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ABSTRACT

Limited asset market participation (LAMP) and trade openness are crucial features that characterize all real-world economies. We study equilibrium determinacy and optimal monetary policy in a model of a small open economy with LAMP. With low enough participation in asset markets, conventional wisdom concerning the stabilizing benefits of policy inertia can be overturned, irrespective of the constraint of a zero lower bound on the nominal interest rate. In contrast to recent studies, trade openness can play an important stabilizing role in LAMP economies. Optimal monetary policy is derived as a robust timeless rule, where the optimal level of interest-rate inertia depends on the degree of trade openness. The optimal rule is shown to be super-inertial for standard economies, whereas the degree of inertia is significantly lower and not super-inertial for LAMP economies.

1. Introduction

Limited asset market participation (LAMP) is a well documented feature of all economies. While there has been recent work studying the implications of LAMP for monetary policy, the focus has largely been on closed economies. This paper seeks to address this gap. Our results suggest that trade openness and LAMP have important consequences for the design of monetary policy. First, we challenge the conventional wisdom on the benefits of policy inertia in monetary policy rules for the prevention of indeterminacy. Second, we show that LAMP alters the trade-offs faced by a welfare-maximizing policymaker, such that super-inertial policy is no longer optimal. In standard economies, we find that the optimal rule places a weaker response to both domestic inflation and output, and a lower degree of super-inertia, as the economy becomes more open to trade. However, in LAMP economies, the optimal policy

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coefficients become more strongly negative as trade openness increases, with a lower (possibly negative) weight for the degree of interest-rate smoothing.

LAMP is commonly introduced into two-agent New Keynesian (TANK) models by allowing for a share of hand-to-mouth households who differ from Ricardian consumers in that they hold no assets and consume all current income (see, e.g., Kaplan et al., 2014).[1] The empirical evidence supports the inclusion of a large share of hand-to-mouth behavior. Aguiar et al. (2020) estimate that 40% of US households are hand-to-mouth. For low and middle-income countries, financial exclusion is estimated to be significantly higher.[2]

This paper makes two main contributions to the literature. First, we examine the determinacy properties of a small open economy (SOE) with LAMP, focusing on the role of monetary policy inertia and trade openness for indeterminacy of popular Taylor-type feedback rules with and without the zero lower bound (ZLB) on the nominal interest rate. Then, we replace the feedback rule with a microfounded welfare criterion and examine the implications of LAMP and trade openness for optimal monetary policy under commitment, where the central bank is concerned about nominal interest rate volatility.

Monetary policy inertia and trade openness As shown by Bilbiie (2008), LAMP can overturn the contractionary aggregate demand effect of a real interest rate increase in a closed economy. This results in an 'inverted aggregate demand logic' (IADL) that requires an 'inverted Taylor principle' to ensure determinacy of rational expectations equilibrium. The emergence of IADL depends on the relative strengths of the profit and labor income channels. Under LAMP, the increase in firm profits via a fall in marginal cost can dominate the effect of lower wages, leading to an expansionary effect of increasing interest rates.[3] Bilbiie and Straub (2013) and Bhattarai et al. (2016) present evidence of IADL in the pre-Volker U.S., but it is likely to be a greater concern in developing economies with high rates of financial exclusion and labor-market informality.[4]

While Boerma (2014) and Buffie and Zanna (2018) extend Bilbiie (2008) to explore the determinacy implications of LAMP in the open economy, they limit attention to simple inflation targeting (IT) rules.[5] We add to this literature by focusing on the role of interest-rate smoothing, including price-level targeting (or *Wicksellian*) rules.[6] Our findings reveal contrasting effects of monetary policy inertia on determinacy in standard and IADL economies. In the standard case, policy inertia reduces indeterminacy, whereas it increases its likelihood under IADL. This underscores an important caveat concerning the reliance on interest-rate smoothing to address potential indeterminacy. In the absence of LAMP, Woodford (2003a) and Bullard and Mitra (2007) show that super-inertial rules eliminate indeterminacy, and several studies find that price-level targeting improves stability compared to inflation targeting.[7] While price-level targeting always ensures determinacy in standard economies, we find that under IADL there are many degrees of LAMP for which determinacy is not possible.

Boerma (2014) and Buffie and Zanna (2018) show that the Taylor principle can be restored in high-LAMP open economies through the terms of trade channel of monetary policy, which exerts contractionary pressure following a rise in the real interest rate. However, we find that the benefits of openness can be offset by the destabilizing effects of policy inertia and factors such as imperfect exchange rate pass-through (ERPT) and foreign currency pricing. Imperfect ERPT dampens the terms of trade response to real interest rate changes, preventing determinacy in IADL economies under the Taylor principle. With a similar outcome, dominant currency pricing mitigates and can even reverse the export demand effects of the terms-of-trade channel. Finally, we show that the implications of trade openness for local stability crucially depend on the timing of the interest-rate rule. Under forward- and backward-looking IT, openness can serve as a stabilizing force in LAMP economies by mitigating the destabilizing effects of policy inertia. However, under contemporaneous IT, trade openness can reinforce the degree of policy inertia, leading to equilibrium indeterminacy in both closed and open LAMP economies.

Following the analysis of the linear model, we also examine determinacy in the presence of a ZLB. We find that policy inertia and trade openness both increase indeterminacy under a ZLB, regardless of the degree of LAMP. We also show that policy rules that are determinate under IADL in the linear model can be highly unstable with a ZLB.

Optimal monetary policy Our second contribution is to extend the optimal monetary policy analysis of Bilbiie (2008) to the open economy with interest-rate inertia. We derive optimal monetary policy under an equitable allocation using government transfers.[8] Similar to Woodford (2003a, chap. 6), Giannoni and Woodford (2003) and Levine et al. (2008), we allow for the costs of interest-rate volatility to enter the loss function of the policymaker. We show how these costs arise in the welfare-consistent loss function due to an incentive to avoid hitting the ZLB. Under commitment, the implicit instrument rule is shown to be robustly optimal and timeless.

[1] Introduced by Mankiw (2000) as 'rule-of-thumb' consumers and further popularized by Galí et al. (2004). It has been shown that for many purposes TANK models provide an appropriate theoretical shortcut to fully heterogeneous-agent New Keynesian models (see, e.g., Bilbiie, 2020).

[2] See Figure A12 in the online appendix.

[3] Colciago (2011) and Ascari et al. (2017) argue that nominal wage stickiness dampens the profit channel and can help restore the Taylor principle in closed economies. Buffie (2013) shows, however, that real wage rigidity is key for preventing the emergence of IADL.

[4] Recent evidence (e.g., Bosch and Maloney, 2010; Coşkun, 2022) suggests that the informal sector adds to the wage flexibility of the formal sector in emerging market economies. Estimating the effects of monetary policy in developing countries poses many challenges, and as Mishra and Montiel (2013) show in their survey, results are often inconclusive. However, they do list several studies finding evidence of price puzzles (i.e., an inflationary impact of monetary contractions), which emphasizes the importance of our study.

[5] For the literature highlighting the importance of trade openness for equilibrium determinacy in non-LAMP SOE economies, see Zanna (2003), De Fiore and Liu (2005), and Linnemann and Schabert (2006).

[6] This is a well-document feature of central bank behavior, often described in terms of "make-up" strategies for central banks (e.g., Powell, 2020).

[7] See, e.g., Vestin (2006), Gaspar et al. (2007), Dib et al. (2013), Giannoni (2014), McKnight (2018). Holden (2023) shows these benefits extend to a ZLB setting.

[8] In doing so, our normative analysis relates to empirical studies which find that consumption inequality closely tracks income inequality (Aguiar and Bils, 2015).

Optimal policy is found to be super-inertial for standard economies, whereas the degree of inertia is significantly lower and not super-inertial for IADL economies. In both cases, the optimal level of inertia depends on the degree of trade openness.

There are two policy trade-offs that welfare-maximizing policymakers face. The first is between interest-rate stability and the stability of domestic-price inflation and output. The second is the standard trade-off between inflation and output stabilization. We find that the penalty on interest-rate variability only affects the degree of activeness of the optimal rule and not the degree of inertia. In standard economies, the optimal IT rule is shown to be super-inertial with positive coefficients for inflation and the output gap. All policy coefficients are increasing with LAMP and decreasing with trade openness. In contrast, the policy coefficients are negative for IADL economies and become more strongly negative as LAMP decreases and trade openness increases. For empirically-plausible ranges of LAMP, the optimal degree of interest-rate smoothing is low, and can be negative if the economy becomes sufficiently open.

Our analysis contributes to a small literature that characterizes optimal monetary policy in the presence of financially-excluded households. For the closed economy, Bilbiie (2008) shows that the optimal monetary policy under commitment is robustly optimal, where the optimal response to inflation is decreasing in the share of LAMP. For open economies, Iyer (2016) finds that when the degree of LAMP is high, the policymaker, in addition to domestic inflation, should also stabilize the nominal exchange rate by putting more weight on stabilizing output.[9] We extend the analysis of both Bilbiie (2008) and Iyer (2016) by deriving the optimal monetary policy for LAMP economies under interest-rate inertia.

Road-map The rest of the paper is structured as follows. Section 2 sets out the baseline SOE model with LAMP; section 3 considers the issue of equilibrium determinacy both with and without a ZLB; section 4 studies optimal monetary policy; and section 5 concludes. Detailed derivations, proofs, and additional results are provided in an online appendix.

2. A small open-economy model with LAMP

This section presents our theoretical setup. It nests both the influential representative-agent SOE framework of Galí and Monacelli (2005) and the closed-economy LAMP model of Bilbiie (2008). The economy is comprised of perfectly competitive wholesale firms that produce a final good and monopolistically competitive retailers that sell intermediate tradable goods under Calvo (1983) price setting. There are two types of households in the economy. In addition to standard Ricardian households, we include an exogenous fraction of constrained households that do not have access to asset markets.

2.1. Households

Households are divided into two types. A fraction of households, $\lambda \in [0,1]$, participate in domestic and international financial markets; these are referred to as Ricardian households and are denoted by superscript R. The remaining households $1-\lambda$, referred to as constrained households and denoted by superscript C, consume only out of wage income, and have no assets or access to financial markets. For both household types, $i = \{C, R\}$, single-period utility is assumed to be:

$$U_t^i = U(C_t^i, N_t^i) = \frac{\left(C_t^i\right)^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi}, \tag{2.1}$$

where C_t^i is real consumption by household type i, σ is the coefficient of relative risk aversion (CRRA), N_t^i is labor supply of type i, and φ is the inverse of the Frisch elasticity.

2.1.1. Ricardian households

Ricardian households solve an intertemporal consumption problem:

$$\max_{C_t^R, N_t^R} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s U(C_{t+s}^R, N_{t+s}^R) \right] \tag{2.2}$$

subject to a sequence of nominal budget constraints given by:

$$P_t^B B_{H,t} + P_t^{B^*} \mathcal{E}_t B_{F,t}^* = B_{H,t-1} + \mathcal{E}_t B_{F,t-1}^* + P_t W_t N_t^R - P_t C_t^R + \Gamma_t. \tag{2.3}$$

$B_{H,t}$ and $B_{F,t}^*$ are domestic and foreign bonds, denominated in the respective currencies, bought at the nominal price $P_t^B = 1/R_t$ and $P_t^{B^*} = 1/R_t^*$, where R_t and R_t^* denote the domestic and foreign nominal gross interest rate, respectively. P_t is the consumer price index (CPI) and $\mathcal{E}_t$ is the nominal exchange rate, measured as the domestic price of a unit of foreign currency. W_t and Γ_t denote the real wage rate and nominal profits, respectively. Maximizing (2.2) subject to the budget constraint we obtain:

$$P_t^B = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}^R}{\Pi_{t,t+1}} \right], \tag{2.4}$$

[9] Lahiri et al. (2007) consider the implications of LAMP for the optimal exchange rate regime in a flexible-price SOE.

$$P_t^{B*} = \mathbb{E}_t\left[\frac{\Lambda^R_{t,t+1}}{\Pi_{t,t+1}}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right], \tag{2.5}$$

$$\frac{U^R_{N,t}}{U^R_{C,t}} = -\left(C_t^R\right)^{\sigma}\left(N_t^R\right)^{\varphi} = -W_t, \tag{2.6}$$

where $\Pi_{t,t+1} \equiv P_{t+1}/P_t$ denotes the gross CPI inflation rate and $\Lambda^R_{t,t+1} \equiv \beta U^R_{C,t+1}/U^R_{C,t}$ is the stochastic discount factor for Ricardian consumers.

2.1.2. Consumption demand

Households demand consumption goods from domestic H and foreign F retailers (imports):

$$C_t = \left[w_C^{\frac{1}{\mu_C}} C_{H,t}^{\frac{\mu_C-1}{\mu_C}} + (1-w_C)^{\frac{1}{\mu_C}} C_{F,t}^{\frac{\mu_C-1}{\mu_C}}\right]^{\frac{\mu_C}{\mu_C-1}}. \tag{2.7}$$

The weight w_C in the consumption basket attached to domestic consumption demand is a measure of home bias (where $w_C = 1$ is the autarky case). Maximizing total consumption (2.7) subject to a given aggregate expenditure $P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$ yields:

$$C_{H,t} = w_C\left(\frac{P_{H,t}}{P_t}\right)^{-\mu_C} C_t, \tag{2.8}$$

$$C_{F,t} = (1-w_C)\left(\frac{P_{F,t}}{P_t}\right)^{-\mu_C} C_t. \tag{2.9}$$

Substituting these demand schedules into (2.7) gives the corresponding price index:

$$P_t = \left[w_C(P_{H,t})^{1-\mu_C} + (1-w_C)(P_{F,t})^{1-\mu_C}\right]^{\frac{1}{1-\mu_C}}. \tag{2.10}$$

Foreign aggregate consumption C_t^* is given by an exogenous process. The real exchange rate is defined as the relative aggregate consumption price $Q_t \equiv P_t^*\mathcal{E}_t/P_t$. Then the foreign counterpart of the import demand schedule (2.9), which determines the export demand of the home good, is

$$C^*_{H,t} = (1-w_C^*)\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\mu_C^*} C_t^* = (1-w_C^*)\left(\frac{P_{H,t}}{P_tQ_t}\right)^{-\mu_C^*} C_t^*. \tag{2.11}$$

$P^*_{H,t}$ and P^*_t denote the respective prices of home-produced consumption goods and of aggregate consumption goods in the rest of the world (RoW) in foreign currency, and we have used the law of one price for differentiated goods, $\mathcal{E}_tP^*_{H,t} = P_{H,t}$. We impose perfect exchange rate pass-through for imports and because the home country is small, the law of one price implies that $P_t^* = P^*_{F,t}$, $\mathcal{E}_tP_t^* = P_{F,t}$, so $Q_t = P_{F,t}/P_t$. We denote the terms of trade $S_t \equiv P_{F,t}/P_{H,t}$ and define total exports per capita as $EX_t \equiv C^*_{H,t}$.

2.1.3. Constrained consumers

Constrained consumers have no income from monopolistically competitive retail firms and must consume out of wage income. Their nominal consumption is given by:

$$P_tC_t^C = P_tW_tN_t^C. \tag{2.12}$$

Constrained consumers choose C_t^C and N_t^C to maximize an analogous utility function to (2.2) but subject to (2.12). The first order conditions can be written as:

$$\frac{U^C_{N,t}}{U^C_{C,t}} = -\left(C_t^C\right)^{\sigma}\left(N_t^C\right)^{\varphi} = \frac{U^R_{N,t}}{U^R_{C,t}} = -W_t, \tag{2.13}$$

which has the same form as eq. (2.6) for the Ricardian consumers, but as we shall discuss further below, C_t^C and N_t^C are not the same as C_t^R and N_t^R in general.

With both Ricardian and constrained households, aggregate consumption and hours supplied are given by:

$$C_t = \lambda C_t^R + (1-\lambda)C_t^C, \tag{2.14}$$

$$N_t = \lambda N_t^R + (1-\lambda)N_t^C. \tag{2.15}$$

2.2. Firms

There are wholesale and retail firms. The former act in perfect competition producing a homogeneous final good, whereas the latter produce and sell differentiated intermediate goods under monopolistic competition.

2.2.1. Wholesale sector

Wholesale firms hire labor N_t to produce homogeneous output Y_t^W using the standard labor-augmenting constant returns to scale production technology:

$$Y_t^W = F(N_t, A_t) = A_t N_t. \tag{2.16}$$

Profit maximization implies:

$$P_t W_t = P_t^W F_{N,t} = P_t^W \frac{Y_t^W}{N_t} \quad \Rightarrow W_t = MC_t \left(\frac{P_{H,t}}{P_t}\right) \frac{Y_t^W}{N_t}, \tag{2.17}$$

where $MC_t \equiv P_t^W / P_{H,t}$ is real marginal cost in units of domestic retail output.

2.2.2. Retail sector

A retail firm m converts an amount of wholesale output $Y_t^W(m)$ into a differentiated good of amount $Y_t(m) - F(m)$, where $F(m) = F$ are fixed costs assumed to be equal across retail firms. The retail differentiated goods are combined into the final good Y_t using a CES-aggregator production technology: $Y_t \equiv \left[\int_0^1 Y_t(m)^{\frac{\varsigma-1}{\varsigma}} dm\right]^{\frac{\varsigma}{\varsigma-1}}$. The CES technology implies demand schedules for each intermediate input j given by $Y_t(m) = \left[\frac{P_{H,t}(m)}{P_{H,t}}\right]^{-\varsigma} Y_t$.

Following Calvo (1983), in every period each retail firm m faces a fixed probability $1-\xi$ of being able to optimally set their price to $P_{H,t}^0(m)$. If the price is not re-optimized, then it is held fixed. The objective of a retail producer m at time t is to choose $P_{H,t}^0(m)$ to maximize discounted real profits:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(m) \left[P_{H,t}^0(m) - P_{H,t+k} MC_{t+k}\right] \tag{2.18}$$

subject to the good-m demand schedule, where $\Lambda_{t,t+k} \equiv \beta^k U_{C,t+k}/U_{C,t}$ is the stochastic discount factor over the interval $[t, t+k]$. This leads to the usual optimal price condition and law of motion for domestic-price inflation.

Aggregate output Y_t is given by $Y_t = \left[A_t N_t - F\right]/\Delta_t$ where $\Delta_t \equiv \int_0^1 \left(\frac{P_{H,t}(m)}{P_{H,t}}\right)^{-\varsigma} dm \geq 1$ is the degree of price dispersion of retail goods.

2.3. Output market clearing

Output market clearing for retail firm m is: $Y_t(m) = C_{H,t}(m) + C_{H_t}^*(m)$. Aggregating yields the resource constraint: $Y_t = C_{H,t} + C_{H_t}^* = C_{H,t} + EX_t$. Using the demand conditions (2.8) and (2.11) yields: $Y_t = w_C \left(P_{H,t}/P_t\right)^{-\mu_C} C_t + (1 - w_C^*)\left(1/S_t\right)^{-\mu_C^*} C_t^*$.

2.4. Monetary policy

The nominal interest rate R_t is a policy variable given by the explicit instrument rule:

$$\log\left(R_t\right) = \left(1-\rho_r\right)\log(R) + \rho_r \log\left(R_{t-1}\right) + \theta_\pi \left(\mathbb{E}_t\left[\log\left(\Pi_{t,t+1}\right)\right] - \log(\Pi)\right) \tag{2.19}$$

where $\rho_r, \theta_\pi \geq 0$, and $\Pi_{t-1,t} \equiv P_t/P_{t-1}$. We initially focus on a forward-looking rule as many central banks target forecasted inflation in practice due to the observed time delay in the transmission mechanism of monetary policy.[10] In section 4, we consider optimal targeting rules (i.e., implicit instrument rules) under commitment.

2.5. Foreign bond accumulation

In nominal terms and measured in the home country currency, foreign bond holdings evolve according to: $P_t^{B^*} \mathcal{E}_t B_{F,t}^* = \mathcal{E}_t B_{F,t-1}^* + P_t TB_t$, where the nominal trade balance $P_t TB_t = P_{H,t} Y_t - P_t C_t$ is the difference between domestic output and private consumption. Defining $B_{F,t} \equiv \mathcal{E}_t B_{F,t}^*/P_t$ to be the stock of foreign bonds in home country consumption units, it follows that

$$P_t^{B^*} B_{F,t} = \frac{\Pi_{t-1,t}^{\mathcal{E}}}{\Pi_{t-1,t}} B_{F,t-1} + TB_t, \tag{2.20}$$

where $\Pi_{t-1,t}^{\mathcal{E}} \equiv \mathcal{E}_t/\mathcal{E}_{t-1}$ is the (gross) nominal depreciation of the SOE currency.

[10] For further discussion, see Batini and Haldane (1999) and McKnight and Mihailov (2015).

2.6. Equilibrium

An equilibrium is defined in the model variables given the conditions outlined above together with the interest-rate rule (2.19). Section A of the online appendix provides a summary of this equilibrium.

2.7. Symmetric equilibrium of small open economies with risk sharing

So far we have modeled the SOE in an environment consisting of the RoW, which from its own viewpoint is closed. For the optimal policy analysis, we amend the environment to consist of a continuum of $i \in [0,1]$ identical open economies of which the 'home' economy is just one. We assume there is international risk-sharing in this version of the model so the risk premium is zero. Equations (2.5) lead to the standard risk-sharing condition:

$$C_t^R = (C_t^R)^i Q_{i,t}^{\frac{1}{\sigma}}, \tag{2.21}$$

where $Q_{i,t} \equiv \mathcal{E}_{i,t} P_t^i / P_t$ is the home country vis-à-vis country i bilateral real exchange rate, with $\mathcal{E}_{i,t}$ now the corresponding bilateral nominal exchange rate between these two countries (both identical SOEs). Naturally, the risk-sharing only applies to Ricardian and not constrained households. Then using (2.8) and (2.11), in a symmetric equilibrium with $C_t = C_t^*$,[11] $\mu_C = \mu_C^*$, $\lambda_i = \lambda$, $\sigma = \sigma^*$ and $Q_t = \mathcal{E}_t P_t^* / P_t$ we have:

$$Y_t = Y_t^* = C_{H,t} + C_{H,t}^* = C_t \left(\frac{P_{H,t}}{P_t}\right)^{-\mu_C} \left(\mathrm{w}_C + (1-\mathrm{w}_C) Q_t^{\mu_C - \frac{1}{\sigma}}\right). \tag{2.22}$$

3. Stability and determinacy analysis

The model is linearized around a non-stochastic steady state where net inflation is zero, i.e., $\Pi = 1$, and prices $P = P_H = P_F = P^* = 1$. Then by definition the steady-state terms of trade and real exchange rate are $\varepsilon = Q = 1$. For the LAMP aspects of the model, we follow Bilbiie (2008) and impose an equitable outcome $C^R = C^C$ and $N^R = N^C$, which can be achieved by assuming that free entry drives profits to zero in an equilibrium in the steady state with $F/Y = (1 - MC) = 1/\zeta$.[12] Since the focus of this section is on (local) stability and equilibrium determinacy, we consider the deterministic perfect foresight case with all shocks set equal to zero. In what follows, all lower-case variables in this section denote percentage deviations from the steady state.

We can describe the non-policy aspects of the model using a New Keynesian Phillips Curve (NKPC) and an intertemporal IS curve, both expressed in terms of consumption by Ricardian consumers[13]:

$$\pi_{H,t} = \beta \pi_{H,t+1} + \Psi \Upsilon c_t^R, \tag{3.1}$$

$$c_t^R = c_{t+1}^R - \frac{w_C}{\sigma}\left(r_t - \pi_{H,t+1}\right), \tag{3.2}$$

$$y_t = \Xi c_t^R, \tag{3.3}$$

where the parameters are defined as:

$$\Upsilon \equiv \frac{\sigma(1-w_C)}{w_C} + \frac{\lambda(\varphi+\sigma)\left[w_C\varphi + \sigma\left(1+\frac{1}{\zeta}\right)\right] + \varphi(1-w_C)(\varphi+\sigma)\left[1+\frac{\omega\sigma}{w_C}\right]}{\lambda(\varphi+\sigma)\left(1+\frac{1}{\zeta}\right) - (1-\lambda)\varphi\left[w_C(1+\varphi) + (\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]}, \tag{3.4}$$

$$\Xi \equiv w_C\lambda + (1-w_C)\left[1+\frac{\omega\sigma}{w_C}\right] + w_C(1-\lambda)\frac{1+\varphi}{\varphi+\sigma}\left(\Upsilon - \frac{\sigma}{w_C} + \sigma\right), \tag{3.5}$$

$\Psi \equiv \frac{(1-\xi)(1-\beta\xi)}{\xi} > 0$, and $\omega \equiv w_C(\mu_C - 1/\sigma) + \mu_C^* = \mu_C(1+w_C) - w_C/\sigma > 0$ if $\mu_C = \mu_C^*$.

The threshold for the proportion of Ricardian households λ below which the inverted aggregate demand logic (IADL) occurs is the point at which Υ changes sign. From (3.4) this is given by:

$$\lambda = \lambda^* = \frac{\varphi\left[w_C(1+\varphi) + (\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]}{\varphi\left[w_C(1+\varphi) + (\sigma-1)\left(1+\frac{1}{\zeta}\right)\right] + (\varphi+\sigma)\left(1+\frac{1}{\zeta}\right)}. \tag{3.6}$$

Then replacing $\lambda^* = \lambda^*(w_C)$ we have the following result:

[11] Note that all macroeconomic quantities are in per capita form.

[12] As we discuss later in the paper, alternatively a subsidy scheme for the optimal equitable allocation in Proposition 3 in section 4.1 can support this outcome.

[13] Alternative NKPC and IS expressions, written in terms of total consumption in deviations from baseline allocations, and hence in standard output gap terms, are discussed in section 4.

Proposition 1 *(IADL threshold). The threshold below which IADL occurs, $\lambda^* = \lambda^*(w_C)$, increases with w_C and therefore decreases with trade openness $1 - w_C$.*

Proof. See online appendix C.3. □

Consequently, trade openness *decreases the possibility of IADL.*[14] To understand the IADL, notice that we can write Ricardian labor supply as: $n_t^R = \varphi^{-1}\left(\Upsilon - \sigma/w_C\right)c_t^R$, which implies that hours fall in consumption for Ricardian households provided $\Upsilon < \sigma/w_C$. When asset market participation is sufficiently low, the profit channel dominates the wage effect, and increases in the real interest rate $r_t - \pi_{t+1} = w_C\left(r_t - \pi_{H,t+1}\right)$ can have an expansionary effect on output y_t. For example, in the closed economy ($w_C = 1$) it follows from (3.5) that $\Xi^{w_C=1} = \frac{\Upsilon(1+1/\zeta)}{\varphi+\sigma(1+1/\zeta)} < 0$ under IADL. From (3.2) and (3.3), a rise in the real interest rate increases output by reducing Ricardian consumption c_t^R, exerting upward pressure on inflation from the NKPC. This contrasts with the standard aggregate demand logic (SADL) where both output and consumption respond negatively to real interest rate rises. In open economies, it follows from (3.4) and (3.5) that $\Xi > 0$ when $\Upsilon > 0$, so that c_t^R always increases in y_t under SADL. However, under IADL, c_t^R can either increase or decrease in y_t depending on the degree of LAMP.

The parameter Υ is a function of λ and other parameters w_C, φ, σ, ω, but is independent of the monetary policy rule. We initially assume a simple inertial rule of the form:

$$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{t+1}, \tag{3.7}$$

where $\rho_r \geq 0$ is the degree of interest-rate inertia and $\theta_\pi \geq 0$ is the inflation response coefficient. Note that the central bank adopts a super-inertial policy if $\rho_r > 1$, and the integral rule with $\rho_r = 1$ yields a price-level (Wicksellian) rule. The interest-rate rule (3.7) can be expressed as:

$$r_t = \rho_r r_{t-1} - \frac{\sigma(1-w_C)\theta_\pi}{w_C}c_t^R + \frac{\sigma(1-w_C)\theta_\pi}{w_C}c_{t+1}^R + \theta_\pi \pi_{H,t+1}. \tag{3.8}$$

Equations (3.1), (3.2) and (3.8) imply the minimal state-space representation of the model:

$$\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t, \quad \mathbf{z}_t = \begin{bmatrix} c_t^R & \pi_{H,t} & r_{t-1} \end{bmatrix}', \tag{3.9}$$

where the coefficient matrix $\mathbf{A}$ is given in online appendix C.1.

3.1. Determinacy analysis

We start by examining the stability properties of the model for the policy rule (3.7).

Proposition 2.

(a) ***(Role of interest-rate inertia)*** *For the standard SADL case $\lambda > \lambda^*$, interest rate inertia increases the policy space for θ_π for which there is determinacy. An equilibrium exists for all $\lambda \in (\lambda^*, 1]$ with an appropriate choice of θ_π. Under IADL, there exists some values of $\lambda \in [0, \lambda^*)$ for which a unique stable equilibrium exists. Interest rate inertia in this case reduces the policy space for θ_π, and for some values $\lambda \in [0, \lambda^*)$ if*

$$-\frac{2\sigma(1+\beta)}{\Psi w_C} < \Upsilon < -\frac{2\sigma(1+\beta)(1-w_C)}{\Psi w_C}$$

then a unique stable equilibrium does not exist for $\theta_\pi > 0$.

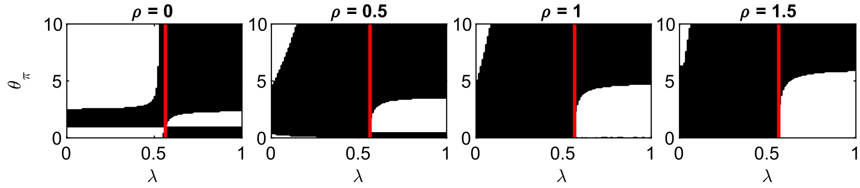
(b) ***(Role of trade openness)*** *For the standard SADL case $\lambda > \lambda^*$, trade openness $1 - w_C$ decreases the policy space for θ_π for which there is determinacy. Under IADL, the determinate policy space for θ_π increases with $1 - w_C$ for some values $\lambda \in [0, \lambda^*)$ if*

$$-\frac{2\sigma(1+\beta)(1-w_C)}{\Psi w_C} < \Upsilon < 0.$$

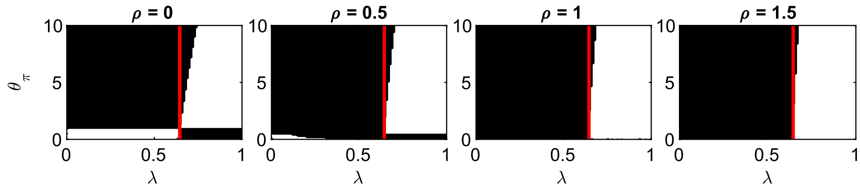
Proof. See online appendix C.4. □

The results given in Proposition 2 follow from the necessary and sufficient conditions for equilibrium determinacy outlined in the online appendix. In the absence of interest-rate inertia ($\rho_r = 0$), the Taylor principle ($\theta_\pi > 1$), which implies an 'active' policy feedback to future inflation, is a necessary condition for determinacy in the SADL case ($\Upsilon > 0$). In contrast, for the IADL case ($\Upsilon < 0$) a 'passive' policy stance ($\theta_\pi < 1$), or the *inverted Taylor principle*, is consistent with determinacy for closed economies. The determinacy conditions indicate that increasing interest-rate inertia *increases* the range of determinacy under SADL, while it *reduces* the determinate policy

[14] This result is consistent with the findings of Boerma (2014). Buffie and Zanna (2018) find trade openness can reduce the threshold value λ^* close to zero in an imperfect capital mobility model with multiple sticky-price and flexible-price sectors.



(a) Open economy



(b) Closed economy

Fig. 1. Determinacy regions (white areas) for the baseline LAMP model. Parameter values are $\Psi = 0.086$, $\varphi = \sigma = 2$, $\beta = 0.99$, $\zeta = 7$, and $\mu_C = 0.62$, $w_C = 0.6$ for the open economy (top panel) and $w_C = 1$ for the closed (bottom panel). The red vertical line gives λ^* below which IADL holds. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

space under IADL. Trade openness has contrasting effects. By reducing an upper bound on the inflation response coefficient, denoted Γ_1, the determinacy region shrinks in open SADL economies. However, the region of determinacy can actually increase under IADL, as the economy becomes more open. For sufficiently low values of λ, determinacy arises under the Taylor principle provided the inflation response coefficient is set sufficiently high $\theta_\pi > \max\left\{\left(1-w_C\right)^{-1}, \Gamma_1\right\}$.

The above results are illustrated in Fig. 1 for a standard quarterly parameterization. We set the discount factor $\beta = 0.99$, the CRRA coefficient $\sigma = 2$, $\zeta = 7$, implying a markup of 16 percent, and the real marginal cost elasticity of inflation $\Psi = 0.086$, consistent with an average price duration of one year. The open economy parameters are set with home bias $w_C = 0.6$ and an elasticity of substitution $\mu_C = 0.62$ in line with the estimates of Boehm et al. (2019). By inspection, while policy inertia has a stabilizing effect on the SADL economy, trade openness has a destabilizing effect. Under IADL, determinacy can also arise under the Taylor principle, as openness exerts a stabilizing effect, whereas policy inertia now destabilizes the economy. Under super-inertial policy ($\rho_r \geq 1$), determinacy is easily achieved in the SADL case. For instance, consider the closed-economy version of the model ($w_C = 1$) with price-level targeting ($\rho_r = 1$). The necessary and sufficient condition for determinacy is given by $0 < \theta_\pi < 2 + \frac{4\sigma(1+\beta)}{\Psi\Upsilon}$, where the upper bound is non-binding within the empirically-relevant interval $\theta_\pi \in [0, 10]$, except for λ very close to the threshold λ^*.[15] In contrast, determinacy is only possible under IADL if $\left(1 - \left[1 + \frac{2\sigma(1+\beta)}{\Psi}\frac{1+\varphi}{\sigma+\varphi}\right]^{-1}\right)\lambda^* < \lambda < \lambda^*$. For the baseline parameter values, there exists a very small interval of λ for which determinacy is possible ($0.986\lambda^* < \lambda < \lambda^*$). This determinacy region is barely visible in Fig. 1 under a price-level rule (and super-inertial policy in general) for both closed and open IADL economies. This is in stark contrast to the case of no hand-to-mouth consumers ($\lambda = 1$), where determinacy is easily induced.[16]

For some intuition, first consider a sunspot-induced increase in inflationary expectations in a closed economy. For the SADL case, the Taylor principle induces a rise in the real interest rate, resulting in a fall in consumption and output. This exerts downward pressure on real marginal cost, which lowers inflation from the NKPC, contradicting the initial inflationary expectations. Similar to Bullard and Mitra (2007), policy inertia helps to enlarge the determinacy region, as the long-run nominal interest rate is $1/\left(1-\rho_r\right)$ times more responsive to permanent changes in inflation compared to the non-inertial case. Under super-inertial rules, any increase in inflation results in a rise in both the nominal and real interest rate. For any $\theta_\pi > 0$, the Taylor principle is always satisfied and indeterminacy is not possible.

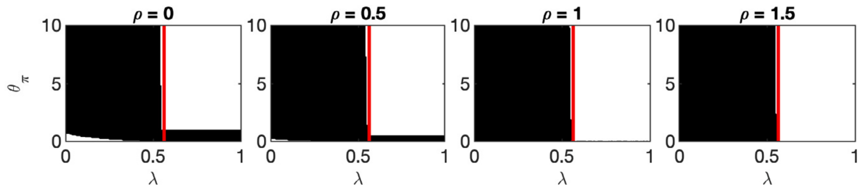
In IADL economies, Ricardian consumption falls but output rises in response to a higher real interest rate. Consequently, real marginal cost increases and the initial inflationary belief becomes self-fulfilling under the Taylor principle. In this case, a passive policy response by letting the real interest fall in response to higher expected inflation, leads to lower demand and deflation from the NKPC, contradicting the initial inflationary expectations. However, interest rate inertia reduces the determinacy region under the inverted Taylor principle, and determinacy becomes nearly impossible under super-inertial rules.

In open economies, the next-period consumer-price inflation rate depends on both the rate of future domestic price inflation and changes in the terms of trade:

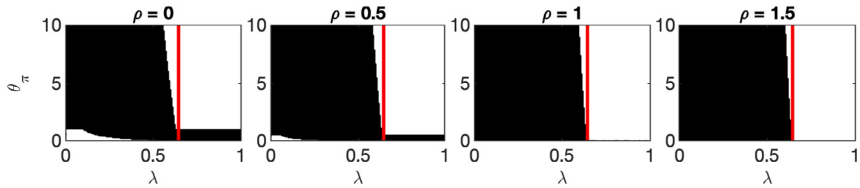
$$\pi_{t+1} = \pi_{H,t+1} + (1-w_C)\left(s_{t+1} - s_t\right) = \pi_{H,t+1} + \sigma\left(\frac{1-w_C}{w_C}\right)\left(c^R_{t+1} - c^R_t\right). \tag{3.10}$$

[15] With our baseline parameter values, the upper bound on θ_π lies in the interval [2, 10] for $\lambda \in [0.644, 0.684]$.

[16] E.g., a unique stable equilibrium exists in the closed economy under $\lambda = 1$ iff $0 < \theta_\pi < 2\left[1 + \frac{2\sigma(1+\beta)}{\Psi(\sigma+\varphi)}\right]$.



(a) Open economy



(b) Closed economy

Fig. 2. Determinacy regions (white areas) under a contemporaneous IT rule. Parameter values are the same as in Fig. 1. The red vertical line gives λ^* below which IADL holds.

For the SADL case, a real interest rate rise results in an expected deterioration in the terms of trade $s_{t+1} - s_t > 0$. Consequently, indeterminacy can arise under the Taylor principle provided the upward pressure on consumer-price inflation, generated by the adjustments in the terms of trade, is sufficiently strong to offset the reduction in domestic-price inflation generated from lower domestic demand. As the degree of trade openness $1 - w_C$ increases, the economy becomes more prone to indeterminacy. However, in stark contrast to closed economies, determinacy can be consistent with the Taylor principle under IADL. While rises in the real interest rate now result in an increase in domestic-price inflation, the upward pressure exerted on consumer-price inflation can be more than offset via a reduction in Ricardian consumption ($c^R_{t+1} - c^R_t < 0$) arising from the adjustment in the terms of trade.

3.2. Timing of interest-rate rules

We now examine the robustness of our findings under alternative timing specifications for the policy rule (3.7).[17] Specifically, we consider simple inertial rules of the form:

$$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{t+k}, \tag{3.11}$$

where $k = 0, -1$. If $k = 0$, the interest-rate rule responds to contemporaneous inflation, whereas, if $k = -1$, the policy rule is lagged or backward-looking.

Remark 1 *(Current-looking rule).* For the standard SADL case $\lambda > \lambda^*$, interest-rate inertia increases the determinate policy space for θ_π. Trade openness has no effect. Under IADL, both interest-rate inertia and openness decrease the determinate policy space for θ_π.

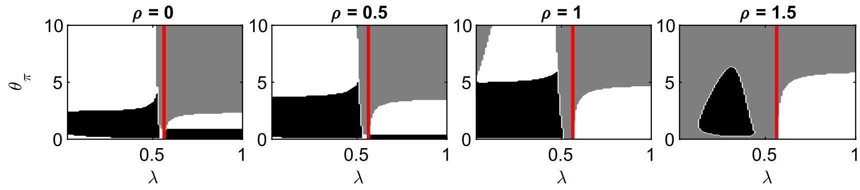
Remark 1 follows directly from the determinacy analysis outlined in online appendix C.5. Under a current-looking inflation rule, the necessary condition for determinacy in the SADL case is given by $\theta_\pi > 1 - \rho_r$. It follows that under a super-inertial policy ($\rho_r \geq 1$), the Taylor principle is always satisfied and indeterminacy is not possible. While policy inertia exerts a stabilizing effect on the SADL economy, in the absence of an upper bound on θ_π trade openness no longer has a destabilizing effect. To understand why, first note that in the baseline model the UIP condition is satisfied (i.e., an exchange rate depreciation is equal to the lagged interest rate). Consequently, the policy rule (3.11) can be written as:

$$r_t = \left[\rho_r + \left(1 - w_C\right)\theta_\pi\right] r_{t-1} + w_C \theta_\pi \pi_{H,t}.$$

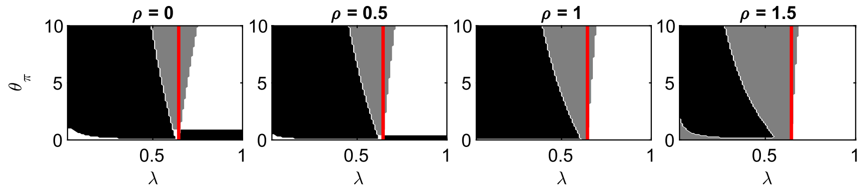
Under a contemporaneous IT rule, policy inertia arises in open economies even in the absence of interest-rate smoothing ($\rho_r = 0$), where the degree of inertia is increasing in openness. Consequently, open SADL economies are no longer prone to indeterminacy.

As illustrated in Fig. 2, targeting contemporaneous inflation has important implications for IADL economies. First, the determinacy region associated with the inverted Taylor principle is significantly reduced (in both closed and open economies), only holding for a small interval of λ. Second, since trade openness reinforces the degree of policy inertia under contemporaneous IT, the large determinacy region that arises under the Taylor principle in open economies is no longer available. Third, the region of determinacy

[17] In the online appendix, we additionally examine the robustness of the results if the policy rule targets domestic price inflation (appendix C.6) and responds to output (appendix C.7).



(a) Open economy



(b) Closed economy

Fig. 3. Determinacy regions (white areas) under backward-looking IT. Black areas indicate indeterminacy and grey areas indicate equilibrium explosiveness. Parameter values are the same as in Fig. 1. The red vertical line gives λ^* below which IADL holds.

that arises for values of λ close to the threshold λ^* is significantly enlarged, particularly in the closed economy. However, in open economies the lower bound $\theta_\pi > -(1+\rho_r)\frac{2\sigma(1+\beta)+\Psi w_C \Upsilon}{2\sigma(1+\beta)(1-w_C)+\Psi w_C \Upsilon}$ is increasing in trade openness and quickly rises to very large values.[18]

In sum, under a current IT rule, trade openness no longer exerts a destabilizing effect on SADL economies and the indeterminacy problem is resolved under interest-rate inertia. However, since policy inertia and openness both now have a destabilizing effect under LAMP, the likelihood of determinacy is greatly reduced in IADL economies.

We now turn our attention to a backward-looking interest-rate rule (i.e., $k = -1$ in (3.11)). In this case, the dynamic system is four dimensional and analytical results are not possible.

Remark 2 *(Backward-looking rule).* For the standard SADL case $\lambda > \lambda^*$, interest-rate inertia increases the determinate policy space for θ_π, whereas trade openness decreases determinacy. Under IADL, interest-rate inertia decreases the determinate policy space for θ_π, whereas trade openness increases determinacy.

Fig. 3 summarizes the key results under a backward-looking rule. While the Taylor principle remains a necessary condition for determinacy in SADL economies, an excess response to lagged inflation renders the equilibrium locally explosive. Under price-level targeting, the determinacy region in open IADL economies is larger compared to both forward- and current-looking rules. Nonetheless, the conclusions are very similar to forward-looking IT policy. Interest-rate inertia has a stabilizing (destabilizing) effect on the economy in SADL (IADL) economies, whereas trade openness has a destabilizing (stabilizing) effect.

3.3. Model extensions

We now explore the robustness of our results for the inertial policy rule (3.7) in three important directions by considering the determinacy implications of incomplete asset markets, dominant (or local) currency pricing, and imperfect exchange rate pass-through. Since analytical results are no longer possible, we run numerical computations of local stability.

3.3.1. Incomplete asset markets

Under incomplete asset markets, the risk-sharing condition (2.21) is replaced by:

$$\mathbb{E}_t\left[\frac{\Lambda^R_{t,t+1}}{\Pi_{t,t+1}}\right] R_t = \phi_B\left(\frac{\mathcal{E}_t B^*_{F,t}}{P_{H,t}Y_t}\right)\mathbb{E}_t\left[\frac{\Lambda^R_{t,t+1}}{\Pi_{t,t+1}}\Pi^{\mathcal{E}}_{t,t+1}\right] R^*_t, \tag{3.12}$$

where $\phi_B > 0$ controls the risk premium on foreign bonds. Incomplete asset markets introduce an additional state variable $B^*_{F,t}$ into the analysis and a law of motion for foreign bond holdings given by equation (2.20). The rest of the model is unchanged.

Our results show that the international risk-sharing assumption has a negligible effect for the determinacy regions arising under the Taylor principle (SADL case) and the inverted Taylor principle (IADL case), suggesting that the terms-of-trade channel discussed in section 3.1 is unaffected by market incompleteness.[19] In contrast, the quantitative analysis suggests that policy inertia is strength-

[18] In the closed economy with $\rho = 0$, the lower bound on θ_π lies in the interval $[1, 10]$ for $\lambda \in [0.556, 0.626]$, whereas the lower bound lies in the interval $[1, 100]$ for $\lambda \in [0.552, 0.535]$ for $w_C = 0.6$.

[19] See Figure A4 in online appendix C.8.

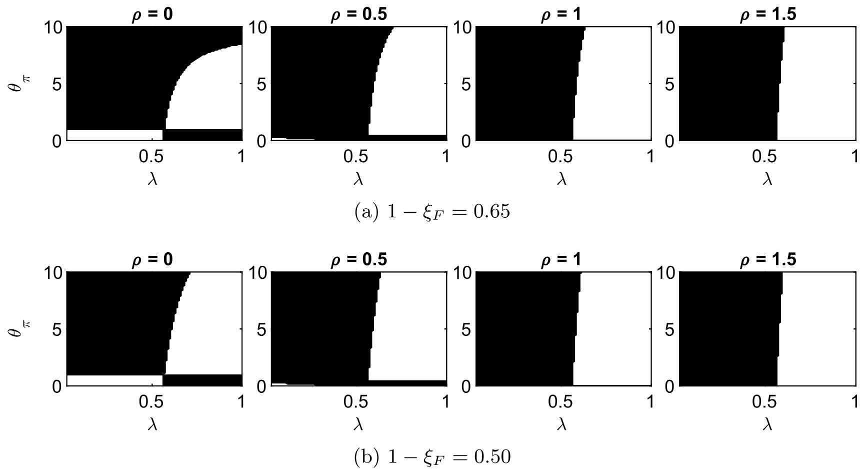


Fig. 4. Determinacy regions (white areas) under incomplete exchange rate pass-through, $1-\xi_F$. Parameter values are the same as in Fig. 1 for the open economy.

ened under incomplete markets, reducing the determinacy region that arises under the Taylor principle in open IADL economies.[20] The larger the value of ϕ_B, the more Ricardian consumption is determined by domestic conditions, and the smaller this region of determinacy becomes.

3.3.2. Imperfect exchange rate pass-through

In the baseline model, the price of foreign goods was assumed to be fully flexible implying perfect exchange rate pass-through (ERPT). We now investigate the implications of imperfect ERPT for our results. Following Monacelli (2005), we introduce a retail sector for imported goods where import prices are now subject to the Calvo price-setting friction ξ_F, which governs the degree of ERPT, $1-\xi_F$.[21]

Fig. 4 presents the determinacy plots for two values of ERPT: $1-\xi_F = \{0.65, 0.5\}$, broadly consistent with the empirical evidence.[22] Incomplete ERPT has dramatic implications for determinacy in both SADL and IADL economies, as the stability properties of the open economy converge to the closed economy. From (3.10), trade openness determines the weight of imported inflation (via adjustments in the terms of trade) on the CPI inflation rate of the SOE. When import prices are fully flexible ($\xi_F = 0$), all exchange rate fluctuations are passed on to the prices of foreign goods, which can result in CPI inflation diverging in the opposite direction from domestic-price inflation. Stickier import prices help to dampen this terms-of-trade effect: real interest rate changes now generate lower adjustments in the terms of trade, thereby reducing the importance of imported inflation for the CPI inflation rate. Consequently, imperfect ERPT exerts a stabilizing effect on SADL economies, but decreases the stability of IADL economies.

3.3.3. Dominant currency pricing

As a final sensitivity, we explore the determinacy implications when exports of the SOE are priced in a dominant or local currency. In this model version, there are two retail sectors, domestic and export, where the export retail sector sets prices in the foreign currency.[23] Fig. 5 presents the results under dominant currency pricing (DCP) setting an average export price duration of four quarters ($\xi^* = 0.75$).[24] In the absence of policy inertia ($\rho_r = 0$) the determinacy region under the Taylor principle is greatly reduced in SADL economies. To understand why, consider that the expected exchange-rate depreciation caused by a rise in the domestic interest rate will shift up the expected domestic price of exports. To counter this, exporters lower foreign-currency prices. This crowds in foreign demand, raising real marginal costs, and leads to higher domestic-price inflation. By helping to reinforce self-fulling beliefs, this reduces the upper bound on θ_π, shrinking the determinacy region. Interest-rate inertia, however, dampens the expected depreciation of the exchange rate, offsetting this effect sufficiently to restore determinacy.

In the IADL case, the determinacy region that previously appeared under the Taylor principle now vanishes under DCP for the same reason.[25] However, under DCP, policy inertia continues to exert a stabilizing effect in SADL economies and a destabilizing effect in IADL economies. Thus, we obtain very similar results to the baseline model with $\rho_r > 0$.

[20] In the absence of interest-rate inertia, the determinacy regions are nearly identical under both complete and incomplete asset markets regardless of the value of ϕ_B.

[21] Specific details of this model version are given in online appendix B.1.

[22] The degree of ERPT on import prices is estimated by Campa and Goldberg (2005) and Dees et al. (2013) to be around 50% for small advanced economies. Although the degree of ERPT has been falling over time in emerging economies (Frankel et al., 2012), in general it remains higher than in advanced economies. See, e.g., Gagnon and Ihrig (2004) and Ca' Zorzi et al. (2007).

[23] Details of this model version are given in online appendix B.2.

[24] Additional results for alternative values of ξ^* are presented in online appendix C.9.

[25] As shown in online appendix C.9, more flexible export prices can restore the determinacy region under the Taylor principle. Following a real interest rate rise, exporters in IADL economies raise foreign currency prices in response to an expected exchange-rate depreciation. By reducing export demand, this exerts downward pressure on real marginal costs and domestic-price inflation, helping induce determinacy.

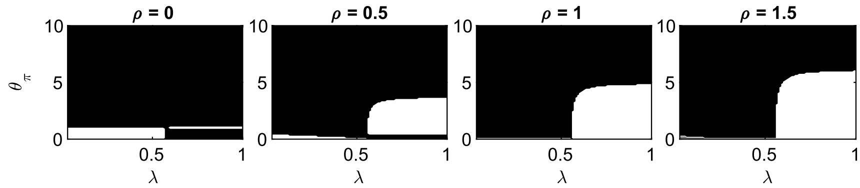


Fig. 5. Determinacy regions (white areas) under dominant currency pricing setting $\xi^* = 0.75$. The remaining parameter values are the same as in Fig. 1 for the open economy.

3.4. Indeterminacy at the zero lower bound

We now suppose that the interest rate is subject to a zero lower bound (ZLB) such that:

$$r_t + \bar{r} = \max\left\{0, \bar{r} + \rho_r r_{t-1} + \theta_\pi \pi_{t+1}\right\}. \tag{3.13}$$

The presence of a ZLB affects the determinacy properties of the model, introducing the potential for both dynamic and steady-state indeterminacy.[26] Consider the intuition for a sunspot shock induced by the ZLB. The expectation that the ZLB will bind in the future implies that, for some period, the nominal interest rate is expected to exceed the level dictated by the policy rule. The higher future interest rate will exert a deflationary effect prompting a cut in the current interest rate. If the deflation is sufficiently severe or the current monetary policy response strong enough, the interest rate can reach zero, making the ZLB episode self-fulfilling.

To test for equilibrium determinacy at time t, we choose a future horizon $t + T$ where agents expect the economy to have escaped the ZLB. Using the tests proposed by Holden (2023), we evaluate the necessary and sufficient conditions for determinacy across different horizons, T. If a sunspot-induced ZLB equilibrium is possible at t despite expectations of exiting the ZLB in the next period, then the economy always suffers from indeterminacy regardless of future beliefs. While it may be possible to rule out sunspot equilibria at t if agents expect the economy to be away from the ZLB by $t + T$, this does not rule out sunspot equilibria in general. However, in principle, T could be chosen large enough that the ZLB's future risk of binding should not plausibly affect current inflation.[27]

While fully verifying all the necessary and sufficient conditions is computationally prohibitive for large T, we can test sufficient conditions with $T = 200$, equivalent to agents expecting to have escaped the ZLB within 50 years. This exercise shows that, under IADL, uniqueness is always guaranteed when a determinate policy rule exists, except for high values of $\theta_\pi > \max\left\{\left(1 - w_C\right)^{-1}, \Gamma_1\right\}$. However, under SADL, multiplicity cannot be ruled out unless interest-rate inertia is absent. By reducing the horizon to $T = 20$, where agents expect the economy to have escaped the ZLB within 5 years, we can employ the recursive test proposed in Tsatsomeros and Li (2000) to check the full set of necessary and sufficient conditions.[28]

Fig. 6 presents the results of these tests. In open IADL economies, the blue region arising from a sufficiently large inflation response θ_π faces a risk of equilibrium multiplicity. Here, the nominal interest rate reacts negatively to a positive contemporaneous monetary policy shock, making the risk of self-fulfilling ZLB episodes inevitable due to the aggressiveness of the policy rule. The red region illustrates the parameter space where indeterminacy arises from the ZLB in open SADL economies. While less unstable than the blue region, multiple equilibria occur unless the economy is expected to be away from the ZLB the following period. As a result, the determinate policy space shrinks in open SADL economies with policy inertia, where news of a future ZLB episode can cause multiple equilibria.

This might seem at odds with the existing literature, which shows that price-level targeting and other make-up strategies can prevent sunspot equilibria. However, note that the lagged interest rate in (3.13) becomes zero at the ZLB, erasing any stored price-level information. To preserve this, either a defined price-level target is required, or the interest-rate rule must include a shadow interest rate $r_t^* = \rho_r r_{t-1}^* + \theta_\pi \pi_{t+1}$:

$$r_t + \bar{r} = \max\left\{0, r_t^* + \bar{r}\right\}. \tag{3.14}$$

Under this policy rule, determinacy is restored in the red regions shown in Fig. 6. This demonstrates that policy inertia alone is insufficient to mitigate ZLB risk.

We further analyze how policy inertia and trade openness affect determinacy under a ZLB using indicative statistics. For the interest-rate rule (3.13), we find that higher policy inertia worsens determinacy in both the closed and open economies, except for small values of ρ_r. However, with the lagged shadow rate in (3.14), policy inertia tends to improve the determinacy conditions.[29] Consider the following intuition: self-fulfilling ZLB episodes depend on the current impact of future monetary policy news shocks. Policy inertia has two competing effects. On one hand, it increases the persistence of monetary policy shocks, making the binding

[26] It is easily verified that two deterministic steady states exist in the standard model with a ZLB; one steady state exists when the interest rate is at zero and inflation is below target, and a second steady state arises under a positive interest rate and inflation on target. See Benhabib et al. (2002) and Fernández-Villaverde et al. (2015) for a detailed analysis of dynamic indeterminacy under a ZLB.

[27] A detailed discussion of the tests is provided in the online appendix. For further reading, see Holden (2023) who outlines the necessary and sufficient conditions for determinacy with a ZLB.

[28] We rely on the implementation of these tests in the dynareOBC toolkit (see https://github.com/tholden/dynareOBC) as described in Holden (2023).

[29] See online appendix C.10.1 for details.

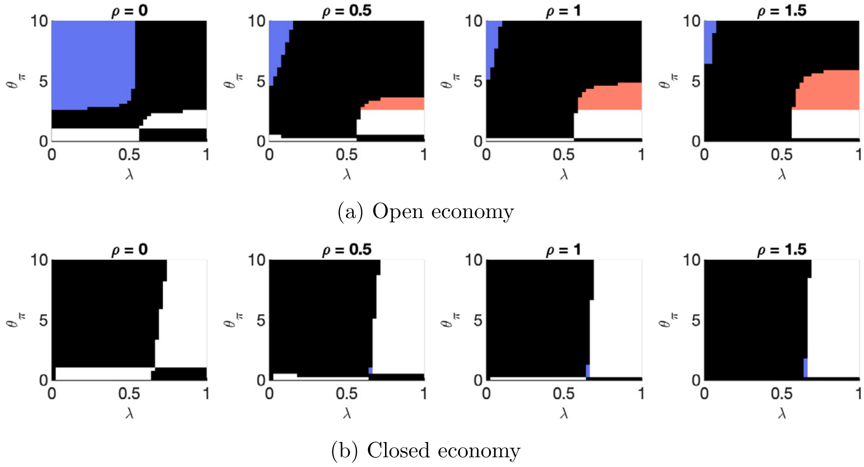


Fig. 6. Uniqueness results for the baseline LAMP model with a ZLB. Black areas represent indeterminacy in the linear model. White areas indicate there is always a unique equilibrium conditional on agents expecting to be away from the ZLB in 20 quarters. Uniqueness can only be guaranteed in the red areas when the economy escapes the ZLB in the following period. In the blue areas, self-fulfilling ZLB episodes are always possible. Parameterization is the same as in Fig. 1.

ZLB more contractionary and raising the risk of sunspots. On the other hand, inertia influences long-term interest rates, amplifying the impact on current inflation through the expectations channel. With a shadow rate rule, the inflation expectations generated by policy inertia offset the contractionary effect of future ZLB episodes.

4. Optimal monetary policy

This section considers optimal policy using a (slightly) restricted form of the model for reasons of analytical tractability. The optimal policy problem is defined in section 4.2. We confine ourselves to the case where the policymaker can commit (as for the Taylor-type rules examined above). As is standard in the literature, in order to derive analytical results, we define an approximate linear-quadratic optimal policy problem. Section 4.3 sets out the commitment solution where the policymaker is concerned about interest rate variance.

We follow Galí and Monacelli (2005), among others, and restrict the welfare analysis to the special case where $\sigma = \mu_C = \mu_C^* = 1$.

Assumption 1. We hereafter assume: (i) log utility in consumption ($\sigma = 1$); (ii) unit elasticity of substitution between home and foreign goods in both the SOE and RoW ($\mu_C = \mu_C^* = 1$); (iii) no fixed costs ($F = 0$) so without subsidies, the steady state is not equitable.

In our model there are three market distortions. In addition to market power arising from monopolistic competition and relative price dispersion arising from nominal price stickiness, the terms of trade can be influenced to the benefit of domestic consumers. Moreover, with LAMP, there is an additional behavioral distortion which creates inequality across household types.[30]

Under the restricted parameterization of Assumption 1, it follows from (2.10) that $Q_t = S_t^{\mathrm{w}_C}$ and equations (2.21) and (2.22) combine to give:

$$C_t^{\mathrm{w}_C}(C_t^R)^{1-\mathrm{w}_C} = (C_t^{R*})^{1-\mathrm{w}_C} Y_t^{\mathrm{w}_C}. \tag{4.1}$$

4.1. The social planner's equitable allocation problem

The social planner's problem for the SOE with LAMP is to choose C_t^i and N_t^i for $i = C, R$ to maximize aggregate utility $\lambda U(C_t^R, N_t^R) + (1-\lambda)U(C_t^C, N_t^C)$ subject to the resource constraint. We seek an equitable allocation that removes the LAMP distortion to give constrained consumers the benefit of risk-sharing. Then the optimal equitable allocation with $C_t = C_t^R = C_t^C$ and $N_t = N_t^R = N_t^C$ follows from optimizing the same aggregate utility function but subject to the risk-sharing and output equilibrium constraint given by (4.1) and the technology constraint $Y_t = A_t N_t/\Delta_t$. In the optimal allocation prices are flexible so the price dispersion $\Delta_t = 1$. In terms of C_t and N_t and given C_t^{R*} the relevant constraint becomes

$$C_t = (A_t N_t)^{\mathrm{w}_C}(C_t^{R*})^{1-\mathrm{w}_C}. \tag{4.2}$$

The first-order condition then is given by

[30] This distortion arises from three sources preventing constrained households from (i) owning domestic shares, (ii) owning foreign shares, and (iii) trading in international state-contingent securities.

$$U_{N,t} + U_{C,t} C_t^{R*} A_t^{\mathrm{w}_C} \mathrm{w}_C N_t^{\mathrm{w}_C - 1} = 0. \tag{4.3}$$

With our choice of preferences (2.1) (with $\sigma = 1$), $U_{N,t} = N_t^{\varphi}$ and $U_C = \frac{1}{C_t}$. Then combining (4.2) and (4.3) we arrive at $N_t = N = \mathrm{w}_C^{\frac{1}{1+\varphi}}$. Thus the socially optimal labor supply is constant. The deterministic steady state of this equilibrium is the baseline about which the first-order solution of the model and the second-order approximation of the welfare criterion are conducted, in accordance with the determinacy analysis of section 3.

How then can the decentralized equilibrium sticky-price SOE with LAMP support this optimal equitable allocation in our baseline steady state? We seek two tax instruments, a firm subsidy τ_f that eliminates the sticky-price distortion and a household subsidy τ_h that eliminates the LAMP distortion. Both are financed out of lump-sum taxation of Ricardian consumers. These payments must then satisfy

$$W(1-\tau_f) = -\frac{U_{N^i}}{U_{C^i}}; \qquad i = R, C, \tag{4.4}$$

$$C^C = W(1+\tau_h) N^C. \tag{4.5}$$

From the sticky-price decentralized equilibrium this requires tax subsidies that satisfy:

$$\mathrm{w}_C(1-\tau_f) = 1 - \frac{1}{\zeta}, \tag{4.6}$$

$$1 + \tau_h = \frac{1}{\mathrm{w}_C}. \tag{4.7}$$

Thus the next proposition directly follows.

Proposition 3 *(Subsidies for an optimal equitable allocation). Given the sticky-price LAMP equilibrium, in our baseline steady state an optimal equitable flexi-price allocation is sustained following* (4.6) *and* (4.7) *which determine tax subsidies for the firm τ_f and household τ_h. These subsidies are financed by lump-sum taxes, introduced in the budget constraint for Ricardian households* (2.3)*.*

Note that the LAMP dimension, via λ, does not appear in either (4.6) and (4.7). The optimal employment subsidy paid to the firm is influenced (negatively) by the degree of trade openness, $1 - \mathrm{w}_C$, in addition to its standard (positive) dependence on the inverse of the markup, $1 - 1/\zeta$. In contrast, the optimal wage subsidy paid to all households is positively related to the degree of trade openness. These results generalize the results of Bilbiie (2008) for the closed LAMP economy ($\mathrm{w}_C = 1$), where no household subsidy is required, and Galí and Monacelli (2005) for the open economy case without LAMP ($\lambda = 1$).

A result we have established here is that the optimal equitable hours of work (4.3) depend on the degree of trade openness: the more open is the economy, the less R and C agents work. This result arises because of the same risk-sharing condition (4.1) across Ricardian consumers in the SOE and the RoW. Our interpretation is linked to the role of the open-economy dimension in risk-sharing seen clearly here: the benefit of foreign profits and international risk-sharing, originally going only to the R-types, now gets shared between both R and C agents via the redistribution that makes the allocation equitable. The more open an economy, the wider the range of risk-sharing.

4.2. The optimal policy problem with commitment

The optimal policy problem consists of minimizing the second-order approximation to social welfare loss, given the constraints of the model economy, summarized by the intertemporal IS equation (NKIS) and the NKPC of section 3. For the remainder of the optimal policy analysis, along with the restrictions in Assumption 1, we choose the steady state of the determinacy analysis of section 3 corresponding to the optimal equitable allocation. As is standard in the literature, we rewrite these equations in terms of the output gap x_t and the natural rate of interest r_t^n. From online appendix C.1 we have:

$$x_t = \mathbb{E}_t x_{t+1} - w_C \Xi \left(r_t - \mathbb{E}_t \pi_{H,t+1} - r_t^n \right), \tag{4.8}$$

$$\pi_{H,t} = \beta \mathbb{E}_t \pi_{H,t+1} + \kappa x_t + u_t, \tag{4.9}$$

where the parameters $\Xi = \Xi(\lambda, w_C)$ and $\kappa = \kappa(\lambda, w_C)$ determine the slopes of the NKIS and NKPC curves given by:

$$\Xi = \frac{\lambda}{w_C(\lambda - (1-\lambda) w_C \varphi)}, \tag{4.10}$$

$$\kappa \equiv \frac{\Psi \Upsilon}{\Xi} = \Psi \frac{(1 + w_C \varphi)\lambda + (1 - w_C)\varphi}{\lambda}, \tag{4.11}$$

$$\Upsilon = \frac{(1 + w_C \varphi)\lambda + (1 - w_C)\varphi}{w_C(\lambda - (1-\lambda) w_C \varphi)}, \tag{4.12}$$

$$\Psi \equiv \frac{(1-\xi)(1-\beta\xi)}{\xi}. \tag{4.13}$$

In (4.9) we have added a cost-push mark-up shock process u_t, which in logs is of AR(1) form, $u_t = \rho_u u_{t-1} + \epsilon_t$, where ϵ_t is i.i.d. Note also that given the restrictions of Assumption 1 equation (3.4) becomes (4.12).[31] It follows that the threshold λ^* at which IADL occurs:

$$\lambda < \lambda^* = \frac{w_C \varphi}{1 + w_C \varphi}. \tag{4.14}$$

Recall from section 3 that the slopes of the NKIS and NKPC are affected both by λ and w_C, which operate via the composite parameters κ and Ξ, where $\partial\kappa/\partial w_C$, $\partial\kappa/\partial\lambda < 0$ and $\partial\Xi/\partial w_C$, $\partial\Xi/\partial\lambda < 0$. For any given w_C and inverse Frisch elasticity φ, when gradually increasing the degree of LAMP from nil (at $\lambda = 1$), at some point the sign of the NKIS curve becomes positive (as Ξ becomes negative).[32] The intuition is that the more open the economy or the higher the degree of LAMP, the less domestic output depends on the domestic real interest rate. The latter is because constrained consumers spend their current income *irrespective* of the interest rate. Observe that for this restricted parameterization the slope, but now not the sign, of the NKPC is affected by λ and w_C, so that the output gap exerts greater influence on domestic inflation in the SOE with LAMP. The intuition is straightforward: the more open the economy, the more domestic inflation depends on the domestic output gap via the aggregate demand effect of increased spending on imports; and the higher the degree of LAMP, the more domestic inflation depends on the domestic output gap due to a greater share of C-type households consuming all current income, thus strengthening the link between output and inflation.

In light of Proposition 3 we now choose our social welfare criterion and by implication the welfare-relevant output gap.

Assumption 2 *(Social welfare criterion).* Our social welfare criterion is a second-order approximation of the sum of the Ricardian and constrained households utility weighted by their mass in the region of the optimal equitable flexible-price allocation $\bar{Y}_t$ with a welfare-relevant output gap $x_t = \frac{Y_t - \bar{Y}_t}{\bar{Y}_t}$ supported by the subsidy scheme of Proposition 3.

The form of this welfare criterion is given by the following proposition:

Proposition 4 *(Social welfare loss with a flexi-price equilibrium). For the non-linear model of section 2 and welfare-relevant output gap x_t, given Proposition 3, the micro-founded social welfare loss criterion for the LAMP SOE is approximated as:*

$$\Omega_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}(\pi_{H,t}^2 + \varpi x_t^2) - \Lambda_x x_t \right], \tag{4.15}$$

where $\varpi = \varpi(\lambda) = \frac{\Psi(1+\varphi)}{\zeta\lambda}$, $\Psi \equiv \frac{(1-\beta\xi)(1-\xi)}{\xi}$, $\Lambda_x = \frac{(1-w_C)(1-\lambda)\varphi}{\lambda}$.

Proof. See online appendix D.1. □

The linear term in x_t captures the fact that any marginal increase in the output gap relative to its steady state value has a positive first-order effect on social welfare, since output is below its efficient level at that steady state.

We now turn to the policy implications of our results with respect to the central bank operating under commitment, before deriving the corresponding optimal targeting rule.

4.3. Optimal policy with interest rate inertia

Nominal interest rate inertia can be introduced by penalizing its variance. Following Woodford (2003a), this penalty can be formalized in terms of an approximation to the ZLB on the nominal interest rate. Writing (4.15) as

$$\Omega_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t L_t, \tag{4.16}$$

first define discounted average values of R_t and R_t^2 as, respectively:

$$m_1 = \mathbb{E}_0 \left[(1-\beta) \sum_{t=0}^{\infty} \beta^t R_t \right], \tag{4.17}$$

$$m_2 = \mathbb{E}_0 \left[(1-\beta) \sum_{t=0}^{\infty} \beta^t R_t^2 \right]. \tag{4.18}$$

Then the standard deviation of R_t also in discounted average form is given by:

[31] In particular, setting $\sigma = 1$ and $F = 0$ implies $\zeta = \infty$.

[32] See Figure A13 in the online appendix, illustrating the role of trade openness and LAMP for the NKIS and the NKPC, setting $\sigma = \mu_C = 1$, $\beta = 0.99$, $\varphi = 2$, and $\Psi = 0.086$.

$$\mathrm{sd} = \sqrt{\mathbb{E}_0\left[(1-\beta)\sum_{t=0}^{\infty}\beta^t(R_t - m_1)^2\right]} = \sqrt{\mathbb{E}_0\left[(1-\beta)\sum_{t=0}^{\infty}\beta^t(R_t^2 - m_1^2)\right]} = \sqrt{m_2 - m_1^2}. \tag{4.19}$$

Woodford (2003a, chap. 6) proposes an approximate effect of the ZLB on R_t. The mean of R_t, m_1, must be positive and be at least k standard deviations above the lower bound, where k is large enough to ensure its violation is infrequent. This is achieved by requiring $m_1 \geq k,\ \mathrm{sd} = k\sqrt{m_2 - m_1^2}$. Squaring both sides of the latter, we arrive at the constraints:

$$m_1 \geq 0, \tag{4.20}$$

$$m_2 \leq K m_1^2, \tag{4.21}$$

where $K \equiv 1 + k^{-2}$.

The optimization problem is now to minimize Ω_0 given by (4.16) subject to (4.20) and (4.21). Using the Kuhn-Tucker theorem, Woodford (2003a, chap. 6) shows that the optimal policy modifies the welfare criterion by replacing L_t in (4.16) with the modified form:

$$L_t^{mod} = L_t + w_r(r_t - r^*)^2, \tag{4.22}$$

where we recall $r_t \equiv \log\left(R_t/R\right)$ with R the nominal interest rate in a zero-net inflation deterministic state. In (4.22), $w_r > 0$ if (4.21) binds in which case $r^* = \log(R^*/R) > 0$ associated with a target interest $R^* > R$ and a non-zero positive target net inflation rate.

Levine et al. (2008) and Deak et al. (2024) use this approach to study optimal monetary policy with a ZLB in an estimated closed economy model with interest-rate rules in the form of optimized Taylor-type rules. In the rest of this section we follow Giannoni and Woodford (2003) by constraining optimal policy to target a zero net inflation target so $r^* = 0$ in (4.22) and the tractable linearization of the NK model up to now is retained.[33]

Proceeding with the optimization problem with the modified loss criterion, the Lagrangian function for the optimization problem under commitment is now given by:

$$\begin{aligned}\mathcal{L}^C\left(\{x_t,\pi_t\}_{t=0}^{\infty};\{\mu_t\}_{t=0}^{\infty};\{\nu_t\}_{t=0}^{\infty}\right) \equiv \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[\tfrac{1}{2}\left(\pi_{H,t}^2 + \varpi x_t^2 + w_r r_t^2\right) - \Lambda_x x_t \\ + \mu_t\left(\pi_{H,t} - \kappa x_t - \beta\pi_{H,t+1}\right) + \nu_t\left(x_t - x_{t+1} + \Xi w_C\left(r_t - \pi_{H,t+1} - r_t^n\right)\right)\Big] + t.i.p.,\end{aligned} \tag{4.23}$$

for the welfare-relevant output gap x_t, where $\{\mu_t\}_{t=0}^{\infty}$ and $\{\nu_t\}_{t=0}^{\infty}$ are sequences of Lagrange multipliers for $t = 0, 1, 2, \ldots$, and the law of iterated expectations has been applied to eliminate the conditional expectation that appeared in each constraint.

Differentiating the Lagrangian function with respect to the decision variables, we obtain the respective first-order conditions:

$$\frac{\partial\mathcal{L}^C\left(\{x_t,\pi_{H,t}\}_{t=0}^{\infty};\{\mu_t\}_{t=0}^{\infty};\{\nu_t\}_{t=0}^{\infty}\right)}{\partial x_t} = \varpi x_t - \kappa\mu_t - \Lambda_x + \nu_t - \frac{1}{\beta}\nu_{t-1} = 0, \tag{4.24}$$

$$\frac{\partial\mathcal{L}^C\left(\{x_t,\pi_{H,t}\}_{t=0}^{\infty};\{\mu_t\}_{t=0}^{\infty};\{\nu_t\}_{t=0}^{\infty}\right)}{\partial \pi_{H,t}} = \pi_{H,t} + \mu_t - \mu_{t-1} - \frac{\Xi w_C}{\beta}\nu_{t-1} = 0, \tag{4.25}$$

$$\frac{\partial\mathcal{L}^C\left(\{x_t,\pi_{H,t}\}_{t=0}^{\infty};\{\mu_t\}_{t=0}^{\infty};\{\nu_t\}_{t=0}^{\infty}\right)}{\partial r_t} = w_r r_t + \Xi w_C \nu_t = 0, \tag{4.26}$$

which must hold for $t = 0, 1, 2, \ldots$, where $\mu_{-1} = 0$ and $\nu_{-1} = 0$.[34] Equations (4.24)–(4.26) plus the NKIS and NKPC equations yield an equilibrium in the multipliers μ_t and ν_t, and endogenous variables r_t, $\pi_{H,t}$, and x_t.

Proposition 5. *Define*

$$d_t = \pi_{H,t} + \frac{1}{\kappa}(1-L)\left(\varpi x_t - \Lambda_x\right) = \pi_{H,t} + \frac{1}{\kappa}\varpi(x_t - x_{t-1}) \tag{4.27}$$

as a departure from the 'leaning against the wind' optimal condition (or 'wedge'), where $\pi_{H,t} + \frac{1}{\kappa}\varpi(x_t - x_{t-1}) = 0$ in the case of no penalizing of the interest rate variance when $w_r = 0$. Then the optimal policy with commitment is given by:

$$r_t = \left(1 + \frac{1}{\beta} + \frac{\kappa\Xi w_C}{\beta}\right)r_{t-1} - \frac{1}{\beta}r_{t-2} + \frac{\kappa\Xi w_C}{w_r}d_t. \tag{4.28}$$

[33] We can also motivate the loss from interest-rate volatility as stemming from non-modeled features other than the ZLB that provide incentive to smooth interest rates. See, e.g., Rudebusch and Svensson (1999), Woodford (2003b), Givens (2012).

[34] The last equality results because the inflation first-order condition corresponding to period -1 is not an effective constraint to the monetary authority when choosing its optimal policy plan in period 0.

Table 1
Coefficients for the inertial Taylor-type rule approximation to optimal policy. Parameterization is the same as in Fig. 7. For $\lambda = 0.9, 0.7 > \lambda^*$, whereas for $\lambda = 0.5, 0.3 < \lambda^*$.

	$\lambda = 0.9$			$\lambda = 0.7$			$\lambda = 0.5$			$\lambda = 0.3$		
$w_C =$	1	0.8	0.6	1	0.8	0.6	1	0.8	0.6	1	0.8	0.6
$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{H,t} + \theta_x (x_t - x_{t-1})$												
ρ_r	1.33	1.32	1.31	2.82	1.87	1.60	0.74	0.51	-0.65	0.93	0.88	0.77
θ_π	0.33	0.32	0.31	1.80	0.87	0.59	-0.26	-0.49	-1.63	-0.07	-0.12	-0.23
θ_x	0.05	0.05	0.05	0.37	0.17	0.11	-0.07	-0.12	-0.37	-0.03	-0.04	-0.07

Optimal policy can be implemented by the following approximate first-order dynamic inertial Taylor-type rule that responds only to the wedge[35]*:*

$$r_t = \left(1 + \frac{\kappa \Xi w_C}{\beta}\right) r_{t-1} + \frac{\kappa \Xi w_C}{w_r} d_t. \tag{4.29}$$

Proof. See online appendix D.2. □

Rule (4.28) is the open-economy version of the *robustly optimal* and *timeless* implicit instrument rule derived in Giannoni and Woodford (2003) for closed economies. It is robust in the sense that it is independent of the nature of the exogenous disturbances. It is timeless in the sense that is the same for all periods $t \geq 0$, as opposed to the optimal time-variant, once-and-for-all commitment to (4.28) from $t \geq 2$ with $\mu_{-1} = \nu_{-1} = 0$ implying a different setting in periods $t = 0$ and $t = 1$. Under the restricted parameterization of the model, the optimal rule (4.28) takes the form of an inertial Taylor-type interest-rate rule (4.29) targeting domestic-price inflation and output gap growth.[36]

It follows that both the optimal rule (4.28) and the inertial Taylor-type rule (4.29) is super-inertial under SADL (since $\Xi > 0$), whereas the degree of inertia is significantly lower and not super-inertial under IADL. The degree of (super) inertia is independent of the weight w_r on the variance of the nominal interest rate r_t, and depends on the discount factor β, the degree of openness $1 - w_C$, and the intrinsic dynamics driving the NKPC and NKIS equations via the parameters κ and Ξ, respectively. It is the latter that determines the transmission of changes to r_t on outcomes, whilst the optimal choice of such changes only depends on w_r through the response to the wedge d_t.

How does the inertial Taylor-type rule given by (4.29) compare to the fully optimal policy rule (4.28) with a variance penalty? Because of the second-order interest rate lag, the optimal policy rule (4.28) can generate oscillatory behavior under IADL following a mark-up shock. This oscillatory behavior becomes more pronounced as the rule deviates further from $d_t = 0$.[37] This is particularly true for either high levels of LAMP or high values of w_r, where the interest rate is determined less by inflation and output and more by history dependence. In these cases, the Taylor-type rule (4.29) smooths out the oscillations, while still delivering very similar outcomes for output and inflation as the optimal policy rule.[38]

Two optimal policy trade-offs arise under (4.29). In the first trade-off, determined by w_r, greater interest-rate stability comes at the cost of reduced stabilization of the wedge d_t. The second is the standard trade-off, where lower inflation variability comes at the expense of greater output gap volatility, the outcome of which is determined by the standard 'leaning against the wind' policy, and the relative weight of these variables given in equation (4.27). Below, we consider how these monetary policy trade-offs are affected by the degree of trade openness and LAMP.

To evaluate the first trade-off, we turn to simulations in response to a positive mark-up shock with persistence $\rho_u = 0.7$ under the approximation rule (4.29). Using the restricted parameterization of Assumption 1, Fig. 7 displays the effect of higher interest-rate stabilization brought about by increasing the penalty parameter w_r for the cases of $\lambda > \lambda^*$ (SADL) and $\lambda < \lambda^*$ (IADL), where λ^* is the IADL threshold given by (4.14), which equals $\lambda^* = 0.546$ with $w_C = 0.6$ and $\varphi = 2$.[39] In both the SADL and IADL cases, an increase in the penalty dampens the monetary policy response. While domestic-price (and CPI) inflation is accommodated to limit the fall in output,[40] the wedge d_t moves further from its optimal value (i.e., $d_t = 0$) as the penalty parameter increases, resulting in a significant difference between the SADL and IADL cases.

Table 1 reports the optimal policy coefficients ρ_r, θ_π, θ_x of the rule (4.29) for different values of λ and w_C. Under SADL an increase in the share of hand-to-mouth behavior calls for higher values for θ_π and θ_x and a stronger degree of super-inertia. Under

35 Another approximation to the optimal rule, only available under SADL, which can be generalized to higher-order responses to the wedge, is shown in online appendix D.2.

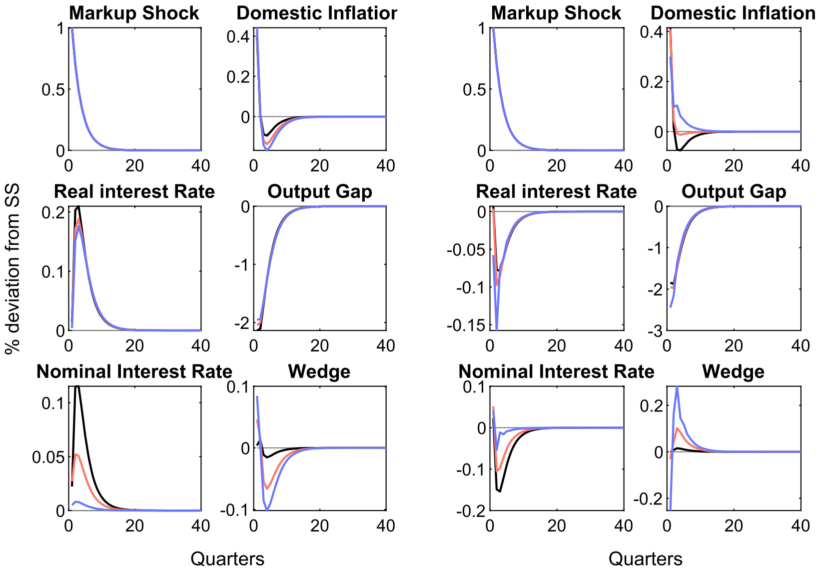
36 Online appendix D.3 shows how the implicit instrument rule (4.29) can be implemented in the form shown in the determinacy analysis of section 3. With uncertainty, the forward-looking form of the optimal rule would also include a linear combination of the shocks. These would not affect the determinacy results.

37 The impulse response functions for each rule are given by Figure A9 in online appendix D.4.

38 This is demonstrated by Figures A10 and A.11 in online appendix D.4 that depict the simulations under the optimal policy rule (4.28) for variations in both the variance penalty and trade openness.

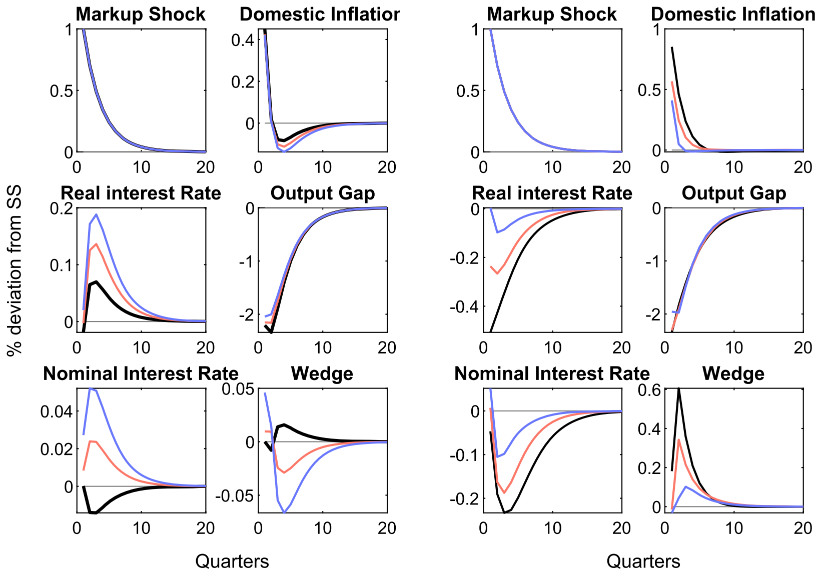
39 We use a value of $w_r = 1$ for our central case, consistent with the optimized weight computed by Deak et al. (2024).

40 Since no efficiency shocks in the model are used in this section, the natural rate of output remains at its steady state ($x_t = y_t - y_t^n = y_t$). Therefore, the output gap equals output.



(a) SADL ($\lambda = 0.7 > \lambda^*$) (b) IADL ($\lambda = 0.5 < \lambda^*$)

Fig. 7. Optimal policy (approximation rule) and variance penalty for $w_r = 0.1$ (black line), $w_r = 1$ (red line) and $w_r = 10$ (blue line). Parameter values are $\lambda = 0.5, 0.7$, $\Psi = 0.086$, $\varphi = 2$, $\beta = 0.99$, $\sigma = \mu_C = 1$, $w_C = 0.6$.



(a) SADL ($\lambda = 0.7 > \lambda^*$) (b) IADL ($\lambda = 0.5 < \lambda^*$)

Fig. 8. Optimal policy (approximation rule) and openness for $w_C = 1$ (black line), $w_C = 0.8$ (red line) and $w_C = 0.6$ (blue line). Parameter values are $\lambda = 0.5, 0.7$, $\Psi = 0.086$, $\varphi = 2$, $\beta = 0.99$, and $\sigma = \mu_C = w_r = 1$.

IADL, super-inertial policy is not optimal, and the coefficients for θ_π and θ_x turn negative. While the parameter governing policy inertia in (4.28) or (4.29) is also increasing with the degree of LAMP in the IADL case, for values of λ below but close to λ^*, the degree of policy inertia can become negative and so in this region an increase in the share of hand-to-mouth behavior can lower the degree of history dependence.

Fig. 8 examines the effect of trade openness on optimal policy for the SADL and IADL cases, again using the approximation rule (4.29) setting $w_r = 1$.[41] In open economies, the exchange rate provides an additional channel for the transmission of monetary policy.

[41] Under Assumption 1, the IADL threshold λ^* given by (4.14) equals $\lambda^*(w_C) = 0.546, 0.615, 0.667$ for $w_C = 0.6, 0.8, 1.0$. We choose $\lambda = 0.7$ (SADL) and $\lambda = 0.5$ (IADL) to represent the two cases.

For the SADL case, the exchange rate appreciation further reinforces the monetary contraction, reducing the optimal interest rate response to higher inflation. Consequently, both the optimal rule (and its approximation) exhibit a smaller response to inflation and output, and a lower degree of super-inertia, the more open the economy becomes. At the same time, trade openness weakens the link between output and the domestic real interest rate, which eases the trade-off in open economies under SADL and permits the larger monetary policy response seen in Fig. 8. The opposite is true under IADL. Since the real interest-rate falls in this case, the optimal policy generates a depreciation of the exchange rate, which requires a larger response to inflation and output, as the degree of openness increases. At the same time, increasing trade openness gives real interest rate adjustments a larger impact on output in the IADL case. This magnifies the impact of policy adjustments and leads to the smaller policy response in open economies shown in Fig. 8.

Finally, regarding the trade-off between inflation and output stabilization, we find that this trade-off is unaffected by the IADL threshold. In both SADL and IADL economies, a higher degree of trade openness requires greater stabilization of domestic inflation relative to the output gap, whereas a higher degree of LAMP requires the opposite. The intuition is as follows. The more open is the economy, the less the output gap depends on domestic inflation, which eases the trade-off allowing the central bank to focus more on inflation. In contrast, the higher the degree of LAMP, the larger the movements in demand, which worsens the trade-off with the opposite effect. This reinforces the earlier work of Bilbiie (2008) and Iyer (2016) who consider optimal monetary policy but in the absence of inertia.[42]

5. Concluding remarks

This paper examines the role of limited asset market participation and trade openness in the design of monetary policy. These features are empirically-relevant and are shown to have important considerations for policy.

Our main findings challenge the conventional wisdom that policymakers should engage in interest-rate smoothing in two important ways. First, it is well established that policy inertia helps increase the likelihood of determinacy in the absence of LAMP. In contrast, we have shown for IADL economies that determinacy is actually undermined if the central bank follows a policy rule with excessive interest-rate inertia. Therefore, the commonly-advocated use of super-inertial feedback rules, including price-level (Wicksellian) rules, as potential remedies for indeterminacy, are strongly ill-advised under LAMP.

Second, in the absence of hand-to-mouth households, optimal monetary policy is robust and timeless with super-inertia, the latter arising from the costs of interest-rate volatility. We have shown that super-inertial policy is not optimal in IADL economies and, for empirically-plausible values of LAMP, a negative weight for the interest-rate smoothing coefficient can be optimal. It is important to stress the key role trade openness plays in our analysis. It exerts a stabilizing effect in IADL economies, reducing the possibility of self-fulfilling beliefs. Moreover, as emphasized in the optimal monetary policy analysis, the inertial coefficient of the optimal targeting rule crucially depends on the degree of trade openness.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jedc.2025.105047.

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[42] Bilbiie (2008) finds the optimal response to inflation is decreasing in the degree of LAMP for a closed economy. Iyer (2016) shows that the output gap should be stabilized relatively more in a SOE with higher LAMP, in order to smooth the disposable income of hand-to-mouth agents.

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