

# *On the (almost) stochastic dominance of cryptocurrency factor portfolios and implications for cryptocurrency asset pricing*

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# On the (almost) stochastic dominance of cryptocurrency factor portfolios and implications for cryptocurrency asset pricing

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## Abstract

Cryptocurrency returns are highly nonnormal, casting doubt on the standard performance metrics. We apply almost stochastic dominance, which does not require any assumption about the return distribution or degree of risk aversion. From 29 long–short cryptocurrency factor portfolios, we find eight that dominate our four benchmarks. Their returns cannot be fully explained by the three-factor coin model of Liu et al. So we develop a new three-factor model where momentum is replaced by a mispricing factor based on size and risk-adjusted momentum, which significantly improves pricing performance.

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**KEYWORDS**

almost stochastic dominance, asset pricing, cryptocurrencies, mispricing

**JEL CLASSIFICATION**

G11, G12

## 1 | INTRODUCTION

Cryptocurrencies are built on blockchain technology (Abadi & Brunnermeier, 2018; Biais et al., 2019) which permits transactions without central supervision. Since Bitcoin, the most famous cryptocurrency, appeared in 2009, more than 50 million investors have traded cryptocurrencies on more than 100 global exchanges; and over 100,000 companies worldwide accept payment in Bitcoins and Bitcoin debit cards (Easley et al., 2019; Makarov & Schoar, 2020). As attention to cryptocurrencies rises, whether they have investment value becomes an important issue, and both academics and practitioners are examining their properties for uses other than speculation. If we treat cryptocurrencies as financial assets, it is vital to understand their characteristics for several reasons. First, to find an appropriate performance metric for comparing cryptocurrencies with other financial classes. Second, to investigate whether factors can be used to form outperforming cryptocurrency portfolios. Third, to determine whether the cross-sectional variation in returns on outperforming cryptocurrency portfolios can be captured by a cryptocurrency pricing model.

A problem in this area is that the empirical distributions of cryptocurrency returns are highly nonnormal, raising doubts about the metrics for pricing and performance measurement. Specifically, due to speculation and excessive active trading carried out by investors, the probability distribution of cryptocurrency returns is depicted by asymmetric and leptokurtic behavior with fat tails induced by event risks, such as sudden price swing and hacks (Borri, 2019; Phillip et al., 2018). Consequently, investors may suffer from the overestimated risks and underestimated returns of cryptocurrencies if investors only rely on two moments—mean and variance—since the occurrence of events on right tails is more pronounced than that on their left tails (Nguyen et al., 2020). To circumvent this, we use almost stochastic dominance (ASD) (Leshno & Levy, 2002; Levy, 2006). ASD has several notable characteristics that are particularly appropriate for cryptocurrencies. First, unlike the mean–variance (MV) approach, ASD does not rely on the first two moments (e.g., mean and variance) and assumptions of return distributions when evaluating two uncertain prospects because it directly assesses the empirical cumulative distribution functions (CDFs) of these prospects, which also considers the higher moments. Thus, ASD has a natural advantage to tackle the assets with highly skewed and leptokurtic distributions in comparison to the potentially misleading MV method (Bali et al., 2013; Farinelli et al., 2008), and produce unbiased decisions. Second, ASD is a nonparametric approach and does not require a specific form of utility functions of investors (e.g., quadratic utility function) (Holcblat et al., 2022). These properties relax the conditions of the MV approach that a finite set of parameters must be predetermined, and allows a more complex form of utility functions (with different risk preferences) rather than only relying on a combination of the first two moments and the coefficient of risk aversion. Third, ASD allows for the pathological utility function (i.e., investors with such utility functions are indifferent

between a small and large amount of payoffs), which is superior to ordinary stochastic dominance and MV approach (Bali et al., 2009). Meanwhile, ASD can also draw conclusions from the case that one asset has a higher expected return along with higher variance than those of another asset, whereas the MV approach does not.<sup>1</sup> Therefore, ASD is a proper method to measure the performance of cryptocurrencies against benchmarks since cryptocurrencies are deemed as a highly fluctuated asset but with potentially high returns.

We examine cryptocurrency portfolios based on different risk factors for the 2353 cryptocurrencies with a minimum market capitalization of \$1 million, accounting for over 90% of the total market capitalization. In forming factor portfolios, we use each cryptocurrency's open price, close price, volume and market capitalization, which comprise the only public information for each cryptocurrency. Following Feng et al. (2020) and Liu et al. (2022), we classify these cryptocurrencies into four categories: size, momentum, volume and volatility, which we then subdivide into 29-factor portfolios to study whether cryptocurrency returns respond to the same factors as shares, and examine each factor's relative performance against four benchmarks.

We employ almost first-order stochastic dominance (AFSD) and almost second-order stochastic dominance (ASSD) to compare relative performance over five different horizons for each factor portfolio against the S&P 500, US T-Bonds, US T-Bills and Bitcoin. Using AFSD, we find that eight of the 29 factors are dominant against our four benchmarks, demonstrating outperformance for investors, irrespective of their level of risk aversion. We then use the coin three-factor model of Liu et al. (2022) to explain returns on our eight dominant factor portfolios. We find that seven of these dominant factor portfolios have positive alphas, indicating abnormal positive returns, which implies mispricing by the factor model.

Since portfolio managers use the capital asset pricing model and a range of multifactor models to compute expected returns (Ahmed et al., 2019; Ang, 2014; Fischer & Wermers, 2012), understanding which pricing model provides more the accurate estimates is important for both academics and practitioners. For equities, Stambaugh and Yuan (2017) note the proliferation of studies identifying anomalies that violate the standard (pre-cryptocurrency) three-factor model and how, with parsimony a virtue, these anomalies are only rarely included as additional factors. They press this conundrum in the following way: “given the proliferation of anomalies, however, the need for an alternative factor model that can accommodate more anomalies has become increasingly clear.” To tackle this mispricing issue, Hou et al. (2015) constructed two factors from the set of 11 anomalies, and combined the market and size factors into a single factor in a four-factor model for equities.

Since cryptocurrencies lack pricing models that accommodate cryptocurrency anomalies, this motivated us to examine whether combining the returns of factor models that generate anomalies into a single extra factor—the mispricing factor—in the coin market three-factor pricing model for cryptocurrencies leads to superior performance. Inspired by Stambaugh and Yuan (2017), we form a mispricing factor by calculating the equally weighted average of the returns on those factor portfolios that generate anomalies, to approximate the average anomalous effect of these factor portfolios. When we do this, we find that the number of significant alphas drops substantially, and the  $R^2$  values increase.

Liu and Tsyvinski (2020), among others, document the importance of network factors and investor attention in explaining and forecasting returns of the major cryptocurrencies. Thus, we

<sup>1</sup>We document a straightforward example with discussion in Section 4.

also include cryptocurrency-related fundamental factors, such as network and investor attention, in the coin market three-factor model, along with our mispricing factor. Using seven performance metrics that are widely used in the empirical asset pricing literature (Ahmed et al., 2019; Fama & French, 1993, 2015, 2017; Stambaugh & Yuan, 2017) to evaluate the performance of the coin three-factor model plus the seven combinations of the mispricing, attention and network factors, we find that the best results are achieved when the mispricing and network factors are included. This gives us an augmented coin factor model with five factors. To check that all five factors are needed, we apply spanning regression to identify redundant factors, and conclude that the momentum and network factors can be removed from the augmented model.

Next, we investigate the composition of our mispricing factor. This is a combination of seven dominant factor portfolios based on size, momentum, risk-adjusted momentum and return volatility. We again apply spanning regression, and discover that the momentum and volatility factors are redundant, leaving size and risk-adjusted momentum. This gives us our three-factor coin factor model which relies on the coin market index, coin size and our mispricing factor based on the size and risk-adjusted momentum factors.

This paper is the first to use the ASD approach when examining the relative performance of cryptocurrency factor portfolios. This addresses the nonnormality of cryptocurrency returns, and AFSD does not require any assumption about investor's risk aversion. We are unaware of any literature that focuses on cryptocurrency factor portfolio performance augmented pricing models. Liu et al. (2022) were the first to study the cross-sectional returns of a large number of cryptocurrencies from an asset pricing perspective. Our research has some similarities, but with important differences. Liu et al. (2022) find risk factors that capture variations in the cross-sectional returns of factor portfolios, and develop a cryptocurrency three-factor model. In contrast, we focus on factor portfolio performance. We evaluate the investment value of cryptocurrencies by comparing their performance with that of different asset classes using a nonparametric approach. Although influential studies have explored the performance of several popular asset pricing models (e.g., Fama & French, 2016; Hou et al., 2017), none accounts for sampling and model misspecification uncertainty due to the lack of a formal statistical procedure (Harvey & Liu, 2021; Kan & Robotti, 2009), and we rectify this omission by investigating the metrics related to alphas and model variations. We investigate the ability of a mispricing factor to explain returns on cryptocurrency factor portfolios by incorporating it into a new three-factor coin pricing model. Thus, we are the first to study the performance of cryptocurrency factors, along with constructing a model to capture the variation in returns of the dominant factor strategies.

For robustness and completeness, we present eight additional findings in Section A13 of the Supporting Information Appendix. First, we re-evaluate the empirical critical values for AFSD and ASSD proposed by Levy et al. (2010). Second, to eliminate the concern that shorting is expensive or unavailable for most of the coins, we evaluate the performance of our augmented model by regressing our eight dominant long-short factor portfolios shorting just Bitcoin, on our augmented model.<sup>2</sup> Third, we examine whether the coin market three-factor model of Liu

<sup>2</sup>Shorting Bitcoin can be effected by selling derivatives contracts. The Chicago Mercantile Exchange, Bakkt (part of Intercontinental Exchange), Deribit, Binance, FTX Trading Ltd., Quedex, Bitmex and Bitfinex trade futures and options on Bitcoin; and Kraken and Poloniex trade futures on Bitcoin. Investors can also trade Bitcoin contracts for difference on IG Index, eToro and Plus500; or bet against Bitcoin on Predictionis. Futures and options on other cryptocurrencies are also available

et al. (2022) and our augmented model (the coin market three-factor model incorporating a mispricing factor) capture the variation in returns of the 21 nondominant factor portfolios. Fourth, we explore whether well-known equity pricing models can explain the cryptocurrency anomalies indicated by our eight dominant factor portfolios. Fifth, we add the mispricing factor to coin market one- and two-factor models. Sixth, we investigate the effect of electricity on the performance of our augmented pricing model. Seventh, using ASD we investigate whether the long and short legs have different effects on the performance of our eight dominant factor portfolios. Eighth, we test the long leg of our eight dominant factor portfolios against three equity benchmarks (i.e., equity portfolios based on size, momentum and book-to-market ratio).

This paper is organized as follows. Section 2 discusses the related literature. Section 3 describes our data and provides summary statistics. Section 4 presents the methodology of the ASD approach. Section 5 describes the formation of the factor portfolios, with their summary statistics. Section 6 evaluates the empirical results using ASD. In Section 7, using spanning regression, we develop a new three-factor model (NTFM) featuring a mispricing factor with better performance than existing models. Section 8 explores the driving force behind the pricing power of the proposed mispricing factor. Section 9 concludes the paper.

## 2 | RELATED LITERATURE

The academic finance literature on the application of blockchain has developed rapidly in recent years, especially in the areas of ethics, initial coin offerings, token-weighted Crowdsourcing and analysis of the most popular coins (Benhaim et al., 2021; Cachon et al., 2021; Chod et al., 2020; Foley & Karlsen, 2019; Gan et al., 2020, 2021; Liu, Sheng, et al., 2021; Tsoukalas & Falk, 2020). Although numerous papers have studied cryptocurrencies, most focus on the major cryptocurrencies, especially Bitcoin (e.g., Atsalakis et al., 2019; Qiu et al., 2021). Only a limited literature explores the factors that influence cryptocurrency returns, and even fewer focus on the performance of cryptocurrency factor portfolios. Borri (2019) and Liu and Tsyvinski (2020) study the relationship between popular cryptocurrencies, finding exposure to only cryptocurrency-related factors. A coin market three-factor pricing model for cryptocurrencies that captures much of the cross-sectional variation in a range of cryptocurrency factors is presented by Liu et al. (2022). Similarly, Liu, Tsyvinski et al. (2021) find that information regarding the number of new addresses is valuable to the cryptocurrency market, where they find a negative relationship between price-to-new address ratios and future cryptocurrency returns.

Our paper is related to the literature that examines financial asset performance using ASD. Levy and Levy (2019) apply stochastic dominance (SD) to compare the performance of different portfolios, and find that US stocks dominate US bonds. Other papers (Bali et al., 2013; Board & Sutcliffe, 1994; Post, 2003) apply SD to examine the performance of hedge funds, relative to benchmarks. This literature evaluates equity, bond and hedge fund performance using a nonparametric approach. In addition, a series of studies have considered ASD from a methodological perspective (Arvanitis et al., 2020; Liesjö et al., 2020; Lok & Tabri, 2021; Post et al., 2018, among others). Our research is also related to the literature that explores asset pricing models and mispricing factors. There is a vast literature on equity asset pricing models, and a list of possible equity risk factors has been assembled by both academics and practitioners (Barillas & Shanken, 2018; Dong et al., 2022; Han et al., 2022, 2013; Hou et al., 2015; Huang



et al., 2014, 2020; Kan et al., 2013; Neely et al., 2014; Rapach et al., 2009; Stambaugh & Yuan, 2017; Zhou & Zhu, 2012).

Liu and Tsyvinski (2020) examine the relationship between risk and return for three mainstream cryptocurrencies (Bitcoin, Ripple and Ethereum). They find little or no evidence that cryptocurrencies have exposure to the Fama–French factors, major currencies, precious metal and macroeconomic factors. However, these three cryptocurrencies do have exposure to crypto-related factors, such as cryptocurrency momentum, investor attention and the cost of mining. Their findings provide crucial information for selecting potential explanatory variables when constructing a cryptocurrency asset pricing model.

### 3 | DATA

We collected cryptocurrency data via a free application programming interface (API) from [Coingecko.com](https://www.coingecko.com), which is a source of cryptocurrency prices, volumes and market capitalizations. [Coingecko.com](https://www.coingecko.com) aggregates information from over 400 major cryptocurrency exchanges on the opening price, closing price, volume and market capitalization. A cryptocurrency must meet certain criteria to be listed on this website, such as trading on a public exchange with an API which shows closing prices and nonzero trading volume during the previous 24 h. [Coingecko.com](https://www.coingecko.com) includes both defunct and active cryptocurrencies, which mitigates survivorship bias. We analyze all listed cryptocurrencies with a market capitalization larger than \$1 million on a daily basis during the sample period. Because data on trading volume first became available in the last week of 2013, we use cryptocurrency data from the beginning of 2014. After filtering, we analyze data for 2353 cryptocurrencies over the period January 1, 2014–June 30, 2021. The market capitalization of these cryptocurrencies accounts for over 90% of the total cryptocurrency capitalization on June 30, 2021. We also use data for the S&P 500 and 10-year US T-Bonds from CRSP, and the risk-free rate from Kenneth French's website.<sup>3</sup>

To gain a better insight into cryptocurrency pricing models and factor portfolios, we construct a cryptocurrency market index using data on 2353 cryptocurrencies. First, we calculate the daily log returns across the sample period for each cryptocurrency and allocate market capitalization weights to each asset every day. Then we sum these market capitalization-weighted returns to obtain the daily returns on our coin market index (CMKT). The purpose of constructing CMKT is to help build a cryptocurrency pricing model. The summary statistics of the weekly returns of Bitcoin, CMKT and the three other asset classes we use as benchmarks in our analysis, are in Table 1. The average weekly return on Bitcoin is 0.0101, which is an order of magnitude higher than the average return on the S&P 500 of 0.0025, 0.0001 for T-Bills and 0.0008 for T-Bonds. However, Bitcoin's risk is also much larger than for equities, bills and bonds. Bitcoin returns are negatively skewed, as are S&P 500 returns; whereas T-Bill and T-Bond returns are positively skewed. The kurtosis of Bitcoin, CMKT, S&P 500 and T-Bond returns is higher than for the normal distribution, which indicates that these distributions are leptokurtic. The Jarque–Bera (J-B) test statistics and corresponding *p* values indicate that these distributions are all highly nonnormal. Therefore, comparisons of risk-adjusted returns across asset classes using standard performance measures such as the Sharpe ratio (SR) (Sharpe, 1966) are unreliable.

<sup>3</sup>[https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)



**TABLE 1** Descriptive statistics.

This table presents descriptive statistics of weekly returns (not percentages) on Bitcoin, CMKT, S&P 500 index, T-Bills and T-Bonds for 2014 to the first half of 2021.

|                  | Mean   | Median | Std    | Skewness | Kurtosis | J-B test | <i>p</i> Value |
|------------------|--------|--------|--------|----------|----------|----------|----------------|
| Bitcoin return   | 0.0101 | 0.0104 | 0.1039 | −0.2402  | 4.9013   | 62.8161  | 0.001          |
| CMKT             | 0.0145 | 0.0140 | 0.1069 | −0.0381  | 6.0799   | 155.0315 | 0.001          |
| S&P 500          | 0.0025 | 0.0042 | 0.0236 | −0.644   | 13.7708  | 1921.937 | 0.001          |
| One-month T-Bill | 0.0001 | 0.0001 | 0.0002 | 0.8154   | 2.1456   | 55.3662  | 0.001          |
| Ten-year T-Bond  | 0.0008 | 0.0009 | 0.0082 | 0.5974   | 8.6441   | 543.6354 | 0.001          |

Abbreviations: CMKT, coin market index; J-B, Jarque–Bera; S&P 500, Standard & Poor's 500.

To construct cryptocurrency factor models, we seek additional explanatory variables. Numerous studies document network factors contributing to the valuation of cryptocurrencies (Biais et al., 2020; Cong et al., 2021; Pagnotta & Buraschi, 2018). Among network factors, the number of users plays a crucial role in explaining returns on cryptocurrencies since user adoption of cryptocurrencies provides a positive network externality. We use four metrics to proxy for network factors. They are the number of wallet users, the number of active addresses, the number of transactions and the number of payments. The data on wallet users were collected from Blockchain.info, and the numbers of active addresses, transactions and payments were obtained from Coinmetrics.io. A growing literature on cryptocurrencies finds that investor attention (approximated by Tweet counts of a keyword) predicts cryptocurrency returns (Cong et al., 2021; Liu & Tsyvinski, 2020; Sockin & Xiong, 2020). We use tweet counts for the keyword “blockchain” as a proxy for investor attention as it has been proved to have predictive power for returns.

## 4 | METHODOLOGY

We test the performance of cryptocurrencies against four benchmarks: S&P 500, 10-year US T-Bonds, 30-day US T-Bills and Bitcoin. Since the distributions of cryptocurrencies are nonnormal, the usual performance metrics based on the normal distribution are problematic, and we apply ASD, which does not require a parametric specification of investor preferences or the assumption of a normal distribution.

### 4.1 | Unsuitability of the MV approach

Although the MV approach, which is a cornerstone of modern portfolio theory, has been widely employed in both academia and industry, a number of studies document that SD is superior to MV (Gandhi & Saunders, 1981; Malavasi, 2021). MV analysis relies on returns following a normal distribution. However, there is a substantial literature indicating that this is not the case (Aharony & Loeb, 1977; Bali et al., 2009; Gotoh & Konno, 2000; Yitzhaki, 1982). First, SD does not assume returns have a normal distribution, and so does not require estimates of its

parameters. Second, SD uses all the individual observations in the data set, while MV uses only the first and second moments of returns. Finally, SD only assumes that investors prefer more to less when outcomes have the same probability (Gandhi & Saunders, 1981).

Leshno and Levy (2002) note that the classical MV decision rule can fail to show dominance between two portfolios when almost all investors would choose one portfolio over another. The few investors who fail to recognize dominance have extreme utility functions. To illustrate, consider two portfolios,  $H$  and  $L$ :

$$\begin{aligned} H : \mu_H &= 1000\%, \quad \sigma_H = 5.1\%, \\ L : \mu_L &= 1\%, \quad \sigma_L = 5\%, \end{aligned}$$

where  $H$  and  $L$  represent portfolios with high and low expected returns, respectively. The returns on portfolios  $H$  and  $L$  are  $\mu_H$  and  $\mu_L$ , respectively, and the standard deviations are  $\sigma_H$  and  $\sigma_L$ . If portfolio  $H$  dominates portfolio  $L$  by MV, then the condition that  $\mu_H > \mu_L$  and  $\sigma_H < \sigma_L$  must be met. In this example, the expected return on portfolio  $H$  is dramatically higher than that of portfolio  $L$  (1000 times) but with only a slightly higher standard deviation (0.1%). Most investors would surely choose portfolio  $H$  over portfolio  $L$  because the decrease in expected utility due to the slightly higher risk is much less than the increase in expected utility from the much higher expected return. Nonetheless, MV fails to identify any dominance.

## 4.2 | Stochastic dominance

SD provides an alternative perspective to MV when comparing the performance of two assets. Following Leshno and Levy (2002), first-order stochastic dominance (FSD) and second-order stochastic dominance (SSD) are defined as follows.

**FSD:** Suppose there are two risky portfolios,  $H$  and  $L$ , and the cumulative return distributions of  $H$  and  $L$  are denoted by  $F_H$  and  $F_L$ , respectively. Portfolio  $H$  dominates portfolio  $L$  by first-order stochastic dominance ( $H$  FSD  $L$ ) if  $F_H(r) \leq F_L(r)$  for all return values  $r$ , and a strict inequality holds for at least one  $r$ .

**SSD:** Suppose there are two risky portfolios,  $H$  and  $L$ .  $H$  represents a portfolio with a high expected return, and  $L$  represents a portfolio with a low expected return. The cumulative returns distributions of  $H$  and  $L$  are denoted by  $F_H$  and  $F_L$ , respectively. Portfolio  $H$  dominates portfolio  $L$  by second-order stochastic dominance ( $H$  SSD  $L$ ) if  $\int_{-\infty}^r [F_L(s) - F_H(s)] ds \geq 0$  for all return values  $r$ , and a strict inequality holds for at least one  $r$ .

The key difference between FSD and SSD is what they assume about investors' utility functions (Daskalaki et al., 2017). FSD only requires that investors prefer more to less (mathematically,  $\mu' > 0$ , where  $\mu$  is the utility function), and makes no assumption about their attitude towards risk. SSD also assumes that investors are risk averse, which implies a concave utility function (mathematically,  $\mu' > 0$  and  $\mu'' < 0$ ). Therefore, SSD finds more dominance relationships than FSD.

### 4.2.1 | Almost first-order stochastic dominance

Our numerical example above in Section 4.1 demonstrated a lack of MV dominance which requires investors to have an extreme or pathological utility function. AFSD addresses the

problem of pathological utility functions by excluding a few extreme utility functions, and examining whether a small violation of FSD can be “ignored” (Leshno & Levy, 2002). AFSD exists when one distribution is “close to” a distribution that dominates another distribution in the classical sense of FSD.

AFSD relies on the concept of a violation area. Figure 1 plots the cumulative distributions of portfolios  $H$  and  $L$ , where the plot for  $H$  lies below that for  $L$ , except for the shaded area denoted  $M$ . So there is no FSD in this case. When considering whether portfolio  $H$  (solid line) dominates  $L$  (dashed line) by AFSD, the area of the cumulative distribution of  $H$  that is above the cumulative distribution of  $L$  is called the violation area (denoted by  $M$  in Figure 1).

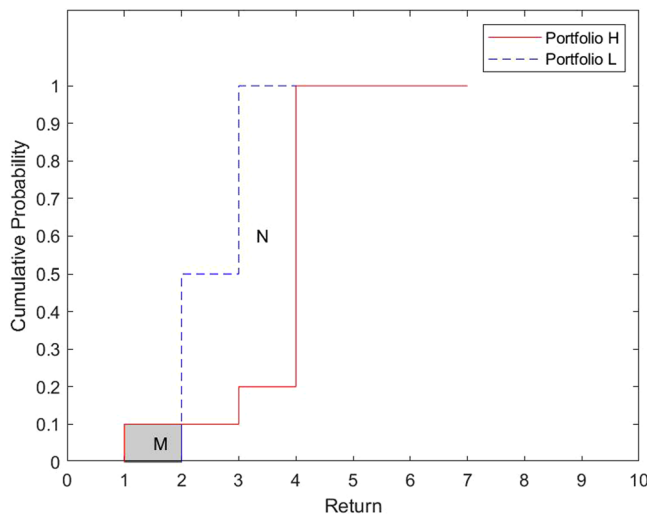
According to Leshno and Levy (2002), the violation area  $M$  may be defined as  $\int_{r_1}^{r_2} [F_H(s) - F_L(s)] ds$ , where the FSD violation range is given by

$$R_1(F_H, F_L) = \{s \in (r_1, r_2) : F_L(s) < F_H(s)\}, \quad (1)$$

where  $r_1$  and  $r_2$  are the bounds of the range where portfolio  $H$  has a higher CDF value than portfolio  $L$ . The term  $\varepsilon_1$  (the ratio of the violation area to the total area of difference) is defined as

$$\varepsilon_1 = \frac{\int_{R_1} [F_H(s) - F_L(s)] ds}{\int_{\min}^{\max} |F_H(s) - F_L(s)| ds}, \quad (2)$$

where  $F_H$  and  $F_L$  have a finite support  $[\min, \max]$ . Equation (1) measures the violation range where the probability of  $H$  is larger than  $L$  (range  $[1, 2]$  in Figure 1). According to Equation (2),  $\varepsilon_1$  is defined as the area  $M$  in Figure 1 divided by the total absolute area enclosed between  $F_H$  and  $F_L$



**FIGURE 1** Almost first-order stochastic dominance. This figure reports the cumulative distribution of the high-return portfolio ( $H$ ) and the low-return portfolio ( $L$ ). Because of the violation area  $M$ ,  $H$  fails to dominate by FSD, SSD or MV. There are some extreme utility functions that assign a large weight to area  $M$ , and a small or zero weight to area  $N$ . However, most investors would choose  $H$  over  $L$ , which indicates the existence of AFSD if the violation area  $M$  is small enough. AFSD, almost first-order stochastic dominance; FSD, first-order stochastic dominance; MV, mean–variance; SSD, second-order stochastic dominance. [Color figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

(area  $M + N$  in Figure 1). It is clear that FSD exists if  $\varepsilon_1 = 0$ , which implies no violation area. However, for  $\varepsilon_1 > 0$ , although  $H$  fails to dominate  $L$  by FSD, AFSD may exist if  $\varepsilon_1$  is small enough to be “ignored.” Levy et al. (2010) conducted an empirical test to set the minimum tolerance, and propose that a violation can be ignored if  $\varepsilon_1$  is smaller than, or equal to, the critical  $\varepsilon_1^*$  value of 5.9%. As the investment horizon lengthens, Leshno and Levy (2002) show that the value of  $\varepsilon_1$  required for investors with pathological utility functions to recognize FSD becomes smaller.

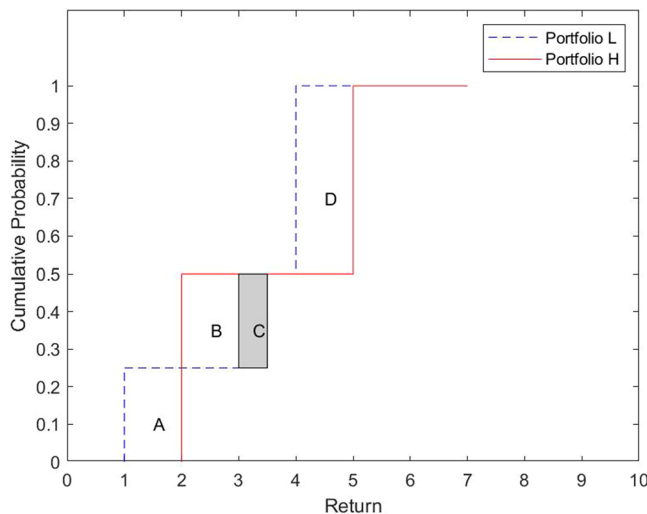
#### 4.2.2 | Almost second-order stochastic dominance

Similar to AFSD, ASSD may exist when there is only a small violation of SSD. Provided the area under the CDF from minus infinity to  $r$  is the same or lower for  $H$  than for  $L$  for every value of  $r$ ; and at some point is lower, then  $H$  SSD  $L$ . Therefore, unlike FSD, SSD permits the CDFs to cross several times. This is illustrated in Figure 2, where area  $A$  equals area  $B$ . If area  $C$  did not exist,  $H$  SSD  $L$ , but the presence of area  $C$  violates SSD. The range of the SSD violation area (area  $C$  in Figure 2) may be defined as

$$R_2(F_H, F_L) = \left\{ s \in R_1(F_H, F_L) : \int_0^s [F_L(s) - F_H(s)] ds < 0 \right\}. \quad (3)$$

The empirical  $\varepsilon_2$  (the ratio of the violation area to total area difference) is defined as

$$\varepsilon_2 = \frac{\int_{R_2} [F_H(s) - F_L(s)] ds}{\int_{\min}^{\max} |F_H(s) - F_L(s)| ds}, \quad (4)$$



**FIGURE 2** Almost first-order stochastic dominance. This figure demonstrates a case where portfolio  $H$  has a higher mean than portfolio  $L$ , but there is no SSD of  $H$  over  $L$  due to the presence of the shaded area  $C$ , which makes the SSD condition fail. If area  $C$  is relatively small, ASSD may exist. ASSD, almost second-order stochastic dominance; SSD, second-order stochastic dominance. [Color figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

where  $\varepsilon_2$  is the violation area (area  $C$  in Figure 2) divided by the total absolute area enclosed between  $F_H$  and  $F_L$ , that is,  $C/(A + B + C + D)$ . Levy et al. (2010) suggest that the threshold value  $\varepsilon_2^*$  of ASSD is 3.2%, which is the minimum tolerance for most investors. Hence, if  $\varepsilon_2$  is smaller than or equal to 3.2%, ASSD exists.

## 5 | PORTFOLIO FORMATION

To investigate anomalies in cryptocurrency returns, we build zero-investment long–short portfolios based on the factors compiled by Liu et al. (2022) and Feng et al. (2020) that can be constructed using the available information. We identify four main factors—size, momentum, volume and volatility, which we divide into 29 the zero-investment long–short factor portfolios which are listed in Table 2.

To form the 29-factor portfolios, each week we sort the 2353 individual cryptocurrencies into quintiles in ascending order of the factor under consideration, and then we track the return on each portfolio in the following week. The weekly portfolio returns are the market capitalization-weighted averages of the individual cryptocurrency returns, and we calculate the excess mean return over the risk-free rate of each quintile portfolio for each factor portfolio. We then compute the excess return on a long–short strategy based on the difference between the fifth and first quintiles (fifth quintile minus first quintile). For the long–short strategies that produce negative returns, we use the first quintile minus the fifth quintile. In the following subsections, we explain the factor portfolios and their mean excess returns in detail (see Table 3).

### 5.1 | Size related portfolios

The size factor portfolios are constructed based on market capitalization, closing price and age factors. Panel A of Table 3 has the expected excess return for each quintile portfolio for each of the three size factors and the long–short strategy. The negative signs before the values of the long–short strategies are not important because a positive return could be achieved by reversing the long–short strategy.

The *AGE* long–short portfolio generates the highest absolute return of 0.0272, while the *LPRC* portfolio has an absolute return of 0.0215 on the long–short strategy, and is the worst performing of the portfolios in the size group. Compared with the mean returns in Table 1, the long–short portfolios have much higher expected returns than *CMKT*, Bitcoin and the other three benchmarks.

### 5.2 | Momentum related portfolios

We formed the momentum factor portfolios based on 1-, 2-, 3-, 4-, 8- and 26-week momentum and risk-adjusted momentum (based on the SR) factors. We use risk-adjusted momentum strategies as our factors because portfolios based on risk-adjusted strategies can provide higher risk-adjusted returns, and have lower tail risks compared with portfolios based on ordinary momentum strategies (Choi et al., 2015; Rachev et al., 2007).

**TABLE 2** Zero-investment long–short factor portfolios.

This table reports the construction of each factor portfolio based on a specific factor. For instance, the procedure for creating a portfolio based on MARCAP is that each week the 2353 cryptocurrencies were sorted into quintiles using market capitalization. All the portfolios were rebalanced weekly. Coin market index (CMKT) refers to our coin market index, which is the value-weighted return of the full sample of cryptocurrencies.

| Category   | Factor used       | Definition  |
|------------|-------------------|---|
| Size       | MARCAP            | Log last-day market capitalization in the portfolio formation week  |
| Size       | LPRC              | Log last-day price in the portfolio formation week  |
| Size       | Age               | The number of existent weeks that listed on <a href="https://coinmarketcap.com">Coinmarketcap.com</a>   |
| Momentum   | MOM1              | One-week momentum   |
| Momentum   | MOM2              | Two-week momentum   |
| Momentum   | MOM3              | Three-week momentum   |
| Momentum   | MOM4              | Four-week momentum  |
| Momentum   | MOM8              | Eight-week momentum   |
| Momentum   | MOM26             | Twenty-six-week momentum  |
| Momentum   | RMOM1             | One-week risk-adjusted momentum based on the Sharpe ratio   |
| Momentum   | RMOM2             | Two-week risk-adjusted momentum based on the Sharpe ratio   |
| Momentum   | RMOM3             | Three-week risk-adjusted momentum based on the Sharpe ratio   |
| Momentum   | RMOM4             | Four-week risk-adjusted momentum based on the Sharpe ratio  |
| Momentum   | RMOM8             | Eight-week risk-adjusted momentum based on the Sharpe ratio   |
| Momentum   | RMOM26            | Twenty-six-week risk-adjusted momentum based on the Sharpe ratio  |
| Volume     | VOL               | Log average daily volume in the portfolio formation week  |
| Volume     | VOLPRC            | Log average daily volume times price in the portfolio formation week  |
| Volume     | VOLSCALE          | Log average daily volume times price then divided by market capitalization in the portfolio formation week  |
| Volatility | RETVOL            | The standard deviation of daily returns in the portfolio formation week   |
| Volatility | RETSKEW           | The skewness of daily returns in the portfolio formation week   |
| Volatility | RETKURT           | The kurtosis of daily returns in the portfolio formation week   |
| Volatility | MAXRET            | The maximum daily return of the portfolio formation week  |
| Volatility | STDPRCVOL         | Log standard deviation of dollar volume in the portfolio formation week   |
| Volatility | MEANABS           | The mean absolute daily return divided by dollar volume in the portfolio formation week   |
| Volatility | BETA              | The regression coefficient of $\beta_{MKT,i}$ in $R_i - R_f = \alpha^i + \beta_{MKT,i}MKT + \epsilon_i$ . The model is estimated using the daily returns of the previous 365 days before formation week |
| Volatility | BETA <sup>2</sup> | Beta squared  |

TABLE 2 (Continued)

| Category   | Factor used    | Definition  |
|------------|----------------|---|
| Volatility | <i>IDIOVOL</i> | The IDIOVOL volatility is measured as the standard deviation of the residuals after estimating $R_i - R_f = \alpha^i + \beta_{MKT,i}MKT + \epsilon_i$ . The model is estimated using the daily returns of the previous 365 days before the formation week   |
| Volatility | <i>DELAY</i>   | The improvement of $R^2$ in $R_i - R_f = \alpha^i + \beta_{MKT,i}MKT + \beta_{MKT-1,i}MKT_{-1} + \beta_{MKT-2,i}MKT_{-2} + \epsilon_i$ compared with a regression that only uses <i>MKT</i> , where $MKT_{-1}$ and $MKT_{-2}$ are lagged 1- and 2-day market index returns. The model is estimated using the daily returns of the previous 365 days before formation week |
| Volatility | <i>LIQ</i>     | The average absolute daily return divided by price volume in the portfolio formation week   |

Panel A of Table 3 reports the expected excess return for each quintile portfolio created using each of the 10 momentum factors. The highest expected excess return of 0.0359 on a long–short momentum portfolio (*MOM1*) uses a 1-week momentum factor, and the portfolio of 26-week momentum (*MOM26*) has the lowest expected return of  $-0.0177$ . The expected excess return on long–short portfolios monotonically decreases from *MOM1* to *MOM26*, which is broadly similar to the pattern for equities (Jegadeesh & Titman, 1993). For the risk-adjusted long–short momentum portfolios, *RMOM3* has the highest excess return of 0.0302, and *RMOM26* has the lowest return of  $-0.0110$ . As for momentum, the risk-adjusted long–short momentum portfolios exhibit an almost monotonically decreasing pattern as the horizon is extended from 1 to 26 weeks. These results suggest that long–short momentum-based strategies with shorter horizons (1–3 weeks) perform best.

5.3 | Volume related portfolios

The volume factor portfolios are constructed based on volume, volume times price and scaled volume times price. Panel A of Table 3 reports the expected excess return for each quintile portfolio created using each of the three volume factors. For long–short portfolios based on volume, the highest absolute excess return of 0.0236 is generated by portfolios based on volume times price (*VOLPRC*), and the portfolio of scaled volume times price (*VOL*) delivers the lowest excess absolute return of 0.0016. For all three volume factors, the first quintile portfolio (low volume) generates the highest excess return, which is greater than the excess absolute return on the long–short portfolios.

5.4 | Volatility related portfolios

The volatility factor portfolios are constructed based on the standard deviation of excess returns, skewness of excess returns, kurtosis of excess returns, maximum excess return, log of the standard deviation of dollar volume, the mean absolute daily excess return scaled by dollar volume and liquidity. Panel A of Table 3 shows the expected excess return for each quintile portfolio created using these 11 factors. The long–short portfolio *STDPRCVOL* has the highest



TABLE 3 Descriptive statistics for each factor portfolio.

Panel A reports weekly mean excess returns on the 29 quintile factor portfolios. The mean excess returns are defined as the excess value-weighted expected returns, and “5–1” represents the long–short strategy. Panel B illustrates the summary statistics of (5–1) mean (where any negative (5–1) mean return can be converted to positive by reversing the differencing), median, standard deviation, skewness, kurtosis, Jarque–Bera (J-B) test and corresponding *p* values.

|        | Panel A: Quintiles |         |         |         |          | Panel B: Statistics for “5–1” Portfolios |         |        |          |          |            |         |
|--------|--------------------|---------|---------|---------|----------|--|---------|--------|----------|----------|------------|---------|
|        | 1 (Low)            | 2       | 3       | 4       | 5 (High) | 5–1 (Mean)                               | Median  | Std    | Skewness | Kurtosis | J-B test   | p Value |
| MARCAP | 0.0203             | 0.0026  | −0.0127 | −0.0100 | −0.0066  | −0.0269                                  | 0.0213  | 0.1046 | 0.8481   | 6.3198   | 226.43     | 0.001   |
| LPRC   | 0.0264             | 0.0273  | 0.0091  | 0.0115  | 0.0049   | −0.0215                                  | −0.0016 | 0.1675 | 3.3295   | 23.5690  | 7615.15    | 0.001   |
| AGE    | 0.0333             | 0.0123  | 0.0025  | 0.0092  | 0.0060   | −0.0272                                  | 0.0008  | 0.2193 | −2.6346  | 16.8386  | 3572.28    | 0.001   |
| MOM1   | −0.0045            | −0.0017 | 0.0084  | 0.0150  | 0.0314   | 0.0359                                   | 0.0226  | 0.1876 | 1.6170   | 15.5703  | 2744.66    | 0.001   |
| MOM2   | −0.0052            | −0.0018 | 0.0017  | 0.0145  | 0.0296   | 0.0349                                   | 0.0246  | 0.1790 | 1.3031   | 13.8838  | 2035.29    | 0.001   |
| MOM3   | −0.0016            | −0.0081 | 0.0079  | 0.0171  | 0.0246   | 0.0262                                   | 0.0143  | 0.2063 | 1.1433   | 18.3854  | 3921.41    | 0.001   |
| MOM4   | 0.0024             | −0.0016 | 0.0035  | 0.0237  | 0.0161   | 0.0137                                   | 0.0067  | 0.1477 | 0.8768   | 7.3223   | 351.75     | 0.001   |
| MOM8   | 0.0098             | 0.0047  | 0.0083  | 0.0186  | 0.0025   | −0.0072                                  | −0.0060 | 0.1628 | 1.7656   | 17.9600  | 3780.36    | 0.001   |
| MOM26  | 0.0194             | 0.0112  | 0.0029  | 0.0116  | 0.0017   | −0.0177                                  | −0.0069 | 0.1768 | −1.7661  | 18.7137  | 3955.79    | 0.001   |
| RMOM1  | −0.0020            | 0.0009  | 0.0106  | 0.0090  | 0.0247   | 0.0268                                   | 0.0133  | 0.1521 | 1.8871   | 20.4177  | 5174.58    | 0.001   |
| RMOM2  | −0.0056            | −0.0010 | 0.0135  | 0.0116  | 0.0207   | 0.0264                                   | 0.0254  | 0.1667 | 0.8223   | 22.7316  | 6337.96    | 0.001   |
| RMOM3  | −0.0032            | 0.0021  | 0.0078  | 0.0081  | 0.0270   | 0.0302                                   | 0.0273  | 0.1780 | 2.8670   | 27.9396  | 10,614.22  | 0.001   |
| RMOM4  | −0.0038            | 0.0052  | 0.0119  | 0.0087  | 0.0190   | 0.0228                                   | 0.0158  | 0.1424 | 0.4663   | 7.4890   | 339.83     | 0.001   |
| RMOM8  | 0.0065             | 0.0132  | 0.0140  | 0.0127  | 0.0100   | 0.0035                                   | 0.0096  | 0.1425 | 0.4462   | 7.6440   | 357.82     | 0.001   |
| RMOM26 | 0.0151             | 0.0138  | 0.0104  | 0.0099  | 0.0040   | −0.0110                                  | −0.0036 | 0.1530 | −0.4496  | 9.7310   | 703.26     | 0.001   |
| VOL    | 0.0155             | 0.0266  | 0.0233  | 0.0059  | 0.0139   | −0.0016                                  | 0.0040  | 0.2256 | −9.5698  | 154.7759 | 381,261.46 | 0.001   |
| VOLPRC | 0.0344             | 0.0228  | 0.0177  | 0.0139  | 0.0108   | −0.0236                                  | 0.0063  | 0.1605 | 3.7847   | 28.7442  | 11,731.03  | 0.001   |

TABLE 3 (Continued)

|                          | Panel A: Quintiles |        |        |        |          | Panel B: Statistics for “5–1” Portfolios |         |        |          |          |           |         |
|--------------------------|--------------------|--------|--------|--------|----------|--|---------|--------|----------|----------|-----------|---------|
|                          | 1 (Low)            | 2      | 3      | 4      | 5 (High) | 5–1 (Mean)                               | Median  | Std    | Skewness | Kurtosis | J-B test  | p Value |
| <i>VOLSCALE</i>          | 0.0223             | 0.0226 | 0.0093 | 0.0078 | 0.0106   | −0.0117                                  | −0.0025 | 0.1412 | 2.7424   | 18.8200  | 4567.44   | 0.001   |
| <i>RETVOL</i>            | 0.0092             | 0.0090 | 0.0129 | 0.0128 | 0.0106   | 0.0014                                   | −0.0107 | 0.2007 | 1.6729   | 18.0143  | 3854.99   | 0.001   |
| <i>RETSKEW</i>           | 0.0051             | 0.0089 | 0.0075 | 0.0097 | 0.0172   | 0.0121                                   | 0.0050  | 0.1527 | 1.7954   | 20.9129  | 5437.60   | 0.001   |
| <i>RETKURT</i>           | 0.0057             | 0.0142 | 0.0128 | 0.0092 | 0.0093   | 0.0036                                   | 0.0044  | 0.1270 | 0.2689   | 7.1715   | 288.21    | 0.001   |
| <i>MAXRET</i>            | 0.0003             | 0.0109 | 0.0119 | 0.0156 | 0.0204   | 0.0201                                   | 0.0078  | 0.1464 | 0.9446   | 5.8347   | 189.06    | 0.001   |
| <i>STDPRCVOL</i>         | 0.0325             | 0.0223 | 0.0157 | 0.0210 | 0.0103   | −0.0222                                  | 0.0049  | 0.1587 | 3.7806   | 29.2807  | 12,183.67 | 0.001   |
| <i>MEANABS</i>           | 0.0107             | 0.0083 | 0.0222 | 0.0231 | 0.0281   | 0.0174                                   | −0.0055 | 0.1845 | 4.2588   | 30.2452  | 13,275.29 | 0.001   |
| <i>BETA</i>              | 0.0062             | 0.0119 | 0.0050 | 0.0127 | 0.0034   | −0.0029                                  | 0.0013  | 0.1492 | 0.1955   | 13.8392  | 1661.68   | 0.001   |
| <i>BETA</i> <sup>2</sup> | 0.0048             | 0.0104 | 0.0092 | 0.0101 | 0.0034   | −0.0014                                  | 0.0019  | 0.1296 | −0.3705  | 20.3684  | 4268.71   | 0.001   |
| <i>IDIOVOL</i>           | 0.0071             | 0.0150 | 0.0193 | 0.0184 | 0.0034   | −0.0037                                  | 0.0123  | 0.1298 | −0.8429  | 21.3632  | 4803.19   | 0.001   |
| <i>DELAY</i>             | 0.0162             | 0.0160 | 0.0187 | 0.0221 | 0.0047   | −0.0115                                  | 0.0169  | 0.1239 | 0.2481   | 8.3735   | 411.33    | 0.001   |
| <i>LIQ</i>               | 0.0107             | 0.0083 | 0.0222 | 0.0231 | 0.0281   | 0.0174                                   | −0.0055 | 0.1845 | 4.2588   | 30.2452  | 13,275.29 | 0.001   |

excess absolute mean return of 0.0222, and *RETVOL* has the lowest excess return of 0.0014. *BETA* and *BETA*<sup>2</sup> have the second lowest excess return of around 0.0002, which is counterintuitive, as portfolios with a higher beta (systematic risk) should be compensated with a higher expected excess return. Hence, beta might not be an appropriate factor to rank cryptocurrencies to form portfolios. The excess returns on quintile portfolios of the maximum return (*MAXRET*) increase as *MAXRET* increases, and excess returns on the quintile portfolios for the log standard deviation (*STDPRCVOL*) of excess returns, with one exception, decrease from the first to fifth quintiles, that is, excess returns drop as risk increases, which is also unexpected.

## 5.5 | Summary statistics of the factor portfolios

Panel B of Table 3 shows the median, standard deviation, skewness, kurtosis, J-B test and their *p* values for the 29-factor portfolios. The *MOM1* portfolio has the highest mean weekly return of 0.0359, and *RETVOL* and *BETA*<sup>2</sup> have the lowest absolute weekly return of 0.0014. The *VOL* portfolio is the most volatile portfolio with a standard deviation of 0.2256, while the *MARCAP* portfolio has the smallest standard deviation of 0.1046. As demonstrated by the J-B test in Panel B, there are dramatic departures from normality in the excess return distributions of all the factor portfolios, and our performance comparisons need to allow for this nonnormality.

## 6 | EMPIRICAL ANALYSIS

In this section, we analyze the empirical results of MV, AFSD and ASSD for zero-investment 4-, 13-, 26-, 52- and 78-week horizon portfolios based on the factors listed in Table 2. The *n*-week horizon portfolios are based on the rolling-window technique, where *n* is the window size. The 29 factors are now represented by their long-short portfolios.

### 6.1 | MV dominance for factor portfolios

We examine the MV dominances for our 29-factor portfolios for later comparison with those of ASD. First we consider the simplest case that compares the mean and standard deviation of returns with those of the four benchmarks. Tables 1 and 3 show the mean returns and standard deviations of our benchmarks and factor portfolios, respectively. We find that, although most of factor portfolios have higher absolute returns than our four benchmarks, there is no outperformance against the benchmarks under the MV framework. Even the long-short factor portfolio with the smallest standard deviation (namely, *MARCAP* with a standard deviation of 0.1046) is higher than the largest standard deviation of the four benchmarks, that is, Bitcoin with a standard deviation of 0.1039.

Even though cryptocurrency factor portfolios have highly nonnormal distributions, as a further illustration of the inadequacies of MV performance measures for cryptocurrency factor portfolios, we investigate the risk-adjusted returns for each factor portfolio using SRs and certainty equivalent returns (CERs) for 4–78-week horizons. CERs require a value for the risk aversion parameter ( $\lambda$ ). We set  $\lambda = 1, 3, 5$  as these values represent investors with different risk

preferences with respect to risk-seeking, risk-neutral and risk-averse investors, respectively (Chen et al., 2022; Tu & Zhou, 2010).

Table 4 contains the number of factor portfolios that dominate or outperform the S&P 500, T-Bond and Bitcoin for horizons of 4–78 weeks, where each count has a maximum of 29. We exclude T-Bills from Table 4 because T-Bills are considered to be the risk-free asset, and it is pointless to evaluate its risk-adjusted return. Panel A reports the number of dominances by each factor portfolio evaluated using SRs, and Panel B has the results using CERs. The number of factor portfolios dominances using the SR is an order of magnitude greater than those for factor portfolios tested using the CER with  $\lambda$  values of 3 and 5. MV requires dominance for all levels of risk aversion, while CERs only require out-performance for a particular level of risk aversion. This leads to CERs finding more occasions of out-performance than does MV dominance.

6.2 | AFSD

To test whether AFSD exists, we need to calculate the ratio of the violation area ( $M$  in Figure 1) to the total enclosed area for each portfolio, and compare these ratios ( $\varepsilon_i$ ) with a critical value of 5.9%, as discussed in Section 4. We define  $M$  as the area between the cumulative distributions when the cumulative distribution of a factor portfolio plots above the cumulative distribution of a benchmark (e.g., S&P 500, T-Bills, T-Bonds and Bitcoin). Likewise, we define  $N$  as the area between the cumulative distributions. The value of  $\varepsilon_i$  for AFSD is  $\varepsilon_i = \frac{M}{M+N}$ . If a portfolio has an  $\varepsilon_i$  value that is smaller than the critical value for AFSD, we conclude there is AFSD domination.

Table 5 shows the AFSD values of  $\varepsilon_i$  for each factor portfolio compared with the S&P 500 index, T-Bills, T-Bonds and Bitcoin for 4–78-week investment horizons. For most factor

TABLE 4 Sharpe ratio (SR) and certainty equivalent return.

This table reports the counts of outperformance by factor portfolios against S&P 500, T-Bonds and Bitcoin for different horizons. The entries in Panel B represent certainty equivalent return (CER) values with  $\lambda = 1, 3, 5$ , respectively. For brevity, the values of these tests are in the Supporting Information Appendix. See Section A12 in Supporting Information Appendix.

| Panel A: Counts of dominance by factor portfolios using SR |        |         |         |         |         | Panel B: Counts of outperformance by factor portfolios using CER ( $\lambda = 1, 3, 5$ ) |         |         |         |         |
|--|--------|---------|---------|---------|---------|--|---------|---------|---------|---------|
| Panel A1: Portfolios against S&P 500 Index                 |        |         |         |         |         | Panel B1: Portfolios against S&P 500 Index   |         |         |         |         |
|  | 4-week | 13-week | 26-week | 52-week | 78-week | 4-week   | 13-week | 26-week | 52-week | 78-week |
| Counts   | 5      | 6       | 3       | 4       | 2       | 7/1/0  | 7/1/1   | 10/1/1  | 11/2/1  | 11/3/1  |
| Panel A2: Portfolios against 10-year T-Bond                |        |         |         |         |         | Panel B2: Portfolios against 10-year T-Bond  |         |         |         |         |
|  | 4-week | 13-week | 26-week | 52-week | 78-week | 4-week   | 13-week | 26-week | 52-week | 78-week |
| Counts   | 7      | 9       | 12      | 11      | 11      | 7/1/0  | 11/1/1  | 11/2/1  | 11/2/1  | 11/3/1  |
| Panel A3: Portfolios against Bitcoin                       |        |         |         |         |         | Panel B3: Portfolios against Bitcoin   |         |         |         |         |
|  | 4-week | 13-week | 26-week | 52-week | 78-week | 4-week   | 13-week | 26-week | 52-week | 78-week |
| Counts   | 7      | 8       | 10      | 11      | 9       | 7/1/1  | 8/4/2   | 9/4/3   | 10/5/4  | 11/4/4  |

portfolios as the investment horizon lengthens  $\varepsilon_i$  decreases monotonically. The generally decreasing  $\varepsilon_i$  values indicate that AFSD dominance by the factor portfolios of the four benchmarks tends to increase as the horizon lengthens, as predicted by Leshno and Levy (2002).

Panel A of Table 5 reports the empirical  $\varepsilon_i$  values for each factor portfolio compared with the S&P 500 index for 4–78-week investment horizons. No portfolios dominate the S&P 500 until the holding period is extended to 13 weeks. For a 13-week horizon, only *MARCAP* dominates the S&P 500 by AFSD, as its  $\varepsilon_i$  value is less than 5.9%. As the holding period extends to 26 weeks, four out of the 29 portfolios exhibit AFSD. Similar to the results for a 13-week horizon, *MARCAP* is the best-performing portfolio with the smallest  $\varepsilon_i$  value of 0.44%, whereas *RMOM4* has the largest  $\varepsilon_i$  value of 5.85%, which is the worst of the four dominant portfolios. These four-factor portfolios have much higher volatility than the S&P 500, but their high returns compensate for their high risk. At a 52-week horizon, 10 of the 29-factor portfolios dominate the S&P 500 benchmark by AFSD, as six new dominant factor portfolios (*MOM2*, *RMOM1-2*, *VOLPRC*, *RETSKEW* and *MAXRET*) emerge. In this case portfolios such as *MARCAP* and *MOM1* perform well against the S&P 500 with zero  $\varepsilon_i$  values. *RETSKEW* is the worst-performing dominant portfolio with an  $\varepsilon_i$  value of 5.44%. When the investment horizon is extended to 78 weeks, the *LPRC* and *STDPRCVOL* factor portfolios now also dominate the S&P 500 by AFSD, resulting in 12 dominant factor portfolios.

Panel B of Table 5 reports  $\varepsilon_i$  values for each factor portfolio compared with 10-year T-Bonds for 4–78-week investment horizons. One more dominant portfolio (*AGE*) appears compared with Panel A at 26–78-week horizons. At a 26-week horizon, *MARCAP* still has the smallest  $\varepsilon_i$  value of 0.74%, and *RMOM4* is the worst of the 13 dominant portfolios with an  $\varepsilon_i$  of 5.62%. As the investment horizon extends to 52 weeks, the same factor portfolios as in Panel A (10 out of 29 portfolios) and *STDPRCVOL* are dominant. As the holding period lengthens to 78 weeks, the number of dominant portfolios increases by one compared with Panel A.

Panel C of Table 5 has  $\varepsilon_i$  values for each factor portfolio compared with 1-month T-Bills for 4–78-week investment horizons. Unlike Panels A and B, dominance appears at the 4-week horizon, and nine-factor portfolios dominate 1-month T-Bills using FSD, rather than just AFSD. An important reason for this is that T-Bills have a small weekly return of 0.0001. As the investment horizon extends to 13, 26, 52 and 78 weeks, the number of factor portfolios that are dominant according to AFSD are 15, 15, 10 and 13, respectively.

Panel D of Table 5 contains the  $\varepsilon_i$  values of factor portfolios against Bitcoin. Unlike the S&P 500 and T-Bond benchmarks, AFSD first appears at the 4-week horizon. As the horizon increases from 4 to 78 weeks the number of AFSD dominances also increases (5, 6, 6, 9 and 10). At 78 weeks it has fewer AFSD dominances than in Panels A–C. We also find that Bitcoin FSD dominates five-factor portfolios (*MOM8*, *MOM26*, *RMOM8*, *RMOM26* and *RETSKEW* at 78 weeks).

To sum up, the same eight-factor portfolios AFSD dominate all four of our benchmarks at 52- and 78-week horizons. They are factor portfolios based on *MARCAP*, *MOM1* and 2, *RMOM1-4* and *MAXRET*. This suggests that portfolios based on these factors may generate excess returns, irrespective of investor risk preferences.

### 6.3 | ASSD

The results from applying ASSD to each factor portfolio, relative to our four benchmarks at 4–78-week horizons, are in Table 6. The key difference between AFSD and ASSD is the critical



TABLE 5 (Continued)

| Portfolios        | Panel A: Portfolios against S&P 500 Index |         |         |         |         |        |         |         |         |         | Panel B: Portfolios against 10-year T-Bond |         |         |         |         | Panel C: Portfolios against 1-month T-Bill |         |         |         |         | Panel D: AFSD for portfolios against Bitcoin |  |  |  |  |
|-------------------|---|---------|---------|---------|---------|--------|---------|---------|---------|---------|--|---------|---------|---------|---------|--|---------|---------|---------|---------|--|--|--|--|--|
|                   | 4-week                                    | 13-week | 26-week | 52-week | 78-week | 4-week | 13-week | 26-week | 52-week | 78-week | 4-week                                     | 13-week | 26-week | 52-week | 78-week | 4-week                                     | 13-week | 26-week | 52-week | 78-week |  |  |  |  |  |
|                   | week                                      | week    | week    | week    | week    | week   | week    | week    | week    | week    | week                                       | week    | week    | week    | week    | week                                       | week    | week    | week    | week    |  |  |  |  |  |
| VOLPRC            | 0.2973                                    | 0.1944  | 0.0986  | 0.016*  | 0.0071* | 0.1794 | 0.4032  | 0.3483  | 0.2840  | 0.1670  | 0.1455                                     | 0.0053* | 0.3025  | 0*      | 0*      | 0*   | 0.1592  | 0.1110  | 0.1552  | 0.0965  | 0.0528*                                      |  |  |  |  |
| VOLSCALE          | 0.4131                                    | 0.3710  | 0.3169  | 0.2032  | 0.1794  | 0.4032 | 0.3483  | 0.2840  | 0.1670  | 0.1455  | 0.4028                                     | 0.3473  | 0.2831  | 0.1684  | 0.1433  | 0.4667                                     | 0.5028  | 0.5319  | 0.4486  | 0.4705  |  |  |  |  |  |
| RETVOL            | 0.5227                                    | 0.5425  | 0.5347  | 0.4824  | 0.4456  | 0.5098 | 0.5195  | 0.5039  | 0.4413  | 0.3949  | 0.5052                                     | 0.5119  | 0.4938  | 0.4271  | 0.3765  | 0.6468                                     | 0.7998  | 0.8983  | 0.9450  | 0.9143  |  |  |  |  |  |
| RETSKEW           | 0.3954                                    | 0.2703  | 0.1527  | 0.0544* | 0.0167* | 0.3875 | 0.2542  | 0.1328  | 0.0458* | 0.0141* | 0.3881                                     | 0*      | 0*      | 0*      | 0*      | 0.3748                                     | 0.1453  | 0.3166  | 0.1591  | 0.0945  |  |  |  |  |  |
| RETKURT           | 0.4884                                    | 0.4838  | 0.4557  | 0.4512  | 0.4987  | 0.4700 | 0.4473  | 0.4050  | 0.3954  | 0.4321  | 0.4652                                     | 0.4397  | 0.3962  | 0.3802  | 0.4067  | 0.9125                                     | 0.9986  | 0.9604  | 1.0000  | 1.0000  |  |  |  |  |  |
| MAXRET            | 0.3333                                    | 0.2196  | 0.1307  | 0.0069* | 0.0064* | 0.3312 | 0.2114  | 0.1187  | 0.007*  | 0.005*  | 0.3347                                     | 0*      | 0*      | 0*      | 0*      | 0.1935                                     | 0.0309* | 0.0133* | 0*      | 0*      |  |  |  |  |  |
| STDPRCV-OL        | 0.3130                                    | 0.2192  | 0.1449  | 0.0704  | 0.0452* | 0.3115 | 0.2077  | 0.1263  | 0.055*  | 0.0279* | 0.3160                                     | 0.2151  | 0.1366  | 0.0619  | 0.0319* | 0.2004                                     | 0.1521  | 0.1931  | 0.1724  | 0.1542  |  |  |  |  |  |
| MEANABS           | 0.3774                                    | 0.3142  | 0.2502  | 0.1555  | 0.1416  | 0.3702 | 0.2979  | 0.2286  | 0.1345  | 0.1226  | 0.3714                                     | 0.2994  | 0.2307  | 0.1369  | 0.1222  | 0.3904                                     | 0.3519  | 0.3157  | 0.2228  | 0.2249  |  |  |  |  |  |
| BETA              | 0.5059                                    | 0.4928  | 0.4706  | 0.4309  | 0.3753  | 0.4901 | 0.4666  | 0.4373  | 0.3919  | 0.3354  | 0.4851                                     | 0.4602  | 0.4294  | 0.3810  | 0.3238  | 0.6839                                     | 0.7346  | 0.7832  | 0.7809  | 0.8213  |  |  |  |  |  |
| BETA <sup>2</sup> | 0.5218                                    | 0.5151  | 0.5011  | 0.4758  | 0.4258  | 0.5024 | 0.4853  | 0.4661  | 0.4333  | 0.3788  | 0.4960                                     | 0.4772  | 0.4565  | 0.4193  | 0.3632  | 0.8738                                     | 0.9476  | 0.8776  | 0.8469  | 0.9368  |  |  |  |  |  |
| IDIOVOL           | 0.4936                                    | 0.4711  | 0.4477  | 0.3844  | 0.2831  | 0.4763 | 0.4424  | 0.4131  | 0.3412  | 0.2243  | 0.4715                                     | 0.4366  | 0.4063  | 0.3321  | 0.2153  | 0.8974                                     | 0.9541  | 0.8873  | 0.8380  | 0.9788  |  |  |  |  |  |
| DELAY             | 0.3911                                    | 0.3171  | 0.2368  | 0.1833  | 0.1388  | 0.3835 | 0.2965  | 0.2117  | 0.1603  | 0.1149  | 0*   | 0*      | 0*      | 0.1617  | 0.1153  | 0.3377                                     | 0.5211  | 0.5572  | 0.4618  | 0.4016  |  |  |  |  |  |
| LIQ               | 0.3774                                    | 0.3142  | 0.2502  | 0.1555  | 0.1416  | 0.3702 | 0.2979  | 0.2286  | 0.1345  | 0.1226  | 0.3714                                     | 0.2994  | 0.2307  | 0.1369  | 0.1222  | 0.3904                                     | 0.3519  | 0.3157  | 0.2228  | 0.2249  |  |  |  |  |  |



TABLE 6
 Almost second-order stochastic dominance (ASSD).

This table presents the empirical estimates of  $\varepsilon_i$  for 4–78-week investment horizons.  $A_1$  is the area between the cumulative distributions when the cumulative distribution of a portfolio plot above the cumulative distribution of a benchmark (i.e., S&P 500, T-Bonds, T-Bills and Bitcoin), and the aggregate area becomes positive.  $A_2$  is the absolute value of the area enclosed between the cumulative distribution functions (CDFs) of the two assets. The measure of  $\varepsilon_i$  for ASSD is  $\varepsilon_i = \frac{A_1}{A_2}$ . The critical value for ASSD is  $\varepsilon^* = 3.2\%$ . If the  $\varepsilon_i$  for any portfolio is less than the critical value, then the result indicates outperformance by that portfolio. \*\* denotes portfolios where  $\varepsilon_i$  is less than the critical value. The  $\varepsilon_i$  value of 1 indicates that benchmark dominates the corresponding portfolio.

| Portfolios | Panel A: Portfolios against S&P 500 |        |           |          |            |          | Panel B: Portfolios against 10-year T-Bond |          |            |          |           |        | Panel C: Portfolios against 1-month T-Bill |        |           |          |            |          | Panel D: AFSD for portfolios against Bitcoin |          |            |      |           |      |
|------------|-------------------------------------|--------|-----------|----------|------------|----------|--|----------|------------|----------|-----------|--------|--|--------|-----------|----------|------------|----------|--|----------|------------|------|-----------|------|
|            | Index                               |        | 4-78-week |          | 26-52-week |          | 4-78-week                                  |          | 26-52-week |          | 4-78-week |        | 26-52-week                                 |        | 4-78-week |          | 26-52-week |          | 4-78-week                                    |          | 26-52-week |      | 4-78-week |      |
|            | week                                | week   | week      | week     | week       | week     | week                                       | week     | week       | week     | week      | week   | week                                       | week   | week      | week     | week       | week     | week   | week     | week       | week | week      | week |
| MARCAP     | 0.1976                              | 0.0417 | 0.0044**  | 0**      | 0**        | 0.0044** | 0**  | 0.0074** | 0.0006**   | 0**      | 0**       | 0**    | 0**  | 0**    | 0**       | 0**      | 0**        | 0**      | 0**  | 0**      | 0**        | 0**  | 0**       | 0**  |
| LPRC       | 0.4955                              | 0.3402 | 0.2161    | 0.0853   | 0.0577     | 0.4890   | 0.3179                                     | 0.1897   | 0.0689     | 0.0395   | 0.4970    | 0.3272 | 0**  | 0.0761 | 0.0433    | 0.4034   | 0.3132     | 0.0460   | 0**  | 0.0760   |            |      |           |      |
| AGE        | 0.5127                              | 0.2766 | 0.1848    | 0.0971   | 0.0648     | 0.5062   | 0.2657                                     | 0.1666   | 0.0824     | 0.0504   | 0.5114    | 0**    | 0**  | 0.0882 | 0.0537    | 0.5725   | 0.1649     | 0.1583   | 0**  | 0**      |            |      |           |      |
| MOM1       | 0.2809                              | 0.0639 | 0.0232**  | 0**      | 0**        | 0.2942   | 0.0733                                     | 0.0244** | 0.0003**   | 0**      | 0**       | 0**    | 0**  | 0**    | 0**       | 0.0776   | 0**        | 0**      | 0**  | 0**      | 0**        |      |           |      |
| MOM2       | 0.3247                              | 0.1606 | 0.1127    | 0.0125** | 0.0088**   | 0.3346   | 0.1635                                     | 0.1083   | 0.0124**   | 0.0081** | 0**       | 0**    | 0**  | 0**    | 0**       | 0.1317   | 0.1030     | 0.0687   | 0**  | 0.0009** |            |      |           |      |
| MOM3       | 0.4944                              | 0.3453 | 0.2716    | 0.1744   | 0.1705     | 0.4916   | 0.3324                                     | 0.2494   | 0.1580     | 0.1486   | 0**       | 0**    | 0**  | 0.1605 | 0.1483    | 0.5683   | 0.2612     | 0.2039   | 0.0845                                       | 0.2415   |            |      |           |      |
| MOM4       | 0.6150                              | 0.4322 | 0.3478    | 0.2520   | 0.1184     | 0.5978   | 0.3975                                     | 0.2961   | 0.1996     | 0.0831   | 0.6012    | 0**    | 0**  | 0.2028 | 0.0867    | 0.6257   | 0.3779     | 0.7207   | 0.7763                                       | 0.5003   |            |      |           |      |
| MOM8       | 0.7495                              | 0.6775 | 0.5977    | 0.5477   | 0.5202     | 0.7843   | 0.7137                                     | 0.6217   | 0.5590     | 0.5249   | 0.8017    | 0.7326 | 0.6378                                     | 0.5694 | 0.5321    | 0.7844   | 0.6859     | 0.5068   | 0.5036                                       | 1.0000   |            |      |           |      |
| MOM26      | 0.6664                              | 0.5786 | 0.5465    | 0.5093   | 1.0000     | 0.6883   | 0.5954                                     | 0.5589   | 0.5131     | 0.5000   | 0.7007    | 0.6067 | 0.5688                                     | 0.5171 | 0.5002    | 0.7594   | 1.0000     | 1.0000   | 1.0000                                       | 1.0000   |            |      |           |      |
| RMOM1      | 0.2997                              | 0.1183 | 0.0698    | 0.005**  | 0**        | 0.3160   | 0.1258                                     | 0.0661   | 0.0054**   | 0**      | 0**       | 0**    | 0**  | 0**    | 0**       | 0.003**  | 0.005**    | 0.0019** | 0**  | 0**      |            |      |           |      |
| RMOM2      | 0.3599                              | 0.1955 | 0.1339    | 0.0169** | 0**        | 0.3705   | 0.1965                                     | 0.1283   | 0.0167**   | 0**      | 0**       | 0**    | 0**  | 0**    | 0**       | 0.2370   | 0.1060     | 0.1124   | 0**  | 0**      |            |      |           |      |
| RMOM3      | 0.2934                              | 0.1174 | 0.0396    | 0.0013** | 0.0003**   | 0.3102   | 0.1255                                     | 0.0394   | 0.002**    | 0.0007** | 0**       | 0**    | 0**  | 0**    | 0**       | 0.0867   | 0.0021**   | 0**      | 0**  | 0**      |            |      |           |      |
| RMOM4      | 0.3682                              | 0.1321 | 0.0621    | 0.0089** | 0**        | 0.3775   | 0.1388                                     | 0.0595   | 0.0089**   | 0**      | 0**       | 0**    | 0**  | 0**    | 0**       | 0.0151** | 0**        | 0**      | 0**  | 0**      |            |      |           |      |
| RMOM8      | 0.9838                              | 0.9763 | 0.9955    | 0.8094   | 0.6856     | 0.9266   | 0.8741                                     | 0.8517   | 0.9482     | 0.7693   | 0.9105    | 0**    | 0.8164                                     | 0.9698 | 0.8311    | 0.9119   | 0.7314     | 0.6667   | 1.0000                                       | 1.0000   |            |      |           |      |
| RMOM26     | 0.7042                              | 0.6227 | 0.5808    | 0.5431   | 0.5319     | 0.7333   | 0.6508                                     | 0.6038   | 0.5564     | 0.5403   | 0.7486    | 0.6670 | 0.6197                                     | 0.5689 | 0.5495    | 0.6834   | 0.6682     | 1.0000   | 1.0000                                       | 1.0000   |            |      |           |      |
| VOL        | 0.9320                              | 0.9842 | 0.9342    | 0.7855   | 0.6871     | 0.9883   | 0.8976                                     | 0.8387   | 0.6989     | 0.5972   | 0.9907    | 0.8764 | 0.8154                                     | 0.6736 | 0.5702    | 0.8804   | 0.5996     | 0.6138   | 0.6555                                       | 0.6419   |            |      |           |      |

(Continues)

TABLE 6 (Continued)

| Portfolios        | Panel A: Portfolios against S&P 500 Index |         |         |         |          |        |         |         | Panel B: Portfolios against 10-year T-Bond |          |        |         |         |         |         |        | Panel C: Portfolios against 1-month T-Bill |          |          |         |        |          |          |          | Panel D: AFSD for portfolios against Bitcoin |        |          |          |          |         |        |          |          |          |        |          |          |          |          |        |
|-------------------|---|---------|---------|---------|----------|--------|---------|---------|--|----------|--------|---------|---------|---------|---------|--------|--|----------|----------|---------|--------|----------|----------|----------|--|--------|----------|----------|----------|---------|--------|----------|----------|----------|--------|----------|----------|----------|----------|--------|
|                   | 4-week                                    | 13-week | 26-week | 52-week | 78-week  | 4-week | 13-week | 26-week | 52-week                                    | 78-week  | 4-week | 13-week | 26-week | 52-week | 78-week | 4-week | 13-week                                    | 26-week  | 52-week  | 78-week | 4-week | 13-week  | 26-week  | 52-week  | 78-week                                      | 4-week | 13-week  | 26-week  | 52-week  | 78-week |        |          |          |          |        |          |          |          |          |        |
|                   | week                                      | week    | week    | week    | week     | week   | week    | week    | week                                       | week     | week   | week    | week    | week    | week    | week   | week                                       | week     | week     | week    | week   | week     | week     | week     | week   | week   | week     | week     | week     | week    | week   |          |          |          |        |          |          |          |          |        |
| VOLPRC            | 0.4230                                    | 0.2413  | 0.1094  | 0.0320  | 0.0072** | 0.4227 | 0.2263  | 0.0960  | 0.0147**                                   | 0.0053** | 0.4336 | 0**     | 0**     | 0**     | 0**     | 0.1382 | 0**  | 0.0027** | 0**      | 0**     | 0.1382 | 0**      | 0.0027** | 0**      | 0**  | 0.1382 | 0**      | 0.0027** | 0**      | 0**     | 0.1382 | 0**      | 0.0027** | 0**      | 0**    |          |          |          |          |        |
| VOLSCALE          | 0.7038                                    | 0.5897  | 0.4640  | 0.2549  | 0.2186   | 0.6756 | 0.5343  | 0.3966  | 0.2005                                     | 0.1703   | 0.6745 | 0.5322  | 0.3949  | 0.2025  | 0.1673  | 0.8741 | 0.9963                                     | 0.9078   | 0.7707   | 0.9410  | 0.8741 | 0.9963   | 0.9078   | 0.7707   | 0.9410                                       | 0.8741 | 0.9963   | 0.9078   | 0.7707   | 0.9410  | 0.8741 | 0.9963   | 0.9078   | 0.7707   | 0.9410 |          |          |          |          |        |
| RETVOL            | 0.9200                                    | 0.8646  | 0.8851  | 0.9321  | 0.8036   | 0.9629 | 0.9303  | 0.9846  | 0.7900                                     | 0.6527   | 0.9798 | 0.9557  | 0.9757  | 0.7456  | 0.6039  | 0.6878 | 0.5715                                     | 0.7715   | 0.6787   | 0.5246  | 0.6878 | 0.5715   | 0.7715   | 0.6787   | 0.5246                                       | 0.6878 | 0.5715   | 0.7715   | 0.6787   | 0.5246  | 0.6878 | 0.5715   | 0.7715   | 0.6787   | 0.5246 |          |          |          |          |        |
| RETSKEW           | 0.6540                                    | 0.3704  | 0.1803  | 0.0575  | 0.0169** | 0.6325 | 0.3408  | 0.1532  | 0.0480                                     | 0.0143** | 0.6341 | 0**     | 0**     | 0**     | 0**     | 0.8152 | 0.0252**                                   | 0**      | 0**      | 0**     | 0.8152 | 0.0252** | 0**      | 0**      | 0**  | 0.8152 | 0.0252** | 0**      | 0**      | 0**     | 0.8152 | 0.0252** | 0**      | 0**      | 0.8152 | 0.0252** | 0**      | 0**      |          |        |
| RETKURT           | 0.9548                                    | 0.9374  | 0.8372  | 0.8222  | 0.9949   | 0.8870 | 0.8091  | 0.6805  | 0.6539                                     | 0.7608   | 0.8700 | 0.7848  | 0.6562  | 0.6134  | 0.6856  | 0.7941 | 0.9986                                     | 0.6292   | 1.0000   | 1.0000  | 0.7941 | 0.9986   | 0.6292   | 1.0000   | 1.0000                                       | 0.7941 | 0.9986   | 0.6292   | 1.0000   | 1.0000  | 0.7941 | 0.9986   | 0.6292   | 1.0000   | 1.0000 | 0.7941   | 0.9986   | 0.6292   | 1.0000   | 1.0000 |
| MAXRET            | 0.5000                                    | 0.2814  | 0.1504  | 0.007** | 0.0064** | 0.4953 | 0.2681  | 0.1347  | 0.0071**                                   | 0.005**  | 0.5031 | 0**     | 0**     | 0**     | 0**     | 0.3512 | 0.0319**                                   | 0.0015** | 0**      | 0**     | 0.3512 | 0.0319** | 0.0015** | 0**      | 0**  | 0.3512 | 0.0319** | 0.0015** | 0**      | 0**     | 0.3512 | 0.0319** | 0.0015** | 0**      | 0**    | 0.3512   | 0.0319** | 0.0015** | 0**      | 0**    |
| STDPRCV-OL        | 0.4556                                    | 0.2808  | 0.1694  | 0.0757  | 0.0473   | 0.4524 | 0.2621  | 0.1446  | 0.0582                                     | 0.0287** | 0.4620 | 0.2740  | 0.1582  | 0.0660  | 0.0329  | 0.2657 | 0.0874                                     | 0.1008   | 0.0145** | 0.1454  | 0.2657 | 0.0874   | 0.1008   | 0.0145** | 0.1454                                       | 0.2657 | 0.0874   | 0.1008   | 0.0145** | 0.1454  | 0.2657 | 0.0874   | 0.1008   | 0.0145** | 0.1454 | 0.2657   | 0.0874   | 0.1008   | 0.0145** | 0.1454 |
| MEANABS           | 0.6061                                    | 0.4582  | 0.3337  | 0.1841  | 0.1649   | 0.5879 | 0.4242  | 0.2963  | 0.1554                                     | 0.1398   | 0.5908 | 0.4273  | 0.2999  | 0.1586  | 0.1393  | 0.6341 | 0.5429                                     | 0.6313   | 0.2756   | 0.2901  | 0.6341 | 0.5429   | 0.6313   | 0.2756   | 0.2901                                       | 0.6341 | 0.5429   | 0.6313   | 0.2756   | 0.2901  | 0.6341 | 0.5429   | 0.6313   | 0.2756   | 0.2901 | 0.6341   | 0.5429   | 0.6313   | 0.2756   | 0.2901 |
| BETA              | 0.9771                                    | 0.9715  | 0.8890  | 0.7572  | 0.6007   | 0.9610 | 0.8749  | 0.7771  | 0.6445                                     | 0.5047   | 0.9421 | 0.8524  | 0.7526  | 0.6155  | 0.4789  | 0.6502 | 0.6102                                     | 0.5803   | 0.5814   | 0.5611  | 0.6502 | 0.6102   | 0.5803   | 0.5814   | 0.5611                                       | 0.6502 | 0.6102   | 0.5803   | 0.5814   | 0.5611  | 0.6502 | 0.6102   | 0.5803   | 0.5814   | 0.5611 | 0.6502   | 0.6102   | 0.5803   | 0.5814   | 0.5611 |
| BETA <sup>2</sup> | 0.9228                                    | 0.9447  | 0.9977  | 0.9515  | 0.7415   | 0.9906 | 0.9429  | 0.8731  | 0.7646                                     | 0.6098   | 0.9842 | 0.9129  | 0.8400  | 0.7220  | 0.5704  | 0.6844 | 0.8392                                     | 0.5375   | 0.5495   | 0.5175  | 0.6844 | 0.8392   | 0.5375   | 0.5495   | 0.5175                                       | 0.6844 | 0.8392   | 0.5375   | 0.5495   | 0.5175  | 0.6844 | 0.8392   | 0.5375   | 0.5495   | 0.5175 | 0.6844   | 0.8392   | 0.5375   | 0.5495   | 0.5175 |
| IDIOVOL           | 0.9746                                    | 0.8906  | 0.8105  | 0.6245  | 0.3948   | 0.9095 | 0.7936  | 0.7039  | 0.5180                                     | 0.2892   | 0.8922 | 0.7750  | 0.6845  | 0.4973  | 0.2744  | 0.9346 | 0.7786                                     | 0.5339   | 0.5535   | 0.6705  | 0.9346 | 0.7786   | 0.5339   | 0.5535   | 0.6705                                       | 0.9346 | 0.7786   | 0.5339   | 0.5535   | 0.6705  | 0.9346 | 0.7786   | 0.5339   | 0.5535   | 0.6705 | 0.9346   | 0.7786   | 0.5339   | 0.5535   | 0.6705 |
| DELAY             | 0.6422                                    | 0.4644  | 0.3102  | 0.2245  | 0.1611   | 0.6220 | 0.4214  | 0.2685  | 0.1908                                     | 0.1298   | 0**    | 0**     | 0**     | 0.1928  | 0.1303  | 0.8584 | 0.8090                                     | 0.6850   | 0.5925   | 0.5925  | 0.8584 | 0.8090   | 0.6850   | 0.5925   | 0.5925                                       | 0.8584 | 0.8090   | 0.6850   | 0.5925   | 0.5925  | 0.8584 | 0.8090   | 0.6850   | 0.5925   | 0.5925 | 0.8584   | 0.8090   | 0.6850   | 0.5925   | 0.5925 |
| LIQ               | 0.6061                                    | 0.4582  | 0.3337  | 0.1841  | 0.1649   | 0.5879 | 0.4242  | 0.2963  | 0.1554                                     | 0.1398   | 0.5908 | 0.4273  | 0.2999  | 0.1586  | 0.1393  | 0.6341 | 0.5429                                     | 0.6313   | 0.2756   | 0.2901  | 0.6341 | 0.5429   | 0.6313   | 0.2756   | 0.2901                                       | 0.6341 | 0.5429   | 0.6313   | 0.2756   | 0.2901  | 0.6341 | 0.5429   | 0.6313   | 0.2756   | 0.2901 | 0.6341   | 0.5429   | 0.6313   | 0.2756   | 0.2901 |

Abbreviation: AFSD, almost first-order stochastic dominance.

value, as ASSD has a lower critical value of 3.2%, as opposed to the value of 5.9% for AFSD. This reveals that ASSD permits smaller violations of SD than AFSD. Panel A of Table 6 documents  $\varepsilon_i$  values for each factor portfolio compared with the S&P 500 for 4–78-week investment horizons. For 4- and 13-week horizons, none of portfolios dominate the S&P 500 index. As the investment horizon extends to 26 weeks, two out of 29-factor portfolios dominate this benchmark. *MARCAP* is the best-performing portfolio with an  $\varepsilon_i$  value of 0.44%. For a 52-week horizon, eight-factor portfolios dominate the benchmark in the sense of ASSD. *MARCAP* and *MOM1* are the best-performing portfolios with  $\varepsilon_i$  values of zero. As the investment horizon extends to 78 weeks, the number of ASSD dominant portfolios increases to 10, and five of these factor portfolios exhibit SSD dominance.

Panel B in Table 6 has  $\varepsilon_i$  values for each factor portfolio compared with T-Bonds for 4–78-week investment horizons. There is no dominance at 4 and 13 weeks. For a 26-week horizon, the number of dominant portfolios (three) is the same as for Panel A. The best-performing factor portfolio is still *MARCAP*, with an  $\varepsilon_i$  value of 0.74%. As the investment horizon extends to 52 and 78 weeks, the number of dominant portfolios increases to 9 and 11, respectively—slightly more than the number of dominant portfolios in Panel A.

Panel C in Table 6 reports the  $\varepsilon_i$  values for each factor portfolio compared with T-Bills for 4–78-week investment horizons. All the dominances are SSD, rather than just ASSD. Unlike Panels A and B, dominance first appears at a 4-week horizon, with nine-factor portfolios exhibiting SSD dominance. As the investment horizon extends to 13, 26, 52 and 78 weeks the number of SSD dominant portfolios increases to 15, 15, 10 and 10, respectively.

Panel D in Table 6 shows the  $\varepsilon_i$  value for each portfolio against Bitcoin. Similar to Panel D in Table 5, ASSD of three portfolios occurs at a 4-week horizon. As the investment horizon becomes longer (13, 26, 52 and 78 weeks), the number of ASSD factor portfolios increases to 8, 8, 14 and 11, respectively. As for AFSD, when the horizon is 78 weeks Bitcoin SSD dominates the same five-factor portfolios.

We have tested the 29-factor portfolios for AFSD and ASSD against four benchmarks—the S&P500, T-Bonds, T-Bills and Bitcoin for 4–78-week horizons. For 52- and 78-week horizons, eight of the 29-factor portfolios dominate the four benchmarks by both AFSD and ASSD. They are factor portfolios based on *MARCAP*, *MOM1* and 2, *RMOM1–4* and *MAXRET*. Therefore no additional factor portfolios exhibit dominance when investors are assumed to be risk averse, and the following sections focus on these eight factors.

## 7 | CRYPTOCURRENCY ASSET PRICING AND FACTOR MODELS

After identifying eight ASD dominant factor portfolios, we now turn to regression analysis. Specifically, we aim to examine whether the excess cross-sectional returns of these eight dominant factor portfolios can be explained by the existing coin market three-factor model of Liu et al. (2022), and how the existing model can be improved. The performance of the existing coin market three-factor model may be unsatisfactory for at least two reasons. First, the dominant factors established by ASD may behave differently, compared with factors determined by regression. So we evaluate whether the coin market three-factor model can account for the dominant factor portfolios we identify. Second, if the coin market three-factor model does not have adequate explanatory power, it is important to find a more comprehensive model to better explain the dominant factor portfolios.

## 7.1 | Coin market three-factor model

We start with the construction of three cryptocurrency market factors: market, size and momentum. The cryptocurrency market excess return (*CMKT*) was discussed in Section 2. For the size factor, we follow the Fama and French three-factor model procedure. We sort the 2353 coins into three size groups by market capitalization: the bottom 30% (Small group), the middle 40% (Middle group) and the top 30% (Big group). We then form value-weighted portfolios of each of the three groups. The cryptocurrency size factor (*CSMB*) is the return on the small group minus that on the big group. As the return on the long-short 1-week momentum portfolio is the highest among the momentum factors (see Table 3), we construct the momentum factor (*CMOM*) using the intersection of 1-week momentum and coin size factor under dependent sorting. Specifically, we first sort the coins into two groups, namely, Small and Big. We then establish three momentum portfolios (Low, Medium and High which correspond to the bottom 30%, the middle 40% and the top 30%) based on 1-week returns for each size group. Finally we compute weekly value-weighted returns on these six groups. *CMOM* is formed by longing the High portfolios and shorting the Low portfolios across the Big- and Small-size groups. The mathematical formulation is

$$CMOM = \frac{1}{2}(Small\ High + Big\ High) - \frac{1}{2}(Small\ Low + Big\ Low). \quad (5)$$

Therefore, the coin market three-factor model (Liu et al., 2022) is

$$R_i - R_f = \alpha_i + \beta_{i,CMKT} CMKT + \beta_{i,CSMB} CSMB + \beta_{i,CMOM} CMOM + \varepsilon_i, \quad (6)$$

where  $R_i$  is the return on the factor portfolio,  $R_f$  is the risk-free rate, *CMKT* is the cryptocurrency market index excess return, *CSMB* is the cryptocurrency size factor and *CMOM* is the cryptocurrency momentum factor.

Table 7 reports the regression results for our eight dominant factor portfolios. We find that the coin market three-factor model only accounts for one of the eight dominant factors. With the exception of *MAXRET*, all the dominant factor portfolios have statistically significant alphas. Seven of the eight dominant factor portfolios have positive exposure to the momentum factor (*CMOM*), including the nonmomentum factor portfolios based on size and volatility. The market and size factors have fewer significant loadings than the momentum factor. Both the nonmomentum dominant factor portfolios (*MARCAP* and *MAXRET*) have statistically significant loadings on *CMKT* and *CSMB*, whereas the six momentum factor portfolios have less significant exposure to these two risk factors. The alpha values for seven of the factor portfolios are positive and significant at the 1%–5% levels, and so the three-factor model suggests these dominant factor portfolios offer abnormal returns.

To conclude, we find that the coin market three-factor model cannot explain 75%–99% of the variation in excess returns for these eight-factor portfolios. The three-factor coin model also finds that seven-factor portfolios have significantly positive alphas, indicating there are abnormal returns. The three-factor model does have significant explanatory power for the 21 nondominant portfolios. As shown in Table A.18 in Supporting Information Appendix A5, the three-factor model provides a good explanation for 15 of the 21 nondominant factor portfolios. These results indicate that Liu et al.'s (2022) model explains factor portfolios which do not dominate our four benchmarks, but cannot fully explain our eight dominant factor portfolios.

**TABLE 7** Coin market three-factor model.

This table reports the results of regression analysis for the coin market three-factor model and our eight dominant factor portfolios. The dependent variables are the long-short factor portfolios constructed in Section 5. \*, \*\* and \*\*\* represent significance at 10%, 5% and 1% levels, respectively. The *t* statistics are reported in parentheses.

|               | $\alpha$              | $\beta_{CMKT}$         | $\beta_{CSMB}$        | $\beta_{CMOM}$         | Adjusted $R^2$ |
|---------------|-----------------------|------------------------|-----------------------|------------------------|----------------|
| <i>MARCAP</i> | 0.0189***<br>(3.8616) | −0.386***<br>(−8.9708) | 0.2233***<br>(7.8978) | 0.0373***<br>(2.6239)  | 0.2523         |
| <i>MOM1</i>   | 0.0234**<br>(2.5164)  | 0.0458<br>(0.5601)     | 0.0854<br>(1.5909)    | 0.2357***<br>(8.7357)  | 0.1617         |
| <i>MOM2</i>   | 0.025***<br>(2.6457)  | 0.0206<br>(0.2484)     | 0.1036*<br>(1.8966)   | 0.1285***<br>(4.6911)  | 0.0530         |
| <i>RMOM1</i>  | 0.0201***<br>(2.6408) | 0.1257*<br>(1.8793)    | −0.0081<br>(−0.1849)  | 0.1761***<br>(7.9663)  | 0.1448         |
| <i>RMOM2</i>  | 0.033***<br>(3.6661)  | −0.0504<br>(−0.6378)   | −0.0775<br>(−1.4899)  | −0.0564**<br>(−2.1435) | 0.0106         |
| <i>RMOM3</i>  | 0.0223**<br>(2.3975)  | 0.1407*<br>(1.7196)    | 0.0303<br>(0.5622)    | 0.1419***<br>(5.2470)  | 0.0686         |
| <i>RMOM4</i>  | 0.0247***<br>(3.2328) | 0.2027***<br>(3.0230)  | −0.089**<br>(−2.0144) | 0.0051<br>(0.2305)     | 0.0240         |
| <i>MAXRET</i> | 0.0087<br>(1.1612)    | 0.2938***<br>(4.4429)  | 0.0739*<br>(1.7014)   | 0.1***<br>(4.5794)     | 0.0988         |

Therefore, to better understand the coin market and its performance, it is important to develop a cryptocurrency asset pricing model that can explain the dominant factor portfolios we have identified.

## 7.2 | Augmented coin factor models

Section 7.1 found that the coin market three-factor model can provide some explanatory power, but seven of the eight dominant factor portfolios have significant alphas. In this section, we propose an augmented coin factor model that provides a better explanation of returns on the dominant factor portfolios. Stambaugh and Yuan (2017) propose that an asset pricing model might increase its explanatory power by incorporating a mispricing factor. They constructed a mispricing factor by computing the equally weighted average of returns on the anomalous assets. We draw on the idea of incorporating a mispricing factor, but use a different approach to evaluate the performance of the augmented model. We use the unweighted average of returns on the mispriced factor portfolios because we cannot determine which factor portfolios are more important; and the unweighted average is considered to be an effective and easily

interpretable way of aggregating returns on the seven mispriced factor portfolios (Kotsiantis et al., 2006; Opitz & Maclin, 1999). Our mispricing factor can be easily replicated by  $\frac{1}{N}$  investing in the anomalous factor portfolios, and the correlations between these factor portfolios are relatively low.

We now augment the three-factor model with two additional factors, in addition to a mispricing factor. Since cryptocurrency is an application of blockchain, we use Twitter counts of the keyword “blockchain” (normalized to zero mean and unit standard deviation) as a proxy for investor attention (*Attention*). We also collected four measures to create a proxy for network effects, complying with Liu and Tsyvinski (2020). These are the number of wallet users for Bitcoin, the number of active addresses for Bitcoin, the number of transactions for Bitcoin, and the number of payments for Bitcoin. We calculate the growth rate for each of these four network proxies, and use their first principal component (*PC1*), which accounts for 88.2% of the total variation in four network proxies, as the network proxy (*Network*). These additional variables create seven augmented coin market models, which are listed in Table 8.

The value “Y” in Table 8 means that the corresponding coin market model includes this factor. For example, Model 1 comprises the coin market three-factor model and the mispricing factor *Mispricing*, as in the equation below. The rest of the models are constructed in the same manner.

$$R_i - R_f = \alpha_i + \beta_{i,CMKT} CMKT + \beta_{i,CSMB} CSMB + \beta_{i,CMOM} CMOM + \beta_{i,Mispricing} Mispricing + \varepsilon_i. \quad (7)$$

Table 9 presents the regression results for Model 6 of Table 8. The number of strategies with a significant exposure to size and momentum is slightly lower than for the original model in Table 7, while the number of significant coefficients for the coin market factor remains unchanged. In addition, only three of the alphas are significant. Since all the mispricing factor coefficients are significant, the mispricing factor appears to be absorbing part of the size and momentum effects, as well as some of the mispricings. The network factor included in Model 6 is significant for only one dominant factor portfolio (*MARCAP*). We archive the complete regression results for seven augmented models in Section A2 of Supporting Information Appendix.

**TABLE 8** Potential combinations.

This table reports the seven augmented models based on the mispricing and two nonfinancial factors. A “Y” means that the corresponding model incorporates the factor.

|   | <i>Coin market three-factor model</i> | <i>Mispricing</i> | <i>Attention</i> | <i>Network</i> |
|---|---------------------------------------|-------------------|------------------|----------------|
| 1 | Y                                     | Y                 | –                | –              |
| 2 | Y                                     | –                 | Y                | –              |
| 3 | Y                                     | –                 | –                | Y              |
| 4 | Y                                     | Y                 | Y                | –              |
| 5 | Y                                     | –                 | Y                | Y              |
| 6 | Y                                     | Y                 | –                | Y              |
| 7 | Y                                     | Y                 | Y                | Y              |

**TABLE 9** Augmented coin market model (Model 6).

This table reports the results of regression analysis of our eight dominant factor portfolios using Model 6 in Table 8. \*, \*\* and \*\*\* represent significance at the 10%, 5% and 1% levels, respectively. The  $t$  statistics are reported in parentheses.

|               | $\alpha$               | $\beta_{CMKT}$          | $\beta_{CSMB}$         | $\beta_{CMOM}$          | $\beta_{Mispricing}$   | $\beta_{Network}$    | Adjusted $R^2$ |
|---------------|------------------------|-------------------------|------------------------|-------------------------|------------------------|----------------------|----------------|
| <i>MARCAP</i> | 0.0162***<br>(3.2120)  | −0.3813***<br>(−8.8870) | 0.2212***<br>(7.8422)  | 0.0252*<br>(1.6684)     | 0.1072*<br>(1.8911)    | 0.0156*<br>(1.7361)  | 0.2620         |
| <i>MOM1</i>   | −0.0134**<br>(−2.0234) | 0.0299<br>(0.5314)      | 0.0272<br>(0.7353)     | 0.0874***<br>(4.4056)   | 1.5405***<br>(20.7143) | 0.0150<br>(1.2665)   | 0.6050         |
| <i>MOM2</i>   | −0.0102<br>(−1.4355)   | −0.0013<br>(−0.0216)    | 0.0456<br>(1.1409)     | −0.0122<br>(−0.5710)    | 1.4811***<br>(18.5189) | −0.0023<br>(−0.1786) | 0.4980         |
| <i>RMOM1</i>  | −0.0072<br>(−1.2085)   | 0.1069**<br>(2.1108)    | −0.0540<br>(−1.6224)   | 0.0674***<br>(3.7786)   | 1.1501***<br>(17.1946) | −0.0067<br>(−0.6271) | 0.5140         |
| <i>RMOM2</i>  | 0.0211**<br>(2.3268)   | −0.0581<br>(−0.7532)    | −0.0976*<br>(−1.9209)  | −0.1035***<br>(−3.7836) | 0.5022***<br>(4.9349)  | −0.0012<br>(−0.0766) | 0.0652         |
| <i>RMOM3</i>  | −0.0094<br>(−1.2515)   | 0.1144*<br>(1.7900)     | −0.0254<br>(−0.6019)   | 0.0166<br>(0.7374)      | 1.3407***<br>(15.8978) | −0.0217<br>(−1.6148) | 0.4362         |
| <i>RMOM4</i>  | 0.0038<br>(0.5555)     | 0.19***<br>(3.2667)     | −0.124***<br>(−3.2338) | −0.0777***<br>(−3.7672) | 0.8821***<br>(11.4881) | −0.0003<br>(−0.0286) | 0.2717         |
| <i>MAXRET</i> | 0.0038<br>(0.4870)     | 0.2921***<br>(4.4234)   | 0.0663<br>(1.5264)     | 0.0799***<br>(3.4335)   | 0.2069**<br>(2.3711)   | 0.0033<br>(0.2381)   | 0.1075         |

In Table 10 we compare the performance of the coin market three-factor model in Table 7, and the seven augmented five-factor coin market models in Table 8. We apply seven formal asset pricing metrics (Ahmed et al., 2019; Fama & French, 2015). The first metric is  $AlR^2$ , representing the average  $R^2$  for each augmented model—the higher is  $AlR^2$  the higher is the explanatory power. Second, the number of significant alphas indicates how many of the dominant factor portfolios appear to have anomalous returns according to the model. Our third metric is the gifted rating scale (GRS) statistic, which has the null hypothesis that all alphas are jointly indistinguishable from zero (Gibbons et al., 1989). Hence, a small GRS statistic is preferable. Our fourth metric is the average absolute value of the eight alphas, denoted as  $Al|\alpha_i|$ . As significant alphas are viewed as a measure of model mispricing, a model with a smaller value of this metric performs better. Our fifth metric is the ratio of the average absolute value of the alphas to the average absolute value of  $\bar{r}_i$ , denoted by  $Al|\alpha_i|/Al\bar{r}_i|$ , where  $\bar{r}_i$  is the average excess return on a dominant factor portfolio, minus the average excess return in the market portfolio. Our sixth metric is the ratio of the average squared alpha to the average squared value of  $\bar{r}_i$ , denoted by  $A\alpha_i^2/A\bar{r}_i^2$ . Both  $Al|\alpha_i|/Al\bar{r}_i|$  and  $A\alpha_i^2/A\bar{r}_i^2$  evaluate the variation of the alphas, relative to the variation of the average excess returns on each dominant factor portfolio; with low values of these ratios representing a good performance of the model. Our final metric is  $As^2(\alpha_i)/A\alpha_i^2$ , which is the ratio of the average variance of the sampling errors of the estimated



**TABLE 10** Comparison of model performance.

This table reports the performance measures for the coin market three-factor model, the seven augmented coin market five-factor models, and the coin market three-factor NTFM and NTFM-SM models. The last row contains results for the NTFM-RMOM model analyzed in Section 8. Columns two to the penultimate column are measures of model performance, and the last column illustrates the overall scores for each model, the higher the better. \* and \*\* demonstrate the highest score within the panel, and the highest score across the panels.

| Model  | $A R^2 $ | Significant<br>alphas | GRS    | $A \alpha_i $ | $A \alpha_i /A \bar{r}_i $ | $A\alpha_i^2/A\bar{r}_i^2$ | $As^2(\alpha_i)/A\alpha_i^2$ | Score |
|--|----------|-----------------------|--------|---------------|----------------------------|----------------------------|------------------------------|-------|
| <i>Panel A: Coin market three-factor model</i>                               |          |                       |        |               |                            |                            |                              |       |
| Three-factor   | 0.10     | 7                     | 8.3486 | 0.0220        | 1.6188                     | 0.0149                     | 0.1284                       | 0     |
| <i>Panel B: Performance metrics for potential models</i>                     |          |                       |        |               |                            |                            |                              |       |
| 1  | 0.35     | 3                     | 3.3648 | 0.0106        | 0.3803                     | 0.1793                     | 0.3454                       | 2     |
| 2  | 0.10     | 7                     | 7.3838 | 0.0213        | 0.7621                     | 0.6281                     | 0.1465                       | 0     |
| 3  | 0.10     | 7                     | 8.2794 | 0.0219        | 0.7827                     | 0.6434                     | 0.1296                       | 0     |
| 4  | 0.34     | 2                     | 3.5880 | 0.0113        | 0.4031                     | 0.2161                     | 0.3132                       | 1     |
| 5  | 0.10     | 7                     | 7.3828 | 0.0213        | 0.7603                     | 0.6254                     | 0.1472                       | 0     |
| 6  | 0.34     | 3                     | 3.3678 | 0.0106        | 0.3801                     | 0.1792                     | 0.3458                       | 4     |
| 7  | 0.34     | 2                     | 3.6111 | 0.0113        | 0.4042                     | 0.2171                     | 0.3120                       | 1     |
| <i>Panel C: Performance metrics for the reduced model</i>                    |          |                       |        |               |                            |                            |                              |       |
| NTFM   | 0.33     | 3                     | 3.4828 | 0.0107        | 0.3821                     | 0.1810                     | 0.3503                       | 0     |
| NTFM-SM  | 0.21     | 1                     | 1.2127 | 0.0051        | 0.3754                     | 0.0012                     | 1.5940                       | 6     |
| <i>Panel D: Performance metrics for the augmented model with MispricingM</i> |          |                       |        |               |                            |                            |                              |       |
| NTFM-RMOM  | 0.24     | 1                     | 3.1132 | 0.0066        | 0.4878                     | 0.0025                     | 0.7038                       | 1     |

Abbreviations: GRS, gifted rating scale; NTFM, new three-factor model; RMOM, risk-adjusted momentum; SM, simplified mispricing.

alphas to  $A\alpha_i^2$ . This final metric measures the proportion of the variance of the alphas due to sampling errors, rather than the dispersion of the true alphas. Therefore, a higher value of this ratio indicates a better model. The right-hand column (*Score*) is the number of metrics on which the factor model is the best.

Panels A and B show that Model 6 is the best performing of the three-factor model and the seven models in Table 8 for four reasons. First, the number of significant alphas decreases to three, instead of seven for the original coin market three-factor model. Second, the  $R^2$  values for the dominant factor portfolios are significantly improved, especially for the momentum factor portfolios. For instance, compared with the results for the original model in Table 7, the  $R^2$  value for *MOM2* increases from 5.3% to 49.8%. Third, Model 6 has the highest score for the performance comparison among the seven models. Finally, in Model 6 all eight dominant factor portfolios have a significant exposure to the mispricing factor, indicating that the average effect of the seven anomalous factor portfolios helps capture variation in the excess returns of the eight dominant factor portfolios. The models including the mispricing factor (Models 1, 4 and 7) outperform those without the mispricing factor (Models 2, 3 and 5), highlighting the importance of the mispricing factor.

7.3 | Identifying redundant independent variables and revising the augmented model

In this section, we examine whether Model 6 has redundant risk factors, and whether we can simplify the mispricing factor. Our empirical test is largely similar to Fama and French (2015, 2018), for example, by applying spanning regression. Spanning regression is a right-hand-side approach to determine whether an independent variable contributes to an asset pricing model's explanatory power. We regress each independent variable on the model's other independent variables in turn. If the intercept of these spanning regressions is statistically different from zero, this individual factor is necessary to the model.

First we investigate Model 6 that includes the coin market, size, momentum, mispricing and network factors. Table 11 shows the results of spanning regressions for Model 6. We find that the coin market (*CMKT*), size (*CSMB*) and mispricing factors (*Mispricing*) have statistically significant intercepts, which reveals that these three factors contribute to the explanatory power of Model 6. In contrast, the momentum (*CMOM*) and network (*Network*) factor spanning regressions have insignificant intercepts, and so these factors make only a limited contribution to our augmented coin market factor model, and will be dropped from further consideration. For the momentum factor, the probable reason is that the mispricing factor contains six types of momentum-based portfolio, and the effect of *CMOM* is subsumed by the mispricing factor. Although the network factor has some predictive power in explaining returns for several mainstream coins (Liu & Tsyvinski, 2020), it cannot account for the variation in returns of our eight dominant factor portfolios. Therefore, we use the remaining three factors—the market, size and mispricing factors—to form an NTFM, and its performance is shown in panel C of Table 10.

Next we evaluate whether we can reduce the components of the mispricing factor. The mispricing factor is based on six momentum-based components, and so incorporates similar

TABLE 11 Spanning regressions for Model 6.

This table reports the results of spanning regression analysis of each individual factor on the other factors of Model 6 in Table 8. \*, \*\* and \*\*\* represent significance at the 10%, 5% and 1% levels, respectively. The *t* statistics are reported in parentheses.

|                   | <i>Intercept</i>      | <i>CMKT</i>           | <i>CSMB</i>            | <i>CMOM</i>            | <i>Mispricing</i>     | <i>Network</i>        | <i>Adjusted R<sup>2</sup></i> |
|-------------------|-----------------------|-----------------------|------------------------|------------------------|-----------------------|-----------------------|-------------------------------|
| <i>CMKT</i>       | 0.0101*<br>(1.6905)   | —                     | 0.0512<br>(1.5346)     | 0.015<br>(0.8363)      | 0.0322<br>(0.4793)    | −0.0177*<br>(−1.6611) | 0.006                         |
| <i>CSMB</i>       | 0.0507***<br>(5.8222) | 0.1185<br>(1.5346)    | —                      | −0.0536**<br>(−1.9735) | 0.1639<br>(1.6079)    | −0.0156<br>(−0.9606)  | 0.011                         |
| <i>CMOM</i>       | 0.0008<br>(0.0456)    | 0.1207<br>(0.8363)    | −0.1864**<br>(−1.9735) | —                      | 1.3179***<br>(7.3799) | 0.0274<br>(0.9064)    | 0.127                         |
| <i>Mispricing</i> | 0.0236***<br>(5.4235) | 0.0185<br>(0.4793)    | 0.0406<br>(1.6079)     | 0.0938***<br>(7.3799)  | —                     | 0.011<br>(1.3596)     | 0.125                         |
| <i>Network</i>    | −0.001<br>(−0.0367)   | −0.4017*<br>(−1.6611) | −0.1531<br>(−0.9606)   | 0.0775<br>(0.9064)     | 0.4349<br>(1.3596)    | —                     | 0.010                         |

information up to six times. To address this issue, we disaggregate the mispricing factor, and replace it in the NTFM model with its components. In other words, the independent variables to be investigated are the coin market and size factors, and the seven anomalous factor portfolios in Table 7. We apply spanning regression to these risk factors, and the results are in Section A11 of the Supporting Information Appendix. We find that the coin market and size factors still have significant intercepts. The mispricing components *MARCAP*, *RMOM1-2* and *RMOM4* also have nonzero intercepts, but the momentum-based portfolios (*MOM1* and *2*) do not contribute to improving model performance. This indicates that the risk-adjusted momentum factor portfolios are superior to the momentum factor portfolios in capturing mispricings. Therefore, we use only the four remaining components—*MARCAP*, *RMOM1*, *RMOM2* and *RMOM4*—to compute a simplified mispricing factor (*Mispricing2*). We refer to the model (see Equation 8) which includes *Mispricing2* as the new three-factor model with the simplified mispricing factor (NTFM-SM).

$$R_i - R_f = \alpha_i + \beta_{i,CMKT} CMKT + \beta_{i,CSMB} CSMB + \beta_{i,Mispricing2} Mispricing2 + \varepsilon_i. \quad (8)$$

Table 12 displays the results of using our NTFM-SM three-factor model with *Mispricing2* to explain returns on our eight dominant factor portfolios, and find that this model outperforms

**TABLE 12** Three-factor model with a new mispricing factor (NTFM-SM).

This table reports the results of regression analysis of the eight dominant portfolios using the NTFM-SM model. \*, \*\* and \*\*\* represent significance at the 10%, 5% and 1% levels, respectively. The *t* statistics are reported in parentheses.

|               | $\alpha$             | $\beta_{CMKT}$          | $\beta_{CSMB}$          | $\beta_{Mispricing2}$  | Adjusted $R^2$ |
|---------------|----------------------|-------------------------|-------------------------|------------------------|----------------|
| <i>MARCAP</i> | 0.0106**<br>(2.1128) | −0.3725***<br>(−8.9465) | 0.2149***<br>(7.8760)   | 0.3797***<br>(5.7573)  | 0.2990         |
| <i>MOM1</i>   | −0.0006<br>(−0.0624) | 0.1075<br>(1.3548)      | 0.0397<br>(0.7643)      | 1.2799***<br>(10.1842) | 0.2085         |
| <i>MOM2</i>   | 0.0079<br>(0.8137)   | 0.0572<br>(0.7093)      | 0.0784<br>(1.4783)      | 0.851***<br>(6.6656)   | 0.1023         |
| <i>RMOM1</i>  | −0.0049<br>(−0.6777) | 0.1777***<br>(2.9463)   | −0.0440<br>(−1.1130)    | 1.2334***<br>(12.9109) | 0.3042         |
| <i>RMOM2</i>  | −0.0012<br>(−0.1421) | −0.0315<br>(−0.4607)    | −0.0773*<br>(−1.7217)   | 1.2575***<br>(11.6365) | 0.2598         |
| <i>RMOM3</i>  | −0.0008<br>(−0.0898) | 0.1858**<br>(2.4180)    | 0.0013<br>(0.0259)      | 1.1032***<br>(9.0769)  | 0.1779         |
| <i>RMOM4</i>  | −0.0043<br>(−0.6301) | 0.2287***<br>(4.0712)   | −0.0967***<br>(−2.6180) | 1.133***<br>(12.7401)  | 0.3139         |
| <i>MAXRET</i> | 0.0106<br>(1.3029)   | 0.31***<br>(4.5720)     | 0.0575<br>(1.2950)      | 0.0716<br>(0.6664)     | 0.0511         |

Abbreviations: NTFM, new three-factor model; SM, simplified mispricing.

all the other models. The number of significant alphas drops from the three of Model 6 in Table 9 to one, and all of the dominant factors except *MAXRET* have exposure to the new mispricing factor.

The performance of the NTFM-SM is the best across Panels A–C in Table 10, with a score of 6. Although its  $R^2$  is 0.21, which is only the sixth best, it accounts for most of the variation on average excess returns on the dominant factor portfolios, since the values of the four metrics evaluating this property (e.g., GRS,  $Al\alpha_i$ ,  $Al\alpha_i/Al\bar{r}_i$  and  $A\alpha_i^2/A\bar{r}_i^2$ ) are minimum in all panels.

## 8 | DISSECTING THE DRIVING FORCE AND MECHANISM OF PRICING MODELS

In this section, we first study whether the explanatory power of the *Mispricing2* factor is due to the inclusion of size (*MARCAP*), risk-adjusted momentum (*RMOM1-2*, *RMOM4*) or both. Then we investigate how size, momentum and the investment horizon interact. This analysis is important for two reasons. First, we aim to design a tradable mispricing factor which will allow both academics and practitioners to replicate the mispricing factor for their research or investing. Second, it will show stylized facts for cryptocurrencies, which are comparable to traditional asset classes, like, equities and bonds.

### 8.1 | Assessing the inherent driving force within the mispricing factor

To evaluate their separate contribution to the NTFM-SM model, we decompose the simplified mispricing factor (*Mispricing2*) into two new factors—a size mispricing factor (*MispricingS*, based on *MARCAP*) and a risk-adjusted momentum mispricing factor (*MispricingM*, based on *RMOM1-2*, *RMOM4*). We replace *Mispricing2* in the NTFM-SM model and create two new models, which we refer to as NTFM-SIZE and NTFM-RMOM, where the terms *SIZE* and *RMOM* specify the factors used to replace *Mispricing2*. *MispricingM* is the simple average of returns on the risk-adjusted momentum portfolios *RMOM1-2* and *RMOM4*. *MispricingS* is the returns on *MARCAP*, and so the *MispricingS* factor is just *MARCAP*. We detail the results for these two models in Tables 13 and 14. We exclude *MARCAP* from the dominant portfolios when examining the NTFM-SIZE model because it is not sensible to include the same variable on both sides of the regression. The NTFM-SIZE model cannot explain the cross-sectional variation because all the dominant factor portfolios have significant alphas, along with small  $R^2$  values. In contrast, the NTFM-RMOM model captures much more of the variation, as only one dominant factor portfolio (*MARCAP*) has a significant alpha, and the  $R^2$  values are much higher than for NTFM-SIZE, ranging from 5.8% for *MAXRET* to 36.7% for *RMOM1*. All the dominant factor portfolios, except for *MARCAP*, have significant coefficients for the *MispricingM* factor.

Although the NTFM-RMOM model provides a good explanation for the dominant factor portfolios, it is inferior to the NTFM-SM model. Panels C and D of Table 10 show that on five of the tests NTFM-SM is superior to NTFM-RMOM, and inferior on only one—the average  $R^2$ . Thus we conclude that there is a combined effect between the *MispricingS* and *MispricingM* factors, which improves the model's performance.

TABLE 13 NTFM-SIZE.

This table reports the results of regression analysis of the eight dominant portfolios using the NTFM-SIZE model. \*, \*\* and \*\*\* represent significance at the 10%, 5% and 1% levels, respectively. The *t* statistics are reported in parentheses.

|        | $\alpha$              | $\beta_{CMKT}$        | $\beta_{CSMB}$         | $\beta_{MispricingS}$   | Adjusted $R^2$ |
|--------|-----------------------|-----------------------|------------------------|-------------------------|----------------|
| MOM1   | 0.032***<br>(3.1052)  | 0.0818<br>(0.8364)    | 0.0472<br>(0.7511)     | 0.0034<br>(0.0320)      | −0.0036        |
| MOM2   | 0.029***<br>(2.9436)  | 0.0502<br>(0.5389)    | 0.0778<br>(1.2950)     | 0.0292<br>(0.2929)      | −0.0008        |
| RMOM1  | 0.0297***<br>(3.5807) | 0.0935<br>(1.1893)    | −0.0029<br>(−0.0572)   | −0.153*<br>(−1.8169)    | 0.0130         |
| RMOM2  | 0.0336***<br>(3.6660) | −0.1071<br>(−1.2354)  | −0.0419<br>(−0.7483)   | −0.1256<br>(−1.3511)    | 0.0035         |
| RMOM3  | 0.0258***<br>(2.6389) | 0.1903**<br>(2.0559)  | −0.0074<br>(−0.1245)   | 0.0744<br>(0.7512)      | 0.0034         |
| RMOM4  | 0.0226***<br>(2.9244) | 0.2466***<br>(3.3727) | −0.1147**<br>(−2.4293) | 0.1134<br>(1.4468)      | 0.0291         |
| MAXRET | 0.0173**<br>(2.2332)  | 0.2173***<br>(2.9616) | 0.1101**<br>(2.3358)   | −0.2397***<br>(−3.0512) | 0.0723         |

Abbreviation: NTFM, new three-factor model.

## 8.2 | Mechanism between the effects of size and risk-adjusted momentum

We investigate the interactive effects of the *MispricingS* and *MispricingM* factors. Most previous studies treat size and momentum effects separately, and do not investigate their combined effects. The influential studies of Chordia and Shivakumar (2002) and Stivers and Sun (2010) emphasize that momentum effects become stronger in bull markets, and are procyclical. Similarly, the literature documents that the size effect accounts for cross-sectional variation in returns (e.g., Fama & French, 1992, 1993, 2017). In contrast, recent papers (Hur et al., 2014; hyun Ahn, 2019) find that the size effect is counter-cyclical, and more pronounced in bear markets. To investigate the interactive effect of the *MispricingS* and *MispricingM* factors, we disaggregate the *Mispricing2* factor into the *MispricingS* and *MispricingM* factors to create a new model (*New Mispricing Model*) that regresses our seven dominant factor portfolios (excluding MARCAP as a dependent variable) on *CMKT*, *CSMB*, *MispricingS* and *MispricingM*. We then divide our data in half, as have other papers (Avramov et al., 2016; Stivers & Sun, 2010), into January 2014–September 2016 (3.75 years) and October 2016–June 2021 (3.75 years), where Bitcoin experienced mild growth during the first period, and rapid growth during the second period.

Table 15 contains the regression results for the seven dominant factor portfolios using the new model (*New Mispricing Model*). Table 15 only displays the coefficients for *MispricingS* and

TABLE 14 NTFM-RMOM.

This table reports the results of regression analysis of the eight dominant portfolios using the NTFM-RMOM model. \*, \*\* and \*\*\* represent significance at the 10%, 5% and 1% levels, respectively. The *t* statistics are reported in parentheses.

|        | $\alpha$             | $\beta_{CMKT}$          | $\beta_{CSMB}$        | $\beta_{MispricingM}$  | Adjusted $R^2$ |
|--------|----------------------|-------------------------|-----------------------|------------------------|----------------|
| MARCAP | 0.022***<br>(4.3012) | −0.3743***<br>(−8.5850) | 0.2133***<br>(7.4635) | −0.0626<br>(−1.1691)   | 0.2416         |
| MOM1   | 0.0038<br>(0.4079)   | −0.0219<br>(−0.2768)    | 0.1142**<br>(2.2046)  | 1.0372***<br>(10.6941) | 0.2253         |
| MOM2   | 0.0110<br>(1.1610)   | −0.0280<br>(−0.3453)    | 0.1276**<br>(2.3948)  | 0.6796***<br>(6.8428)  | 0.1073         |
| RMOM1  | −0.0023<br>(−0.3352) | 0.0473<br>(0.8166)      | 0.0315<br>(0.8287)    | 1.0575***<br>(14.8706) | 0.3665         |
| RMOM2  | 0.0015<br>(0.1910)   | −0.1631**<br>(−2.4546)  | 0.0002<br>(0.0036)    | 1.0697***<br>(13.1083) | 0.3083         |
| RMOM3  | 0.0036<br>(0.3987)   | 0.0767<br>(0.9935)      | 0.0642<br>(1.2640)    | 0.8675***<br>(9.1656)  | 0.2309         |
| RMOM4  | 0.0005<br>(0.0687)   | 0.1183**<br>(2.0866)    | −0.0324<br>(−0.8685)  | 0.8799***<br>(12.6348) | 0.3105         |
| MAXRET | 0.0084<br>(1.0565)   | 0.294***<br>(4.3210)    | 0.0674<br>(1.5110)    | 0.1469*<br>(1.7595)    | 0.0575         |

Abbreviations: NTFM, new three-factor model; RMOM, risk-adjusted momentum.

*MispricingM* as these are the variables of interest (we document the complete table in Section A9 of Supporting Information Appendix). The number of significant *MispricingS* coefficients is three in the first period, increasing to five in the second period. Likewise, the number of significant *MispricingM* coefficients rises from three to six. Our results are consistent with the literature documenting that the momentum effect is larger in bull markets (e.g., Chordia & Shivakumar, 2002; Stivers & Sun, 2010). Our finding is also consistent with Liu et al. (2022) who find that outperforming cryptocurrency portfolios tend to be based on momentum. Nevertheless, our findings are in contrast to the studies proposing that the equity market size effect is counter-cyclical, as more significant coefficients appear for *MispricingS* in the second period.

We also evaluate the interactive effect of the *MispricingS* and *MispricingM* factors by exploring the excess returns on risk-adjusted momentum portfolios while controlling for size; and then the excess returns for portfolios of coins of a different size, while controlling for risk-adjusted momentum. Numerous studies (e.g., Fama & French, 2008, 2012) have found that the momentum effect is strong for large-size equity portfolios, and weak for small-size equity portfolios in the US and most international markets. To form size and risk-adjusted momentum portfolios, we rely on the double sorting technique. First, we sort the cryptocurrencies into two

TABLE 15 Betas for two sample periods using the *NewMispricing* model.

This table reports partial results for the regression of seven dominant factor portfolios using the *NewMispricing* model. Only the results for *MispricingS* and *MispricingM* are displayed as they are the variables of interest.

$$R_i - R_f = \alpha_i + \beta_{i,CMKT} CMKT + \beta_{i,CSMB} CSMB + \beta_{i,MispricingS} MispricingS + \beta_{i,MispricingM} MispricingM + \epsilon_i.$$

\*, \*\*, and \*\*\* represent significance at the 10%, 5% and 1% levels, respectively. The *t* statistics are reported in parentheses.

|        | Panel A: First Period (2014.01–2016.09) |                        |                | Panel B: Second Period (2016.10–2021-06) |                        |                |
|--------|---|------------------------|----------------|--|------------------------|----------------|
|        | $\beta_{MispricingS}$                   | $\beta_{MispricingM}$  | Adjusted $R^2$ | $\beta_{MispricingS}$                    | $\beta_{MispricingM}$  | Adjusted $R^2$ |
| MOM1   | 0.1001<br>(0.7045)                      | 1.1131***<br>(7.3710)  | 0.2148         | 0.0167<br>(0.1562)                       | 0.8764***<br>(7.9835)  | 0.2421         |
| MOM2   | −0.413***<br>(−2.8510)                  | −0.0442<br>(−0.2869)   | 0.0448         | 0.3979***<br>(3.3065)                    | 0.4359***<br>(3.5282)  | 0.1497         |
| RMOM1  | −0.1474<br>(−1.4597)                    | 1.0906***<br>(10.1612) | 0.3625         | 0.0457<br>(0.5286)                       | 0.9359***<br>(10.5269) | 0.3688         |
| RMOM2  | −0.3891***<br>(−2.8168)                 | −0.1012<br>(−0.6890)   | 0.0214         | −0.1959*<br>(−1.8701)                    | 0.7242***<br>(6.7881)  | 0.2108         |
| RMOM3  | 0.1888<br>(1.2537)                      | −0.2809*<br>(−1.7550)  | 0.0246         | 0.326***<br>(3.8425)                     | 1.0578***<br>(12.2424) | 0.4700         |
| RMOM4  | 0.0770<br>(0.7061)                      | −0.0456<br>(−0.3932)   | −0.0160        | 0.1559**<br>(1.9626)                     | 1.3387***<br>(16.5407) | 0.6010         |
| MAXRET | −0.1865*<br>(−1.6836)                   | 0.1259<br>(1.0696)     | 0.0433         | −0.3047***<br>(−2.6233)                  | 0.1419<br>(1.1879)     | 0.1242         |



groups (small and big) using their median value as the division point. Then we sort each of these size groups into three groups (high, medium and low) based on risk-adjusted momentum at the 30th and 70th percentiles. Finally we sort each of these six groups by their horizon period of 1, 2 and 4 weeks, which are corresponding to the statistically significant components of a simplified mispricing factor—*Mispricing*2. Therefore, after dropping the medium-size momentum group, we form 12 portfolios using size, risk-adjusted momentum and horizon.

Table 16 displays the excess returns on these 12 portfolios. All of the small-size portfolios generate statistically significant returns. In contrast, only the big-size and high-risk-adjusted momentum portfolios have statistically significant returns, as the big-size and low-risk-adjusted momentum portfolios produce small returns that are insignificantly different from zero. Returns on the nine significant portfolios decrease as the horizon increases. For the same size of portfolio, all the high-risk-adjusted momentum portfolios have greater returns than the low-risk-adjusted momentum portfolios. For the same risk-adjusted momentum, all the big-size portfolios have lower returns than the small-size portfolios. These results suggest that portfolios of small market capitalization coins with high-risk-adjusted momentum outperform, particularly over a 1-week horizon. Thus, our findings are inconsistent with the papers of Fama and French (2008, 2012), and are in line with Hong et al. (2000) who emphasize that the momentum effect is more pronounced in small-size portfolios. We also find support for a small coin, short horizon, effect.

We summarize the above findings related to the interactive effects of the *Mispricing*S and *Mispricing*M factors. Forming a mispricing factor that combines both size and risk-adjusted momentum is superior to using either alone. Both *Mispricing*S and *Mispricing*M capture more variation in the returns of our dominant factor portfolios during bull than bear markets. Portfolios of big coins (large market capitalization) deliver lower returns than portfolios of small coins, and small coins generate statistically positive excess returns. Similarly, cryptocurrencies with a high-risk-adjusted momentum tend to have larger returns. Since the cryptocurrency market includes coins with large and small market capitalization, and high and low-risk-adjusted momentum, it is important to take the size and momentum effects into account when constructing pricing models.

**TABLE 16** Interactive effect of size, RMOM and horizon.

This table reports the excess return on each size and momentum portfolio for different risk-adjusted portfolios over three horizons that were used to construct *Mispricing*M. \*, \*\* and \*\*\* represent significance at the 10%, 5% and 1% levels, respectively. The *t* statistics are reported in parentheses.

|                       | <i>RMOM</i> 1         | <i>RMOM</i> 2         | <i>RMOM</i> 4         |
|-----------------------|-----------------------|-----------------------|-----------------------|
| <i>Small and Low</i>  | 0.0449***<br>(5.0294) | 0.0320***<br>(4.0404) | 0.0211***<br>(2.6680) |
| <i>Small and High</i> | 0.0535***<br>(4.8883) | 0.0509***<br>(4.7636) | 0.0417***<br>(3.3019) |
| <i>Big and Low</i>    | −0.0043<br>(−0.7056)  | −0.0044<br>(−0.7313)  | 0.0007<br>(0.1098)    |
| <i>Big and High</i>   | 0.0276***<br>(3.6521) | 0.0250***<br>(3.2836) | 0.0198***<br>(2.9973) |

Abbreviation: RMOM, risk-adjusted momentum.

## 9 | CONCLUSIONS

We explore the factors that influence the returns and performance of cryptocurrency factor portfolios. A problem in this area is that the empirical distributions of cryptocurrency returns are highly nonnormal, undermining the usual metrics for measuring performance. To circumvent this we use ASD, a nonparametric method which does not require any assumptions about the return distribution. This paper is the first to use the ASD approach when examining the relative performance of cryptocurrency factor portfolios.

To investigate anomalies in cryptocurrency returns, we build zero-investment long-short portfolios using factors that can be constructed using the available information: opening price, closing price, trading volume and market capitalization. We identify four main factors—size, momentum, volume and volatility, which we use to create 29-factor portfolios. Using portfolios based on these 29 cryptocurrency factors, we investigate whether they generate superior returns over horizons of 4, 13, 26, 52 and 78 weeks, benchmarked against US equities, US Treasury bills, US Treasury bonds and Bitcoin. We find eight dominant factor portfolios, in the sense of almost first-degree stochastic dominance (AFSD) and almost second-degree stochastic dominance (ASSD). These dominant factor portfolios are based on market capitalization, momentum (1 and 2 weeks), risk momentum (1, 2, 3 and 4 weeks), and maximum return. Benchmarking these eight dominant factor portfolios against our four benchmarks and equity portfolios based on size, momentum and book-to-market, we find that the long-only strategies contribute more to performance than the short-only strategies.

We test whether the eight dominant factor portfolios can be explained by a coin market three-factor model, and find that this model has limited success in explaining their returns. For these eight dominant portfolios, the alphas are statistically significant, implying that their dominance is due to mispricing. We then test the explanatory power of the coin market three-factor model versus an augmented three-factor model which incorporates a mispricing factor (the combined average effect of the eight dominant factors), investor attention and network factors. We find that the momentum factor in our augmented model is redundant, and that the mispricing factor can be refined to include just the size and risk-adjusted momentum dominant factor portfolios. We call the result the three-factor NTFM-SM coin model, and the number of nonzero alphas drops from seven to one, and almost all the coefficients of the refined mispricing factor are statistically significant. Since short selling is unavailable for most coins, we also test whether our augmented model can explain returns on our eight dominant factor portfolios, but shorting only Bitcoin. We find it captures variations in returns better than the coin market three-factor model. To address the possibility that equity asset pricing models might have explanatory power in explaining the returns on our eight dominant strategies, we test the performance of nine widely used equity asset pricing models. None of these equity asset pricing models can explain returns on the eight dominant factor portfolios.

Our work supports an augmented three-factor model (NTFM-SM) for cryptocurrencies. To highlight the importance of the mispricing factor, we test the performance of a coin market one-factor model with only a cryptocurrency market factor, and two versions of a coin market two-factor model versus the performance of these models augmented with a mispricing factor based on the size and risk-adjusted momentum dominant factor portfolios. The two two-factor models comprise first, a cryptocurrency market factor and a cryptocurrency size factor; and second a cryptocurrency market factor and a momentum factor. We find that the mispricing factor always improves the performance of the original model. Hence, our work establishes a

collection of stylized facts on cryptocurrency factor portfolios which may promote further studies in evaluating cryptocurrencies and developing theoretical models.

## DATA AVAILABILITY STATEMENT

Data are available from the authors on request, with the caveat that requestors should also be subscribers to parts of the dataset that are derived from commercial providers that require subscription, such as CRSP.

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## SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

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