factor (SMB^{HXZ}), an investment factor (INV), a profitability factor (ROE), and an expected growth/investment factor (EG). The correlation of 0.505 between ROE and EG is noteworthy.

mann (2020) anomaly long–short returns available over the full sample period, market-adjust and re-scale them to have equal volatilities, and obtain the first five principal components (PCs).[20] We then use these five PCs along with the market as another possible model. The results are in Table 12. There is no big difference to the other factor models. The intercepts are of similar magnitudes and the majority of PC-factors generate statistically insignificant risk premium estimates.

V Conclusion

Testing the Fama & French (2015) 5-factor model in the cross-section, we uncover three main findings. (i) Intercept coefficients are way too large to be consistent with the APT; (ii) the estimated risk premia of all factors are generally way below their historical average returns; and (iii) they are often not significantly different from zero. These findings also extend well beyond the Fama & French (2015) 5-factor model to the models of Stambaugh & Yuan (2017), Fama & French (2018), Daniel et al. (2020a,b), Hou et al. (2021), and one based on PCA. Thus, all factor models are inconsistent with no-arbitrage pricing.

[20] Note that we have to use full-sample information to obtain these PCs. This is necessary because the economic properties of the different PCs would likely not be constant over time when using a rolling- or expanding-window to obtain out-of-sample PCs, impairing the model performance. Thus, in a realistic environment using real-time data only, the PCs would likely load more on anomalies that become irrelevant later on and the PCA model would likely fare worse than shown here.

Appendix

A Alternative Beta Estimators

- **Shrinkage Beta** Shrinkage beta estimators shrink each element of the historical beta estimate vector ($\beta_{j,t}$) toward one or multiple priors ($b_{i,j,t}$)

$$\beta_{j,t}^{\text{Shr}} = \frac{\beta_{j,t} + \sum_i \frac{\sigma^2_{\beta_{j,t}}}{s^2_{b_{i,j,t}}} b_{i,j,t}}{1 + \sum_i \frac{\sigma^2_{\beta_{j,t}}}{s^2_{b_{i,j,t}}}}. \tag{A1}$$

  $\sigma^2_{\beta_{j,t}}$ and $s^2_{b_{i,j,t}}$ are the squared standard errors of the historical estimates and the prior(s), respectively. Thus, the degree of shrinkage depends on the relative precision of the historical estimates vs. those of the priors. We use two different sets of priors: (i) the cross-sectional average beta (Vasicek, 1973) (HIST^{V}) and (ii) the cross-sectional average beta, the cross-sectional average beta of firms in the same Global Industry Classification Standard (GICS) industry sector, and the cross-sectional average beta of firms in the same size decile (Karolyi, 1992) (HIST^{K}). Note that we compute Equation (A1) separately for each factor of a model.

- **LW Beta** Levi & Welch (2017) argue that the Vasicek (1973) approach does not sufficiently shrink the estimates to create good forecasts for beta. They suggest further shrinkage using

$$\beta_{j,t}^{\text{LW}} = (1 - s)\beta_{j,t}^{V} + s\beta_{j,t}^{\text{target}}, \tag{A2}$$

  where $\beta_{j,t}^{V}$ is the HIST^{V} Vasicek (1973) beta estimate for stock j at time t. Levi & Welch (2017) suggest using $s = 0.25$ for the market factor as well as β_i^{target} to 0.5 for the smallest market capitalization terzile, 0.7 for the middle terzile, and 0.9 for the highest market capitalization terzile. For other factors, they suggest $s = 0.30$ and $\beta_i^{\text{target}} = 0$. We follow these recommendations.

- **EWMA Beta** The EWMA beta estimates also use Equation (4). However, the historical return observations receive an exponentially decaying weight. We use the weighted least squares (WLS) estimator, placing the highest weights on the most recent observations. The weights are $\frac{exp(-|t-\tau|h)}{\sum_{\tau=1}^{t-1} exp(-|t-\tau|h)}$ with $h = \frac{log(2)}{\iota}$. ι characterizes the horizon, to which the half-life of the weights converges for large samples. Following

Hollstein et al. (2019), we set ι to two-thirds of the number of observations of the initial estimation window. We use both a specification with a rolling (EWMA) and one with an expanding (EWMA^{ex}) estimation window. To reduce the computational burden, we limit the maximum historical window to 10 years when using daily data.

- **Dimson Beta (Dim)** Dimson (1979) aims to account for non-synchronous trading by additionally adding betas toward lagged factor returns. For this approach, we use $N = 4$ lags and the regression equation

$$r_{j,\tau} - r_{f,\tau} = \alpha_{j,t} + \beta_{j,t}^{\prime(0)} F_\tau + \beta_{j,t}^{\prime(1)} F_{\tau-1} + \beta_{j,t}^{\prime(2)} \left(\sum_{n=2}^{N} F_{\tau-n} \right) + \epsilon_{j,\tau}. \tag{A3}$$

The Dimson beta estimator is $\beta_{j,t}^{\text{Dim}} = \sum_{i=0}^{\min\{2,N\}} \beta_{j,t}^{(i)}$, where $\min\{\cdot\}$ is the minimum operator.

- **Scholes–Williams Beta (SW)** We also examine the beta estimator of Scholes & Williams (1977). That is, we estimate three separate regressions. The first regression uses the contemporaneous factor returns, exactly as in Equation (4). The second regression uses the lagged factor returns as explanatory variables, that is $r_{j,\tau} - r_{f,\tau} = \alpha_{j,t} + \beta_{j,t}^{\prime -} F_{\tau-1} + \epsilon_{j,\tau}$, and the third regression uses the leaded factor returns, i.e., $r_{j,\tau} - r_{f,\tau} = \alpha_{j,t} + \beta_{j,t}^{\prime +} F_{\tau+1} + \epsilon_{j,\tau}$. Note also that $\beta_{j,t}^{+}$ uses only information available at time t. Finally, for each factor, the final estimator is:

$$\beta_{j,t}^{\text{SW}} = \frac{\beta_{j,t}^{-} + \beta_{j,t} + \beta_{j,t}^{+}}{1 + 2\rho}, \tag{A4}$$

where ρ is the first-order autocorrelation of the respective factor returns. Note that we use Equation (A4) separately for each factor of the model. Asymptotic standard errors for the estimator are provided by Scholes & Williams (1977).

- **Combination Beta (COMB)** Finally, we use a simple equally weighted average of the betas from all approaches. That is, for each stock, we estimate the factor sensitivities for all estimation methods. The COMB estimate for each factor beta is the average across all estimates. Note that, lacking standard error estimates, we cannot use the COMB beta estimate for the bias-corrected OLS regression for individual stocks.

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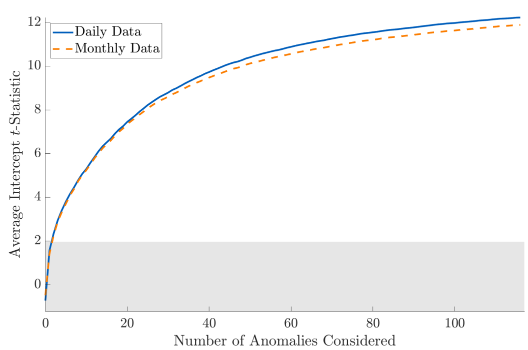
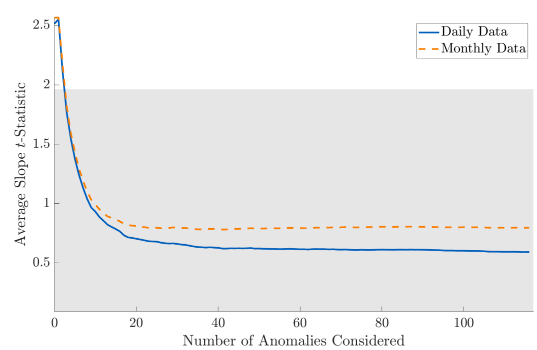
Figure 1: Intercept and Slopes vs. the Number of Anomalies Considered

This figure plots the average t-statistics of the intercept and the extra-market slopes depending on the number of anomalies used. We use as base case the 8 separate long and short portfolios underlying the Fama & French (2015) factors as well as the market portfolio (number of anomalies considered = 0). Then we iteratively add long–short portfolios from the CZ anomaly set. That is, we randomly draw the anomalies without replacement and run the cross-sectional test. We repeat the procedure 1,000 times. The reported lines are the average t-statistics across the 1,000 repetitions of the intercepts (Panels A and C) and the extra-market slope factors (Panels B and D). For the slope t-statistics, we first average over the different factors within one sample and then over the 1,000 random samples. We present the results for betas based on daily (blue solid line) and monthly (orange dashed line) data over the full sample (Panels A and B) and with rolling windows (Panels C and D). The rolling-window length for daily data is 12 months and that for monthly data is 60 months. The shaded gray areas are the 95% non-rejection regions. All t-statistics are based on Kan et al. (2013) standard errors with 5 Newey–West lags.

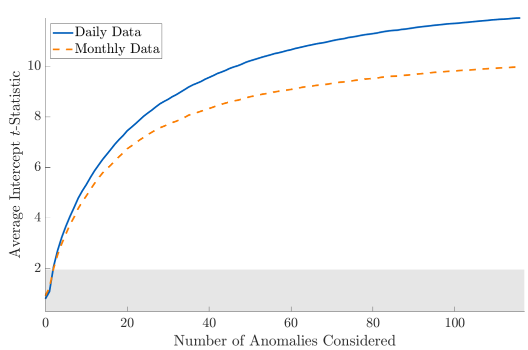
A. Intercept with Full-Sample Betas



B. Slopes with Full-Sample Betas



C. Intercept with Rolling-Window Betas



D. Slopes with Rolling-Window Betas

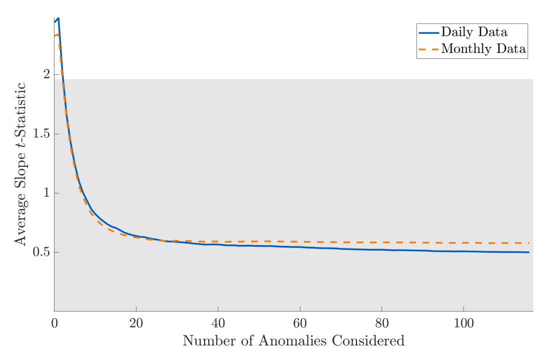


Table 1: Summary Statistics

This table presents summary statistics for the (monthly) returns of (Panel A) and the sensitivities to (Panels B and C) the 5 Fama & French (2015) factors. We present summary statistics of annualized returns in percentage points. "$Mean$" is the average of the factor in the time-series (in Panel A) or the time-series average of the equally weighted cross-sectional mean of beta estimates (in Panels B and C). "SD" is the standard deviation. "$Skew$", "$Kurt$", and "$AR(1)$" are the skewness, kurtosis, and first-order autocorrelation. In Panels B and C, all reported numbers are time-series averages of the cross-sectional statistics. "$Mean_{vw}$" is the value-weighted average. "$q^{0.10}$", "$Median$", and "$q^{0.90}$" denote the 10%, 50%, and 90% quantiles of the cross-sectional beta distributions, respectively. The correlations in Panels B and C are also time-series averages of cross-sectional correlations.

Panel A. Factor Return Summary Statistics

	Summary Statistics					Correlations			
	$Mean$	SD	$Skew$	$Kurt$	$AR(1)$	SMB	HML	RMW	CMA
MKT	6.152	15.21	−0.545	4.980	0.071	0.276	−0.258	−0.225	−0.384
SMB	2.827	10.46	0.365	6.073	0.060		−0.070	−0.347	−0.107
HML	3.898	9.684	0.169	4.970	0.175			0.070	0.696
RMW	3.084	7.571	−0.345	15.28	0.148				−0.026
CMA	3.400	6.918	0.300	4.637	0.121				

Panel B. Summary Statistics of FF Portfolio Beta Estimates

	Summary Statistics								Correlations			
	$Mean_{vw}$	$Mean$	SD	$Skew$	$Kurt$	$q^{0.10}$	$Median$	$q^{0.90}$	SMB	HML	RMW	CMA
12-Month Daily												
MKT	0.999	0.887	0.720	0.221	7.458	0.052	0.851	1.779	0.352	0.150	0.079	0.077
SMB	−0.001	0.710	0.989	0.515	10.14	−0.350	0.611	1.939		0.115	0.212	0.045
HML	−0.001	0.079	1.303	−0.270	12.90	−1.389	0.105	1.514			0.315	−0.361
RMW	0.006	−0.093	1.483	−0.435	16.67	−1.787	−0.013	1.486				0.101
CMA	−0.001	0.019	1.477	0.229	19.70	−1.615	0.032	1.617				
60-Month Monthly												
MKT	0.958	0.898	0.476	0.135	3.186	0.272	0.891	1.513	0.441	0.048	−0.032	−0.028
SMB	0.004	0.712	0.599	0.337	3.582	−0.002	0.676	1.492		0.053	0.110	0.040
HML	0.014	0.085	0.657	−0.235	6.377	−0.676	0.107	0.816			0.384	−0.254
RMW	0.022	−0.086	0.741	−0.842	7.255	−0.977	−0.007	0.693				0.123
CMA	−0.015	−0.003	0.697	−0.232	6.650	−0.805	0.024	0.760				

Table 1: Summary Statistics (continued)

Panel C. Summary Statistics of Stock Beta Estimates

	Summary Statistics								Correlations			
	$Mean_{vw}$	$Mean$	SD	$Skew$	$Kurt$	$q^{0.10}$	$Median$	$q^{0.90}$	SMB	HML	RMW	CMA
12-Month Daily												
MKT	0.999	0.887	0.720	0.221	7.458	0.052	0.851	1.779	0.352	0.150	0.079	0.077
SMB	−0.001	0.710	0.989	0.515	10.14	−0.350	0.611	1.939		0.115	0.212	0.045
HML	−0.001	0.079	1.303	−0.270	12.90	−1.389	0.105	1.514			0.315	−0.361
RMW	0.006	−0.093	1.483	−0.435	16.67	−1.787	−0.013	1.486				0.101
CMA	−0.001	0.019	1.477	0.229	19.70	−1.615	0.032	1.617				
60-Month Monthly												
MKT	0.958	0.898	0.476	0.135	3.186	0.272	0.891	1.513	0.441	0.048	−0.032	−0.028
SMB	0.004	0.712	0.599	0.337	3.582	−0.002	0.676	1.492		0.053	0.110	0.040
HML	0.014	0.085	0.657	−0.235	6.377	−0.676	0.107	0.816			0.384	−0.254
RMW	0.022	−0.086	0.741	−0.842	7.255	−0.977	−0.007	0.693				0.123
CMA	−0.015	−0.003	0.697	−0.232	6.650	−0.805	0.024	0.760				

Table 2: Factor Base Portfolios

This table presents test results for the 75 Fama–French factor base portfolios. In Panel A, we present the annualized time-series average returns of the Fama & French (2015) factors. Newey & West (1987) standard errors with 5 lags are in parentheses. In addition, we show the full-sample average absolute alphas of the portfolios ($avg\ |\overline{\alpha}|$), the share of individually significant portfolio alphas (at 10%) in brackets, and the joint statistical significance of the alphas based on the GRS test (indicated by the stars). In Panel B, we present average coefficient estimates from monthly Fama & MacBeth (1973) regressions. Each month, we use Equation (5) to regress the annualized portfolio excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the Fama & French (2015) 5-factor model. The betas are estimated using Equation (4) along with data from the full sample period or over previous historical windows ("Rolling OLS"). "M" and "D" without superscript indicate that betas are estimated from monthly and daily data, respectively. For the rolling OLS case, the betas are generally based on 60-month windows, while "D^{12mon}" indicates the use of 12 months of daily data. In parentheses, we report the errors-in-variables, model misspecification, as well as heteroskedasticity and autocorrelation robust standard errors of Kan et al. (2013) (with 5 Newey–West lags). *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Panel A. Time-Series Test

	MKT	SMB	HML	RMW	CMA	$avg\ \lvert\overline{\alpha}\rvert$
Monthly	6.152***	2.827*	3.898**	3.084***	3.400***	0.949***
	(2.240)	(1.505)	(1.588)	(1.169)	(1.100)	[30.67]
Daily	6.194***	1.946	4.084***	3.248***	3.453***	1.099***
	(2.148)	(1.198)	(1.217)	(0.915)	(0.883)	[38.67]

Panel B. Cross-Sectional Test

	Full Sample		Rolling OLS		
	M	D	M	D	D^{12mon}
Intercept	7.669**	9.373***	10.18***	10.43***	9.659***
	(3.253)	(3.189)	(2.366)	(2.730)	(2.560)
MKT	−1.210	−2.694	−3.789	−3.963	−3.186
	(3.705)	(3.196)	(2.716)	(2.650)	(2.570)
SMB	2.592*	2.483	1.640	1.460	2.682*
	(1.530)	(1.573)	(1.501)	(1.518)	(1.539)
HML	3.406**	3.347**	3.608**	3.737**	3.190**
	(1.626)	(1.592)	(1.699)	(1.754)	(1.508)
RMW	2.392*	2.060	2.887**	2.801**	2.940**
	(1.349)	(1.516)	(1.243)	(1.423)	(1.210)
CMA	3.019**	2.172	3.013**	2.476*	1.990
	(1.215)	(1.445)	(1.166)	(1.498)	(1.358)

Table 3: Factor Base Portfolios: Different Estimation Methods

This table presents test results for the 75 Fama–French factor base portfolios. We report average coefficient estimates from monthly Fama & MacBeth (1973) regressions. Each month, we use Equation (5) to regress the annualized portfolio excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the Fama & French (2015) 5-factor model. We use three different methods to estimate Equation (5): (i) "OLS-GJ" (where the test assets are repackaged to have a higher dispersion in their market betas), (ii) "GLS" (where we use the full-sample return covariance matrix to weight the observations), and (iii) a specification that omits the common intercept. The betas are estimated using Equation (4) along with data over previous historical windows (rolling OLS). "M" and "D" without superscript indicate that betas are estimated from 60-month windows of monthly and daily data, respectively, while "D^{12mon}" indicates the use of 12 months of daily data. In parentheses, we report the errors-in-variables, model misspecification, as well as heteroskedasticity and autocorrelation robust standard errors of Kan et al. (2013) (with 5 Newey–West lags). For the "OLS No Intercept" specification, we report the average absolute model errors in the intercept line. Significance is based on a joint χ^2 test with the null hypothesis that all average errors are jointly zero. The brackets contain the shares of single significant average pricing errors (at 10%; in percentage points). For the "No Intercept" case, we use Shanken (1992) standard errors. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	OLS-GJ			GLS			OLS No Intercept		
	M	D	D^{12mon}	M	D	D^{12mon}	M	D	D^{12mon}
$Intercept$	7.485***	7.914***	8.575***	8.644***	9.249***	9.686***	0.916***	1.090***	1.125***
	(2.560)	(2.525)	(2.434)	(1.694)	(1.826)	(1.707)	[22.67]	[28.00]	[30.67]
MKT	−1.085	−1.516**	−1.803**	−2.043	−2.705	−2.684	6.009***	6.142***	6.205***
	(0.956)	(0.755)	(0.787)	(1.627)	(1.781)	(1.881)	(2.295)	(2.311)	(2.168)
SMB	1.673	0.780	1.273	0.515	0.260	0.448	2.029	1.749	2.901*
	(1.172)	(1.105)	(1.106)	(0.931)	(1.141)	(1.163)	(1.516)	(1.539)	(1.556)
HML	2.316**	2.130*	2.249**	2.379**	2.358*	1.800	3.984**	4.179**	3.372**
	(1.128)	(1.109)	(1.018)	(1.155)	(1.254)	(1.277)	(1.605)	(1.746)	(1.518)
RMW	1.775*	1.183	1.198	1.612*	1.701*	1.836**	2.977***	3.461***	3.291***
	(1.004)	(1.108)	(0.975)	(0.874)	(0.957)	(0.899)	(1.132)	(1.318)	(1.108)
CMA	1.840*	1.238	0.948	1.663**	1.330	1.271	2.933***	2.923***	2.068*
	(0.985)	(1.175)	(0.982)	(0.770)	(0.937)	(0.966)	(1.035)	(1.100)	(1.088)

Table 4: Alternative Test Portfolios

This table presents test results for the 100 factor-beta-sorted portfolios (Panel A) and 116 anomaly long–short portfolios of Chen & Zimmermann (2020) (Panel B). We present average coefficient estimates from monthly Fama & MacBeth (1973) regressions. Each month, we use Equation (5) to regress the annualized portfolio excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the Fama & French (2015) 5-factor model. The betas are estimated using Equation (4) along with data from the full sample period ("Full") or over previous historical windows ("Roll"). For the rolling OLS case, the betas are generally based 12-month windows of daily data, while Roll-M uses 60 months of monthly data. We generally use OLS but also consider three alternative methods to estimate Equation (5): (i) "OLS-GJ" (where the test assets are repackaged to have a higher dispersion in their market betas), (ii) "GLS" (where we use the full-sample return covariance matrix to weight the observations), and (iii) a specification that omits the common intercept. In parentheses, we report the errors-in-variables, model misspecification, as well as heteroskedasticity and autocorrelation robust standard errors of Kan et al. (2013) (with 5 Newey–West lags). For the "No Intercept" specification, we report the average absolute model errors in the intercept line. Significance is based on a joint χ^2 test with the null hypothesis that all average errors are jointly zero. The brackets contain the shares of single significant average pricing errors (at 10%; in percentage points). For the "No Intercept" case, we use Shanken (1992) standard errors. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Panel A. Beta-Sorted Portfolios

	Full	Roll-M	Roll-D	OLS-GJ	GLS	No Intercept
Intercept	6.609***	4.798**	4.637**	6.810***	5.152***	0.954***
	(1.754)	(2.002)	(1.871)	(2.309)	(1.444)	[26.00]
MKT	−0.594	1.281	1.451	0.003	2.924**	6.068***
	(2.059)	(2.535)	(2.214)	(0.616)	(1.468)	(2.186)
SMB	2.794*	0.980	2.497	1.312	2.129**	2.252
	(1.586)	(1.625)	(1.578)	(1.055)	(0.993)	(1.549)
HML	1.293	3.429**	1.858	1.218	0.326	2.044*
	(1.533)	(1.601)	(1.403)	(0.994)	(0.989)	(1.204)
RMW	4.095***	1.059	1.777	1.185	1.770*	1.440
	(1.413)	(1.274)	(1.377)	(0.931)	(0.916)	(1.113)
CMA	−0.337	2.698*	1.625	0.883	0.516	2.234**
	(1.484)	(1.399)	(1.211)	(0.836)	(0.726)	(1.054)

Table 4: Alternative Test Portfolios (continued)

Panel B. CZ Anomaly Portfolios

	Full	Roll-M	Roll-D	OLS-GJ	GLS	No Intercept
Intercept	4.282***	3.833***	3.821***	4.164***	1.403***	3.518***
	(0.402)	(0.393)	(0.388)	(0.448)	(0.215)	[50.00]
MKT	2.900	0.407	1.703	0.943	0.414	−0.469
	(2.053)	(2.369)	(2.274)	(1.460)	(1.625)	(1.788)
SMB	−3.036	−2.968*	−2.222	−2.039	0.403	−1.252
	(3.594)	(1.701)	(2.409)	(1.848)	(1.331)	(1.932)
HML	−2.388	0.948	0.580	0.590	1.242	2.046
	(1.990)	(1.784)	(1.854)	(1.242)	(0.968)	(1.588)
RMW	2.287	2.361*	2.410	1.904	1.557*	3.311***
	(2.247)	(1.364)	(1.715)	(1.221)	(0.943)	(1.116)
CMA	2.645*	1.795	0.523	0.763	0.418	2.142*
	(1.465)	(1.280)	(1.475)	(1.052)	(0.711)	(1.185)

Table 5: CZ Anomaly Portfolios: The Role of Beta Estimators

This table presents test results for the 116 anomaly long–short portfolios of Chen & Zimmermann (2020). We present average coefficient estimates from monthly Fama & MacBeth (1973) regressions. Each month, we use Equation (5) to regress the annualized portfolio excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the Fama & French (2015) 5-factor model. The betas are estimated with various beta estimation methods (which are described in detail in Appendix A) along with data over previous historical windows. The estimators use daily data over the previous 12 months (the "EWMA^{ex}" estimator gradually expands this window). In parentheses, we report the errors-in-variables, model misspecification, as well as heteroskedasticity and autocorrelation robust standard errors of Kan et al. (2013) (with 5 Newey–West lags). *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	HIST^V	HIST^K	LW	EWMA	EWMA^{ex}	Dim	SW	COMB
Intercept	3.698***	3.581***	3.364***	3.767***	3.778***	3.901***	3.813***	3.691***
	(0.434)	(0.495)	(0.730)	(0.372)	(0.412)	(0.341)	(0.354)	(0.429)
MKT	2.005	2.194	2.674	1.700	1.702	2.709	4.068	2.706
	(2.629)	(2.989)	(4.078)	(2.278)	(2.351)	(2.353)	(2.964)	(2.730)
SMB	−2.209	−2.223	−3.156	−2.067	−2.259	−2.172	−3.158	−2.579
	(2.770)	(3.117)	(4.703)	(2.335)	(2.629)	(1.812)	(1.967)	(2.790)
HML	0.718	0.873	1.025	0.715	0.319	0.899	−0.433	0.209
	(2.127)	(2.396)	(3.524)	(1.800)	(1.953)	(1.583)	(1.963)	(2.190)
RMW	2.764	3.088	3.949	2.535	2.402	2.275*	2.464	2.847
	(2.077)	(2.449)	(3.629)	(1.628)	(2.033)	(1.284)	(1.515)	(2.151)
CMA	0.733	0.958	1.048	0.695	1.237	0.281	0.282	1.030
	(1.804)	(2.160)	(2.969)	(1.421)	(1.473)	(1.175)	(1.670)	(1.799)

Table 6: Individual Stocks

This table presents average coefficient estimates from monthly Fama & MacBeth (1973) regressions for individual stocks. Each month, we use Equation (5) to regress the annualized stock excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the Fama & French (2015) 5-factor model. We use three different methods to estimate Equation (5): (i) "OLS", (ii) "WLS" (where we weight the stocks by their inverse market capitalization for each cross-sectional regression), and (iii) "OLS^{BC}" (where we use the bias-corrected OLS estimator of Equation (6)). The betas are estimated using Equation (4) along with data over previous historical windows. "M" and "D" without superscript indicate that betas are estimated from 60-month windows of monthly and daily data, respectively, while "D^{12mon}" indicates the use of 12 months of daily data. In parentheses, we generally report errors-in-variables as well as heteroskedasticity and autocorrelation robust standard errors of Kan et al. (2013) (with 5 Newey–West lags). For the "OLS^{BC}" approach, we report Fama & MacBeth (1973) standard errors. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	OLS			WLS			OLS^{BC}		
	M	D	D^{12mon}	M	D	D^{12mon}	M	D	D^{12mon}
Intercept	8.529***	9.452***	8.962***	6.429***	7.745***	5.985***	8.663***	9.282***	8.975***
	(1.930)	(1.938)	(2.021)	(1.672)	(1.807)	(1.705)	(1.806)	(1.953)	(1.975)
MKT	−0.524	−1.540	−0.793	−0.282	−1.766	0.032	−0.735	−1.315	−0.735
	(1.329)	(1.623)	(1.410)	(1.724)	(2.271)	(1.874)	(1.415)	(1.654)	(1.444)
SMB	−0.414	−0.328	−0.517	0.951	2.097*	1.858*	−0.319	−0.371	−0.567
	(0.810)	(1.298)	(0.841)	(1.052)	(1.271)	(1.069)	(0.916)	(1.418)	(1.105)
HML	1.825**	2.197	1.834**	1.552	1.336	1.030	2.160**	2.328	2.000**
	(0.788)	(1.382)	(0.826)	(1.150)	(1.426)	(1.144)	(0.872)	(1.480)	(0.906)
RMW	0.342	1.086	1.503**	0.218	1.093	1.723**	0.361	1.322	1.999**
	(0.565)	(1.067)	(0.742)	(0.800)	(1.149)	(0.832)	(0.601)	(1.262)	(0.859)
CMA	0.841	1.867*	1.032	0.726	1.323	0.122	0.795	2.267**	1.161
	(0.569)	(1.008)	(0.737)	(0.822)	(1.120)	(0.925)	(0.628)	(1.089)	(0.818)

Table 7: The Stambaugh & Yuan (2017) 4-Factor Model

This table summarizes the cross-sectional regression results for the Stambaugh & Yuan (2017) 4-factor model. In Panel A, we present the annualized time-series average returns of the Stambaugh & Yuan (2017) factors. Newey & West (1987) standard errors with 5 lags are in parentheses. In addition, we show the full-sample average absolute alphas of the portfolios ($avg\ |\overline{\alpha}|$), the share of individually significant portfolio alphas (at 10%) in brackets, and the joint statistical significance of the alphas based on the GRS test (indicated by the stars). In Panel B, we present average coefficient estimates from monthly Fama & MacBeth (1973) regressions. Each month, we use Equation (5) to regress the annualized portfolio and stock excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the Stambaugh & Yuan (2017) 4-factor model. We use various different methods to estimate Equation (5) as previously described. The betas are generally estimated using Equation (4) along with data over the full sample period or previous historical windows. The estimators use daily data over the previous 12 months (the "EWMA^{ex}" estimator gradually expands this window). The alternative beta estimation methods are described in detail in Appendix A. In parentheses, we generally report errors-in-variables, model misspecification, as well as heteroskedasticity and autocorrelation robust standard errors of Kan et al. (2013) (with 5 Newey–West lags). For the "No Intercept" specification, we report the average absolute model errors in the intercept line. Significance is based on a joint χ^2 test with the null hypothesis that all average errors are jointly zero. The brackets contain the shares of single significant average pricing errors (at 10%; in percentage points). For the "No Intercept" case, we use Shanken (1992) standard errors. For the "OLS^{BC}" approach, we report Fama & MacBeth (1973) standard errors. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Panel A. Time-Series Test

	MKT	SMB^{SY}	$MGMT$	$PERF$	$avg\ \|\overline{\alpha}\|$
Monthly	6.252***	5.376***	6.983***	8.080***	3.578***
	(2.279)	(1.496)	(1.620)	(2.023)	[40.74]
Daily	6.280***	5.486***	6.909***	7.558***	3.405***
	(2.184)	(1.194)	(1.289)	(1.795)	[46.30]

Panel B. Cross-Sectional Test

	Anomaly Portfolios					Beta Adjustment Methods			Stocks		
	Full	OLS	OLS-GJ	GLS	No Intercept	LW	EWMA^{ex}	Dim	OLS	WLS	OLS^{BC}
Intercept	3.737***	3.627***	4.331***	1.404***	3.281***	3.216***	3.356***	3.904***	9.353***	5.429***	9.370***
	(0.356)	(0.347)	(0.415)	(0.226)	[49.07]	(0.619)	(0.344)	(0.336)	(2.108)	(1.761)	(2.041)
MKT	3.640*	2.030	0.744	0.572	0.655	3.009	1.147	2.322	−0.944	0.683	−0.895
	(2.018)	(2.035)	(1.529)	(1.652)	(1.746)	(3.265)	(2.092)	(2.320)	(1.479)	(1.851)	(1.487)
SMB^{SY}	−0.061	0.592	0.180	1.350	2.661	1.040	1.106	−0.710	0.509	2.771**	0.731
	(3.036)	(2.445)	(1.919)	(1.360)	(2.238)	(4.277)	(2.570)	(1.854)	(0.936)	(1.135)	(1.247)
MGMT	3.879**	3.558**	2.525*	1.451	6.385***	5.131**	4.034**	3.117**	3.229***	1.733	4.040***
	(1.633)	(1.653)	(1.413)	(1.083)	(1.605)	(2.457)	(1.728)	(1.557)	(1.044)	(1.166)	(1.178)
PERF	8.229***	7.704***	4.571**	3.384**	8.894***	12.18***	8.742***	5.223**	1.519	2.929*	2.563*
	(2.406)	(2.231)	(1.769)	(1.671)	(2.213)	(3.513)	(2.333)	(2.070)	(1.359)	(1.643)	(1.537)

Table 8: The Fama & French (2018) 6-Factor Model

This table summarizes the cross-sectional regression results for the Fama & French (2018) 6-factor model. In Panel A, we present the annualized time-series average returns of the Fama & French (2018) factors. Newey & West (1987) standard errors with 5 lags are in parentheses. In addition, we show the full-sample average absolute alphas of the portfolios ($avg\ |\overline{\alpha}|$), the share of individually significant portfolio alphas (at 10%) in brackets, and the joint statistical significance of the alphas based on the GRS test (indicated by the stars). In Panel B, we present average coefficient estimates from monthly Fama & MacBeth (1973) regressions. Each month, we use Equation (5) to regress the annualized portfolio and stock excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the Fama & French (2018) 6-factor model. We use various different methods to estimate Equation (5) as previously described. The betas are generally estimated using Equation (4) along with data over the full sample period or previous historical windows. The estimators use daily data over the previous 12 months (the "EWMA^{ex}" estimator gradually expands this window). The alternative beta estimation methods are described in detail in Appendix A. In parentheses, we generally report errors-in-variables, model misspecification, as well as heteroskedasticity and autocorrelation robust standard errors of Kan et al. (2013) (with 5 Newey–West lags). For the "No Intercept" specification, we report the average absolute model errors in the intercept line. Significance is based on a joint χ^2 test with the null hypothesis that all average errors are jointly zero. The brackets contain the shares of single significant average pricing errors (at 10%; in percentage points). For the "No Intercept" case, we use Shanken (1992) standard errors. For the "OLS^{BC}" approach, we report Fama & MacBeth (1973) standard errors. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Panel A. Time-Series Test

	MKT	SMB	HML^M	RMW^{Cash}	CMA	WML	$avg\ \|\overline{\alpha}\|$
Monthly	6.152***	2.827*	3.341*	4.352***	3.400***	7.959***	3.852***
	(2.240)	(1.505)	(1.810)	(0.983)	(1.100)	(2.122)	[48.28]
Daily	6.136***	2.026*	3.056**	4.505***	3.474***	7.748***	3.708***
	(2.150)	(1.200)	(1.415)	(0.806)	(0.884)	(1.810)	[57.76]

Panel B. Cross-Sectional Test

	Anomaly Portfolios					Beta Adjustment Methods			Stocks		
	Full	OLS	OLS-GJ	GLS	No Intercept	LW	EWMA^{ex}	Dim	OLS	WLS	OLS^{BC}
Intercept	3.608***	3.272***	4.030***	1.356***	2.980***	2.940***	3.162***	3.506***	9.227***	6.137***	9.273***
	(0.340)	(0.314)	(0.446)	(0.219)	[48.28]	(0.508)	(0.324)	(0.312)	(2.042)	(1.820)	(2.001)
MKT	1.876	0.650	0.187	0.210	−1.468	1.047	−0.115	2.083	−1.106	−0.127	−1.041
	(2.024)	(1.988)	(1.535)	(1.603)	(1.773)	(3.224)	(2.030)	(2.167)	(1.398)	(1.860)	(1.415)
SMB	−3.912	−1.489	−1.893	0.233	−0.627	−2.129	−1.551	−1.692	−0.215	1.513	−0.302
	(2.904)	(2.087)	(1.708)	(1.277)	(1.865)	(3.800)	(2.194)	(1.668)	(0.849)	(1.044)	(1.021)
HML^M	−0.817	1.391	0.360	1.825	2.558	2.358	0.854	1.206	2.195**	1.496	2.284**
	(2.019)	(1.600)	(1.147)	(1.351)	(1.742)	(3.286)	(1.733)	(1.632)	(1.066)	(1.342)	(1.143)
RMW^{Cash}	0.874	3.263***	2.584**	1.917**	4.212***	5.133**	3.141**	2.825***	1.779***	1.528**	2.304***
	(1.631)	(1.183)	(1.145)	(0.844)	(0.888)	(2.362)	(1.217)	(1.060)	(0.661)	(0.723)	(0.724)
CMA	1.259	0.958	0.828	0.628	2.412**	1.772	0.919	1.137	1.074	−0.024	1.215
	(1.373)	(1.280)	(0.976)	(0.742)	(1.176)	(2.305)	(1.289)	(1.232)	(0.743)	(0.907)	(0.769)
WML	8.499***	5.432***	2.980**	0.721	6.968***	9.020**	6.356***	4.056**	−0.067	1.402	0.153
	(1.984)	(1.915)	(1.378)	(1.536)	(1.909)	(3.514)	(2.005)	(1.813)	(1.334)	(1.563)	(1.448)

Table 9: The Daniel et al. (2020a) 3-Factor Model

This table summarizes the cross-sectional regression results for the Daniel et al. (2020a) 3-factor model. In Panel A, we present the annualized time-series average returns of the Daniel et al. (2020a) factors. Newey & West (1987) standard errors with 5 lags are in parentheses. In addition, we show the full-sample average absolute alphas of the portfolios ($avg\ |\overline{\alpha}|$), the share of individually significant portfolio alphas (at 10%) in brackets, and the joint statistical significance of the alphas based on the GRS test (indicated by the stars). In Panel B, we present average coefficient estimates from monthly Fama & MacBeth (1973) regressions. Each month, we use Equation (5) to regress the annualized portfolio and stock excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the Daniel et al. (2020a) 3-factor model. We use various different methods to estimate Equation (5) as previously described. The betas are generally estimated using Equation (4) along with data over the full sample period or previous historical windows. The estimators use daily data over the previous 12 months (the "EWMA^{ex}" estimator gradually expands this window). The alternative beta estimation methods are described in detail in Appendix A. In parentheses, we generally report errors-in-variables, model misspecification, as well as heteroskedasticity and autocorrelation robust standard errors of Kan et al. (2013) (with 5 Newey–West lags). For the "No Intercept" specification, we report the average absolute model errors in the intercept line. Significance is based on a joint χ^2 test with the null hypothesis that all average errors are jointly zero. The brackets contain the shares of single significant average pricing errors (at 10%; in percentage points). For the "No Intercept" case, we use Shanken (1992) standard errors. For the "OLS^{BC}" approach, we report Fama & MacBeth (1973) standard errors. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Panel A. Time-Series Test

	MKT	FIN	$PEAD$	$avg\ \lvert\overline{\alpha}\rvert$
Monthly	6.442**	8.954***	7.426***	3.896***
	(2.505)	(2.278)	(1.215)	[42.65]
Daily	6.477***	8.397***	7.318***	3.982***
	(2.427)	(1.660)	(0.927)	[52.21]

Panel B. Cross-Sectional Test

	Anomaly Portfolios					Beta Adjustment Methods			Stocks		
	Full	OLS	OLS-GJ	GLS	No Intercept	LW	EWMA^{ex}	Dim	OLS	WLS	OLS^{BC}
Intercept	3.605***	3.522***	4.299***	1.217***	3.518***	2.893***	3.500***	3.649***	9.583***	7.494***	9.526***
	(0.532)	(0.383)	(0.412)	(0.221)	[44.12]	(0.712)	(0.407)	(0.401)	(2.408)	(1.907)	(2.333)
MKT	2.624	2.659	2.585	1.017	−0.003	3.898	3.418	3.036	−0.293	−0.573	0.603
	(2.038)	(2.502)	(1.694)	(1.615)	(2.182)	(4.284)	(2.604)	(2.694)	(1.683)	(2.215)	(1.832)
FIN	5.142*	5.586*	3.260	1.920	7.873***	9.128	6.984**	3.729	3.494**	1.733	6.044**
	(2.690)	(2.945)	(2.299)	(1.416)	(2.508)	(6.014)	(3.384)	(2.455)	(1.579)	(1.592)	(2.375)
PEAD	8.532**	6.459***	4.052**	1.536	7.185***	12.19*	8.624***	5.392***	0.685	1.851*	3.605**
	(3.748)	(2.397)	(1.602)	(1.000)	(1.965)	(6.232)	(2.674)	(1.570)	(0.602)	(1.037)	(1.535)

Table 10: The Daniel et al. (2020b) 5-Factor Model

This table summarizes the cross-sectional regression results for the Daniel et al. (2020b) version of the Fama & French (2015) 5-factor model. In Panel A, we present the annualized time-series average returns of the Daniel et al. (2020b) factors. Newey & West (1987) standard errors with 5 lags are in parentheses. In addition, we show the full-sample average absolute alphas of the portfolios ($avg\ |\overline{\alpha}|$), the share of individually significant portfolio alphas (at 10%) in brackets, and the joint statistical significance of the alphas based on the GRS test (indicated by the stars). In Panel B, we present average coefficient estimates from monthly Fama & MacBeth (1973) regressions. Each month, we use Equation (5) to regress the annualized portfolio and stock excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the Daniel et al. (2020b) 5-factor model. We use various different methods to estimate Equation (5) as previously described. The betas are generally estimated using Equation (4) along with data over the full sample period or previous historical windows. The estimators use daily data over the previous 12 months (the "EWMA^{ex}" estimator gradually expands this window). The alternative beta estimation methods are described in detail in Appendix A. In parentheses, we generally report errors-in-variables, model misspecification, as well as heteroskedasticity and autocorrelation robust standard errors of Kan et al. (2013) (with 5 Newey–West lags). For the "No Intercept" specification, we report the average absolute model errors in the intercept line. Significance is based on a joint χ^2 test with the null hypothesis that all average errors are jointly zero. The brackets contain the shares of single significant average pricing errors (at 10%; in percentage points). For the "No Intercept" case, we use Shanken (1992) standard errors. For the "OLS^{BC}" approach, we report Fama & MacBeth (1973) standard errors. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Panel A. Time-Series Test

	MKT	*SMB*	*HML*	*RMW*	*CMA*	$avg\ \|\overline{\alpha}\|$
Monthly	5.828***	2.036**	2.602***	2.630***	2.394***	4.926***
	(1.766)	(0.960)	(0.998)	(0.741)	(0.710)	[54.31]
Daily	6.194***	1.946	4.084***	3.248***	3.453***	4.515***
	(2.148)	(1.198)	(1.217)	(0.915)	(0.883)	[58.62]

Panel B. Cross-Sectional Test

	Anomaly Portfolios					Beta Adjustment Methods			Stocks		
	Full	OLS	OLS-GJ	GLS	No Intercept	LW	EWMA^{ex}	Dim	OLS	WLS	OLS^{BC}
Intercept	4.255***	3.706***	4.649***	1.447***	3.473***	2.555*	3.804***	3.832***	8.373***	6.153***	8.372***
	(0.527)	(0.512)	(0.573)	(0.227)	[48.28]	(1.504)	(0.540)	(0.458)	(2.354)	(1.864)	(2.250)
MKT	2.332	4.056	2.834	−0.050	3.016	6.961	2.105	3.999	1.598*	0.245	1.975*
	(6.278)	(4.594)	(3.116)	(1.458)	(2.484)	(12.53)	(5.163)	(2.762)	(0.925)	(1.247)	(1.117)
SMB	−5.566	−2.828	−2.400	−0.601	−1.354	−4.624	−3.974*	−3.104**	−0.011	0.143	0.005
	(4.129)	(2.146)	(1.623)	(0.963)	(1.432)	(4.842)	(2.205)	(1.350)	(0.613)	(0.857)	(0.722)
HML	0.364	1.450	1.401	1.527	2.596	2.273	1.272	0.497	0.529	0.002	0.781
	(3.199)	(2.716)	(1.893)	(0.986)	(1.687)	(10.56)	(2.500)	(1.382)	(0.541)	(0.789)	(0.679)
RMW	0.121	1.385	0.777	0.464	2.295*	2.000	1.916	1.258	0.642	0.898	0.773
	(3.460)	(2.199)	(1.397)	(0.961)	(1.333)	(5.895)	(2.452)	(1.153)	(0.506)	(0.605)	(0.605)
CMA	5.658**	1.359	0.929	0.871	2.952**	3.113	2.046	0.183	0.486	0.482	0.836*
	(2.514)	(1.948)	(1.178)	(0.760)	(1.267)	(7.270)	(2.338)	(1.136)	(0.343)	(0.569)	(0.439)

Table 11: The Hou et al. (2021) 5-Factor Model

This table summarizes the cross-sectional regression results for the Hou et al. (2021) 5-factor model. In Panel A, we present the annualized time-series average returns of the Hou et al. (2021) factors. Newey & West (1987) standard errors with 5 lags are in parentheses. In addition, we show the full-sample average absolute alphas of the portfolios ($avg\ |\overline{\alpha}|$), the share of individually significant portfolio alphas (at 10%) in brackets, and the joint statistical significance of the alphas based on the GRS test (indicated by the stars). In Panel B, we present average coefficient estimates from monthly Fama & MacBeth (1973) regressions. Each month, we use Equation (5) to regress the annualized portfolio and stock excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the Hou et al. (2021) 5-factor model. We use various different methods to estimate Equation (5) as previously described. The betas are generally estimated using Equation (4) along with data over the full sample period or previous historical windows. The estimators use daily data over the previous 12 months (the "EWMA^{ex}" estimator gradually expands this window). The alternative beta estimation methods are described in detail in Appendix A. In parentheses, we generally report errors-in-variables, model misspecification, as well as heteroskedasticity and autocorrelation robust standard errors of Kan et al. (2013) (with 5 Newey–West lags). For the "No Intercept" specification, we report the average absolute model errors in the intercept line. Significance is based on a joint χ^2 test with the null hypothesis that all average errors are jointly zero. The brackets contain the shares of single significant average pricing errors (at 10%; in percentage points). For the "No Intercept" case, we use Shanken (1992) standard errors. For the "OLS^{BC}" approach, we report Fama & MacBeth (1973) standard errors. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Panel A. Time-Series Test

	MKT	SMB^{HXZ}	INV	ROE	EG	$avg\ \lvert\overline{\alpha}\rvert$
Monthly	6.275***	3.455**	4.469***	6.530***	9.982***	3.395***
	(2.359)	(1.519)	(1.085)	(1.347)	(1.393)	[39.68]
Daily	6.324***	2.612**	4.523***	6.750***	10.18***	3.317***
	(2.266)	(1.269)	(0.894)	(1.066)	(0.896)	[45.24]

Panel B. Cross-Sectional Test

	Anomaly Portfolios					Beta Adjustment Methods			Stocks		
	Full	OLS	OLS-GJ	GLS	No Intercept	LW	EWMA^{ex}	Dim	OLS	WLS	OLS^{BC}
Intercept	3.043***	3.086***	3.776***	1.264***	2.908***	2.699***	2.939***	3.259***	9.055***	7.278***	9.190***
	(0.483)	(0.315)	(0.373)	(0.230)	[41.27]	(0.620)	(0.340)	(0.310)	(2.101)	(1.711)	(2.089)
MKT	1.948	0.602	0.168	0.092	−1.361	1.110	0.456	1.461	−1.303	−1.229	−1.267
	(2.130)	(2.122)	(1.650)	(1.736)	(1.906)	(3.594)	(2.195)	(2.154)	(1.521)	(1.986)	(1.543)
SMB^{HXZ}	1.881	−1.243	−1.127	0.635	0.272	−1.550	−0.665	−1.605	−1.048	1.091	−1.059
	(3.189)	(2.426)	(1.823)	(1.196)	(1.729)	(5.272)	(2.385)	(1.890)	(0.804)	(1.071)	(1.057)
INV	2.384	2.180	1.052	0.831	3.913***	3.714	2.649	1.831	1.043	0.822	1.463
	(1.451)	(1.656)	(1.275)	(0.748)	(1.440)	(2.975)	(1.786)	(1.333)	(0.740)	(0.943)	(0.910)
ROE	5.107**	4.218**	3.014**	0.716	5.244***	7.279*	5.148**	2.790**	0.792	0.554	0.937
	(2.135)	(1.873)	(1.533)	(1.282)	(1.729)	(3.711)	(2.106)	(1.390)	(0.837)	(0.987)	(1.025)
EG	8.114**	6.060***	4.302***	1.457	8.189***	10.91***	7.431***	4.830***	2.729***	2.567***	4.384***
	(3.555)	(1.693)	(1.564)	(0.962)	(1.262)	(4.024)	(1.758)	(1.170)	(0.735)	(0.814)	(0.935)

Table 12: Principal Components Model

This table summarizes the cross-sectional regression results for the principal components factor model. In Panel A, we present the annualized time-series average returns of the PC factors. Newey & West (1987) standard errors with 5 lags are in parentheses. In addition, we show the full-sample average absolute alphas of the portfolios ($avg\ |\overline{\alpha}|$), the share of individually significant portfolio alphas (at 10%) in brackets, and the joint statistical significance of the alphas based on the GRS test (indicated by the stars). In Panel B, we present average coefficient estimates from monthly Fama & MacBeth (1973) regressions. Each month, we use Equation (5) to regress the annualized portfolio and stock excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the principal components factor model. We use various different methods to estimate Equation (5) as previously described. The betas are generally estimated using Equation (4) along with data over the full sample period or previous historical windows. The estimators use daily data over the previous 12 months (the "EWMA^{ex}" estimator gradually expands this window). The alternative beta estimation methods are described in detail in Appendix A. In parentheses, we generally report errors-in-variables, model misspecification, as well as heteroskedasticity and autocorrelation robust standard errors of Kan et al. (2013) (with 5 Newey–West lags). For the "No Intercept" specification, we report the average absolute model errors in the intercept line. Significance is based on a joint χ^2 test with the null hypothesis that all average errors are jointly zero. The brackets contain the shares of single significant average pricing errors (at 10%; in percentage points). For the "No Intercept" case, we use Shanken (1992) standard errors. For the "OLS^{BC}" approach, we report Fama & MacBeth (1973) standard errors. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Panel A. Time-Series Test

	MKT	*PC1*	*PC2*	*PC3*	*PC4*	*PC5*	$avg\ \lvert\overline{\alpha}\rvert$
Monthly	6.274***	5.684*	12.26***	11.64***	−3.127**	5.697***	3.470***
	(2.227)	(3.038)	(2.696)	(2.006)	(1.407)	(1.297)	(49.07)
Daily	6.311***	6.643**	11.66***	13.69***	−1.041	5.626***	3.162***
	(2.131)	(2.772)	(2.581)	(1.861)	(1.413)	(1.199)	(49.07)

Panel B. Cross-Sectional Test

	Anomaly Portfolios					Beta Adjustment Methods			Stocks		
	Full	OLS	OLS-GJ	GLS	No Int	LW	EWMA^{ex}	Dim	OLS	WLS	OLS^{BC}
Intercept	3.518***	3.323***	4.035***	1.419***	2.937***	2.665***	3.315***	3.428***	9.155***	5.633***	9.163***
	(0.400)	(0.312)	(0.364)	(0.216)	(46.30)	(0.559)	(0.322)	(0.300)	(2.069)	(1.668)	(1.994)
MKT	1.305	3.387	3.472**	1.167	1.070	4.322	3.437	3.120	−0.242	0.540	−0.101
	(3.319)	(2.097)	(1.717)	(1.659)	(1.835)	(3.245)	(2.293)	(2.093)	(1.407)	(1.680)	(1.407)
PC1	−0.297	2.102	2.139	1.917	5.006*	2.885	1.953	2.298	4.029**	2.067	4.079**
	(3.303)	(3.038)	(2.312)	(2.254)	(3.018)	(4.308)	(3.100)	(3.075)	(1.892)	(2.463)	(1.987)
PC2	9.051**	9.313***	7.564***	5.002**	8.784***	13.68**	9.593***	8.578***	5.357**	2.390	5.928**
	(3.985)	(3.325)	(2.639)	(2.371)	(3.151)	(5.525)	(3.424)	(2.971)	(2.219)	(2.255)	(2.352)
PC3	5.829**	6.328***	4.553***	2.638*	9.256***	9.854***	6.705***	5.516***	1.108	2.999*	1.271
	(2.559)	(1.939)	(1.691)	(1.521)	(1.776)	(3.209)	(2.068)	(1.943)	(1.259)	(1.588)	(1.370)
PC4	−4.172	−0.491	−0.180	−0.210	−2.586*	−1.170	−0.872	−1.387	0.568	0.605	0.632
	(3.350)	(1.730)	(1.787)	(1.192)	(1.458)	(3.275)	(1.943)	(1.749)	(0.878)	(1.119)	(0.925)
PC5	1.337	1.749	0.785	1.475	3.916***	2.916	1.572	1.176	1.168	1.399	1.190
	(1.625)	(1.437)	(1.081)	(1.201)	(1.155)	(2.473)	(1.526)	(1.266)	(0.775)	(1.016)	(0.844)

Testing Factor Models in the Cross-Section

Online Appendix

JEL classification: G12, G11, G17

Keywords: Factor models, cross-sectional tests, no-arbitrage pricing, beta estimation

Table A1: Portfolios: Further Sampling Windows and Frequencies

This table presents test results for the 75 Fama–French factor base portfolios. We present average coefficient estimates from monthly Fama & MacBeth (1973) regressions. Each month, we use Equation (5) to regress the annualized portfolio excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the Fama & French (2015) 5-factor model. We use three different methods to estimate Equation (5): (i) "OLS", (ii) "OLS-GJ" (where the test assets are repackaged to have a higher dispersion in their market betas), and (iii) "GLS" (where we use the full-sample return covariance matrix to weight the observations). The betas are estimated using Equation (4) along with data over previous historical windows. "D^{6mon}" and "D^{36mon}" indicate that betas are estimated from 6 and 36 months of daily data, respectively, "Q" denotes a 60-month window of quarterly data. In parentheses, we report the errors-in-variables, model misspecification, as well as heteroskedasticity and autocorrelation robust standard errors of Kan et al. (2013) (with 5 Newey–West lags). *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	OLS			OLS-GJ			GLS		
	D^{6mon}	D^{36mon}	Q	D^{6mon}	D^{36mon}	Q	D^{6mon}	D^{36mon}	Q
$Intercept$	10.91***	10.68***	10.03***	8.676***	8.375***	7.645***	10.57***	10.71***	8.395***
	(2.342)	(2.665)	(2.255)	(2.414)	(2.477)	(2.527)	(1.691)	(1.898)	(1.640)
MKT	−4.359*	−4.193	−3.653	−1.699**	−1.806**	−0.889	−2.878	−3.828*	−1.194
	(2.507)	(2.610)	(2.466)	(0.765)	(0.810)	(0.863)	(1.847)	(1.977)	(1.367)
SMB	2.508*	2.293	1.604	1.228	1.412	1.011	−0.053	0.316	0.922
	(1.498)	(1.557)	(1.458)	(1.007)	(1.124)	(1.067)	(1.089)	(1.133)	(0.918)
HML	3.151**	3.577**	3.662**	1.571	2.062*	2.417**	1.247	2.526**	1.872*
	(1.524)	(1.636)	(1.645)	(0.981)	(1.068)	(1.035)	(1.138)	(1.285)	(0.984)
RMW	2.545**	2.896**	2.726**	1.665*	1.527	1.472	1.702*	1.628*	1.682**
	(1.248)	(1.331)	(1.243)	(1.010)	(1.088)	(1.187)	(0.869)	(0.929)	(0.767)
CMA	1.864	2.218	2.452**	0.776	1.247	1.315	0.799	1.572	0.896
	(1.316)	(1.364)	(1.122)	(0.937)	(1.050)	(0.937)	(0.838)	(0.961)	(0.647)

Table A2: Individual Stocks: The Role of Beta Estimators (OLS^{BC})

This table presents average coefficient estimates from monthly Fama & MacBeth (1973) regressions for individual stocks. Each month, we use Equation (5) to regress the annualized stock excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the Fama & French (2015) 5-factor model. We use the bias-corrected OLS estimator of Equation (6) to estimate Equation (5). The betas are estimated with various beta estimation methods (which are described in detail in Appendix A) along with data over previous historical windows. The estimators use daily data over the previous 12 months (the "EWMA^{ex}" estimator gradually expands this window). In parentheses, we report Fama & MacBeth (1973) standard errors. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	HIST^V	HIST^K	LW	EWMA	EWMA^{ex}	Dim	SW
Intercept	8.841***	8.853***	9.018***	9.331***	9.262***	8.636***	8.817***
	(1.888)	(1.860)	(2.005)	(1.992)	(1.906)	(1.963)	(1.979)
MKT	−1.099	−1.426	−1.228	−0.703	−0.789	−0.361	−0.358
	(1.841)	(2.106)	(2.224)	(1.451)	(1.560)	(1.395)	(1.282)
SMB	−0.280	0.001	−0.621	−1.005	−0.810	−0.585	−0.806
	(1.489)	(1.859)	(2.243)	(1.082)	(1.239)	(0.817)	(0.783)
HML	2.847**	3.814**	4.064**	1.868**	2.033**	2.144***	1.677**
	(1.446)	(1.854)	(2.037)	(0.890)	(1.029)	(0.814)	(0.743)
RMW	3.085**	4.224**	4.543**	2.132**	1.869**	0.807	1.551**
	(1.407)	(1.875)	(2.095)	(0.851)	(0.943)	(0.603)	(0.654)
CMA	1.669	2.213	2.288	1.071	1.483*	0.832	0.860
	(1.241)	(1.615)	(1.772)	(0.809)	(0.838)	(0.657)	(0.635)

Table A3: Individual Stocks: The Role of Beta Estimators (OLS and WLS)

This table presents average coefficient estimates from monthly Fama & MacBeth (1973) regressions for individual stocks. Each month, we use Equation (5) to regress the annualized stock excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the Fama & French (2015) 5-factor model. We use two different methods to estimate Equation (5): (i) "OLS" (Panel A) and (ii) "WLS" (Panel B; where we weight the stocks by their inverse market capitalization for each cross-sectional regression). The betas are estimated with various beta estimation methods (which are described in detail in Appendix A) along with data over previous historical windows. The estimators use daily data over the previous 12 months (the "EWMA^{ex}" estimator gradually expands this window). In parentheses, we report the errors-in-variables as well as heteroskedasticity and autocorrelation robust standard errors of Kan et al. (2013) (with 5 Newey–West lags). *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

Panel A. OLS

	HIST^V	HIST^K	LW	EWMA	EWMA^{ex}	Dim	SW	COMB
Intercept	8.709***	8.712***	8.820***	9.191***	9.264***	8.425***	8.647***	9.034***
	(1.911)	(1.860)	(2.021)	(2.031)	(1.928)	(2.041)	(2.021)	(1.913)
MKT	−1.093	−1.266	−1.196	−0.774	−0.975	−0.168	−0.298	−0.897
	(1.802)	(2.032)	(2.168)	(1.422)	(1.542)	(1.260)	(1.239)	(1.781)
SMB	−0.082	0.032	−0.255	−0.795	−0.605	−0.466	−0.651	−0.673
	(1.286)	(1.705)	(2.077)	(0.816)	(1.047)	(0.627)	(0.714)	(1.289)
HML	2.531*	3.262*	3.585*	1.733**	1.988**	1.758**	1.660**	2.551**
	(1.316)	(1.712)	(1.920)	(0.814)	(0.962)	(0.704)	(0.735)	(1.231)
RMW	2.457**	3.381**	3.503**	1.557**	1.557*	0.813	1.359**	2.230**
	(1.048)	(1.381)	(1.559)	(0.732)	(0.842)	(0.570)	(0.645)	(0.973)
CMA	1.451	1.818	2.038	0.993	1.337*	0.683	0.783	1.592
	(1.188)	(1.570)	(1.704)	(0.717)	(0.784)	(0.560)	(0.631)	(1.084)

Panel B. WLS

	HIST^V	HIST^K	LW	EWMA	EWMA^{ex}	Dim	SW	COMB
Intercept	5.747***	5.705***	5.724***	6.003***	6.694***	5.626***	5.295***	5.985***
	(1.769)	(1.928)	(2.148)	(1.697)	(1.752)	(1.590)	(1.638)	(1.839)
MKT	0.063	−0.017	0.087	0.035	−0.705	0.418	0.318	−0.179
	(2.121)	(2.474)	(2.822)	(1.887)	(2.087)	(1.626)	(1.665)	(2.265)
SMB	2.309*	2.903*	3.300*	1.757*	2.128*	1.025	0.603	2.358*
	(1.290)	(1.568)	(1.877)	(1.047)	(1.177)	(0.860)	(0.929)	(1.366)
HML	1.357	1.817	1.937	1.037	1.169	1.246	0.573	1.274
	(1.418)	(1.691)	(1.990)	(1.133)	(1.254)	(0.976)	(1.012)	(1.463)
RMW	2.137**	2.705**	3.055**	1.715**	1.747*	1.059	1.607**	2.204**
	(1.043)	(1.263)	(1.430)	(0.816)	(0.944)	(0.696)	(0.762)	(1.059)
CMA	0.124	0.264	0.174	0.158	0.461	0.130	−0.015	0.320
	(1.188)	(1.478)	(1.721)	(0.906)	(1.039)	(0.754)	(0.803)	(1.206)

Table A4: Individual Stocks: Further Sampling Windows and Frequencies

This table presents average coefficient estimates from monthly Fama & MacBeth (1973) regressions for individual stocks. Each month, we use Equation (5) to regress the annualized stock excess returns (in percentage points) during that month on a constant as well as factor sensitivities (betas) to the Fama & French (2015) 5-factor model. We use three different methods to estimate Equation (5): (i) "OLS", (ii) "WLS" (where we weight the stocks by their inverse market capitalization for each cross-sectional regression), and (iii) OLS^{BC} (where we use the bias-corrected OLS estimator of Equation (6)). The betas are estimated using Equation (4) along with data over previous historical windows. "D^{6mon}" and "D^{36mon}" indicate that betas are estimated from 6 and 36 months of daily data, respectively, "Q" denotes a 60-month window of quarterly data. In parentheses, we generally report errors-in-variables as well as heteroskedasticity and autocorrelation robust standard errors of Kan et al. (2013) (with 5 Newey–West lags). For the "OLS^{BC}" approach, we report Fama & MacBeth (1973) standard errors. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	OLS			WLS			OLS^{BC}		
	D^{6mon}	D^{36mon}	Q	D^{6mon}	D^{36mon}	Q	D^{6mon}	D^{36mon}	Q
Intercept	8.538***	9.570***	8.274***	5.645***	7.600***	5.968***	8.914***	9.494***	8.360***
	(2.157)	(1.911)	(1.968)	(1.659)	(1.771)	(1.695)	(2.081)	(1.923)	(1.847)
MKT	−0.321	−1.487	−0.218	0.487	−1.370	0.295	−0.282	−1.407	−0.270
	(1.291)	(1.560)	(1.105)	(1.692)	(2.101)	(1.452)	(1.376)	(1.584)	(1.169)
SMB	−0.567	−0.014	−0.264	0.938	1.968	0.741	−0.891	0.040	−0.348
	(0.595)	(1.225)	(0.708)	(0.857)	(1.212)	(0.925)	(0.883)	(1.379)	(0.782)
HML	1.296*	1.950	1.593**	0.497	1.356	1.462	1.590*	1.940	1.875***
	(0.708)	(1.184)	(0.641)	(0.972)	(1.332)	(0.989)	(0.823)	(1.264)	(0.672)
RMW	1.253*	1.333	0.081	1.442**	1.424	−0.032	2.276**	1.620	0.034
	(0.653)	(0.962)	(0.488)	(0.710)	(1.010)	(0.692)	(0.894)	(1.110)	(0.507)
CMA	0.772	1.717*	0.645	0.129	0.745	0.383	1.007	1.885**	0.766
	(0.610)	(0.902)	(0.459)	(0.815)	(1.057)	(0.686)	(0.749)	(0.951)	(0.493)