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The Big Mac index: An exact multilateral clarification

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ABSTRACT

The Economist's Big Mac index measures the external purchasing power of a currency relative to a common numéraire currency. The Big Mac index is based on the economic theory of absolute purchasing power parity. There are two versions of the index, namely, the Raw Big Mac index and the GDP-adjusted Big Mac index. The bilateral valuations for both versions are conditional on the common numéraire currency. For example, the common numéraire currency is always at fair value, whereas the valuations of the other currencies are measured relative to the common numéraire currency. This paper extends previous research by providing a theoretical framework to calculate exact multilateral valuations for a system of currencies. The methodological extension sheds light on a number of linkages between the bilateral valuations and the multilateral valuations for both versions of the Big Mac index.

1. Introduction

The Economist magazine's Big Mac index measures the external purchasing power of a system of currencies relative to a common numéraire currency based on the economic theory of absolute purchasing power parity (The Economist, 2025). There are two versions of the index, namely, the Raw Big Mac index (invented in 1986) and the GDP-adjusted Big Mac index (invented in 2011). The average-basket approach of O'Brien & Ruiz de Vargas (2017) created an approximation of the multilateral (overall) valuation of the common numéraire currency by using the inverse of the arithmetic average of the bilateral valuations of the other currencies relative to the common numéraire currency for the GDP-adjusted Big Mac index. The motivation of this paper is to improve on this approximation by providing a theoretical framework to calculate exact multilateral valuations for all currencies for both versions of the Big Mac index.

The economic theory of absolute purchasing power parity (PPP) measures the external purchasing power of currencies relative to either a common numéraire currency for bilateral measurements or a multicurrency numéraire currency for multilateral measurements. There are many variants of PPP, such as the law of one price, absolute and relative purchasing power parity, together with indices for measuring purchasing power parity (Rogoff, 1996). PPP is not expected to hold in the short run, but most international economists expect some variant of purchasing power parity to provide an anchor in the long run (Dornbusch & Krugman, 1976; Rogoff, 1996). The valuations used to test PPP are also known as real exchange rates, which are exchange rates adjusted for bilateral price ratios (Taylor & Sarno, 1998). The usual hypothesis of PPP is that real exchange rates are stationary and mean-reverting (Xie et al., 2021; Vo & Vo, 2023). The PPP puzzle is associated with the half-lives of real exchanges being inconsistently slow, with observed values between 3–5 years (Rogoff, 1996). Attempts to explain and resolve the PPP puzzle use unit root tests, co-integration methods, panel tests, non-linear models, models with structural breaks, and others (see Xie et al., 2021; Vo & Vo, 2023). The interested reader is referred to MacDonald (2007) for a thorough PPP review and Vo and Vo (2023) for an up-to-date PPP review.

The Big Mac indexes are based on two variants of absolute PPP, namely, the law of one price for the Raw Big Mac index and the

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Harrod-Balassa-Samuelson model for the GDP-adjusted Big Mac index. The law of one price compares the ratio of two local baskets of goods and services to the associated bilateral exchange rate, where absolute PPP holds when the ratio equals the associated bilateral exchange rate (Taylor & Taylor, 2004). The Harrod-Balassa-Samuelson model, named after Harrod (1933), Balassa (1964), and Samuelson (1964), accounts for the expectation that non-tradeable goods and services tend to be cheaper in lower GDP per capita countries compared to higher GDP per capita countries, which is ultimately caused by productivity growth differentials in tradable goods (Taylor & Taylor, 2004). Click (1996) found that deviations from PPP were explained by the Balassa-Samuelson effect. Valuations are biased if the Harrod-Balassa-Samuelson effect is not taken into account (O'Brien & Ruiz de Vargas, 2017). For example, that the overvaluation implied by the Raw Big Mac index for the Chinese renminbi relative to the US dollar was driven by China having a significantly lower GDP per capita compared to the United States (Yang, 2004). In addition, Clements et al. (2012) concluded that the Raw Big Mac index was biased and that the mispricing in previous valuations should be incorporated in current valuations.

The Raw Big Mac index and the GDP-adjusted Big Mac index both use locally produced McDonald's Big Mac hamburgers as the local baskets of goods and services. One advantage of using Big Mac hamburgers is that they are produced in different countries using the same recipe (Annaert & De Ceuster, 1997). The Big Mac index assumes that the local hamburgers are identical for different currencies, which is palatable as a consistent basket of goods and services (Ong, 1997; Ong, 2003). However, Haidar (2011) argued that PPP tests would be biased if the local baskets of goods and services were not identical, and provided ten reasons why locally produced hamburgers are not identical baskets of goods and services, such as demand variability, product comparability, transportation costs, trade restrictions, and productivity differences. Some of these underlying reasons have resulted in recommendations

Table 1
Table of definitions.

Terms	Definitions
General	
N	Number of currencies
i	The i th currency $i = 1, \dots, N$
j	The j th currency $j = 1, \dots, N$
η	A common single-currency numéraire, where η is one of the N currencies
\mathcal{M}	An equally-weighted multicurrency numéraire of N currencies
Local hamburger prices	
$H_{i/i}$	Local hamburger price of the i th currency in terms of the i th currency
$H_{i/\eta}$	Local hamburger price of the i th currency in terms of the common numéraire (η th) currency
$H_{\eta/\eta}$	Local hamburger price of the η th currency in terms of the common numéraire (η th) currency
$H_{i,\mathcal{M}}$	Local hamburger price of the i th currency in terms of the multicurrency numéraire (\mathcal{M})
$H_{\eta,\mathcal{M}}$	Local hamburger price of the η th currency in terms of the multicurrency numéraire (\mathcal{M})
$H_{\mathcal{M},\mathcal{M}}$	Geometric average of the N local hamburger prices in terms of the multicurrency numéraire (\mathcal{M})
Local GDP per capita	
$G_{i/i}$	Local GDP per capita for the i th currency in terms of the i th currency
$G_{i/\eta}$	Local GDP per capita for the i th currency in terms of the common numéraire (η th) currency
$G_{i,\mathcal{M}}$	Local GDP per capita for the i th currency in terms of the multicurrency numéraire (\mathcal{M})
Exchange Rates	
$I_{i/\eta}$	The i th/ η th bilateral implied rate
$I_{i,\mathcal{M}}$	Multilateral implied rate for the i th currency
$S_{i/j}$	The i th/ j th bilateral exchange rate
$S_{i/\eta}$	The i th/ η th bilateral exchange rate
$S_{i,\mathcal{M}}$	Multilateral exchange rate for the i th currency
$S_{\eta,\mathcal{M}}$	Multilateral exchange rate for the common numéraire (η th) currency
Raw Valuations (R superscript)	
$V_{i/\eta}^R$	Bilateral valuation of the i th currency relative to the common numéraire (η th) currency
$V_{i,\mathcal{M}}^R$	Multilateral valuation of the i th currency relative to the multicurrency numéraire (\mathcal{M})
$V_{\eta,\mathcal{M}}^R$	Multilateral valuation of the common numéraire (η th) currency relative to the multicurrency numéraire (\mathcal{M})
GDP-adjusted Valuations (A superscript)	
$V_{i/\eta}^A$	Bilateral valuation of the i th currency relative to the common numéraire (η th) currency
$F_{i/\eta}^A$	Bilateral adjustment factor of the i th currency relative to the common numéraire (η th) currency
$V_{i,\mathcal{M}}^A$	Multilateral valuation of the i th currency relative to the multicurrency numéraire (\mathcal{M})
$F_{i,\mathcal{M}}^A$	Multilateral adjustment factor of the i th currency relative to the multicurrency numéraire (\mathcal{M})
$V_{\eta,\mathcal{M}}^A$	Multilateral valuation of the common numéraire (η th) currency relative to the multicurrency numéraire (\mathcal{M})
$F_{\eta,\mathcal{M}}^A$	Multilateral adjustment factor of the η th currency relative to the multicurrency numéraire (\mathcal{M})
Cross-sectional Regressions	
$\hat{H}_{i/\eta}$	Fitted hamburger price of the i th currency in terms of the common numéraire (η th) currency
$\hat{H}_{\eta/\eta}$	Fitted hamburger price of the η th currency in terms of the common numéraire (η th) currency
$\hat{H}_{i,\mathcal{M}}$	Fitted hamburger price of the i th currency in terms of the multicurrency numéraire (\mathcal{M})
$\hat{H}_{\eta,\mathcal{M}}$	Fitted hamburger price of the η th currency in terms of the multicurrency numéraire (\mathcal{M})
$\hat{H}_{\mathcal{M},\mathcal{M}}$	Geometric average of the N fitted hamburger prices in terms of the multicurrency numéraire (\mathcal{M})

Notes: Table 1 reports a list of definitions for the main terms used throughout this paper.

for caution when using the Big Mac index (Taylor & Taylor, 2004; Haidar, 2011).

In addition, pricing-to-market is another reason for caution when using the Big Mac index. Pricing-to-market was developed from evidence that foreign firms did not adjust export prices into the United States subsequent to large changes in the US dollar (Dornbusch, 1987; Krugman, 1987). In general, a firm would be practicing pricing-to-market if it sold products for different prices in different markets (Krugman & Obstfeld, 2009). For example, there is nearly full pass through of increases in minimum wages to higher local Big Mac hamburger prices at McDonald's restaurants in the United States (Ashenfelter & Jurajda, 2022). Valuation measurements are based on local hamburger prices, so any bias in local hamburger prices would be expected to bias the valuations. In this situation, misvaluations may be due to other factors, rather than currencies.

The Raw Big Mac index is based on the law of one price and compares the ratio of two local hamburger prices to the associated bilateral exchange rate. In contrast, the GDP-adjusted Big Mac index extends the Raw Big Mac index by accounting for the Harrod-Balassa-Samuelson effect. O'Brien & Ruiz de Vargas (2017) clarified the methodology of the GDP-adjusted Big Mac index and used an approximation of the multilateral valuation of the common numéraire currency: the inverse of the arithmetic average of the bilateral valuations of the other currencies relative to the common numéraire currency. Subsequently, O'Brien & Ruiz de Vargas (2019) used the approximation to estimate the differences in multilateral valuation estimates. Furthermore, Clements & Si (2017) provided an alternative approximation of the multilateral valuation of the common numéraire currency by modelling in natural logarithms.

Both the Raw Big Mac index and the GDP-adjusted Big Mac index create bilateral valuations that are conditional on a chosen common numéraire currency. The common numéraire currency is always fixed at fair value, whereas other currencies are measured relative to the common numéraire currency. An alternative common numéraire currency can be used, but the new bilateral valuations are conditional on the alternative common numéraire currency: and the conditional bilateral valuation problem recurs.

This paper contributes to the literature by improving on the approximation of O'Brien & Ruiz de Vargas (2017). This is achieved by providing a theoretical framework that utilises an equally-weighted multicurrency numéraire to calculate exact multilateral valuations for a system of currencies for both versions of the Big Mac index. In addition, this paper sheds light on a number of linkages between the bilateral valuations and the multilateral valuations for both versions of the Big Mac index. It is shown that the geometric average of the multilateral valuations for a system of currencies is fixed at parity (fair value), where currencies are at fair value, overvalued, or undervalued, relative to a whole system of currencies. The multilateral valuations are conditional on the whole system of currencies, rather than a single currency. Consequently, the multilateral valuations provide a clearer picture of the whole system of currencies, such as post-validation analysis.

This paper is organized as follows: Section 2 presents the methodology; Section 3 reports some results with a discussion; and Section 4 concludes.

2. Material and methods

2.1. Definition of terms

In an effort to avoid repetition and provide clarity, Table 1 reports a list of definitions for the main terms used throughout this paper.

2.2. Multilateral exchange rates

Multilateral exchange rates represent currencies priced in terms of a basket of currencies. The basket of currencies, also known as a multicurrency numéraire, is a weighted basket, where the weights are positive and sum to one. In contrast, bilateral exchange rates represent currencies priced in terms of a single-currency numéraire. For example, $S_{EUR/USD}$ is the EUR/USD bilateral exchange rate, which represents the Eurozone euro (EUR) priced in terms of the US dollar (USD).

In log terms, Mahieu and Schotman (1994) showed that a system of $N-1$ bilateral exchange rates priced in terms of a common numéraire (η th) currency could be decomposed into a system of N multilateral exchange rates:

$$s_{i/\eta} = s_{i/\mathcal{N}} - s_{\eta/\mathcal{N}} \quad (1)$$

where $i \neq \eta$; $s_{i/\eta} = \ln(S_{i/\eta})$ is the natural logarithm of the i th/ η th bilateral exchange rate; $s_{i/\mathcal{N}} = \ln(S_{i/\mathcal{N}})$ is the natural logarithm of the multilateral exchange rate for the i th currency; and $s_{\eta/\mathcal{N}} = \ln(S_{\eta/\mathcal{N}})$ is the natural logarithm of the multilateral exchange rate for the common numéraire (η th) currency. The common numéraire (η th) currency can be any one of the N currencies. Note that the η th/ η th bilateral exchange rate for the common numéraire (η th) currency is excluded from the system of $N-1$ bilateral exchange rates in Eq. (1).

The decomposition in Eq. (1) can be written in nominal terms:

$$S_{i/\eta} = \frac{S_{i/\mathcal{N}}}{S_{\eta/\mathcal{N}}} \quad (2)$$

The system of $N-1$ bilateral exchange rates in Eq. (2) are conditional on the chosen common numéraire (η th) currency. For example, a bilateral exchange rate of a currency priced in terms of the common numéraire (η th) currency can be derived from the multilateral

exchange rate of the same currency divided by the multilateral exchange rate of the common numéraire (η th) currency.

Assuming that the first currency is the common numéraire currency ($\eta = 1$), the decomposition of the system of $N - 1$ bilateral exchange rates in log terms in Eq. (1) can be written in matrix notation:

$$\begin{bmatrix} s_{2/1} \\ s_{3/1} \\ \vdots \\ s_{N/1} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ -1 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots \\ -1 & 0 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} s_{1/\mathcal{M}} \\ s_{2/\mathcal{M}} \\ s_{3/\mathcal{M}} \\ \vdots \\ s_{N/\mathcal{M}} \end{bmatrix} \tag{3}$$

Note that the bilateral exchange rate $s_{1/1}$ for the common numéraire currency ($\eta = 1$) in terms of the common numéraire currency is excluded in the system of $N - 1$ bilateral exchange rates in Eq. (3).

There are $N - 1$ equations (bilateral exchange rates) in Eq. (3), which is one less than the number of N unknown variables (multilateral exchange rates). In this situation, the system of equations is underdetermined with no solution or an infinite number of solutions. Kunkler and MacDonald (2015) resolved this issue by adding an extra equation (equilibrium, or no-arbitrage, condition):

$$\sum_{i=1}^N s_{i/\mathcal{M}} = 0 \tag{4}$$

where $s_{i/\mathcal{M}} = \ln(S_{i/\mathcal{M}})$ is the natural logarithm of the multilateral exchange rate for the i th currency. The no-arbitrage condition in log terms shows that the arithmetic average of the system of N multilateral exchange rates is centred at zero.

The no-arbitrage condition in Eq. (4) can be written in nominal terms:

$$\prod_{i=1}^N S_{i/\mathcal{M}} = 1 \tag{5}$$

The no-arbitrage condition in nominal terms shows that the geometric average of the system of N multilateral exchange rates is centred at one, which highlights the relative nature of the exchange rate market. For example, multilateral exchange rates move relative to the whole system of N multilateral exchange rates, but the system remains centred and fixed at one.

In log terms, the no-arbitrage condition in Eq. (4) can be included into the decomposition of the system of $N - 1$ bilateral exchange rates in Eq. (1). The combined system of N equations can be written in matrix notation:

$$\begin{bmatrix} s_{2/1} \\ s_{3/1} \\ \vdots \\ s_{N/1} \\ 0 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ -1 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots \\ -1 & 0 & \cdots & 0 & 1 \\ 1 & 1 & \cdots & 1 & 1 \end{bmatrix} \begin{bmatrix} s_{1/\mathcal{M}} \\ s_{2/\mathcal{M}} \\ s_{3/\mathcal{M}} \\ \vdots \\ s_{N/\mathcal{M}} \end{bmatrix} \tag{6}$$

There are N equations ($N - 1$ bilateral exchange rates and one no-arbitrage condition) in Eq. (6), which is the same number as the N unknowns (multilateral exchange rates). In this situation, there is a unique solution to the system of N equations:

$$\begin{bmatrix} s_{1/\mathcal{M}} \\ s_{2/\mathcal{M}} \\ s_{3/\mathcal{M}} \\ \vdots \\ s_{N/\mathcal{M}} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ -1 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots \\ -1 & 0 & \cdots & 0 & 1 \\ 1 & 1 & \cdots & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} s_{2/1} \\ s_{3/1} \\ \vdots \\ s_{N/1} \\ 0 \end{bmatrix} \tag{7}$$

Kunkler and MacDonald (2015) showed that the unique solution is a system of N multilateral exchange rates priced in terms of an equally-weighted multicurrency numéraire, represented by \mathcal{M} :

$$s_{i/\mathcal{M}} = \frac{1}{N} \sum_{j=1}^N s_{ij} \tag{8}$$

where $s_{i/\mathcal{M}} = \ln(S_{i/\mathcal{M}})$ is the natural logarithm of the multilateral exchange rate for the i th currency; and $s_{ij} = \ln(S_{ij})$ is the natural logarithm of the i th/ j th bilateral exchange rate.

The multilateral exchange rates in Eq. (8) can be written in nominal terms as a geometric average of N bilateral exchange rates:

$$S_{i/\mathcal{M}} = \prod_{j=1}^N (S_{ij})^{\frac{1}{N}} \tag{9}$$

The multilateral exchange rates in nominal terms in Eq. (9) are also known as *invariant currency indexes*, which are normalised bilateral exchange rates and are independent to the choice of the common numéraire (η th) currency (Hovanov et al., 2004).

2.3. Local hamburger prices

Local hamburger prices can be converted from one currency to another currency by using the appropriate bilateral exchange rate. More specifically, to convert the local hamburger price of the i th currency from the i th currency to the common numéraire (η th)

currency can be achieved by multiplying by the i th/ η th bilateral exchange rate:

$$H_{i/\eta} = S_{i/\eta} H_{i/i} \quad (10)$$

Similarly, local hamburger prices can be converted from one currency to the multicurrency numéraire (\mathcal{M}) by using the appropriate multilateral exchange rate. More specifically, to convert the local hamburger price of the i th currency from the i th currency to the multicurrency numéraire (\mathcal{M}) can be achieved by multiplying by the multilateral exchange rate of the i th currency to give:

$$H_{i/\mathcal{M}} = S_{i/\mathcal{M}} H_{i/i} \quad (11)$$

Interestingly, for the system of N currencies, the geometric average of the local hamburger prices in terms of the multicurrency numéraire (\mathcal{M}) is equal to the geometric average of the local hamburger prices in terms the associated local currencies:

$$\begin{aligned} H_{\mathcal{M}/\mathcal{M}} &= \prod_{i=1}^N (H_{i/\mathcal{M}})^{\frac{1}{N}} \\ &= \prod_{i=1}^N (S_{i/\mathcal{M}} H_{i/i})^{\frac{1}{N}} \\ &= \prod_{i=1}^N (S_{i/\mathcal{M}})^{\frac{1}{N}} \prod_{i=1}^N (H_{i/i})^{\frac{1}{N}} \\ &= \prod_{i=1}^N (H_{i/i})^{\frac{1}{N}} \end{aligned} \quad (12)$$

where $H_{i/\mathcal{M}} = S_{i/\mathcal{M}} H_{i/i}$ from Eq. (11); and $\prod_{i=1}^N (S_{i/\mathcal{M}})^{\frac{1}{N}} = 1$ from Eq. (5).

2.4. The Raw Big Mac index

The system of interest for the Raw Big Mac index consists of local hamburger prices and bilateral exchange rates in terms of a common numéraire (η th) currency for a system of N currencies. The common numéraire (η th) currency can be any one of the N currencies.

2.4.1. Implied rates

Implied rates are exchange rates that are implied by purchasing power parity (Solnik & McLeavey, 2009). More specifically, the i th/ η th bilateral implied rate is the ratio of the local hamburger price of the common numéraire (η th) currency in terms of the common numéraire (η th) currency to the local hamburger price of the i th currency in terms of the i th currency:

$$I_{i/\eta} = \frac{H_{\eta/\eta}}{H_{i/i}} \quad (13)$$

Similarly, a *multilateral implied rate* for the i th currency is the ratio of the geometric average of N local hamburger prices in terms of the multicurrency numéraire (\mathcal{M}) to the local hamburger price of the i th currency in terms of the i th currency:

$$I_{i/\mathcal{M}} = \frac{H_{\mathcal{M}/\mathcal{M}}}{H_{i/i}} \quad (14)$$

2.4.2. Law of one price

The law of one price is a variant of absolute purchasing power parity. The law of one price compares the bilateral implied rate (ratio of two local hamburger prices) to the associated bilateral exchange rate. The law of one price holds for bilateral exchange rates using local hamburger prices when the bilateral exchange rate equals the associated bilateral implied rate:

$$S_{i/\eta} = I_{i/\eta} \quad (15)$$

Similarly, the law of one price holds for multilateral exchange rates using local hamburger prices when the multilateral exchange rate equals the associated multilateral implied rate:

$$S_{i/\mathcal{M}} = I_{i/\mathcal{M}} \quad (16)$$

2.4.3. Bilateral valuations

The law of one price that uses local hamburgers as the basket of goods and services is not expected hold continuously. As a consequence, there is a potential misalignment between the bilateral exchange rates and the bilateral implied rates. When the law of one price does not hold, bilateral valuations are used to measure whether currencies are at fair value, overvalued, or undervalued, relative to the common numéraire (η th) currency. The bilateral valuations for the Raw Big Mac index measure the absolute purchasing power of a system of N currencies relative to a common numéraire (η th) currency:

$$V_{i/\eta}^R = \frac{S_{i/\eta}}{I_{i/\eta}} = \frac{S_{i/\eta}H_{i/i}}{H_{\eta/\eta}} = \frac{H_{i/\eta}}{H_{\eta/\eta}} \tag{17}$$

where $I_{i/\eta} = H_{\eta/\eta}/H_{i/i}$ from Eq. (13); and $H_{i/\eta} = S_{i/\eta}H_{i/i}$ from Eq. (10). The bilateral valuations in Eq. (17) are also known as *bilateral real exchange rates*, which are bilateral exchange rates adjusted for bilateral price ratios (Taylor & Sarno, 1998).

Bilateral valuations are conditional on the chosen common numéraire (η th) currency. For example, the bilateral valuation of the common numéraire (η th) currency is always fixed at parity when $i = \eta$ ($V_{i/\eta} = 1$), since the local hamburger prices are the same ($H_{i/\eta} = H_{\eta/\eta}$) when $i = \eta$ and the i th/ η th bilateral exchange rate is always one ($S_{i/\eta} = 1$) when $i = \eta$. The other currencies are measured relative to the common numéraire (η th) currency, where each currency is: at fair value relative to the common numéraire (η th) currency when $V_{i/\eta} = 1$; overvalued relative to the common numéraire (η th) currency when $V_{i/\eta} > 1$; or undervalued relative to the common numéraire (η th) currency when $V_{i/\eta} < 1$.

2.4.4. *Multilateral valuations*

It is assumed that the multilateral valuations are measured relative to the same equally-weighted multicurrency numéraire (\mathcal{M}) that was used for the multilateral exchange rates in Eq. (9). The multilateral valuations for the Raw Big Mac index measure the absolute purchasing power parity of a system of N currencies relative to an equally-weighted multicurrency numéraire (\mathcal{M}):

$$V_{i/\mathcal{M}}^R = \frac{S_{i/\mathcal{M}}}{I_{i/\mathcal{M}}} = \frac{S_{i/\mathcal{M}}H_{i/i}}{H_{\mathcal{M}/\mathcal{M}}} = \frac{H_{i/\mathcal{M}}}{H_{\mathcal{M}/\mathcal{M}}} \tag{18}$$

where $I_{i/\mathcal{M}} = H_{\mathcal{M}/\mathcal{M}}/H_{i/i}$ from Eq. (14); and $H_{i/\mathcal{M}} = S_{i/\mathcal{M}}H_{i/i}$ from Eq. (11). The multilateral valuations in Eq. (18) are also known as *multilateral real exchange rates*, which are multilateral exchange rates adjusted for multilateral price ratios.

The geometric average of the N multilateral valuations in Eq. (18) is equal to one:

$$\prod_{i=1}^N \left(V_{i/\mathcal{M}}^R \right)^{\frac{1}{N}} = \prod_{i=1}^N \left(\frac{H_{i/\mathcal{M}}}{H_{\mathcal{M}/\mathcal{M}}} \right)^{\frac{1}{N}} = \frac{1}{H_{\mathcal{M}/\mathcal{M}}} \prod_{i=1}^N (H_{i/\mathcal{M}})^{\frac{1}{N}} = 1 \tag{19}$$

where $V_{i/\mathcal{M}}^R = H_{i/\mathcal{M}}/H_{\mathcal{M}/\mathcal{M}}$ from Eq. (18); and $H_{\mathcal{M}/\mathcal{M}} = \prod_{i=1}^N (H_{i/\mathcal{M}})^{\frac{1}{N}}$ from Eq. (12).

In summary, the geometric average of the multilateral valuations for a system of N currencies is fixed at parity (valuation of one). In this situation, the multilateral valuations are conditional on the entire system of N currencies, where currencies are at fair value, overvalued, or undervalued, relative to an equally-weighted basket of currencies.

2.4.5. *Bilateral valuations revisited*

Bilateral valuations can be decomposed into two multilateral valuations. More specifically, the bilateral valuation of a currency relative to the common numéraire (η th) currency can be derived from the multilateral valuation of the same currency divided by the multilateral valuation of the common numéraire (η th) currency:

$$V_{i/\eta}^R = \frac{S_{i/\eta}}{I_{i/\eta}} = \frac{S_{i/\eta}H_{i/i}}{H_{\eta/\eta}} = \left(\frac{S_{i/\mathcal{M}}H_{i/i}}{H_{\mathcal{M}/\mathcal{M}}} \right) / \left(\frac{S_{\eta/\mathcal{M}}H_{\eta/\eta}}{H_{\mathcal{M}/\mathcal{M}}} \right) = \left(\frac{S_{i/\mathcal{M}}}{I_{i/\mathcal{M}}} \right) / \left(\frac{S_{\eta/\mathcal{M}}}{I_{\eta/\mathcal{M}}} \right) = \frac{V_{i/\mathcal{M}}^R}{V_{\eta/\mathcal{M}}^R} \tag{20}$$

where $V_{i/\eta}^R = S_{i/\eta}/I_{i/\eta}$ from Eq. (17); $I_{i/\eta} = H_{\eta/\eta}/H_{i/i}$ from Eq. (13); $S_{i/\eta} = S_{i/\mathcal{M}}/S_{\eta/\mathcal{M}}$ from Eq. (2); $V_{i/\mathcal{M}}^R = S_{i/\mathcal{M}}/I_{i/\mathcal{M}}$ from Eq. (18); and $V_{\eta/\mathcal{M}}^R = S_{\eta/\mathcal{M}}/I_{\eta/\mathcal{M}}$ from Eq. (18). In general, Eq. (20) can be written in vector notation:

$$\mathbf{V}_{\eta}^R = \frac{1}{V_{\eta/\mathcal{M}}^R} \mathbf{V}_{\mathcal{M}}^R \tag{21}$$

where \mathbf{V}_{η}^R is a $N \times 1$ vector of bilateral valuations relative to the common numéraire (η th) currency with the i th element being $V_{i/\eta}^R$; and $\mathbf{V}_{\mathcal{M}}^R$ is a $N \times 1$ vector of multilateral valuations relative to the multicurrency numéraire (\mathcal{M}) with the i th element being $V_{i/\mathcal{M}}^R$.

The N bilateral valuations relative to common numéraire (η th) currency are perfectly correlated with the N multilateral valuations relative to the multicurrency numéraire (\mathcal{M}):

$$\text{cor}(\mathbf{V}_{\eta}^R, \mathbf{V}_{\mathcal{M}}^R) = \frac{\text{cov}(\mathbf{V}_{\eta}^R, \mathbf{V}_{\mathcal{M}}^R)}{\sqrt{\text{var}(\mathbf{V}_{\eta}^R)\text{var}(\mathbf{V}_{\mathcal{M}}^R)}} = \frac{\text{cov}\left(\mathbf{V}_{\mathcal{M}}^R / V_{\eta/\mathcal{M}}^R, \mathbf{V}_{\mathcal{M}}^R\right)}{\sqrt{\text{var}\left(\mathbf{V}_{\mathcal{M}}^R / V_{\eta/\mathcal{M}}^R\right)\text{var}(\mathbf{V}_{\mathcal{M}}^R)}} = \frac{\text{var}(\mathbf{V}_{\mathcal{M}}^R) / V_{\eta/\mathcal{M}}^R}{\text{var}(\mathbf{V}_{\mathcal{M}}^R) / V_{\eta/\mathcal{M}}^R} = 1 \tag{22}$$

where $\mathbf{V}_{\eta}^R = \frac{1}{V_{\eta/\mathcal{M}}^R} \mathbf{V}_{\mathcal{M}}^R$ from Eq. (21); $\text{cov}(\mathbf{V}_{\eta}^R, \mathbf{V}_{\mathcal{M}}^R)$ is the covariance between the bilateral valuations relative to the common numéraire (η th) currency and the multilateral valuations relative to the multicurrency numéraire (\mathcal{M}); $\text{var}(\mathbf{V}_{\eta}^R)$ is the variance of the bilateral valuations relative to the common numéraire (η th) currency; and $\text{var}(\mathbf{V}_{\mathcal{M}}^R)$ is the variance of the multilateral valuations relative to the multicurrency numéraire (\mathcal{M}).

In summary, the bilateral valuations are conditional on the chosen common single-currency numéraire, where currencies are at fair value, overvalued, or undervalued, relative to the common numéraire (η th) currency. In contrast, the multilateral valuations are conditional on the entire system of N currencies, where currencies are at fair value, overvalued, or undervalued, relative to an equally-weighted basket of currencies.

2.4.6. Exact multilateral valuation of the common numéraire currency

The exact multilateral valuation of the common numéraire (η th) currency for the Raw Big Mac index can be calculated by first calculating the geometric average of the bilateral valuations for the system of N currencies:

$$\prod_{i=1}^N \left(V_{i/\eta}^R \right)^{\frac{1}{N}} = \prod_{i=1}^N \left(\frac{V_{i/\mathcal{M}}^R}{V_{\eta/\mathcal{M}}^R} \right)^{\frac{1}{N}} = \frac{1}{V_{\eta/\mathcal{M}}^R} \prod_{i=1}^N \left(V_{i/\mathcal{M}}^R \right)^{\frac{1}{N}} = \frac{1}{V_{\eta/\mathcal{M}}^R} \tag{23}$$

where $V_{i/\eta}^R = V_{i/\mathcal{M}}^R / V_{\eta/\mathcal{M}}^R$ from Eq. (20); and $\prod_{i=1}^N \left(V_{i/\mathcal{M}}^R \right)^{\frac{1}{N}} = 1$ from Eq. (19). Rewriting Eq. (23) in terms of the multilateral valuation of the common numéraire (η th) currency:

$$V_{\eta/\mathcal{M}}^R = \prod_{i=1}^N \left(V_{i/\eta}^R \right)^{-\frac{1}{N}} \tag{24}$$

Thus, the multilateral valuation of the common numéraire (η th) currency is simply the inverse of the geometric average of the bilateral valuations relative to the common numéraire (η th) currency for the system of N currencies.

In summary, there are two methods to calculate the multilateral valuation of the common numéraire (η th) currency. The first method uses the ratio of the local hamburger price of the (η th) currency in terms of the multicurrency numéraire (\mathcal{M}) to the geometric average of the N local hamburger prices in Eq. (18). The second method uses the inverse of the geometric average of the N bilateral valuations relative to the common numéraire (η th) currency in Eq. (24). As expected for consistency, both methods produce the same result and are equivalent. For example, starting with Eq. (24) produces Eq. (18):

$$V_{\eta/\mathcal{M}}^R = \prod_{i=1}^N \left(V_{i/\eta}^R \right)^{-\frac{1}{N}} = \prod_{i=1}^N \left(\frac{S_{i/\eta}}{I_{i/\eta}} \right)^{-\frac{1}{N}} = \prod_{i=1}^N \left(\frac{S_{i/\mathcal{M}} H_{i/i}}{S_{\eta/\mathcal{M}} H_{\eta/\eta}} \right)^{-\frac{1}{N}} = H_{\eta/\mathcal{M}} \prod_{i=1}^N \left(H_{i/\mathcal{M}} \right)^{-\frac{1}{N}} = \frac{H_{\eta/\mathcal{M}}}{H_{\mathcal{M}/\mathcal{M}}} = \frac{S_{\eta/\mathcal{M}}}{I_{\eta/\mathcal{M}}} \tag{25}$$

where $V_{\eta/\mathcal{M}}^R = \prod_{i=1}^N \left(V_{i/\eta}^R \right)^{-\frac{1}{N}}$ from Eq. (24); $V_{i/\eta}^R = S_{i/\eta} / I_{i/\eta}$ from Eq. (17); $I_{i/\eta} = H_{\eta/\eta} / H_{i/i}$ from Eq. (13); $S_{i/\eta} = S_{i/\mathcal{M}} / S_{\eta/\mathcal{M}}$ from Eq. (2); $H_{i/\mathcal{M}} = S_{i/\mathcal{M}} H_{i/i}$ from Eq. (11); $H_{\eta/\mathcal{M}} = S_{\eta/\mathcal{M}} H_{\eta/\eta}$ from Eq. (11); $H_{\mathcal{M}/\mathcal{M}} = \prod_{i=1}^N \left(H_{i/\mathcal{M}} \right)^{\frac{1}{N}}$ from Eq. (12); and $V_{\eta/\mathcal{M}}^R = S_{\eta/\mathcal{M}} / I_{\eta/\mathcal{M}}$ from Eq. (18).

2.5. GDP-adjusted Big Mac index

The bilateral valuations for the GDP-adjusted Big Mac index provide an alternative measurement of the absolute purchasing parity by accounting for the expectation that local hamburger prices tend to be cheaper in lower GDP per capita countries compared to higher GDP per capita countries (The Economist, 2025). For the GDP-adjusted Big Mac index, the local basket of goods and services is once again a McDonald’s Big Mac hamburger. The system of interest for the GDP-adjusted Big Mac index consists of the same data as the Raw Big Mac index, but also includes the local GDP per capita for the system of N currencies.

2.5.1. Local GDP per capita

The local GDP per capita in terms of one currency can be converted into terms of the common numéraire (η th) currency by using the appropriate bilateral implied rate, rather than the appropriate bilateral exchange rate. For example, to convert the local GDP per capita of the i th currency in terms of i th currency to the common numéraire (η th) currency can be achieved by multiplying by the appropriate bilateral implied rate:

$$G_{i/\eta} = I_{i/\eta} G_{i/i} = \frac{H_{\eta/\eta}}{H_{i/i}} G_{i/i} \tag{26}$$

where $I_{i/\eta} = H_{\eta/\eta} / H_{i/i}$ from Eq. (13).

Similarly, the local GDP per capita in terms of one currency can be converted into terms of the multicurrency numéraire (\mathcal{M}) by using the appropriate multilateral implied rate, rather than the appropriate multilateral exchange rate. For example, to convert the local GDP per capita of the i th currency in terms of i th currency to the multicurrency numéraire (\mathcal{M}) can be achieved by multiplying by the appropriate multilateral implied rate:

$$G_{i/\mathcal{M}} = I_{i/\mathcal{M}} G_{i/i} = \frac{H_{\mathcal{M}/\mathcal{M}}}{H_{i/i}} G_{i/i} \tag{27}$$

where $I_{i|\mathcal{M}} = H_{i|\mathcal{M}}/H_{i|i}$ from Eq. (14).

2.5.2. Bilateral valuations

For the system of N currencies, the bilateral valuations for the GDP-adjusted Big Mac index use the fitted values from a cross-sectional regression of the local hamburger prices in terms of the common numéraire (η th) currency against the local GDP per capita in terms of the common numéraire (η th) currency. The cross-sectional regression model is estimated using ordinary least squares (OLS). The fitted cross-sectional regression model can be written as:

$$\widehat{H}_{i|\eta} = \widehat{\alpha}_\eta + \widehat{\beta}_\eta G_{i|\eta} \tag{28}$$

where $\widehat{\alpha}_\eta$ is the OLS-estimated *bilateral intercept term*; and $\widehat{\beta}_\eta$ is the OLS-estimated *bilateral slope term*. In the literature, the cross-sectional regression model in Eq. (28) is typically fitted in log terms, rather than in nominal terms (O'Brien & Ruiz de Vargas, 2019).

The bilateral valuations for the GDP-adjusted Big Mac index measure the absolute purchasing power of a system of N currencies relative to a common numéraire (η th) currency:

$$V_{i|\eta}^A = \frac{H_{i|\eta}/H_{\eta|\eta}}{\widehat{H}_{i|\eta}/\widehat{H}_{\eta|\eta}} = \frac{V_{i|\eta}^R}{F_{i|\eta}^A} \tag{29}$$

where $V_{i|\eta}^R = H_{i|\eta}/H_{\eta|\eta}$ from Eq. (17); and:

$$F_{i|\eta}^A = \frac{\widehat{H}_{i|\eta}}{\widehat{H}_{\eta|\eta}} \tag{30}$$

is the *bilateral adjustment factor* for the i th currency relative to the common numéraire (η th) currency. An alternative name for the bilateral adjustment factors could be *bilateral fitted valuations*, since they are essentially a fitted version of the bilateral valuations of the Raw Big Mac index. For example, the bilateral valuations of the Raw Big Mac index are $V_{i|\eta}^R = H_{i|\eta}/H_{\eta|\eta}$ from Eq. (17) and the bilateral adjustment factors are $F_{i|\eta}^A = \widehat{H}_{i|\eta}/\widehat{H}_{\eta|\eta}$ from Eq. (30).

In the literature, it is standard to interpret the bilateral valuations in Eq. (29) in terms of a residual type of analysis. For example, $H_{i|\eta}/\widehat{H}_{i|\eta}$ is the *relative residual ratio* of the local hamburger price for the i th currency against the fitted hamburger price for the i th currency (see O'Brien & Ruiz de Vargas, 2017). However, the focus in this paper is on the bilateral adjustment factors, rather than the relative residual ratios. As a consequence, Eq. (29) shows that the bilateral valuation of a currency relative to the common numéraire (η th) currency for the GDP-adjusted Big Mac index is simply a scaled version of the bilateral valuation of the same currency relative to the common numéraire (η th) currency for the Raw Big Mac index. Thus, the GDP-adjusted Big Mac index is aptly named, since it is an adjusted (scaled) version of the Raw Big Mac index.

2.5.3. Multilateral valuations

For the system of N currencies, the multilateral valuations for the GDP-adjusted Big Mac index use the fitted values from a cross-sectional regression of the local hamburger prices in terms of the multicurrency numéraire (\mathcal{M}) against the local GDP per capita in terms of the multicurrency numéraire (\mathcal{M}). The cross-sectional regression model is estimated using ordinary least squares (OLS). The fitted cross-sectional regression model can be written as:

$$\widehat{H}_{i|\mathcal{M}} = \widehat{\alpha}_\mathcal{M} + \widehat{\beta}_\mathcal{M} G_{i|\mathcal{M}} \tag{31}$$

where $\widehat{\alpha}_\mathcal{M}$ is the OLS-estimated *multilateral intercept term*; and $\widehat{\beta}_\mathcal{M}$ is the OLS-estimated *multilateral slope term*.

The multilateral valuations for the GDP-adjusted Big Mac index measure the absolute purchasing power parity of a system of N currencies relative to an equally-weighted multicurrency numéraire (\mathcal{M}):

$$V_{i|\mathcal{M}}^A = \frac{H_{i|\mathcal{M}}/H_{\mathcal{M}|\mathcal{M}}}{\widehat{H}_{i|\mathcal{M}}/\widehat{H}_{\mathcal{M}|\mathcal{M}}} = \frac{V_{i|\mathcal{M}}^R}{F_{i|\mathcal{M}}^A} \tag{32}$$

where $V_{i|\mathcal{M}}^R = H_{i|\mathcal{M}}/H_{\mathcal{M}|\mathcal{M}}$ from Eq. (18);

$$F_{i|\mathcal{M}}^A = \frac{\widehat{H}_{i|\mathcal{M}}}{\widehat{H}_{\mathcal{M}|\mathcal{M}}} \tag{33}$$

is the *multilateral adjustment factor* for the i th currency relative to the multicurrency numéraire (\mathcal{M}); and:

$$\widehat{H}_{\mathcal{M}|\mathcal{M}} = \prod_{j=1}^N (\widehat{H}_{j|\mathcal{M}})^{\frac{1}{N}} \tag{34}$$

is the geometric average of the fitted hamburger prices for the system of N currencies in terms of the multicurrency numéraire (\mathcal{M}). An

alternative name of the multilateral adjustment factors could be *multilateral fitted valuations*, as they are essentially a fitted version of the multilateral valuations of the Raw Big Mac index. For example, the multilateral valuations of the Raw Big Mac index are $V_{i|\mathcal{M}}^R = H_{i|\mathcal{M}}/H_{\mathcal{M}|\mathcal{M}}$ from Eq. (18) and the bilateral adjustment factors are $F_{i|\mathcal{M}}^A = \widehat{H}_{i|\mathcal{M}}/\widehat{H}_{\mathcal{M}|\mathcal{M}}$ from Eq. (33).

Once again, the focus in this paper is on the multilateral adjustment factors, rather than the relative residual ratios. As a consequence, Eq. (32) shows that the multilateral valuation of a currency for the GDP-adjusted Big Mac index is simply a scaled version of the multilateral valuation of the same currency for the Raw Big Mac index. Once again, the GDP-adjusted Big Mac index is aptly named, since it is an adjusted (scaled) version of the Raw Big Mac index.

The geometric average of the N multilateral valuations in Eq. (32) is:

$$\prod_{i=1}^N \left(V_{i|\mathcal{M}}^A \right)^{\frac{1}{N}} = \prod_{i=1}^N \left(\frac{H_{i|\mathcal{M}}/H_{\mathcal{M}|\mathcal{M}}}{\widehat{H}_{i|\mathcal{M}}/\widehat{H}_{\mathcal{M}|\mathcal{M}}} \right)^{\frac{1}{N}} = \frac{\widehat{H}_{\mathcal{M}|\mathcal{M}}}{H_{\mathcal{M}|\mathcal{M}}} \prod_{i=1}^N \left(\frac{H_{i|\mathcal{M}}}{\widehat{H}_{i|\mathcal{M}}} \right)^{\frac{1}{N}} = 1 \tag{35}$$

where $V_{i|\mathcal{M}}^A = \frac{H_{i|\mathcal{M}}/H_{\mathcal{M}|\mathcal{M}}}{\widehat{H}_{i|\mathcal{M}}/\widehat{H}_{\mathcal{M}|\mathcal{M}}}$ from Eq. (32); $H_{\mathcal{M}|\mathcal{M}} = \prod_{j=1}^N (H_{j|\mathcal{M}})^{\frac{1}{N}}$ from Eq. (12); and $\widehat{H}_{\mathcal{M}|\mathcal{M}} = \prod_{j=1}^N (\widehat{H}_{j|\mathcal{M}})^{\frac{1}{N}}$ from Eq. (34).

In addition, the geometric average of the N multilateral adjustment factors in Eq. (33) is:

$$\prod_{i=1}^N \left(F_{i|\mathcal{M}}^A \right)^{\frac{1}{N}} = \prod_{i=1}^N \left(\frac{\widehat{H}_{i|\mathcal{M}}}{\widehat{H}_{\mathcal{M}|\mathcal{M}}} \right)^{\frac{1}{N}} = \frac{1}{\widehat{H}_{\mathcal{M}|\mathcal{M}}} \prod_{i=1}^N (\widehat{H}_{i|\mathcal{M}})^{\frac{1}{N}} = 1 \tag{36}$$

where $F_{i|\mathcal{M}}^A = \widehat{H}_{i|\mathcal{M}}/\widehat{H}_{\mathcal{M}|\mathcal{M}}$ from Eq. (33); and $\widehat{H}_{\mathcal{M}|\mathcal{M}} = \prod_{i=1}^N (\widehat{H}_{i|\mathcal{M}})^{\frac{1}{N}}$ from Eq. (34).

2.5.4. Bilateral valuations revisited

This section revisits the bilateral valuations to show that they can be decomposed into two multilateral valuations. For the GDP-adjusted Big Mac index, O'Brien & Ruiz de Vargas (2019) stated that a bilateral valuation of a currency can be derived from the multilateral valuation of the same currency divided by the multilateral valuation of the common numéraire (η th) currency. In an earlier paper, the same authors also stated that it was too complex to provide further details of the multilateral valuations (O'Brien & Ruiz de Vargas, 2017). In this paper, the derivation of this relationship is provided in the Appendix (see Eq. (A16)).

The bilateral valuation of a currency relative to the common numéraire (η th) currency can be derived from the multilateral valuation of the same currency divided by the multilateral valuation of the common numéraire (η th) currency (see Eq. (A16) for details):

$$V_{i|\eta}^A = \frac{V_{i|\mathcal{M}}^A}{V_{\eta|\mathcal{M}}^A} \tag{37}$$

In general, Eq. (37) can be written in vector notation:

$$\mathbf{V}_{\eta}^A = \frac{1}{V_{\eta|\mathcal{M}}^A} \mathbf{V}_{\mathcal{M}}^A \tag{38}$$

where \mathbf{V}_{η}^A is a $N \times 1$ vector of bilateral valuations relative to the common numéraire (η th) currency with the i th element being $V_{i|\eta}^A$; and $\mathbf{V}_{\mathcal{M}}^A$ is a $N \times 1$ vector of multilateral valuations relative to the multicurrency numéraire (\mathcal{M}) with the i th element being $V_{i|\mathcal{M}}^A$.

The N bilateral valuations relative to common numéraire (η th) currency are perfectly correlated with the N multilateral valuations relative to the multicurrency numéraire (\mathcal{M}):

$$\text{cor}(\mathbf{V}_{\eta}^A, \mathbf{V}_{\mathcal{M}}^A) = \frac{\text{cov}(\mathbf{V}_{\eta}^A, \mathbf{V}_{\mathcal{M}}^A)}{\sqrt{\text{var}(\mathbf{V}_{\eta}^A)\text{var}(\mathbf{V}_{\mathcal{M}}^A)}} = \frac{\text{cov}(\mathbf{V}_{\mathcal{M}}^A/V_{\eta|\mathcal{M}}^A, \mathbf{V}_{\mathcal{M}}^A)}{\sqrt{\text{var}(\mathbf{V}_{\mathcal{M}}^A/V_{\eta|\mathcal{M}}^A)\text{var}(\mathbf{V}_{\mathcal{M}}^A)}} = \frac{\text{var}(\mathbf{V}_{\mathcal{M}}^A)/V_{\eta|\mathcal{M}}^A}{\text{var}(\mathbf{V}_{\mathcal{M}}^A)/V_{\eta|\mathcal{M}}^A} = 1 \tag{39}$$

where $\mathbf{V}_{\eta}^A = \mathbf{V}_{\mathcal{M}}^A/V_{\eta|\mathcal{M}}^A$ from Eq. (38); $\text{cov}(\mathbf{V}_{\eta}^A, \mathbf{V}_{\mathcal{M}}^A)$ is the covariance between the bilateral valuations relative to the common numéraire (η th) currency and the multilateral valuations relative to the multicurrency numéraire (\mathcal{M}); $\text{var}(\mathbf{V}_{\eta}^A)$ is the variance of the bilateral valuations relative to the common numéraire (η th) currency; and $\text{var}(\mathbf{V}_{\mathcal{M}}^A)$ is the variance of the multilateral valuations relative to the multicurrency numéraire (\mathcal{M}).

In summary, the bilateral valuations are conditional on a single currency, where currencies are at fair value, overvalued, or undervalued, relative to the chosen common numéraire (η th) currency. In contrast, the multilateral valuations are conditional on the entire system of N currencies, where currencies are at fair value, overvalued, or undervalued, relative to an equally-weighted basket of currencies.

2.5.5. Adjustment factors

For the GDP-adjusted Big Mac index, the bilateral adjustment factors in Eq. (30) can be decomposed into two multilateral adjustment factors. More specifically, the bilateral adjustment factor of a currency relative to the common numéraire (η th) currency can be derived from the multilateral adjustment factor of the same currency divided by the multilateral adjustment factor of the common numéraire (η th) currency:

$$F_{i/\eta}^A = \frac{\widehat{H}_{i/\eta}}{\widehat{H}_{\eta/\eta}} = \frac{S_{\eta/\mathcal{M}} \widehat{H}_{i/\eta}}{S_{\eta/\mathcal{M}} \widehat{H}_{\eta/\eta}} = \frac{\widehat{H}_{i/\mathcal{M}} / \widehat{H}_{\mathcal{M}/\mathcal{M}}}{\widehat{H}_{\eta/\mathcal{M}} / \widehat{H}_{\mathcal{M}/\mathcal{M}}} = \frac{F_{i/\mathcal{M}}^A}{F_{\eta/\mathcal{M}}^A} \tag{40}$$

where $F_{i/\eta}^A = \widehat{H}_{i/\eta} / \widehat{H}_{\eta/\eta}$ from Eq. (30); $F_{i/\mathcal{M}}^A = \widehat{H}_{i/\mathcal{M}} / \widehat{H}_{\mathcal{M}/\mathcal{M}}$ from Eq. (33); $F_{\eta/\mathcal{M}}^A = \widehat{H}_{\eta/\mathcal{M}} / \widehat{H}_{\mathcal{M}/\mathcal{M}}$ from Eq. (33); $\widehat{H}_{i/\mathcal{M}} = S_{\eta/\mathcal{M}} \widehat{H}_{i/\eta}$ is the fitted hamburger price for the i th currency in terms of the multicurrency numéraire (\mathcal{M}); $\widehat{H}_{\eta/\mathcal{M}} = S_{\eta/\mathcal{M}} \widehat{H}_{\eta/\eta}$ is the fitted hamburger price for the η th currency in terms of the multicurrency numéraire (\mathcal{M}). In general, Eq. (40) can be written in vector notation:

$$\mathbf{F}_{\eta}^A = \frac{1}{F_{\eta/\mathcal{M}}^A} \mathbf{F}_{\mathcal{M}}^A \tag{41}$$

where \mathbf{F}_{η}^A is a $N \times 1$ vector of bilateral adjustment factors relative to the common numéraire (η th) currency with the i th element being $F_{i/\eta}^A$; and $\mathbf{F}_{\mathcal{M}}^A$ is a $N \times 1$ vector of multilateral adjustment factors relative to the multicurrency numéraire (\mathcal{M}) with the i th element being $F_{i/\mathcal{M}}^A$.

The N bilateral adjustment factors relative to common numéraire (η th) currency are perfectly correlated with the N multilateral adjustment factors relative to the multicurrency numéraire (\mathcal{M}):

$$\text{cor}(\mathbf{F}_{\eta}^A, \mathbf{F}_{\mathcal{M}}^A) = \frac{\text{cov}(\mathbf{F}_{\eta}^A, \mathbf{F}_{\mathcal{M}}^A)}{\sqrt{\text{var}(\mathbf{F}_{\eta}^A) \text{var}(\mathbf{F}_{\mathcal{M}}^A)}} = \frac{\text{cov}(\mathbf{F}_{\mathcal{M}}^A / F_{\eta/\mathcal{M}}^A, \mathbf{F}_{\mathcal{M}}^A)}{\sqrt{\text{var}(\mathbf{F}_{\mathcal{M}}^A / F_{\eta/\mathcal{M}}^A) \text{var}(\mathbf{F}_{\mathcal{M}}^A)}} = \frac{\text{var}(\mathbf{F}_{\mathcal{M}}^A) / F_{\eta/\mathcal{M}}^A}{\text{var}(\mathbf{F}_{\mathcal{M}}^A) / F_{\eta/\mathcal{M}}^A} = 1 \tag{42}$$

where $F_{\eta}^A = \mathbf{F}_{\mathcal{M}}^A / F_{\eta/\mathcal{M}}^A$ from Eq. (41); $\text{cov}(\mathbf{F}_{\eta}^A, \mathbf{F}_{\mathcal{M}}^A)$ is the covariance between the bilateral adjustment factors relative to the common numéraire (η th) currency and the multilateral adjustment factors relative to the multicurrency numéraire (\mathcal{M}); $\text{var}(\mathbf{F}_{\eta}^A)$ is the variance of the bilateral adjustment factors relative to the common numéraire (η th) currency; and $\text{var}(\mathbf{F}_{\mathcal{M}}^A)$ is the variance of the multilateral adjustment factors relative to the multicurrency numéraire (\mathcal{M}).

In summary, the bilateral adjustment factors are conditional of a single currency. In contrast, the multilateral adjustment factors are conditional on the entire system of N currencies.

2.5.6. Exact multilateral valuation of the common numéraire currency

The average-basket approach of O'Brien & Ruiz de Vargas (2017) approximated the multilateral valuation of the common numéraire (η th) currency with the inverse of the arithmetic average of the $N - 1$ bilateral valuations of the other currencies (excluding the common numéraire (η th) currency) relative to the common numéraire (η th) currency:

$$V_{\eta/\mathcal{M}}^A \approx \frac{1}{\frac{1}{N-1} \sum_{i=1}^N V_{i/\eta}^A} \tag{43}$$

The approximation of the multilateral valuation of the common numéraire (η th) currency can be improved upon by using the geometric average of the bilateral valuations for the system of N currencies:

$$\prod_{i=1}^N (V_{i/\eta}^A)^{\frac{1}{N}} = \prod_{i=1}^N \left(\frac{V_{i/\mathcal{M}}^A}{V_{\eta/\mathcal{M}}^A} \right)^{\frac{1}{N}} = \frac{1}{V_{\eta/\mathcal{M}}^A} \prod_{i=1}^N (V_{i/\mathcal{M}}^A)^{\frac{1}{N}} = \frac{1}{V_{\eta/\mathcal{M}}^A} \tag{44}$$

where $V_{i/\eta}^A = V_{i/\mathcal{M}}^A / V_{\eta/\mathcal{M}}^A$ from Eq. (37); and $\prod_{i=1}^N (V_{i/\mathcal{M}}^A)^{\frac{1}{N}} = 1$ from Eq. (35). Rearranging Eq. (44) in terms of the multilateral valuation of the common numéraire (η th) currency:

$$V_{\eta/\mathcal{M}}^A = \prod_{i=1}^N (V_{i/\eta}^A)^{-\frac{1}{N}} \tag{45}$$

Thus, the multilateral valuation of the common numéraire (η th) currency is simply the inverse of the geometric average of the bilateral valuations relative to the common numéraire (η th) currency for the system of N currencies. Note that the inverse of the geometric average in Eq. (45) also includes the common numéraire (η th) currency, which is excluded from the inverse of the arithmetic average in Eq. (43).

The multilateral valuation of the common numéraire (η th) currency in Eq. (45) is an exact measure. More generally, geometric

averages are typically used by central banks for exchange rate indexes (Loretan, 2005). Thus, the geometric average should be used, rather than the arithmetic average. For example, the arithmetic average would be appropriate if natural logarithms were used. Taking natural logarithms of Eq. (45) gives:

$$v_{\eta|\mathcal{M}}^A = -\frac{1}{N} \sum_{i=1}^N v_{i|\eta}^A \tag{46}$$

where $v_{\eta|\mathcal{M}}^A = \ln(V_{\eta|\mathcal{M}}^A)$ is the natural logarithm of the multilateral valuation of the common numéraire (η th) currency for the GDP-adjusted Big Mac index; and $v_{i|\eta}^A = \ln(V_{i|\eta}^A)$ is the natural logarithm of the bilateral valuation of the i th currency relative to the common numéraire (η th) currency for the GDP-adjusted Big Mac index.

In summary, there are two methods to calculate the multilateral valuation of the common numéraire (η th) currency. The first method uses the ratio of the local hamburger price of the common numéraire (η th) currency to the geometric average of the local hamburger prices divided by the ratio of the fitted hamburger price of the common numéraire (η th) currency to the geometric average of the fitted hamburger prices in Eq. (32). The second method uses the inverse of the geometric average of the N bilateral valuations of the GDP-adjusted Big Mac index in Eq. (45). As expected for consistency, both methods produce the same result and are equivalent. For example, starting with Eq. (45) produces Eq. (32):

$$\begin{aligned} V_{\eta|\mathcal{M}}^A &= \prod_{i=1}^N (V_{i|\eta}^A)^{-\frac{1}{N}} = \prod_{i=1}^N \left(\frac{H_{i|\eta}/H_{\eta|\eta}}{\widehat{H}_{i|\eta}/\widehat{H}_{\eta|\eta}} \right)^{-\frac{1}{N}} = \prod_{i=1}^N \left(\frac{S_{i|\eta}H_{i|i}/H_{\eta|\eta}}{S_{i|\eta}\widehat{H}_{i|i}/\widehat{H}_{\eta|\eta}} \right)^{-\frac{1}{N}} = \prod_{i=1}^N \left(\frac{S_{i|\mathcal{M}}H_{i|i}/S_{\eta|\mathcal{M}}H_{\eta|\eta}}{S_{i|\mathcal{M}}\widehat{H}_{i|i}/S_{\eta|\mathcal{M}}\widehat{H}_{\eta|\eta}} \right)^{-\frac{1}{N}} \\ &= \frac{H_{\eta|\mathcal{M}}}{\widehat{H}_{\eta|\mathcal{M}}} \prod_{i=1}^N \left(\frac{H_{i|\mathcal{M}}}{\widehat{H}_{i|\mathcal{M}}} \right)^{-\frac{1}{N}} = \frac{H_{\eta|\mathcal{M}}/H_{\mathcal{M}|\mathcal{M}}}{\widehat{H}_{\eta|\mathcal{M}}/\widehat{H}_{\mathcal{M}|\mathcal{M}}} \end{aligned} \tag{47}$$

where $V_{\eta|\mathcal{M}}^A = \prod_{i=1}^N (V_{i|\eta}^A)^{-\frac{1}{N}}$ from Eq. (45); $V_{i|\eta}^A = \frac{H_{i|\eta}/H_{\eta|\eta}}{\widehat{H}_{i|\eta}/\widehat{H}_{\eta|\eta}}$ from Eq. (29); $S_{i|\eta} = S_{i|\mathcal{M}}/S_{\eta|\mathcal{M}}$ from Eq. (2); $H_{i|\mathcal{M}} = S_{i|\mathcal{M}}H_{i|i}$ from Eq. (11); $H_{\eta|\mathcal{M}} = S_{\eta|\mathcal{M}}H_{\eta|\eta}$ from Eq. (11); $H_{\mathcal{M}|\mathcal{M}} = \prod_{i=1}^N (H_{i|\mathcal{M}})^{\frac{1}{N}}$ from Eq. (12); $\widehat{H}_{\mathcal{M}|\mathcal{M}} = \prod_{i=1}^N (\widehat{H}_{i|\mathcal{M}})^{\frac{1}{N}}$ from Eq. (34); $\widehat{H}_{i|\mathcal{M}} = S_{\eta|\mathcal{M}}\widehat{H}_{i|\eta}$ is the fitted hamburger price for the i th currency in terms of the multicurrency numéraire (\mathcal{M}); $\widehat{H}_{\eta|\mathcal{M}} = S_{\eta|\mathcal{M}}\widehat{H}_{\eta|\eta}$ is the fitted hamburger price for the η th currency in terms of the multicurrency numéraire (\mathcal{M}).

3. Results and discussion

3.1. Data sample

The data sample is sourced from The Economist and consists of local hamburger prices, US dollar bilateral exchange rates, and local GDP per capita for a system of 53 currencies for the 1st of January 2024. Note that the original data sample included 55 countries. However, there were no GDP per capita values from both Lebanon and Venezuela. As a consequence, Lebanon and Venezuela were excluded from the analysis since the cross-sectional regression was not possible with missing values. Although the data sample is for a single period, it is sufficient to show the numerous interrelationships between the bilateral valuations and multilateral valuations for both versions of the Big Mac index.

The common numéraire currency for the bilateral valuations is the US dollar ($\eta = USD$). Table 2 reports the local hamburger prices, US dollar bilateral exchange rates, local GDP per capita prices, bilateral valuations, multilateral valuations, adjustment factors, and the other calculated data for the system of 53 currencies. In terms of the US dollar (USD), the local hamburger price is USD 5.69 in the United States, the least expensive hamburger is USD 2.39 in Taiwan, and the most expensive hamburger is USD 8.17 in Switzerland. In terms of the common multicurrency numéraire (\mathcal{M}), the local hamburger price is \mathcal{M} 94.4917 in the United States, the least expensive hamburger is \mathcal{M} 39.7320 in Taiwan, and the most expensive hamburger is \mathcal{M} 135.6276 in Switzerland. Note that the order of the local hamburger prices is the same in terms of the US dollar (USD) and in terms of the multicurrency numéraire (\mathcal{M}).

3.2. Raw Big Mac index

3.2.1. Multilateral valuation of the US dollar

The exact multilateral valuation of the US dollar is $V_{USD|\mathcal{M}}^R = 1.3584$, which shows that the US dollar is overvalued by 35.84% ($1.3584 - 1$) relative to the system of $N = 53$ currencies. There are two methods to calculate the multilateral valuation of the US dollar as the common numéraire currency. The first method uses the ratio of the local hamburger price of the US dollar in terms of the multicurrency numéraire (\mathcal{M}) to the geometric average of the 53 local hamburger prices (see Eq. (18)):

$$V_{USD|\mathcal{M}}^R = \frac{H_{USD|\mathcal{M}}}{H_{\mathcal{M}|\mathcal{M}}} = \frac{94.4917}{69.5625} = 1.3584 \tag{48}$$

and the second method uses the inverse of the geometric average of the 53 bilateral valuations relative to the US dollar (see Eq. (24)):

$$V_{USD/\mathcal{M}}^R = \prod_{i=1}^{53} \left(V_{i/USD}^R \right)^{-\frac{1}{53}} = \frac{1}{0.7362} = 1.3584 \tag{49}$$

where $V_{USD/\mathcal{M}}^R$ is the multilateral valuation of the US dollar; $V_{i/USD}^R$ is the bilateral valuation of the i th currency relative to the US dollar; $H_{USD/\mathcal{M}} = 94.4917$ is the local hamburger price for the US dollar in terms of the multicurrency numéraire; and $H_{\mathcal{M}/\mathcal{M}} = 69.5625$ is the geometric average of the 53 local hamburger prices in terms of the multicurrency numéraire (\mathcal{M}).

3.2.2. Bilateral valuations and the multilateral valuations

Fig. 1 (a) displays the bilateral valuations for the Raw Big Mac index, and Fig. 1 (b) displays the multilateral valuations for the Raw Big Mac index. Apart from the location of the party line, the figures look identical, with the same ordering. It was shown in Eq. (22) that the 53 bilateral valuations relative to the US dollar are perfectly correlated with the 53 multilateral valuations relative to the multicurrency numéraire (\mathcal{M}).

The bilateral valuation of the US dollar relative to US dollar is fixed at fair value with $V_{USD/USD}^R = 1$. The bilateral valuations are conditional on the US dollar, where currencies are at fair value, overvalued, or undervalued, relative to the US dollar. In contrast, the geometric average of the multilateral valuations for a system of 53 currencies is fixed at parity (valuation of one). The multilateral valuations are conditional on the entire system of 53 currencies, where currencies are at fair value, overvalued, or undervalued, relative to an equally-weighted basket of 53 currencies.

The bilateral valuation of a currency relative to the common numéraire currency can be decomposed into two multilateral valuations: the multilateral valuation of the same currency divided by the multilateral valuation of the US dollar:

$$V_{i/USD}^R = \frac{V_{i/\mathcal{M}}^R}{V_{USD/\mathcal{M}}^R} = \frac{V_{i/\mathcal{M}}^R}{1.3584} = 0.7361 \times V_{i/\mathcal{M}}^R \tag{50}$$

where $V_{i/USD}^R$ is the bilateral valuation of the i th currency relative to the US dollar; $V_{i/\mathcal{M}}^R$ is the multilateral valuation of the i th currency; and $V_{USD/\mathcal{M}}^R = 1.3584$ is the multilateral valuation of the US dollar. The large overvaluation of 35.84% for the US dollar scales all of the bilateral valuations downward. As a consequence, the bilateral valuations relative to the US dollar are lower than the associated multilateral valuations by a factor of 0.7361 (1/1.3584).

For example, the bilateral valuation of the Eurozone euro relative to the US dollar is $V_{EUR/USD}^R = 1.0312$, which is a factor of 0.7361 of the multilateral valuation of the Eurozone euro is $V_{EUR/\mathcal{M}}^R = 1.4007$:

$$V_{EUR/USD}^R = 0.7361 \times V_{EUR/\mathcal{M}}^R = 0.7361 \times 1.4007 = 1.0312 \tag{51}$$

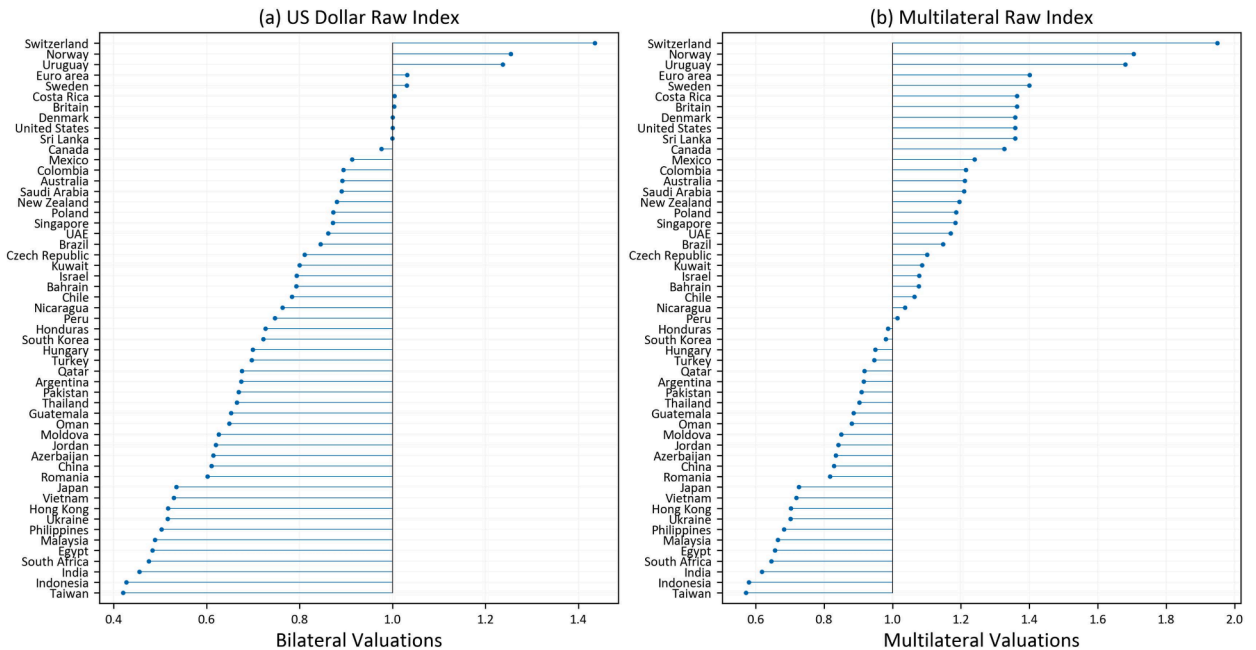


Fig. 1. Raw Big Mac index.

Notes: Fig. 1 (a) displays the bilateral valuations of each currency relative to the US dollar (common numéraire currency) for the 1st of January 2024. Fig. 1 (b) displays the multilateral valuations of each currency for the 1st of January 2024.

Thus, the Eurozone euro is overvalued by 3.12% (1.0312 – 1) relative to the US dollar and is overvalued by 40.07% (1.4007 – 1) relative to the system of 53 currencies.

The multilateral valuations for the Raw Big Mac index are conditional on the entire system of N currencies, which results in a complete picture. In contrast, the bilateral valuations for the Raw Big Mac index are conditional on the chosen common numéraire currency. The bilateral valuation of a currency relative to the US dollar is derived from the multilateral valuation for the same currency divided by the multilateral valuation of the US dollar. This provides clarity on the conditional nature of the bilateral valuations for the Raw Big Mac index.

3.3. GDP-adjusted Big Mac index

3.3.1. Fitted cross-sectional regression models

The bilateral valuations for the GDP-adjusted Big Mac index use the fitted values from a cross-sectional regression of the 53 local hamburger prices in terms of the US dollar against the 53 local GDP per capita in terms of the US dollar in Eq. (28). In contrast, the multilateral valuations for the GDP-adjusted Big Mac index use the fitted values from a cross-sectional regression of the 53 local hamburger prices in terms of the common multicurrency numéraire against the 53 local GDP per capita in terms of the common multicurrency numéraire in Eq. (31).

The cross-sectional regression models are estimated using ordinary least squares (OLS). Table 3 reports the OLS parameter estimates for the intercept and slope terms. The OLS-estimated bilateral intercept term is related to the OLS-estimated multilateral intercept term (see Eq. (A14)):

$$\hat{\alpha}_{USD} = \frac{1}{S_{USD/\mathcal{M}}} \hat{\alpha}_{\mathcal{M}} = \frac{1}{16.60663} \times 64.8717 = 3.9064 \tag{52}$$

where $\hat{\alpha}_{USD} = 3.9064$ is the OLS-estimated bilateral intercept term; $\hat{\alpha}_{\mathcal{M}} = 64.8717$ is the OLS-estimated multilateral intercept term; and $S_{USD/\mathcal{M}} = 16.60663$ is the multilateral exchange rate for the US dollar. The bilateral slope term is related to the multilateral slope term in Eq. (A13):

$$\hat{\beta}_{USD} = \frac{1}{S_{USD/\mathcal{M}} (H_{USD/USD} / H_{\mathcal{M}/\mathcal{M}})} \hat{\beta}_{\mathcal{M}} = \frac{1}{16.60663 \times (5.6900 / 69.5625)} \times 0.0187 = 0.0138 \tag{53}$$

where $\hat{\beta}_{USD} = 0.0138$ is the OLS-estimated bilateral slope term; $\hat{\beta}_{\mathcal{M}} = 0.0187$ is the OLS-estimated multilateral slope term; $S_{USD/\mathcal{M}} = 16.60663$ is the multilateral exchange rate for the US dollar; $H_{USD/USD} = 5.6900$ is the local hamburger price for the US dollar in terms of the US dollar; and $H_{\mathcal{M}/\mathcal{M}} = 69.5625$ is the geometric average of the 53 local hamburger prices in terms of the multicurrency numéraire (\mathcal{M}). Note that that the GDP per capita used in the cross-sectional regression are divided by 1000, which impacts the slope value (see Eq. (A19) and Eq. (A20)). The t -statistics are significant at 2.42 for both the bilateral slope term $\hat{\beta}_{USD}$ and the multilateral slope term $\hat{\beta}_{\mathcal{M}}$. The R-squared is 10.27 % for both cross-sectional regressions.

Fig. 2 (a) displays the 53 local Big Mac prices in terms of the US dollar against the 53 local GDP per capita in terms of the US dollar, together with the fitted regression line. Fig. 2 (b) displays the 53 local Big Mac prices in terms of the multicurrency numéraire against the 53 local GDP per capita in terms of the multicurrency numéraire, together with the fitted regression line. Apart from the axis scales, the figures look identical. It was shown in Eq. (A12) that the 53 *bilateral ratios* of the local hamburger prices in terms of the US dollar to the associated local GDP per capita values in terms of the US dollar are perfectly correlated with the 53 *multilateral ratios* of the local hamburger prices in terms of the multicurrency numéraire (\mathcal{M}) to the associated local GDP per capita values in terms of the multicurrency numéraire (\mathcal{M}).

Table 3
Ordinary Least Squares (OLS) regression results.

A: Bilateral Regression			
	Coefficient	Std. Error	t-statistic
$\hat{\alpha}_{USD}$	3.9064	0.251	15.55
$\hat{\beta}_{USD}$	0.0138	0.006	2.42
B: Multilateral Regression			
	Coefficient	Std. Error	t-statistic
$\hat{\alpha}_{\mathcal{M}}$	64.8717	4.172	15.55
$\hat{\beta}_{\mathcal{M}}$	0.0187	0.008	2.42

Notes: Table 3 reports the cross-sectional regression results for the bilateral regression in Panel A and the cross-sectional regression results for the multilateral regression in Panel B for the 1st of January 2024. The cross-sectional regression results consist of the OLS estimated coefficients, standard errors, and t -statistics.

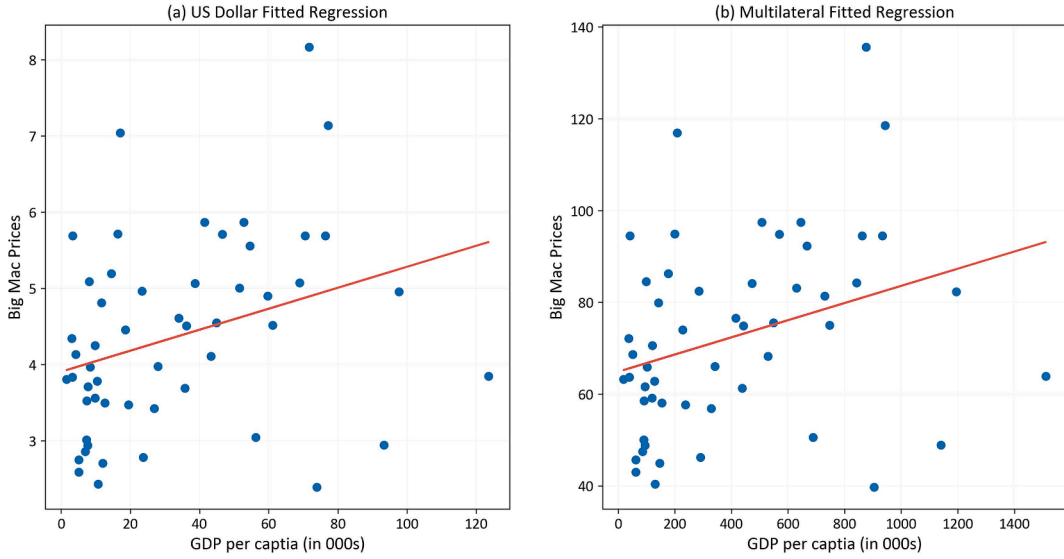


Fig. 2. GDP-adjusted Big Mac fitted cross-sectional regressions.

Notes: Fig. 2 (a) displays the local Big Mac prices in terms of the US dollar against the local GDP per capita in terms of the US dollar, together with the fitted regression line, for the 1st of January 2024. Fig. 2 (b) displays the local Big Mac prices in terms of the multicurrency numéraire against the local GDP per capita in terms of the multicurrency numéraire, together with the fitted regression line, for the 1st of January 2024.

3.3.2. Multilateral valuation of the US dollar

The exact multilateral valuation of the US dollar for the GDP-adjusted Big Mac index is $V_{USD/\mathcal{M}}^A = 1.1904$, which shows that the US dollar is overvalued by 19.04% ($1.1904 - 1$) relative to the system of $N = 53$ currencies. There are two methods to calculate the multilateral valuation of the common numéraire currency. The first method uses the ratio of the local hamburger price of the i th currency to the geometric average local hamburger price divided by the ratio of the fitted hamburger price of the i th currency to the geometric average fitted hamburger price in Eq. (32):

$$V_{USD/\mathcal{M}}^A = \frac{V_{USD/\mathcal{M}}^R}{F_{USD/\mathcal{M}}^A} = \frac{1.3584}{1.1411} = 1.1904 \tag{54}$$

and the second method uses the inverse of the geometric average of the 53 bilateral valuations in Eq. (45):

$$V_{USD/\mathcal{M}}^A = \prod_{i=1}^{53} \left(V_{i/USD}^A \right)^{-\frac{1}{53}} = 1 / 0.8400 = 1.1904 \tag{55}$$

where $V_{USD/\mathcal{M}}^A$ is the multilateral valuation of the US dollar for the GDP-adjusted Big Mac index; and $V_{i/USD}^A$ is the bilateral valuation of the i th currency relative to the US dollar; $V_{USD/\mathcal{M}}^R = 1.3584$ is the multilateral valuation of the US dollar for the Raw Big Mac index; and $F_{USD/\mathcal{M}}^A = 1.1411$ is the multilateral adjustment factor for the US dollar relative to the multicurrency numéraire (\mathcal{M}).

The multilateral adjustment factor for the US dollar is:

$$F_{USD/\mathcal{M}}^A = \frac{\hat{H}_{USD/\mathcal{M}}}{\hat{H}_{\mathcal{M}/\mathcal{M}}} = \frac{82.3289}{72.1489} = 1.1411 \tag{56}$$

where $F_{USD/\mathcal{M}}^A$ is the multilateral adjustment factor for the US dollar relative to the multicurrency numéraire (\mathcal{M}); $\hat{H}_{USD/\mathcal{M}} = 82.3289$ is the local fitted hamburger price for the US dollar in terms of the multicurrency numéraire (\mathcal{M}); and $\hat{H}_{\mathcal{M}/\mathcal{M}} = 72.1489$ is the geometric average of the 53 local fitted hamburger prices in terms of the multicurrency numéraire (\mathcal{M}).

The approximation of the multilateral valuation of the US dollar by O'Brien & Ruiz de Vargas (2017) used the inverse of the arithmetic average of the $(53 - 1)$ bilateral valuations of the other currencies relative to the US dollar (excluding the US dollar):

$$V_{USD/\mathcal{M}}^A \approx \frac{1}{\frac{1}{53-1} \sum_{i \neq USD}^{53} V_{i/USD}^A} = \frac{1}{0.8682} = 1.1517 \tag{57}$$

where $V_{USD/\mathcal{M}}^A$ is the multilateral valuation of the US dollar; and $V_{i/USD}^A$ is the bilateral valuation of the i th currency relative to the US

dollar. The approximation of the multilateral valuation of the US dollar is approximately 1.1517, which shows that the US dollar is overvalued by 15.17% ($1.1517 - 1$). The difference between the exact measurement of 1.1904 in Eq. (54) and the approximation of 1.1517 in Eq. (57) is 3.87% ($1.1904 - 1.1517$).

3.3.3. Bilateral valuations and the multilateral valuations

Fig. 3 (a) displays the bilateral valuations for the GDP-adjusted Big Mac index, and Fig. 3 (b) displays the multilateral valuations for the GDP-adjusted Big Mac index. Apart from the location of the party line, the figures look identical, with the same ordering. It was shown in Eq. (39) that the 53 bilateral valuations relative to the US dollar are perfectly correlated with the 53 multilateral valuations relative to the multicurrency numéraire (\mathcal{M}).

The bilateral valuation of the US dollar is fixed at fair value with $V_{USD/USD}^A = 1$. The bilateral valuations are conditional, where currencies are at fair value, overvalued, or undervalued, relative to the US dollar. In contrast, the geometric average of the 53 multilateral valuations is fixed at parity (valuation of one). The multilateral valuations are conditional on the entire system of 53 currencies, where currencies are at fair value, overvalued, or undervalued, relative to an equally-weighted basket of 53 currencies.

For the GDP-adjusted Big Mac index, the bilateral valuation of a currency relative to the US dollar for the GDP-adjusted Big Mac index can be derived from the multilateral valuation for the same currency divided by the multilateral valuation of the US dollar:

$$V_{i/USD}^A = \frac{V_{i/\mathcal{M}}^A}{V_{USD/\mathcal{M}}^A} = \frac{V_{i/\mathcal{M}}^A}{1.1904} = 0.8401 \times V_{i/\mathcal{M}}^A \tag{58}$$

where $V_{i/USD}^A$ is the bilateral valuation of the i th currency relative to the US dollar; $V_{i/\mathcal{M}}^A$ is the multilateral valuation of the i th currency; and $V_{USD/\mathcal{M}}^A = 1.1904$ is the multilateral valuation of the US dollar. The large overvaluation of 19.04% for the US dollar scales the bilateral valuations relative to the US dollar downward. As a consequence, the bilateral valuations relative to the US dollar are lower than the associated multilateral valuations by a factor of 0.8401 ($1/1.1904$).

For example, the bilateral valuation of the Eurozone euro relative to the US dollar is $V_{EUR/USD}^A = 1.1418$, which is a factor of 0.8401 of the multilateral valuation of the Eurozone euro is $V_{EUR/\mathcal{M}}^A = 1.3591$:

$$V_{EUR/USD}^A = 0.8401 \times V_{EUR/\mathcal{M}}^A = 0.8401 \times 1.3591 = 1.1418 \tag{59}$$

Thus, the Eurozone euro is overvalued by 14.18% ($1.1418 - 1$) relative to the US dollar and is overvalued by 35.91% ($1.3591 - 1$) relative to the system of 53 currencies.

The multilateral valuations for the GDP-adjusted Big Mac index are conditional on the entire system of 53 currencies, which provides a complete picture. In contrast, the bilateral valuations for the GDP-adjusted Big Mac index are conditional on the US dollar.

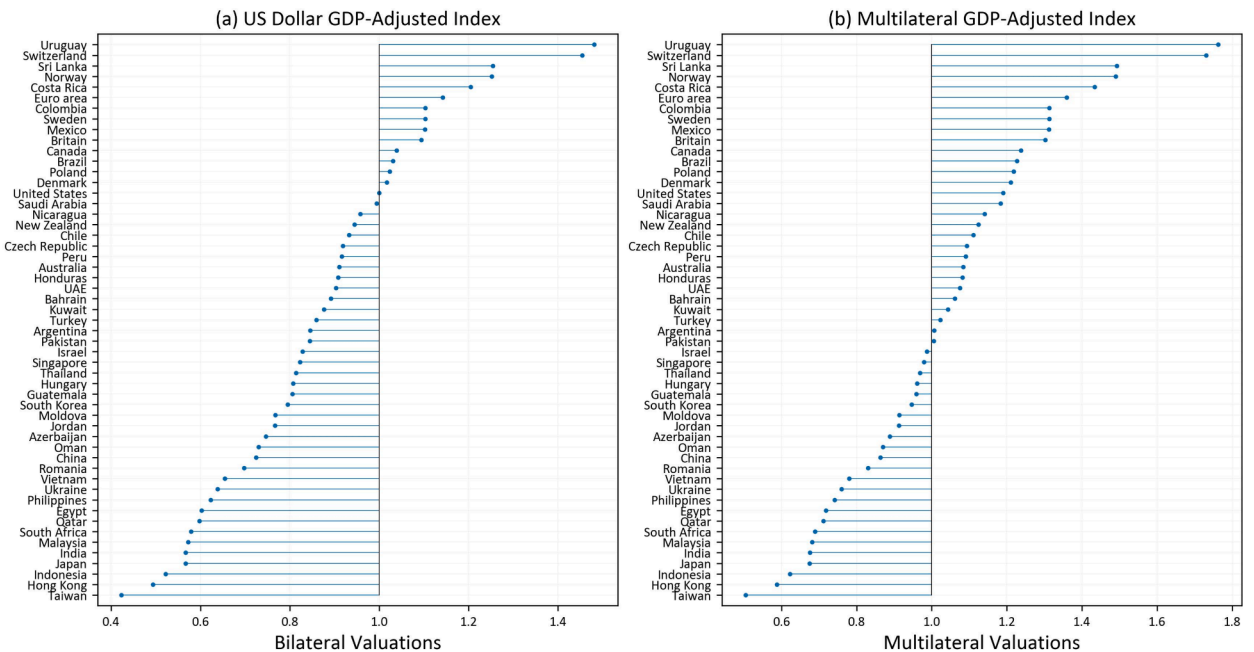


Fig. 3. GDP-adjusted Big Mac indexes.

Notes: Fig. 3 (a) displays the bilateral valuations relative to the US dollar for the 1st of January 2024. Fig. 3 (b) displays the multilateral valuations relative to the multicurrency numéraire (\mathcal{M}) for the 1st of January 2024.

The bilateral valuation of a currency relative to the US dollar is derived from the multilateral valuation of the same currency divided by the multilateral valuation of the US dollar. This result provides clarity on the conditional nature of the bilateral valuations for the GDP-adjusted Big Mac index.

3.3.4. Adjustment factors

Fig. 4 (a) displays the bilateral adjustment factors relative to the US dollar for the GDP-adjusted Big Mac index and Fig. 4 (b) displays the multilateral adjustment factors for the GDP-adjusted Big Mac index. Apart from the location of the party line, the figures look identical, with the same order. It was shown in Eq. (42) that the 53 adjustment factors relative to the US dollar are perfectly correlated with the 53 multilateral adjustment factors relative to the multicurrency numéraire (€).

The bilateral adjustment factor of a currency relative to the US dollar can be derived from the multilateral adjustment factor of the same currency divided by the multilateral adjustment factor for the US dollar:

$$F_{i/USD}^A = \frac{F_{i,€}^A}{F_{USD,€}^A} = \frac{F_{i,€}^A}{1.1411} = 0.8763 \times F_{i,€}^A \tag{60}$$

where $F_{i/USD}^A$ is the bilateral adjustment factor for the i th currency relative to the US dollar; $F_{i,€}^A$ is the multilateral adjustment factor for the i th currency; and $F_{USD,€}^A = 1.1411$ is the multilateral adjustment factor for the US dollar. The multilateral adjustment factor for the US dollar scales the bilateral adjustment factors relative to the US dollar downward. As a consequence, the bilateral adjustment factors relative to the US dollar are lower than the associated multilateral adjustment factors by a factor of 0.8763 (1/1.1411). For example, the bilateral adjustment factor of the Eurozone euro relative to the US dollar is $F_{EUR/USD}^A = 0.9031$, which is a factor of 0.8763 of the multilateral adjustment factor of the Eurozone euro is $F_{EUR,€}^A = 1.0306$:

$$F_{EUR/USD}^A = 0.8763 \times F_{EUR,€}^A = 0.8763 \times 1.0306 = 0.9031 \tag{61}$$

The multilateral adjustment factors are conditional on the entire system of 53 currencies, whereas the bilateral adjustment factors are conditional on the US dollar. The bilateral adjustment factor for a currency is derived from the multilateral adjustment factor for the same currency divided by the multilateral adjustment factor of the US dollar. This provides clarity on the conditional nature of the bilateral adjustment factors.

3.4. Comparison of the Big Mac indexes

The bilateral valuations for the GDP-adjusted Big Mac index are simply scaled (adjusted) versions of the bilateral valuations for the

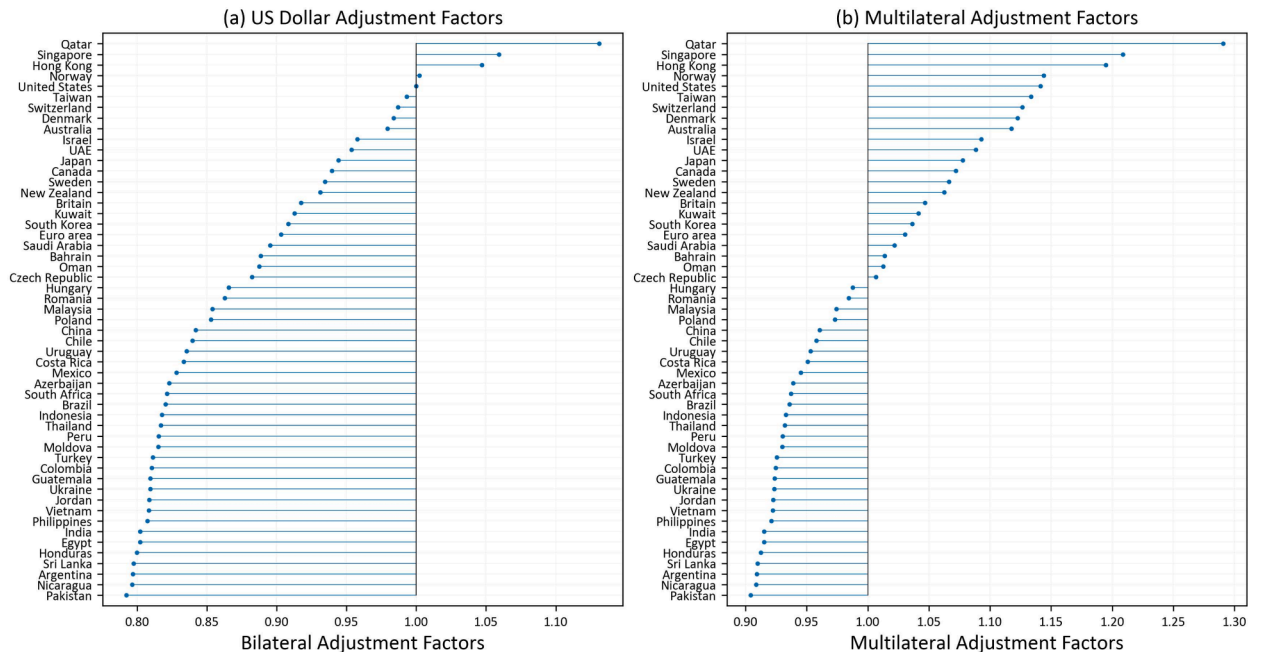


Fig. 4. Adjustment factors.

Notes: Fig. 4 (a) displays the bilateral adjustment factors relative to the US dollar for the GDP-adjusted Big Mac index for the 1st of January 2024, and Fig. 4 (b) displays the multilateral adjustment factors for the GDP-adjusted Big Mac index for the 1st of January 2024.

Raw Big Mac index. For example, the bilateral valuation of the Eurozone euro relative to the US dollar for the GDP-adjusted Big Mac index is derived from the bilateral valuation of the Eurozone euro relative to the US dollar for the Raw Big Mac index divided by the bilateral adjustment factor of the Eurozone euro relative to the US dollar:

$$V_{EUR/USD}^A = \frac{V_{EUR/USD}^R}{F_{EUR/USD}^A} = \frac{1.0312}{0.9031} = 1.1418 \tag{62}$$

where $V_{EUR/USD}^A$ is the bilateral valuation of the Eurozone euro relative to the US dollar for the GDP-adjusted Big Mac index; $V_{EUR/USD}^R$ is the bilateral valuation of the Eurozone euro relative to the US dollar for the Raw Big Mac index; and $F_{EUR/USD}^A$ is the bilateral adjustment factor of the Eurozone euro relative to the US dollar for the GDP-adjusted Big Mac index.

Similarly, the multilateral valuations for the GDP-adjusted Big Mac index are simply scaled (adjusted) versions of the multilateral valuations for the Raw Big Mac index. For example, the multilateral valuation of the Eurozone euro for the GDP-adjusted Big Mac index is derived from the multilateral valuation of the Eurozone euro relative for the Raw Big Mac index divided by the multilateral adjustment factor of the Eurozone euro:

$$V_{EUR/\$}^A = \frac{V_{EUR/\$}^R}{F_{EUR/\$}^A} = \frac{1.4007}{1.0306} = 1.3591 \tag{63}$$

where $V_{EUR/\A is the multilateral valuation of the Eurozone euro for the GDP-adjusted Big Mac index; $V_{EUR/\R is the multilateral valuation of the Eurozone euro for the Raw Big Mac index; and $F_{EUR/\A is the multilateral adjustment factor of the Eurozone euro.

Fig. 5 (a) displays the bilateral valuations relative to the US dollar for the Raw Big Mac index against the bilateral valuations relative to the US dollar for the GDP-adjusted Big Mac index. Fig. 5 (b) displays the multilateral valuations for the Raw Big Mac index against the multilateral valuations for the GDP-adjusted Big Mac index. The correlation is 0.9508 in both figures. Apart from the axis values, the figures look identical, with the same order. This result is driven by the relationship between the bilateral adjustment factors and the multilateral adjustment factors.

For example, rearranging Eq. (29) so that the ratio of the bilateral valuations relative to the US dollar for the GDP-adjusted Big Mac index to the associated bilateral valuations relative to the US dollar for the Raw Big Mac index can be written as:

$$\frac{1}{F_{i/USD}^A} = \frac{V_{i/USD}^A}{V_{i/USD}^R} \tag{64}$$

Similarly, rearranging Eq. (32) so that the ratio of the multilateral valuations for the GDP-adjusted Big Mac index to the associated multilateral valuations for the Raw Big Mac index can be written as:

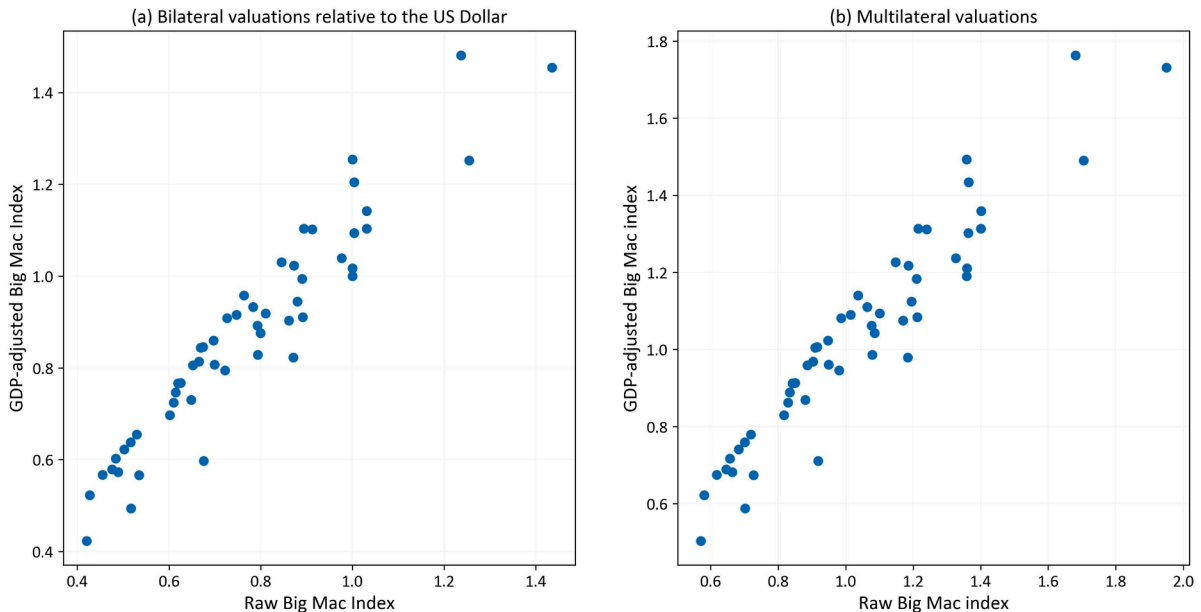


Fig. 5. Comparison of Raw Big Mac index versus GDP-adjusted Big Mac index.

Notes: Fig. 5 (a) displays the bilateral valuations relative to the US dollar for the GDP-adjusted Big Mac index against the bilateral valuations relative to the US dollar for the Raw Big Mac index for the 1st of January 2024. Fig. 5 (b) displays the multilateral valuations for the GDP-adjusted Big Mac index against the multilateral valuations for the Raw Big Mac index for the 1st of January 2024.

$$\frac{1}{F_{i/\eta}^R} = \frac{V_{i/\eta}^A}{V_{i/\eta}^R} \tag{65}$$

Finally, it was shown in Eq. (42) that the 53 adjustment factors relative to the US dollar are perfectly correlated with the 53 multilateral adjustment factors relative to the multicurrency numéraire (\mathcal{M}).

3.5. Recommendations for caution

Some authors in the literature have recommended for caution when using the Big Mac index (Taylor & Taylor, 2004; Haidar, 2011). Even The Economist asserts that the Big Mac index was never meant to be an accurate measurement of currency valuations (The Economist, 2025). The Raw Big Mac index and the GDP-adjusted Big Mac index use locally produced McDonald’s Big Mac hamburgers as the local baskets of goods and services, which are assumed to be identical for different currencies. However, PPP tests are biased if the local baskets of goods and services are not identical (Haidar, 2011). It is beyond the scope of this paper to fully investigate this aspect of the Big Mac index. However, this section briefly highlights some discrepancies between different markets of the system of 53 currencies.

Table 4 reports the geometric averages for each MSCI market classification in terms of both the US dollar (USD) and the multicurrency numéraire (\mathcal{M}) of the local hamburger prices, local fitted hamburger prices, valuations of the Raw Big Mac index, adjustment factors, and valuations of the GDP-adjusted Big Mac index for a system of 53 currencies. The data is grouped into four MSCI country classifications, namely, DM (Developed Market) currencies, EM (Emerging Market) currencies, FM (Frontier Market) currencies, and NC (Not Classified) currencies (see MSCI, 2025).

For the local hamburger prices in terms of the US dollar, Developed Market currencies have the highest geometric average of 5.1974, compared to geometric averages of 3.7710 for Emerging Market currencies, 3.8740 for Frontier Market currencies, and 4.1625 for Not Classified currencies. Similarly, for the local hamburger prices in terms of the multicurrency numéraire (\mathcal{M}), Developed Market currencies have the highest geometric average of 86.3112, compared to geometric averages of 62.6242 for Emerging Market currencies, 64.3349 for Frontier Market currencies, and 69.1259 for Not Classified currencies. Thus, the local hamburger prices appear to be biased, where local hamburger prices are more expensive for Developed Market currencies compared to the local hamburger prices for Emerging Market, Frontier Market, and Not Classified currencies.

Fig. 6 (a) displays the geometric averages of the bilateral valuations for both the Raw Big Mac index and the GDP-adjusted Big Mac index grouped by the MSCI country classification. Fig. 6 (b) displays the geometric averages of the multilateral valuations for both the Raw Big Mac index and the GDP-adjusted Big Mac index grouped by the MSCI country classification.

For the multilateral valuations for the Raw Big Mac index, Developed Market currencies have the highest geometric average of 24.08% (1.2408 – 1), compared to geometric averages of –9.97% (0.9003 – 1) for Emerging Market currencies, –7.52% (0.9248 – 1) for Frontier Market currencies, and –0.63% (0.9937 – 1) for Not Classified currencies. Similarly, For the multilateral valuations for the GDP-adjusted Big Mac index, Developed Market currencies have the highest geometric average of 12.16% (1.1216 – 1), compared to geometric averages of –8.42% (0.9158 – 1) for Emerging Market currencies, –2.83% (0.9717 – 1) for Frontier Market currencies, and 7.11% (1.0711 – 1) for Not Classified currencies.

In summary, there appears to be a bias in the local hamburger prices, which are more expensive in Developed Market currencies compared to Emerging Market, Frontier Market, and Not Classified currencies. The bias feeds through to the valuations of both the Raw Big Mac index and the GDP-adjusted Big Mac index, where Developed Market currencies are, on average, overvalued compared to Emerging Market, Frontier Market, and Not Classified currencies. The GDP-adjusted Big Mac index reduces the size of differences between the MSCI groups, but the similar differences remain. In contrast, interpretations of the geometric averages of the bilateral valuations relative to the US dollar are less clear. For example, the average bilateral valuations for all MSCI country classifications are negative for both the Raw Big Mac index and the GDP-adjusted Big Mac index.

3.6. A note on the system of currencies

The multilateral valuations are conditional on the chosen system of 53 currencies. In the results above, the data sample consisted of

Table 4
Geometric averages by MSCI market classifications.

Market	Count	$H_{i/USD}$	$\widehat{H}_{i/USD}$	$V_{i/USD}^R$	$F_{i/USD}^A$	$V_{i/USD}^A$	$H_{i/\mathcal{M}}$	$\widehat{H}_{i/\mathcal{M}}$	$V_{i/\mathcal{M}}^R$	$F_{i/\mathcal{M}}^A$	$V_{i/\mathcal{M}}^A$
DM	14	5.1974	4.8060	0.9134	0.9694	0.9422	86.3112	79.8123	1.2408	1.1062	1.1216
EM	23	3.7710	4.2710	0.6627	0.8615	0.7693	62.6242	70.9273	0.9003	0.9831	0.9158
FM	7	3.8740	4.1352	0.6809	0.8341	0.8163	64.3349	68.6717	0.9248	0.9518	0.9717
NC	9	4.1625	4.0308	0.7316	0.8131	0.8998	69.1259	66.9376	0.9937	0.9278	1.0711

Notes: Table 4 reports the geometric averages for each MSCI market classification in terms of both the US dollar (USD) and the multicurrency numéraire (\mathcal{M}) of the local hamburger prices, local fitted hamburger prices, the Raw valuations, adjustment factors, and GDP-adjusted valuations of 53 currencies for the 1st of January 2024. The MSCI market classifications are DM (Developed Market), EM (Emerging Market), FM (Frontier Market), and NC (Not Classified).

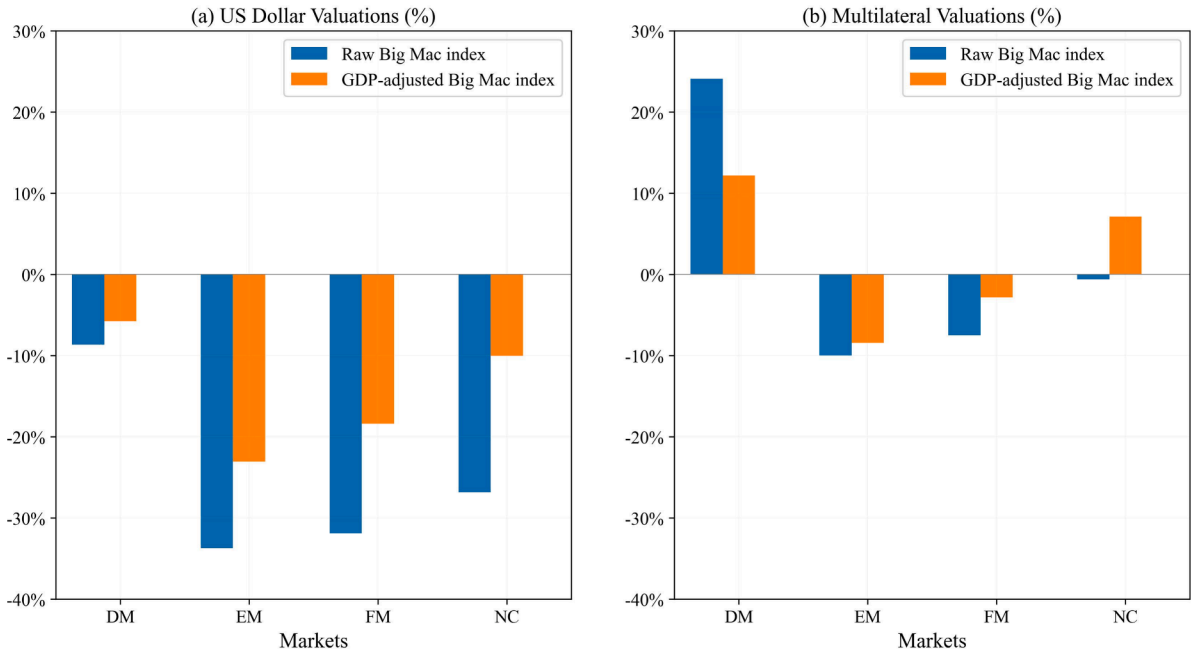


Fig. 6. Geometric valuation averages by market classification.

Notes: Fig. 6 (a) displays the geometric average of the bilateral valuations relative to the US dollar for both the Raw Big Mac index and the GDP-adjusted Big Mac index for the 1st of January 2024. Fig. 6 (b) displays the geometric average of the multilateral valuations for both the Raw Big Mac index and the GDP-adjusted Big Mac index for the 1st of January 2024. The MSCI market classifications are DM (Developed Market), EM (Emerging Market), FM (Frontier Market), and NC (Not Classified).

local hamburger prices, US dollar bilateral exchange rates, and local GDP per capita for a system of 53 currencies for the 1st of January 2024. In this section, 14 Developed Market currencies are chosen from the system of 53 currencies by using the MSCI country classification. The common numéraire currency for the bilateral valuations is the US dollar ($\eta = USD$). Table 5 reports the local hamburger prices, US dollar bilateral exchange rates, local GDP per capita prices, bilateral valuations, multilateral valuations, adjustment factors, and the other calculated data for the system of 14 Developed Market currencies. Fig. 7 (a) displays the multilateral valuations of each currency for the Raw Big Mac index. Fig. 7 (b) displays the multilateral valuations of each currency for the GDP-adjusted Big Mac index.

For the Raw Big Mac index, the multilateral valuation of the US dollar is $V_{USD/\$}^R = 1.0948$, which shows that the US dollar is overvalued by 9.48% ($1.0948 - 1$) relative to the system of 14 Developed Market currencies. This is very different from the multilateral valuation of the US dollar of 35.80% relative to the system of 53 currencies. Similarly, for the GDP-adjusted Big Mac index, the multilateral valuation of the US dollar is $V_{USD/\$}^A = 1.1156$, which shows that the US dollar is overvalued by 11.56% ($1.1156 - 1$) relative to the system of 14 Developed Market currencies. Once again, this is very different from the multilateral valuation of the US dollar of 19.04% relative to the system of 53 currencies.

In summary, multilateral valuations are conditional on the chosen system of currencies. If the number of currencies in the system changes, the multilateral valuations change. This is similar behaviour to the cross-sectional regression models used in the GDP-adjusted Big Mac index, where the OLS estimates change when the number of currencies in the system change.

4. Conclusion

Both versions of the Big Mac index are popular applications of absolute purchasing power parity. The bilateral valuations are relative to a common single-currency numéraire, whereas the multilateral valuations are relative to a multicurrency numéraire. This paper contributed to the literature by extending the approximation of O'Brien & Ruiz de Vargas (2017) to provide an exact multilateral valuation of the common numéraire currency, as well as exact multilateral valuations for all currencies for both versions of the Big Mac index.

The bilateral valuation of a currency relative to a single-currency numéraire can be derived from the multilateral valuation of the same currency divided by the multilateral valuation of the common numéraire currency. This shows the restrictive conditional nature of the bilateral valuations on the chosen common numéraire currency. In contrast, by providing a theoretical framework that utilises an equally-weighted multicurrency numéraire results in exact multilateral valuations that are conditional on the entire system of currencies.

This paper highlighted on a number of linkages between the valuations of the Raw Big Mac index and the valuations of the GDP-

Table 5
Data sample, bilateral valuations, and multilateral valuations.

Big Mac Data			Bilateral										Multilateral						
Country	MSCI	i	$G_{i/i}$	$H_{i/i}$	$S_{USD/i}$	$I_{i/USD}$	$S_{i/USD}$	$H_{i/USD}$	$\hat{H}_{i/USD}$	$V_{i/USD}^R$	$F_{i/USD}^A$	$V_{i/USD}^A$	$I_{i/\$}$	$S_{i/\$}$	$H_{i/\$}$	$\hat{H}_{i/\$}$	$V_{i/\R	$F_{i/\A	$V_{i/\A
United States	DM	USD	76,343.25	5.69	1.00000	1.00000	1.00000	5.6900	5.2706	1.0000	1.0000	1.0000	2.87172	3.14390	17.8888	16.5702	1.0948	0.9813	1.1156
Australia	DM	AUD	93,263.34	7.70	1.51745	0.73896	0.65900	5.0743	5.3422	0.8918	1.0136	0.8798	2.12209	2.07183	15.9531	16.7954	0.9763	0.9947	0.9816
Canada	DM	CAD	71,633.05	7.47	1.34465	0.76171	0.74369	5.5553	5.4807	0.9763	1.0399	0.9389	2.18743	2.33808	17.4654	17.2307	1.0689	1.0204	1.0475
Switzerland	DM	CHF	89,426.97	7.10	0.86935	0.80141	1.15028	8.1670	5.3157	1.4353	1.0086	1.4232	2.30142	3.61638	25.6763	16.7120	1.5714	0.9897	1.5877
Denmark	DM	DKK	483,265.14	39.00	6.85195	0.14590	0.14594	5.6918	5.3269	1.0003	1.0107	0.9897	0.41898	0.45883	17.8945	16.7472	1.0951	0.9918	1.1042
Euro area	DM	EUR	39,283.59	5.39	0.91865	1.05566	1.08855	5.8673	5.6070	1.0312	1.0638	0.9693	3.03155	3.42230	18.4462	17.6277	1.1289	1.0440	1.0814
Britain	DM	GBP	36,748.57	4.49	0.78629	1.26726	1.27180	5.7104	5.5578	1.0036	1.0545	0.9517	3.63922	3.99839	17.9528	17.4731	1.0987	1.0348	1.0618
Hong Kong	DM	HKD	377,117.20	23.00	7.81800	0.24739	0.12791	2.9419	5.1071	0.5170	0.9690	0.5336	0.71044	0.40214	9.2491	16.0561	0.5660	0.9509	0.5953
Israel	DM	ILS	182,550.82	17.00	3.76425	0.33471	0.26566	4.5162	5.4176	0.7937	1.0279	0.7722	0.96118	0.83520	14.1984	17.0324	0.8689	1.0087	0.8614
Japan	DM	JPY	4,451,712.34	450.00	147.86000	0.01264	0.00676	3.0434	5.4640	0.5349	1.0367	0.5159	0.03631	0.02126	9.5682	17.1783	0.5856	1.0173	0.5756
Norway	DM	NOK	1,017,428.09	75.00	10.50570	0.07587	0.09519	7.1390	5.2624	1.2547	0.9985	1.2566	0.21787	0.29926	22.4442	16.5446	1.3736	0.9798	1.4019
New Zealand	DM	NZD	74,261.88	8.20	1.63814	0.69390	0.61045	5.0057	5.5099	0.8797	1.0454	0.8415	1.99269	1.91919	15.7374	17.3227	0.9631	1.0259	0.9388
Sweden	DM	SEK	568,302.83	61.29	10.44965	0.09284	0.09570	5.8653	5.4981	1.0308	1.0432	0.9882	0.26660	0.30086	18.4398	17.2854	1.1285	1.0237	1.1024
Singapore	DM	SGD	114,164.15	6.65	1.34150	0.85564	0.74543	4.9571	5.0648	0.8712	0.9609	0.9066	2.45715	2.34357	15.5847	15.9231	0.9538	0.9430	1.0114
Geometric Avg			183,155.13	16.34	3.14390	0.34822	0.31808	5.1974	5.3709	0.9134	1.0190	0.8964	1.00000	1.00000	16.3401	16.8856	1.0000	1.0000	1.0000

Notes: Table 5 reports the local hamburger prices, the US dollar bilateral exchange rates, and local GDP per capita of 14 developed market currencies for the 1st of January 2024.

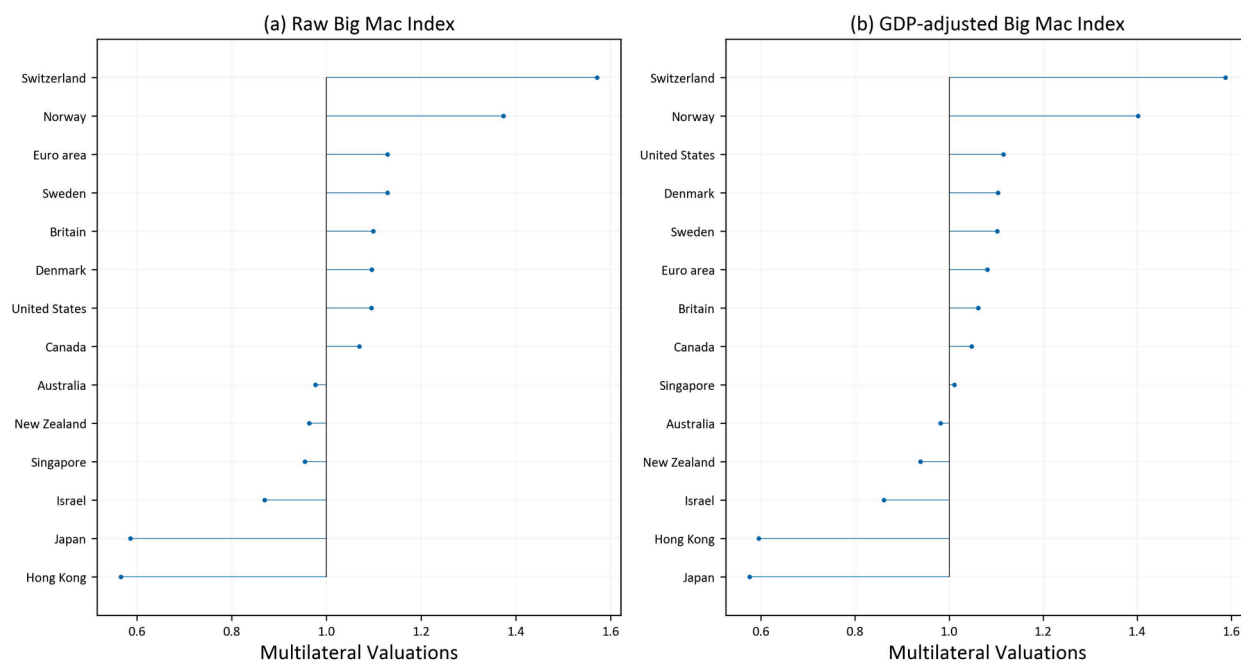


Fig. 7. Multilateral valuations.

Notes: Fig. 7 (a) displays the multilateral valuations of each currency for the Raw Big Mac index for the 1st of January 2024. Fig. 7 (b) displays the multilateral valuations of each currency for the GDP-adjusted Big Mac index for the 1st of January 2024.

adjusted Big Mac index. For example, the bilateral valuations for the GDP-adjusted Big Mac index are simply scaled (adjusted) versions of the bilateral valuations for the Raw Big Mac index. Similarly, the multilateral valuations for the GDP-adjusted Big Mac index are scaled (adjusted) versions of the multilateral valuations for the Raw Big Mac index.

This paper also highlighted on a number of linkages between the bilateral valuations and the multilateral valuations for both versions of the Big Mac index. For example, the multilateral valuations are perfectly correlated with the associated bilateral valuations for the Raw Big Mac index. Similarly, the multilateral valuations are perfectly correlated with the associated bilateral valuations for the GDP-adjusted Big Mac index. Furthermore, the multilateral adjustment factors are perfectly correlated with the associated bilateral adjustment factors for the GDP-adjusted Big Mac index.

The international economy is gradually becoming more multipolar (Eichengreen, 2024). The Economist allows readers to select from five numéraire (or base) currencies, namely, the US dollar, the Eurozone euro, the British pound, the Japanese yen, and the Chinese yuan (The Economist, 2025). These five currencies are the same five currencies used in the International Monetary Fund's Special Drawing Right, which is an international reserve asset (IMF, 2025). This paper has shown that once multilateral valuations are calculated for a system of N currencies, any bilateral valuation can be derived from the multilateral valuations. This reduces the requirement for separate cross-sectional regressions for all possible N numéraire currencies for the GDP-adjusted Big Mac index.

CRediT authorship contribution statement

Michael Kunkler: Writing – review & editing, Writing – original draft, Software, Methodology, Investigation, Formal analysis, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

A1 Introduction

For the GDP-adjusted Big Mac index, O'Brien & Ruiz de Vargas (2019) stated that a bilateral valuation of a currency can be derived from the multilateral valuation of the same currency divided by the multilateral valuation of the common numéraire currency. In an

earlier paper, the same authors stated that it was too complex to provide further details of the multilateral valuations (O'Brien & Ruiz de Vargas, 2017). This appendix derives the decomposition of a bilateral valuation of a currency into the multilateral valuation of the same currency divided by the multilateral valuation of the common numéraire currency.

A2 Local hamburger prices

The N local hamburger prices in terms of the common numéraire (η th) currency (bilateral dependent variables) in the cross-sectional regression model in Eq. (28) are related to the N local hamburger prices in terms of the multicurrency numéraire (\mathcal{M}) (multilateral dependent variables) in the cross-sectional regression model in Eq. (31):

$$\begin{aligned} H_{i/\eta} &= S_{i/\eta} H_{i/i} \\ &= \frac{1}{S_{\eta/\mathcal{M}}} S_{i/\mathcal{M}} H_{i/i} \\ &= \frac{1}{S_{\eta/\mathcal{M}}} H_{i/\mathcal{M}} \end{aligned} \tag{A1}$$

where $H_{i/\eta} = S_{i/\eta} H_{i/i}$ from Eq. (10); $S_{i/\eta} = S_{i/\mathcal{M}} / S_{\eta/\mathcal{M}}$ from Eq. (2); and $H_{i/\mathcal{M}} = S_{i/\mathcal{M}} H_{i/i}$ from Eq. (11). In general, Eq. (A1) can be written in vector notation:

$$\mathbf{H}_\eta = \frac{1}{S_{\eta/\mathcal{M}}} \mathbf{H}_\mathcal{M} \tag{A2}$$

where \mathbf{H}_η is a $N \times 1$ vector of local hamburger prices in terms of the common numéraire (η th) currency with the i th element being $H_{i/\eta}$; and $\mathbf{H}_\mathcal{M}$ is a $N \times 1$ vector of local hamburger prices in terms of the multicurrency numéraire (\mathcal{M}) with the i th element being $H_{i/\mathcal{M}}$.

In addition, the average of the local hamburger prices in terms of the common numéraire (η th) currency is related to the average of the local hamburger prices in terms of the multicurrency numéraire (\mathcal{M}):

$$\begin{aligned} \bar{H}_\eta &= \frac{1}{N} \sum_i^N H_{i/\eta} \\ &= \frac{1}{S_{\eta/\mathcal{M}}} \frac{1}{N} \sum_i^N H_{i/\mathcal{M}} \\ &= \frac{1}{S_{\eta/\mathcal{M}}} \bar{H}_\mathcal{M} \end{aligned} \tag{A3}$$

where $H_{i/\eta} = H_{i/\mathcal{M}} / S_{\eta/\mathcal{M}}$ from Eq. (A1); \bar{H}_η is the average of the N local hamburger prices in terms of the common numéraire (η th) currency; and $\bar{H}_\mathcal{M}$ is the average of the N local hamburger prices in terms of the multicurrency numéraire (\mathcal{M}).

A3 Local GDP per capita

The local GDP per capita in terms of the common numéraire η th currency (bilateral independent variables) in the cross-sectional regression model in Eq. (28) are related to the local GDP per capita in terms of the multicurrency numéraire (\mathcal{M}) (multilateral independent variables) in the cross-sectional regression model in Eq. (31):

$$\begin{aligned} G_{i/\eta} &= \frac{H_{\eta/\eta}}{H_{i/i}} G_{i/i} \\ &= \frac{H_{\eta/\eta}}{H_{\mathcal{M}/\mathcal{M}}} \frac{H_{\mathcal{M}/\mathcal{M}}}{H_{i/i}} G_{i/i} \\ &= \frac{H_{\eta/\eta}}{H_{\mathcal{M}/\mathcal{M}}} G_{i/\mathcal{M}} \end{aligned} \tag{A4}$$

where $G_{i/\eta} = (H_{\eta/\eta} / H_{i/i}) G_{i/i}$ from Eq. (26); and $G_{i/\mathcal{M}} = (H_{\mathcal{M}/\mathcal{M}} / H_{i/i}) G_{i/i}$ from Eq. (27). In general, Eq. (A4) can be written in vector notation:

$$\mathbf{G}_\eta = \frac{H_{\eta/\eta}}{H_{\mathcal{M}/\mathcal{M}}} \mathbf{G}_\mathcal{M} \tag{A5}$$

where \mathbf{G}_η is a $N \times 1$ vector of local GDP per capita in terms of the common numéraire (η th) currency with the i th element being $G_{i/\eta}$; and $\mathbf{G}_\mathcal{M}$ is a $N \times 1$ vector of local GDP per capita in terms of the multicurrency numéraire (\mathcal{M}) with the i th element being $G_{i/\mathcal{M}}$.

In addition, the average of the local GDP per capita in terms of the common numéraire (η th) currency is related to the average of the local GDP per capita in terms of the multicurrency numéraire (\mathcal{M}):

$$\begin{aligned}\bar{G}_\eta &= \frac{1}{N} \sum_i^N G_{i/\eta} \\ &= \frac{H_{\eta/\eta}}{H_{\mathcal{M}/\mathcal{M}}} \frac{1}{N} \sum_i^N G_{i/\mathcal{M}} \\ &= \frac{H_{\eta/\eta}}{H_{\mathcal{M}/\mathcal{M}}} \bar{G}_\mathcal{M}\end{aligned}\quad (\text{A6})$$

where \bar{G}_η is the average of the N local GDP per capita in terms of the common numéraire (η th) currency; $G_{i/\eta} = (H_{\eta/\eta}/H_{\mathcal{M}/\mathcal{M}})G_{i/\mathcal{M}}$ from Eq. (A4); and $\bar{G}_\mathcal{M}$ is the average of the N local GDP per capita in terms of the multicurrency numéraire (\mathcal{M}).

Furthermore, the variance of the local GDP per capita in terms of the common numéraire (η th) currency is related to the variance of local GDP per capita in terms of the multicurrency numéraire (\mathcal{M}):

$$\begin{aligned}\text{var}(\mathbf{G}_\eta) &= \sqrt{\frac{1}{N-1} \sum_i^N (G_{i/\eta} - \bar{G}_\eta)^2} \\ &= \left(\frac{H_{\eta/\eta}}{H_{\mathcal{M}/\mathcal{M}}} \right)^2 \sqrt{\frac{1}{N-1} \sum_i^N (G_{i/\mathcal{M}} - \bar{G}_\mathcal{M})^2} \\ &= \left(\frac{H_{\eta/\eta}}{H_{\mathcal{M}/\mathcal{M}}} \right)^2 \text{var}(\mathbf{G}_\mathcal{M})\end{aligned}\quad (\text{A7})$$

where $\text{var}(\mathbf{G}_\eta)$ is the variance of the N local GDP per capita in terms of the common numéraire (η th) currency; $G_{i/\eta} = (H_{\eta/\eta}/H_{\mathcal{M}/\mathcal{M}})G_{i/\mathcal{M}}$ from Eq. (A4); $\bar{G}_\eta = (H_{\eta/\eta}/H_{\mathcal{M}/\mathcal{M}})\bar{G}_\mathcal{M}$ from Eq. (A6); and $\text{var}(\mathbf{G}_\mathcal{M})$ is the variance of the N local GDP per capita in terms of the multicurrency numéraire (\mathcal{M}).

A4 Ratios of local hamburger prices to local GDP per capita values

A *bilateral ratio* is the ratio of a local hamburger price of a currency in terms of the common numéraire (η th) currency to the local GDP per capita of the same currency in terms of the common numéraire (η th) currency:

$$R_{i/\eta} = \frac{H_{i/\eta}}{G_{i/\eta}} \quad (\text{A8})$$

where $R_{i/\eta}$ is the bilateral ratio of the local hamburger price of the i th currency in terms of the common numéraire (η th) currency to the local GDP per capita of the i th currency in terms of the common numéraire (η th) currency. Similarly, a *multilateral ratio* is the ratio of a local hamburger price of a currency in terms of the multicurrency numéraire (\mathcal{M}) to the local GDP per capita of the same currency in terms of the multicurrency numéraire (\mathcal{M}):

$$R_{i/\mathcal{M}} = \frac{H_{i/\mathcal{M}}}{G_{i/\mathcal{M}}} \quad (\text{A9})$$

where $R_{i/\mathcal{M}}$ is the multilateral ratio of the local hamburger price of the i th currency in terms of the multicurrency numéraire (\mathcal{M}) to the local GDP per capita of the i th currency in terms of the multicurrency numéraire (\mathcal{M}).

The bilateral ratio in Eq. (A8) is related to the multilateral ratio in Eq. (A9):

$$\begin{aligned}R_{i/\eta} &= \frac{H_{i/\eta}}{G_{i/\eta}} \\ &= \frac{H_{i/\mathcal{M}}/S_{\eta/\mathcal{M}}}{H_{\eta/\eta}G_{i/\mathcal{M}}/H_{\mathcal{M}/\mathcal{M}}} \\ &= \frac{H_{\mathcal{M}/\mathcal{M}}}{H_{\eta/\eta}S_{\eta/\mathcal{M}}} \frac{H_{i/\mathcal{M}}}{G_{i/\mathcal{M}}} \\ &= \frac{I_{\eta/\mathcal{M}}}{S_{\eta/\mathcal{M}}} R_{i/\mathcal{M}}\end{aligned}\quad (\text{A10})$$

where $R_{i/\eta} = H_{i/\eta}/G_{i/\eta}$ from Eq. (A8); $H_{i/\mathcal{M}} = H_{i/\eta}/S_{\eta/\mathcal{M}}$ from (A1); $G_{i/\eta} = (H_{\eta/\eta}/H_{\mathcal{M}/\mathcal{M}})G_{i/\mathcal{M}}$ from Eq. (A4); $I_{\eta/\mathcal{M}} = H_{\mathcal{M}/\mathcal{M}}/H_{\eta/\eta}$ from Eq. (14); and $R_{i/\mathcal{M}} = H_{i/\mathcal{M}}/G_{i/\mathcal{M}}$ from Eq. (A9). In general, Eq. (A10) can be written in vector notation:

$$\mathbf{R}_\eta = \frac{I_{\eta/\mathcal{M}}}{S_{\eta/\mathcal{M}}} \mathbf{R}_\mathcal{M} \tag{A11}$$

where \mathbf{R}_η is a $N \times 1$ vector of bilateral ratios of a local hamburger price in terms of the common numéraire (η th) currency to the local GDP per capita in terms of the common numéraire (η th) currency with the i th element being $R_{i/\eta}$; and $\mathbf{R}_\mathcal{M}$ is a $N \times 1$ vector of multilateral ratios of the local hamburger prices in terms of the multicurrency numéraire (\mathcal{M}) to the local GDP per capita values in terms of the multicurrency numéraire (\mathcal{M}) with the i th element being $R_{i/\mathcal{M}}$.

The N bilateral ratios relative to common numéraire (η th) currency are perfectly correlated with the N multilateral ratios relative to the multicurrency numéraire (\mathcal{M}), so that:

$$\begin{aligned} \text{cor}(\mathbf{R}_\eta, \mathbf{R}_\mathcal{M}) &= \frac{\text{cov}(\mathbf{R}_\eta, \mathbf{R}_\mathcal{M})}{\sqrt{\text{var}(\mathbf{R}_\eta)\text{var}(\mathbf{R}_\mathcal{M})}} \\ &= \frac{\text{cov}(I_{\eta/\mathcal{M}}\mathbf{R}_\mathcal{M}/S_{\eta/\mathcal{M}}, \mathbf{R}_\mathcal{M})}{\sqrt{\text{var}(I_{\eta/\mathcal{M}}\mathbf{R}_\mathcal{M}/S_{\eta/\mathcal{M}})\text{var}(\mathbf{R}_\mathcal{M})}} \\ &= \frac{I_{\eta/\mathcal{M}}\text{var}(\mathbf{R}_\mathcal{M})/S_{\eta/\mathcal{M}}}{I_{\eta/\mathcal{M}}\text{var}(\mathbf{R}_\mathcal{M})/S_{\eta/\mathcal{M}}} \\ &= 1 \end{aligned} \tag{A12}$$

where $\mathbf{R}_\eta = I_{\eta/\mathcal{M}}\mathbf{R}_\mathcal{M}/S_{\eta/\mathcal{M}}$ from Eq. (A11); $\text{cov}(\mathbf{R}_\eta, \mathbf{R}_\mathcal{M})$ is the covariance between the bilateral ratios relative to the common numéraire (η th) currency and the multilateral ratios relative to the multicurrency numéraire (\mathcal{M}); $\text{var}(\mathbf{R}_\eta)$ is the variance of the bilateral ratios relative to the common numéraire (η th) currency; and $\text{var}(\mathbf{R}_\mathcal{M})$ is the variance of the multilateral ratios relative to the multicurrency numéraire (\mathcal{M}).

A5 Estimates of the slope terms

The cross-sectional regression models in Eq. (28) and Eq. (31) are both estimated using ordinary least squares (OLS). The OLS estimate of the bilateral slope term in the cross-sectional regression model in Eq. (28) is related to the OLS estimate of the multilateral slope term in the cross-sectional regression model in Eq. (31):

$$\begin{aligned} \widehat{\beta}_\eta &= \frac{\text{cov}(\mathbf{H}_\eta, \mathbf{G}_\eta)}{\text{var}(\mathbf{G}_\eta)} \\ &= \frac{\text{cov}\left(\frac{1}{S_{\eta/\mathcal{M}}}\mathbf{H}_\mathcal{M}, (H_{\eta/\eta}/H_{\mathcal{M}/\mathcal{M}})\mathbf{G}_\mathcal{M}\right)}{\text{var}\left((H_{\eta/\eta}/H_{\mathcal{M}/\mathcal{M}})\mathbf{G}_\mathcal{M}\right)} \\ &= \frac{1}{S_{\eta/\mathcal{M}}(H_{\eta/\eta}/H_{\mathcal{M}/\mathcal{M}})} \frac{\text{cov}(\mathbf{H}_\mathcal{M}, \mathbf{G}_\mathcal{M})}{\text{var}(\mathbf{G}_\mathcal{M})} \\ &= \frac{1}{S_{\eta/\mathcal{M}}(H_{\eta/\eta}/H_{\mathcal{M}/\mathcal{M}})} \widehat{\beta}_\mathcal{M} \end{aligned} \tag{A13}$$

where $\widehat{\beta}_\eta = \text{cov}(\mathbf{H}_\eta, \mathbf{G}_\eta)/\text{var}(\mathbf{G}_\eta)$ is the OLS estimate of the bilateral slope term in the cross-sectional regression model in Eq. (28); $\mathbf{H}_\eta = \mathbf{H}_\mathcal{M}/S_{\eta/\mathcal{M}}$ from Eq. (A2); $\mathbf{G}_\eta = (H_{\eta/\eta}/H_{\mathcal{M}/\mathcal{M}})\mathbf{G}_\mathcal{M}$ from Eq. (A5); $\widehat{\beta}_\mathcal{M} = \text{cov}(\mathbf{H}_\mathcal{M}, \mathbf{G}_\mathcal{M})/\text{var}(\mathbf{G}_\mathcal{M})$ is the OLS estimate of the multilateral slope term in the cross-sectional regression model in Eq. (31); $\text{cov}(\mathbf{H}_\eta, \mathbf{G}_\eta)$ is the covariance between the N local hamburger prices in terms of the common numéraire (η th) currency and the N local GDP per capita in terms of numéraire (η th) currency; $\text{var}(\mathbf{G}^\eta)$ is the variance of the N local GDP per capita in terms of the common numéraire (η th) currency; $\text{cov}(\mathbf{H}_\mathcal{M}, \mathbf{G}_\mathcal{M})$ is the covariance between the N local hamburger prices in terms of the multicurrency numéraire (\mathcal{M}) and the N local GDP per capita in terms of multicurrency numéraire (\mathcal{M}); and $\text{var}(\mathbf{G}_\mathcal{M})$ is the variance of the N local GDP per capita in terms of the multicurrency numéraire (\mathcal{M}).

A6 Estimates of the intercept terms

The OLS estimate of the bilateral intercept term in the cross-sectional regression model in Eq. (28) is related to the OLS estimate of the multilateral intercept term in the cross-sectional regression model in Eq. (31):

$$\begin{aligned}
 \hat{\alpha}_\eta &= \bar{H}_\eta - \hat{\beta}_\eta \bar{G}_\eta \\
 &= \frac{1}{S_{\eta|\mathcal{M}}} \bar{H}_\mathcal{M} - \frac{(H_{\eta|\eta}/H_{\mathcal{M}|\mathcal{M}})}{S_{\eta|\mathcal{M}}(H_{\eta|\eta}/H_{\mathcal{M}|\mathcal{M}})} \hat{\beta}_{\mathcal{M}} \bar{G}_\mathcal{M} \\
 &= \frac{1}{S_{\eta|\mathcal{M}}} (\bar{H}_\mathcal{M} - \hat{\beta}_{\mathcal{M}} \bar{G}_\mathcal{M}) \\
 &= \frac{1}{S_{\eta|\mathcal{M}}} \hat{\alpha}_\mathcal{M}
 \end{aligned}
 \tag{A14}$$

where $\hat{\alpha}_\eta$ is the OLS estimate of the bilateral intercept term in the cross-sectional regression model in Eq. (28); $\hat{\alpha}_\mathcal{M}$ is the OLS estimate of the multilateral intercept term in the cross-sectional regression model in Eq. (31); $\bar{H}_\eta = S_{\eta|\mathcal{M}} \bar{H}_\mathcal{M}$ from Eq. (A3); $\bar{G}_\eta = (H_{\eta|\eta}/H_{\mathcal{M}|\mathcal{M}}) \bar{G}_\mathcal{M}$ from Eq. (A6); and $\hat{\beta}_\eta = \hat{\beta}_{\mathcal{M}} / (S_{\eta|\mathcal{M}} H_{\eta|\eta}/H_{\mathcal{M}|\mathcal{M}})$ from Eq. (A13).

A7 Fitted prices

The fitted (GDP-adjusted) hamburger price for a currency in terms of the common numéraire currency from the cross-sectional regression model in Eq. (28) is related to the fitted GDP-adjusted hamburger price for the same currency in terms of the multicurrency numéraire from the cross-sectional regression model in Eq. (31):

$$\begin{aligned}
 \hat{H}_{i|\eta} &= \hat{\alpha}_\eta + \hat{\beta}_\eta G_{i|\eta} \\
 &= \frac{1}{S_{\eta|\mathcal{M}}} \hat{\alpha}_\mathcal{M} + \frac{(H_{\eta|\eta}/H_{\mathcal{M}|\mathcal{M}})}{S_{\eta|\mathcal{M}}(H_{\eta|\eta}/H_{\mathcal{M}|\mathcal{M}})} \hat{\beta}_{\mathcal{M}} G_{i|\mathcal{M}} \\
 &= \frac{1}{S_{\eta|\mathcal{M}}} (\hat{\alpha}_\mathcal{M} + \hat{\beta}_{\mathcal{M}} G_{i|\mathcal{M}}) \\
 &= \frac{1}{S_{\eta|\mathcal{M}}} \hat{H}_{i|\mathcal{M}}
 \end{aligned}
 \tag{A15}$$

where $\hat{H}_{i|\eta}$ is the fitted hamburger price for the i th currency in terms of the common numéraire (η th) currency; $\hat{H}_{i|\mathcal{M}}$ is the fitted hamburger price for the i th currency in terms of the multicurrency numéraire (\mathcal{M}); $\hat{\alpha}_\eta = \hat{\alpha}_\mathcal{M} / S_{\eta|\mathcal{M}}$ from Eq. (A14); $\hat{\beta}_\eta = \hat{\beta}_{\mathcal{M}} / (S_{\eta|\mathcal{M}} H_{\eta|\eta}/H_{\mathcal{M}|\mathcal{M}})$ from Eq. (A13); and $G_{i|\eta} = (H_{\eta|\eta}/H_{\mathcal{M}|\mathcal{M}}) G_{i|\mathcal{M}}$ from Eq. (A4).

A8 Valuation decomposition

The bilateral valuation of a currency relative to the common numéraire currency can be decomposed into two multilateral valuations: the multilateral valuation of the same currency and the multilateral valuation of the common numéraire currency:

$$\begin{aligned}
 V_{i|\eta}^A &= \left(\frac{H_{i|\eta}}{\hat{H}_{i|\eta}} \right) / \left(\frac{H_{\eta|\eta}}{\hat{H}_{\eta|\eta}} \right) \\
 &= \left(\frac{H_{i|\mathcal{M}}/S_{\eta|\mathcal{M}}}{\hat{H}_{i|\mathcal{M}}/S_{\eta|\mathcal{M}}} \right) / \left(\frac{H_{\eta|\mathcal{M}}/S_{\eta|\mathcal{M}}}{\hat{H}_{\eta|\mathcal{M}}/S_{\eta|\mathcal{M}}} \right) \\
 &= \left(\frac{H_{i|\mathcal{M}}}{\hat{H}_{i|\mathcal{M}}} \right) / \left(\frac{H_{\eta|\mathcal{M}}}{\hat{H}_{\eta|\mathcal{M}}} \right) \\
 &= \left(\frac{H_{i|\mathcal{M}}/H_{\mathcal{M}|\mathcal{M}}}{\hat{H}_{i|\mathcal{M}}/\hat{H}_{\mathcal{M}|\mathcal{M}}} \right) / \left(\frac{H_{\eta|\mathcal{M}}/H_{\mathcal{M}|\mathcal{M}}}{\hat{H}_{\eta|\mathcal{M}}/\hat{H}_{\mathcal{M}|\mathcal{M}}} \right) \\
 &= \frac{V_{i|\mathcal{M}}^A}{V_{\eta|\mathcal{M}}^A}
 \end{aligned}
 \tag{A16}$$

where $V_{i|\eta}^A = \frac{H_{i|\eta}/H_{\eta|\eta}}{H_{i|\eta}/H_{\eta|\eta}}$ from Eq. (29); $H_{i|\eta} = H_{i|\mathcal{M}}/S_{\eta|\mathcal{M}}$ from Eq. (A1); $H_{\eta|\eta} = H_{\eta|\mathcal{M}}/S_{\eta|\mathcal{M}}$ from Eq. (A1); $\hat{H}_{i|\mathcal{M}} = \hat{H}_{i|\mathcal{M}}/S_{\eta|\mathcal{M}}$ from (A15); $\hat{H}_{\eta|\mathcal{M}} = \hat{H}_{\eta|\mathcal{M}}/S_{\eta|\mathcal{M}}$ from Eq. (A15); $V_{i|\mathcal{M}}^A = \frac{H_{i|\mathcal{M}}/H_{\mathcal{M}|\mathcal{M}}}{H_{i|\mathcal{M}}/H_{\mathcal{M}|\mathcal{M}}}$ from Eq. (32); and $V_{\eta|\mathcal{M}}^A = \frac{H_{\eta|\mathcal{M}}/H_{\mathcal{M}|\mathcal{M}}}{H_{\eta|\mathcal{M}}/H_{\mathcal{M}|\mathcal{M}}}$ from Eq. (32).

In summary, the bilateral valuation of a currency can be decomposed into a multilateral valuation of the same currency divided by the multilateral valuation of the common numéraire currency.

A9 Scaling local GDP per capita

The local GDP per capita are large values relative to the associated local hamburger prices. To make the cross-sectional slope term

easier to interpret, local GDP per capita are scaled, typically divided by 1000. The scaling the N bilateral local GDP per capita values by 1000 can be written in vector notation:

$$\mathbf{G}_\eta^* = \frac{1}{1000} \mathbf{G}_\eta \quad (\text{A17})$$

where \mathbf{G}_η^* is a $N \times 1$ vector of *scaled* local GDP per capita in terms of the common numéraire (η th) currency with the i th element being $G_{i/\eta}^*$; and \mathbf{G}_η is a $N \times 1$ vector of *unscaled* local GDP per capita in terms of the common numéraire (η th) currency with the i th element being $G_{i/\eta}$.

Similarly, the scaling the N multilateral local GDP per capita values by 1000 can be written in vector notation:

$$\mathbf{G}_\#^* = \frac{1}{1000} \mathbf{G}_\# \quad (\text{A18})$$

where $\mathbf{G}_\#^*$ is a $N \times 1$ vector of *scaled* local GDP per capita in terms of the multicurrency numéraire ($\#$) with the i th element being $G_{i/\#}^*$; and $\mathbf{G}_\#$ is a $N \times 1$ vector of *unscaled* local GDP per capita in terms of the multicurrency numéraire ($\#$) with the i th element being $G_{i/\#}$.

Although the scale does not impact the fitted values, alpha term, or t-statistics, it does impact the slope term. The OLS estimator of the bilateral slope term in cross-sectional regression model in Eq. (28) using \mathbf{G}_η^* , rather than \mathbf{G}_η , is:

$$\hat{\beta}_{\eta^*} = \frac{\text{cov}(\mathbf{H}_\eta, \mathbf{G}_\eta^*)}{\text{var}(\mathbf{G}_\eta^*)} = \frac{\text{cov}(\mathbf{H}_\eta, \mathbf{G}_\eta/1000)}{\text{var}(\mathbf{G}_\eta/1000)} = 1000 \frac{\text{cov}(\mathbf{H}_\eta, \mathbf{G}_\eta)}{\text{var}(\mathbf{G}_\eta)} = 1000 \hat{\beta}_\eta \quad (\text{A19})$$

where $\hat{\beta}_{\eta^*} = \text{cov}(\mathbf{H}_\eta, \mathbf{G}_\eta^*)/\text{var}(\mathbf{G}_\eta^*)$ is the scaled OLS estimator of the bilateral slope term; $\mathbf{G}_\eta^* = \mathbf{G}_\eta/1000$ from Eq. (A17); and $\hat{\beta}_\eta = \text{cov}(\mathbf{H}_\eta, \mathbf{G}_\eta)/\text{var}(\mathbf{G}_\eta)$ is the unscaled OLS estimator of the bilateral slope term.

Similarly, the OLS estimator of the multilateral slope term for a cross-sectional regression model in Eq. (31) using $\mathbf{G}_\#^*$, rather than $\mathbf{G}_\#$, is:

$$\hat{\beta}_{\#^*} = \frac{\text{cov}(\mathbf{H}_\#, \mathbf{G}_\#^*)}{\text{var}(\mathbf{G}_\#^*)} = \frac{\text{cov}(\mathbf{H}_\#, \mathbf{G}_\#/1000)}{\text{var}(\mathbf{G}_\#/1000)} = 1000 \frac{\text{cov}(\mathbf{H}_\#, \mathbf{G}_\#)}{\text{var}(\mathbf{G}_\#)} = 1000 \hat{\beta}_\# \quad (\text{A20})$$

where $\hat{\beta}_{\#^*} = \text{cov}(\mathbf{H}_\#, \mathbf{G}_\#^*)/\text{var}(\mathbf{G}_\#^*)$ is the scaled OLS estimator of the multilateral slope term; $\mathbf{G}_\#^* = \mathbf{G}_\#/1000$ from Eq. (A18); and $\hat{\beta}_\# = \text{cov}(\mathbf{H}_\#, \mathbf{G}_\#)/\text{var}(\mathbf{G}_\#)$ is the unscaled OLS estimator of the multilateral slope term.

Data availability

Data will be made available on request.

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