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A stable aggregate currency revisited: Highlighting some fundamental issues

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ABSTRACT

A multicurrency numéraire is a weighted basket of currencies, where the weights sum to zero and all weights are positive. A stable aggregate currency is an optimal multicurrency numéraire, which is estimated using a minimum-variance portfolio optimisation. The variables of the portfolio optimisation are invariant currency indexes, which are multilateral exchange rates and are priced in terms of an equally-weighted basket of the currencies. This paper shows that there are some fundamental issues with the concept of a stable aggregate currency. Once these issues have been resolved, it is shown that the portfolio optimisation problem is infeasible.

1. Introduction

A bilateral exchange rate is the price of a currency in terms of a numéraire currency. Invariant currency indexes are normalised bilateral exchange rates that are independent of the choice of numéraire currency (Hovanov et al., 2004). The normalisation changes the numéraire of the currency from a single-currency numéraire to a multicurrency numéraire (Kunkler, 2022). As a consequence, invariant currency indexes are multilateral exchange rates, which provide transparency on the underlying currency weights. In contrast, a stable aggregate currency is an optimal multicurrency numéraire, which is a weighted basket of currencies. The optimal multicurrency numéraire is estimated using a minimum-variance portfolio optimisation subject to two optimisation constraints: that the portfolio weights sum to one and that all of the portfolio weights are nonnegative (Hovanov et al., 2004). The motivation of this paper is to highlight some fundamental issues with the concept of a stable aggregate currency.

The concept of a stable aggregate currency is a novel approach that estimates an optional multicurrency numéraire using portfolio optimisation. More specifically, a stable aggregate currency is estimated using a minimum-variance portfolio optimisation with two optimisation constraints: the portfolio weights sum to one and each portfolio weight is nonnegative. The result of the portfolio optimisation is an optimal portfolio of invariant currency indexes, rather than an optimal portfolio of currencies. Thus, it is important to understand the relationship between the portfolio weights of the optimal portfolio of invariant currency indexes and the currency weights of the optional multicurrency numéraire. For example, when the portfolio weights sum to one, the portfolio weights are not equal to the currency weights (Kunkler, 2022).

Stable aggregate currencies have been applied several times in the literature. Hovanov et al. (2004) introduced the concept of a stable aggregate currency as a proxy for a world currency, which was more stable than any individual currency. Pontines (2009) estimated a stable aggregate currency for an optimal common currency basket in East Asia. In the cryptocurrency literature, Giudici et al. (2022) estimated a stable aggregate currency to create a digit currency whose movements are minimised relative to several international currencies.

This paper contributes to the literature by showing that there are some fundamental issues with the concept of a stable aggregate currency. These issues include a spurious covariance matrix, Seigel's paradox, a singular covariance matrix, and assuming that the portfolio weights of the optimal portfolio of invariant currency indexes are equal to the currency weights of the optional multicurrency numéraire. Once these issues have been resolved, it is shown that the portfolio optimisation problem is infeasible.

2. Methodology

2.1. Data

The data sample consists of four US dollar exchange rates from 1st January 1981 to 31st December 1998 and is sourced from Bloomberg. The five currencies ($N = 5$) are: US dollar (USD), German deutsche mark (DEM), French franc (FRF), British pound (GBP), and Japanese yen (JPY). For ease of comparison to Hovanov et al. (2004), this paper considers the same five currencies and the same pre-1999 data.

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2.2. Bilateral exchange rates

A bilateral exchange rate is the price of a fixed currency in terms of a variable currency, which is a single-currency numéraire. The variable currency is also known as the numéraire currency. In nominal terms, for a group of N currencies, the ith/jth bilateral exchange rate can be written in terms of currency positions (quantities) as:

$$S_t^{i/j} = P_t^j / P_t^i \tag{1}$$

where $i, j = 1, \ldots, N$; $t = 0, \ldots, T$; $S_t^{i/j}$ is the ith/jth bilateral exchange rate; P_t^j is the currency position for the jth currency; and P_t^i is the currency position for the ith currency. For example, to buy one unit of the GBP/USD bilateral exchange rate at $S_t^{GBP/USD} = P_t^{USD}/P_t^{GBP} = 1.20$ involves a long currency position of $P_t^{GBP} = 1.00$ British pounds (GBP) and a short currency position of $P_t^{USD} = 1.20$ US dollars (USD). In log terms, the ith/jth bilateral exchange rate in Eq. (1) can be written as:

$$s_t^{i/j} = p_t^j - p_t^i \tag{2}$$

where $i, j = 1, \ldots, N$; $t = 0, \ldots, T$; $s_t^{i/j}$ is the ith/jth exchange rate; p_t^j is the currency position for the jth currency; and p_t^i is the currency position for the ith currency. When a currency is priced in terms of itself, the bilateral exchange rate is deterministic (constant), with $S_t^{i/i} = 1$ in nominal terms and $s_t^{i/i} = 0$ in log terms. Thus, the risk of any currency priced in terms of itself is zero.

2.3. Seigel's paradox

Bilateral exchange rates are usually modelled in log terms to overcome Siegel's Paradox, which is based on Jensen's Inequality (Taylor and Sarno, 1998). In nominal terms, the inverse of the expectation of the ith/jth bilateral exchange rate is less than or equal to the expectation of the inverse of the ith/jth bilateral exchange rate:

$$1/E(S_t^{i/j}) \neq E(1/S_t^{i/j}) \tag{3}$$

where $i, j = 1, \ldots, N$; $t = 0, \ldots, T$; $S_t^{i/j}$ is the ith/jth bilateral exchange rate; and the inverse is a convex function (Taylor and Sarno, 1998). In log terms, there is no convex function for the ith/jth bilateral exchange rate so that:

$$E(-s_t^{i/j}) = -E(s_t^{i/j}) \tag{4}$$

where $i, j = 1, \ldots, N$; $t = 0, \ldots, T$; and $s_t^{i/j}$ is the ith/jth bilateral exchange rate (Taylor and Sarno, 1998).

2.4. Invariant currency indexes

Invariant currency indexes are normalised bilateral exchange rates (Hovanov et al., 2004). In nominal terms, an invariant currency index for a group of N currencies with the kth currency as the numéraire currency, is given by[1]:

$$\begin{aligned} H_t^i &= S_t^{i/k} \Big/ \prod_{j=1}^{N} \left(S_t^{j/k}\right)^{\frac{1}{N}} \\ &= \prod_{j=1}^{N} \left(S_t^{i/k} S_t^{k/j}\right)^{\frac{1}{N}} \\ &= \prod_{j=1}^{N} \left(S_t^{i/j}\right)^{\frac{1}{N}} \end{aligned} \tag{5}$$

where $i = 1, \ldots, N$; $t = 0, \ldots, T$; H_t^i is the ith invariant currency index for the ith currency; $S_t^{i/k}$ is the ith/kth bilateral exchange rate, $S_t^{j/k}$ is the jth/kth bilateral exchange rate; $\prod_{j=1}^{N} \left(S_t^{j/k}\right)^{\frac{1}{N}}$ is the normalisation term; and $S_t^{k/j} = 1/S_t^{j/k}$ is the standard relationship for bilateral exchange rates (Hovanov et al., 2004). In log terms, the invariant currency index in Eq. (5) can be written as:

$$\begin{aligned} h_t^i &= s_t^{i/k} - \frac{1}{N} \sum_{j=1}^{N} s_t^{j/k} \\ &= \frac{1}{N} \sum_{j=1}^{N} s_t^{i/k} + \frac{1}{N} \sum_{j=1}^{N} s_t^{k/j} \\ &= \frac{1}{N} \sum_{j=1}^{N} s_t^{i/j} \end{aligned} \tag{6}$$

where $i = 1, \ldots, N$; $t = 0, \ldots, T$; h_t^i is the ith invariant currency index; $s_t^{i/k}$ is the ith/kth bilateral exchange rate; $s_t^{j/k}$ is the jth/kth bilateral exchange rate; and $s_t^{k/j} = -s_t^{j/k}$ is the standard relationship for log bilateral exchange rates.

The invariant currency indexes are independent of the choice of numéraire currency, which is clear by the absence of the numéraire currency (the kth currency) in the last line of both Eqs. (5) and (6). The normalisation term for each invariant currency index changes the numéraire of the currency from a single-currency numéraire to a multicurrency numéraire (Kunkler, 2022). In other words, the normalisation term for each invariant currency index converts a bilateral exchange rate to a multilateral exchange rate. Thus, invariant currency indexes are multilateral exchange rates, which are priced in terms of an equally-weighted basket of currencies.

2.5. Currency positions for invariant currency indexes

In nominal terms, the underlying currency positions that result from trading the ith invariant currency index can be found by substituting Eq. (1) into Eq. (5) to give:

$$H_t^i = \prod_{j=1}^{N} \left(P_t^j / P_t^i\right)^{\frac{1}{N}} = \prod_{j=1}^{N} \left(P_t^j\right)^{\frac{1}{N}} / P_t^i = P_t^M / P_t^i \tag{7}$$

where $i = 1, \ldots, N$; $t = 0, \ldots, T$; H_t^i is the ith invariant currency index in Eq. (5); P_t^j is the jth currency position for the jth currency; P_t^i is the ith currency position for the ith currency; and $P_t^M = \prod_{j=1}^{N} \left(P_t^j\right)^{\frac{1}{N}}$ is an equally-weighted basket of currencies. Similarly, in log terms, the underlying currency positions that result from trading the ith invariant currency index can be found by substituting Eq. (2) into Eq. (6) to give:

$$h_t^i = \frac{1}{N} \sum_{j=1}^{N} (p_t^j - p_t^i) = \frac{1}{N} \sum_{j=1}^{N} p_{C,t}^j - p_t^i = p_t^M - p_t^i \tag{8}$$

where $i = 1, \ldots, N$; $t = 0, \ldots, T$; h_t^i is the ith invariant currency index in Eq. (6); p_t^j is the jth currency position for the jth currency; p_t^i is the ith currency position for the ith currency; and $p_t^M = \frac{1}{N} \sum_{j=1}^{N} p_{C,t}^j$ is an equally-weighted basket of currencies.

Table 1 reports the nominal currency positions in US dollars for buying USD 500 worth of the US dollar invariant currency index ($H_{C,t}^{USD}$), which consists of a long currency position of USD 500 in the US dollar (P_t^{USD}) and a short currency position of USD 500 in the equally-weighted basket of currencies (P_t^M). The result is a positive USD 400 currency position in the US dollar

[1] Eq. (5) appears as Eq. (3.2) in Hovanov et al. (2004).

Table 1
US dollar invariant currency index.

	DEM	FRF	GBP	JPY	USD
P_t^{USD}					500
P_t^M	100	100	100	100	100
H_t^{USD}	−100	−100	−100	−100	400

Table 2
No-arbitrage condition for the invariant currency indexes.

	DEM	FRF	GBP	JPY	USD
$H_{C,t}^{DEM}$	400	−100	−100	−100	−100
$H_{C,t}^{FRF}$	−100	400	−100	−100	−100
$H_{C,t}^{GBP}$	−100	−100	400	−100	−100
$H_{C,t}^{JPY}$	−100	−100	−100	400	−100
$H_{C,t}^{USD}$	−100	−100	−100	−100	400
Total	0	0	0	0	0

(USD) and a negative USD 100 currency position in the other four currencies. Note that the currency position in US dollars can be simply converted to local currency positions by using the appropriate US dollar exchange rate.

2.6. No-arbitrage condition

In nominal terms, there is a no-arbitrage condition on the group of N invariant currency indexes given by:

$$\prod_{i=1}^{N} H_t^i = 1 \tag{9}$$

where $t = 0, \ldots, T$; and H_t^i is the ith invariant currency index. The no-arbitrage condition in Eq. (9) is a generalisation of the triangle arbitrage relationship between three bilateral exchange rates. Similarly, in log terms, the no-arbitrage condition in Eq. (9) can be written as:

$$\sum_{i=1}^{N} h_t^i = 0 \tag{10}$$

where $t = 0, \ldots, T$; and h_t^i is the ith invariant currency index (see Kunkler and MacDonald, 2015). In log terms, the no-arbitrage condition in Eq. (10) also applies to the log returns of the group of invariant currency indexes given by:

$$\sum_{i=1}^{N} \Delta h_t^i = 0 \tag{11}$$

where $t = 1, \ldots, T$; $\Delta h_t^i = h_t^i - h_{t-1}^i$ is the log return of the ith invariant currency index; and Δ is a first difference operator. Table 2 reports the nominal currency positions in US dollars for buying USD 500 worth of the all five invariant currency indexes. The result is zero currency positions in all five currencies.

2.7. Singular covariance matrix

In log terms, there is a linear dependency between the group of N invariant currency indexes in Eq. (10) and a linear dependency between the log returns of the group of N invariant currency indexes in Eq. (11). As a consequence, both the $N \times N$ covariance matrix of the group of invariant currency indexes and the $N \times N$ covariance matrix of log returns of the group of invariant currency indexes are singular, where the ordinary inverse does not exist. This can also be seen from the variance of the sum of the group of N invariant currency indexes in Eq. (10):

$$var(\sum_{i=1}^{N} h_t^i) = 0 \tag{12}$$

and from the variance of the sum of the log returns of the group of N invariant currency indexes in Eq. (11):

$$var(\sum_{i=1}^{N} \Delta h_t^i) = 0 \tag{13}$$

where $t = 1, \ldots, T$; h_t^i is the ith invariant currency index; and Δh_t^i is the log return of the ith invariant currency index.

2.8. Spurious covariance matrix

Hovanov et al. (2004) used the levels of rebased invariant currency indexes, which leads to a spurious covariance matrix. One reason that the covariance matrix is spurious is due to significant autocorrelated errors that occur when modelling the levels of exchange rates, rather than the log returns (see Granger and Newbold, 1974). In nominal terms, the rebased invariant currency indexes are calculated by[2]:

$$\tilde{H}_t^i = H_t^i / H_0^i \tag{14}$$

where $i = 1, \ldots, N$; $t = 0, \ldots, T$; $\tilde{H}_t^i$ is the ith rebased invariant currency index; H_t^i is the ith invariant currency index at time t; and H_0^i is the ith invariant currency index at time 0. In log terms, the rebased invariant currency indexes in Eq. (14) are calculated by:

$$\tilde{h}_t^i = h_t^i - h_0^i = \sum_{s=1}^{t} \Delta h_s^i \tag{15}$$

where $i = 1, \ldots, N$; $t = 0, \ldots, T$; $\tilde{h}_t^i$ is the ith rebased invariant currency index; h_t^i is the ith invariant currency index at time t; and h_0^i is the ith invariant currency index at time 0; $\Delta h_s^i = h_s^i - h_{s-1}^i$ is the log return of the ith invariant currency index at time s. Thus, the rebased invariant currency indexes are the cumulative sums of the log returns from time 0 until time t. In log terms, the no-arbitrage condition also applies to the rebased invariant currency indexes in Eq. (15) by:

$$\sum_{i=1}^{N} \tilde{h}_t^i = \sum_{i=1}^{N} h_t^i - \sum_{i=1}^{N} h_0^i = 0 \tag{16}$$

where $t = 0, \ldots, T$; $\tilde{h}_t^i$ is the ith rebased invariant currency index; h_t^i is the ith invariant currency index at time t; and h_0^i is the ith invariant currency index at time 0. As a consequence, the $N \times N$ covariance matrix of the group of rebased invariant currency indexes is singular, where the ordinary inverse does not exist.

Table 3 reports the annualised volatilities, Augmented Dickey–Fuller (ADF) statistics, and correlation matrices for both the levels of the rebased invariant currency indexes (Panel A) and the log returns of the invariant currency indexes (Panel B). The annualised volatilities for the levels of the rebased invariant currency indexes are unintuitive. For example, the annualised volatility for the Japanese yen is 94.98%. In contrast, the annualised volatilities for the log returns of the invariant currency indexes are more in line with expectations. For example, the annualised volatilities range from a low of 4.77% for the German Deutsche mark and a high of 8.18% for the US dollar.

[2] Eq. (14) appears as Eq. (3.3) in Hovanov et al. (2004).

The augmented Dickey–Fuller (ADF) statistics are not highly significant for the levels of the rebased invariant currency indexes, which indicate that they have a unit root. In contrast, the ADF statistics are all highly significant for the log returns of the invariant currency indexes, which indicate that they do not have a unit root.

The correlations for the log returns of the invariant currency indexes are also in line with expectations. For example, the German Deutsche mark and the French franc have an observed correlation of 0.80, which is caused by both currencies being members of the European Monetary System. In contrast, the correlations for the levels of the rebased invariant currency indexes are extreme. For example, the British pound and the Japanese yen have an observed correlation of −0.93. The extreme correlations are due to significant autocorrelated errors that occur when modelling the rebased levels of multilateral exchanges rate, rather than the returns.

In summary, the covariance matrix used in Hovanov et al. (2004) was spurious, which was caused by modelling the levels of the rebased invariant currency indexes, rather than the returns of the invariant currency indexes. The spurious nature of the results was evident in both the variance terms and the correlation matrix.

2.9. Stable aggregate currency

A stable aggregate currency (SAC) is an optimal multicurrency numéraire that is estimated by a minimum-variance portfolio optimisation on a group of rebased invariant currency indexes (Hovanov et al., 2004). The objective function to minimise the variance of the portfolio of invariant currency indexes can be written as[3]:

$$\sigma^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w^i \sigma^{i,j} w^j \tag{17}$$

subject to a constraint that the portfolio weights sum to one[4]:

$$\sum_{i=1}^{N} w^i = 1 \tag{18}$$

and a constraint that the portfolio weights are all nonnegative[5]:

$$w^i \geq 0 \tag{19}$$

where $i = 1, \ldots, N$; σ^2 is the variance of the portfolio; w^i is the ith portfolio weight associated with the ith invariant currency index; w^j is the jth portfolio weight associated with the jth invariant currency index; and $\sigma^{i,j}$ is the covariance term between the ith and jth invariant currency indexes.

2.10. Currency weights for a portfolio of invariant currency indexes

Once an optimal portfolio of invariant currency indexes has been estimated, the next step is to calculate the currency weights for the optimal multicurrency numéraire. In log terms, a portfolio of invariant currency indexes can be written as:

$$f = \sum_{i=1}^{N} w^i h_T^i \tag{20}$$

where f is the portfolio; w^i is the ith portfolio weight for the ith invariant currency index; and h_T^i is the ith invariant currency index. Note that the portfolio weights sum to one from Eq. (18), with $\sum_{i=1}^{N} w^i = 1$.

The currency weights and currency positions of a portfolio of invariant currency indexes are found by substituting $p_T^i - p_T^M$ from Eq. (8) for h_T^i into Eq. (20) to give[6]:

$$\begin{aligned} f &= \sum_{i=1}^{N} w^i (p_T^i - p_T^M) \\ &= \sum_{i=1}^{N} w^i p_T^i - p_T^M \sum_{i=1}^{N} w^i \\ &= \sum_{i=1}^{N} w^i p_T^i - \sum_{i=1}^{N} \frac{1}{N} p_T^i \\ &= \sum_{i=1}^{N} (w^i - \eta^i) p_T^i \\ &= \sum_{i=1}^{N} \delta^i p_T^i \end{aligned} \tag{21}$$

where p_t^i is the ith currency position for the ith currency; the portfolio weights sum to one with $\sum_{i=1}^{N} w^i = 1$; $p_T^M = \frac{1}{N} \sum_{i=1}^{N} p_T^i$ is the equally-weighted basket of currencies in Eq. (8);

$$\delta^i = w^i - \eta^i \tag{22}$$

where δ^i is the ith currency weight for the ith currency; w^i is the ith portfolio weight for the ith invariant currency index; and $\eta^i = \frac{1}{N}$ is the ith numéraire weight for the ith currency.

In summary, the portfolio weights of the optimal portfolio of invariant currency indexes are not equal to the currency weights of the optional multicurrency numéraire. More specifically, the currency weights are equal to the difference between the portfolio weights and the numéraire weights. The numéraire weights are associated with the multicurrency numéraire of the invariant currency indexes, where the multicurrency numéraire is an equally-weighted basket of currencies. When a portfolio weight is equal to the associated numéraire weight ($w^i = \eta^i$), the associated currency weight is zero ($\delta^i = 0$). In general, when all of the portfolio weights are equal to the numéraire weights, all of the currency weights are zero and the risk of the portfolio is zero. Thus, the risk of any multicurrency numéraire priced in terms of itself is zero.

2.11. Missing numéraire weights

The numéraire weights are unintentionally missing in Eq. (4.12) in Hovanov et al. (2004), where it was assumed that the currency weights of the optional multicurrency numéraire were equal to the portfolio weights of the optimal portfolio of invariant currency indexes. Table 4 reports the portfolio weights (w^i) of the optimal portfolio from Hovanov et al. (2004),[7] together with the numéraire weights (η^i), and the currency weights (δ^i). Note that although the portfolio weights sum to one, the currency weights sum to 0.0274, which is significantly less than one. Furthermore, 0.0274 represents a 2.74% capital allocation to the stable aggregate currency. As a consequence, the volatility of the portfolio is very low, which is a direct result of a small capital allocation. Also note that three of the five currency weights are negative, where the French franc (FRF) has a currency weight of −0.0845,

[3] Eq. (17) appears as Eq. (4.11) in Hovanov et al. (2004).

[4] Eq. (18) appears as Eq. (4.10) in Hovanov et al. (2004).

[5] Eq. (19) appears as Eq. (4.10) in Hovanov et al. (2004).

[6] Note that $p_T^i - p_T^M$ is substituted for h_T^i, rather than $p_T^M - p_T^i$ from Eq. (8). A positive weight w^i is associated with a positive currency position with $p_T^i > 0$ (Kunkler, 2022).

[7] The portfolio weights in Eq. (5.1) in Hovanov et al. (2004).

Table 3
Volatilities, ADF, and correlation matrix.

Panel A: Levels							
	Vol	ADF	DEM	FRF	GBP	JPY	USD
DEM	30.20%	0.5	1.00	0.03	−0.79	0.76	−0.87
FRF	17.51%	−1.6	0.03	1.00	0.39	−0.32	−0.26
GBP	34.32%	−1.9*	−0.79	0.39	1.00	−0.93	0.59
JPY	94.98%	1.1	0.76	−0.32	−0.93	1.00	−0.73
USD	58.66%	−0.2	−0.87	−0.26	0.59	−0.73	1.00
Panel B: Log returns							
	Vol	ADF	DEM	FRF	GBP	JPY	USD
DEM	4.77%	−14.3***	1.00	0.80	−0.17	−0.36	−0.56
FRF	4.79%	−14.4***	0.80	1.00	−0.17	−0.39	−0.52
GBP	6.66%	−12.4***	−0.17	−0.17	1.00	−0.41	−0.21
JPY	7.99%	−14.0***	−0.36	−0.39	−0.41	1.00	−0.20
USD	8.18%	−13.9***	−0.56	−0.52	−0.21	−0.20	1.00

Table 4
Currency weights of the stable aggregate currency.

	DEM	FRF	GBP	JPY	USD
w^i	0.2595	0.1155	0.3259	0.1265	0.1726
$\eta^i = \frac{1}{N}$	0.2000	0.2000	0.2000	0.2000	0.2000
δ^i	0.0595	−0.0845	0.1259	−0.0735	−0.0274

Table 5
Currency weights of the stable aggregate currency.

	DEM	FRF	GBP	JPY	USD
w^i	0.20	0.20	0.20	0.20	0.20
$\eta^i = \frac{1}{N}$	0.20	0.20	0.20	0.20	0.20
δ^i	0.00	0.00	0.00	0.00	0.00

the Japanese yen (JPY) has a currency weight of −0.0735, and the US dollar (USD) has a currency weight of −0.0274.

Not accounting for the numéraire weights in the calculation of the currency weights of the optimal multicurrency numéraire produces misleading results for both the capital allocation and the variance of the portfolio. In addition, the portfolio weights of the optimal portfolio are not representative of the currency weights of the optimal multicurrency numéraire (see Table 4).

2.12. Resolving the fundamental issues

There are four fundamental issues to resolve: Seigel's Paradox, the spurious covariance matrix, singular covariance matrix, and incorrectly calculating the currency weights. Seigel's Paradox and the spurious covariance are resolved by modelling the log returns of the invariant currency indexes. The singular covariance matrix is resolved by using generalised inverse of the covariance matrix, rather than using an ordinary inverse. Finally, calculating the currency weights is resolved by correctly including the numéraire weights.

Table 5 reports the portfolio weights (w^i) of the optimal portfolio subsequent to resolving all four fundamental issues, together with the numéraire weights (η^i) and the currency weights (δ^i). Note that although the portfolio weights sum to one, the currency weights sum to 0.00. Now that the fundamental issues have been resolved, there is a further fundamental issue with the concept of a stable aggregate currency. The optimal portfolio has currency weights that are all zero and has zero risk.

2.13. Infeasible portfolio optimisation problem

The only situation when the portfolio weights of a portfolio of invariant currency indexes are equal to the currency weights of the optimal multicurrency numeraire is when the portfolio weights sum to zero, rather than to one (Kunkler, 2022). For example, when the portfolio weights sum to zero, the currency weights are found by substituting $p^i_T - p^M_T$ from Eq. (8) for h^i_T into Eq. (20) to give:

$$\begin{aligned} f &= \sum_{i=1}^{N} w^i (p^i_T - p^M_T) \\ &= \sum_{i=1}^{N} w^i p^i_T - p^M_T \sum_{i=1}^{N} w^i \\ &= \sum_{i=1}^{N} w^i p^i_T \\ &= \sum_{i=1}^{N} \delta^i p^i_T \end{aligned} \tag{23}$$

where $\sum_{i=1}^{N} w^i = 0$ is the sum of the portfolio weights; p^M_T is the equally-weighted basket of currencies in Eq. (4); and:

$$\delta^i = w^i \tag{24}$$

where δ^i is the ith currency weight for the ith currency; and w^i is the ith portfolio weight for the ith invariant currency index.

Only when portfolio weights sum to zero do the portfolio weights equal the currency weights. However, if the portfolio weights sum to zero, there must exist currency weights that are negative. Thus, a stable aggregate currency is an infeasible minimum-variance portfolio optimisation problem. An optimal multicurrency numéraire is expected to have nonnegative currency weights and the currency weights must sum to one, which is impossible using the minimum-variance portfolio optimisation for a stable aggregate currency.

3. Conclusion

Multicurrency numéraires are weighted baskets of currencies, where the weights are expected to sum to one and to be positive. A stable aggregate currency is a novel approach to using portfolio optimisation to estimate an optimal multicurrency numéraire. However, there are some fundamental issues with the concept of a stable aggregate currency. Ultimately, the concept of a stable aggregate currency suffers from an infeasible portfolio optimisation problem, where it is impossible to find an optimal portfolio where the currency weights of the optimal multicurrency numéraire are all positive and that they sum to one.

There are different applications of multicurrency numéraires. For example, the ability to create a digit currency whose movements are minimised relative to several international currencies, which has potential applications for companies, such as Facebook (see Giudici et al., 2022). In addition, multicurrency numéraires are used in the Frankel–Wei regression framework, which measures the relationship between the comovements of currencies (Frankel and Wei, 1994). However, future research is required to find alternative methods for choosing multicurrency numéraires. A policy implication from these results is that care needs to be taken when using portfolios of multilateral exchange rates.

Data availability

Data will be made available on request.

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