Logic Science and Engineering in Wittgenstein's *Tractatus*

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Abstract

The *Tractatus* is widely held as showing that logic is a *sui generis* discipline, independent of the empirical and the psychological. Logic deals in *tautologies*, independent of and not representative of the world, and true in all circumstances. The logic proposed is however deemed to be irreparably flawed because the key notion *object* is inconsistent/unsatisfiable, and because elementary propositions are not independent of one another (colour exclusion problem).

The argument presented aims to reconstruct the logic offered within the resources available in the text to show that Wittgenstein was aware of these difficulties and that he had responses to them.

Chapter 1 presents logic as Wittgenstein inherited it from Russell, as universalist, non-psychologistic, and somehow related to science. Chapters 2 and 3 consider the logical system as formal/uninterpreted, but as intended to be applicable and thus consistent. Tensions between logic as wholly *sui generis* or as somehow involved in the empirical are discussed. What is formal and what is empirical, and what is necessary and what accidentally general, are clearly demarcated.

Chapter 4 relates to tensions between objects as nameable individuals, and the context principle. It is argued that Wittgenstein does not think in familiar terms of quantification/first-order logic. A critical difference between Russellian propositional functions and Tractarian functions, and between quantification and generalisation, is brought out.

Chapters 5, 6, and 7 discuss logical space as a space of possibilities, and how this together with the logical treatment of probability enables a probabilistic account of laws of nature. The account relies on totalities of objects and propositions, deploying S5B.

In Chapter 8 models are presented based on nineteenth century structural chemistry (for objects) and a relational view of space (exclusion problems) to show how the logical system can be regarded as consistent.

Contents

Introduction

There are many ways of reading the *Tractatus*. In the following I pursue one particular way which involves taking seriously Wittgenstein's claim at 5.557 that "logic has to be in contact with its application", the application of logic being, I suggest, its use in thinking and reasoning about what there is, where by what there is I have in mind objects, construed as physical entities. What follows is a realist interpretation with objects construed, ultimately, as entities discovered by scientists, and not by logicians, or metaphysicians. The position advanced has an implicit antecedent; if objects are physical entities, then ...; where the formal/logical aspects of objects unfold over the course of the argument, with the physical realisation filled out in Chapter 8.

This approach also involves taking entirely seriously 2.025, that "It [substance] is form *and* content" (emphasis added), where *substance* is the objects that there are; "Objects make up the substance of the world." (2.021) Logic, I suggest, as espoused in the *Tractatus*, requires objects that have forms, where the form of an object is its range of combinatorial possibilities into states of affairs. Logic so construed deals essentially with *possibility*; all else beyond this relates to the *content* of objects, and thus falls without the purview of the logician. For the logician it is enough that there are objects, and that objects have forms. A number of expository difficulties arise because commentators sometimes fail to appreciate what, exactly, Wittgenstein places within the purview of the logician, and what he thinks is somebody else's undertaking.

Wittgenstein's notion of the application of logic involves him in, I think, a commitment to *logical realism*, that there is some sense in which validity and consequence are underwritten by *something* about what there is. This, I take it, lies behind the claim at 6.124, that "something about the world must be indicated [*anzeigen*] by the fact that certain combinations of symbols—whose essence involves the possession of a determinate character—

are tautologies."

6.124 is, along with 2.025 and 5.557, central to my reading of the *Tractatus*. The key question is whether logic has a subject-matter of its own. The orthodoxy, if such there is, is that Wittgenstein shows that logic does not have a subject-matter of its own, that the propositions of logic are tautologous, effectively turning logic into a *sui generis* discipline that is not involved in any direct sense in the things of the world. But this leaves logic awkwardly situated, because it is not obvious that logic can be both an autonomous or *sui generis* discipline answerable only to its own internal standards *and* that it can be deployed in thinking about and reasoning about what there is; that the propositions of logic *show* something about the world.

The *sui generis* conception is the first of three broad ways of thinking about logic. It assumes that logic needs no external justification or explication or grounding, that it is sufficient unto itself. Behind this lies the intuition that to think illogically is not really to think at all, and that if someone does think illogically, on having this pointed out they will accept such correction and amend their reasoning practices accordingly. What is pointed out is not a mistake, because to make a mistake is to be wrong about something. A failure in logic is not to be wrong *about* something. If somebody persists in illogicality then sooner or later we give up, and decide that they are not one of us.

I do not want to deny the attractions of this line of thinking, but it features in *On Certainty* and not, I suggest, in the *Tractatus*. In his early work Wittgenstein would, I think, have regarded this as *psychologistic*, this being the second broad way of thinking about logic. The psychological logician regards thinking as somehow bound up with the structures, functions, faculties or processes of the human mind or brain. Wittgenstein would I think have endorsed Frege's rejection of psychologism in *Grundgesetze*, i pp.xvff, reflected in 4.1121. For present purposes I assume psychologism can be discounted.

The third broad way of thinking about logic is *logical realism*. But to be a realist about logic one need not be committed to a full-blooded empiricism about logic, that logic is an empirical science. *Prima facie*, though, it is not obvious that one can be a realist about logic without logic collapsing into physics, perhaps taking mathematics with it.

The project Wittgenstein pursues in the *Tractatus* is, I think, to engineer

a *sui generis* conception of logic that has nothing to do with psychologism, on the one hand, and is not prone to collapse into physics, on the other. This balancing act is sketched in 5.557:

> The *application* of logic decides what elementary propositions there are. What belongs to its application, logic cannot anticipate. It is clear that logic must not clash with its application. But logic has to be in contact with its application. Therefore logic and its application must not overlap. (5.557)

For the first proposition, what counts as an elementary proposition is a concatenation of names of objects (4.22) such that the combination represents a concatenation of objects that can occur (2.03), a state of affairs that can be a fact, such that the elementary proposition in question is at any given moment either true, or false. This is effectively definitional. However "what" is better translated as "which" (the German is *welche*), and "decides" (*entscheidet*) should be thought of as recognition; which strings of names should be acknowledged, by us, as propositional. Logic in this context does not *do* anything, it has no executive function.

In the subsequent propositions of 5.557 Wittgenstein draws a line and says, on this side we have logic—the acknowledgement of the propositional—and beyond that we have what pertains to the application of logic, this being the empirical, whatever there is to be said about the *content* of objects. Logic deals solely with the possible, with the *forms* of objects, their range of combinatorial possibilities into states of affairs. The *names* of objects have concomitant combinatorial possibilities into elementary propositions; at a further stage, states of affairs can stand to one another to form situations, with the latter represented by complex propositions (truth-functional combinations of elementary propositions). The application of logic involves the acknowledgment of what is propositional, and implicitly the rejection of what is not propositional.

For logic and its application to be in contact, without overlapping, there has to be a precisely drawn boundary between the logical, and the empirical; in the context of language, between the senseless and the meaningful. The denial that logic has a subject-matter of its own is bound up with the claim that the former *shows* (*anzeigen*) something about the latter (6.124), but does not *say* anything.

To make good on this conception of logic as *sui generis*, as not having a subject-matter of its own, as being autonomous and not responsive to or accountable to any external considerations, Wittgenstein has to make good on a hard distinction between what is, and what is not, logical. In particular given the colour exclusion problem (6.3751) it is, I think, incumbent on any interpretation that it shows how Wittgenstein thought he could address this logically, within the resources available in the text. Hacker suggests that the colour exclusion problem "might appear [for Wittgenstein] a matter of detail" (Hacker 2021, p.108), but it isn't. It is I think critical that Wittgenstein had a way of addressing this, because it is the point in the text where the hard distinction between the logical and the empirical, the point where logic is in contact with its application, is located. A defence that ultimately fails is good enough, but no defence at all is not.

When Wittgenstein wrote the *Tractatus* logic was in flux, between the universalist systems of Frege, and Russell and Whitehead, and the first-order model-theoretic conception that emerged in the 1930s. This is discussed in Chapters 2 and 3. Wittgenstein's *sui generis* conception of logic as having no subject-matter of its own perhaps influenced this. The full-blooded model-theoretic conception of logic is a *sui generis* discipline, responsive only to internal metalogical criteria (soundness, completeness, etc.), but this goes together with a sharp separation between logical systems, and application. The logician devises systems with nice formal properties and offers them to others to find uses, or applications.

This is not the project of the *Tractatus*, but one way of thinking of the logic of the *Tractatus*, of rounding out the conception of logic as *sui generis*, is to regard it as an uninterpreted calculus, in need of an interpretation. Much of Chapter 2 engages with a suggestion of Grayling's, along these lines. I accept that regarding the *Tractatus* as expounding a logical system, and then treating this as an uninterpreted calculus, is a not unreasonable way of going about things, but I do want to emphasise that on this basis one cannot simply help oneself to *truth* (and *falsity*) as properties realised by propositions. Applying the logic of the *Tractatus* involves recognising the propositional as *bipolar*, as either true or false and, at any given moment, either one or the other. It is not a matter of recognising or acknowledging some formal property, defined within a formal system. In order not to foreclose on the issues I have, particularly in Chapter 2, used $\top\bot$ rather than

TF, in considering the logic of the *Tractatus*, where the former indicates the propositional in terms of an implicitly defined property prior to interpretation (the possession of sense), the latter, the propositional in terms of being *actually* true (being the case in the world) or being *actually* false (what could be the case but is not at present).

The way in which Wittgenstein thinks of logic as *sui generis* is not the full-blooded model-theoretic way. But the reason I do not want to foreclose on this is because without the *sui generis* psychologistic approach sketched earlier, the full-blooded model-theoretic approach is the only obvious alternative. What I want to argue is that once one gets embroiled in truth, and falsity, the Tractarian *sui generis* approach becomes unstable. The question then becomes, if not psychologism, can one maintain applicability in the terms stated, without logic going empirical?

The logic of the *Tractatus* is, in the main, a propositional calculus (there are, as will be argued, few vestiges of predicate calculus and quantification). What is needed for the application of this calculus is elementary propositions. As noted elementary propositions are combinations of names for objects, and since there are no logical objects, elementary propositions do not contain logical constants. An elementary proposition says, this is how things (objects) stand, and it is true if they do so stand, false otherwise; consequently one of the burdens on interpretation is to show how a concatenation of names is propositional.

The critical point becomes, what is involved in an elementary proposition expressing a sense, that is, being bipolar, capable of being true, or of being false. The point of contact between language and reality is, I think, conveyed by 3.203; "A name means an object. The object is its meaning." What Wittgenstein intends is, in a strong sense, that names and objects do not come apart. In thinking or speaking a proposition one thinks or speaks the objects themselves. There is no question of interpretation as a further step or process on top of thinking and speaking a proposition. A proposition is not a combination of the things themselves, but in a sense that is not intended to be metaphorical it involves or carries the objects named along with it. This I suggest lies behind the curious parenthetical conclusion of 3.203, that "'A' is the same sign as 'A'." That one cannot get outside of or beyond language goes without saying, but I think it is a mistake to expect or ask for any further elucidation than "A" means "A"; the object is already

present and correct.

For Hacker, "[S]imple objects are ... the final residue of analysis, the indecomposable elements that are the meanings of the unanalysable names that occur in elementary propositions. There *must* be such things." (Hacker 2021, p.65) He then discusses what he terms the " 'naming-relation', the association of a name ... with its meaning, viz. an object" (Hacker 2021, p.73), quoting from *Notebooks 1914–16*; "Names are necessary ... They link the propositional form with quite definite objects. And if the general description of the world is like a stencil of the world, the names pin [*nageln*] it to the world." (*Notebooks 1914–16*, p.53)

This is fine so far, but Hacker then asks, "What did he conceive to be the nature of the correlation of name and object? How is it to be effected?" (Hacker 2021, p.73) But there is nothing in the *Tractatus* to indicate an *act* of naming. What Wittgenstein says is this:

> In a proposition a thought can be expressed in such a way that elements of the propositional sign correspond to the objects of thought. (3.2)
> I call such elements 'simple signs' ... the simple signs employed in propositions are called names. (3.201, 3.202)

There is no obvious sense in which attaching or assigning a name is something that we do. In this context Pears makes the logical point, that if the naming of objects is the crux of application—where logic and language gets started—then naming is somehow prior to all else; "In the *Tractatus* the beginning of language is the naming of objects" (Pears 1987, p.9), that "representation ... requires an initial correlation between name and object" (*op. cit.* p.75), and of what follows "when a name has been attached to an object" (*op. cit.* p.111).

But to see this as something that we do is, I think, a mistake. When Wittgenstein says, "all the propositions of our everyday language, just as they stand, are in perfect logical order" (5.5563), he means that if we could carry out an analysis all the way down to names, then we would see that the objects are all there, present and correct. It would of course be "a lunatic account", as Diamond points out (Diamond 1985, p.193), to claim that a speaker of a language must be in command of all of this apparatus (all the "little wires", as Diamond puts it (Diamond 1985, p.193)) in order to speak meaningfully. Wittgenstein presumes, nevertheless, that in making use of

everyday propositions all of this, all the way down to objects, is in place.[1] The contact that is involved in application is, I suggest, that there is no meaningful or statable gap or distinction to be drawn between a name and an object. If a proposition is true, whatever might be involved in analysis and in naming, the objects do so stand. Objects are, I suggest, real; they are what there is, they are the substance of the world. My aim in the following is in large part to substantiate this claim.

Although the title notes logic, science, and engineering, the coverage in the following is weighted towards the first, and then the second. What I have to *say* about Wittgenstein the engineer is mostly limited to the introduction. There is, though, a great deal of *showing* of Wittgenstein's sensibility as an engineer in the following, paying due respect to von Wright's remark, "the two most important facts to remember about Wittgenstein were, firstly, that he was Viennese, and secondly, that he was an engineer" (quoted at Janik and Toulmin 1973, pp.58–9).

There are two respects in which Wittgenstein's approach is fundamentally that of the engineer. The first is a profound sense of visualisation, a way of seeing things. In the *Tractatus*, Wittgenstein says, "It is obvious that a proposition of the form '*aRb*' strikes us as a picture" (4.012), and in later reflections, "I meant that I could insert a picture, literally a drawing, into a proposition ... I could accordingly use a picture in the same way as a proposition." (*Ludwig Wittgenstein and the Vienna Circle*, p.185) Unless one thinks in such fashion it is, I suggest, not obvious at all. A fundamental mode of thinking for the engineer is of turning what one envisages in mind into drawings, where these encode all the information that is required for somebody else to reproduce whatever-it-is. For the engineer the transition from one medium to another such that all the requisite information is encoded comes naturally. This way of thinking goes together with determinacy of sense, that vagueness is precluded. There cannot be anything vague or undetermined in an engineering drawing.[2] This facility at moving between different forms of representation while encoding all the requisite information

[1] Cf. Carruthers 1989, p.127.

[2] So I disagree with Hamilton's remark that "engineer's drawings ... communicate more or less precise information." (Hamilton 2001, p.66) Engineering drawings communicate precise information, to enable the shop floor to make exactly what is intended; they epitomise determinacy of sense. Imprecision is a design error, triggering a shop floor query for the drawing to be rendered precise.

is shot through the 4.1s, with its talk in particular of Wittgenstein's other great passion, for music (the transitions from the musical notation of the score to the performance to the recording thereof).

The picture theory of the proposition with the concomitant notion of a "method of projection" as how one "think[s] the sense of the proposition" (3.11) is, I think, an expression of the sensibility of the engineer. When Wittgenstein says, "Reality is compared with propositions" (4.05), what he has in mind is, I suggest, that in thinking propositionally we think what there is; and we can reorganise what we think in order to represent not only what is the case, but also what can be the case. We can run through the possibilities in thought as a reorganising and reconstructing, but always as essentially a visualising, a what can be projected as a way in which things can so stand. But in such thinking we are guided or constrained by objective possibilities. There need not be any essentially private psychological contribution involved. This, I suggests, furnishes a way of thinking about logical space and the non-representational nature of the logical constants (circumventing the *Satzverband* problem, a central topic in Chapter 5).

When Wittgenstein says that "logical form ... is mirrored in propositions" (4.121), that logic is "all-embracing ... an infinitely fine network, the great mirror" (5.511), that the sense of the rules for the logical is "mirrored" in the symbols for them (5.514), that logic is "a mirror-image of the world" (6.13), what he is expressing, I think, is a literal seeing of an ordering and structuring immanent in what there is. A seeing, that is, with the mind's eye of how things can be moved and ordered with respect to one another, an envisaging of possibilities and a running through of them in the mind.

This leads into the second point, that the task of the engineer is to come up with an optimum solution within given parameters, where the parameters or constraints arise elsewhere, external to the task at hand. This is, I think, exemplified in the Preface, where Wittgenstein says, "I therefore believe myself to have found on all essential points the final solution of the problems." (p.4) The problems are those set by, principally, Frege, and Russell, and what Wittgenstein says is, given *these* problems, i.e., accepting *these* parameters, I have engineered the optimum solution.[3] It may be, as he goes on to say, that very little is achieved thereby, but that is not of concern to the engineer (and in the following the topics of solipsism, mysticism, ethics

[3]Cf. L.W. to B.R. 13 March 1919, at *Wittgenstein in Cambridge*, p.89.

and aesthetics are not addressed, not because they are not important, but because I have nothing to say on these worth saying).

The parameters are, I take it, that sense is determinate (3.23), that states of affairs obtain entirely independent of one another (2.061), that logic is applicable to what there is (5.557), that the propositions of our everyday language are in perfectly good logical order just as they stand (5.5563), that the propositions of logic are tautologies, are not representative (4.462). The solution that is meticulously engineered aims to show how these can be reconciled, that while logic is immanent in our thinking and our reasoning and is, it will be argued, consequently immanent in what there is, it is nevertheless also somehow independent of what there is. I do not dispute that Wittgenstein ostensibly regards logic as a *sui generis* discipline with no subject-matter of its own, but I will argue that what Wittgenstein took logic to be, and what his views commit him to, ultimately come apart.

Of 6.124, the significant passage is, "Something about the world must be indicated by the fact that certain combinations of symbols—whose essence involves the possession of a determinate character—are tautologies." This follows 6.1, "The propositions of logic are tautologies." Wittgenstein is thus committed, I think, to logical realism, that what makes tautologies true (and contradictions false) is, in some sense that stands in need of explication, bound up with what there is; "*logical realism* [is] the doctrine that statements attributing logical properties and relations, such as '"0 = 0" is logically true' or '"0 = 0" does not imply "1 = 0"', are true or false independently of our holding them to be true, our psychology, our linguistic and inferential conventions, and other facts about human beings." (Resnik 1997, p.162) On this basis the Wittgenstein of the *Tractatus* is not, I hold, an evaluationist, a non-factualist, or a non-cognitivist about logical propositions, or about inference (consequence).[4] In particular I do not think Wittgenstein would accept Resnik's "restrained *logical non-cognitivism*", as this characterises logical propositions as "neither true nor false." (Resnik 1997, p.167) Wittgenstein is clear that tautologies are true, and contradictions false (4.46). Difficulties with this are discussed in §2.9

The creative tension in and, I think, the source of the fascination of the *Tractatus* is that it is an optimised solution within tightly constrained

[4]For evaluationism see Field 2000, for non-factualism, Boghossian 2000, particularly pp.236–8, and Shapiro's discussion of Resnik at Shapiro 2000, pp.344–57.

parameters, a thought captured, perhaps, in a 1931 remark:

> I think there is some truth in my idea that I am really only reproductive in my thinking. I think I have never *invented* a line of thinking but that it was always provided for me by someone else & I have done no more than passionately take it up for my work of clarification. (*Culture and Value*, p.16)

This is, I suggest, an acute description of the *Tractatus*. The present aim is to explore the system of the *Tractatus* with an eye on the predetermined parameters, to try to work out exactly why these became intolerable; why the "final solution" in 1918 could come to be regarded as irreparably broken barely a decade later.

As far as science goes, this, particularly the 6.3s, is addressed in detail in Chapter 7. For the moment I note that my interpretation is, in general, light on metaphysics. This follows on Wittgenstein's hard distinction between the logical (the necessary, the tautological, the senseless) and what can be termed *Wissenschaft*, where the English word *science* doesn't fully capture the sense of this as any ordered, systematic inquiry. The propositions of science are contingent, bipolar (true or false, whether or not we can ascertain which), and meaningful. This hard distinction squeezes out metaphysics, so in general I hold that what may appear to be ontological or metaphysical is better thought of as science, in the sense of *Wissenschaft*.

From a methodological point of view I concentrate as far as possible on the *Tractatus* itself and make minimal use of other texts. It is reasonable to use any available texts as an aid, but nevertheless we know that Wittgenstein intended his pre-*Tractatus* notebooks to be burnt.[5]

Naked numbers throughout refer to numbered propositions in the *Tractatus*. Naked numbers prefixed § with a decimal point refer to chapters/sections in the present work, without a decimal point, to sections in Hertz's *Principles of Mechanics*.

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[5]Postscript to L.W. to B.R. 1 November 1919, at *Wittgenstein in Cambridge*, pp.105/106; cf. von Wright 1971, p.4.

Chapter 1

Logic as Universal

1.1 The Russellian inheritance

One would think that, in 1912, the author of *Principia Mathematica* had a pretty good idea as to what logic is. *Principia Mathematica* was intended as the definitive statement of *logicism*, "typically defined as the thesis that mathematics reduces to, or is an extension of, logic." (Klement 2019, p.151) But when Russell tried to set out his views the result was 'What is Logic?', "an abortive attempt ... All that remains is a rather short manuscript in which Russell does little more than reveal his own confused state of mind." (Klement 2015, p.215) "Logic," we are told, "is the study of the forms of complexes." (Russell 1912c, p.55) A form is what is left after the constituents of a proposition are replaced by variables, until the remaining constituents cannot be so replaced without the form ceasing to generate a viable complex on suitable substitution for one or more of the variables; but this, as Russell realises, can only work if one has beforehand a means of distinguishing logical and nonlogical vocabulary.[1] As he wrote in a letter:

> I can't get on with "What is Logic?", The subject is hopelessly difficult, and for the present I am stuck. I feel very much inclined to leave it to Wittgenstein. (Russell 1992, p.54)

This was "the morass in Russell's philosophy of logic awaiting Wittgenstein, when he arrived in Cambridge in 1911." (Ricketts 2002, p.228) Nevertheless logic as Wittgenstein inherited it from Russell, however inchoate, was

[1] As Sullivan points out, "turning *everything* in a proposition into a variable would give us just a shapeless mush" (Sullivan 2000, p.183; cf. Proops 2007, p.11).

nigh-on unrecognisable compared to logic as Russell had inherited it from Bradley, Bosanquet, and Sigwart.[2] In his intellectual autobiography Russell notes Joachim's recommendation of Bradley's *Logic* ("good but hard") and Bosanquet's *Logic* ("better but harder").[3]

The salient point is that Russell's approach to logic is not wholly formal. The influence of his forebears is shown in a conception of logic as, in some sense, the ultimate intellectual discipline that stands behind and is involved in all others. His conception of logic is implicitly universal, in that the principles of thinking and reasoning, the inferential moves that can be made, are universally applicable. This is somewhat vague, as Proops notes:

> [A]lthough the universalist interpretation has been widely endorsed, its precise content remains elusive, and its accuracy, consequently, open to question. One sign of this elusiveness is the proliferation of glosses on the claim that for Russell "logic is universal." Some commentators have meant by this that for Russell logic is a "universally applicable theory" [Urquhart 1988, p.83], others, that it constitutes a "universal language" [Hylton 1990a, p.200], still others, that its laws are "maximally general truths" [Ricketts 1996, p.59], or that its principles are "all-encompassing" [Goldfarb 1989, p.27]. (Proops 2007, p.1)

To say that logic is universal is not, though, to specify which universe is in question. Is what is intended a platonistic universe of abstract logical concepts, or a mental universe that we as individual possessors of minds more or less participate in, or a physical universe, that somehow realises or instances logical structure? Rescher terms the first of these *absolutist platonism*, with logic dictated by the abstract conceptual objects of logic, and the second *absolutist psychologism*, with logic dictated by the empirical realities of human reasoning processes. (Rescher 1969, p.222) The third is discussed by Almog; "Logic investigates the structural traits of the *world*." (Almog 1989, p.204) This tripartite distinction is considered further in §1.8. That Frege's, Russell's, and Wittgenstein's positions are not clearly delineated lies behind Proops' further observation:

[2] Russell's logical influences as listed in the preface to *Foundations of Geometry*.

[3] Russell 1959, p.29. He read the former in September 1893, the latter in June 1895 (Russell 1891/1902, pp.352, 355).

> A sign of the lack of clarity about the content of universalism is disagreement among commentators about who the *other* universalists are [other than Frege and Russell]. Thus Jaakko Hintikka locates Wittgenstein within what he calls "the universalist tradition" (Hintikka 1988, pp.104–5), while Thomas Ricketts (Ricketts 1996, p.59) and Warren Goldfarb (Goldfarb 2001, p.29) take him to have repudiated it. (Proops 2007, p.25n.2)

So it is immediately unclear exactly what is being argued about. One has first to get clear about what universalism is before one can decide whether or not Wittgenstein is an universalist.

If there is a distinctive feature of the universalist position, it is, to adopt a common metaphor, that there is no outside, no metaperspective.[4] Universalists are not supposed to be able to talk about, let alone theorise about, what they are doing, at least not in language that is as meaningful as everyday scientific discourse carried on within language. It seems reasonable to regard the Wittgenstein of the *Tractatus* as an universalist, on the basis of 4.001, that "The totality of propositions is language", this totality excluding "propositions" that ostensibly talk about language. Whatever one might want to say from a metaperspective cannot be propositional and hence does not fall within language so defined. With reference to Proops' remarks the onus looks to be on Goldfarb and Ricketts.

Proops refers to the following passages in Ricketts, and in Goldfarb:

> Wittgenstein rejected Frege's and Russell's universalist conception of logic—what he disparaged as the old logic—while retaining their inchoate guiding assumptions first that logic frames all thought, and second that it is possible to give a clear, completely explicit, and unambiguous expression to the contents judged true or false. (Ricketts 1996, p.59)[5]
>
> In the *Tractatus*, Wittgenstein breaks with the universalist conception in order to arrive at a view in which the propositions of logic are empty. (Goldfarb 2001, p.29)

Rejecting the view that logic has a subject-matter of its own, embodied in asserting that the propositions of logic are tautologies (6.1) is, it seems,

[4] Cf. Moore 1988, p.111.
[5] Cf. Ricketts 2002, pp.228, 233.

a break with universalism as it is ascribed to Frege and Russell, but it is not obviously a break with universalism *per se*. In terms of the "no meta-perspective" doctrine universalism is, according to Hintikka, characterised by the "ineffability of semantics", that "I cannot discuss in my language the relationships that connect it with the world." (Hintikka 1997b, p.3) Goldfarb says that he is in favour of readings of the *Tractatus*, notably Diamond's, that endorse the view that "In Wittgenstein we find explicit remarks about the unavailability of a stance from which to do semantic theorising." (Goldfarb 2002, p.186) Despite the earlier protestation quoted above Goldfarb thus seems to endorse, on Wittgenstein's behalf, one of the central doctrines of universalism.

What stands in need of clarification, then, is the sense in which Frege and Russell are universalists, where this involves attributing a subject-matter to logic, and Wittgenstein is also an universalist, but without attributing a subject-matter to logic. It is not simply a matter of whether one is, or is not, an universalist about logic. What will be argued is that Wittgenstein is an universalist about language and about logic, but this does not, by itself, decide whether Wittgenstein holds a realist or a *sui generis* conception of logic.

1.2 Universalism

The *locus classicus* for the universalism debate is van Heijenoort 1967, in which he draws a Leibnizian distinction between logic as a *lingua characterica*[6], an universal language, and logic as a *calculus ratiocinator*, a calculus of, perhaps, limited application. Van Heijenoort associates the former view with Frege, Peano, and Russell, the latter with Boole, Schröder, and Löwenheim (Wittgenstein is barely mentioned, in association with Frege; van Heijenoort 1967, pp.11, 13n.5). For present purposes the former view is more salient.

Tappenden sets out, in a 1997 paper, a series of nine passages, from Goldfarb, Conant, Ricketts, van Heijenoort, Weiner, and Hylton, addressing the central universalist thesis, that "there can be no external standpoint which one may view and discuss the system" (Goldfarb 1979, p.353, quoted

[6]Peckhaus notes that Leibniz does not use the term "lingua characterica", and that Frege got this term from Trendelenberg. Leibniz did use the expression "characteristica universalis" (cf. Peckhaus 2004, p.5n.5). Both van Heijenoort and Frege use the phrase "lingua characterica", so this expression is used here.

at Tappenden 1997, pp.220–1). In the main Tappenden discusses Ricketts but, he says, "The views in the family examined here emerge from what has been until recently [1997] largely an oral and 'underground' tradition of seminars, conversations, and correspondence, with few detailed published elaborations. The source waters for the interpretation were series of seminars by Burton Dreben at Harvard in the 1970s and onward." (Tappenden 1997, p.220)

The work referred to is principally addressed at Frege, except for the passage from Hylton, from the latter's seminal work on Russell (Hylton 1990a). There is, it would seem, some distinction attributable, broadly, to Dreben-inspired interpreters of Frege, Russell, and Wittgenstein, a distinction bound up in some fashion with universalism.[7] At bottom the claim that Wittgenstein holds *an* universalist conception of logic is not, I take it, controversial. The dispute is over what such a conception amounts to. The main points at issue are these:

1. Whether one's logical system is a *lingua characterica*, or whether it is a *calculus ratiocinator* that is in some sense restricted,
2. Whether or not there is an "external perspective" on language and/or logic,
3. Whether or not there are restrictions on variables,
4. Whether or not the domain is fixed, comprising all that there is, or whether it can be varied,
5. Whether one thinks that the propositions of logic are the most general truths that there are, addressing a subject-matter, or whether they are tautologies, having no subject-matter.[8]

Of these, (1) and (2) are core commitments of universalism, although the precise commitments involved are less than pellucid. (3) is more specific to discussions of Russell and his notion of the unrestricted variable, although it carries over to the Tractarian context with respect to objects and names. (4) can be discounted because Wittgenstein operates with a fixed domain of objects. (5) is specific to Wittgenstein's distancing himself from Frege and from Russell.

[7] A key text here being Dreben and van Heijenoort 1986.

[8] Hintikka sets out a not dissimilar listing at Hintikka 1990, pp.227–8. The list here draws on van Heijenoort and on Hintikka, tailored to the Tractarian context.

1.3 *Lingua characterica* or *calculus ratiocinator*

Van Heijenoort's critical distinction between a *lingua characterica* and a *calculus ratiocinator* is that the former is not just *a*, but *the* universal language, whereas the latter is a calculus that may not be so universal. According to van Heijenoort, Frege and Russell in particular, and universalists generally, intend to create an "universal formal language [that] supplants the natural language", that will "exhaust the intuitive modes of reasoning actually used in science." (van Heijenoort 1967, p.14) This is perhaps more palatable if one thinks of *rendering coextensive with*, rather than *supplanting*. If we identify the universal language with the only language that we speak, *the* language that we use in thinking and reasoning generally (as 5.62), then it has to be taken seriously that there is no outside or beyond. We can only think, reason, and talk within this language. Whatever reflecting we might do on language and our linguistic practices has to be carried out by deploying this language.

If such coextensiveness obtains, we can perhaps speak and think in terms of a language akin to that of *Begriffschrift* or *Principia Mathematica*, where the full range of such an universal language is systematised or formalised. If this could be carried out we would have a *calculus ratiocinator*, a calculus for reasoning, capturing the entirety of the universal language.[9] In Tractarian terms, this is the quest for a complete notation, as intended by Frege; "I wished to produce, not a mere *calculus ratiocinator*, but a *lingua characterica* in the Leibnizian sense . . . I wish to blend together the few symbols which I introduce and the symbols already available in mathematics to form a single formula language." (Frege 1882, pp.91, 93)

One might reject this wholesale, as manifestly absurd and not even beginning to approximate to a natural language.[10] Such a language deals only in what is true or false, in what is involved in thinking and reasoning about what is, or might be, or might have been, the case, so a critical corollary is

[9] Van Heijenoort "interprets the distinction as standing for two kinds of logic" (Peckhaus 2004, p.5) but, as Anellis points out, the distinction between the algebraic (*calculus ratiocinator*) and the quantification theoretic (*lingua characterica*) traditions is "artificial at best" (cf. Anellis 2012, p.345n.8, Peckhaus 2004, p.7n.8).

[10] Engelmann's remark, though, is striking, that "The *Tractatus* is not a treatise on the nature of human language . . . everything 'said' about language applies to any possible language, even a transhuman language (should such exist), so long as it is a mode of depiction." (Engelmann 1967, p.99) If so, Wittgenstein had vaulting universalist ambitions.

an account of truth, and of falsity. In Tractarian terms the issue is whether a natural language is a *calculus ratiocinator* that is effectively coextensive with a *lingua characterica*, where this is to be shown by a suitable process of analysis. The language at stake is propositional, as 4.001, these propositions being those of our everyday language (5.5563). The characteristic of the propositional is bipolarity, as p.4.

If this is granted, the difference between a *lingua characterica* and a *calculus ratiocinator* is a difference in degree, and not of kind. In the limit, with all patterns of reasoning expressed in formal terms, a *calculus ratiocinator* is a *lingua characterica*; the two become coextensive.

1.4 The logocentric predicament

In his introduction to the *Tractatus* Russell says, "In the part of [Wittgenstein's] theory which deals with Symbolism he is concerned with the conditions which would have to be fulfilled by a logically perfect language." (Russell 1922a, p.ix) Russell has been castigated for this, but it isn't clear what is so wrong with this attribution.[11] Wittgenstein says, after all, this:

> [A]ll the propositions of our everyday language, just as they stand, are in perfect logical order. (5.5563)

The reader could be forgiven for thinking that if we stick to the propositions of our natural language, that is, uses of language that are involved in fact-stating discourse—are in the truth business—then this fragment of natural language *is* a logically perfect language. The conditions for this are presumably shown by analysis down to truth-functional combinations of elementary propositions (5), with the latter consisting of names (4.22). There is nothing obviously false in holding that Wittgenstein's notation in the *Tractatus* is intended to show the conditions a natural language must meet if it is to be thought of as a logically perfect language.[12] Ramsey says that Russell's statement "seems to be a very doubtful generalisation", which is hardly damning (Ramsey 1923, p.9). He then says:

[11] This is from a passage added, at Ogden's request, to the original version printed by Ostwald; cf. headnote to Russell 1922b, Iglesias 1977, p.29, Faulkner 2008/09, pp.145–7. For Russell's later defence see Russell 1959, p.123.

[12] Hence Copi defends Russell, noting that "Wittgenstein does not seem to maintain a completely consistent attitude towards ordinary language ... The tendency to reject ordinary language seems to me to predominate." (Copi 1958, p.168)

> [T]here are ... passages in which Mr Wittgenstein is explicitly concerned with the logically perfect, and not with any language, e.g. the discussion of "logical syntax" in 3.325ff.; but in general he seems to maintain that his doctrines apply to ordinary languages in spite of the appearance of the contrary (see especially 4.002 ff.). (Ramsey 1923, pp.9–10)

To say that one is setting out the conditions that a logically perfect language must meet is not obviously different from saying that on analysis the propositions of our everyday language can be seen to meet the conditions required to be a logically perfect language, the salient condition being an ultimate correlation between names and objects. What Wittgenstein could have legitimately objected to is Russell's further statement, "not that any language is logically perfect" (Russell 1922a, p.x), because *prima facie* this runs counter to 5.5563. Nevertheless 5.5563 presents the propositions of natural language as an universal logical language, however well disguised, and if this is granted, much else follows on rails. It is key to Wittgenstein's thinking that our everyday language of propositions *is* a *lingua characterica*, and that the point of the project of analysis is to discover within it and render clearly an embedded universal *calculus ratiocinator*, with the latter presented as a complete notation.[13] However, as Goldfarb points out:

> If the system constitutes the universal logical language, then there can be no external standpoint from which one may view and discuss the system. Metasystematic considerations are illegitimate rather than simply undesirable. This is what Henry M. Sheffer called "the logocentric predicament" (Sheffer 1926), and forms a large part, I think, of the motivations behind Wittgenstein's *Tractatus*. (Goldfarb 1979, p.353)[14]

The notion of a "logocentric predicament" can, though, all too quickly look like a platitude. Collins English dictionary says that a predicament is "a perplexing, embarrassing, or difficult situation." It is unclear why any of these apply. As language-using animals the logocentric predicament is how it is, and not obviously a "predicament" at all. That we can only think and say what we can think and say is hardly momentous. If the notion of the

[13] A project pursued in a number of publications by Kuusela.

[14] Not illegitimate because, it would seem, any rules or laws have been broken, but rather because to take up an external standpoint is, somehow, to transgress against universalism.

logocentric predicament is not to be platitudinous there must be more to it than this.

The key point is bound up with Goldfarb's "metasystematic considerations". If one's language—the only language that one understands, that is, one's entire linguistic resources, however expressed—if this language, regarded as an universal language, a *lingua characterica*, is realised as a *calculus ratiocinator* (a complete notation)—how can one think about, and reflect on, one's cognitive/linguistic practices? And how is one to formulate such a calculus *as if* one can stand outside it and discuss and explain it?

A critical distinction is needed between informal reflection on, and consideration of, one's thinking and reasoning practices, and formal metasystematic or metatheoretical work. Sheffer introduces the "logocentric predicament" with a specific formal point in mind; if one's logical system holds that from a false proposition, any proposition is implied, what is the force of *reductio* as a proof procedure?[15] This is a very different context from informal reflection on one's thinking and reasoning practices. Tappenden points out that "mathematics includes, as a crucial part of that very activity, the critical analysis and scrutiny of primitives and techniques." (Tappenden 1997, p.219) So whatever account is given had better leave space for such reflective activity.

In a similar vein Proops says:

> Another metalogical question that Russell would surely have found intelligible is that of completeness. Given that Russell (at one stage) conceived the truths of logic as precisely those expressible in purely logical vocabulary he would have been able to ask whether all such truths were provable within his logical system. (Proops 2007, p.20)[16]

One might think that such reflections constitute a conservative extension of language, but as they are not expressed propositionally it is hard to see what basis there is for assessing them. It is not obviously unreasonable to

[15]The difficulty Sheffer discusses is the conflation of "$\supset$" and "implies" in *Principia Mathematica* (Sheffer 1926, pp.230–1); cf. Lewis 1919, p.328, Quine 1962, p.177.

[16]Proops perhaps alludes to Hylton, who claims that for Russell, "the question of the completeness of a system in the modern sense simply could not arise." (Hylton 1990b, p.62) Completeness is hinted at informally in a 1905 paper; "If it is our purpose to make all our assumptions explicit, and to effect the deduction of all our other propositions from these assumptions, it is obvious that the first assumptions we need are those that are required to make deduction possible." (Russell 1905, p.22)

reject such reflections, on the grounds that they do not fall within language as delimited by 4.001. Such thinking can be seen in Weiner's discussion of Frege:

> With the exception of the proofs of *Begriffschrift* and *Grundgesetze*, the views expressed in Frege's writings cannot be expressed in Begriffschrift. The bulk of Frege's writings ... seem to consist primarily of claims and arguments from the standpoint of some meta-perspective that does not exist ... my aim ... will be to argue that it follows from Frege's general epistemological views that his discursive work has the status of elucidation rather than of objective statements of facts. (Weiner 1990, pp.228, 229)

The question is whether the "propositions" in which one conducts one's reflective thinking on one's practices can be expressed within the *lingua characterica* that is on offer. In the case of both Frege and Russell it would seem that the answer is no, and this is, perhaps, what Tappenden intends in saying that "something in Frege's conception of logic precludes any appeal to a 'metaperspective'." (Tappenden 1997, p.221) In Wittgenstein's case, when he states that the propositions of the *Tractatus* are nonsense, at 6.54, a status that is not applicable to the propositions of our everyday language, this looks like a straightforward consequence of 4.001 and 5.5563. Of course this raises difficult issues, because if one thinks that any use of language that is not meaningful, that cannot be expressed in the language of *Begriffschrift* or *Principia Mathematica* or the logical notation of *Tractatus*, is nonsense, then *prima facie* one is committed to significant nonsense. Unless one really wants to say that the prose interludes in *Begriffschrift*, *Grundgesetze*, and *Principia Mathematica*, and the elucidations of the *Tractatus*, are plain nonsense.

If Wittgenstein is an universalist, then there should be a sharp distinction between uses of language for fact-stating discourse, that is, the propositions that comprise the totality of language (4.001), and plain, flat-out nonsense. One cannot have one's elucidatory cake and eat it unless one prescinds from universalism *in this respect*. A certain sort of weaseling out can, I think, be seen in Ricketts 1996. In Section V, entitled "Throwing away the ladder", with reference to Diamond 1985, Diamond 1991a, and McGuinness 1981, Ricketts ostensibly treats the 1s and 2s as presenting a metaphysics of ob-

jects and facts, before warning his reader that "Wittgenstein's rhetoric ... is dangerously misleading" (Ricketts 1996, p.89):

> Wittgenstein's rhetoric in the 2.0's is carefully calculated both to limn a metaphysical picture and simultaneously to cancel the incompatible implicatures that any presentation of this metaphysics carries with it. What I have called careful calculation may, however, with equal justice, be labelled philosophical incoherence. (Ricketts 1996, pp.89–90)

In discussing the internal/external properties of objects (2.01231) he says, "We have here another instance of Wittgenstein's unavoidably deceptive rhetoric" (Ricketts 1996, p.92), and of the saying/showing distinction, that "this talk of what is said and what is shown itself misleads ... the attempt to say what is shown leads to nonsense, to what we on reflection recognise to be plain gibberish" (Ricketts 1996, p.93). It is hard to see, though, how one can "carefully calculate" nonsense, or why one would ostensibly take the text seriously only to denounce it as "plain gibberish".[17] If one thinks that universalism involves not being able to take up a position from the outside or talk meaningfully about what one is doing, then Wittgenstein is clearly an universalist with respect to language; he clearly holds that language is propositional, an affair of fact-stating discourse, and that any other "uses" of language are nonsense. It is not a matter of being "philosophically incoherent", it is a matter of being incoherent *tout court*. But this does, of course, entail that the elucidations of the *Tractatus* itself are plain, flat-out nonsense (this is discussed further in §1.10).

The point is, though, that on *this* basis—any attempt to take up an external perspective on the language of everyday propositions (5.5563)—Wittgenstein is as committed to universalism as Frege and Russell. This rather undermines Ricketts' and Goldfarb's contention that Wittgenstein broke with Fregean and Russellian universalism.

[17] The unsympathetic reader might see, for example, Conant 2007 and Conant and Bronzo 2017 as last-ditch efforts. It is hard to know what to make of Conant and Bronzo's remark, "if someone tells you that they or someone else is a resolute reader of Wittgenstein ... our advice is to get that person first to tell you what they mean by the term" (Conant and Bronzo 2017, p.192). This looks rather like bidding against oneself at an auction.

1.5 Logic as super-science

The move Ricketts and Goldfarb claim is decisive occurs in 6.124, that "the propositions of logic . . . have no 'subject-matter'." What is distinctive of the propositions of logic is that they are tautologies (6.1), this being what Goldfarb presumably has in mind in asserting that "the propositions of logic are empty." (Goldfarb 2001, p.29) Both Ricketts and Goldfarb are clear that this is crucial to Wittgenstein's rejection of the "old logic" of Frege and Russell, that the propositions of logic "say nothing" (4.461), that "they do not represent any possible situations" (4.462), that "All theories that make a proposition of logic appear to have content are false." (6.111) Wittgenstein holds that the propositions of Frege's and of Russell's logics, appearances to the contrary notwithstanding—as in many cases they are truth-functional tautologies—are not of the same nature as Tractarian tautologies, because the former somehow have content or make substantive claims and are thus not empty or senseless. But it is far from obvious how this mooted distinction is to be established, that is, whether or not logic as expressed by the propositions of logic has a subject-matter of its own.

Ricketts' and Goldfarb's claims that the universalist logics of Frege and Russell commit them to a conception of logic as having a subject-matter of its own comprise, I think, the critical distinction they seek to draw between Frege and Russell on one hand, and Wittgenstein on the other. The implication is that Frege and Russell are, while Wittgenstein is not, committed to the view that logic is somehow responsive to *something* that is external to and independent of us. The question is what this *something* is. In more recent terms such commitment is to some form of *logical realism*, that "there is a fact of the matter of whether something is a logical truth, a logical inconsistency or logically implies something else" (Resnik 2000, p.181; cf. p.9).

Logical realism is not, though, a clear-cut, precisely defined notion. McSweeney distinguishes between ontological and ideological metaphysical structural realism (McSweeney 2019, pp.3–4), a distinction refined by Tahko as *realism about logical constants* and *realism about logical structure*. (Tahko 2021, p.4784) The first of these is clearly inappropriate for the *Tractatus*, given Wittgenstein's "fundamental idea . . . that the 'logical constants' are not representatives." (4.0312) But realism about logical structure is not ob-

viously inappropriate, and may be a way of cashing out the remarks at 6.124 that "something about the world" is indicated by the propositions of logic, given that they "describe the scaffolding of the world". For the moment I want to hold open the option of ascribing logical realism to all of Frege, Russell, and Wittgenstein, while accepting that Wittgenstein thinks that there is *some* way in which he distances himself from Frege and from Russell.

In generating such distance Ricketts and Goldfarb pick up on Hacker's claim, that "both Frege and Russell thought of [the laws of logic] as general *a priori* truths about logical entities, laws of a kind of super-physics" (Hacker 2021, p.44):[18]

> On the [Frege/Russell] universalist view ... logic is thus a science in its own right, one that is directed at reality in the same way that physics is, but at reality's more general features. (Ricketts 1996, p.60)

Similarly Goldfarb asserts:

> [O]n the [Frege/Russell] universalist conception logic sits squarely at the object level, issuing laws that are simply statements about the world ... On Frege's view, as on Russell's, it is precisely reality that obeys the laws of logic. (Goldfarb 2001, pp.28, 29)

Hence Ricketts and Goldfarb follow Hacker in holding that Wittgenstein's break with Frege and Russell is contained in claims that logical propositions as tautologies are not in any way representative, that logic has no subject-matter of its own, and is not any form of super-physics. *If* this is a fair characterisation of Frege and of Russell, this represents *a* break with their particular form of universalism. But logic can still be universalist in the sense of being a *lingua characterica*, so there is scope for Wittgenstein to subscribe to universalism about logic, unless one takes subscribing to logic having a subject-matter of its own as necessary and sufficient for universalism.

[18] Hacker ascribes to Frege and to Russell a conception of logic in terms of a "mystifying picture of a kind of super-physics" at *Insight and Illusion*, 1st edn., p.6. In later editions the phrase appears twice, at pp.44–5, and at p.50. At Baker and Hacker 1983b, p.23 Russell's type-theory is said to "constitute a sort of ultra-physics." For the expression "super-science", cf. quote from Ryle 1957, p.257 at Baker and Hacker 1984, p.27; Ryle discusses "philosophers as scientists or *a fortiori* as super-scientists", but the article is avowedly popular and Ryle does not expand on this.

In an endnote Ricketts says that Frege's version of universalism can be found at *Grundgesetze*, i p.xv, and at Frege 1897, p.128, and Russell's at *Introduction to Mathematical Philosophy*, p.169 (Ricketts 1996, pp.94–5n.2). The passage in *Grundgesetze* that Ricketts has in mind is presumably this:

> Every law stating what is the case can be conceived as prescriptive, one should think in accordance with it, and in that sense it is accordingly a law of thought. This holds for geometrical and physical laws no less than for the logical. The latter better deserve the title "laws of thought" only if thereby it is supposed to be said that they are the most general laws, prescribing how to think wherever there is thinking at all. But the phrase "laws of thought" seduces one to form the opinion that these laws govern thinking in the same way that the laws of nature govern events in the external world. In that case they can be nothing other than psychological laws; for thinking is a mental process. And if logic had to do with psychological laws, it would be a part of psychology. (*Grundgesetze*, i p.xv)

This looks to make a distinction between laws of nature and laws of logic, because without such a distinction, laws of logic would be empirical laws and thus, if psychology is the empirical science of the mind, psychological laws. But Frege holds that laws of logic are normative, and hence that it is a mistake to look for a grounding or a justification for such laws. Nothing at *Grundgesetze*, i p.xv looks to make Ricketts' case, that Frege regards logic as super-physics.

The passage in Frege 1897 is this:

> [L]ogic can also be called a normative science. How must I think in order to reach the goal, truth? . . . [T]he task we assign to logic is . . . of saying what holds with the utmost generality for all our thinking, whatever its subject-matter. We must assume that the rules for our thinking and for our holding something to be true are prescribed by the laws of truth. The former are given along with the latter. Consequently we can also say: logic is the science of the most general laws of truth. (Frege 1897, p.128)

This certainly embraces logic as ubiquitous in thinking and reasoning, but given the portmanteau character of the word *Wissenschaft* it is not

obvious that Frege is committed to logic as super-physics. But, as will be emphasised shortly, what matters here is not what Frege thought, but what Wittgenstein took him to be thinking.

The passage in *Introduction to Mathematical Philosophy* is readily identifiable, directed at Meinong:

> In such theories, it seems to me, there is a failure of that feeling for reality which ought to be preserved even in the most abstract studies. Logic, I should maintain ... is concerned with the real world just as truly as zoology, though with its more abstract and general features. (*Introduction to Mathematical Philosophy*, p.169)

Logic as super-physics is, it seems, the science of *logical forms*, where these are arrived at by generalisation:

> We can ... turn *all* the constituents of a proposition into variables, while keeping the form unchanged. This is what we do when we use such a schema as "xRy," which stands for any one of a certain class of propositions, namely, those asserting relations between two terms ... We are left with pure *forms* as the only possible constituents of logical propositions ... We may thus lay down, as a necessary (though not sufficient) characteristic of logical or mathematical propositions, that they are to be such as can be obtained from a proposition ... by turning every constituent into a variable and asserting that the result is always true or sometimes true. (*Introduction to Mathematical Philosophy*, pp.198ff)

Wittgenstein had not read *Introduction to Mathematical Philosophy* in composing the *Tractatus*, but Russell's views on forms follow those set out in *Theory of Knowledge*, with which Wittgenstein was familiar. Perhaps the most salient remark is this:

> "[S]omething has some relation to something" contains no constituent at all. It is, therefore, suitable to serve as the "form" of dual complexes. In a sense, it is simple, since it cannot be analysed. At first sight, it seems to have a structure, and therefore to be not simple; but it is more correct to say that it *is* a structure. (*Theory of Knowledge*, p.114)

The aim of Russellian logical analysis is to reveal logical forms, an inventory of these forms serving as the subject-matter of logic. It is not clear whether Russell regarded this as super-physics. He notes that "Philosophy is not a shortcut to the same kind of results as those of the other sciences: if it is to be a genuine study, it must have a province of its own, and aim at results which the other sciences can neither prove nor disprove." (*Our Knowledge of the External World*, p.17)

Frege's and Russell's positions would seem, then, to be more nuanced than the claims put forward by Hacker, by Ricketts, and by Goldfarb suggest. The critical question is how one interprets Frege's remark that "logic is the science of the most general laws of truth", and Russell's contention that logical propositions as "pure forms" are arrived at by generalisation, and how one interprets Wittgenstein's moves to distance his views from theirs.

1.6 Logical propositions as general truths

The difficulty here is that one finds oneself faced with an evanescent distinction, because what matters is not the result, but the manner of getting to it. As Kienzler points out, with respect to Frege, "Wittgenstein is not accusing Frege of getting any *false results* ... his point is merely philosophical: Frege will get the correct results but his conception cannot explain to us *how a sentence works* ... he has no notational device for the difference between cases where something is by coincidence true of all objects and those whose truth is grounded in some essential, or conceptual, relation." (Kienzler 2011, p.97n.57) The difficulty lies in establishing that a Tractarian logical proposition as a tautology is different in principle from a Fregean or a Russellian logical proposition in cases where the latter are also tautologous.

Ricketts and Goldfarb characterise Wittgenstein's decisive shift away from the "old logic" of Frege and Russell, the rejection of logic as "super-physics", as rejecting the claim that logic has a subject-matter of its own. Tractarian logical propositions are tautologies and thus empty of content. This looks straightforward, except for the fact that the axioms of *Begriffschrift* are truth-functional tautologies (props.1, 2, 8, 28), and arguably likewise, if one accepts double negation elimination (props.31, 41), second-order quantification over properties (prop.52), and universal instantiation (prop.58). Similarly the primitive propositions of *Principia Mathematica*, $*1$

are in the main truth-functional tautologies.[19] So wherein lies the mooted distinction?

Wittgenstein's central claim is that the propositions of logic are *toto caelo* different from propositions that are in the business of fact-stating discourse. It isn't enough to be a truth-functional tautology, identifiable as such from the symbol alone (6.113); the propositions of logic must have "a unique status among all propositions" (6.112). But what exactly does Wittgenstein have in mind?

One can begin by considering his rejection of the approach he ascribes to Frege, and to Russell. Wittgenstein thinks that their logical propositions are arrived at by a process of generalisation or formalisation, beginning with propositions that are either empirical or at least drawn from other disciplines. Even if the result is a truth-functional tautology it is somehow tainted by its origins, such that it is only accidentally general. As McGinn puts it, with reference to *Notebooks 1914–16*, p.16, "A completely generalised proposition that is arrived at through a process of generalisation has not cut its representational links to reality." (McGinn 2006, p.63)[20] But this begs the question, wherein lies the difference?

It seems to me that interpreters of Wittgenstein have accepted, largely uncritically, the charge levelled by Wittgenstein at Frege and Russell, that the latter failed to distinguish logic from empirical science. Unless one thinks that Frege's and Russell's commitment to logicism also entails a commitment to the applicability of logic via mathematics, and thereby, somehow, to logic as super-science. It is not clear that such a set of commitments can be pinned on Frege and Russell. The question becomes, what exactly *is* involved in pinning on Frege and on Russell the claim that the propositions of logic are the most general truths there are? And at this point there seems to be a slide, with Ricketts asserting, *à propos* Frege's remark that "logic is the science of the most general laws of truth" (Frege 1897, p.128), that:

> To say that the laws of logic are the most general laws of truth is to say that they are the most general truths. (Ricketts 1986, p.80)

[19] As pointed out by Ramsey; "The real primitive propositions [of *Principia Mathematica*] ... are, with one exception [∗1.1], tautologies in Wittgenstein's sense." (Ramsey 1925a, p.174)

[20] Or, as Spinney notes, "Maximal generality is, in Wittgenstein's view, not a criterion we ought to apply when inquiring after the nature of logical truth, for maximally general claims may well be contingent." (Spinney 2023, p.188)

Of this slide, Heck asks, "But whence the identification of the most general laws of truth with the most general truths?" (Heck 2011, p.353n.34) The answer is, I think, that Ricketts has uncritically taken over Wittgenstein's criticisms of Frege, and of Russell, with Wittgenstein running their positions together to ensure that his criticism hits the mark.

There is, though, one last line of defence open to Ricketts and Goldfarb, pointed out by Proops. This is to argue that because Russellian and Fregean logical propositions are truths, they fail "to have a *unique* status among propositions" (Proops 2000, p.2), because they are not adequately distinguished from accidental generalisations (truths in all possible worlds). But to make this argument work Wittgenstein has to show how his logical propositions are true, but not true in the same way that the propositions of Fregean and Russellian logic are true (this is discussed further in §2.9). The difficulty that won't go away is that Wittgenstein's logical propositions don't look to be, and in many cases are not, in *any* obvious sense, different from those put forward by Frege, and by Russell.

1.7 Applicability

What this perhaps comes down to is the issue of *applicability*, that although many of the propositions of Fregean/Russellian logic are *prima facie* truth-functional tautologies, their being generalisations of empirical propositions entails being implicitly involved in their application. By contrast Tractarian tautologies are so to speak externally involved in their application. Their function is to serve as bridges from propositions to propositions without getting their feet wet (cf. 6.211).

The critical Fregean texts occur in *Grundgesetze*, and in *Foundations of Arithmetic*:

> [I]t is applicability alone which elevates arithmetic above a game to the rank of a science. (*Grundgesetze*, ii §91)[21]

[21] This appears in Frege's criticism of formalism. Dummett notes, "It is when he is criticising empiricism that Frege insists on the gulf between the senses of mathematical propositions and their applications; it is when he criticises formalism that he stresses that applicability is essential to mathematics." (Dummett 1991, p.60)

> The basis of arithmetic lies deeper, it seems, than that of any of the empirical sciences, and even than that of geometry. The truths of arithmetic govern all that is numerable. This is the widest domain of all; for to it belongs not only the actual, not only the intuitable, but everything thinkable. (*Foundations of Arithmetic*, §14)[22]

> Arithmetic thus becomes simply a development of logic, and every proposition of arithmetic a law of logic, albeit a derivative one. To apply arithmetic in the physical sciences is to bring logic to bear on observed facts . . . The laws of number, therefore, are not really applicable to external things; they are not laws of nature. They are, however, applicable to judgements holding good of things in the external world: they are laws of the laws of nature. (*Foundations of Arithmetic*, §87)

At a stretch one could construe the "laws of the laws of nature" as superphysics, but this would need further argument. Goldfarb says:

> On Frege's *universalist conception* . . . the concern of logic is the articulation and proof of logical laws, which are universal truths. Since they are universal, they are applicable to any subject-matter, as application is carried out by instantiation. For Frege, the laws of logic are general, not in being about nothing in particular (about forms), but in using topic-universal vocabulary to state truths about everything. (Goldfarb 2001, p.28)

In conflating *logical laws* and *universal truths*, the idea is that the step from logical laws to their application in any of the sciences is a matter of substituting terms for particulars or concepts for the "topic-universal" vocabulary in the statements of the logical laws. It isn't clear (and we aren't told) what this topic-universal vocabulary is or how instantiation is executed. What can be argued for is a certain *directness* of application, that there is some formal commonality between the laws of logic as universal

[22] This is arguably a defining statement of universalism, of the universal applicability of logic. Proops discusses this with reference to Russell at Proops 2007, §1 pp.2ff. Korhonen has such a conception in mind in characterising universalism; "[O]n the conception of logic as language, logic is *universal* or ubiquitous in the dual sense that *there is only one logic* and *this one logic is present everywhere*, whenever there is thinking, reasoning, and so on." (Korhonen 2012, p.600)

truths and what there is, such that the application of laws involves instantiation (for example, going from a law of the form $\forall x \exists F \exists G.Fx \supset Gx$ to an instance $fa \wedge ga$). Such application is perhaps the instantiation that Goldfarb has in mind.[23]

Turning to the *Tractatus* one finds a rather different conception, stated in 5.557, quoted at p.3. In 5.557 Wittgenstein is, I think, deliberately distancing himself from Frege and, on the basis that both are committed to super-physics, from Russell. But it is hard not to think that Wittgenstein simply takes Frege's remarks on the applicability of mathematics, and argues that the latter's logical propositions are tainted by accidental generality precisely because they are applicable. Russellian logical propositions are tainted because they are arrived at by generalisation from empirical propositions. Frege/Russell logic is thus not merely *in contact with* its application, it is wholly and directly *involved in* its application. By contrast a Tractarian logic of tautologies is only *in contact* with its application, because although it is equally applicable, and although the propositions of logic as tautologies in *Begriffschrift*, *Grundgesetze*, *Principia Mathematica* and *System Tractatus* (the *Tractatus* thought of as a formal system) are ostensibly formally indistinguishable, the propositions of the latter are somehow without empirical taint. It should be appreciated that this is what Wittgenstein intends. Whether or not he succeeds is a matter of how far he has distanced himself not so much from Frege, as from Russell.

1.8 Differences between Frege and Russell

The reason why this distancing matters is because there are significant differences between Frege's and Russell's views on logic that have yet to be addressed, bound up with the tripartite distinction mentioned in §1.1.

Of the three positions noted I will discount *absolutist psychologism*, as this is not salient for any of Frege, Russell, or Wittgenstein. This leaves *absolutist platonism*, and the view discussed by Almog, that "logic investigates the structural traits of the *world*". It is distinguishing between these that, perhaps, separates Frege and Russell, a matter of what one thinks the

[23] On the basis that neither Frege nor Russell is squeamish about going second-order. As Moore points out, at this date there was "no generally accepted classification of the different kinds of logic." (Moore 1988, p.96) Such distinctions first emerge in Hilbert and Ackermann 1928 (cf. Goldfarb 1979, pp.359–60).

truths of logic as the most general truths are truths *of*. The argument is, with reservations, that Frege's approach is more akin to absolutist platonism, whereas Russell's approach is more akin to an investigation of the most general or structural/formal features of the world. In both cases this generates a subject-matter for logic, and both approaches are, ostensibly, at least, rejected by Wittgenstein.

There are three respects in which one might think of Frege as an absolutist platonist; his notion of numbers as objects (*Foundations of Arithmetic*, §§55ff), his postulation of thoughts as entities in a "third realm" ('Thoughts', p.69), and his assertion that logic is normative (*Grundgesetze*, i pp.xv–xvi). Frege is, though, more interested in securing objectivity than in ontology and metaphysics. He also wants to ensure that arithmetic is applicable but without, I think, committing himself as to whether logic has a subject-matter of its own. So perhaps the best way to characterise this is logic as *sui generis*, a normative science in its own right that is not further grounded or justified (and may not be in need of grounding or justifying). Wittgenstein's sympathy for this comes out in his remark that "Logic must take care of itself" (5.473), and in 5.551:

> Our fundamental principle is that whenever a question can be decided by logic at all it must be possible to decide it without more ado.
> (And if we get into a position where we have to look at the world for an answer to such a problem, that shows that we are on a completely wrong track.) (5.551)

Russell's approach is, though, rather different. In his case the charge of logic as super-physics is more pressing. The question is, if logic is somehow involved in what is *prima facie* extralogical—"The logical traits of the *world*", perhaps, as Almog puts it—is this physics, or is it metaphysics? Surveying Russell's writings shows, I suggest, a profound interest in science, and little interest in traditional metaphysical questions. The one paper in which Russell clearly essays a traditional metaphysical distinction is Russell 1911. But the bulk of this paper is devoted not to metaphysical distinctions, but to the "two logical distinctions which are relevant in the present enquiry ... the distinction between relations and entities which are not relations ... [and] the distinction between verbs and substantives, or, more correctly, between the objects denoted by verbs and the objects denoted by

substantives." (Russell 1911, p.107) The slide between words and entities is all too Russellian, the point for present purposes being that the distinction between universals and particulars is drawn not in terms of a metaphysical/conceptual framework of substance and attributes, but in terms of an (admittedly loose) logico-linguistic framework.[24]

Even if, though, Russell's outlook is predominantly scientific and not metaphysical, it remains the case that he is not wholly innocent of the charge of super-physics. There is *some* sense in which logic is involved in the things of this world, even if only at an abstract level of structure. Frege's commitment to the applicability of mathematics, and of logic, looks to commit him to a structural form of logical realism, but his *sui generis* approach to logic enables him to sidestep the charge of super-physics. Russell on the other hand is, I think, full-bloodedly committed to structural logical realism, based on his notion of logical form:

> It is obvious ... that when all the constituents of a complex have been enumerated, there remains something which may be called the form of the complex, which is the way in which the constituents are combined in the complex. It is such pure forms that occur in logic. (Russell 1913, p.98)[25]

A logical proposition is a pure form, with variables as required to *show* the way in which the form orders the constituents of a proposition. Logical propositions of such form can be used to *show* inference; "All inferences, when stated fully, are instances of propositions having this kind of generality." (*Our Knowledge of the External World*, p.44) It is the process of generalisation that leaves, for Wittgenstein, Russellian logical propositions somehow sullied by their worldly origins.

Whether a Russellian logical form, insofar as it is any kind of entity, is scientific or metaphysical, is moot. The point is that there are differences between Frege's and Russell's positions, and traces of both of their positions can be found in the *Tractatus*. What will be argued is that it is difficult to

[24] Similar remarks apply to Russell 1959, Chapter 14, entitled "Universals and Particulars and Names".

[25] Wittgenstein was of course familiar with the views expressed in Russell 1913, and much of Russell's position in *op. cit.* Book I Chapter IX can be found in *Our Knowledge of the External World*, Lecture II. For evidence that Wittgenstein read the latter in 1915, cf. Levine 2018, p.330n.1.

maintain a *sui generis* approach to logic, *and* hold that it is applicable, on a principled basis. In other words, that Wittgenstein's ostensibly *sui generis*-but-applicable position may, ultimately, be unstable.

1.9 The unrestricted variable, and objectuality

This issue is introduced as it is common in the literature, and also to dispatch it as quickly as possible. This is bound up with variability, whether "The *laws* of logic are 'universally applicable' in virtue of containing only variables that range without restriction over everything there is." (Proops 2007, p.3)

Proops' discussion of the unrestricted variable is focused largely on Russell. In the present context there are, I think, no salient difficulties for Wittgenstein. From a logical point of view restrictions on variables reflect logical segmentation or categorisation. In the case of the *Tractatus* there is only one formal category, that of *object*. Wittgenstein's position is, I think, that objects are uniform, in that they instantiate or realise logical form. Hence I think MacBride's claim that Wittgenstein is committed to categorial pluralism and not to categorial monism (MacBride 2018, p.201) is not quite right. MacBride's argument is based on 5.55, that we cannot give *a priori* "the composition of elementary propositions." I agree with MacBride that giving such composition is an *a posteriori* matter. Where I disagree is that this entails *logically* a commitment to categorial pluralism. Rather logically the commitment is to categorial monism, on the grounds that objects have forms. MacBride wants to argue on the basis of 5.55 that Wittgenstein is pushed towards categorial pluralism on metaphysical and thus *a priori*/logical grounds. This it seems to me involves an unwarranted imposition of metaphysics, overlooking the form/content distinction of 2.025. Campbell's observation that "All that is necessary for an account of logic is the very *idea* of categorial combination" (Campbell 2011, p.138) is I think on the right lines. The difficulty is that allowing for categorial combination forces one in the direction of accepting that there is more than one category, even if this is, strictly, an empirical and not a logical matter. Since the world is not a varying density distribution of uniform stuff (i.e., stuff of uniform content), the present reading assumes that objects are partitioned by form into at least two kinds. Nevertheless in the context of elementary propositions Wittgenstein needs only one kind or type of *variable*, relating to names

for objects (the *variable name* of 4.1272) (noting that this is a *propositional variable* and not just a single letter standing for a name, as 3.313/3.314; the reasons for this caveat will become clear in due course). Any symbols appearing in the notation will, on analysis, prove to be variable names, or a non-representative logical apparatus, that is, an apparatus of logical signs, T, F, (,), $\sim$, $\supset$, and so on. There are no logical *objects*, in the sense that logical objects are not existents (4.441, 5.4).

Two corollaries follow. The task Russell assigns to the theory of types with its correlative notion of significance is performed for Wittgenstein by the forms of objects. Russellian logical forms for propositions are wholly absorbed into the forms of Tractarian objects.[26] It will be argued that unrestricted combinatorialism does not obtain for objects into states of affairs, and thus for names into elementary propositions. Although it may not as yet be clear, the issue of unrestricted combinatorialism is alluded to in 5.5541, as to whether or not one should be able to say *a priori* if one has any need in one's logic for a 27-termed relation. The answer is that one's logic has to be able to deal with any n-termed relation; whether or not one needs a 27-termed relation is then an empirical matter.

A second corollary is a doctrine that will be called *objectuality*. This is that the world as we experience it is consequent on the objects that there are, and their forms (their range of combination into states of affairs). Further, for Wittgenstein, what there is—the objects that there are, and hence, as a conservative extension, their names—is fixed. Hence I agree with Morris's *fixed-form* interpretation, made up of two claims: "There must be a fixed form which is common to all possible worlds ... There can only be a fixed form common to all possible worlds if there are objects common to all possible worlds." (Morris 2016, p.2)[27]

The key proposition for objectuality is 2.0231; "It is only by means of propositions that material properties are represented—only by the configuration of objects that they are produced." Taken absolutely at face value this

[26] As Campbell puts it, in discussing Russell's *Theory of Knowledge*, Wittgenstein "locat[es] form in the *objects themselves*" (Campbell 2011, p.149). This is qualified somewhat but not withdrawn at *op. cit.* p.155n.25. Cf. Palmer 1996, Zalabardo 2015, Chapter 1.

[27] See also Morris 2008, pp.49–50. Page presents effectively the same doctrine, asserting that such a view "will be familiar; certainly nothing in it is intended to be controversial" (Page 1997, pp.40–1). This makes the Wittgenstein of the *Tractatus* a *necessitist*, as Williamson 2013, pp.1–2. The antecedent of this view—that objects are eternal, unchanging, timeless—is Anscombe 1971, p.43, discussed at Keyt 1963, pp.293ff.

claim that material properties *can only arise from configurations of objects* is central to the present reading of the *Tractatus*.

1.10 Truth and nonsense

In characterising a formal system an author typically specifies a property realised by the sentences/propositions or formulas of the language in question. If truth is taken as the property of the propositional, and if an universal language is composed of propositions, then it is hard to see what one is to say about any uses of language that are not propositional.

In the *Tractatus* Wittgenstein offers a tripartite distinction; there is the propositional (the meaningful, expressions that have sense), tautologies and contradictions (the senseless), and, broadly, anything else (the nonsensical). Where this is *prima facie* a programmatic division.

This is programmatic because, I suggest, given the assumption that we can unfailingly identify what is propositional, all else follows. This is not a small assumption but it is, I think, one that Wittgenstein makes. By his standards any further discussion of this would be a matter for psychology. A corollary is that we can identify tautologies and contradictions "from the symbol alone" (6.113); we can see that a tautology is such that it can only ever be true, or if a contradiction, false. As will become clear later (§3.5) there is an effective decision procedure for this. All other "uses" of language are nonsense, on the basis that what does not express sense and is not senseless can only be nonsense.

Standing back for a moment, non-propositional uses of language can be divided, perhaps, into two categories; a first, that is of present concern, consisting of hints, gestures, pinches of salt and elucidations, intended to convey somehow mastery of or familiarity with language as a *lingua characterica* (what Korhonen calls "logical propadeutic" (Korhonen 2012, p.600)), and a second, not of present concern, encompassing any other non-declarative uses of language; commands, questions, and so on.

To start with, for the universalist, truth is not a problem in need of a solution. With the benefit of post-Gödelian/Tarskian hindsight it has to be accepted that there is a blindspot in Frege, in Russell, and in early Wittgenstein. If one has a *lingua characterica* then one has the resources to state all truths; to express all and any stretches of reasoning about what there

is (i.e., fact-stating discourse). The blindspot alluded to is manifest in, for example, Frege's correspondence with Hilbert, on sets of axioms and the consistency thereof; "I call axioms propositions that are true ... From the truth of the axioms it follows that they do not contradict one another" (Frege to Hilbert 27 December 1899, at Frege 1980, p.37). For Frege, what is true, taken together, cannot be contradictory, so there is no need for a proof of consistency. This goes together with the assertion that what is true, has nothing to do with us; "being true is different from being taken to be true, be it by one, be it by many, be it by all, and is in no way reducible to it." (*Grundgesetze*, i p.xv) Truth, it seems, for axioms, can only be cashed out in terms of self-evidence, as a quality that resists further analysis. If, as all of Frege, Russell, and Wittgenstein plausibly do, one subscribes to such a realist conception of truth, then while it may be hard to attain in a given case, if one is in possession of it then one need have no fear of inconsistency. At bottom it is not so much that there is no external perspective from which to discuss metasystematically what one is doing, it is rather that there is no point.

A way of putting this is, when we say "p"—a sentence that expresses a proposition—what we say takes us directly to what there is; there is no meaningful word-world relation to speak of. This absence comes out in Frege's remark, "The sentence 'I smell the scent of violets' has just the same content as the sentence 'It is true that I smell the scent of violets'. So it seems, then, that nothing is added to the thought by my ascribing to it the property of truth." ('Thoughts', p.61) This is as one would expect, if one thinks that language takes one directly to what there is.

The bottom line assumption is that if one gets the syntax of a *calculus ratiocinator* that amounts to a *lingua characterica* right, then the semantics can be left to take care of itself; semantics becomes *transparent*. For the universalist, there is nothing meaningful to say about semantics once the syntax is in place.

In a similar vein Etchemendy considers *representational semantics*:

> [W]e are engaged in *representational semantics* [if] ... Our theory provides an account of a relation, "x is true in y," and what the theory takes to satisfy the "y" position are, for all intents, just ordinary objects of some sort or other—chunks of the *actual* world. (Etchemendy 1990, p.20)

He goes on to remark that "it may seem perverse to view [this] as a *semantics* at all." (Etchemendy 1990, p.21) When the representationalist says "'p' is true iff p"—or, to sidestep truth for the moment, and with perhaps a small pinch of salt, "'p' means p"—what is intended is the things themselves, that it is not a metaphor to claim that "language [is] transparent" (Russell 1959, pp.11, 108), that one somehow "sees" through uses of language and is thereby taken directly to the things themselves. This is not exactly to do semantics, it is to say it as it is.[28] For the universalist nothing meaningful can be said about a "relation" between words and things, between propositions and what can be the case. When Wittgenstein says, "A name means an object. The object is its meaning. ('A' is the same sign as 'A'.)" (3.203), what he means is exactly what he says.[29] One can talk of an intended or an implicit interpretation as a *façon de parler* provided one does not allow oneself to be misled. There is no notion of de-semantification and reinterpretation, of logic as model theory, in play.[30] We, one might say, speak what there is, and in so doing sense is already present, and if things stand as our propositions say they stand, then truth is equally present. It follows immediately, though, that our awareness of what is true as true consists in passive recognition, it is not a matter of any sort of act of determination or evaluation. Such recognition is, I think, what Frege has in mind in talking of "advanc[ing] from a thought to a truth-value." (Frege 1892, p.35) Such "advancing" just is the recognising of the truth-value that obtains for a given proposition, here and now (and unlike Fregean propositions, as will be argued, Tractarian propositions may have different truth-values in different times and places). Whether or not we can so recognise in a given case is another matter.

This has a twofold significance. First, an assumption of truth as unproblematic leads to an assumption of the propositional as unproblematic. As noted, however remarkable it may seem, Wittgenstein simply assumes that we can unfailingly identify what is propositional and distinguish what is not propositional. Second, in presuming that the expression of sense, of being

[28] See also Hylton 1990a, p.171. Similarly Korhonen talks of "switching from semantics to 'blind syntax' " (Korhonen 2012, p.600), along the lines of what he calls the *semantic reading of universality*, that "logic is universal because there is in the end only one set of fixed semantic relations." (*op. cit.* p.601) Hintikka's version of this comes out in his talk of "the ineffability of semantics." (Hintikka 1997a)

[29] For a discussion of this in a Russellian context, see Hay 2022b.

[30] De-semantification is discussed in Dutilh Novaes 2012, Chapter 6 pp.198ff.

bipolar, is the property of the propositional, Wittgenstein is setting himself up for a pervasive problem; of getting clear about the exact nature of tautologies and contradictions.

Concerning the latter, in treating tautologies and contradictions as *true*, and as *false*, respectively, as if they are true, or false, in the same manner as the propositional, Wittgenstein operates with an univocal notion of truth, that a proposition is true if things stand as it says they stand, and false otherwise. But given that tautologies and contradictions are not representative (4.462) it is not obvious that Wittgenstein is entitled to such an univocal notion, because tautologies and contradictions are not bipolar, and it is far from clear that they are true, or false, respectively, in the same way as the propositional is true, or false. This is pursued further in §2.9

A further difficulty, noted at the end of §1.4, is that the "propositions" of the *Tractatus* itself, along with the prose interludes in *Begriffschrift*, *Grundgesetze*, and *Principia Mathematica*, all come out as nonsense. Metasystematic theorising may be fairly ruled out, but when it comes to informal reflection then I think one has to bite the bullet and accept that what can be called *programmatic nonsense*—"nonsense" intended to convey a capacity to do something, construable perhaps as know-how, or expressible in a computer program written in a programming language—Korhonen's "logical propadeutic"—is not nonsense in the way that "There is a buggle-wuggle" (Diamond 1991b, p.2) or "Socrates is frabble" (Diamond 1985, p.197) or "piggly wiggle tiggle" (Diamond 1991a, p.151) are nonsense.[31] I find myself in broad agreement with Carruthers' approach, of paying "scant attention to Wittgenstein's official doctrine that all philosophical . . . statements are nonsensical", not least because "the official doctrine is . . . violated systematically throughout the *Tractatus* itself". (Carruthers 1990, p.xi) My reasons, though, are different from Carruthers'. Whereas Carruthers thinks that Wittgenstein makes an "easily corrected" mistake in the domain of semantics, my reason is that if it can be presumed that we have a capacity to identify what is propositional, then there is no obvious reason to deny ourselves a capacity to discriminate what is sheer nonsense from what is elucidatory or clarificatory (programmatic nonsense). On this basis saying that we can throw

[31] "Programmatic nonsense" is, I think, better than "important" or "ineffable" or "significant nonsense", because it carries the implication of, so to speak, installing capacities that can then be exercised more or less unthinkingly.

the ladder away and see the world aright is not obviously different from advancing from doing elementary arithmetic by counting one's fingers to doing complicated calculations in one's head, or unthinkingly riding a bicycle. One simply leaves behind what has become programmatically ingrained.

1.11 Fixed domain

A critical distinction for van Heijenoort is whether or not one is dealing with a fixed domain (van Heijenoort 1967, pp.12–13). Van Heijenoort associates variation in domain with the algebraic (*calculus ratiocinator*) tradition, and a fixed domain with the universalist (*lingua characterica*) conception. For present purposes it is the consequences of ascribing a fixed domain to the universalist conception that is at stake.

In the context of the *Tractatus* Wittgenstein simply assumes, I think, a fixed domain of objects as necessary existents, as Morris's "fixed-form" interpretation (p.34). Hintikka calls this the "one-domain assumption", describing it as "hold[ing] a large number of philosopher-logicians in its iron grip." (Hintikka 1992, p.200) This I take it is what Wittgenstein means when he says that "Objects make up the substance of the world" (2.021), and that "Objects, the unalterable, and the subsistent are one and the same." (2.027) On this basis the *Tractatus* precisely meets this criterion for universalism as laid down by van Heijenoort.

A corollary is a fixed extent to language. Hintikka makes the point that "Wittgenstein's idea of the limits of language in the *Tractatus* ... [is] constituted by the totality of actually existing individuals [objects]." (Hintikka 1989, p.35) Assuming that objects are named (accepting that this is problematic, and has not as yet been explained), the totality of language follows (4.001), because the totality of language is a function of the combinatorial possibilities of names into elementary propositions; and thus a function of the forms of the objects that there are (a corollary of objectuality, §1.9).

This goes together with the discussion in §1.9 of the unrestricted variable. The language of the *Tractatus* expressed formally, in terms of the bottom level of analysis—names for objects, elementary propositions as combinations of names—is implicitly, as we would now think of it, first-order, noting that this will be heavily qualified. The structure of elementary propositions is a function of the forms of objects and as such is nothing over and above

the objects themselves; hence, in particular, there is no categorial distinction into properties or relations. The approach taken is thus broadly nominalistic, as universality, repeated instances of *material* properties, is a function of the forms of the objects that there are. There are nevertheless very real problems over naming, that is, how objects can be named, given the characteristics that are ascribed to them, and over thinking in terms of quantification. Given that objects have form, and that logically, the notion of form is univocal, and that material properties arise from combinations of objects, it is I think clear that the apparatus of quantification is not readily applicable. And given that objects are necessary existents there is no gainsaying ontological commitments. In line with the text, though, what will be argued is that objects are at least *in principle* nameable, that names are not redundant or eliminable in favour of objectual quantifiers and variables ranging over a domain of objects, and that substitutional approaches to names in the context of generalisations is the least bad way of thinking about this *pro tem.*[32]

1.12 *Tatsache*, *Sachverhalt*, *Sachlage*, *Bestehen*

Before proceeding further there are issues arising from these terms, and translations thereof, that stand in need of clarification.

In terms of translation, Pears and McGuinness's terms are used for *Tatsache* as *fact*, *Sachverhalt* as *state of affairs*, and *Sachlage* as *situation*. For *Bestehen* I will use *obtaining*, and not *existing*.[33] Wittgenstein himself suggested "holding" as a translation of *Bestehen*, with reference to 4.122, at *Letters to C.K. Ogden*, p.28 (a suggestion accepted in the Ogden/Ramsey translation).

The approach taken runs as follows. Given the objects that there are, and given that objects have forms, these being their combinatorial possibil-

[32] Cf. Marcus 1978, in particular, pp.117, 118–9. Carruthers terms Wittgenstein's approach "quasi-substitution" (Carruthers 1989, p.115; cf. *op. cit.* pp.171–3). See also Friedman 1997, pp.28–9, Potter 2024, p.113.

[33] Black points out that *Bestehen* "can perhaps best be rendered as 'the holding' (of a fact). It is awkward to speak of the 'existence' of a fact." (Black 1964, p.39) Translation as "holding" or "obtaining" is endorsed by Palmer, with reference to Stenius and Anscombe (Palmer 1996, pp.171–4), and by Shwayder (Shwayder 1963, pp.308–9). "Obtaining" is also used to align with Glock's ascription of an obtainment theory of truth to the *Tractatus* (Glock 2006).

ities into states of affairs together with other objects of suitable form(s), a combination of objects is a state of affairs (2.01).

If there are states of affairs standing in truth-functional relations to one another, the result is a situation.

A fact is the obtaining of a state of affairs, and/or the obtaining of a situation.[34]

The approach is, as noted in §1.11, broadly nominalistic. The objects that there are, are given, and have the forms that they have; the form of an object is thus necessary to that object. Hence in being given objects one is given, at the same time and by the same token, all states of affairs. There is nothing mysterious about possibility, or possibilities. If one is given a set of, say, 100 Lego pieces, partitioned into different shapes, then one is thereby (mathematically/permutationally) given all the ways in which such pieces can be combined with one another. If one makes, say, a house and a car, such that these are spatially independent of one another, then one has an analogue for a situation. And this is I think the sort of thing that Wittgenstein had in mind, along the lines of the model of the Paris traffic accident; but as to what a "truth-functional relation" is, that is another matter. The aim at present is to set out terminology and not to define, as these terms are defined implicitly by the roles they play in the logical system of the *Tractatus*. This is bound up with the logical constants not being representative (4.0312); if one stands the car next to the house there is no "and". Beyond such intuitive gesturing it has to be admitted that there are real difficulties here, with stating what a truth-functional relation is such that we get situations (*Sachlagen*). This is the *Satzverband* problem, discussed in Chapter 5.

The next step is straightforward. If one makes a car out of Lego bricks, then it is *a fact that* there is a car made out of Lego bricks. The critical point is that it being *a fact that* is nothing over and above, it is simply a matter of things so standing.[35]

[34]This, I think, is what Frege quotes back to Wittgenstein in a letter of 16 September 1919: "what corresponds to the elementary proposition when it is true is the obtaining [Bestehen] of a [state of affairs]" (quoted at Klagge 2022, p.321). That a fact is *the obtaining of* or *the holding of* was perhaps first pointed out by Schwyzer; "If a combination of things 'obtains', i.e. if things are combined in such-and-such a way—if the square is red, then it is a fact that it is; if the square is not red, then it is a fact that it is not. Wittgenstein's use of 'fact' is far closer to what we usually mean by the word than has been supposed." (Schwyzer 1962, pp.275–6)

[35]The terminology of *supervenience* is useful here. Armstrong calls this the "ontological

Reverting to *Bestehen*, the opening sentence of 2.06 reads „Das Bestehen und Nichtbestehen von Sachverhalten ist die Wirklichkeit“, translated by Ogden and Ramsey as “The existence and non-existence of atomic facts is the reality”, and by Pears and McGuinness as “The existence and non-existence of states of affairs is reality.” What it means is that reality is *all* the ways in which the objects that there are can combine into states of affairs. This is equivalent to all the ways in which objects are at present combined (the obtaining states of affairs), and all the ways in which they can combine but are not so combined at present (the non-obtaining states of affairs). “Obtaining” makes the point better than “existing”.

“Existence” is furthermore unhelpful, because talk of the “non-existence of states of affairs” makes it look as if there is something substantive, only it doesn't exist, whereas it is simply that there is a way in which objects can stand to one another that at present is not the case—that does not obtain. There is no possible fat man in the doorway, it is merely the case that the objects that could have so stood such that he would have been there are doing duty elsewhere. “Reality” is a capacious term, embracing all the ways in which objects can stand to one another, whether or not they do so stand at present.

It is unfortunate that Wittgenstein continues, in a parenthetical addition to 2.06, to talk of negative facts. This was most likely directed at Russell, but either way, it can be simply defused. One can call an obtaining fact a positive fact if one chooses, and one can call a state of affairs that does not obtain (is *Nichtbestehen*) a negative fact, if one so chooses. The locutions are harmless provided nothing more is read into them. There is nothing ontologically interesting going on.[36]

In the context of language, a state of affairs is represented by an elemen-

free lunch”, that “what supervenes is no addition of being.” (Armstrong 1997, p.12) If things are rearranged according to ways in which they can stand to one another then there is no ontological addition or subtraction. MacBride disputes this as involving some additional commitment to complex universals (MacBride 2018, p.219n.23), but it seems to me that Wittgenstein is committed only to objects so standing and not thereby to complex universals, because if one countenances complex universals as e.g. conjunctions or disjunctions there is a risk of turning the logical constants into representatives.

[36] As Floyd and Shieh note, “Distinct unrealised possibilities individuate distinct falsehoods. This allows Wittgenstein to reinstate *talk* of ‘negative facts’ ... These facts are unrealised possibilities.” (Floyd and Shieh 2024, p.37) McGinn discusses negative facts at McGinn 2006, pp.157–8, making the confusing comment that “A negative fact is what is represented by a true negation of an elementary proposition” (*op. cit.* p.157). A “true negation” of an elementary proposition says what is not the case, it does not represent.

tary proposition, and a situation by a complex proposition. The "everyday propositions" of 5.5563 are complex propositions. Being so representative is to have sense; *having sense* is the defining characteristic of the propositional.

If a state of affairs or a situation obtains—is a fact—then the proposition that represents this state of affairs, or this situation, is true. If the state of affairs or the situation does not obtain the proposition that represents it is false.

It is crucial that the truth-value of a proposition, elementary or complex, is not something over and above the truth-values of its constituent elementary propositions. Filling out earlier remarks at p.4, propositions are bivalent in that any given elementary proposition, at any given moment, is either true or false, *tertium non datur*. This is what it is to have sense. But just as *being a fact* is a matter of obtaining or not obtaining, so truth-value is a matter of obtaining—being true—and non-obtaining—being false. Hence we get *bipolarity*, that an elementary proposition is bivalent *and* at any given moment it is either true, or it is false.[37] An elementary proposition has, of necessity, one of the two truth-values, at any given moment. Which truth-value obtains is contingent. From hereon the term *bivalent* will not be used, and the term *bipolar* will be used as stated.

The totality of states of affairs is reality (*die Wirklichkeit*), and this embraces all possible states of affairs. The states of affairs that obtain, that are facts—what is the case at present—constitute the world (*die Welt*).

1.13 Possibility and reality

In this section the position taken will be more or less programmatically set out. It is assumed that objects are necessary existents, given as a totality, and that they have the forms that they have necessarily.

It follows, given the totality of objects with their forms, that we are given the totality of states of affairs (considered irrespective of which do, and which

[37]Put differently, the step from bivalence to bipolarity is of not countenancing truth-value gaps; as Williamson points out, "Truth-value gaps . . . are irrelevant to necessitism" (Williamson 2013, p.75), and on the present reading, the *Tractatus* is committed to necessitism. Von Wright points out that the term *bipolarity* is used only at *Notes on Logic*, p.94, but is nevertheless one of the "features which can be said to pervade the whole of Wittgenstein's philosophy" (von Wright 1982, p.174; see also *op. cit.* p.192). Spinney notes that "Admitting the possibility of non-bipolar elementary propositions threatens to undermine the success of Wittgenstein's truth-tabular procedure." (Spinney 2023, p.193)

do not, obtain at present). Assuming in principle, for the moment, a 1:1 correlation between objects and names—this will be qualified in §5.10—it follows that we are given all elementary propositions. A possible world, a way in which objects can stand to one another, can be thought of as a consistent (indeed maximally consistent) totality of true elementary propositions (talk of possible worlds is not intended to carry any commitments beyond the heuristic or expository).

If A and B are names for objects a and b respectively, and if a and b can combine into a state of affairs, then AB is an elementary proposition (the possible objection that AB is a list of names is addressed in §2.4). At any given moment the elementary proposition AB is actually true, or actually false, *tertium non datur*; the totality of possible worlds can be partitioned into a possibly empty set in which the state of affairs ab obtains and AB is true, and a complementary possibly empty set in which the state of affairs ab does not obtain (a and b are otherwise combined, in accord with their forms) and AB is false. A proposition has of necessity *a* truth-value in any possible world. Contingency is a matter of *which* truth-value obtains here and now. It may be the case that any given proposition (*not* tautologous or contradictory) is never actually true, or never actually false. If so this is merely accidental and not logically significant.

It is assumed, for the sake of the argument, that this approach extends recursively to complex propositions.

The critical point concerning truth-value is the *nothing over and above* claim, based on the presumption noted in §1.10, that we can identify the propositional as such.

In a 1997 paper Goldfarb asserts that "reification of possibilities" is a common feature of readings of the *Tractatus*, by those who do not countenance resolutism:

> In the 2's of the *Tractatus*, Wittgenstein helps himself to talk of possible situations; his point, apparently, is that any conception of fact, of what is the case, requires as a backdrop a conception of what is not the case but might be. Irresolute readings of the *Tractatus* often rely on a reification of possibilia (e.g., it is central to both Hacker and Pears). Yet, I would claim, Wittgenstein does not countenance possibilia in his ontology. For this would make the obtaining of a state of affairs a property of the combination of objects, whereas Wittgenstein

> is explicit that the combining of objects *is* the obtaining of the state of affairs. (Goldfarb 1997, p.65)[38]

Goldfarb does not provide references to such reification in Pears or in Hacker, and it is not clear that the charge sticks. What I do agree with is that for objects, there is nothing to possibility over and above having forms, that objects can combine into other states of affairs at other times, in accord with their forms. Hence Wittgenstein does countenance possibilities, but only in terms of form and thus not as anything over and above the objects that there are. This is not reification.

Goldfarb also seems to accuse readers who don't subscribe to resolutism of reifying logical form, at the level of elementary propositions, but the argument is unclear. The purported achievement of the resolutist lies in observing that "All we are doing is noting that names have to be put together in one way or another in order to make sentences." (Goldfarb 1997, p.66) This, also, follows on the present reading. There is no need to reify logical form as this is, of course, nothing over and above the forms of objects (cf. p.34).

Goldfarb also emphasises what he calls the "contrastive view of meaning" in the *Tractatus*:

> [F]or a sentence to have content requires a contrast between what would make the proposition true and what would make it false, and hence requires that there be conditions under which the proposition is true and under which it is false. If there is no such contrast, then there is no claim being made by the sentence; it lacks meaning. (Goldfarb 1997, p.67)

The only way, it seems, that non-resolutists can get the contrastive view to work is by somehow standing outside the world, taking up "a stance external to language" (Goldfarb 1997, p.68) such that one can see whether

[38] Also Ricketts; "[I]t is clear that Wittgenstein does not countenance possibilia in his ontology." (Ricketts 1996, p.89) The origin of such "reification" is, I think, Hart 1971; "[I]f Wittgenstein is to have actually existing senses for all propositions whether true or false, then it would seem that he must infer ... that a proposition could be true if and only if there is a possible fact to which that proposition corresponds; and here he must regard possible facts as a variety of facts just as spaniels are a variety of dogs." (Hart 1971, p.277) This is I think wholly mistaken, because *senses* do not *exist*; only *objects* exist. Wittgenstein knew of the difficulties here from Russell's struggles with falsehood and negative facts.

or not such combinations occur; and this looks to be where the "reification of possibilities" comes in, with what Goldfarb calls a "full-blooded notion of states of affairs" (Goldfarb 1997, p.68). The implication is that reifying the possible leaves the non-resolutist with no contrast between what is true and what is false, short of taking up an impossible external stance. Goldfarb then points out the connection between the contrastive view of meaning and bipolarity, pointing out that "Put as starkly as possible, a proposition *is* a contrast, between what makes it true and what makes it false. Hence if there is no contrast, there is no proposition" (Goldfarb 1997, p.69):

> Consider Wittgenstein's characterisation "Objects are what is unalterable and subsistent; their configuration is what is changing and unstable." (2.0271) Once the notion of "changing" is scrutinised, it becomes clear that configurations don't change—configurations of objects are what we express by what Quine calls eternal sentences. (Goldfarb 1997, p.70)

It is true that the *possible* configurations don't change, and neither do propositions thought of in terms of *sense*. But what does change is which configurations do, or do not, *obtain*; which propositions are *actually true*, and which propositions are *actually false*. The "backdrop" that Goldfarb talks about is propositions thought of in Fregean or in Quinean fashion, as true, or false, once and for all. But this is not how Wittgenstein thinks of propositions. Against this backdrop (reality, *die Wirklichkeit*) we have *a* world (*die*, or better, *eine Welt*), it being contingent which elementary propositions are true and thus which world out of all possible worlds obtains. The backdrop is the objects that there are as eternal and unchanging, this being reality, that is, the totality of possible worlds. The contrastive is a function of what is the case, the facts that obtain. The point, here as elsewhere, is to distinguish clearly between the logical, the possible, the formal, on the one hand, and the empirical, the actual (the obtaining), and the contingent/contentual, on the other. In particular there is a reason to distinguish between the *sense* and the *actual truth-value* of a proposition.

1.14 Independence

The last issue to be addressed in this chapter is Wittgenstein's use of the notion of *independence*, translating either *Selbständigkeit*, or *Unabhängigkeit*. Use of these different terms marks the two principal ways in which Wittgenstein talks of independence.

The first use will be called the *Independence Thesis*. This is the assertion that any state of affairs can obtain or not obtain independently (*unabhängig*) of any other (2.061). It follows immediately that an elementary proposition is true, or false, independently of any other (4.211).

The second use concerns the idea that objects are, in some sense, independent of the states of affairs/situations they can be constituents of (2.0122, 2.024), that a picture can be thought independently of what it is a picture of (2.22), that a proposition has sense independently of the facts (4.061). The critical point is stated in 2.0122:

> Things [objects, as 2.01] are independent [*selbständig*] in so far as they can occur in all *possible* situations [*Sachlagen*], but this form of independence [*Selbständigkeit*] is a form of connexion with states of affairs [*Sachverhalten*], a form of dependence [*Unselbständigkeit*]. (2.0122)

Wittgenstein's point is that independence, the word here being *Selbständigkeit*, reflects the assertion at 2.0271 that "Objects are what is unalterable and subsistent; their configuration is what is changing and unstable." Objects are persistent, and the capacity has to be allowed to them to fall out of states of affairs when the latter disintegrate, and then to combine into different (at least numerically different) states of affairs. Objects are not in principle *selbständig*, any more than the names that stand for them; this I take it is the thinking behind the use of *selbständig* at 3.261, in the parenthetical remark "Nor can any sign that has a meaning [signify] independently [*selbständig*] and on its own." An object can no more stand outside the context of a state of affairs than a name can be propositional. Hence the parenthetical addition to 2.0122, "It is impossible [*unmöglich*] for words to appear in two different roles: by themselves, and in propositions", although I think this is somewhat overstated, because impossibility here cannot preclude a compositional account of language. Unless this is so we can hardly understand or make ourselves understood by means of propositions (4.026).

In talking of independence as *Selbständigkeit* it matters that Wittgenstein denies any such thing over and above the *possibility* of change of context for objects and thus of names, what is changeable being which states of affairs obtain, and hence the truth-values of elementary propositions; and superveniently which situations obtain, and hence the truth-values of complex propositions. When he talks of independence as *unabhängig* at 2.024, 2.22, and 4.061, his point is that while we can *think* of objects (substance) independently of what is the case (of which states of affairs obtain), of pictures (2.22) and of propositions (4.061) independently of what is true, they are nevertheless dependent. To use Goldfarb's term, this is the backdrop—reality—against which all plays out. Independence is tightly constrained.

This can go wrong if one holds a Cartesian view of independence, that what can be *thought of* as being independent of other things can *exist* independently of other things:

> [O]n the one hand I have a clear and distinct idea of myself, in so far as I am simply a thinking, non-extended thing; and on the other hand I have a distinct idea of body, in so far as this is simply an extended, non-thinking thing. And accordingly, it is certain that I am really distinct from my body, and can exist without it. (Descartes 1642, AT VII p.78)

This claim, that what one can conceive of independently of something else can therefore exist independently of whatever-it-is, has cast a long shadow, termed by Gendler and Hawthorne the *conceivability-possibility principle*:

> [W]hen we conceive (or imagine) one of two non-overlapping things without the other, we establish the thing's possible distinct existence, thereby establishing their actual distinctness. (Gendler and Hawthorne 2002, p.21)

However, if the only notion of possibility in play is logical possibility, then "conceivability seems somewhat superfluous ... the activity of conceiving seems largely irrelevant ... to the determination of possibility." (Gendler and Hawthorne 2002, p.5) Given that the only necessity in the *Tractatus* is logical necessity (6.37), it follows immediately that the only possibility is logical possibility. And this has nothing to do with what we can and

cannot *conceive*, but only with what we can *think*, this being the sense of a proposition. (3.11)[39]

When Wittgenstein says, "Things are independent in so far as they can occur in all possible situations" (2.0122), that "Substance is what subsists independently [*unabhängig*] of what is the case" (2.024), what he means is that in virtue of their forms objects can be, and are, constituents of different states of affairs at different times. A corollary is that we can think the sense of propositions whether they are actually true, or actually false; consequently Russell's difficulties with negated propositions evaporate.[40]

It also follows that while names and propositions can be thought independently of the objects, states of affairs, or situations they represent, they are nevertheless dependent on the latter. Levine furnishes, I think, an interesting example of how things can go astray. After arguing that Frege can ascribe sense to instances of singular thought that lack a reference, by containing an empty name, Levine argues that this commits Frege to two doctrines; "[A-T_1] We apprehend thoughts, *a priori*, and [A-T_2] In apprehending a thought, we do not thereby have any knowledge of the world." (Levine 1998, p.35) Setting aside questions of Frege interpretation[41], Levine claims that Wittgenstein accepts both of these in the *Tractatus*, and further, that Wittgenstein is committed to an epistemic Russellian principle of singular thought; "[ST_E] Apprehending a singular thought requires knowing (being acquainted with) the object which that thought is about". It follows, he argues, that "Wittgenstein is thereby forced to accept [W_1] We know (*kennen*) objects *a priori*." (Levine 1998, p.38) But there are no Tractarian empty names, so the attribution of [A-T_2] and hence [A-T_1] is not well mo-

[39]The use of the word "imagine" by Pears and McGuinness to translate *denken* at 2.0121 (four times) and at 2.013 (twice), and "imagined" for *gedachte* at 2.022, is perhaps unfortunate, because it suggests that our imagination has some play over and above thinking the sense of a proposition, an activity in which we have no creative scope. After pointing out the translation issue Cerezo says, "there is no room in the *Tractatus* for an idea of *imagining something* different from *depicting something* and, thus, *thinking something*." (Cerezo 2012, p.657n.6). However the translation is arguably warranted by a passage at *Letters to C.K. Ogden*, p.23, concerning 3.001; "I have rendered it by 'we can imagine it' because 'imagine' comes from 'image' and this is something like a picture. In German it is a sort of pun you see." What is imaginable has, though, to be constrained by what is thinkable (6.361).

[40]For Russell's difficulties, see Candlish 1996. In this respect Wittgenstein's views are a substantial improvement on Russell's, provided one is prepared to stomach a fixed totality of objects, of fixed form.

[41]Levine notes that Evans (Evans 1982, pp.27–30) and McDowell (McDowell 1986, p.234) do not agree with his rendering of Frege (Levine 1998, p.20n.4).

tivated, and Wittgenstein's countenancing of sense for propositions does not commit him to the sort of independence that Levine postulates. Levine is partially right in attributing *a priori* knowledge of objects, but he doesn't note that this knowledge is limited to form, and does not extend to content.

It is such misconstruals of independence that lead to the "reification" discussed in §1.13. Ricketts presents the 2s, and then says:

> This discussion of the metaphysics of the 2.0's, a discussion that draws heavily on Wittgenstein's own rhetoric, is dangerously misleading. It ineluctably suggests by its very grammar that the determination of the range of possibilities by the form of objects is itself some sort of fact. Furthermore, talk of [states of affairs] as obtaining or not obtaining—see 2, 2.04–2.06, and 4.21—reifies possibilities ... The reification of possible [states of affairs] would make them independent of what is the case. (Ricketts 1996, p.89)

It is only misleading, and such reification only occurs, if one thinks that what one can *conceive of* as independent of something else entails that it can *exist* independently. Once one appreciates that one can only think logically, i.e. propositionally, and holds to the fact that one is dealing with "a form of dependence" (2.0122) („eine Form der Unselbständigkeit"), any temptation to think thus evaporates.

Chapter 2

The *Tractatus* as a formal system

If Wittgenstein is committed to a *sui generis* conception of logic, as having no subject-matter of its own, as p.31, then it should be possible for him to develop a logical system *independently* of its application (bearing in mind the consideration of independence in §1.14). This is the subject of Chapters 2 and 3.

2.1 An uninterpreted calculus

In his introductory book on Wittgenstein, Grayling says this:

> In certain respects the *Tractatus* is like a game of chess. One cannot imagine thinking that the *Tractatus* might be true any more than one can imagine thinking that a game of chess might be true. This is because the *Tractatus* is an uninterpreted calculus. The key notions 'object', 'name', and so on are formal devices like the pieces in chess: the 'queen' in chess is not in any sense a queen, even a toy one, but is a purely formal entity defined by its permitted moves alone. This is what the 'objects' and the 'names' of the *Tractatus* are; they are elements of abstract parallel structures, defined only by their roles and mutual relations. (Grayling 2001, p.58)[1]

[1]For the chess analogy, see Kenny 1973, pp.74–7, Schroeder 2006, pp.49–51.

This is, I think, a reasonable way of approaching the *Tractatus* albeit backed, as will be argued shortly, by a way of thinking about logic that was not available to Wittgenstein at the time. In the first place, Grayling's assertion that truth is not a concept that is readily applicable to the *Tractatus* is incompatible with Wittgenstein's remark in the Preface that "the *truth* of the thoughts that are here communicated seems to me unassailable and definitive." (p.4) Nevertheless the impression of conveying an uninterpreted calculus in formal terms, with a definite separation between the apparatus of such a calculus ("names"), and what there is ("objects"), with the postulation of some sort of common structure, is not unfounded.

What needs to be explained, then, is why this approach is misguided. Such explanation involves the notions of truth, and of naming and interpretation; in particular, that how Wittgenstein thinks about naming is at a distant remove from thinking about naming in terms of an interpretation, that is, a mapping between elements of categorically different domains. The difference is between Frege's and Russell's thinking about naming, as influencing Wittgenstein's approach, and the conception of naming that first emerges clearly in Hilbert and Ackermann 1928. This text marks the emergence of the modern conception of first-order and higher-order logics, with a clear-cut notion of first-order logic as dealing with a domain of individually identified elements that can be named (by means of an interpretation, a mapping from names to discrete, identifiable entities) and quantified over.

There are, in other words, two fundamentally different ways of thinking about naming that stand in need of explanation, and along with this, an account of how commentators on the *Tractatus*, self-consciously or otherwise, treat naming in their discussions of the text. Some commentators, I suggest, notably Hacker and Pears, are committed to anachronistic ways of thinking about naming that are, wittingly or otherwise, carried over from ways of thinking about first-order logic that have become more or less embedded in our philosophical thinking.

This is bound up with what will be called the *Independent Access* problem, that is, whether naming involves having, in some fashion, independent access to names on the one hand, and elements of the domain—objects, in the present context—on the other, such that one can, somehow, attach or ascribe names to objects. Independent access has to be considered both in the earlier conception, pre-dating the clear-cut emergence of first-order

logic, and in the later conception, the latter involving an interpretation (a mapping between domains).

The notion of Independent Access is, I think, central to interpreting the *Tractatus*, because the key interpretive problem is that of objects and their names. The charge sheet brought by anti-realist interpreters against metaphysical/realist readings begins with Independent Access, as seen in McGuinness:

> We are naturally inclined to attribute to Wittgenstein . . . a theory on the following lines. Propositions have sense, i.e. are either true or false, because the following is a conceivable series of events for every one of them: first, a set of names or simple signs . . . is correlated with a set of objects by ostensive definition. Second, some of these names are put into a relation which is possible for them. It happens that any relation which is possible for the names is possible also for the objects with which the names have been correlated. The fact that the names stand in the relation in question will then be a proposition to the effect that the objects also stand in that relation. Now, one of two states of affairs holds, either the objects are so related or they are not. Whichever of these two is the fact must be compared with the proposition, and if it corresponds, the proposition will be true. However, it will be seen that it can only be either true or false. Thus given that every proposition could in theory be constructed in the way described, it will be seen that every proposition will be either true or false and what its being true or false consists in. (McGuinness 1981, pp.82–3)

As Grayling, this presents *a* way in which one might think, reasonably, about the text. Because Wittgenstein does talk about naming, in particular, in the 3.2s. But these propositions are followed immediately by his statement of the context principle, at 3.3; "Only propositions have sense; only in the nexus of a proposition does a name have meaning." *Prima facie* there is a tension between meaning as a direct correlation of name and object—"A name means an object. The object is its meaning" (3.203)—and the context principle, that a name has meaning *only* in the context of a proposition. And one can give very different accounts of the *Tractatus* by emphasising one or the other of these. That Wittgenstein's thinking was in flux is pointed out by Kremer, in his discussion of the elevation of the context principle between

the *Prototractatus* and the *Tractatus* itself (Kremer 1997, p.90). This is discussed further in §4.4.

The point about naming and interpretation concerns *application*, how logic gets to be applied in thinking about and talking about the world, because different ways of thinking about naming correlate with different conceptions of logic. In this chapter and the next the logic of the *Tractatus* will be presented along the lines of a formal system, termed *System Tractatus*. Such a system can, I think, be found in the text. It will be argued, though, that this system is immanent in the propositions of our everyday language, discoverable by analysis. It is not a free or *sui generis* construction of the mind. That the system is applicable is assumed, in the sense that the "parallel structures" noted by Grayling are realised isomorphically by language and by the world, that this is what it is for logic to have the correct multiplicity (4.04), to be "a mirror image of the world" (6.13), and for the "propositions of logic [to] describe the scaffolding of the world" (6.124). *System Tractatus* is a *calculus ratiocinator* that amounts to a *lingua characterica*, because it is all-encompassing and all-embracing. It is not, though, an uninterpreted calculus, because Tractarian names name objects. Put differently, Grayling's approach treats the logic of the *Tractatus* as *sui generis*, but fails to respect this logic as consequent on a view of reality (specifically, of the forms of the objects that there are).

2.2 The problem of naming

The critical point raised by McGuinness is that talk of naming involves talk of *meaning*. This is where things get difficult. I will concentrate first on notions of naming as they were available to Wittgenstein at the time, before now-familiar separations between syntax and semantics. Frege's conception is tripartite, that for names, there is the *idea*, the *sense* (*Sinn*), and the *meaning* (*Bedeutung*) ('Sense and Meaning', pp.26, 29).[2] The idea is wholly subjective, what an individual has in mind, whereas sense is an wholly objective way of thinking about whatever-it-is, available to any thinking subject. The meaning is the thing itself, the object of thought. It is an obvious move to think of Frege as a descriptivist, that there are one or more

[2]This is Frege's post-1891 account. Wittgenstein's approach in the *Tractatus* is I think influenced by this account and not by the earlier *Begriffschrift* account, lacking the distinction between sense and meaning.

ways in which one might think about something; the *Morning Star*, or *the man washed ashore at Ithaca*, and so on. For Frege, every name has at least one sense associated with it, because this is how names function as names. Naming is indirect, going via sense.

Russell's account is less easily stated. Ostensibly it looks to be Millian, a bipartite account in which names have denotation but no connotation; they denote directly whatever-it-is without saying anything further (connotation) about whatever-it-is to enable one to ascertain what is being talked about (what is denoted). And on one level this looks right. One either knows that "Walter Scott" names Walter Scott, or one doesn't, and on being introduced to Walter Scott all one acquires thereby is recognitional information which, presumptively, results in a recognitional capacity, an ability to recognise Walter Scott on later re-encounter. This is the "Fido"/Fido account of naming.

If one is a thorough-going anti-psychologist, Frege's account appears stronger because it does not rely on a postulated recognitional capacity. Sense as an explication of meaning says what Russell can, it seems, only gesture at, by invoking some such capacity. But Russell rejects Frege's account because he thinks that interposing sense between name and entity named renders the latter somehow or somewhat unknowable. Russell thinks that it is only by direct acquaintance with whatever-it-is that one can thereby gain *knowledge* of it; anything that mediates such direct acquaintance precludes knowledge. This is his main point in the *Mont Blanc* correspondence (Frege 1980, pp.163, 169). Knowing that Mont Blanc is over 4000 metres high is knowledge of a description of Mont Blanc, and this is a piece of knowledge *about* Mont Blanc. It is not knowledge *of* Mont Blanc, because this can only arise from acquaintance. For Russell, Mont Blanc is not a constituent of the proposition, "Mont Blanc is over 4000 metres tall". One does not have the mountain itself in mind, but a presentation that meets Russell's standards for knowledge of the thing itself. It follows that one can be said to understand the proposition fully only if one's knowledge of Mont Blanc is by acquaintance, and not by description. This is not a distinction that can be readily stated, but it is one that Russell clearly makes.[3] It is in fact a distinction that can only fully be *shown*, by how one responds to propositions.

Faced with these two very different views, Wittgenstein looks to come

[3]This goes against the orthodox interpretation of Russell; see further Hay 2022b.

up with a way of retaining what is most attractive from each. 3.3 says, "Only propositions have sense", but this is better stated as, only propositions instance sense, because we "think the sense of the proposition" (3.11, amended translation). The sense of a proposition is a way in which objects can stand to one another, and it is true if they do so stand, false if they do not. The critical point is that sense addresses possibility. It is a further step to ascertain truth-value, to "advance from a thought [a proposition] to a truth-value" (cf. p.37).

After taking a broadly Fregean approach to propositions, Wittgenstein ostensibly takes a Russellian approach to names, treating them as having meaning/denotation and lacking sense (or, for that matter, any sort of analogue or correlate to sense). This, outwardly, is the force of 3.203, "A name means an object". But this cannot straightforwardly be the case, as account has to be taken of the second phrase of 3.3, that "only in the nexus of a proposition does a name have meaning." There is no allusion to sense, rather it is *only* in the nexus of . . . ; and here the reader should recall 2.0231, that "it is only by means of propositions that material propositions are represented—only by the configuration of objects that they are produced." There is, I suggest, a close correlation between 2.0231, and 3.3. The sense of a proposition is bound up with the realisation of material properties, because this is the *content* of propositions, what it is that we think and perceive. That names have meaning only in the context of a proposition is because names, somehow, name objects, where objects are not in any straightforward sense individually identifiable and nameable; because they do not realise *as individuals* perceivable material/empirical properties.

If we think of naming as "Fido"/Fido then on the Russellian account naming requires entities that are individually identifiable and nameable. But this is exactly what Tractarian objects are not. So however naming works in the *Tractatus* it is not a matter of acquaintance followed by "Fido"/Fido dubbing. McGuinness is right that Tractarian semantics should enable us to grasp what a proposition says (its sense) such that we can proceed to ascertain what its truth-value actually is. But however naming operates in the *Tractatus*, it cannot be along the lines of a straightforward correlation or mapping between names and objects.[4] But consideration first needs to be

[4]Carruthers, I think, picks up on the difficulties here in denying a straightforward correlation account in favour of a "quasi-Fregean" account, deploying *semantic values* for

given to changes in logic between 1879, when Frege published *Begriffschrift*, and the early 1930s, when the conception of logic that emerged in Hilbert and Ackermann 1928 came to maturity. The issue of naming can then be reconsidered against this altered background.

2.3 Logic in flux

Issues around naming involve universalism, because the defining doctrine is that there is no external perspective from which one can talk about and theorise about language (§1.2). But this cannot preclude informal thinking and reasoning about what logicians and mathematicians do (p.19). Such talk can be elucidatory or clarificatory, but it cannot be propositional. One cannot for example state *truths* within language as to how language does what it does. One cannot state semantic facts, as truths; one can only hint and gesture, ask for pinches of salt and hope for meetings of minds. Equally one cannot state facts about and theorise propositionally about syntax. As Wittgenstein makes clear, the theory of types cannot be *stated*, it can only be *shown*, in a correct notation (3.332). As Hintikka points out, if one subscribes to universalism, one "cannot step outside language so as to be able to view its relations to the world. Hence *semantics is ineffable* according to the thesis of the universality of language." (Hintikka 1990, p.215) In the Tractarian context this is where one falls back on talk of *showing* rather than *saying*. The difficulty in the background is that the distinction is, *prima facie*, too coarse, failing to distinguish within the non-propositional between what is *formally* nonsense (ill-formed) and what succeeds in being elucidatory.

When it comes to naming, proponents of universalism can of course distinguish between a name, and what is named. The universalist can say, in an appropriate setting, "I name this ship *Beelzebub*", and assert that it is *a fact* that this ship is called *Beelzebub*, that it is *true* that this ship is called *Beelzebub*. A name has been attached to an entity by baptism or dubbing, and is thereafter maintained by usage and custom. There is no obvious connotation or description attached to or conveyed thereby. So for present purposes what is needed is an account of how things change in the later, post-Hilbert and Ackermann 1928 model.

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names (Carruthers 1989, pp.120–2). Child criticises this as "implausible", but he looks to revert to a straightforward correlation account (Child 1991, p.355).

The difference can, I suggest, be stated thus. On the latter conception, that this ship is called *Beelzebub* is not true in the same way as *Beelzebub* is 180 foot long is true, and it is not a fact that this ship is called *Beelzebub* in the same way as it is a fact that *Beelzebub* has an armour-plated hull. "Truths" and "facts" about naming and referring relate to our customs and practices and not, univocally, to how things are in the world. Invoking the notion of interpretation breaks the implicit Tractarian identification of name and object named.

The salient move in breaking this identification is a clear-cut distinction between object-language and meta-language, a distinction that emerges in work by Gödel, Tarski, and Carnap, in the early 1930s.[5] This goes together with the classification of logic into first-order and higher-order that emerges in Hilbert and Ackermann 1928 (a clear-cut distinction between „Der engere Funtionenkalkül" and „Der erweiterte Funtionenkalkül"). When both of these come together one gets the modern full-blooded model-theoretic conception of logic that stands, I think, behind the notion of logic as an uninterpreted calculus that Grayling has in mind in the quote at p.51.[6]

The picture that emerges is of a well-defined domain of individuals, each of which can be individually identified and named in "Fido"/Fido fashion. This is the basis for first-order logic, as without such a domain of individuals one does not have, for example, the sequences that are needed for Tarski's account of truth, based on satisfaction. Such a well-defined domain of individuals is also needed for familiar accounts of quantification. Goldfarb emphasises this in discussing parallels between quantification and choice functions; "The connection between quantifiers and choice functions ... is the heart of how classical logicians in the twenties viewed the nature of quantification." (Goldfarb 1979, p.357) Both Hilbert and Skolem rely on this, effectively construing the domains they were addressing as composed of well-defined individuals (Goldfarb 1979, p.360). Noting that in the Tractarian context, the familiar *differences* between objectual and substitutional

[5] Cf. Carnap 1963, p.54.

[6] In discussing Russell, Proops says, "Some advocates of the universalist interpretation have maintained that Russell's conception of logic differs from 'the' modern one in conceiving of logic as a body of truths as opposed to schemas" (Proops 2007, p.7), with Quine taken as exemplifying the modern conception. Proops argues that "the attempt to draw a contrast on this point is confused" (*op. cit.* p.7). In the Tractarian context, though, it seems to me that there is a distinction to be drawn, between names as taking one directly to the things themselves, and as going via an interpretation.

accounts of quantification are not salient, because the parallelism noted by Grayling, along with multiplicity (and thus isomorphism) for objects is effectively read over to an individuation of names (coupled with the implicit assumption that there is a name for every object).

What goes together with the later conception is the notion of an interpretation, a mapping of names to objects. While this may look to be much the same as naming by baptism or dubbing the way of thinking about naming is, I suggest, fundamentally different, but this difference can only be drawn by contrast.

On the mapping conception names are assigned to elements of the domain by deliberate statements of the form, "'a' names a", or "'a' means a", and so on for other elements of the domain, as required. This may look to be the same move that Wittgenstein makes at 3.203, when he says that "A name means an object. The object is its meaning", but there is a difference, to do with how these expressions function. On the full-blooded model-theoretic conception a deliberate decision is made, to assign *this* name (on the one hand) to *this* object (on the other). It is thus a metalinguistic statement, that *this* mapping between name and object is established, for present purposes; and this mapping may be changed at some future date (a different name may be mapped to this object).[7] In the metalanguage one can sensibly ask, is it true that *this* name is mapped to *that* object? The expressions that one uses are distinguished; one uses, in the metalanguage a translation of words in the object language (the translation may of course be homophonic) ("a"), words in the metalanguage itself (*names*, *means*), and reference, somehow, to the object itself that is named (a). There is a great deal of theoretical apparatus that is not present in 3.203, which functions as an elucidation of how Tractarian naming works and does not purport to state truths or facts.

The point, on the earlier conception, is that there is no attempt at theorising, and there is no object language/metalanguage distinction. Names are much more tightly tied to what they name than on the model-theoretic approach; there is, in short, a fundamentally different but largely tacit notion of naming and referring in play. That this tacit conception ineluctably involves ourselves is true, but our involvement has to be regarded as incidental. As p.37, language functions as a transparent medium that simply

[7] This is Putnam's push-through construction; cf. Putnam 1977, Button 2013, Chapter 2, Button and Walsh 2018, Chapter 2.

takes one directly to the things themselves.

This is, I suggest, even more difficult in the Tractarian context than it is with respect to Frege, and to Russell. Whether or not it is correct to ascribe descriptivism to Frege, he does hold that sense functions as a route to the reference associated with a name. Hence there is some conception of name, and thing named. In Russell's case, by the time of his interactions with the young Wittgenstein he held, I think, that only objects of acquaintance can be properly said to be named. The doctrine of logically proper names ('The Philosophy of Logical Atomism', pp.200ff) is predicated on names being tightly tied to, and taking one directly to, what they name.

Wittgenstein, as we have seen, rejected Frege's account of ascribing senses to names. But he did not adopt Russell's approach wholesale. While he holds that, somehow, objects are named, he does not cash this out in terms of acquaintance and dubbing. This of course leaves his notion of naming and indeed of object obscure. It also imposes a burden on commentators to make sense of this unpromising material.

A corollary is that Wittgenstein does not hold to a full-blooded model-theoretic conception of logic, and he does not hold to a first-order/higher-order conception of logic with a domain of well-defined individuals that can be quantified over. But Wittgenstein's approach with a single category of names as the constituents of elementary propositions exercised, I think, some influence, however indirect, on the emergence of first-order logic, and it is tempting to project such a conception of logic onto the *Tractatus*. The idea that material properties arise from combinations of objects is also in principle not incompatible with treating predicates and relations extensionally, as sets of instances. But if one goes down this path the notion of naming in play becomes more rather than less obscure.

It may be that Wittgenstein thought that a syntactic specification of the category of name is sufficient, and that the semantics could be left to take care of themselves; much as for logic it is enough that objects have forms, that the realisation of form as content can be left to others. But what does matter for the present approach is that if the *Tractatus* can be thought of as a formal system, and if the notion of naming is central to application, then the notion of object and thus of name has at least to be consistent. Anachronistically, it has to be possible for the system to have a model.

It would, then, be an immediate problem if Hacker's claim can be upheld,

that "if one searches for an example of something that will satisfy Wittgenstein's specifications [for an object], one will search in vain. As Wittgenstein himself realised in 1929, the specifications are inconsistent, and there can be no such thing as a simple object as conceived in the *Tractatus*" (Hacker 2021, p.70). Because given that the notion of *object* is key to *System Tractatus*, if the notion of *object* is inconsistent then there can be no model.

Hacker assumes, I think, that the specification given is inconsistent, without offering much by way of argument or explanation beyond Wittgenstein's later rejection of his earlier views.[8] He suggests, to aid the reader, that "it may be helpful to think of spatiotemporal points, of unanalysable colours or notes as 'objects' " (Hacker 2021, p.21) On the basis of 2.0231, that "it is only by means of propositions that material properties are represented—only by the configuration of objects that they are produced", Hacker's hints constitute exactly what objects cannot be; objects cannot be individually identifiable instances of material properties. Ishiguro is right, I think, when she says that whatever properties Tractarian objects have (i.e., form), the "properties concerned are not material properties like being of a particular colour." (Ishiguro 1969, p.45) One has to be able to do justice to 2.0232, that "objects are colourless" (even if this is parenthetically qualified, "in a manner of speaking"). Goddard and Judge pick up on 2.032, but take it to an opposite extreme: "In the sense of 'property' which excludes relations, objects have no properties on this view—not merely no perceptible properties, but no properties at all." (Goddard and Judge 1982, p.9) Consequently they argue that the only relation that can obtain between objects is an univocal "*relation of combination*" (*op. cit.* p.10). Their objects are thus Copi's "absolutely bare particulars", possessing neither formal nor material properties, and not his "bare particulars", possessing formal but not material properties (Copi 1958, p.184). But the variegated world we experience cannot be composed solely of absolutely bare particulars standing in an uniform relation to one another. One has to do justice to both form *and* content, leaving room for the world as we actually experience it.

2.0231 and 2.0232 also rule out any account that identifies objects as individually perceptible, in particular, as sense-data (Hintikka and Hintikka

[8]MacBride also thinks that Tractarian objects cannot be specified; "The objects of the *Tractatus* can neither be interpreted as particulars, such as the material particles of physics, nor universals, such as the colour blue." (MacBride 2018, p.201) He doesn't state whether he thinks the Tractarian notion of object is itself inconsistent.

1986, p.55) or as qualia (Frascolla 2007, pp.78, 100). Regarding objects as specks of colour immediately falls foul of the colour exclusion problem; Wittgenstein knew of this problem at the time he wrote the text and, I think, had, or at least thought he had, an answer to this, an answer that precludes objects being specks of colour (this is discussed further in Chapter 8). What is also ruled out is a "Fido"/Fido treatment of names, and any straightforward 1:1 correlation of names and objects. There has to be scope for a different account; to give it a name, we can call this *Tractarian naming*.

By way of preview, what will be suggested is that objects are ultimate material particles, entities to be discovered by science. Nevertheless objects can be considered logically, in terms of Grayling's "abstract parallel structures". One thus gets objects that can be considered either logically, that is, in terms of form, or physically, that is, in terms of content. This is I think the crux of 2.025, that objects (as substance) are "form and content". The promissory nature of Wittgenstein's approach is expressed by Pears, that Wittgenstein "relied on his *a priori* argument for his extreme version of logical atomism and expected the details to be discovered later", to which he adds a telling footnote, "*As if this were science*" (Pears 1987, p.84, emphasis added). The present reading can be seen as following out Pears' footnote.

In terms of meaning, that is, naming, Hacker says:

> The last residue of analysis consists of simple unanalysable names . . . It is they that 'pin' language to reality, for their meanings *are* the simple, sempiternal objects in reality for which they stand. (Hacker 2021, p.20)[9]

However in giving an account "of the correlation of name and object" (Hacker 2021, p.73), such correlation arising as "the result of some mental act of meaning or intending a certain word to signify an object one has in mind" (*op. cit.* p.73), Hacker refers overwhelmingly to the *Notebooks 1914–16*, and not to the *Tractatus*. This mental act is an act of the will; "The idea that the skeleton of language only takes on flesh and blood through the mediation of the mind, in particular the will, is implicit in the *Notebooks 1914–16*." (Hacker 2021, p.74)[10] But such an account is not to be found in

[9] For the metaphor of "pinning" see *Notebooks 1914–16*, p.53, quoted at p.6.

[10] Winch regards some such mental act as a "natural . . . further step" if one thinks there is, or at least should be, an account of a name/object relation of meaning in the *Tractatus* (Winch 1987, p.6).

the *Tractatus*.

Writing after Hacker, Pears suggests "as a vaguely formulated possibility" that "If [objects] were not material points it may have seemed to [Wittgenstein] possible that they should belong to the world as we find it without being Russellian sense-data and their properties" (Pears 1987, p.90), but this doesn't make any obvious progress. This suggestion is not filled out, Pears asserting that "[I]t was surely a mistake to leave the category of the objects underpinning factual discourse unspecified in the *Tractatus*." (Pears 1987, p.194)

Faced with this, Zalabardo asserts that "facts are the only ultimate items of Tractarian ontology ... objects are not self-standing items from whose combination states of affairs arise ... Obtaining states of affairs are indivisible units." (Zalabardo 2015, pp.116, 117) This is a way out, but it is incompatible with the text. It is unclear how indivisibility can allow for the possibility of different states of affairs obtaining at different times/in different possible worlds, where it is required, by 2.0271, that the configuration of objects into states of affairs "is what is changing and unstable".[11]

Zalabardo's approach acquits Wittgenstein of any need to solve Russell's problems with propositional unity (Zalabardo 2015, p.108), because such problems evaporate if a state of affairs and by extension an elementary proposition is an indivisible unity. But this comes at a price. 2.03 says, "In a state of affairs objects fit into one another like the links of a chain"; the correlate for elementary propositions being 4.22, "An elementary proposition ... is a nexus, a concatenation, of names." There is no obvious way in which this can be construed as saying that a state of affairs "is an indivisible unity". In reviewing Zalabardo 2015, Proops says, "as if in defiance of 2.03, Zalabardo claims that the *Tractatus*'s approach to the problem of factual unity takes an altogether different form. Wittgenstein, he maintains, is dissolving rather than solving the problem ... The stubborn truth ... is that the reading's textual fit is poor." (Proops 2017, p.534) Whatever account one gives has to do justice to objects as constituents of different states of affairs at different times, that they have *some* degree of independence from states of affairs (2.0122). Zalabardo's account does not, I think, meet this

[11] Concomitantly it is hard to find a treatment of negation in Zalabardo's text, beyond some inconclusive remarks at Zalabardo 2015, p.117, and a discussion of picturing and negative propositions (*op. cit.* pp.217–9, an account dismissed at *op. cit.* p.226.)

consideration.

The notion he adopts of *common* or *shared features* (Zalabardo 2015, pp.124–5), where this relates to logical form (Zalabardo 2015, p.53), is cashed out in terms of the use we make of propositions (Zalabardo 2015, pp.206ff). But this leads into an account of analysis that is based on showing what Zalabardo calls our "inferential inclinations", that "our main source for determining the truth-functional structure of our everyday propositions" (and concomitantly, I take it, the common or shared features of propositions) involves "referring to our inclinations concerning the logical relations that they bear to each other—for example, when two propositions are incompatible or when a proposition logically entails another proposition" (Zalabardo 2015, p.206). But this is I think far too psychologistic. There is nothing close to "inferential inclinations" in the text, rather it is made clear that inference is *shown* by propositional structure (5.11ff). When Wittgenstein says, "All deductions are made *a priori*" (5.133) this is because deductions are *shown* by propositional structure, not because we can make them prior to experience.

Wittgenstein undeniably makes some very large and deeply uncomfortable assumptions; that there are objects that do not exhibit empirical properties individually, but do so in combination, and that such objects can be named. It may be that any attempt to arrive at such objects, by logical or by empirical means, ultimately runs into the sand. The thing is not to throw in the towel too early. For reasons already noted Hacker's hints do not accord with the text, because of colour exclusion; if this object here, now, is an unanalysable colour—red, say—then it cannot also be a different unanalysable colour—green, say. Given that Wittgenstein was aware of the colour exclusion problem at the time (6.3751) he must have thought that he had a suitably *logical* solution to it, and any account needs to give a plausible account of this. Hacker, I think, throws in the towel too early.

Nevertheless I do not want to deny the plausibility of Hacker's account in outline. We are able to talk about what there is, propositionally, and the account given is plausible against the background of the *Notebooks 1914–16* and in the context of how little is said in the *Tractatus*. Given that Hacker regards the *Tractatus* as an "austerely beautiful edifice" that nevertheless "crumbled ... in ruins" shortly after Wittgenstein's return to philosophy in 1929, he may not be averse to seeing the *Tractatus* as flawed from the outset. But it seems to me that one can reasonably refuse any account of

the correlation between name and object that involves independent access to the name and to the object named, because this implies an independence of names and objects and not "a form of dependence" (2.0122). Indeed to get the text to work one has no choice but to reject any such account. The burden is to try and find a different approach that ascribes to Wittgenstein a plausible response to the colour exclusion problem at the time of composing the work, and plausible grounds for later rejecting it. This is addressed *seriatim* in the present work and ultimately in Chapter 8.

Perhaps unsurprisingly there is a tendency to assimilate Wittgenstein to Russell, to regard Tractarian naming as relevantly similar to Russell's logically proper names in 'The Philosophy of Logical Atomism', names emptied of sense or connotation ("this", "that" and, perhaps, "I") that apply only to perceptions maximally emptied of empirical/descriptive content. This is not how Tractarian naming works, not just because objects taken individually have no empirical/perceptual content, but also because they are not independently available. And this is the critical point, bound up, of course, with the context principle.

This is where, I think, the later pervasiveness of first-order logic and the concomitant notion of a domain of well-defined individuals to which names can be attached by a mapping is salient. It is not just the idea that there is a flexibility about naming, that we could, if we wanted, propose different mappings or interpretations. It is the critical notion of there being discrete, identifiable individuals to which names can be applied, that these individuals can be the values of bound variables or their name substituted for schematic letters, and consequently, that we are and show ourselves to be ontologically committed to such individuals. And this, I think, with its concomitant "Fido"/Fido account of naming is exactly what Wittgenstein rejects, by trading in states of affairs as concatenations of objects, where these are represented propositionally, and where it is only propositions that have sense (that say how things stand). So the moment one starts to think in terms of objects and names, of a correlation between objects and names—as both Hacker and Pears do—then one has lost sight of Tractarian naming. But pointing out that Hacker and Pears do not fully respect the text, in particular, the context principle (3.3), is not yet to offer a positive account.

The demand on Tractarian naming, then, is to delineate meaning as a "relation" of some sort between a name and an object, side-stepping the

problem of independent access, with "name" and "object" functioning as theoretical terms, implicitly defined within the theory. For names conventionally regarded a list of names is not a proposition, but if a list of Tractarian names collapses into an unity, then it becomes itself a name and not a proposition with, as a corollary, a loss of any grasp of compositionality and thus of a learnable language.[12] Reverting to Grayling, he notes the key notions of the *Tractatus* as *object* and *name*, as "elements of abstract parallel structures." Assuming such parallelism, we can begin with and concentrate on *names*, on the basis that (at a later stage) an account can be given for our access to objects, consonant with 3.203, that "A name means an object. The object is its meaning." But before that it may be helpful, in advance of the account to be given in Chapter 8, to offer a preliminary sketch of the proposed model.

2.4 Sketch of the proposed model

The demands on Tractarian objects are these:

1. Not individually identifiable and nameable in "Fido"/Fido fashion.
2. Not exhibiting material properties individually.
3. Having form, a capacity to combine with other objects to generate a range of states of affairs exhibiting a range of material properties.
4. Given as a totality partitioned by form (where empirical enquiry will show that the number of forms is at least two).
5. Thinkable independently of states of affairs but in some sense dependent on states of affairs.

The demands on Tractarian naming are these:

1. Not individually meaningful (context principle).
2. Not saying anything individually (the "impossibility" of 2.0122).
3. Having a combinatorial capacity to combine with other Tractarian names to form a proposition and not a list of names (an elementary proposition is "articulate" (3.141), reflecting the fact that a state of affairs has a "structure" (2.032)).

[12] And of course an unwanted commitment in some fashion to Frege's later treatment of propositions as names of truth-values (e.g., Frege 1892, p.34).

4. Given as a totality (5.5561), functioning primarily as tokens and not as types (applying primarily to forms and only secondarily to individual objects).
5. Thinkable independently of propositions but in some sense dependent on propositions.

Given the realist/scientific bent of the present reading, the proposed model draws on structural chemistry as developed in the second half of the nineteenth century. Structural chemistry assumes combinations of elements according to combinatorial principles, resulting in the world as we experience it. This is, I take it, at least scientifically uncontroversial, perhaps little different from Feynman's observation:

> *[A]ll things are made of atoms—little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another.* In that one sentence ... there is an *enormous* amount of information about the world, if just a little imagination and thinking are applied. (Feynman 1963, p.I–2)

In the late nineteenth century this *was* controversial, because the view that matter is ultimately composed of atoms was not universally held.[13] Wittgenstein would have been familiar with this from reading Boltzmann's *Populäre Schriften* and, one would expect, have followed him in accepting the reality of atoms as the ultimate constituents of what there is. The underlying point, though, considered abstractly is that structural chemistry operates with elements that are partitioned into, at the time of writing, 118 different recognised forms, with differing combinatorial capacities (to form bonds) resulting in molecules which can then combine (intermolecular forces) to form the world as we experience it. Atoms of the same element are, broadly, qualitatively indistinguishable, and they are at least not readily individually identifiable. The matter (energy) of the universe is plausibly given as a whole. We can think about and talk about atoms as if they are individuals, but very often in practice we can only do so *en masse*, statistically.

[13]There is a huge literature on this; cf. Schütt 2003, Rocke 2003. A comprehensive account can be found in Russell 1971. The leading proponents in the atomism debate in Wittgenstein's youth were Boltzmann (pro—cf. Cercignani 1998, *Populäre Schriften*, Chaps. 10, 11) and Ostwald (con—cf. Ostwald 1895, Russell 1971, pp.317–8).

I do not want to lay too much emphasis on this. The aim is to propose a model to show that the account of objects and naming in the text is not inconsistent. The proposed model is, I hasten to add, not sophisticated, and probably doesn't go beyond Wittgenstein's chemistry course at Charlottenburg Technische Hochschule.[14]

2.5 Names

In approaching the *Tractatus* along the lines of a formal system, names are assumed as basic or primitive elements.[15] Here, though, we run into an immediate, if not perhaps wholly surprising, difficulty. Given that we do not have a specification for the notion of *object*, we cannot readily have constant expressions that name objects. The point about *object* is that it is a formal concept (or a pseudo-concept, as 4.1272) so it cannot appear in a proposition. One cannot *say* that there are objects, one can only *show* that there are objects by using a notation that includes object expressions. So at 4.1272 Wittgenstein says that "the variable name 'x' is the proper sign for the pseudo-concept *object*", "pseudo" here just meaning "formal" (Black 1964, p.202).

In regarding the variable for objects, that is, object expressions—names—as variable, what Wittgenstein intends is that x functions syntactically as a name variable *in a propositional context* (a prototype, as 3.315) for which other names can be substituted. The critical point is that variability is contextual, because this is how the context principle (3.3) is upheld. This is the force of 3.314; "An expression has meaning only in a proposition. All variables can be construed as propositional variables. (Even variable names.)" Variables do not appear by themselves, they are not somehow entities in their own right that range over whatever-it-is. And this is, I think, critically important in understanding the *Tractatus*. The notion of variability is much harder than it may at first appear, shown in Russell's discussion in *Principles of Mathematics*. This is hedged with uncertainties, the "least objectionable" theory being, "Taking *any* term, a certain member of any class

[14]The lecture course for first-year engineering students at the Charlottenberg Technische Hochschule, reproduced at Hamilton 2001, pp.58–9, notes courses in experimental chemistry.

[15]The presentation follows that of a typical logic textbook, with a four-part division into vocabulary, formation rules for elementary propositions, rules for logical constants/connectives for elementary propositions (complex propositions), rules of inference.

of propositions of constant form will contain that term ... We may say that x is *the* x is *any* ϕx, where ϕx denotes the class of propositions resulting from different values of x" (*Principles of Mathematics*, §86).

The Wittgensteinian twist is to concentrate not on the x but on the class of propositions in which what would conventionally be called the *values* of x appear. Curious though it might seem, for Wittgenstein what is variable is best thought of as what remains constant, this being the *propositional variable*. Provided this contextuality is respected Tractarian names can be thought of as schematic place-holders for substitution. But what this emphasises is that Tractarian names are, effectively, variable names; they function as names of forms, as type symbols rather than tokens.

Concomitantly, it is perhaps unsurprisingly difficult to find a clear-cut instance of a constant name in the *Tractatus*. If small letters from the beginning of the alphabet are assumed to relate to constants, instances appear in 4.1252, 4.1273, 4.241, 4.242, 4.243, 5.1311, 5.2511, 5.2522, 5.441, 5.47, 5.5151, 5.5301, 5.526, 5.531, 5.5321, 5.534, 5.5351, and 6.1201. None of these can, though, I think be regarded straightforwardly as constant names. The propositions noted fall broadly into two classes, a first concerning signs/expressions (4.241, 4.242, 4.243, 5.441, 5.47, 5.5351) and inference ($\forall x.fx \supset fa$) (5.1311, 5.526, 5.5301, 6.1201), and a second concerning numbers (4.1252, 4.1273) and operations (5.2521, 5.2522; signs in [...] expressions, discussed in §4.8). One could also refer to 3.203, where A is perhaps used as a constant name.[16] It is noticeable that in 5.526 Wittgenstein talks of "the customary mode of expression", suggesting that he is merely following established practice. In the *Notebooks 1914–16*, indeed, one can find a view close to that suggested here—that Tractarian names are essentially variable names, relating to the forms of objects rather than to individual objects—"The proposition fa speaks of particular objects" (*Notebooks 1914–16*, p.75); that is, of objects of a particular form, and not of a particular object of the form a.

This absence of constant names should not come as a surprise, given the curious nature of Tractarian naming. Either way, my suggestion is that Tractarian names are essentially variable names—names for forms—that can be thought of as referring, somehow, to *any object of a given form*. Because

[16] Cf. *Notebooks 1914–16*, p.14, where A and B are treated as names for things (*Dingen*). However A is used to refer to a complex, at *Moore Notes*, pp.111–2.

what clearly matters to Wittgenstein is not objects as individuals, but that they can be differentiated by form. This is, I think, shown at 4.24, where Wittgenstein uses not the letters for constant names, from the beginning of the alphabet, but the usual letters for variables, from the end of the alphabet; "Names are the simple symbols: I indicate them by single letters (x, y, z)." If in terms of form or internal properties all objects are the same (as Goddard and Judge, p.61) then it is hard to see how deploying different letters can get a grip. Equally if all objects are formally different (haecceitism) then all names would be constant, not variable. Things come out just right if the totality of objects, the objects that there are, is partitioned according to form, and if Tractarian names function as names of forms, and not—at least, not directly—of individuals.

This thinking lies behind, I think, an August 1919 exchange of letters between Russell and Wittgenstein, after the text had been completed:

> This touches the cardinal question of what can be expressed by a prop[osition] and what can't be expressed but only shown ... what you want to say by the apparent prop[osition] "there are 2 things" is *shown* by there being two names which have different meanings (or by there being one name which may have two meanings). A prop[osition] e.g. $\phi(a, b)$ or $(\exists \phi, x, y) \,.\, (\phi.x, y)$ doesn't say that there are two things ... *it* SHOWS what you want to *express* by saying "there are two things". (*Wittgenstein in Cambridge*, p.99)

For there to be two names with different meanings, there have to be two objects with different forms. For there to be one name with two meanings, there have to be two objects of the same form, where they are only numerically different. So what Wittgenstein states here is that according to the *Tractatus*, there are objects of differing form, *and* that there can be more than one object of the same form. The former distinguishes x, y, z, and so on, the latter ensures that x, y, z, and so on, are *variable* and not *constant* names (noting that Wittgenstein does not distinguish here between a, b, x, and y; they are all implicitly or explicitly bound, because we are dealing with *propositions* and not propositional functions; this is discussed further in §4.3).

Pointing out that there can be multiple objects of the same form is not a

new observation.[17] Hacker states this in objecting to Winch's argument that names are not to be accounted for in terms of referring to objects but solely in terms of the logico-syntactic role of the notion of *name* (Winch 1987, pp.7–8). In terms of the present reading this is consequent on a concentration by Winch on form, at the expense of content. Names are syntactically uniform because objects are uniform, in the sense of having form. Hacker puts it thus:

> Winch says nothing to clarify what distinguishes one name from another, whether the role of '*a*' differs from the role of '*b*', and how the use of '*a*' differs from the use of '*b*'. But, unless he claims that each name has a different logico-syntactic form, a claim that cannot be sustained, then this is a fatal flaw in his interpretation. (Hacker 1999, p.172)[18]

In a postscript Hacker points out:

> Nothing in the *Tractatus* suggests that there are not objects with the same logical form ... different objects may have different external properties—that is be concatenated with different objects. But does that imply that the *only* thing that distinguishes different objects that have the same form is their external properties? That is nowhere asserted. (Hacker 1999, p.190)

The present argument is that the notion of variable names for objects (4.1272) and the use of different variable names (x, y, z, as 4.24), and the fact that the world as we experience it is not a more or less dense distribution of homogeneous stuff (and one can read this out of 2.02331), requires there to be objects of different forms; at least, that logic has to countenance this. What lies beyond this, of course—how many different variable names one needs—is an empirical affair. This I take it is what Hacker has in mind, in allowing for objects with differing forms, and pushing the empirical (material properties) outside of the logical. That there are multiple objects of any given form is also stated, albeit indistinctly, by Geach:

[17] For example, "It is, at the same time, quite clear that Wittgenstein believed that there is, in principle, a plurality of objects of every kind." (Ishiguro 1990, p.30)

[18] This amounts, I take it, to a rejection by Hacker of haecceitism as an interpretation of the *Tractatus*.

> [T]he objects with regard to which sentence-facts are facts cannot just be token-words, they must be type-words; it is fairly clear that without this there are many places where the theory [of the *Tractatus*] would not work. (Geach 1976, p.67)

What I think Geach gets right is that Tractarian names can be thought of as type-words and not as token-words; they are variable names, where objects of the same form can be thought of as the substitution class for a particular name.[19] These objects are numerically and not qualitatively (formally) different from one another. Geach goes interestingly wrong, however, I think, in going on to say that "type-words are not Wittgensteinian simple objects" (Geach 1976, p.67), where it is the implicit conflation of type-words (names) and objects that is interesting. Because it seems to me that if there are objects that are *formally* identical then Tractarian names *are* type-words, tied very tightly to the obects named.

The account given is also compatible with Ishiguro:

> The concept of a simple object in the *Tractatus* is that of an instantiation of an irreducible predicate, where the question of individuation of different instantiations of the same predicate cannot arise. (Ishiguro 1969, p.21)

The "irreducible predicate" is having form; the account given here countenances multiple instances of the same form. What is needed in addition is that different instantiations of the same form can be distinguished, albeit only numerically. On this basis I find myself broadly in agreement with Ishiguro's statement that "A Name [i.e., a Tractarian name] is not like an individual tag or a paper label. A Name is a class of similar token expressions, each of which is used in propositions to refer to the same object." (Ishiguro 1969, p.37) The difference is that on the present reading a name is a variable name that correlates with objects of the same form; that any object of the same form can be substituted *salve sensu*. So I would revise Ishiguro's remark above to conclude, "to refer to *any object of the same form*." Lastly, what Ishiguro calls a "dummy name" (Ishiguro 1969, p.45) is simply a name thought of as a variable name and not as a constant name.

[19] Similarly Black says, "Let us call word-tokens which occur as parts of elementary propositional-tokens by the title of name-tokens. Each 'name' in Wittgenstein's sense is a logical construction out of a class of name-tokens." (Black 1962, p.106)

It is assumed nevertheless that objects are, somehow, in principle nameable, and that there is available a name for every object.[20] If necessary these can be symbolised a_1, a_2, and so on, deploying some such recursive strategy to ensure that the language is learnable. These names are the *simple signs*, the vocabulary, of *System Tractatus* (3.201, 3.202).

2.6 Elementary propositions

The second step in setting up *System Tractatus* is an account of elementary propositions, as the basic formulas. An elementary proposition is a combination of names, as 4.22; "An elementary proposition consists of names. It is a nexus, a concatenation, of names." It is defining of an elementary proposition that its constituents are names, that it does not contain symbols of any other kind (in particular, that it does not contain logical constants).

It is also significant that an elementary proposition has a *structure*, that it is not just a list of names. This structure is a function of the forms of the objects that are named, absorbing Russellian logical forms for propositions, as §1.9. That objects have form—logical form—is stated at 2.021–2.025. *Logical form* is noted at 2.0233, for short the term *form* is used. An immediate corollary is that an elementary proposition has to consist of at least two names; a name by itself cannot have structure, it cannot *say* anything.[21]

The form of an object is its range of possibilities to combine with other objects into states of affairs (2.0141). The combinatorial possibilities of a name to combine with other name(s) to form an elementary proposition exactly parallels the form, the combinatorial possibilities, of the object(s) named (3.21). As Pears puts it, "Once a name has been attached to an object, the nature of the object takes over and controls the logical behaviour of the name, causing it to make sense in some sentential contexts but not in others." (Pears 1987, p.88) A commitment to this does not of itself explain how such attachment comes about, or how it is preserved.

[20] A point made at Carruthers 1989, pp.125, 171. 3.203 could be read as "the *autonomous interpretation*: let every term of [the object language] L be interpreted as denoting itself" (Kripke 1976, p.353). Carnap's expression for this is *autonymous* (*Logical Syntax of Language*, §4).

[21] Hence Mácha is, I think, wrong in suggesting "that there are admissible states of affairs consisting only of a single object." (Mácha 2015, p.64) Copi (Copi 1958, p.185), Sellars (Sellars 1962, pp.255–6), and Keyt (Keyt 1963, pp.290, 294) all argue that a single object cannot be a state of affairs.

It is critical that unrestricted combinatorialism does not obtain, for objects into states of affairs. Intuitively, a world where unrestricted combinatorialism obtains is one where anything can be anywhere at any time, and where transitions from a world to a temporally adjacent world would not be predictable; there could not be laws of nature in such a world. Such absence of laws would be an *a priori* matter for logic and not for empirical enquiry, running counter to 6.33; "We do not have an a priori belief in a law of conservation, but rather a priori *knowledge* of the possibility of a logical form." If unrestricted combinatorialism obtained then there could be no *a priori* knowledge of even the possibility of laws of nature. In a world where anything can happen at any time all we would know *a priori* is that there cannot be any laws at all, and hence there would be no scope for forms that laws might conform to.

Logically, objects are *uniform*. In other words, it is sufficient to state that objects have forms. The correlate for restricted combinatorialism is that the number of different object forms is at least two, and that it is less than the number of objects (because there are objects of the same form). It does not, though, I think help to think of this in metaphysical terms, as a categorial distinction between universals and particulars, or as a metaphysical conception of categorial pluralism (cf. p.33). To say that objects divide into at least two disjoint forms is, at bottom, an empirical claim, that we do not live in a world of varying density distribution of homogeneous stuff.

The number of forms cannot be the same as the number of objects because there would be a 1:1 pairing between objects and forms; every object would be different from any other object. In such a world there cannot be laws or regularities either, because there would be no objectual basis for laws and regularities, as these arise from combination and recombination of the objects that there are. In the Tractarian context this effectively constitutes an argument against haecceitism. The number of forms has to be finite, for similar reasons; a world with an infinite number of different forms would be infinitely complex and consequently beyond laws and regularities. How forms are realised, and how many forms there are, is a matter for empirical enquiry, such enquiry addressing the *content* of an object, as 2.025, that objects are "form *and* content" (emphasis added). Content, the realisation of objects construed from an empirical point of view, falls outside of logic.

It follows that one can think of a name, in an elementary proposition,

as standing for any object of the same form. If we could survey the totality of objects, then it would be partitioned into sets of objects of the same form. In principle there is a name for each object, but it does not in fact matter for an elementary proposition, *abc*, say, which object of form *a*, or which object of form *b*, or which object of form *c*, comprises the state of affairs that is represented by the proposition *abc*. This is because Tractarian names function as variable names, as 3.314.

The form of an object exhausts its internal properties. This is effectively stated in 2.01231, that "If I am to know an object, though I need not know its external properties, I must know its internal properties." For logic, all that matters about an object is that it has a form; it is, simply, an instantiation of a form. That an object has the form that it has is a *necessary* property of the object. It is necessary that it is a constituent of *a* state of affairs, but beyond this, it is contingent which state of affairs, compatible with its form, it is at present a constituent of; it is important to get clear as to exactly what, here, is necessary, and what is contingent. This is the point of 2.0271, that "Objects are what is unalterable and subsistent; their configuration is what is changing and unstable." For the argument to work (for elementary propositions to be bipolar) states of affairs have to be able to disintegrate and objects recombine (in accord with their forms) into similar or different states of affairs.

External (material) properties arise from, are produced by, combination of n ($n \geq 2$) objects of suitable logical form, represented by an elementary proposition, a combination of n names, $n \geq 2$. But this leads into the not obviously compatible claims that each object is named (3.203), and that objects of the same form are somehow different (2.0233).

The crux is that two such objects are distinguishable—are different—only for as long as an observer distinguishes them (noting that objects are not in any way observer-dependent). If one ceases to distinguish them, then they are no longer distinguished. If I drop an unmarked Slazenger No.7 golf ball into a box of unmarked Slazenger No.7 golf balls then all else being equal I can only distinguish it for as long as I can keep an eye on it. This is where Hacker's argument, that naming involves an act of will, perhaps garners some plausibility, because keeping track of particular objects in such fashion is an act of will. This is the force of 2.0233, that "If two objects have the same logical form, the only distinction between them, apart from

their external properties, is that they are different", and 2.02331, "if there is nothing to distinguish a thing, I cannot distinguish it, since otherwise it would be distinguished after all." The point is that if we have two objects of the same logical form then they cannot be distinguished from one another considered purely by themselves, on the basis of their forms; because they are of the same form.[22] But how, exactly, given 2.0233, are two such objects to be distinguished?

The thing is, I think, that if objects of the same logical form can be distinguished only in terms of their external properties, then such distinction can only be numerical. It follows that naming is *essentially* variable, and that Tractarian objects are classically well-behaved, on the basis that classically well-behaved objects are unalterable and subsistent (2.027), whereas non-classical (quantum) particles are not.

Nevertheless it might be argued that Tractarian objects are akin to quantum particles, if one holds that objects of the same form are not even numerically different. Call this *utter indistinguishability*. If we consider the example at *Notebooks 1914–16*, p.14 of a world consisting only of objects A and B and a property F, then if A and B are utterly indistinguishable we have only three possibilities; if we label these $(2)(FA, FB)$, $(1)(FA, FB)$, $(0)(FA, FB)$, these are the circumstances in which both FA and FB, one of FA, FB, neither of FA, FB, is the case. Assuming these are possible worlds, call them W_2, W_1, and W_0 respectively. If we have utter indistinguishability then it may be that Bose-Einstein statistics apply such that $p(W_2) = p(W_1) = p(W_0) = \frac{1}{3}$.[23]

This, however, goes against 2.027, that Tractarian objects are unchangeable and subsistent, so two objects cannot coherently be regarded as if they are one. And, as will become clear, it breaks the account of truth-tables and the logical theories of inference and probability based thereon.[24] Once one sees the *Tractatus* as a system, it becomes clear that Tractarian objects have to be classically well-behaved, that objects of the same form are numerically and not utterly indistinguishable. This brings us back to independence, be-

[22] Taken together with other remarks in the present chapter this constitutes, I think, a rebuttal of the claim advanced by Bradley (Bradley 1989, Bradley 1992) and by Nakano (Nakano 2021) that Wittgenstein adheres to haecceitism.

[23] C.f French 2014, pp.34–5. An argument along these lines with Tractarian objects construed non-classically is proposed in Cheung n.d..

[24] Unless one chooses quantum logic over classical logic, as Putnam 1968, but this rather falls outside the present project.

cause it follows that objects of the same form can be distinguished by us only as long as we keep track of them; independence pertains to our thinking, as §1.14. What Bowley and Sanchez say of classical particles—"in principle, a classical particle can be distinguished by its position for all time" (Bowley and Sánchez 1996, p.122)—applies equally, I think, to Tractarian objects. We distinguish Tractarian objects by identifying where material properties are realised, and in principle, at least, we keep track of individual objects by keeping track of instances of material properties.

It follows, in principle at least—for the logical god of 5.123—that if it were possible to keep track of all objects then, bearing in mind that one can only think of and speak of an object by means of its name, there could be a 1:1 relationship between names and objects. But what matters to us is that if the state of affairs expressed by the proposition *abc* obtains, then there are objects of forms *a*, *b*, and *c* that so stand to one another.

As the aim at present is to set up *System Tractatus*, it is assumed that enough has been said to grasp the first two steps, names as vocabulary, and elementary propositions.

2.7 Propositional calculus

The calculus of elementary propositions begins with the logical constants, as the means whereby complex propositions are generated out of elementary propositions. Wittgenstein deploys his N operator, introduced at 5.502; "$N(\bar{\xi})$ is the negation of all values of the propositional variable $\bar{\xi}$." $\bar{\xi}$ stands for a selection, however chosen, of elementary propositions (5.5ff). It is significant that N is an operator and not a function; in short, a truth-operation is what one applies to a selection of elementary propositions, a truth-function of those elementary propositions is what one gets; complex and everyday propositions simply *are* truth-functions of elementary propositions. This is bound up with Wittgenstein's "fundamental idea", that "the 'logical constants' are not representatives." (4.0312) The claim that they are not representative stands in need of explanation, but not from a formal point of view. It is the formal behaviour of the logical constants that is of present concern.

It is notable that Wittgenstein regards the interdefinability of the familiar logical constants (negation, conjunction, disjunction, and so on) as showing that they are not representative. This is stated in 5.4–5.42, in par-

ticular, 5.42(2); "The interdefinability of Frege and Russell's 'primitive signs' of logic is enough to show that they are not primitive signs, still less signs for relations." That is, that the logical constants do not represent or somehow stand for substantive relations[25] between the states of affairs that are represented by, for example, the elementary propositions p and q, in the complex proposition $p \vee q$.

Although Wittgenstein intends to distinguish his approach from the position he attributes to Frege and Russell, it is hard to see this as an *argument*. That N and any other adequate set of connectives ($\{\sim, \vee\}$, perhaps, as 3.3441) are interdefinable does not obviously show that logical constants are not representative. All it evidently shows is that it may be easier to deploy a different notation (using $\sim$ and $\vee$ may be more perspicuous than using $|$). There is no obvious sense in which any adequate set of connectives is more "fundamental" than any other.

The underlying reasons are programmatic. First, the logical concepts are, in the terminology of the *Tractatus*, *formal concepts*. This is set out in 4.126; "[W]hen something falls under a formal concept as one of its objects, this cannot be expressed by means of a proposition. Instead it is shown in the very sign for this object." Such showing is effected by analysis of complex/everyday propositions down to their constitutive elementary propositions.

Second, one might advert to 5.4, where Wittgenstein says that "there are no 'logical objects' or 'logical constants' ", but what he goes on to say, in parentheses, is "in Frege's and Russell's sense." It does not follow that there aren't logical entities in a non-Fregean or non-Russellian sense. There is a logical apparatus in the *Tractatus*, and there has to be some allowance for this. There is nevertheless a hard distinction between names as constituents of elementary propositions, and symbols for the logical apparatus, where only the former are meaningful (function as names for objects).

Reverting to 4.126, if truth-functional relations between elementary propositions, indicated by logical constants, are construed as material relations, then there is some additional propositional content. This is not allowed in *System Tractatus*, so any construction of a complex proposition that appears to generate substantive content, e.g. "A believes that p is the case" (5.541), has to be suitably defused. That truth-functional relations between

[25] "Substantive" as in, e.g., left and right, as opposed to, e.g., $\vee$ or $\supset$, as 5.42(1).

elementary propositions are operations and not material relations, that is, a matter of formal concepts and not "concepts proper" (4.126 (4)), is shown by a logical sign (e.g. $\vee$, $\supset$), and as noted such signs are not representative. If logical constants are construed as representational, then something has gone wrong.

That the logical constants are not representative goes together with the Independence Thesis (§1.14), that elementary propositions, within or without the context of a complex proposition, are independent of one another; "It is a sign of a proposition's being elementary that there can be no elementary proposition contradicting it." (4.211) This parallels two earlier remarks, that "States of affairs are independent of one another" (2.061) and "From the [obtaining] or [non-obtaining] of one state of affairs it is impossible to infer the [obtaining] or [non-obtaining] of another." (2.062) The point is that elementary propositions do not and, in fact, cannot stand in material relations to one another. The only relations in which they can stand to one another are truth-functional.

At this point it is useful to make explicit a distinction which is largely implicit in the text. This is bound up with thinking of logic as *sui generis*, and with the discussion in §2.3 of logic as being in flux in the period of composition of the text.

If one wants to think of the *Tractatus* as presenting an uninterpreted calculus then one is not, without further argument, entitled to the concept of *truth*, or *falsity*, as the salient property of propositions. What there is a need for is *a* property realised by all and only propositions, a property that can be recognised independently of what there is. That it can be recognised independently of the obtaining states of affairs need not, though, lead to independence *per se*, although for an wholly *sui generis* conception of logic, it might. The property that we have so far for elementary propositions is bipolarity, of expressing sense, of stating how objects *can* stand to one another. Assuming that we have a capacity to recognise elementary propositions as instances of this property, the crux remains that this property is, in the sense of §1.14, *independent* of what is the case. *System Tractatus* can be thought of as an uninterpreted calculus, in terms of a *sui generis* conception, but only by first separating out *a* suitable property for elementary propositions. This is where familiar realist reservations kick in.

At this juncture, as noted (p.37), Frege talks of "advancing" from the

sense of a proposition, to its truth-value, where it is far from clear what this *advance* consists in. The reason why it is unclear, and this, I suggest, applies equally to Wittgenstein's account, is that if one regards the propositional as bipolar, as expressing sense, and if one thinks of sense in terms of truth-conditions, in a realist sense, then any proposition one might want to merely consider or entertain is already *actually* true, or *actually* false. "Advancing" can only be a matter of recognition on our part of what already is, or is not, the case. There is a directional difference between "p" and "not p", whether or not the state of affairs/situation represented by "p" is the case. The difficulty is that building the notion of truth into the notion of sense prejudges the choice between a *sui generis* and a realist conception of logic in favour of the latter.

The obvious way out is to think of the propositional in anti-realist terms, of verifiability or warranted assertibility, or some such similar property. But for Wittgenstein this is overly psychological and it is unclear that the anti-realist is entitled to full truth-tables. The only way, I think, of keeping open the *sui generis*/uninterpreted calculus conception is to cut Wittgenstein some slack and accept that just as it isn't clear that the logician *qua* logician need attribute to objects anything beyond what is needed for logical purposes, so the logician *qua* logician need not have a view as to how truth, and falsity, are *actually* realised.

The needed distinction can, I suggest, be marked in logical contexts by using the symbols $\top$ and $\bot$ instead of T and F, where what is propositional realises the property $\top\bot$, that is, expresses sense, is bipolar, but without thereby committing to a realist conception of truth. This is a largely negative approach, the intention being not to dismiss the idea of an uninterpreted calculus by moving too quickly to truth and falsity, as opposed to *a* formal property realised by elementary propositions. The $\top\bot$ notation also makes the point that negation is an operation, and that double negation brings one back to where one started. With the benefit of hindsight we can perhaps render Wittgenstein's meaning more clearly without, I think, in any significant way deviating from what he actually wrote. The change simply is, *when doing logic*, to use $\top\bot$ instead of TF.[26]

[26]Some such distinction may lie behind Wittgenstein's switch at 5.512/5.513 from *wahr* and *falsch* to *bejahen* and *verneinen*, in addressing the question, "How then can the stroke '$\sim$' make [the proposition p] agree with reality?" Cf. Shieh 2014, pp.323ff, Shieh pointing out that *bejahen* and *verneinen* are used by Frege at *Begriffschrift*, §5. There is though a

That one can think of truth-tables as a mechanical filling-in by rote is I think one of the reasons why it is tempting to think of the *Tractatus* as an uninterpreted calculus and it is also, perhaps, why Grayling suggests that we would not think of it as *true* (p.51). Using ⊤⊥ rather than TF serves to distance the logical, the domain of possibility, from its application, what actually is, or is not, the case. From hereon where what is intended is the formal property of the propositional, that is, bipolarity as the expression of sense, ⊤⊥ will be used; where what is intended is actual truth-value, T and F will be used. That this brings Wittgenstein's approach closer to the post-1930 model-theoretic conception of logic is not only not in dispute, it is, I think, a strength of the present reading. Any account of the logic presented in the *Tractatus* should be able to do justice to logic being in a state of flux at the time of its composition.

At this point more can be said about *independence*, following on from §1.14. This requires careful handling because the distinction between dependency and independency needs to be located precisely. For objects, as stated in §1.14, their independence from states of affairs reflects their having form, that they can combine into different states of affairs at different times. Such independence is most salient at the level of thought. It carries over to pictures (if a proposition is thought of as a picture), as well as to propositions, as these can be thought independently of what is the case:

> What a picture represents it represents independently [*unabhängig*] of its truth and falsity, by means of its pictorial form. (2.22)

> It must not be overlooked that a proposition has a sense that is independent [*unabhängigen*] of the facts: otherwise one can easily suppose that true and false are relations with equal status between signs and what they signify. (4.061)

That propositions and pictures can be thought as representative, independently of the states of affairs/situations represented, leaves untouched the facts, that at any given moment a possible world—a way in which objects can stand to one another—obtains such that some elementary propositions are true, and some are false. This is what is involved in thinking of

risk of veering into the psychological here.

elementary propositions as $\top\bot$. As to what is involved in ascertaining the truth-value of a given proposition, here and now, as noted it is not evident that the logician need have an opinion on this. What is critical is that the move from $\top\bot$ to TF, from something that has sense, to the ascertaining of what its truth-value actually is here and now, does not make any difference either to the proposition or to our thinking the sense of the proposition.

The main point is that for objects, names, pictures, and propositions, independence occurs, and can *only* occur, at the level of thinking. For a picture to be representative, for a proposition to have sense, is a matter of thinking and entertaining. That there are such things to be thought *as if* they are independent arises out of an ultimate dependency.

There are, consequently, two related but distinguishable senses of independent. First, there is the sense in which ultimately dependent entities, propositions and pictures, can be thought independently of what is the case. That we can think propositions independently of what is the case is in the main a useful concession to finite humans. Second, there is the Independence Thesis, which goes together with determinacy of sense, ensuring that an elementary proposition can be thought irrespective of any other. The Independence Thesis is a logical doctrine, ensuring the applicability of truth-tabular methods in all cases.

With this in place, we can move to a formal treatment of complex and generalised propositions. Taking $\{\sim,\vee\}$ as an adequate set of connectives, if p and q are elementary propositions, then $\sim p$, $\sim q$, and $p\vee q$ are the shortest complex propositions that can be formed therefrom. It is definitional that an elementary proposition does not contain either $\sim$ or $\vee$, and that a complex proposition contains one or more instances of either $\sim$ or $\vee$ (cf. 4.211). Since generalised propositions are treated as conjunctions and disjunctions, and since $\wedge$ can be expressed in terms of $\sim$ and $\vee$, this extends to generalised propositions ($\exists$ can be treated as a disjunction, $\forall$ as a conjunction).

It follows, from the Independence Thesis and the non-representational nature of the logical constants, that in generating complex propositions unrestricted truth-functional combinatorialism obtains. The operations $\sim$ and $\vee$ can be applied iteratively to any selection $\bar{\xi}$ of elementary propositions to generate complex propositions. There is no obvious upper bound on this process, except that any iterative process can never get beyond $\aleph_0$. The length of some propositions can be reduced, as any propositional symbol

prefixed by $2n-1 \sim$ symbols can be written as prefixed $\sim$, and any propositional symbol prefixed by $2n \sim$ symbols can have the string of $\sim$s elided, and other pruning methods adopted if applicable. Given unrestricted truth-functional combinatorialism it also follows immediately that tautologies and contradictions are generated mechanically within the system.

2.8 Truth

This brings us to the notions of truth-table, and the propositional sign, at 4.3ff. The first notion one meets is *truth-possibility*; "Truth-possibilities of elementary propositions mean possibilities of [obtaining] and [non-obtaining] of states of affairs." (4.3) The truth-possibilities for an elementary proposition are true and false, but if one is dealing in possibilities, the $\top\bot$ notation is perhaps more appropriate.

As noted it is not immediately obvious that the logician need have a view as to what truth and falsity actually are, or how the truth-value that obtains now of any given elementary proposition is to be ascertained. There is, in fact, good reason to steer clear of giving a conception of truth. According to Glock, "The *Tractatus* is commonly regarded as a *locus classicus* for a correspondence theory [of truth]." (Glock 2006, p.346) This is controversial, because it is hard to pin down exactly what the relation is that would be involved in a correspondence theory of truth, given that it is unclear how one would differentiate the relata to such a relation; how one can have independent access to propositions on the one hand, and facts on the other.

Glock attributes a correspondence conception to Hacker[27], claiming that Hacker later changed his mind on this, albeit only in footnotes.[28] What Hacker writes is, I think, not a statement of a change of view, but a clarification. The key point is this; "In so far as there is a correspondence conception of anything in the *Tractatus*, it is a correspondence conception of sense. The fact that Wittgenstein speaks of a proposition's agreeing with *reality* if it is true does not imply any commitment to a 'truth-relation' or 'correspondence relation' between propositions and facts, of which being true consists. To assert that a proposition 'p' agrees with *reality* is to assert that 'p' says

[27] On the basis of Baker and Hacker 1983a, Exegesis §136, and Hacker 1981, p.100.
[28] Cf. Hacker 1996, pp.291n.16, 318n.13, and Hacker 2000, p.386n.36 (Glock 2006, p.346).

that p and it is in fact the case that p." (Hacker 2000, p.386n.36, emphasis added)

This is where the independence/dependence distinction (§2.7) becomes critically important. It is, I think, significant that Hacker talks of propositions as agreeing with *reality*, and not as agreeing with *facts*. A fact is the obtaining of a state of affairs (2), and *a* world (at any given point in time) can be regarded as a totality of obtaining states of affairs, represented by a set of actually true elementary propositions. To "agree with *reality*" is to express *sense*; one cannot disagree with reality, propositionally, because the attempt would generate nonsense (would not be propositional). Beyond this, for a proposition to be *actually* true, or *actually* false, is a matter of agreement with what is or is not the case in the *world*.

The salient distinction, between the world (*die Welt*) and reality (*die Wirklichkeit*), is introduced at 2.063, in a rather cryptic formulation; „Die gesamte Wirklichkeit ist die Welt", translated by Pears and McGuinness as "The sum-totality of reality is the world." Understanding this has been muddied by the parenthetical 2.06(2), where Wittgenstein says "We also call the [obtaining] of states of affairs a positive fact, and their [non-obtaining] a negative fact." This is unfortunate, because it led to the debate over whether or not Wittgenstein countenanced Russellian negative facts (cf. p.42). Once one grasps what Wittgenstein has in mind the phrase "negative fact" may be useful shorthand, but it is a mistake to think that there is something akin to an obtaining state of affairs (a fact) that is represented by a negated elementary proposition.

What this comes down to is that there are the objects that there are. Objects do not pop into or out of existence, they constitute the "unalterable form" of the world (2.023): "Objects, the unalterable, and the subsistent are one and the same." (2.027) Whatever objects might be, they are necessary existents, the same in all possible worlds (the "fixed form" interpretation, as p.34).

When Wittgenstein says, "The world is the totality of facts, not of things" (1.1), his point is that any possible world is *a* way in which the objects that there are can stand to one another, in accord with their forms. As noted in §2.6, unrestricted combinatorialism does not hold for objects, this reinforcing the point (p.73n.21) that a single object cannot be a state of affairs; objects are eternal, so an object cannot *obtain*, it cannot be a *fact*,

because it just *is*. "Existing" is *de facto* applicable to all objects.

That elementary propositions are bipolar is a corollary of the idea that objects can be, and in fact often are, constituents of different states of affairs in different possible worlds. If objects could not be constituents of qualitatively and not merely quantitatively different states of affairs in different possible worlds, then the world could not change; elementary propositions would no longer be bipolar/contingent. (The distinction between quantitatively and qualitatively different is because the same state(s) of affairs could obtain with numerically different objects of the same forms. Such numerical difference would leave the truth-values of the relevant elementary propositions unchanged.) Whatever account one gives of objects, it has to allow for such disintegration and recombination.

If there are propositions that happen to be true in all possible worlds, they would not be necessary, because Tractarian necessity is not a matter of obtaining in all possible worlds. They would be what von Wright calls "synthetic accidentally general propositions" (von Wright 1984, p.108). Wittgenstein does not, though, I think, need von Wright's *Principle of Plenitude*, that "everything possible sometimes comes true" (von Wright 1984, p.106). It is sufficient that everything possible *may* come to pass.

When Wittgenstein says that the world is the totality of facts, what he means is that the world *now* is the totality of facts that obtains *now*, where the world *now* can be thought of as supervenient on the states of affairs that objects are at present constituents of, expressed by a (maximally) consistent set of elementary propositions, each of which is true *now*. Nevertheless since objects have forms, all possible worlds are in a manner of speaking also present; because there is *nothing* to a possible world over and above the combining of the objects that there are according to the forms that they have (cf. 2.024). One might say, loosely, that there is a *possible* fat man in the doorway, as the objects that there are could have combined such that there is, now, a fat man in the doorway. But he isn't, because the objects that could have so combined are not so combined, and the elementary propositions that would in those circumstances have been true, are at present false. Possibilities are not things or entities in their own right, they are ways in which the objects that there are can combine (into states of affairs). It is misleading to call this a reification of possibilities (cf. §1.14).

As to how many objects there are, considered logically the number is,

at most, $\aleph_0$. It is strictly nonsensical to speak of a total number of objects (4.1272), but this simply reflects the fact that "object" is a formal concept. That there is a totality of objects (and of elementary propositions) is stated at 5.5561, and this rules out a cardinality greater than $\aleph_0$. If the number of objects is $\aleph_0$ then Wittgenstein could have adopted *Cantor's principle of finitism*; "The transfinite is on a par with the finite and mathematically is to be treated as far as possible like the finite." (Hallett 1984, p.7) Either way the number of objects is at most $\aleph_0$, falling into a finite number of disjoint forms. Objects comprise reality (*die Wirchlichkeit*), all the ways in which the world can be; that is, the totality of possible worlds. To be logically tractable these totalities have to be at most countably infinite. If we think of reality as a totality of possible worlds, generated by the objects that there are, taken together with their forms, then the world (*die Welt*) as it is now is a member of this totality. So when Wittgenstein says, at 2.063, "The sum-totality of reality is the world", this is literally true. There is nothing to reality over and above the world. It just happens to be the case that the world as it is now, the facts, the states of affairs that obtain at present, are *a* way in which reality can be.

There are, then, no negative *facts*, on a par with the positive facts (the obtaining states of affairs). There is nothing wrong in thinking of unrealised possibilities as facts that might have obtained in the past, might have been the case now, or as facts that may obtain in the future, but in fact won't. One can call these "negative facts", provided it is understood what they are; possibilities internal to reality, functions of the forms, the internal properties, of the objects that there are.

Consequently when Hacker talks of elementary propositions as agreeing with reality (Hacker 2000, p.386n.36), his point is that elementary propositions have sense, they express ways in which things can stand to one another. From a logical point of view this is all there is to say (this is what the $\top\bot$ notation is intended to capture). But to show how quickly things can go wrong, in the same footnote Hacker says, "It is surprising to see Ricketts attributing to the *Tractatus* a correspondence conception of truth (Ricketts 1996, p.64). This is mistaken." (Hacker 2000, p.386n.36) It is true that Ricketts says, "Wittgenstein's alternative to Russell's [multiple relations] theory [of judgement], nevertheless, shares with it a commitment to a correspondence conception of truth." (Ricketts 1996, p.64) But matters are more

complicated than this.

First, judgement ostensibly takes us from sense to reference, from ⊤⊥ to TF, and at this point one may invoke correspondence. But as noted this is arguably outside the domain of logic. Second, later in the same paper, Ricketts talks of "truth as agreement with reality" (*op. cit.* pp.90, 93). Given how close this is to the wording of 2.222, it is hard to see how one can object to this, as both Hacker and Ricketts echo 4.05, "*Reality* is compared with propositions", and 4.06, "A proposition can be true or false only in virtue of being a picture of *reality*" (emphasis added). Assuming that *being a picture* can be construed as *having sense* (cf. 4.021, 4.022), both Hacker and Ricketts are talking about propositions in terms of *sense* and *reality*, that is, ⊤⊥, and not in terms of the facts, the obtaining of states of affairs, the world, that is, in terms of TF. Once this is clear many of the discussions in the literature can be seen to be confused.

There is perhaps one point where the interpretation given here stumbles. At 2.221, Wittgenstein says, "What a proposition represents [*darstellt*] is its sense." In addressing this run of five propositions from 2.221–2.225 I will, on the basis of 3, assume that the words "elementary proposition" can be substituted for "picture". So 2.22 becomes "What an elementary proposition represents is its sense." 2.224 and 2.225 follow straightforwardly; "It is impossible to tell from the elementary proposition alone whether it is true or false" (2.224), "There are no elementary propositions that are true *a priori*" (2.225). This is expected, as at the level of sense we are operating with elementary propositions in terms of ⊤⊥, and not in terms of TF. We have to go beyond the elementary proposition itself and into the domain of judgement—outside of logic—to ascertain what the truth-value of an elementary proposition actually is. No elementary proposition is true *a priori* because an elementary proposition cannot be a tautology. It is definitionally *a priori* that an elementary proposition is *true or false*, but what we cannot say *a priori* is whether it is *actually* true, or *actually* false. This is where the distinction between ⊤⊥ and TF is helpful, because it enables this distinction to be drawn.

There is, though, a problem with 2.222 and 2.223; "The agreement or disagreement of [an elementary proposition's] sense with reality constitutes its truth or falsity" (2.222), "In order to tell whether an elementary proposition is true or false we must compare it with reality." (2.223) On the present

reading the comparison should be with the *world* and not with *reality*, if it is a matter of ascertaining what is actually true and what is actually false; I think this is what 2.222 and 2.223 intend, although I accept that my reading here is a little strained. Nevertheless, the world, *is* reality; reality is not something over and above the world. The world just is how reality happens to be, now; which states of affairs obtain, and which do not.[29] But if Wittgenstein had talked here of the *world* rather than of *reality* then there is a risk of his reader construing reality as partitioned into positive facts (the world) and negative facts (the complement of the world, so to speak). This way of putting things should I think be avoided, because it is too close to reifying possibilities.

Wittgenstein would have been familiar with the difficulties here from his study of Frege. In his 1918 essay 'Thoughts' Frege says, "What is a fact? A fact is a thought that is true." ('Thoughts', p.74)[30] Frege says, "It is ... worth noticing that the sentence 'I smell the scent of violets' has just the same content as the sentence 'It is true that I smell the scent of violets'. So it seems, then, that nothing is added to the thought by my ascribing to it the property of truth." ('Thoughts', p.61) As already noted, from a logical point of view what matters is that there are elementary propositions, and they express sense. This is what is intended in saying that elementary propositions are $\top\bot$. The point is that supervenient on being propositional, a proposition is, now, actually true, or actually false. In recognising actual truth-value nothing is added or subtracted.

Glock quotes 2.21, 2.222, and 2.223, and claims that "These passages ... feature claims which would grace any correspondence theory. They spell out truth in terms of an agreement between truth-bearer and reality." (Glock 2006, p.354) Yet Rhees can read the same text and say, "And if p is true—the truth is not a relation between the facts and what it says. I say the iron is getting warmer. If this is true, then what it says *is* a fact; not something else which corresponds to it." (Rhees 1960, p.11) The reason, I think, is that the discussions are at cross-purposes, and this is characteristic of much of the

[29] As Zalabardo points out, concerning the identification of world and reality, "By moving from the world to reality, one has not added anything to our ontology: we have only unpacked what was already present in the world." (Zalabardo 2015, p.134)

[30] For the claim that Wittgenstein was probably familiar with these views of Frege at the time of writing the *Tractatus*, see Carruthers 1989, pp.14–15, Sluga 2002, pp.89–90, Klagge 2022, pp.73–5.

literature on truth in the *Tractatus*. Pears argues that "In the interpretation of [Wittgenstein's] writings it is a golden rule to treat as peripheral questions that he himself treats as peripheral." (Pears 1987, p.89) From the logician's point of view all that matters is that elementary propositions are bipolar, that they are ⊤⊥. Everything else about truth, about the ascertaining of truth for any particular proposition, is somebody else's problem. This enables Wittgenstein to sidestep the difficulties raised by Frege. He arguably oversteps this at 4.024, when he says that "To understand a proposition means to know what is the case if it is true." Here Wittgenstein is on the cusp of talking about the self that can think and entertain propositions, as 5.641(1); "There really is a sense in which philosophy can talk about the self in a non-psychological way." This non-psychological way is presumably what is involved in considering the self as capable of engaging in thinking the sense of propositions. In any case this is pulled back by the parenthetical 4.024(2), "One can understand [a proposition], therefore, without knowing whether it is true." This is a statement of independence, that propositions have sense, that they can be thought irrespective of the facts. A proposition (an elementary proposition) is ⊤⊥, and it is either T, or F, where being actually true or actually false is no ontological addition. There is the world *and* reality, all wrapped up into one, the former being the way in which the objects that there are stand to one another at present, the latter entertainable in thought as all the other ways in which they can so stand.[31]

The salient distinction is between the *dependent* and the *independent*. The only thing that is fully independent is reality, that is, the totality of objects, this embracing, of course, the forms of objects *and* their content, the states of affairs/propositions that they can realise (2.0122).

2.9 Truth and tautologies

A second reason for deploying ⊤⊥ when doing logic rather than TF is, I suggest, because there is a profound difficulty with the notions of tautology and contradiction. Von Wright was still puzzling over this at the end of a

[31] This is how Beckermann and Hacker can interpret the *Tractatus* as respectively proposing a semantic/deflationary or a redundancy theory of truth (cf. Glock 2006, pp.353ff). The relevant passages are *Notebooks 1914–16*, p.9 and *Moore Notes*, p.113, but these passages did not make it into the *Tractatus*. These discussions reflect the "nothing over and above" aspect of truth-value with respect to sense.

long life[32], suggesting that tautologies and contradictions (given their duality I will talk of tautologies, on the basis that similar considerations apply to contradictions) should be categorised as "unipolar" (von Wright 2006, p.99), as a way of addressing the fact that they do not express sense (are not bipolar) but are not nonsense (not polar at all) either.

The problem is that a tautology is not obviously true, and a contradiction is not obviously false, *in the same way that a proposition with sense is true, or is false*. With masterly understatement von Wright says, "It is a minor inconsistency when Wittgenstein, in 4.461, nevertheless calls the tautology unconditionally *true* (*bedingunglos wahr*). Much later he gave a better expression to this thought when he said of a tautology that it was a degenerate proposition 'on the side of truth' (Wittgenstein 1978, p.167)." (von Wright 1982, p.192)[33] But being "on the side of truth" is less than pellucid. Von Wright later remarked, "in my opinion—the sense in which necessary sentences are true and contradictory sentences false is very different from the sense in which contingent sentences are true or false—I think it clarifying to drop the terms 'true' and 'false' altogether as attributes of that which is (logically) necessary and impossible." (von Wright 2006, pp.105, 106) One can try and finesse the difficulties by equivocating over the notions of truth and falsity, or one can describe this as "some kind of muddle or, maybe, inconsistency", but the question remains "how we shall deal with it." (von Wright 2006, p.101)[34] However von Wright's remarks end here.

The disanalogy between what has sense (is propositional) and what is senseless becomes clear in the 4.46s:

> Among the possible groups of truth-conditions there are two extreme cases.
>
> In one of these cases the proposition is true for all the truth-possibilities of the elementary propositions. We say that the truth-conditions are *tautological*.

[32] Cf. Pichler and Säätelä 2006, pp.21–25.

[33] Inspired by Ramsey, it seems; "Ramsey quite correctly called tautologies and contradictions 'degenerate propositions' [*degenerierte Sätze*]" (*The Big Typescript*, p.389).

[34] This is what, I think, Goldstein has in mind in postulating that Wittgenstein might have said, "I am committed to the view that tautologies are not *bona fide* statements—they have no sense and so, of course, cannot be true" (Goldstein 1999, p.504). Similarly Link asserts that "a tautology is not true, strictly speaking, for there is no corresponding fact of the matter" (Link 2009, p.45). Floyd and Shieh also question the sense in which tautologies are true, and contradictions false, at Floyd and Shieh 2024, p.42.

> In the second case the proposition is false for all the truth-possibilities; the truth-conditions are *contradictory*.
> In the first case we called the proposition a tautology; in the second, a contradiction. (4.46)
>
> Tautologies and contradictions are not pictures of reality. They do not represent any possible situations. For the former admit *all* possible situations, and the latter *none*. (4.462)

Now if a proposition is *true* if things stand at present as it says, and *false* if things could so stand but do not at present, if a proposition is *representative*, then it is clear that tautologies *cannot be true in this sense*, and equally, that contradictions *cannot be false in this sense*. Because they do not say how things do, or do not, but can, stand to one another.[35] This is obscured by indiscriminate use of *true* and *false*, hence the earlier proposal (§2.7), to mark it notationally, by using $\top\bot$ as well as TF, depending on context, whether one is or is not doing logic, i.e., dealing with reality, or with the world. Tautologies have the value $\top$ and contradictions the value $\bot$, are unipolar, in von Wright's terminology, with this notation marking the distinction that they have these values *irrespective of how things stand*, implying that different conceptions of truth and falsity are in play. With respect to truth-tables, if these are filled in mechanically *irrespective of how things stand*, then there are grounds to think that it is more seemly to use $\top$ and $\bot$ rather than T and F.

A tautology does not represent some sort of super-fact, a way in which things can stand, it happening to be the case that they do so stand in every possible world. If there is such a super-fact then it is not represented by a Tractarian tautology, because a Tractarian tautology is not representative; it does not represent any possible situation (4.462). If there is such a super-fact then the proposition that represents it is a synthetic accidentally general proposition, that just happens to be true in all possible worlds (cf. p.85).

The gist of the present difficulties is stated by Baker, writing of the period 1929/31:

> With hindsight [Wittgenstein] saw that the *Tractatus* had made use of two different criteria for singling out tautologies. One was that a

[35] Carruthers argues that tautologies "are propositions, since they are sentences with a determinate mode of determining a truth-condition." (Carruthers 1989, p.40) The objection is that there are in fact no *conditions*.

> proposition is a tautology if it is a degenerate truth-combination of independent propositions, i.e. if it is true independently of whether the bases of the truth-operation are true or false. Equivalently, it can be expressed in a truth table of 2^n lines in which 'T' is correlated with every line ... The other criterion was that a proposition is a degenerate truth-combination of propositions which are individually significant, i.e. that it is a truth-combination which is true in all circumstances. (Baker 1988, p.137)

To this he appends a footnote:

> As this [second] criterion is not avowed or scrutinised, its content and rationale are not made clear. Presumably the claim that a truth-combination of individually significant propositions cannot be false is not to be grounded in self-evidence, but no explanation is even suggested which would afford any non-psychological grounds for the claim that 'Not: A is red and A is green' cannot be false. (Baker 1988, p.137n.15)

The first criterion is the openly presented account, bound up with one of Wittgenstein's central claims, that the propositions of logic as tautologies are *toto caelo* different from any other kind of proposition. This is shown by their characteristic formal property, that they can be seen to be "*true* from the symbol alone" (6.113, emphasis added); they have a solid row or column of $\top$s in a truth-table, or a propositional sign of the form $(\top\top\top\dots)(p, q, r, \dots)$, with only $\top$s in the first parentheses. One needs nothing over and above the *symbol* (for the propositional *sign*) to see that a tautology (or a contradiction) is a tautology (or a contradiction) and is thus said to be true (or false) irrespective of how things stand in the world; provided, of course, that it is correctly arrived at, and not by means of, for example, Russellian generalisation (p.25). This first criterion is of being a *truth-functional tautology.*[36]

[36] That the *Tractatus* primarily deals in the propositional is significant here; "Strictly speaking, a proposition is a tautology iff it is true for any combination of truth-values of its constituent elementary propositions ... The doctrine of Wittgenstein is flawed, since the notion of tautology only seems to make sense within the realm of propositional logic, as Tarski pointed out in Tarski 1936, pp.419–20." (Ferreirós 2007, p.350n.1) Tarski merely notes that the concept of tautology "to me personally seems rather vague" (Tarski 1936, p.420), referring to Carnap's *Logical Syntax of Language*, where Carnap notes, with reference to the *Tractatus*, that the notion of tautology "is only defined for the sentential calculus" (Carnap 1934, p.44).

In the second case, though, insofar as there is a criterion on offer, we have what can be called an *obvious impossibility*, expressed in propositions whose contradictoriness is *not* shown by the symbol alone, the critical case here being colour exclusion (6.3751). Now of course no rational person would assert, seriously, that a surface can be red and green all over. As Wittgenstein put it, in later remarks:

> "Red and green won't both fit into the same place" doesn't mean that they are as a matter of fact never together, but you can't even say they are together, or, consequently, that they are never together. (Wittgenstein 1975, §78)

The critical point is, though, that *there is no evident way within the resources available in the* Tractatus *to* show *that this is a truth-functional contradiction.* Further, these resources do not permit one to say, that such a proposition is ruled out by grammar, or is not a move in a language game, because these are not available.

The difficulties here lead Lampert to argue that colour exclusion is a logical impossibility on physicalistic grounds (Lampert 2003, pp.310–11), but the cost of this strategy is the claim that "Wittgenstein's statement that there is only *logical impossibility* does *not* mean that there is *only one kind* of logical impossibility ... There are two different kinds of logical impossibility: One [the familiar truth-functional account] ... and one that consists in a conceptual conflict" (Lampert 2003, p.312). Although Lampert does not mention Baker or von Wright his account is, I think, motivated by similar concerns. But it is hard to see how one can cash out a notion of *conceptual conflict* without resort to such non-Tractarian notions as physical impossibility, or the synthetic *a priori.*

In a similar vein, I think, Goldstein attributes to the *Tractatus* a view that "contradictions (and tautologies) are logically quite separate from ordinary statements—they form a 'logical island' ... Geach (from whom I have borrowed the phrase 'logical island') thinks that this insulation doctrine is incorrect and, moreover, that the attribution of it to Wittgenstein is incorrect too." (Goldstein 1986, p.44) Goldstein, however, largely on the basis of the difference between logical proofs and proofs in logic set out in 6.1263, thinks that the "logical islands" view is Wittgenstein's (Goldstein 1986, pp.50–1). It seems to me, though, that there is right on both sides. There

is a sense in which, in terms of truth and falsity, tautologies and contradictions *are* different from contingent propositions, and this is, clearly, what Goldstein has in mind. But there is also a sense in which they aren't, or at least, shouldn't be, because they are generated within the system, because they are deployed in inference, and because there is no *logical* sense in which the top (all-$\top$) and the bottom (all-$\bot$) rows cannot obtain.[37] The problem simply is that a tautology is not true, and a contradiction is not false, in the same way that a proposition with sense is true, or is false. How serious a problem this is remains to be seen.

[37] Goldstein suggests that such difficulties can be evaded "by noting that ... tautologies and contradictions are not *Sätze*" (Goldstein 1986, p.50). It would be helpful if there was a clear distinction here, but there isn't, as tautologies and contradictions are noted as *Sätze* at 4.46 and at 4.466.

Chapter 3

Inference

Inference comprises the fourth and final stage in the construction of *System Tractatus*. The subject of this chapter is *logical*, that is, *truth-functional* inference.

3.1 A single calculus

There are two things one would expect to see in an account of inference in the *Tractatus*. First, an account of the *propositional sign*, because it is here that one gets "a proposition ... in its projective relation to the world" (3.12), this giving us the truth-possibilities for an elementary proposition. Second, an account of truth-tables, and of inferences based thereon; 4.442 identifies a truth-table with a propositional sign, with the concomitant account of inference based on truth-possibilities representable by truth-tables given at 5.11ff. This is sufficiently familiar that it can be effected in short order. More interesting, perhaps, are Russell's remarks in his introduction on Wittgenstein's use of a single truth-functional connective:

> [B]y a very interesting analysis [Wittgenstein] succeeds in extending the process to general propositions, i.e. to cases where the propositions which are arguments to our truth-function are not given by enumeration but are given as all those satisfying some condition. For example, let fx be a propositional function (i.e. a function whose values are propositions) such as 'x is human'—then the various values of fx form a set of propositions. We may extend the idea 'not-p and not-q' so as to apply to simultaneous denial of all the propositions which are

> values of fx. In this way we arrive at the proposition which is ordinarily represented in mathematical logic by the words 'fx is false for all values of x'. The negation of this would be the proposition 'there is at least one x for which fx is true' which is represented by '$(\exists x).fx$'. If we had started with not-fx instead of fx we should have arrived at the proposition 'fx is true for all values of x' which is represented by '$(x).fx$'. Wittgenstein's method of dealing with general propositions (i.e. '$(x).fx$' and '$(\exists x).fx$') differs from previous methods by the fact that the generality comes only in specifying the set of propositions concerned and when this has been done the building up of truth-functions proceeds exactly as it would in the case of a finite number of enumerated arguments p,q,r, ... (Russell 1922a, pp.xiv–xv)

By way of caveat, the careful reader will note that the notion of *propositional function* does not appear in the *Tractatus*.[1] This is addressed in Chapter 4. For the moment I will proceed in the familiar Russellian fashion on the basis, noted at p.40, that a substitutional approach to quantification is, in the present context, the least bad approach. The thing to note is that the bulk of Russell's remarks in the quoted passage relate to *generalisation*, that is, to treating "all" in terms of conjunction and "some" in terms of disjunction. One should be prepared, I suggest, for the possibility that what Wittgenstein is doing is not what one might expect. Russell's observation that "Wittgenstein's method of dealing with general propositions ... differs from previous methods" is an indication that what Wittgenstein is doing differs from his predecessors, in particular, from Frege, and from Russell.

Consequently Wittgenstein treats predicate calculus *as if* it is propositional calculus, by assuming that universal quantifiers can be treated as conjunctions and existential quantifiers as disjunctions of, after suitable but unstated analysis, elementary propositions. In order to do this the domain has to be given *in toto*, and it has to be logically tractable; in principle the number of names and thus of objects cannot be greater than $\aleph_0$. In this way the standard objection that one needs an "and these are all the relevant cases" clause is sidestepped.

Given this conjunction/disjunction analysis, inference in the *Tractatus* is addressed primarily at the level of the propositional. What is striking, and

[1] An omission noted only, I think, by Fisher and McCarty (Fisher and McCarty 2016, p.305).

much less commented on, is the value Wittgenstein later attributed to his account:

> In *Tractatus* I said $(x)fx$ & $(\exists x)fx$ were truth-functions,—first a logical product, 2^{nd} a logical sum.
> My mistake was to think the product, though we couldn't find it now, was contained in it.
> My good point was that I did make one calculus. (*Moore Lecture Notes*, p.219)[2]

He refers to the issues at some length, in a 1935 lecture:

> If from $(x)f(x)$ there followed $f(a)$, if this was really the relation of following, then it had to be the same relation as $p \vee q$ follows from p. What all these latter relations had in common was clear from the T–F scheme.
> I said to myself it had to be a relation between the truth of $(x)f(x)$ and the truth of fa.
> It had to be related in the way one truth-function was contained in another. (*Wittgenstein-Skinner Manuscripts*, p.286, lecture VIII 13 February 1935)

The point, in short, is that if generalised propositions are equivalent to conjunctions or disjunctions of elementary propositions, then inference proceeds on the same propositional basis as it does for any other complex proposition. The critical point to bear in mind is that Tractarian inference is *propositional* and not *sub-propositional*.

It takes a little excavation to find this idea of a single calculus in the text. At 5.451 Wittgenstein says, "If a primitive idea of logic has been introduced it must have been introduced in all the combinations in which it ever occurs." With respect to the negation sign $\sim$, the examples he gives in 5.41 are "$\sim (p \vee q)$" and "$(\exists x). \sim fx$", indicating that generalised propositions are regarded straightforwardly as complex propositions (i.e., arising from the application of truth-operations to elementary propositions). Similar remarks apply to 5.46, beginning "If we introduced logical signs properly, then we should also have introduced at the same time the sense of all combinations

[2]Lecture 8, 28 November 1932. Cf. *Ambrose Lecture Notes*, pp.6, 138.

of them"; there is no question of redefinition for different types or levels or orders. Logical signs operate in the same fashion in the context of generalised and non-generalised propositions. That generalised propositions partake of the general propositional form is effectively stated at 5.47: "One could say that the sole logical constant was what *all* propositions, by their very nature, had in common with one another. But that is the general propositional form." That the N operator can be deployed in the case of generalised propositions as in the case of any other selection of elementary propositions is stated at 5.52:

> If ξ has as its values all the values of a function fx for all values of x, then $N(\bar{\xi}) = \sim (\exists x).fx$. (5.52)

The other critical remark is buried in 6.1201:

> The fact that '$(x).fx :\supset: fa$' is a tautology shows that fa follows from $(x).fx$. (6.1201)

Here we get the notions of $\supset$, tautology, and following, all in the context of an inference from an universal generalisation. There is a single calculus with an univocal treatment of the logical constants once one is sensitised to its presence. But, as will be argued in Chapter 4, despite surface similarities Wittgenstein's approach is very different to Russell's.

Using modern notation, given that Wittgenstein is interested only in $\forall$ and $\exists$ treated respectively as logical products and logical sums of elementary propositions, these can be written as:

$$\forall x.fx \equiv fa_1 \wedge fa_2 \wedge fa_3 \wedge fa_4 \wedge \ldots$$
$$\exists x.fx \equiv fa_1 \vee fa_2 \vee fa_3 \vee fa_4 \vee \ldots$$

Where the dots are not dots of laziness, they indicate all the objects of the relevant form. It follows that $\forall x.fx$ is true iff for all substitutions for x the result is true, and $\exists x.fx$ is true iff there is at least one substitution for which the result is true. And here one has to assume that the notion of Tractarian naming has been made good, and that one is dealing with propositions in terms of sense/bipolarity, without referring to the world to see what is actually true, or actually false. The latter is required to ensure that one is dealing with tautological inference (6.1201) and not with what happens to be merely accidentally universally true (6.1232).

A further unargued assumption, as a corollary of dealing with a fixed domain, is that there is an effective decision procedure for inferences based on generalisation. This is the gist of 6.1262; "Proof in logic is merely a mechanical expedient to facilitate the recognition of tautologies in complicated cases." Inferences involving generalised propositions are tautologous because their truth-value is shown by the propositional sign, of the form $fa_1 \wedge fa_2 \wedge fa_3 \ldots \supset \ldots$ or $fa_1 \vee fa_2 \vee fa_3 \ldots \supset \ldots$ ($\models_{ST}$ might be preferable to $\supset$ here, provided one overlooks any implied metalogical or external perspective, and will be used hereafter). This is only feasible if the number of forms of objects is denumerable. That only the logical god of 5.123 can operate with such a calculus is not a *logical* objection (if the totalities in play are denumerable, there are inferences that we cannot in practice draw). This is discussed further in §3.4.

3.2 Properties of elementary propositions

From a logical point of view the requisite properties for elementary propositions can be stipulated. The Independence Thesis states that elementary propositions are independent of one another, that neither the sense nor the truth-value of an elementary proposition makes a difference to the sense or the truth-value of any other.

Elementary propositions are of course bipolar. For complex propositions, if they have sense then equally they are bipolar. What lacks sense (is nonsense, *unsinnig*) is not propositional. What is senseless (*sinnlos*), is tautologous or contradictory (unipolar, as p.90), and can of course only arise at the level of complex propositions.

Determinacy of sense is a "nothing over and above" notion, because to have sense, coupled with independence and bipolarity, is to have a *determinate* sense. There can be no such thing as an indeterminate sense; an indeterminate sense would not be a sense at all, and thus would be nonsense (*unsinnig*), and certainly not senseless (*sinnlos*). "Determinacy" in this context makes no essential reference to what we can and cannot do. If it is held that there is vagueness at the level of everyday propositions, this disappears on analysis, and if it persists, this shows that analysis is not complete (3.24).

The grounds for stipulating these properties is that they are requisite for the notion of *propositional sign*, and this is key to Wittgenstein's treatment

of logic in the *Tractatus*.

The notion of *propositional sign* (*Satzzeichen*) is introduced at 3.12; "I call the sign with which we express a thought a propositional sign.—And a proposition is a propositional sign in its projective relation to the world." Here it is significant that Wittgenstein talks of the *world* rather than of *reality*, because what we are dealing with is what is and what is not the case, what is actual, and not with the possible. The propositional sign is so to speak the propositional symbol in action; this I take it is the gist of 3.326.

This talk of the "propositional sign in its projective relation to the world" implies that the shift from proposition to propositional sign is somehow bound up with a shift from sense (the domain of $\top\bot$) to reference (the domain of TF). There is no obvious logical need for this, it is only from an expository point of view that it looks to serve a purpose, as with the notion of propositional sign we get the establishing of a link between a proposition and its truth-possibilities that goes beyond the merely definitional (the formal specification in terms of independence and bipolarity).

This link is presented in three different ways, by truth-table (4.442), in row form (4.442), and by bracketing notation (6.1203). These are notational variants, presenting the same content in different visual form. The simplest from a presentational point of view is row form, with p as $(\top\bot)(p)$, $\sim p$ as $(\bot\top)(p)$, $p \supset q$ as $(\top\top\bot\top)(p,q)$, and so on.

If we consider a truth-table for, say, $p \supset q$:

p	q	$p \supset q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\top$

then the need for bipolarity, independence, and determinacy is clear. It is only if all three conditions are in place that we can be sure *a priori* that given any two elementary propositions combined into a complex proposition by $\supset$ then its truth-possibilities are as per the table, that the propositional sign is $(\top\bot\top\top)(p,q)$; that we get the full truth-table.[3]

[3]Hacker notes independence as a "crucial idea", that "the logic of the [*Tractatus*] depended on the independence [thesis]." (Hacker 1999, p.187) Baker points out that "the *Tractatus*' philosophy of logic depend[s] on the doctrine of the independence of elementary propositions" (Baker 1988, pp.124–5), and that one can only have full truth-tables if

Since elementary propositions are given as a totality (4.52, 5.3262, 5.5561), and given unrestricted truth-functional combination there are no evident difficulties here.

On this basis one gets a very simple propositional calculus. Tautologyhood can be regarded as the formal property of axioms, should one want to axiomatise, but there is no need for this because any proposition ϕ that can be expressed as $\models_{ST} \phi$ is a truth-functional tautology and can be regarded as an axiom and/or as a rule of inference. For the sake of the argument one can simply invoke a classical natural deduction system for propositional calculus without identity, with introduction/elimination rules construed semantically.[4] Wittgenstein addresses proof in logic at 6.126–6.1265, in particular, 6.1263:

> [I]t would be altogether too remarkable if a proposition that has sense could be proved *logically* from others, and *so too* could a logical proposition. It is clear from the start that a logical proof of a proposition that has sense and a proof *in* logic must be two entirely different things. (6.1263)

Proof in logic is essentially a matter of beginning with tautologies and generating further tautologies by means of truth-operations (specifically, N), hence 6.1262, that "Proof in logic is merely a mechanical expedient to facilitate the recognition of tautologies in complicated cases". Wittgenstein's point, for proofs in logic, is that a closure principle applies, that such proofs generate tautologies out of tautologies, remaining within the domain of the senseless (*sinnlos*, the formal property of tautologies (4.461)). By contrast logical proofs show the consequences of a selection of meaningful propositions; a logical proof is a proof by means of logic, with undischarged premises. This (as Black notes, at Black 1964, p.339) is clearer in the *Moore Notes*:

> Therefore, if we say the *logical* proposition *follows* logically from another, this means something quite different from saying that a *real*

the Independence Thesis holds (Baker 1988, p.118).

[4]Goldstein says that Wittgenstein has "just such a system [of natural deduction] ... in mind" at Goldstein 1986, p.48. Proops suggests that the account at 5.13–5.1311, in terms of internal relations between the structures of propositions, and that at 5.11–5.121, in terms of a containment of truth-grounds, are somehow different, offering the "conjecture that 5.11 is supposed to function as a guide to the adoption of a perspicuous notation." (Proops 2002, pp.294–5) It is hard to see a substantive rather than a presentational difference between these.

> proposition follows logically from *another*. For so-called *proof* of a logical proposition does not prove its *truth* (logical propositions are neither true nor false) but proves *that* it is a logical proposition = is a tautology. (*Moore Notes*, p.109)

In discussing this Klagge offers a syllogism of the form ((S is P $\supset$ S is M) $\wedge$ S is P) $\supset$ S is M, and a proof of $q \supset (p \supset q)$ (*Principia Mathematica*, i $*2.02$ p.100), remarking that "Wittgenstein is claiming that these proofs are 'two entirely different things' " (Klagge 2022, p.244), without saying how they are supposed to differ. This is I think to make heavy weather of a rather simple point, albeit one obscured by Wittgenstein's near-total emphasis on semantics rather than proof theory, failure to distinguish between sense and actual truth-value, and absence of any metalogical perspective or results. The assertion that these "must be two entirely different things" is misleading.

3.3 Inference as structural

Wittgenstein's views on inference are contained in a run of propositions, from 5.11 to 5.143. These views are in fact very simple, and boil down to 5.13; "When the truth of one proposition follows (*folgen*) from the truth of others, we can see this *from the structure of the propositions*" (emphasis added).[5] It follows immediately that the antecedent to an inference is a complex proposition, and that inference is carried out entirely formally, without reference to the meaning of the complex proposition, or to that of any of its constituent propositions. The antecedent selection of propositions can of course be combined into a disjunction and thence into a logically equivalent conditional. Inference is simply a matter of detachment by modus ponens, or by a shown, but not stated, elimination rule (5.1241). Given the single calculus approach this applies to generalised propositions, as per the parenthetical remark at 5.1311, that "The possibility of inference from $(x).fx$ to fa shows that the symbol $(x).fx$ itself has generality in it."

The structure of a complex proposition is its truth-functional structure. So, for example, the structure of $(p \supset q) \wedge p$ can be written as $(\ldots_i \supset$

[5] A thorough study of inference in the *Tractatus* can be found in Dilman 1973, Chapter 8.

$\ldots_j) \wedge \ldots_i$. From this we can infer p, by modus ponens. Inference is decidable by mechanical procedure (assuming finiteness/denumerability).

3.4 Inference as decidable

To say that inference is in all cases mechanically decidable may, though, be too quick. Landini describes Wittgenstein's postulation of the N operator as adequate for the expression of quantification (on the basis that the formulas for $\forall x.fx$ and $\exists x.fx$ can be rewritten using N) as "little more than bravado based on a belief that logic is decidable." (Landini 2007, p.146) Setting aside the fact that Wittgenstein is not doing first-order logic with quantification in the now-familiar sense it is, I think, possible to say rather more in defence of his position.

That there is no decision procedure for higher-order logic is familiar, but a consequence of objectuality is that Tractarian logic is *in this limited respect* akin to first-order logic, with a single category of names. Properties and relations as they appear in everyday propositions analyse down to truth-functional combinations of elementary propositions, and here decidability is shown by the propositional sign. A proposition so complex that we may not be able to write out or compute a truth-table for it is of course possible, but this is not a logical objection.

At the level of elementary propositions analysis reveals a combination of names. Here again the idea is that all is shown by the propositional sign. So for example we might begin with an elementary proposition that is ostensibly of function-argument form, say, fa. On analysis down to the level of objects we might end up with, say, $rctubbbacba$, this being a combination of names for objects. For convenience $rctubbbacbx$ can be regarded as a (propositional) prototype (*Urbild*, 3.315), with x as a variable name (a propositional variable); so provided the objects concerned are of suitable form we can substitute a or b, say, for x, to get $rctubbbacba$ and $rctubbbacbb$ respectively (for the sake of the argument it is of course easier to stick with fx as long as this is regarded as a prototype and not a propositional function, for reasons that will become clear in Chapter 4).

Concerning generalised propositions we can say *a priori* that for every existential generalisation there is a possible world (a way in which objects can stand to one another) in which it is true, because such a proposition has

sense. This is simply a condition for full truth-tables.

Given that one is dealing with a fixed domain of objects of manageable cardinality whose form—range of combinatorial possibilities—is also fixed, the result is a *Spielraum* (range of possibilities for the totality of elementary propositions, as 4.463, 5.5262) of manageable size, albeit in principle if not in practice. Given such a totality (of objects with forms, states of affairs, elementary propositions, language, as 4.001) then decidability obtains in principle. This is of course an intuitive and not a formal result, consonant with Wittgenstein's approach in the *Tractatus* generally.[6]

The underlying point is that Wittgenstein isn't really in the business of doing predicate calculus. What matters is the distribution of truth-values in truth-tables, and hence what can be seen to follow from what. In practice this is mirrored in reality by the distribution of objects across states of affairs, and the concomitant distribution of truth-values across the totality of elementary propositions. Inference is then a matter of how these distributions change over time. Permissible inferences are a function of objects and their forms, and ideally, these are reflected in the notation. On the matter of whether one can "quantify" into elementary propositions and thus have to hand criteria of identity for names and, consequently, objects, this is a recurring topic in the following chapters.

3.5 Propositions of logic

As for the propositions of logic, this status is accorded to tautologies (and by duality to contradictions) (6.1). The key point is stated at 6.113; "It is the peculiar mark of logical propositions that one can recognise that they are true from the symbol alone." But this is not as innocent as it may appear.

It is true that if a complex proposition is a tautology then, in principle, it can be shown to be so by a purely mechanical process (by constructing a truth-table and noting that the column beneath the main connective is entirely of $\top$s). But it is not clear that this makes it *true*, in the sense of being *actually* true. Formally it certainly comes out as $\top$, and thus as unipolar and not bipolar. But as noted (§2.9) it is not entirely clear how one gets

[6] Fogelin argues that "*given the procedures explicitly stated in the* Tractatus", the N operator is not "adequate to construct all formulas of a standard first-order quantificational theory." (Fogelin 1987, p.78) This will not be addressed, for reasons of space.

from ⊤⊥ to TF and, more to the point, whether this is a matter that logic need have anything to say about.

What Wittgenstein says is, "It is clear that something about the world must be indicated by the fact that certain combinations of symbols—whose essence involves the possession of a determinate character—are tautologies." (6.124) As to what is supposed to be clear, the reader searches more or less in vain.[7] There is more to be said but, concerning inference in a purely *logical* sense, it is enough to say that a tautology realises a formal property, that it's propositional sign is of the form (⊤⊤⊤⊤ ...) (*p*,*q*,*r*,*s*, ...) in row form, or has only ⊤s in the column under the main connective in truth-table form. In bracket form as 6.1203 the outer symbols will all be ⊤. Similar remarks apply to contradictions, with ⊥ in place of ⊤. As to the sense, if any, in which tautologies can be said to be *actually* true, and contradictions *actually* false, this is discussed further in Chapter 5.

3.6 Inference, justification, and application

Concerning rules of inference and the justification thereof, Wittgenstein makes a self-conscious advance on Frege and Russell or, at least, on Frege and Russell as he construes them. This is stated in 5.132; " 'Laws of inference', which are supposed to justify inferences, as in the works of Frege and Russell, [are senseless], and would be superfluous."

The crux is that, as Wittgenstein sees things, while the propositions of logic propounded by Frege and by Russell are supposed to be the most general truths that there are, universally applicable, the propositions of some sort of super-science (§1.6), by contrast, Tractarian logical propositions as tautologies ostensibly do not say anything about the world or, for that matter, about reality, at all. They are not representative.

Waiving further considerations for the moment, there is no obvious basis for a non-circular justification for Tractarian inference, because it is simply a matter of what follows (*folgen*) on the basis of the truth-functional

[7] Black says, "the 'connection' of logic with the world is an identity of logical form, and has nothing to do with the contingent features distinguishing the actual world and the other possible worlds that might have existed in its place." (Black 1964, p.330) But this is a statement of what needs explication. Klagge does not touch on this at all in his discussion of 6.124 (Klagge 2022, pp.242–3), and McGinn jumps from 6.1222 to 6.126, omitting any mention of 6.124 (McGinn 2006, p.249). Cerezo mentions 6.124 three times (Cerezo 2005, pp.141, 149–50, 172), but doesn't address this particular issue.

structures of complex propositions, where such following involves no reference to anything outside of the propositions concerned. All is *shown* by propositional signs. If this can be made to stand up then one could ascribe to Wittgenstein a *sui generis* conception of logic. A proposition with the structure of modus ponens is senseless, that is, tautologous. This is why statements of laws of inference are superfluous, because inference is *shown* by the truth-functional *structure* of propositions, rendered transparent by means of propositional signs. As Wittgenstein said later, "Propositions do not follow from one another as such; they simply are what they are." (*Lee Lecture Notes*, p.57) From the same lecture Moore has, "[I]f you don't see it by looking at them, it [a rule of inference] won't help you." (*Moore Lecture Notes*, p.128) If there is anything to be *said* about justification it falls outside of logic.

Referring back to the Grayling quote at p.51, in terms of his remark concerning the truth of the *Tractatus*—*System Tractatus*, regarded as a formal system—what we have been dealing with is not truth *per se*, but rather with sense, indicated by the distinction between $\top\bot$ and TF. It is by operating at the level of sense that Grayling can talk of the *Tractatus* as an uninterpreted calculus, and it is on this basis that one might think that Wittgenstein holds the *sui generis* conception of logic. To do this a number of assumptions have to be made, principally that everyday propositions can be analysed down to complex and then to independent, bipolar elementary propositions, composed of names for objects, where the notion of bipolarity deployed does not entail a full-blooded realist commitment to truth (cf. p.80), and names (in terms of Tractarian naming) are regarded as a primitive syntactic category. It is also tacitly assumed that we can identify the propositional as such and distinguish it from what is nonsense (and also that we can identify the senseless, the tautological/contradictory). But this does not explain how the logical system is applicable to what there is.

The critical assumption made so far is that if we think of the *Tractatus* as presenting an uninterpreted calculus, whatever calculus it presents is interpretable. That is, the concepts employed are consistent. At bottom, the demand is that the concept "object" is consistent, that there can be a model for the system. But it should be emphasised that the account of inference given in this chapter is wholly *logical*, that is, *truth-functional*, and that inference in this sense is of little consequence. As will be seen in Chapters

6 and 7, probabilistic relations between propositions are of much greater interest.

To say that inference can only be shown, and that there is no meaningful account to be given as justification for the logical system on offer, leaves the issue of application somewhat dangling, where by *applicable* in this context is meant that Tractarian names take one directly to the objects themselves, that there is no notion of an interpretation function (a mapping from names to the basic entities in one's ontology). It is open to the proponent of a *sui generis* approach to logic to say that they are not concerned with application; that the logician's task is to devise systems that exhibit formal properties—consistency, completeness, compactness, and so on—and that it is up to others to see if any use can be made of such systems. In other words, what the logician does is *wholly* independent of application.

It is true that Wittgenstein could take this stance, but nevertheless there are reasons to think that he doesn't. Whatever one might think of the logics of Frege and of Russell as "super-science", there is no doubt that they held their logics to be straightforwardly applicable in thinking and reasoning about what there is. And, I suggest, applicable in a principled and not a merely serendipitous fashion. Anxiety over applicability occurs, I suggest, as soon as one adopts a *sui generis* approach to the constructing or devising of logical systems, because now one is unmoored from application.

In discussing Frege's rejection of the formalism of his day (*Grundgesetze*, ii §§86–137), Dummett considers "an alternative conception" open to the formalist "according to which application consists in supplying a specific interpretation for an intrinsically uninterpreted formal system." (Dummett 1991, p.257) Dummett goes on to suggest that should such a system prove to be applicable:

> [T]he notion of interpretation employed will be that used in standard model theory, rather than that appropriate for someone trying to break an intricate code or decipher an unknown script; if there proved to be a successful interpretation of this latter sort that did not respect the apparent syntax of the formal language, it would not be intuitively regarded as an application of the mathematical theory, but merely as an astounding coincidence. (Dummett 1991, p.257)

The use to be made of this is perhaps not what Dummett has in mind;

because what this illustrates, I suggest, is the usually tacit presupposition in deploying model theory of having independent access to the structures/categories of the theory, *and* to what it is applied to. So for example right at the beginning of a standard text on model theory (Hodges 1993, p.2) one finds a specification of the elements of the domain, and of the functions and relations on that domain. An interpretation that "respects the syntax" implicitly involves independent access to the elements of the domain, as p.52, and a "Fido"/Fido approach to the assigning of names to elements of the domain.

Now, what would be "a successful interpretation ... that [does] not respect the apparent syntax of the formal language"? The answer is, I suggest, a formal system that works, in that reasoning and inference based on the logical system enables us to do what we set out to do, but where the logical system functions as a "black box", with the inner operations thereof opaque to us. And this, I suggest, is what a *sui generis* conception of logic comes down to if we think that given suitable input (meaningful propositions) the deliverances of the system are bipolar—true or false in a realist sense—and not just verifiable or warranted as assertible. For such a "black box" system we have no grounds to claim that applicability is principled, and not just an astounding coincidence.

The logic of the *Tractatus* is of course predicated on the basis of propositions expressing truth-conditions (4.431), where the truth-conditions of a proposition are what is the case if it is actually true, and what is not the case if it is actually false. By contrast on a full-blooded *sui generis* approach the core property of the propositional is *not* the expression of truth-conditions; typically it is expressed as a syntactic/structural property realised by well-formed formulas. It is only if such a system can be interpreted with what there is taken as a model that one can trade in truth, and this raises the question whether such interpretation is an "astounding coincidence" or whether it is on some principled basis.

If one adopts an wholly *sui generis* approach to logic then it is not obvious that there is any external or non-autonomous basis on which to construct a logical system. If one constructs a system by abstraction from what we say and do then arguably one is doing psychology. If one considers what there is, then one is arguably doing empirical science. In neither case is one clearly doing what is wanted. In discussing an analogy between logic and

geometry Rescher argues that there is no pure logic in the way that there is pure geometry, because while "the development of a geometric system is unfettered and free of involvement with presystematic geometric principles" (Rescher 1969, p.219)—requiring nothing more, perhaps, than lines drawn in the sand—logic cannot so originate:

> There can be no "pure" (i.e., uninterpreted) logic. This point merits decisive emphasis . . . In the course of formulating as a system the materials of "a logic" one must inevitably make use of logical principles. To articulate a *systematic* logic we necessarily employ some *presystematic* logical machinery. (Rescher 1969, pp.217–8, 219)

Setting aside the pure/applied geometry analogy, and ignoring the question of whether Rescher is right about geometry, his main point is that logic has to originate somewhere, as an ordering or systematising of something or other.[8] In the present context the most promising way of cashing out the "presystematic logical machinery", of evading absolutist platonism, psychologism, or empiricism about logic, looks to be by locating it in the "something about the world" of 6.124, or the assertion that "Propositions *show* the logical form of reality" of 4.121. And this looks to be moving in the direction of logical realism. The challenge is to do this without going empirical, by identifying something structural that is, so to speak, sufficiently numinous and pervasive to be a plausible candidate for the "mirror-image of the world" (6.13), without offering a black box *sui generis* conception.

Further, the logic of the *Tractatus* should not be too readily assimilated to the first-order/higher order logics with a clear-cut account of quantification that first emerged clearly in Hilbert and Ackermann 1928. Such assimilation brings to a head the tensions between naming and the demands of the context principle, manifested in difficulties over quantification. There is, I suggest, no clear-cut account of quantification as we would now think of it in the *Tractatus*. This is discussed in Chapter 4, but to set the scene, it may be helpful to look at an extreme response. Von Plato remarks, with reference to the *Tractatus*, that:

> Wittgenstein's teaching had a devastating effect on some of his students, who never understood quantificational logic under his guidance.

[8] Rescher's analogy is discussed further at Cohnitz and Estrada-González 2019, pp.16–22.

(von Plato 2017, p.24)[9]

Much of von Plato's discussion centres on the transition period (von Plato 2017, pp.151ff), with generalisation treated in terms of conjunction and disjunction, as per the *Tractatus*. He clearly regards Wittgenstein's work as wholly misguided, particularly set against the achievements of Hilbert's school (the Göttingers), remarking that:

> My method for reading accounts of Wittgenstein's logic in the *Tractatus* was to check ... if Wittgenstein's failure to understand quantificational logic was adequately presented. (von Plato 2017, pp.183–4)[10]

There is a germ of truth in von Plato's reaction but it is not, I suggest, what he has in mind. The rub here is, that *Wittgenstein is not doing what he is generally taken to be doing.* Quite simply, he is not operating with a domain of well-defined individual objects that can be named in "Fido"/Fido fashion and can then be quantified over, whether one thinks objectually or substitutionally. The notion of *object* is elusive, but for the moment, the critical observation is, I think, the generally unnoticed absence of the notion of *propositional function.* This is the topic of the next chapter.

3.7 Logical realism, monism, exceptionalism

As noted (p.109), Wittgenstein is in some sense committed to logical realism, in a structural sense. This is what it is for logic to be a "mirror of the world", for logical propositions to deal with the scaffolding of the world (6.124), to "*show* the logical form of reality" (4.121). Wittgenstein is also, I think, committed to monism about logic, that there is one correct logic; this being, of course, *System Tractatus*. There is no evidence of logical pluralism in the text.

Things are rather more interesting in the context of exceptionalism and anti-exceptionalism about logic, because here the continuities and discontinuities between Frege, Russell, and Wittgenstein come into sharper focus. Hjortland states anti-exceptionalism thus:

[9] According to Hintikka, "[A] full understanding of the most central concept of contemporary logic, the notion of the quantifier ... eluded Wittgenstein." (Hintikka 1996, p.261)

[10] Von Plato's substantive criticism can be found at von Plato 2014, pp.438–45. The polemical remarks were added for book publication.

> Logic isn't special. Its theories are continuous with science; its methods continuous with scientific method. Logic isn't *a priori*, nor are its truths analytic truths. Logical theories are revisable, and if they are revised, they are revised on the same grounds as scientific theories. (Hjortland 2017, p.632)

Of exceptionalism, he says this:

> For the exceptionalist logic *is* special. There are a number of ways in which logic can be special, but for our purposes the central exceptionalist claim is that the justification of logical theories is *a priori*. (Hjortland 2017, p.633)

Exceptionalism is, then, akin to the *sui generis* approach to logic discussed at p.31. On the basis noted by Hjortland, Frege certainly looks to be in the exceptionalist camp. He was prepared to revise his logic, but only under logical/mathematical pressures (the revision of Basic Law V in the Afterword to *Grundgesetze*, ii, to try and circumvent Russell's paradox). His commitment to applicability veers towards logic as super-science, but on balance I think he is not so committed, and that his outlook is exceptionalist.

By contrast Russell looks to be in the anti-exceptionalist camp. He does think that logic is, in some sense, adjacent to if not in fact continuous with science, and that logic is revisable. Whether he thinks logic is revisable on empirical rather than logical or conceptual grounds is less clear.[11] It is, though, hard to find a definite statement of this in the terms set out by Hjortland, because Russell's scientific method in philosophy looks more like the application of what is wholly general, and prior to the special sciences (Russell 1914b, p.109).[12] His openness to revision in the case of the axioms

[11] Russell regards his regressive method of discovering premises in mathematics, and scientific methodology, as a "close analogy" (Russell 1907, p.272) but the discussion in the paper (and the use of the method noted at *Principia Mathematica*, i pp.v–vi) looks to be constrained within the logical/mathematical.

[12] Russell also, I think, would have regarded the concept of apriority as psychological rather than logical, after his remarks on the *a priori* at *Foundations of Geometry*, §5 were heavily criticised by Moore (Moore 1899, pp.399–400). He says however that there "is such a thing as 'logical experience' ... which ... enables us to understand logical terms" (*Theory of Knowledge*, p.97), this being perhaps *a priori*. However Hylton notes that Russell's anti-psychologism left him, circa 1910–13, with "no account of logic which he can accept" (Hylton 1990b, p.80).

of infinity (adopted only as an hypothesis; *Principia Mathematica*, ii p.189) and reducibility is motivated by logical, conceptual, and mathematical rather than empirical reasons.[13]

Prima facie Wittgenstein is fully committed to exceptionalism. Logic must, as he says, "look after itself" (5.473). It is not obviously responsive to empirical considerations; indeed it looks to be wholly *sui generis*, not responsive to any extra-logical considerations at all. But this is, I think, because Wittgenstein believes that he has arrived at the one true logic, which fully captures the structuring and organising of what there is, that is, the logical forms of the objects that there are. The question is whether this position can be maintained when logic comes under pressure, in particular, when it comes to questions about exclusion (colour exclusion).

3.8 Antipsychologism

If one thinks of logic as *sui generis*, in exceptionalist terms, there is a risk of logic becoming wholly unmoored from its application, as akin to a "black box" system. But this smacks of absurdity. We are rational beings, and do not need to be instructed in logic to think rationally. We imbibe logic, it seems, in imbibing language, so there is no obvious reason to seek an account beyond this of what logic is and how we get to know it, or a justification thereof. Reflection on language and linguistic practices looks to be sufficient.

At this juncture I want to bring together two strands in recent discussions, concerning the saying-showing distinction, and the grounds or justification for our capacity to think and speak rationally, that is, logically. The principal figures here are Ricketts, Kremer, and Kuusela.

Notoriously in a letter to Russell dated 19 August 1919, after the text of the *Tractatus* was complete, Wittgenstein says:

> The main point is the theory of what can be expressed (*gesagt*) by prop[osition]s—i.e. by language—(and, which comes to the same, what can be *thought*) and what can not be expressed by prop[osition]s, but only shown (*gezeigt*); which, I believe, is the cardinal problem of philosophy. (*Wittgenstein in Cambridge*, p.98)

[13] For Russell's doubts about reducibility, see Linsky 1999, pp.95–8, Hylton 1990b, pp.74–5. The doubts discussed by Linsky and Hylton are logical/mathematical rather than empirical.

Hacker develops this by mooting a distinction between overt and covert nonsense, where the latter includes "what might ... be called illuminating nonsense, and misleading nonsense":

> Illuminating nonsense will guide the attentive reader to apprehend what is shown by other propositions which do not purport to be philosophical; moreover it will intimate, to those who grasp what is meant, its own illegitimacy. (Hacker 2021, pp.18–9)

This, I suggest, reiterates the familiar universalist point, that there is no external perspective from which one can talk, propositionally, about what logic is and how it does what it does. This is clearly at odds with the resolutist position, that one should come to see that there is only plain or sheer nonsense, that there is no distinction within the nonsensical between "misleading" and "illuminating" nonsense.[14]

Given a programmatic commitment to an univocal conception of plain or austere nonsense the resolutist cannot allow that there is also illuminating or substantive nonsense.[15] So Ricketts says, "Any resolute understanding of saying and showing will have to be non-contrastive: there are not two species of content, sayable content and the ineffable content that can only be shown, as ontology-oriented interpretations maintain" (Ricketts 2023, pp.24–5).[16]

In this context Ricketts notes Kremer 2013, where Kremer seeks to deflate the saying/showing distinction, on the grounds that what is shown is our knowing *how* to deploy logic in a practical sense in our thinking and reasoning, and not that there is anything *that* we can be said to know (i.e., anything propositional). All we need to do is look and see and resist the temptation to theorise:

> I suggest that the distinction between saying and showing must at least sometimes be deployed in a way that involves philosophical confusion—a confusion we seize upon because it seems to let us satisfy our desire to take care of logic. Our temptation, in other words, is to give a justification, or grounding, for logic. (Kremer 2013, p.479)

[14] Cf. Conant's distinction between *substantial* and *austere* conceptions of nonsense at Conant 2000, pp.191–98. The Hacker/resolutist debate is well set out in Kuusela 2011.

[15] Ricketts says, with reference to Diamond, that the "austere view of nonsense is the first characteristic of resolute interpretations" (Ricketts 2023, p.11).

[16] The "classic exponents" of "ontology-oriented interpretations" being Pears, Hacker, Black, and Malcolm (Ricketts 2023, p.8).

The invocation of *know how* does, though, look like a grounding or a justification of at least our capacity to operate with a logic that is immanent in (our uses of) language. Kuusela approaches the text along these lines:

> [W]ith his saying-showing distinction Wittgenstein can be understood as aiming to clarify the nature of the tacit logical knowledge that thinkers and language users must possess in order to be able to think or understand a language, and to use it to speak of the world. This, of course, is not a psychological theory about how people actually manage to use language, but an account of what understanding language (or representation) must in principle involve. (Kuusela 2023b, p.69)

Kuusela's project relies on his claim that:

> [T]he key to the *Tractatus* is Wittgenstein's insight that language users and thinkers possess a tacit, pre-theoretical comprehension of logic which logicians cannot inform them about, or state prescriptions about, because being informed about logic or understanding logical prescriptions already presupposes an understanding of logic. This pre-theoretical comprehension is what Wittgenstein's book seeks to clarify. (Kuusela 2019, p.40)[17]

There is much to be said for Kuusela's underlying idea, that there is a logical notation to be discovered within natural language. This logical notation is disguised by everyday language (4.002) and it is by such techniques as Russell's theory of descriptions (4.0031) that it ought to be discovered. This is the force of 4.1213:

> Now, too, we understand our feeling that once we have a sign-language in which everything is alright, we already have a correct logical point of view. (4.1213)

Where I think Kuusela's account is problematic is in ascribing logic primarily to ourselves, that the aim of the *Tractatus* is to enable us "to clarify the nature of the tacit logical knowledge that thinkers and language users must possess" (Kuusela 2023b, p.69). Because what the *Tractatus* says is that "Logic is . . . a mirror image of the *world*" (6.13, emphasis added), and

[17] Cf. Kuusela 2019, pp.38, 50, 62–4, 72–4, 92–3, Kuusela 2023a, p.42.

not that logic is a technique that we project on to the world. Now of course 6.13 continues, "Logic is transcendental". One can treat this as wholly Kantian, as Kuusela does, I think, when he says that "Logic . . . is internal to and constitutive of all thought and language use" (Kuusela 2023a, p.44). This fits well with Wittgenstein's later work, but as a reading of the *Tractatus* it is I think mistaken, because Kuusela's protestations notwithstanding this is, by Tractarian standards, psychologistic.

Frege's rejection of psychologism in the Foreword to *Grundgesetze* is familiar ("the ruinous incursion of psychology into logic", *op. cit.* p.xiv). Russell is equally forthright:

> The truth is that, throughout logic and mathematics, the existence of the human or any other mind is totally irrelevant; mental processes are studied by means of logic, but the subject-matter of logic does not presuppose mental processes, and would be equally true if there were no mental processes. (Russell 1904, p.478)

> There are in fact many ways, besides logical inference, by which we pass from one belief to another: the passage from the print to its meaning illustrates these ways. These ways may be called 'psychological inference'. We shall, then, admit such psychological inference as a means of obtaining derivative knowledge, provided there is a discoverable logical inference which runs parallel to the psychological inference. (*Problems of Philosophy*, p.78)

The point is that what is true, and the relations between truths that logic aims to capture, are as they are irrespective of our thinking and reasoning. As Frege puts it, "being true is different from being taken to be true, be it by one, be it by many, be it by all, and is in no way reducible to it" (*Grundgesetze*, i p.xv), and this applies as much to inferences drawn in gap-free proofs as it does to the premises thereof. In considering Frege's discussions of proof and the fundamental laws of logic, in *Foundations of Arithmetic*, §3, and in the Foreword to *Grundgesetze*, i, Dummett notes that Frege is explicit that "no justification can be given for accepting those laws of logic which cannot be derived from other laws" (Dummett 1982, p.134), before considering how one might essay a justification:

> Either we assume, or have reason to believe, that what prompts the judgement is a reliable sign that it is true; or we take the judgement as relating solely to the occurrence of that which prompts it. The former is the realist option, the latter the idealist one. (Dummett 1982, p.136)

Both Frege and Russell are realists, and not idealists, so one would expect any attempt at justification to be based on what is external to ourselves, whether platonist or empirical (perceptually available). The point is, however curious this may sound, that Frege, and even more so Russell, were intent on producing logical systems that are applicable (are bodies of truths, not only characterised by formal properties, principally consistency), but are not in any way *dependent on* or *derived from* our practices of and capacities to think and reason about what there is (whether the latter is regarded platonistically or empirically). In this sense they may both intend to arrive at logical systems that are *sui generis* but, as Wittgenstein sees things, they fail to emancipate themselves fully from what there is (construed in a broad sense, i.e., as super-science). This comes out in a commitment to truths that are generalised and thus are still marked, somehow, by an originary taint, where, as we have seen (§1.8), this more obviously applies to Russell than to Frege.

What Wittgenstein aims at, then, is a *Wissenschaft* of pure structure, where the structures in play are in common between a fully-analysed language and the states of affairs/situations that can obtain (structure thus being, at bottom, a function of objectuality). The direction of dependency is, though, from reality, the totality of objects with their forms, to language. Logic has no subject-matter of its own because it is a showing, in a correct notation, of the structuring or ordering of objects into states of affairs/situations. Hence McCarty has a point in comparing language to a mechanism that is largely external to us:

> The operative setting is what I choose to call "the mechanism of language". It is a mechanism over which we exert little control, especially of an intellectual sort. As mechanism, language is an automaton; like the vast subterranean machines of Lang's *Metropolis*, a major part of its working is in operating us. The language mechanism is infinite—boundless and all-embracing. It is complicated beyond intelligibility. We are linked to it and in it but we have no real notion of the details of

> its workings. Certainly, we do not work it by grasping it or by lodging its governing principles in our minds. (McCarty 1991, pp.72–3)

With the caveat, though, that language must not be divorced from its application, its capacity to represent how things stand. Wittgenstein's method is a sustained reflection on structures and patterns in language, but the target in the *Tractatus* is not our presystematic thinking and reasoning, or tacit knowledge. This is why Wittgenstein can dismiss questions about the "psychical constituents" of thoughts, in his 19 August 1919 letter to Russell (*Wittgenstein in Cambridge*, p.99). The target is the "something about the world" that "must be indicated" by the propositions of logic (6.124); how it is that "Propositions *show* the logic of reality" (4.121). It is only if one grasps this, that the later criticisms in *Philosophical Investigations* come into focus, particularly this:

> For there seemed to pertain to logic a peculiar depth—a universal significance. Logic lay, it seemed, at the bottom of all the sciences.—For logical investigation explores the nature of all things. It seeks to see to the bottom of things and is not meant to concern itself whether what actually happens is this or that.—It takes its rise, not from an interest in the facts of nature, nor from a need to grasp causal connexions: but from an urge to understand the basis, or essence, of everything empirical. (*Philosophical Investigations*, §89)

Wittgenstein belatedly came to realise that the *Tractatus* is prone to the same criticisms that he levelled therein against Frege and Russell, of failing to emancipate logic from the empirical. And this suggests that the notion of a logic of pure structure, one that really has no subject-matter of its own, is an illusion.[18] Here again we come back to objects and colour exclusion, the topics of Chapter 8.

[18]Link describes the logic of the *Tractatus* as "pure form without content" (Link 2023, p.78), concluding "I tend to challenge the idea that such a realm, a realm separated from experience, from reality, and from scientific inquiry, can be marked off in a meaningful way" (*op. cit.* p.90).

Chapter 4

Tractarian functions are not propositional functions

4.1 The absence of "propositional function"

When Wittgenstein talks about functions in the *Tractatus*, and in particular, the "function fx" in 5.501, what he has in mind is *not*, I suggest, Russell's notion of propositional function. Conflating Wittgenstein's notion of a function with Russell's notion of propositional function is so widespread that, in quoting 5.501, Rogers and Wehmeier insert the word as if Wittgenstein had, perhaps carelessly, omitted it: "a [propositional] function fx, whose values for all values of x are the propositions to be described (5.501)." (Rogers and Wehmeier 2012, p.563)

Reading the familiar Russellian notion of propositional function into the *Tractatus* is a barrier to understanding what Wittgenstein is doing, not least because despite outward similarities with the notation of *Principia Mathematica*, it isn't there to be found. The Russellian notion goes together with the notions of a well-defined domain of individuals that are independently accessible and nameable, and not the Tractarian conception of objects with forms that constitute states of affairs and, as such, are not straightforwardly nameable. And this of course affects how one thinks about generalisation and quantification. In fact, the expression "propositional function" doesn't appear in the *Notes on Logic*, the *Moore Notes*, the *Notebooks 1914–16*, the *Prototractatus*, the *Tractatus*, or the correspondence (*Wittgenstein in Cambridge*, *Letters to C.K. Ogden*). Wittgenstein's earliest use of the expression

is at *Philosophical Grammar*, p.125.

Rogers and Wehmeier note that Wittgenstein could have used Russell and Whitehead's $f\hat{x}$ notation:

> That Wittgenstein was familiar and comfortable with representing propositional functions by means of the circumflex notation is evidenced by *Prototractatus*, 5.3321: "[T]he proposition, '*Only* one x satisfies $F(\hat{x})$', will read '$(\exists x).Fx :\sim (\exists x, y).Fx.Fy$'." Compare [*Tractatus*] 5.5321, where $F(\hat{x})$ is replaced by $f(\ )$. (Rogers and Wehmeier 2012, p.564n.48)

They also quote from Ramsey 1923, where Ramsey adds Russellian notation to Wittgenstein's; "$\bar{\xi}$ is the set of values of $f\hat{x}$" (Ramsey 1923, p.15). They then say, "Although the review [Ramsey 1923] was written before Ramsey's consultations with Wittgenstein in the summer of 1923, the passage demonstrates how natural it was for someone writing in the *Principia* tradition to assume the availability of circumflex notation." (Rogers and Wehmeier 2012, p.565) But this is a comment about *Ramsey*, not about *Wittgenstein*. The only use of circumflex notation in the *Prototractatus* is in 5.3321. It appears in *Notebooks 1914–16* at pp.11, 16, 19, 28 and 32, at *Notes on Logic*, p.101 (cf. Potter 2009, p.278), and once in the correspondence (L.W. to B.R. November 1913, *Notebooks 1914–16*, pp.128/9, *Wittgenstein in Cambridge*, pp.57/59). It is not used in the *Tractatus* at all.

One can, then, draw the diametrically opposed moral, that Wittgenstein came to be dissatisfied with the Russellian notion of a propositional function when he wrote the *Tractatus*, showing this by choosing not to use the circumflex notation. It is noticeable that Russell attributes to Wittgenstein an account of generality based on "all propositions of the form fx where fx is *a given propositional function*." (Russell 1922a, p.xxi, emphasis added) Wittgenstein refused to have Russell's introduction printed, because on translation into German, "what remained was superficiality and misunderstanding" (L.W. to B.R. 6 May 1920, *Wittgenstein in Cambridge*, p.119). The topic of the present chapter is perhaps an illustration of such divergence.

The presumption that Wittgenstein was operating with the Russellian notion of propositional function in the *Tractatus* is a largely unquestioned orthodoxy. In a 1981 paper, for example, Ishiguro talks almost entirely in Russellian terms, attributing to Wittgenstein the notion of propositional

function; "If we follow the *Tractatus* view of how propositional functions are identified within a proposition and constitute themselves into types, I think Russell's paradox can be avoided ... It is important to remind ourselves here that for both Russell and Wittgenstein the value of a propositional function is not a truth-value but a proposition." (Ishiguro 1981, pp.43, 49) But this is to foist on Wittgenstein a notion that he doesn't use. Referring to 5.501, Hylton says, "At least some of the uses of the word 'function' (*funktion*) in the *Tractatus*, moreover, *must* be taken to mean 'propositional function', if we are to make even *prima facie* sense of them ... 5.5301 speaks of an object as *satisfying* a function (the German is *genügen*), which hardly makes sense unless it is a propositional function that Wittgenstein has in mind." (Hylton 1997, p.141, emphasis in the original) But his first remark is just assertion. Of 5.5301, this concerns how one should think of "$(x) : fx . \supset . x = a$" when "only a satisfies the function f", so this is about how one should talk about the realisation of single cases of a Tractarian function; there is no commitment to Russellian propositional functions *per se*. Ricketts says, referring to Hylton 1997, that Wittgenstein "uses 'function' in the *Tractatus* to mean 'propositional function' " (Ricketts 2013, p.129n.4), without qualification.

Potter says, "In the case of a relational proposition aRb, for instance, Wittgenstein's proposal was that we analyse $\phi([aRb]) \equiv \phi'(a, b).aRb$, where ϕ' is a propositional function appropriately related to the original function ϕ." (Potter 2009, p.44) But this is not to be found as such in *Notes on Logic*. He also says that Wittgenstein "could not simply banish propositional functions from logic completely: they are too useful" (Potter 2009, p.177), but this reflects what a sophisticated logician thinks Wittgenstein should have said, and does not accord with the text. Weiss begins by saying, "Wittgenstein seems to intend this mysterious 'function' fx [in 5.501] to be something like a propositional function in the sense of *Principia Mathematica*" (Weiss 2017, p.5), and discusses a difference between constituents and parts of propositions ("it is not at all clear just what could be meant by 'constituent' (*Bestandteil*), let alone by turning one into a variable"; *op. cit.* p.6), and talks of varying the remainder of a proposition rather than "a singled-out 'part' " (*op. cit.*) p.7), but then proceeds to assimilate Tractarian functions (as I will term the "function fx" of 5.501) to the Russellian notion of propositional function ("propositional functions are canonically

explicated . . . as formulas containing at least one free variable", *op. cit.* p.9). Connelly talks similarly of the xRy of 4.1252 as a propositional function, and of the second and third methods in 5.501 as "procedures . . . to gather together substitution instances of triadic, quadratic or other propositional functions." (Connelly 2021, p.174) As noted (p.96n.1) Fisher and McCarty are, as far as I am aware, the only authors to note the absence of the notion of propositional function in the *Tractatus*, but far from questioning this omission, they make extensive use of the notion in their reconstruction of the logic of the *Tractatus* (in Fisher and McCarty 2016).

4.2 Russell's notion of significance

In *Principia Mathematica*, Russell (on the basis that Russell was largely responsible for this aspect of *Principia*), says:

> Let ϕx be a statement containing a variable x and such that it becomes a proposition when x is given any fixed determined meaning. Then ϕx is called a "propositional function" . . . 'x is hurt' is an ambiguous 'value' of a propositional function. When we wish to speak of the propositional function corresponding to 'x is hurt', we shall write '$\hat{x}$ is hurt'. Corresponding to any propositional function $\phi\hat{x}$, there is a range, or collection of values, consisting of all the propositions (true or false) which can be obtained by giving every possible determination to x in ϕx . . . Any value 'ϕx' of the function $\phi\hat{x}$ can be asserted. Such an assertion of an ambiguous member of the values of $\phi\hat{x}$ is symbolised by '$\vdash \phi x$' . . . This . . . embodies the use of the [real] variable. (*Principia Mathematica*, i pp.17–18)

The critical distinction is between a propositional function, and determinations of the variable that result in a proposition. Since Russell was engineering a way of evading paradoxes, given such separation he needed a way of specifying the range of determination (the "total variation", *Principia Mathematica*, i p.16) of x. Russell sets out his notion of significance in a footnote in 'Mathematical Logic':

> A function is said to be significant for the argument x if it has a value for this argument. Thus we may say shortly 'ϕx is significant',

> meaning 'the function ϕ has a value for the argument x'. The range of significance of a function consists of all the arguments for which the function is true, together with all the arguments for which it is false. ('Mathematical Logic', p.72n.*)

He goes on to make a rather odd remark:

> [I]f the function ceases to be significant when the variable goes outside a certain range, then the variable is *ipso facto* confined to that range, *without the need of any explicit statement to that effect.* ('Mathematical Logic', p.73, emphasis added)

How is such confinement to be effected? For the moment, it is worth noting that the range of significance is equated with a type; "A *type* is defined as the range of significance of a propositional function, i.e., as the collection of arguments for which the said function has values." ('Mathematical Logic', p.75) But how is a type to be specified? Here we get the critical move, to evade vicious circles:

> When we say that "ϕx" ambiguously denotes ϕa, ϕb, ϕc, etc., we mean that "ϕx" means one of the objects ϕa, ϕb, ϕc, etc., though not a definite one, but an undetermined one. It follows that "ϕx" only has a well-defined meaning ... if the objects ϕa, ϕb, ϕc, etc., are well-defined. That is to say, *a function is not a well-defined function unless all its values are already well-defined.* (*Principia Mathematica*, i p.41, emphasis added)

In other words, Russell assumes that the range of significance of a propositional function is already present and correct, given in advance.[1] On this basis, "we should say 'ϕx is a proposition,' but '$\phi\hat{x}$ is a propositional function'." (*Principia Mathematica*, i p.42)[2] Wittgenstein, I think, simply as-

[1] The significance of significance can be seen in its ubiquity, at *Principles of Mathematics*, §497, Russell 1906a, pp.201–2, 205–6, Russell 1906b, p.177, 'Mathematical Logic', pp.72–5, *Principia Mathematica* pp.43–4, 48, 50, $*1.11, *1.72, *3$ summary, $*3.03, *9.14, *10.121, *10.14, *10.4$f, $*11.14, *13.22, *20.631$f, $*21.631$, *Introduction to Mathematical Philosophy*, pp.159, 162, Russell 1924, pp.337–8.

[2] As Quine notes, the x in ϕx (and not in $\phi\hat{x}$) only looks like a free variable; "because the free variable is in this way unhampered, Russell likes to suppress a universal quantifier when it has the whole of a theorem as its scope." (Quine 1967, p.151) This allows Russell to term ϕx a *proposition.*

sumes, in the *Tractatus*, that the fx of 5.501 can be regarded as propositional because it is well-defined, because all the relevant propositions are in some sense already present.

This is bound up with Russell's difficulties over propositional unity, and with judging whether a given proposition is true, or false. Wittgenstein states his position in *Notes on Logic*:

> Symbols are not what they seem to be. In "aRb", "R" looks like a substantive, but is not one ... Similarly in "ϕx", "ϕ" looks like a substantive but is not one. (*Notes on Logic*, p.98)

This is aimed at Russell, because Wittgenstein thinks that the former's approach involves treating signs for properties and relations (the universals of *Problems of Philosophy*, Chapter 9) as names, and thus what they name, as substantives. In this way a Russellian proposition becomes a list of names for substantives such that there isn't a proposition at all, resulting in ever more sophisticated versions of the multiple-relations theory of judgement in, Wittgenstein holds, a vain effort to reassemble what Russell's theoretical commitments have hopelessly sundered.

4.3 Technical and philosophical approaches to propositions

There is nevertheless a profound tension between assuming that the values of a propositional function (a class of propositions) are given in advance—"already well defined" (*Principia Mathematica*, i p.41)—and the idea that a proposition "is not a single entity at all" (*Principia Mathematica*, i p.46), on the basis that its disparate elements—an n-adic propositional function and n determinations of its variable(s)—are only assembled into a proposition in a mental act by a judging subject. Russell ostensibly rides two horses, treating propositions both as somehow given in advance, and thus as suitably unified entities, *and* as somehow constructed in judgement. These will be called respectively the "technical approach" (that propositional functions are well-defined, that propositions can be taken for granted) and the "philosophical approach" (multiple-relations theory). This bifurcation is clearest in *Principia Mathematica*, i Chapter II §§II–III, with the technical approach

figuring in the discussion of propositional functions in §II and the philosophical approach (multiple-relations theory of judgement) in the discussion of truth and falsity in §III.[3]

In his technical work Russell's preference is for the first. He was fully aware of Wittgenstein's objection, that you can't assemble a proposition out of disparate constituents (the philosophical approach) unless you already know that they are such that they can be so assembled, and one can manufacture different responses depending on which text one chooses to concentrate. *Theory of Knowledge* represents the apotheosis of the disparate end of the assembly problem (the multiple-relations theory), with the approach assuming that all values are given in advance (the technical approach) in 'Mathematical Logic' and *Principia Mathematica*, i Chapter II §III constituting the opposite extreme.

That Wittgenstein presumes the technical approach in the *Tractatus* is, I think, anticipated at *Notebooks 1914–16*, p.54; "In these investigations I always seem to be unconsciously taking the elementary proposition as my starting point." Ramsey notes this presupposition of elementary propositions in a record of his 1923 conversations with Wittgenstein.[4]

Beginning with elementary propositions enables Wittgenstein to sidestep both the major problems afflicting Russell's multiple-relations theory, the "wide direction" problem:

> WD is the problem of ensuring that a judgement contains at least one predicate or relation, that the relation occurs in the correct position and thus that the judgement has sense, (Connelly 2021, p.35)

and the "narrow direction" problem:

> ND is simply the problem of distinguishing, for example, Othello's belief that Desdemona loves Cassio from Othello's belief that Cassio loves Desdemona. (Connelly 2021, p.33)[5]

Assuming elementary propositions from the start sidesteps the wide direction problem. The narrow direction problem is sidestepped by assuming

[3]Cf. discussion at Linsky 1999, pp.23–5. Klement notes that after 1907 Russell "used 'proposition' in a variety of ways" (Klement 2018, p.161).

[4]Cf. Ramsey papers #002-27-01 p.24, quoted at Klagge 2022, p.55.

[5]Wittgenstein's views are thoroughly discussed in Connelly 2021; the successive phases of the multiple-relations theory are well set out in Candlish 1996.

that elementary propositions have sense; the issue of the direction of the relation becomes a matter of the actual truth-value of the propositions concerned, ascertained by comparison with the obtaining facts.

By assuming elementary propositions Wittgenstein can reject any approach that involves treating a Russellian propositional function as an entity in its own right, containing one or more free variables, such that a determination of its variables results in a proposition, along with any associated strictures needed to ensure that the determinations are drawn from the range of significance of the function (in addition to the propositional function itself, the Russellian needs as well a statement of its range of significance). This is the critical difference between Russellian propositional functions and Tractarian functions.

In discussing Tractarian functions Soames says, "The function mentioned ... is not really a function, but a formula containing a (free) variable. Such formulas have often been accorded the misleading title 'propositional function'." (Soames 1983, p.578n.14) Soames does not explain what he has in mind by "misleading", referring his reader to the "clear explanation" to be found in Ramsey 1925a:

> A propositional function is an expression of the form $f\hat{x}$, which is such that it expresses a proposition when any symbol (of a certain appropriate logical type depending on f) is substituted for $\hat{x}$. Thus '$\hat{x}$ is a man' is a propositional function. We can use propositional functions to collect together the range of propositions which are all the values of the function for all possible values of x. Thus '$\hat{x}$ is a man' collects together all the propositions 'a is a man', 'b is a man', etc. (Ramsey 1925a, p.171)

Ramsey was clearly familiar with Russell and Whitehead's notation, and that on this approach, there is a need for a specification of appropriate logical type. With this in mind, we can turn back to Rogers and Wehmeier:

> By a *propositional variable*, Wittgenstein understands any variable whose values are propositions. A *variable proposition* is an open sentence, that is, the result of replacing, in a proposition, a constituent by a variable. Finally, a *propositional function* is a function that sends names to propositions in such a way that a name a is mapped to the

> result $\phi(a)$ of replacing the variable x in a variable proposition $\phi(x)$ with a. (Rogers and Wehmeier 2012, p.556)

Similarly Klagge says, "what Wittgenstein here [3.315] calls 'propositional variables' seems to be what Russell means by 'propositional function'." (Klagge 2022, p.91) Glock talks of specifying "the base of an operation ... through a 'propositional variable'—Russell's propositional function—'fx'." (Glock 1996, p.146) Marion also talks in terms of propositional functions, noting Wittgenstein's expression "logical prototype", but not distinguishing these (Marion 1998, pp.35, 37). Ishiguro says, "A propositional function, [Wittgenstein] says, contains a prototype of its argument" (Ishiguro 1981, p.56), with reference to 3.333. But this isn't what he actually says; 3.333 talks of *functions*, and not of *propositional functions*. At no point in the text do we get the phrase "propositional function".

The notion of *propositional variable* is certainly to be found in the *Tractatus* (cf. 3.313), as is that of *variable proposition*:

> If we turn a constituent of a proposition into a variable, there is a class of propositions all of which are values of the resulting variable proposition. (3.315)

The "function fx" of 5.501 is thus a *variable proposition*, "whose values for all values of x are the propositions to be described" (5.501). 3.315 is premonitory of 5.501, Wittgenstein exploiting, I think, Russell's technical approach, that all the instances are given in advance; this is the approach adumbrated in *Principia Mathematica*, that "when we say 'ϕx is a proposition', we mean to state something which is true for every possible value of x, though we do not decide what value x is to have." (*Principia Mathematica*, i p.42) The salient phrasing here is "true for every possible value of x", where what is presumed is all the resulting propositions, and not a working through (a ranging over) every element in the domain to see whether or not the result is a proposition. Because to do so one would first have to decide all the values that x can have, and this is what Russell here, in accord with his technical approach, specifically refuses.

Variable propositions certainly look like open sentences, containing free variables, but this appearance has to be treated with care. In *Principia Mathematica* Russell and Whitehead distinguish *real* and *apparent* variables:

> When we are considering or asserting ϕx, the variable x is called a "real variable" ... The x which occurs in "$(x).\phi x$" or "$(\exists x).\phi x$" is called (following Peano) an "apparent variable". (*Principia Mathematica*, i pp.18, 17)

This certainly looks like the familiar distinction between free and bound variables, but one should be wary. Wittgenstein recurs to this several times in his correspondence with Russell:

> The prop[ositions] of logic contain only APPARENT variables. (L.W. to B.R., June 1912)
> All my progress comes out of the idea that the *indefinables* of logic are of the general kind ...and this again comes from the abolition of the real variable. (L.W. to B.R., 22 July 1913) (*Wittgenstein in Cambridge*, pp.30, 42)

This denial of real variables goes together, for Wittgenstein, with eschewing the notion of propositional function. This is because the notion of a real variable goes together with familiar notions of quantifiers ranging over well-defined individuals, and this is exactly the conception that he refuses. What looks like an open sentence with a real or free variable is not what it might appear to be, because Wittgenstein assumes Russell's technical approach, that we are dealing only with well-defined functions all of whose values are given in advance, and then develops it further, going beyond Russell's conflicted position. On this basis we begin with *propositions*, and then seek variability *within* propositions, this being the force of 3.314:

> An expression has meaning only in a proposition. All variables can be construed as propositional variables. (Even variable names.) (3.314)

It is significant that 3.314 is a comment on the context principle, that "Only propositions have sense; only in the nexus of a proposition does a name have meaning" (3.3). The key point is that what looks like an open sentence, or a propositional function, is not, in the *Tractatus*, an open sentence or a propositional function in the now familiar sense, and to construe the Tractarian function fx of 5.501 as a Russellian propositional function is, I suggest, to go astray from the beginning.

4.4 Top-down and bottom-up

Hanks suggests, as a closing speculative remark in a 2014 paper, that Wittgenstein "held a bottom-up view from *Notes on Logic* through to the composition of the *Notebooks 1914–16*, his view was in transition as he compiled the *Prototractatus*, and that he took a decisive turn in favour of the top-down view in the final composition of the *Tractatus*." (Hanks 2014, p.12) The bottom-up approach correlates names with objects and then assembles propositions therefrom, whereas the top-down approach reverses this, invoking the context principle, that "a name can *only* have a meaning in the context of a proposition" (Hanks 2014, p.10). Hanks also notes Kremer's observation, that the context principle is elevated in the numbering hierarchy, from 3.202 in the *Prototractatus* to 3.3 in the *Tractatus*:

> The [context principle], as *Prototractatus* 3.202, appears as simply one point among many about propositions and propositional signs—which are often not clearly distinguished. In *Tractatus*, the [context principle] heads up the largest group of remarks in the 3's, numbering thirty-three in all, almost half of the seventy-four remarks in the 3's, and more than twice as many as any other major group. (Kremer 1997, p.90)

It is noticeable that of the propositions in the *Tractatus* setting up the notion of propositional variable, as well as being grouped as remarks on the context principle, four of them—3.311–3.314—have no counterpart in the *Prototractatus*. The counterpart to 3.315 is 4.102274 in the *Prototractatus*, with 3.316 and 3.317 spread out over 5.004–5.005 in the earlier work. This is indicative of the shift Hanks notes, a decisive rejection of Russell's philosophical approach to propositions, in favour of one more akin to his technical approach.

With respect to 3.31–3.313 Zalabardo argues, with reference to Palmer and Morris[6], that "these passages put forward a picture in which propositions are not regarded as complex entities, produced by the combination of more simple items. They are treated instead as basic units." (Zalabardo 2015, p.112) On this basis, "the fundamental semantic relation [is] a mapping of (true) propositions onto facts" (Zalabardo 2015, p.139), and is not a

[6]Palmer 1996, Morris 2008.

matter of correlating names and objects. This is a top-down view in spades, and there is much to recommend it. However, as noted at p.63, it overshoots the mark because one can have too much unity, such that one cannot make sufficient sense of the idea that an elementary proposition is a nexus, a concatenation, of names (4.22). Zalabardo argues that objects should be regarded as "features that states of affairs share with one another" (Zalabardo 2015, p.117), and also that "It is perfectly possible to hold that propositions are indivisible units while maintaining at the same time that their ability to represent the world requires that they share with other propositions the kind of features that I have identified with expressions" (Zalabardo 2015, p.125). But this leads Zalabardo into an account that regards us as imposing logico-syntactic structuring onto what there is, leading to a radical indeterminacy:

> We cannot claim that our inferential inclinations define the truth-functional structure of our everyday propositions if our inferential inclinations are compatible with radically different hypotheses concerning truth-functional structure. (Zalabardo 2015, pp.214–5)

Such radically different hypotheses can only arise from disagreement over common or shared features considered wholly independently of what there is, and this shows that Zalabardo's approach is excessively top-down. When Wittgenstein says, "A proposition is articulate" (3.141, 3.251) he does not mean that this is wholly up to us; "names have meaning and elementary propositions sense", because they stand in a "connexion with the world" (6.124).

In other words, the top-down strategy exemplifies the *sui generis* approach, risking losing sight of application (5.557). This comes out particularly well in Zalabardo's anxieties over the possibility of radically different hypotheses because on the top-down *sui generis* approach objects become what we posit them to be. They cease to exercise the constraint noted by Pears, that an object, somehow, "takes over and controls the logical behaviour of [its] name, causing it to make sense in some sentential contexts but not in others" (Pears 1987, p.88). In the absence of such constraints Zalabardo has to opt for metalogical constraints, that our logical/syntactical employment of language has to meet conditions of consistency and coherence, that is, not contradicting ourselves, and adhering to our inferential

inclinations in asserting consequences (cf. Zalabardo 2015, pp.214–6). But this requires a metaperspective that is not available to the universalist in general, and most certainly not to the Wittgenstein of the *Tractatus* in particular. At the end of his discussion of his contextual definition/common or shared features approach Zalabardo says, "on this construal Wittgenstein's approach is intrinsically unsatisfactory as well as incompatible with other aspects of his overall position" (Zalabardo 2015, p.216). The reason is, I think, that while in the *Tractatus* Wittgenstein leans towards an approach that regards propositions as unities in which distinctions can be drawn, rather than as assemblages of previously given constituents, Zalabardo takes this too far. One has to be able to do justice to the articulacy of propositions and the remarks about objects and naming, and to the constraints exercised by what there is on our thinking and reasoning. But this, of course, exacerbates all the issues around objects and naming.

4.5 Tractarian prototypes I

In getting beneath the level of the propositional, that is, within elementary propositions, Wittgenstein's key notion is that of *prototype* (*Urbild*). This can be seen at 3.315, where he talks of variable propositions as resulting from turning a constituent of a proposition into a variable; substituting a variable name for a name, yielding a Tractarian function fx out of a proposition fa.[7] For present purposes the long-standing argument over whether objects include properties and relations can be set aside, on the basis that an elementary proposition is a concatenation of names. For logical purposes *name* is a single category. In concluding 3.315 Wittgenstein says, "It [i.e., a variable proposition] corresponds to a logical form—a logical prototype [*einem logischen Urbild*]." So what is a prototype?

The first difficulty is etymological, as Black observes:

> *Urbild* is perhaps an unfortunate neologism. For the *Urbild* is not a picture of the form in the sense of Wittgenstein's 'picture theory', but rather embodies the form; it shows what every proposition that is an

[7] Given that a Tractarian elementary proposition is a concatenation of names, for example *cdkrrpsa*, it is clearly more convenient to treat *cdkrrps* as f, that is, a Tractarian function, as p.103.

> instance of the *Urbild* has in common with its sense. (Black 1964, p.126)

Black is right that a prototype is a *common characteristic*, something that all instances have in common. Wittgenstein calls such characteristics *formal properties* (4.126), a formal property being an instance of a formal concept. This ties in with the notion of *variable* in the *Tractatus*, because "Every variable is the sign for a formal concept." (4.1271) We have a tight circle, of formal concept, (propositional) variable, prototype, all instanced by propositions. The crux is that a variable standing for a formal concept does not *range over* a domain, looking for entities that can be substituted for it to produce a value (in the most salient context here, ranging over individuals such that a propositional function becomes a proposition). A variable shows a common characteristic, so in the Tractarian function fx the formal concept x *shows* that the instances fa, fb, and so on, are propositional. A variable for a formal concept shows a common characteristic, it is not an unqualified gap in need of satisfaction.

Where matters go awry is in assimilating the notion of *Urbild* with the picture theory of the proposition (*Bildtheorie*). Consequently Anscombe's and von Wright's usual translation of *Urbild* in the *Notebooks 1914–16* as *proto-picture* is unfortunate.[8] As Black points out, Anscombe's brief mention of *Urbild* occurs in an extended discussion of the picture theory; "Anscombe identifies the proto-picture with the 'picture', without individual correlations (Anscombe 1971, p.67)." (Black 1964, p.126) At this point in her text Anscombe discusses how a proposition can function as a picture of a situation, for example, of two men fencing. She is fully alert to the top-down nature of the *Tractatus*:

> The picture of two men fencing was intelligible as a picture, without our making any correlations of the figures with individual men. We might compare to this picture, without individual correlations, what Wittgenstein at 3.24 calls 'the proto-picture' occurring in the generality notion: the 'xRy', for example, in '$(Ex)(y)xRy$'. (Anscombe 1971, p.67)

By usually translating *Urbild* as *proto-picture*, and associating it almost entirely with the picture theory of the proposition Anscombe succeeded, I

[8] Not in all cases; it is translated as *prototype* at *Notebooks 1914–16*, p.29.

think, in largely overlooking the significance of this notion, because what Wittgenstein is after is what is common to the propositional *per se*, and not just what is bound up with the notion of the propositional as pictorial. If there is a general approach to this in the literature it is to see the notion of *prototype* as a Russellian hangover, to be acknowledged, at best, before moving swiftly on. This is typified by Klagge, who asserts that "What Wittgenstein calls a 'logical form' or a 'logical prototype' Russell calls 'the form of the proposition'." (Klagge 2022, p.92)

Quite simply, if one thinks that a Tractarian prototype (*Urbild*) is a Russellian logical form, then one has already gone wrong. A Russellian logical form is "the way in which the constituents are combined in a complex" (*Theory of Knowledge*, p.98) but, as the schema $U(S, x, R, y, \gamma)$ set out therein shows, the logical form γ is in some sense a constituent of the proposition as understood (as assembled in thought and in judgement) by the subject S. Wittgenstein vehemently rejected this whole approach, absorbing Russell's logical forms into the forms of objects (p.34). In short, however much a Tractarian function might look like a Russellian propositional function—whatever use Wittgenstein might make of such notation as xRy (e.g. 4.1252, 4.1273), and however tempting it might be to assimilate a Tractarian *Urbild* to a Russellian propositional form—what Wittgenstein was doing is not what Russell was doing. The temptation to find Russell in the *Tractatus* is strong, though: "In contrast to [Russell] Wittgenstein offers no theory whatsoever about the kind of entity which a propositional function is." (Ishiguro 1981, p.51) This is hardly surprising, given that Wittgenstein eschews the notion altogether. A Tractarian prototype is not a Russellian logical form, but given the notational similarities it is reasonable to ask how they differ.

The critical difference is this. A Tractarian prototype is a *formal concept*, characteristic of, or immanent to, a class of instances. It is not something external to or independent of a proposition, such that pre-given constituents are combined by means thereof into a proposition. So Wittgenstein owes his reader an account of how the formal concept that characterises propositions is a prototype, and thus how we get the notion of *propositional variable*.

4.6 General form of the proposition

This is where the notion of the general form of the proposition, stated at 6, comes to the fore, bearing in mind Wittgenstein's curious remark at 4.53, that "The general propositional form is a variable." What needs to be shown is what Wittgenstein means by this, and by the closing remarks of 4.126:

> [T]he sign for the characteristics of a formal concept is a distinctive feature of all symbols whose meanings fall under the concept. So the expression for a formal concept is a propositional variable in which this distinctive feature alone is constant. (4.126)

That "proposition" is a formal concept is shown by the fact that one cannot meaningfully say "p is a proposition". A proposition can only show that it is such by being used to express a sense, that is, a way in which things—objects—can stand to one another, as per the concluding remark of 4.5, that "The general form of a proposition is: This is how things stand."

Of 4.5, Black says, "[T]he form of words offered is cryptic to the point of unintelligibility ... We might try saying, more ponderously: The essence of a proposition is its capacity to say something about how matters are in reality." (Black 1964, pp.236, 237) But this over-eggs the pudding. The remark at 4.5 is not intended as a profound insight, it is, rather, quite the opposite. Wittgenstein's aim is to identify a characteristic that is common to and distinctive of the propositional, so what is on offer is minimalistic to the point almost of banality. What do we do with propositions? We use them to say what is, or is not, but might have been, the case. One can say, *this* is how things stand, where the proposition deployed to say *this* is true, if they do so stand, and false, if they don't. And that's all there is to it.

The notion of *propositional variable* is effectively presented in 6:

> The general form of a truth-function is $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$.
> This is the general form of a proposition. (6)

In other words, as per 4.53—"The general propositional form is a variable" —$[\bar{p}, \bar{\xi}, N(\bar{\xi})]$ is a variable. This, Wittgenstein tells us, is the common characteristic of the propositional.

It is striking, first, that Wittgenstein implies an equivalence between *proposition* and *truth-function*. The reason is his idiosyncratic terminology.

Here, as elsewhere (cf. p.69), he takes over terms from Frege, or from Russell and Whitehead, and gives them a twist. So a Tractarian function is not a mapping or a correlation that sends names to propositions, and a Tractarian truth-function is not a mapping from propositions to more (or less) complex propositions. For the latter, at least, Wittgenstein has the notion of *operation*. Broadly speaking in the *Tractatus* an operation is what you *do*, a function is what you *get*. Sundholm asserts that "Several *ion*-words show similar ambiguities in the *Tractatus*. *Funktion* is ambiguous between, on the one hand, 'function of' and, on the other, the Fregean unsaturated notion." (Sundholm 1992, p.60n.7) An elementary proposition is the primitive or degenerate case (a truth-function of itself, as 5). Complex propositions are truth-functions, the result of the application of a truth-operation.

Wittgenstein expresses this notion of function at 3.318, that "Like Frege and Russell I construe a proposition as a function of the expressions contained in it." A proposition is a function of its constituent expressions, whether it be names combining to form elementary propositions (a function of names), or whether it be elementary propositions combining by means of truth-operations to form complex propositions (a truth-function of elementary propositions). The latter process can be iterated without limit, resulting in more and more complex propositions, because an operation can take one of its own results as its base, whereas a function cannot be its own argument (5.251).

Wittgenstein offers little explanation of the symbols used in 6, a deficit rectified by Russell in his introduction (Russell 1922a, p.xv). $\bar{p}$ is the totality (5.5561) of all elementary propositions, $\bar{\xi}$ a selection therefrom, N the joint negation truth-operator, so $N(\bar{\xi})$ is the result of applying N to the members of $\bar{\xi}$. After a first application of N $\bar{\xi}$ can of course include complex propositions, as 5.251. According to Russell, "The symbol [i.e. the whole expression $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$] is intended to describe a process by the help of which, given the elementary propositions, all others can be manufactured." (Russell 1922a, p.xv) What Russell says is correct. The [...] expression in 6 does show how to manufacture complex propositions as truth-functions of less complex/elementary propositions. N is *the* truth-operation, and can be applied iteratively without limit. But this is only part of the story and not, for Wittgenstein, I think, the most important part. Because what Russell does not explain is how the [...] expression in 6 is a *variable*.

4.7 What is and what is not variable

This is difficult to grasp because, confronted with the notion of a variable, we naturally look for a range of values for the variable (or a range of substitution instances). But this is not how Wittgenstein thinks of variability. For him, the notion of variability is bound up with a range of instances of a common characteristic, of a formal concept. What can be thought of as variable is what will be called the *realisations* of a common characteristic. In the case of propositions, a proposition is a realisation of the common characteristic—the general form—of the proposition, in that it (the proposition) says, this is how things stand. Hence "To give values for a propositional variable is *to give the propositions* whose common characteristic the variable is." (3.317, emphasis in the original) Every proposition is a realisation of the general form, in the sense that the general form (the [...] expression in 6) can be thought of as capturing the constructional history of the proposition.

What Wittgenstein has in mind is his presumption that all propositions are given in advance. It isn't difficult to trace this back through the claims that all objects are given in advance ("objects are what is unalterable and subsistent", 2.0271) and hence "at the same time all *possible* states of affairs are also given" (2.0124). The totalities of names and elementary propositions follow immediately (4.52, 5.5262, 5.5561) and given 6 we get all possible truth-functional combinations (that is, all complex and all everyday propositions). Since all of this is given any concerns over type theory, impredicativity, and reducibility, take care of themselves (and this is reflected in a correct notation). These are not small presumptions but they can all be found in the text.

It follows that what varies, for Wittgenstein, is the ways in which a common characteristic, a prototype, is realised. The ostensible blandness of 4.5 is not because it is a triviality but because it is the statement of a common characteristic, the lowest common denominator. 6 can be thought of, as it was by Russell, as a way of manufacturing complex propositions, but what Wittgenstein really intends 6 to serve as is a statement of what is fundamental to the propositional, this being truth-functionality (where an elementary proposition is a truth-function of itself, as 5). What one might choose to call the "range" of a variable is the class of instances of a formal concept; the variable—the prototype—is immanent, realised by the instances.

We can now see the force of 3.312, "in this form the expression will be *constant* and everything else *variable*." This can be rephrased as "the expression of the general form of the common characteristic, the prototype, is *constant*, and everything else *variable*." When Wittgenstein says, "The general propositional form is a variable" (4.53), what he means is, however paradoxical it may sound, that the variable is what stays fixed, variation arising from the realisations of the common characteristic. And to understand the *Tractatus* one has to appreciate not just the promotion of the context principle as compared to the *Prototractatus*, and the rearrangement of the 3.1s between the former and the latter, but also that the critically significant 3.311–3.314 in the *Tractatus* have no counterpart in the *Prototractatus*. Wittgenstein moves a long way in a short period of time.

4.8 Definition by abstraction, and [...] expressions

It might seem that Wittgenstein's approach, deploying familiar notation with unfamiliar intent, is aberrant. What lies in the background is, I think, the notion of *definition by abstraction*:

> [W]e can say that a definition by abstraction takes its start from an equivalence relation $\sim$ over a class of entities ... The equivalence relation can be thought of as capturing a common feature, a similarity, among the entities standing in the $\sim$ relation. One then ignores any individual feature of the objects except whether they stand in the $\sim$ relation thereby 'abstracting' from any other feature distinguishing the elements so related. (Mancosu 2016, p.14)

Wittgenstein would have been familiar with this approach from Frege's use of Hume's principle in his account of number in *Foundations of Arithmetic*, and Russell's definition of cardinal number:

> [T]his method ... [which] applies to all the cases in which Peano employs definition by abstraction ... is, to define as the number of the class the class of all classes similar to the given class. (*Principles of Mathematics*, §111)

The approach in the *Tractatus* is to show what it is that all the instances of a formal concept have in common. If pressed as to how, I think the

answer would have been by analysis, rather than by abstraction, because the latter tends to come freighted with psychological baggage.[9] Analysis reveals the "real" logical form of a proposition (cf. 4.0031), and in the end, the general form, the common characteristic of all propositions. Once analysis has stripped away "the outward form of the clothing" (4.002) we are left with a common characteristic, and on this basis we can think of propositions as forming an equivalence class. This is why there is no need for the Russellian prefix "$p \supset p$" (5.5351), because what is propositional shows itself to be so, by realising the general form.

Looking at Wittgenstein's other uses of [...] expressions helps to make this clearer. Such expressions can be found in 5.2522, 6.01, and 6.03, as well as in 6, quoted at p.134:

> $[a, x, 0'x]$ (general term of a series of forms) (5.2522)
>
> $[\bar{\xi}, N(\bar{\xi})]'(\bar{\eta})$ $(= [\bar{\eta}, \bar{\xi}, N(\bar{\xi})])$ (general form of an operation) (6.01)
>
> $[0, \xi, \xi + 1]$ (general form of an integer) (6.03)[10]

Sundholm criticises Wittgenstein on the basis that whereas:

> Wittgenstein is at pains [in 5.2522] to emphasise the homogeneity of arguments ... in 6, on the other hand, homogeneity is violated in the most blatant way: the third argument-place is taken by a result of an application of the joint negation operator N to a certain range of

[9] Cf. Adamson 1901, Dummett 1991, pp.50–2.

[10] It has been pointed out to me that Ramsey translates „ganzen Zahl" as "cardinal number" (Ogden translation), but I think there are good reasons to resist this. First, where Wittgenstein intends cardinal number, at 5.02, *Notebooks 1914–16*, p.11, and *Prototractatus*, 5.012, the expression he uses is „Kardinalzahlen". Second, in one of the very first responses to the work, Russell wrote "You only get finite ordinals. You deny classes, so cardinals collapse. What happens to $\aleph_0$?" (B.R. to L.W. 13 August 1919, *Wittgenstein in Cambridge*, pp.96–7). In eschewing identity it is unclear how Wittgenstein can have the familiar conception of cardinality derived from Cantor, as a double abstraction from *Menge* (Cantor 1895, §1), or the Frege/Russell notions based on equinumerosity. As Landini points out, Wittgenstein didn't answer this (Landini 2007, p.187), and as he also points out, "Wittgenstein is offering a radically eliminativistic approach to logicism which adopts the notion of finite (ordinal) number—more exactly, the notion of recursion (powers of an operation) as a primitive formal notion." (Landini 2007, p.183–4) That Wittgenstein's account deals only with positive integers is implicit in the [...] expression, with 0 as the first term of a series generated by the operation +; "Wittgenstein speaks of the 'general form of an integer' (*allgemeine Form der ganzen Zahl*) (6.03), clearly meaning 'non-negative integer'. As he does not deal with negative numbers, we speak more simply here and afterwards of natural numbers." (Frascolla 1994, p.174n.2)

> *Sätze*, that is, by a *Satz*, whereas the other two places are taken by ranges of *Sätze*, whether elementary or not. It is incumbent on any *Tractatus* interpretation to try to make sense of this deviant use of the square brackets. (Sundholm 1992, p.66)

But this is not, I think, an objection that would have concerned Wittgenstein. What he is trying to do is identify common characteristics of instances of formal concepts. Uses of [...] expressions are intended as elucidatory, and are not comprehensive, as while the "list" of formal concepts at 4.1272 mentions *complex*, *fact*, *function*, and *number*, and clearly applies to *object* and to *proposition*, in most cases not only are we not given [...] expressions, it is hard to see what expression there could be. For elementary propositions we might have $[\bar{x}, \bar{y}, C'(\bar{y})]$, with $\bar{x}$ as the totality of names for objects (cf. 5.556), $\bar{y}$ a selection from these, and C the concatenation relation of 4.22. But this won't work in any useful sense because whereas N can be applied iteratively in unrestricted fashion to propositions, C cannot be so applied to objects, because objects have forms and as such unrestricted combinatorialism does not apply (2.0123).

What is more difficult in the propositional context is the talk in 5.2522 of a *series* of forms (*Formenreihe*); "I call a series that is ordered by an *internal* relation a series of forms (*Formenreihen*)." (4.1252) This is clearly Wittgenstein's technique to get out numbers, as a series (4.1252, 4.1273), although the consequence of the [...] expression in 6.03 is a limitation to the finite.[11] But what is programmatically unclear is what analogue there is for *successor* or *ancestral* when it comes to propositions; in what sense does the successive application of N yield a formal series, where the terms of the series are internally related to one another? What does Wittgenstein mean when he says in 6.001 that "every proposition is a result of successive applications" of $N'(\bar{\xi})$?

That every proposition can be so generated is true, assuming expressive completeness, for the sake of the argument, provided one accepts that *successive* means that there is *some* point in a process of repeated application to selections of propositions as input at which one will get a particular proposition as output. There is no obvious significance to the order in which

[11] Cf. Russell's remarks quoted in the previous footnote, and Russell 1936, p.323. Ramsey points out, "As it stands this is obviously a ridiculously narrow view of mathematics, and confines it to simple arithmetic." (Ramsey 1925a, p.180)

propositions appear, and no obvious sense in which propositions can be ordered in terms of "ancestor of" or "successor of".[12] One could order propositions into disjoint classes by complexity (number of constituent elementary propositions, number of applications of N), or by arithmetisation, but it is hard to see why one would do so.

What is true is that there is a harmony in the Tractarian system between what can be constructed by successive applications of N and what can be deconstructed by inference, as these operate harmoniously as inverse operations, but this is more or less trivial. It is hard not to think that only propositions and numbers have general forms in any interesting sense, and that only numbers form a series in any interesting sense.

4.9 Functions in 5.501

In discussing an argument put forward in Diamond 1985, and with respect to 5.501, Sullivan claims that the argument he (Sullivan) has given "considers only one possible style for a variable ranging over propositions, when the *Tractatus* recognises three." (Sullivan 2004, p.51) It is, I hope, by now clear that there is no such thing in the *Tractatus* as a *variable ranging over propositions*. The general form is a variable that is *realised by* propositions, there is no *ranging over*. With this in mind, we can turn to Sullivan's discussion of the relevant part of 5.501:

> We may distinguish three kinds of description: 1. Direct enumeration ... 2. Giving a function fx, whose values for all values of x are the propositions to be described. 3. Giving a formal law, according to which those propositions are constructed. In this case the [values] are all the terms of a formal series (5.501). (Sullivan 2004, p.51)

This is the Ogden translation, Sullivan replacing the phrase "terms of the expression in brackets" with the word "values". But what is going on in (1)–(3) is not, I think, three different ways in which a variable ranges over propositions. Rather we get three different ways in which a selection $\bar{\xi}$ of propositions can be made from the totality of propositions, such that

[12] "This cannot be the case, since, for example, $p \wedge q$ has more than one immediate predecessor (either p or q) so there cannot be a total ordering." (Marion 1998, p.26) The point is also made at Cheung 2000, p.253, in opposition to Fogelin and Anscombe, and at Landini 2007, p.175–6.

this selection can be the input to N. The wording is difficult, but it becomes a great deal more difficult if one reads it à la Russell rather than à la Wittgenstein.

Since we are dealing with propositions, all of them are covered by 6; they all realise the general form of the proposition. It is a matter now of choosing a selection out of the (at most denumerable) totality of propositions, elementary and complex, as input to N. If we get the selection by (1), then there is no difficulty. If it is given by means of a Tractarian function fx (2), all of whose values—propositions—are given in advance, then we get all propositions that realise the prototype fx. We don't have a propositional function that ranges over values, rather we have a selection from the totality of propositions, whose members realise a prototype. If it is objected that we can't do this, the answer is that we can, in principle, at least, if we can just find a way of emulating the logical god of 5.123. In the case of (3) we have a [...] expression, where this expression is the variable; it is a specification of the common characteristic, the prototype, realised by a selection from the totality of propositions.

The point is that we are not looking for three different styles of variable that range over, rather we are looking for three different ways in which a variable, a common characteristic that is more discriminating than the general form, is realised (the general form of course being wholly indiscriminate as it is realised by all propositions). This is potentially awkward for (1), because the only common characteristic may be *appearing in this direct enumeration*. Stipulation may be enough to carry the day. This is, perhaps, what Wittgenstein has in mind in his talk of "stipulat[ing the] values for a propositional variable" at 3.316 and 3.317. It is noticeable that in the *Prototractatus* such stipulation is discussed at 5.004ff, this series of propositions including the analogue of 5.501 (5.00531 in the *Prototractatus*). The implied link is lost in rearrangement.

In the case of (2), in principle at least, we need to know what prototypes there are. However, "we are ... unable to give the composition of elementary propositions" (5.55), but this simply because we do not know *a priori* how many different prototypes there are. We have, however, "some concept of elementary propositions quite apart from their particular logical forms" (5.555), because we possess "a system by which we can create symbols" (5.555), that is, to create whatever symbols we need for however many pro-

totypes there are (fx, ϕxy, and so on). The members of (2) are given by realising a particular prototype, this picking out a selection from the totality of elementary propositions. In the case of (3) we have of course a [. . .] expression.

The argument of Diamond's that Sullivan addresses is a strategy of replacement, that we will achieve philosophical insight by replacing philosophical vocabulary "by a notation designed to make logical similarities and differences clear." (Diamond 1985, p.183) But in the case of, at least, *proposition* this is not, I think, what Wittgenstein is doing. Rather in saying that a proposition is a truth-function, that the common characteristic of the propositional is a matter of *This is how things stand*, what is on offer for this common characteristic is a definition by abstraction. Once we see that this is so, we no longer need to ask whether or not some candidate expression is a proposition; rather we ask, do things stand thus-and-so, or don't they? We are past the point of sense, because what we are dealing with expresses a sense, and have already advanced to the question of truth-value. If we are unable to make sense then we can only conclude that what we are dealing with is not propositional; appearances notwithstanding, it is senseless, or nonsense.

In the *Tractatus*, then, Wittgenstein has a mitigated top-down approach that assumes that all objects and thus all propositions, elementary, complex, and everyday, are given in advance. What he then sets out to show is what it is that these have in common, what their common characteristic is. This is the general form of the proposition, and it is a variable in terms of definition by abstraction. What is propositional is what realises the general form. There is no Russellian apparatus of propositional functions, types, significance, or a bottom-up strategy generally to bring together disparate elements into propositions. Superficial similarities between Wittgenstein's and Russell's work conceal deep differences.

Chapter 5

Logical Space

What has been addressed so far has involved, in the main, logic as a propositional calculus of elementary propositions, without addressing modal considerations beyond possibility as it relates to the forms of objects, that is, their combinatorial possibilities into states of affairs (and concomitantly of names into elementary propositions). But what has been said so far has, in fact, already involved logical space. Because "logical space" *is* the domain of possibility, relating first to the forms of objects, and second, to the combination, however it occurs, of states of affairs into situations. In parallel there is the combination of names to constitute elementary propositions, and of elementary propositions to be formed into truth-functions (complex/everyday propositions) by truth-operations. After this logical space relates to possible worlds, as totalities of obtaining states of affairs, and concomitantly, as totalities of true elementary propositions; thereby embracing whatever truth-functional combinations can be formed out of the latter.

As a space of possibilities, logical space does not *exist* in any sense over and above what there is, the totality of objects. This is a different way of stating that reality (*die Wirklichkeit*) is nothing over and above the world (*die Welt*) (cf. p.86). What is possible should not be reified (cf. p.44); the possible can be said to "exist" only as a corollary of the existence of objects, and of their having forms.

5.1 Objects and their forms

There is a persistent strand in Tractarian commentary that objects should be thought of not as particulars but as, in some sense, universals. Wedin says, "My claim is that Tractarian objects must be repeatable properties ... they must be items capable of occurring in distinct states of affairs without prejudice to the ontological independence of the states of affairs they occur in." (Wedin 1990, p.59)[1] That is, the Independence Thesis (the independence of elementary propositions) needs to be met, and an account given of how, ostensibly, the same "repeatable property" can occur in differing states of affairs. The reading given meets these conditions while broadly treating objects nominalistically, as *formally* uniform, and contentually (empirically) categorisable.[2]

I do not want to argue that thinking about objects in terms of universals or unsaturatedness is wholly wrong. The issue is of getting the needed distinctions in exactly the right places. It is fine to say that objects have formal properties, in the sense that they have forms. Logically, objects are uniform. In terms of content there is a range of forms, but this is an empirical affair.

In terms of nominalism and realism, on the present reading objects are treated nominalistically on the basis that material properties can be regarded as universals, arising from combinations of objects according to their form (as functions of the forms of objects). Objects can thus fulfil the function commonly ascribed to universals, while remaining particular. What objects are not is haecceities. In terms of idealism and realism, on the present reading objects are regarded realistically as persistent entities that exist independently of us.

If one wants to go beyond the points made here—that there is nothing more to an object than having a form, a capacity to combine with other objects into states of affairs—then such an enquiry will, I think, take one

[1] Frascolla's position is relevantly similar, with objects as "repeatable phenomenal qualities" akin to Goodman's qualia (Frascolla 2007, pp.72, 78). There is also a tendency to invoke Frege's object/concept, saturated/unsaturated distinction but, as the Hintikkas claim, "[T]he usual identification of the 'objects' in Wittgenstein's *Tractatus* with individuals (saturated entities) is not only mistaken, but diametrically wrong. It is less misleading to think of Wittgenstein's 'objects' as all being functions than to think of them all as individuals." (Hintikka and Hintikka 1986, p.42) Cf. Ishiguro 2001, p.28.

[2] Ramsey's remark, "It is clear that in reality there are not individuals and universals but just objects of different forms" (Ramsey 1991, p.144) is I think exactly right, if it was written with the *Tractatus* in mind.

outside of logic. One could proceed to a metaphysical treatment of objects, but I do not think this is helpful. The trajectory of the present reading is towards a characterisation of objects in scientific terms, in Chapter 8, and not a metaphysical treatment, because Wittgenstein was not, I think, particularly interested in much beyond the logical. In this context, Pears says:

> Wittgenstein was studiously agnostic about any features of objects that would make no difference to the occurrence of their names in logically independent elementary propositions. That left a vacuum which commentators felt obliged to fill with dogmatic interpretations, and so there was a proliferation of exegetes offering to unlock the secrets of the ontology of the *Tractatus*. (Pears 1987, pp.91–2)

Johnston has the right line, I think, although his conclusion may be a little strong, in saying:

> To argue whether Wittgenstein intended us to take Tractarian objects to be particulars, or to include relations, is a mistake. The Tractarian Wittgenstein does not think that any such would-be logical category terms as 'particular' or 'relation' has *a priori* application. A suggestion made in advance of the pursuit of truth-functional analysis that Tractarian objects include, or do not include, relations is something Wittgenstein would have seen as "mere playing with words." (Johnston 2009, pp.156–7)

Johnston points out that this is not a novel proposal, referring to Pears (Pears 1987, pp.137, 139), and to Kenny (Kenny 1984, p.16). He also says that this line of thinking can be traced back to Ramsey, in particular, to his paper 'Universals'. Attributing credit to Wittgenstein (Ramsey 1925b, p.30), Ramsey says:

> The truth is that we can know nothing whatever about the forms of elementary propositions; we do not know whether some or all objects can occur in more than one form of atomic proposition; and there is obviously no way of deciding any such question. (Ramsey 1925b, p.29)

Ramsey makes it clear earlier in the paper that he had Tractarian elementary propositions in mind when, after discussing theories proposed by

Russell and by Johnson, he says; "Lastly there is Mr Wittgenstein's theory that neither is there a copula, nor one especially connected constituent, but that, as he expresses it, the objects hang one in another like the links of a chain." (Ramsey 1925b, p.17, referring to 2.03) The view Ramsey proposes (*op. cit* p.28) follows that expressed in the *Tractatus*.

On the basis of the text *all* we can say about states of affairs is that they arise from combinations of objects, that they are a function of the forms of objects. And "If all objects are given, then at the same time all *possible* states of affairs are also given" (2.0124); this is an immediate corollary. About all one can draw from this is that not all possible states of affairs occur simultaneously, because the configuration of objects into states of affairs "is changing and unstable." (2.0271) If all possible states of affairs could occur simultaneously, then there would be nothing left of possibility, and it is essential to the *Tractatus* that there is *a* difference between reality (the realm of possibility) and the world (what there is, the facts) (cf. Goldfarb's "contrastive view of meaning", discussed at p.45). Put loosely, the difference is that for any possible world there are states of affairs that do not obtain in that world; there are actually false as well as actually true elementary propositions.

Doubts may arise concerning use of the term *agnosticism*, implying that there is something to know that we do not and perhaps cannot know. But this is not the case. It is true that we do not know how to carry out an analysis down to objects, and that Wittgenstein doesn't tell us how to do this. We are offered little more than a claim that there must be simple objects because propositions are true or false, irrespective of whether we know or can know what their truth-values are (the *determinacy of sense* argument, as 4.023, that a proposition "must describe reality completely", taken together with the *substance argument* of 2.021–2.0212, that there is something and not nothing).

From a *formal* point of view, though, there is *nothing* that we do not know about objects. Speculating about whether objects are properties, relations, particulars, is beside the point. Objects have forms, and they can combine according to their forms, the result of such combination being the obtaining of states of affairs. Logically, this is all there is to say. Beyond this lies the empirical, that is, the material properties produced by combination (2.0231).

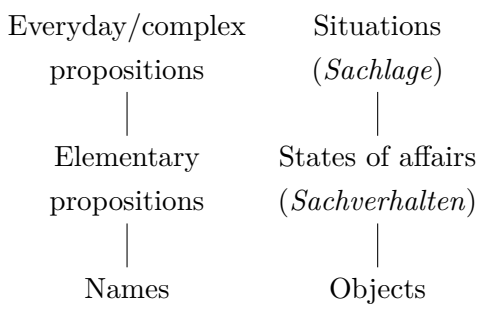
5.2 Facts

The notion of logical space is somewhat controversial[3]; it is, I think, best taken in stages, beginning with the combining of objects into states of affairs/names into elementary propositions, followed by the combining of states of affairs into situations/elementary propositions into complex/everyday propositions, and then by the distributions of truth-values across the totality of elementary propositions, corresponding to possible worlds.

The notion of logical space is introduced early, at 1.13; "The facts in logical space are the world." We have just been told at 1.1 that "The world is the totality of facts, not of things", so it might seem that facts are somehow in space *and* in logical space, where by "space" unqualified I mean, hereafter, physical space. This would be a mistake, because (as §1.12) facts are not *in* space, in the same way that objects, and states of affairs and situations as arising from combinations of objects, are in space. The critical distinction is between what is in space—objects—and what is "in" logical space—*that* objects are so arranged or ordered or structured. Space is the domain of what there is, whereas logical space addresses ways in which things can stand to one another, such ordering or structuring involving possibility and not constituting any addition of ontology. Logical space is best thought of as a mathematical space, as an abstract space with a domain of entities (in the present case, names, and elementary and complex propositions), with operations on the domain (in the present case, combination for names, and truth-operations for propositions).

Whatever an object is, a state of affairs is a combination of objects in accord with the forms of the objects involved, expressed by an elementary proposition. These are isomorphic—"abstract parallel structures", as Grayling puts it (p.51)—as for each object in the state of affairs there is a name in the elementary proposition that expresses it (this is the "logical (mathematical) multiplicity" of 4.04). Elementary propositions consist only of names, they do not have logical constants as constituents. This parallelism can be represented as follows:

[3]Whether or not logical space can be empty (Gale 1976, Cerezo 2005, pp.260ff, Cerezo 2012), and how to understand the bottom or all-$\perp$ row of the truth-table (cf. Page 1997).



To reiterate from §1.12 and §2.7, elementary propositions express sense, they are $\top\bot$. If an elementary proposition is actually true (is T)[4] then *it is a fact that* the state of affairs it expresses obtains (*besteht*). Such a fact is not any sort of thing or entity, it is not something over and above the objects that are combined into the state of affairs. If it is a fact that p, then the state of affairs p obtains, and the proposition that expresses p is true (T). If the state of affairs p does not obtain, then p is not a fact, and the proposition that expresses p is false (F). One might choose to call the non-obtaining of the state of affairs p a negative fact, but as noted (p.42), there is no need or reason to so do.

It is with Wittgenstein's claim that "All the propositions of our everyday language, just as they stand, are in perfect logical order" (5.5563) that the notion of *situation* (*Sachlage*) comes to the fore. At the level of sense, unrestricted truth-functional combination occurs, so we can fill in truth-tables with $\top$ and $\bot$ by rote. In reality, if a complex proposition is T, this is because there are elementary propositions that stand to one another in (a structure of) truth-functional relations, and the distribution of truth-values over these elementary propositions is such that the truth-value of the complex proposition is T. If we think of the elementary propositions that form the basis for a complex proposition as a selection S, then the truth-value of the complex proposition is a function of the truth-functional structure of the complex proposition, taken together with the distribution of truth-values over the elementary propositions in S. A situation is, simply, what is expressed by a complex proposition, a "complex" of states of affairs standing in suitable relations to one another. And just as a fact is an obtaining state of affairs, in the case of the obtaining of a situation, then *it is a fact that*

[4]From hereon the rider "actually" will be dropped, with T and F indicating that a proposition is actually true, or actually false.

the situation obtains. There is nothing over and above the states of affairs, because there is nothing over and above—a truth-functional "structure"—to be represented. This simply restates the familiar point that the logical constants are not representative (4.0312). There is a need only for three terms, because *elementary proposition* goes together with *state of affairs*, *complex proposition* with *situation*, and *fact* for *obtaining*, in either case.

5.3 Picture theory and the *Satzverband* problem

There is, though, a difficulty here, glossed over in the previous paragraph by the phrase "suitable relation". On this I find myself disagreeing with Zalabardo's identification of *a fact* with *an obtaining state of affairs*, and not as *that a state of affairs obtains* (Zalabardo 2015, §4.5 pp.115ff). This may look like a matter of wording, but the difficulty at stake comes out in his statement that "we can make a substantial case in favour of the view that a fact is *any obtaining truth-functional combination* of states of affairs" (Zalabardo 2015, pp.116–7, emphasis added) with, presumably, the obtaining of *a* state of affairs regarded as a limiting case, with no logical constants involved. The point at stake is that the obtaining of a *situation* does not follow immediately and unproblematically from the obtaining of the requisite states of affairs. It is not at all clear how *states of affairs* can stand in truth-functional relations to one another, because truth-functional relations obtain between *propositions*, and not between *states of affairs*. If one wants to assert, as Zalabardo does, that truth-functional relations obtain between *states of affairs*, then one needs to state what a truth-functional relation is in this context.

But Zalabardo doesn't do this, rather he turns to picturing. If truth-functional relations obtaining between states of affairs are termed *mode of combination*, then "if propositions are pictures it becomes very easy to understand why the mode of combination doesn't need to be explained to us. If the proposition is a fact, then in grasping the fact we grasp both its constituents and how they are combined with one another." (Zalabardo 2015, p.65) But while this looks straightforward for elementary propositions, the names of which stand for objects, it does not obviously extend to complex propositions, which contain symbols for truth-operations as, in some sense, constituents. Because as 4.0312 clearly states, these are not representative.

This raises a number of difficulties.

Wittgenstein goes on to assert that truth-functions are not what we would usually call relations; "It is self-evident that $\vee$, $\supset$, etc. are not relations in the sense in which right and left etc. are relations." (5.42) If we think of a proposition as a picture, and if we think of obtaining states of affairs as in any way chunky[5], then it is hard to see how we are to make sense of picture theory for complex/everyday propositions, because it looks to follow immediately that truth-functional relations between states of affairs are going to be cashed out pictorially in terms of spatial relations between chunky states of affairs. But, as Copi points out, with reference to 4.0312, "non elementary [complex] propositions contain elements which do not stand for objects, but all elements of pictures must stand for objects, therefore non elementary propositions cannot be pictures." (Copi 1958, p.171)[6] 2.131 says, "In a picture the elements of the picture are the representatives of objects," this underlining Copi's point, because this cannot obviously be extended beyond the elementary.

This is why, I think, Wittgenstein says that it is propositional *signs* that are facts (3.14; cf. §3.2), where a propositional sign is a proposition "in its projective relation to the world." (3.12) This "projective relation" obtains, I think, when the metaphysical subject—the self thought of in a non-psychological way—engages in "think[ing] the sense of the proposition" (3.11).[7] So there is *a* warrant for glossing "truth-functional relations" between states of affairs by means of picture theory. But this looks to go against 4.0312 and 5.42, because it looks to construe such relations as spatial relations.

The difficulty under consideration—what the logical constants as relations between elementary propositions actually do, given that they are not representative—is discussed at Cerezo 2005, pp.28, 184, 285, termed the *Satzverband question*, with particular reference to 4.221:

> It is obvious that the analysis of propositions must bring us to ele-

[5] Cf. Sullivan and Johnston 2018, pp.165ff.

[6] Copi also notes as a *reductio* argument what happens if one tries to picture $\sim\sim\sim p$, or $\sim\sim\sim\sim\sim p$, and so on (Copi 1958, p.172). Cf. Potter 2009, p.141.

[7] Pears and McGuinness translation emended as Hacker 1999, p.178, Malcolm 1986, p.73. Perhaps first pointed out by Schwyzer; "*das Denken des Satz-Sinnes* is not the thinking *of* the sense—as if we thought *about* it. It is the thinking-the-sense; for the sense is *what* we think." (Schwyzer 1962, p.282n.1)

> mentary propositions which consist of names in immediate combination. This raises the question how such combination [*Satzverband*] into propositions comes about. (4.221)

Cerezo argues that "picture theory cannot justify how the non-elementary propositions, conceived as compound [complex] propositions, depict" (Cerezo 2005, p.184):

> *Wittgenstein does not have any resources to account for the connection of the propositions in order to produce compound propositions. For this reason he wonders about how the* propositional connection (inter-propositional nexus, Satzverband) *comes about.* (4.221) (Cerezo 2005, p.184, emphasis in the original)

4.221 follows 4.22—"An elementary proposition consists of names. It is a nexus, a concatenation of names"—so it could be argued that 4.221 addresses connections between names *within* elementary propositions and not connections *between* elementary propositions into complex/everyday propositions.[8] Even if one thinks this, the *Satzverband* problem persists. Given that we have an account of structuring within elementary propositions in terms of concatenation, paralleling relations between objects in accord with their forms, Cerezo is right, I think, to point out that we have no such account in the case of relations between states of affairs as generating situations, and that *this* is the point addressed by 4.221 (so "which consist of names in immediate combination" merely qualifies "elementary propositions"). And given that the logical constants are not representative it is difficult to see how any such account can be given.

In discussing picturing, Zalabardo says, "the constituents of the picturing fact can be combined with one another, for example, spatially . . . when they are combined spatially, they can represent objects in the world as arranged in that same spatial combination." (Zalabardo 2015, p.48) He then discusses an example, of a bottle being behind a cup, and the isomorphic fact, of a pencil being heavier than a sharpener; the "modes of combination" exhibited by the "represented complex" (the state of affairs) and the "representing complex" (the proposition) "are the same: the way in which the constituents of the represented complex would have to be combined with one another is the way

[8] By Sullivan, at Sullivan 1990, p.77.

in which the constituents of the representing complex are actually combined with one another." (Zalabardo 2015, p.46)

This seems straightforward, but when Zalabardo later returns to it (Zalabardo 2015, pp.186ff) he goes about things differently, because first, bottles and cups and pencils and sharpeners cannot be simple, so they cannot be objects, and second, because independence (and thus exclusion) "rule[s] out treating, in general, something being heavier than something as a state of affairs: the pencil being heavier than the sharpener and the sharpener being heavier than the pencil can't both be states of affairs" (*op. cit.* p.187). It follows that while his earlier model (*op. cit.* pp.48ff) applies to elementary propositions (provided of course we are dealing with objects), it "cannot apply to everyday propositions and their constituents" (*op. cit.* p.187). Nevertheless when Zalabardo returns to this, in a further discussion entitled *Non-Elementary Picturing*, at Zalabardo 2015, pp.217ff, he points out that his first account "was based on the pretence that everyday propositions ['The bottle is behind the cup', 'The pencil is heavier than the sharpener'] are what Wittgenstein calls *elementary*—consisting of 'names in immediate combination' (4.221). Only on this pretence can we suppose, say, that the pencil being heavier than the eraser can be represented by a proposition consisting in the fact that two objects standing for the pencil and the eraser instantiate (in the right order) a relation standing for *heavier than*." (Zalabardo 2015, p.217)

This is followed by discussion primarily of passages in the *Notebooks 1914–16*, concluding that "In the *Tractatus*, like in the *Notebooks 1914–16*, picturing is restricted to elementary propositions, but here [in the *Tractatus*] ways of representing are not mentioned. Their place appears to be occupied [in the *Tractatus*] by what Wittgenstein calls *truth-operations*." (Zalabardo 2015, p.224). But truth-operations are not representatives, so Zalabardo's overall conclusion is that Wittgenstein "can't be understood as characterising propositions as consisting of elementary propositions and truth-operations. He is not, in fact, making any claim about their constitution" (*op. cit.* p.226).[9] Furthermore, as Copi points out, multiplicity—a 1:1 correlation between the constituents of a proposition and elements in a

[9]Zalabardo refers to Cerezo, in a footnote—"If I understand her correctly, María Cerezo raises an objection to the *Tractatus* that is related to the issues we have discussed in this section. See Cerezo 2005, pp.176–201" (Zalabardo 2015, p.226n.29)—but does not engage further with her discussion.

picture (4.04)—breaks down if one regards the symbols for truth-operations as representative. (Copi 1958, pp.171–2) One might think that Wittgenstein deliberately suppressed the earlier discussions in the *Notebooks 1914–16*, without having anything substantive to substitute therefor.

Similar difficulties arise, I think, for Pears' analogy with pointilist painting (Pears 1987, pp.72–6). If one thinks of the dots as objects, then the analogy breaks down, first because objects are ascribed material properties (colour) as individuals, and second, because spatial juxtaposition is not any sort of combination or concatenation. If one thinks of the dots as elementary propositions then the only relation between elementary propositions is, again, spatial juxtaposition, thus treating the mode of combination—the relations between elementary propositions bound up with truth-operations—as spatial relations. This is explicitly ruled out at 5.42. There is, in short, a very real difficulty in making sense of the text here, and of the literature thereon.

5.4 Resolving the *Satzverband* problem

These are the propositions mentioning logical space:

> 1.13 The facts in logical space are the world.
> 2.11 A picture presents a situation [*Sachlage*] in logical space, the obtaining [*Bestehen*] and non-obtaining [*Nichtbestehen*] of states of affairs.
> 2.202 A picture represents a possible situation [*Sachlage*] in logical space.
> 3.4 A proposition determines a place [*Ort*] in logical space. The existence [*Existenz*] of this logical place [*logischen Ortes*] is guaranteed by the mere existence [*Existenz*] of the constituents—by the existence [*Existenz*] of the proposition with a sense.
> 3.42 A proposition can determine only one place in logical space: nevertheless the whole of logical space must already be given by it. (Otherwise negation, logical sum, logical product, etc., would introduce more and more new elements—in coordination.) (The logical scaffolding [*Gerüst*] surrounding a picture determines logical space. The force of a proposition reaches through the whole of logical space.)

> 4.463 A tautology leaves open to reality the whole—the infinite whole—of logical space: a contradiction fills the whole of logical space leaving no point of it for reality.

My suggestion is that Wittgenstein's solution to the *Satzverband* problem, to what it is that the logical constants do *not* represent—and how it is that a complex proposition can be a picture—is enacted in logical and not in physical space. Charitably, one needs to make a leap of the imagination. Uncharitably, this is an awkward conflation of physical space and logical space. The way to regard this is, I think, as a carefully engineered solution within parameters set by other aspects of *System Tractatus*, and an over-reliance by Wittgenstein on a not fully worked-out notion of picturing.

In principle, at least, there isn't a problem with elementary propositions. The correlation is stated at 2.131, that "In a picture the elements of the picture are the representatives of objects." The problem is with complex/everyday propositions.

One needs to think, I suggest, of a complex proposition as representing, and thus as picturing, wholly in *logical* space. In physical space there are only obtaining states of affairs, these being the facts. The imaginative leap is to think of complex propositions as "picturing" facts in logical space. Hyder gets close to this, I think, in distinguishing what he calls "core logical space" as it relates to elementary propositions, and what he calls the "logical superstructure", the latter arising from taking account of the logical constants:

> The introduction of functions such as '$\vee$' leads in a sense to an expansion of the core space, if we take the view that the logical space is the space of all possible propositions, including complex ones. But even if we do take that view, we must still hold to the requirement that this addition be no more than a superstructure, and an arbitrary one at that: the introduction of $p \vee q$ into the elementary language does not add new entities to the ontology (that is, a logical object corresponding to '$\vee$') and it does not introduce new possible facts either. (Hyder 2002, p.120)

The point that there is no addition of ontology is well taken, but that this expansion has to take us from what is in physical space—containing

states of affairs—into what is solely in logical space—situations—needs, I think, to be made.

It is critical that 2.11 and 2.202 talk of *situations* (*Sachlage*), and not of *states of affairs* (*Sachverhalte*). Because the logical constants are not representative, complex propositions, representing situations, cannot straightforwardly be thought of as pictures, in terms pertaining to physical space. One has to take an imaginative leap and think of them as representative in logical space. Here the notion of *Gerüst* (6.124), of structure, comes into play, so long as one doesn't think of this as some sort of physical support apparatus, but rather as a structuring that is immanent to and nothing over and above. One has to perform a Gestalt switch, seeing states of affairs as in physical space and also, at the same time and by the same token, "seeing" the situation (*die Sachlage*) in logical space. The solution to the *Satzverband* problem, the way to grasp Zalabardo's "mode of combination", is to make the imaginative leap from physical space to logical space. A situation is a function of a distribution of truth-values over a selection of elementary propositions, thus constituting, I think, a place (*Ort*, region) in *logical* space. There is thus nothing over and above the obtaining of states of affairs.

Of Wittgenstein's training at the Charlottenburg Technische Hochschule, Kallenberg says, "students spent hundreds of hours drawing mechanisms, for example mechanisms moving by increments through various possible positions." (Kallenberg 2012, p.58) As an example of such thinking, this account is of re-design work for an engine sump/oil pump assembly:

> In the original design the oil pump driving shaft, carrying the gear which mated to the one on the crankshaft, had its axis parallel to, but sloping back towards, the centre of the engine. This brought the oil pump near to the middle of the engine lengthwise but well to one side due to the centres of the two spiral gears. This meant that the pump could be starved of oil if the engine was standing on a severe camber. So I suggested that we swing the pump round the crankshaft gear to bring the oil pump nearer to the centre line of the engine ... [W]e proceeded to alter the sump to take the new position of the oil pump. But the helix angles of the gears were not altered ... Gear experts were called in, more gears were made by another firm, but no-one was able to ... quieten the noise ... Eventually I realised I had made a mistake in assuming that the gear on the oil pump was just being rolled round

> the crankshaft gear, whereas it should have been treated as an exercise in solid geometry with a correction made for the second angle, which was about seven degrees. New gears were made to this corrected figure and all was peace. (Varley 1969, p.121)

The point is that the engineer visualises in three dimensions the effect of rotating the pump (the range of possibilities) and thereby "sees" what correction is needed. It is this training that, I think, leads Wittgenstein to overestimate the value of picturing as an account of the propositional, and to think that picture theory can be deployed to gloss over the transition from the physical to the logical; how one can circumvent the non-representational nature of the logical constants. The substantive function of picturing is, in Tractarian terms, psychological rather than logical. In making to ourselves pictures of situations we think complex/everyday propositions, these including uses of logical constants. The latter are not representative *per se* (4.0312), but the picture theory of the proposition furnishes them with what one might call an ersatz representational role or function; that things are so spatially ordered in the pictures we make to ourselves.[10]

Visualisation in this sense, Kallenberg suggests, lies behind the remark to Turing about "we who have had a certain training" (*Lectures on the Foundations of Mathematics*, p.66). The point is, I think, that those of us who have had a certain training—long hours doing orthogonal projection and exercises in solid geometry—do imbibe certain ways of thinking.

It is not my intention to defend this, but as far as I can see it is about the only way of making sense of the text and of reconciling points made in the literature. If one thinks of the *Tractatus* as a carefully engineered solution within given parameters, and allows Wittgenstein an over-reliance on the trained engineer's capacity for visualisation/picturing, then it is of a piece with the work generally.

Turning back to the text, 1.13 can be seen to be Janus-faced. On the one hand, "The world is the totality of facts" (1.1), that is, the facts—the obtaining states of affairs (*die Tatsachen*)—in physical space. On the other hand, the world is the facts—the obtaining situations (*die Tatsachen*)—in logical space. The duck and the rabbit are one and the same, this being where the distinction between *existence* and *obtaining* is critical.

[10] An account along these lines can, I think, be found in Cerezo 2005, Chapter 4.

3.4 is equally Janus-faced, because it can be construed in terms either of elementary propositions, or of complex/everyday propositions. The first two stages of logical space (objects/names, and states of affairs/elementary propositions) involve a straightforward correlation between physical and logical space. It is with the complex that the physical is left behind, that the imaginative leap is needed.

At 3.42 we get a definite shift to logical space as distinct from physical space, and although it may not be obvious this is consequent on unrestricted truth-functional combinatorialism in logical space.

As far as the topology of logical space goes, Wittgenstein talks of "logical co-ordinates" at 3.41, but this is not filled out and it is not clear how one could go about doing so. The difficulty is, given that the logical constants do not represent, it follows immediately that the sense of a complex/everyday proposition—what it conveys to us, how we understand it—involves ineluctably an excursion into logical space. Wittgenstein I think slurs over this, relying on the intuitive attractiveness of picture theory. It is because of this intuitive attractiveness that it is so difficult to demarcate clearly what is in physical space and what is only in logical space. The unspecific talk of co-ordinates is, I think, a more or less empty gesturing. It is curious to be told that picturing the cat being on the mat involves an excursion into logical space, because there are logical constants involved, unless one also thinks that understanding the particular case involves grasping the range of possibilities open to all the objects involved, along the lines of Evans's *generality constraint*.[11] Bringing this within the ambit of picturing is, I think, the core of Wittgenstein's solution to the *Satzverband* problem.

There is, lastly, the question of how tautologies and contradictions figure in logical space. Wittgenstein clearly thinks, programmatically, that he should be able to say something with respect to tautologies and contradictions, with respect to logical space, as they are at least on the face of it *bona fide* complex propositions. But here we have left physical space far behind. It is true that *System Tractatus* generates tautologies and contradictions as logical possibilities, but it does not follow that they can actually occur. The text in fact has nothing to say on this point. In particular, it should

[11] "[A]ny thought which we can interpret as having the content *that a is F* involves the exercise of an ability—knowledge of what it is for something to be F—which can be exercised in indefinitely many distinct thoughts, and would be exercised in, for instance, the thought that b is F." (Evans 1982, p.103)

not be inferred from 6.375 that because there is only logical necessity and logical impossibility, that there is only logical contingency and logical possibility. Contingency and possibility are manifested in changing distributions of truth-values across the totality of elementary propositions (Chapter 6) in accord with the laws of nature (Chapter 7).

5.5 Form and content

The point that facts are the *obtaining of*, and not *what obtains*, enables a clear-cut distinction between physical space and logical space. Physical space is the domain of objects, that is, *reality*. In terms of 2.025 this is where we get the content of objects, that is, their empirical manifestation as contributing to the realisation of material properties. Logical space is bound up with *form* (referring to 2.025 again), the domain of possibilities that 2.025 comprises being logical space, *that* the objects that there are—reality—can stand to one another such that *a* way in which things can so stand—a world—is realised.

This *having of form* is the internal property of an object. This has already been touched on (at p.62), but more of the detail can now be filled in. Wittgenstein's assertions that "Objects make up the substance of the world" (2.021) and that substance "is form *and* content" (2.025, emphasis added) comprise the background to this.

On the basis of 2.02, "Objects are simple", Goddard and Judge assert:

> [Objects] are all equally simple; and they lack all properties (qualities) in the ordinary [i.e. perceptual] sense. So they cannot stand in ordinary relations to each other, since this would imply that they *do* have ordinary properties. It therefore comes as no surprise to learn that the relation which holds between objects is always the same and is always the featureless 'combination' or 'configuration'. (Goddard and Judge 1982, p.8)

This is of course *a* way of reading the text, and from a purely logical point of view it is not unwarranted. But one has to ask why Wittgenstein says in the middle of a run of remarks about the logical form of objects that "If two objects have the same logical form, any distinction between them, apart from their external properties, is that they are different." (2.0233)

On the Goddard and Judge reading this would go without saying, as *all* objects would have the one and only logical form; objects could only be numerically different. The world would, presumably, only differ locally in terms of differing density distribution of uniform stuff. We could say *a priori* what elementary propositions there are, that is, what forms of elementary propositions there are; if we take γ as a schematic variable name, elementary propositions would be of the form $\gamma_2\gamma_3\gamma_4 \ldots \gamma_n$, or, alternatively, (γ_n), $n \geq 2$ (cf. p.73).

This, however, goes against 5.55, that "Elementary propositions consist of names. Since, however, we are unable to give the number of names with different meanings, we are also unable to give the composition [*a priori*] of elementary propositions." If Goddard and Judge are right, we *can* give all the forms of elementary propositions that are actually realised *a priori*. This unwanted conclusion shows that something has gone wrong.

To get the account in the *Tractatus* to work it has to be the case that logic only addresses form, and that all that can be said about form is that objects are such that they can combine with other objects to form states of affairs. After this we get content, this being somebody else's problem, because it falls outside of logic. Logic nevertheless has to leave space for this, by accepting that the totality of objects is partitioned according to form, where the number of partitions is more than one, and is finite. This is how we get the variegated world that we represent in our everyday propositions. Unrestricted combinatorialism does not obtain for objects, not least because if unrestricted combinatorialism is countenanced, then any string of words would be an elementary proposition, expressing a sense and thus either actually true, or actually false. It is hard to pinpoint the absurdity here, but if unrestricted combinatorialism were the case, then a surface could indeed be red and green all over.

Hence as soon as one starts to think in terms of complex/everyday propositions one has, I suggest, effectively ventured into logical space (cf. p.155). When one thinks a complex proposition as a picture, the explanation is that one is thinking—imagining—in logical space (cf. p.49n.39). This is how the difficulty is resolved, that picture theory as stated applies only to elementary propositions. Wittgenstein extends it to the complex by tacitly assuming what is "in" logical space.

5.6 Logical space and objects

With this background in place, in constructing logical space, we begin with the notion of objectuality (§1.9). While logical space is singular (it is an "infinite whole", as 4.463) it is helpful to think of it in a series of stages. Objectuality assumes that these successive stages in the construction of logical space follow from the forms of the objects that there are, because logical space relates to possibilities.

The combinatorial possibilities of an object is the range of states of affairs it can be a constituent of. If the objects that there are, are partitioned into, for the sake of the argument, forms α, β, and γ, and if the structure of a (possible) state of affairs is $\alpha\beta\gamma$ (a state of affairs that can obtain, that it would be *a fact that* $\alpha\beta\gamma$ if it obtains), then the state of affairs $\alpha\beta\gamma$ can arise from the combination of any three objects one of which is of the form α, one of form β, and one of form γ. All else being equal it doesn't matter which α, which β, and which γ are so combined. So the form of an object β, for example, can be thought of as a set of states of affairs, $\alpha\beta\beta, \beta\gamma\delta\tau, \epsilon\beta\gamma\gamma\tau, \ldots$, if β appears at least once in each of the states of affairs of the forms listed.

A corollary is that in any possible world there are no "loose" objects, all objects are combined into states of affairs (cf. p.73). The linguistic corollary is that a name by itself is not just senseless, it is impossible, as 2.0122; "It is impossible for words to appear in two different roles: by themselves, and in propositions." It follows in fact from 1, that "The world is all that is the case." An object by itself is not "the case", it is not a fact. However as already noted (§1.14) it has to be the case that states of affairs can disintegrate and objects recombine into other states of affairs, to allow for the world to change, for elementary propositions to change their truth-values.

In terms of the question of propositional unity, the question that so dogged Russell, the problem is dissolved by absorbing Russell's logical forms into the forms of objects; with the combinatorial capability ascribed by Russell to logical forms ascribed in the *Tractatus* to objects, as constituents of states of affairs, and thereby to names, as constituents of elementary propositions (cf. §1.9). Elementary propositions thus possess a sufficient degree of propositional unity, with their structure (2.032) consequent on the logical forms of the objects constituting the state of affairs expressed by the elementary proposition. This is the point of 2.03, that "In a state of affairs

objects fit into one another like the links of a chain", and its propositional corollary, 4.22; "An elementary proposition consists of names. It is a nexus, a concatenation of names." Russellian logical forms were introduced to solve *a Satzverband* problem, but for elementary propositions, at least, that problem has disappeared.

5.7 Logical space and states of affairs

Logical space as it pertains to elementary propositions is relatively simple, because bipolarity obtains. For any given state of affairs, and for the elementary proposition that expresses it, in any given possible world, the state of affairs either does or does not obtain, and the elementary proposition expressing it is consequently either true (T) or false (F). In principle this is pretty much all there is to say, because bipolarity is effectively definitional for elementary propositions.

Although it is not immediately obvious, we now have the whole of logical space. This is the essence of objectuality, that all follows from the objects that there are, and the forms of these objects (§1.9). Logical space from hereon (the logically later stages) is supervenient, is "nothing over and above."[12] What stands behind this is the claim that the logical constants are not representative (4.0312). This supervenience is effectively stated at 3.4:

> A proposition determines a place [*Ort*, region] in logical space. The existence [*Existenz*] of this logical place is guaranteed by the mere existence [*Existenz*] of the constituents—by the existence of the proposition with a sense. (3.4)

Places in logical space can be said to exist, because objects exist. Existence here goes together with *sense*, and is prior to obtaining (*Bestehen*). Here the distinction between reality (*die Wirklichkeit*) (the possible, the propositional) and the world (*die Welt*) (the obtaining, the possibilities that are realised, what is true and what is false) is consequential.

It also matters that 3.4 is read as dealing with elementary propositions. It may not be wholly clear at this point in the text that the only propositions we are dealing with are elementary. It is true that the notation for

[12] As Armstrong puts it, "What supervenes is not something ontologically more than what it supervenes on." (Armstrong 1997, p.87)

complex propositions is first mentioned in 3.3441, but the notion of complex proposition is not fully introduced before 5, where we get the core notion, for a complex proposition, of truth-functionality; "A [complex] proposition is a truth-function of elementary propositions. (An elementary proposition is a truth-function of itself.)" (5) So it is reasonable to read 3.4 as addressing elementary propositions because, as will become clear, what is said at 3.4 does not apply straightforwardly to complex propositions.[13]

What 3.4 states is that given suitable constituents, that is, objects of suitable form, one immediately gets a state of affairs. In other words, states of affairs are supervenient on or, in Tractarian terms, *internally related* to the objects they are composed of.[14] Concomitantly a state of affairs is expressed by an elementary proposition, where an elementary proposition somehow inherits its cohesion or unity from the concatenation of objects into the state of affairs it represents.

A corollary of 3.25, that "A proposition has one and only one complete analysis", is that if objects of forms α, β, and γ, say, could combine in different ways, the result of such differing combination would be different propositions ($\alpha\beta\gamma$ is different from $\beta\gamma\alpha$). Because states of affairs are concatenations of objects and not mere juxtapositions, and because propositions are combinations and not lists, different concatenations are represented by different propositions.

So given all the objects that there are, along with their forms, we are effectively given all the states of affairs that there can be. This is reality (*die Wirklichkeit*). Logical space is, then, given as a whole, with each "place" in logical space being an elementary proposition. Once objects are given, together with their forms, logical space is given, this extending to language because the totality of names and of elementary propositions as combinations of names is equally given. If we think of the range of combinatorial

[13] 3.3441 preceding 3.4 is perhaps an oversight in the rearrangement of the text of the *Prototractatus*. In the earlier version what became 3.4 *precedes* what became 3.3441; in the *Prototractatus*, the former (i.e., 3.4) is numbered 3.1201, and is repeated verbatim, whereas the latter (i.e., 3.3441) is an amalgam of 3.2511 and 3.2512, with some changes to the notation between the two texts.

[14] Armstrong says, of internal relations, "that a relation is internal to its terms if and only if it is impossible that the term should exist and the relation not exist, where the joint existence of the terms is possible." (Armstrong 1997, p.12) A state of affairs is not merely a set or a mereological sum. That the relations between objects within states of affairs in the *Tractatus* can be construed as internal, and thus as supervenient, in Armstrong's sense, is clear at Armstrong 1997, p.87.

possibilities available to an object as its degree of freedom, then the degree of freedom of an elementary proposition is two; either it is true ($\top$) or untrue ($\bot$), reflecting the fact that in the world (*die Welt*) it is either true (T) or false (F). The number of places in logical space is the same as the number of true (T) elementary propositions and the number of false (F) elementary propositions regarded as types, and not as tokens (to be qualified in §5.10).[15]

It may be helpful at this point to propose a model for logical space, as a lamp array. The array has a lamp for each elementary proposition. Each lamp has two degrees of freedom; it can be illuminated, or it can be dark. An illumination pattern is a possible world, a distribution of truth-values across the totality of elementary propositions (to be qualified in §5.10, but from a purely logical point of view the account so far is fine).[16] It is true that we cannot survey the lamp array, but this is not an objection in principle. For the sake of the argument it is assumed that we can mimic the "logical god" of 5.123, that we can survey the whole of logical space and consequently "see" an illumination pattern as a possible world.

Here we can think of a giant truth-table (GTT); "Given our talk of possible worlds, we may say that [the] GTT represents everything possible, or all possible worlds, with each row representing an individual possible world." (Page 1997, p.41) We can think in terms of a GTT because we are dealing with totalities. The rows of the truth-table exhaust all the possibilities, from the top all-$\top$ row to the bottom all-$\bot$ row. It is not possible to construct a diagonal argument resulting in a row that differs from any other row in at least the k_{th} place, because all possible rows are effectively generated by the process.

Prima facie there is a difficulty in regarding each row of the GTT as a possible *world*, as a way in which the world can be. Here the distinction between reality and the world, between what is, and what is not, addressed by logic, is salient. The point is that the full extent of the lamp array relates

[15]For discussions of the parallels between Tractarian logical space and phase space, see Preston 2015, Pilch 2017, §2 pp.20–3. While Wittgenstein may have been influenced by ideas in thermodynamics, the influence is at best indirect. Pilch concludes, "The answer to this question [whether the idea of a 'logical' space can be seen as a direct application of the physical framework of phase space or state space] must clearly be 'no' " (Pilch 2017, p.21).

[16]The view being developed is similar to that termed by Friedman "Carnap's reading of the *Tractatus*" (Friedman 1997, pp.27–9); Pilch proposes a somewhat similar switch model at Pilch 2017, p.32, for what he calls "parameter space".

to *reality*, that is, the domain of the possible. It is not inconsistent to argue that what is *logically* possible (and here one has in mind 6.37, that "The only necessity that exists is logical necessity") can outstrip what is *physically* possible. But it needs to be made very clear here, that one goes outside of logic as it is construed in the *Tractatus*.[17]

5.8 Logical space and situations

The third stage of logical space relates to complex propositions, and here, of course, we run into the *Satzverband* problem; in what sense are complex propositions composed of elementary propositions? In what sense are truth-functional relations *relations*, given Wittgenstein's "fundamental idea ... that the 'logical constants' are not representatives; that there can be no representatives of the *logic* [i.e. the truth-functional structure] of facts" (4.0312)?

As we have seen, the world can be represented by a distribution of truth-values over the totality of elementary propositions. Each row of the GTT is a logically possible world, the table representing the full extent of reality. But this is, I think, merely a different way of stating that in combining propositions into more complex propositions, unrestricted combinatorialism obtains. If, say, p and q are true, then $p \wedge q$, $p \supset q$, $p \vee q$, $p \equiv q$, are all true, and $N(p, q)$ is false. If r is false, then $\sim (p \wedge q \wedge r)$, $p \vee q \vee r$ is true, $(p \wedge q) \wedge r$ is false, and so on. All there is to say here can be shown by truth-tables, by propositional signs, or by bracketing notation. In all cases we have an effective decision procedure, and all possible inferences can be presented algorithmically.

The key point is the *nothing over and above claim*. A complex proposition, as representative of a situation, is a truth-function of elementary propositions. If one asks, what does a complex proposition represent, the answer is, nothing over and above the constituent elementary propositions, taking due account of the distribution of truth-values over them. In the world as we experience it there are likely to be *spatial* relations between the states of affairs concerned, but spatial relations in the world are not in any significant sense correlated with "relations" of some sort in logical space.

[17] The logician can thus sidestep the argument as to whether the all-$\top$ or the all-$\bot$ worlds can obtain; cf. Page 1997, pp.42–3, Reinhardt 2005, Geach 2006, Cerezo 2012, Pt.III.

Logical space is a mathematical space, and not a physical space. In logical space an elementary proposition is $\top\bot$. It can be taken as standing in a truth-functional relation to any (number of) other elementary propositions, in terms of a function of truth-values. If one is interested in the joint truth of p and q, then one is interested in $(\top\bot\bot\bot)(p, q)$. All this says is that there is at least one logically possible world in which both p and q can be true (T). And this is *all* that is so stated. That the state of affairs expressed by p and that expressed by q may need to be juxtaposed for $p \wedge q$ to be true (T) is irrelevant from a logical point of view, and is not in any way expressed or otherwise conveyed by the complex proposition $p \wedge q$. All that there is to be said is expressed by the propositional sign $(\top\bot\bot\bot)(p, q)$. If both T, then T; if not, then F.[18]

It is familiar that given n elementary propositions, each of which has two truth-possibilities, we get 2^n truth-possibilities (set out in 5.101). The shift is from K_n (at 4.27) to L_n (4.442). If there is a denumerable number of elementary propositions then *prima facie* there is a non-denumerable number of truth-possibilities. Such combinatorial explosion can, I think, be denied, on the grounds that what we are dealing with—truth-values—is ultimately discrete, and not continuous. The shift, in other words, is not from $\mathbb{N}$ to $\mathbb{R}$, rather it is from $\mathbb{N}$ to $\mathbb{N}^{\mathbb{N}}$; and because exponentiation is primitive recursive, this cannot take us from a totality of elementary propositions of cardinality $\aleph_0$ to a logically intractable totality of truth-possibilities of cardinality $2^{\aleph_0}$. There is, then, a manageable totality of complex propositions and of possible worlds, and the cardinality of L_n is at most $\aleph_0$.

5.9 Logical space and possible worlds

This leads to the fourth, and final, stage of logical space. As noted it is definitional of elementary propositions that they are bipolar, that they are $\top\bot$. For elementary propositions not to be necessary it has to be the case that they can have different truth-values in different possible worlds. In other words, it has to be possible for the truth-value of an elementary proposition

[18] This is where I think there are difficulties with Pilch's reconstruction of logical space, as the transition from parameter space, akin to the lamp array model here, to his state space and then propositional space (Pilch 2017, §§6,7) involves spatially juxtaposing elementary propositions in logical space. This risks presenting purely logical truth-functional relations as if they are spatial.

to change, within its degree of freedom two ($\top$, or $\bot$). In terms of states of affairs either the state of affairs expressed by an elementary proposition obtains, in which case it takes the value T, or it does not, in which case it takes the value F, *tertium non datur*. What matters for present purposes is that some elementary propositions take different values in different possible worlds, and consequently, that there is a totality of possible worlds. My suggestion is that we think of temporally successive worlds as a trajectory through this fourth stage of logical space.[19]

There is no *logical* reason to prefer any possible world over any other. Further, any discussion of how it comes about that elementary propositions take different truth-values in different possible worlds takes us outside of logic and into the domain of the empirical. This is fine as far as it goes, but now difficulties arise consequent on countenancing the existence of multiple objects of the same form. This complicates the tidy position set out so far, discussed in the next section. First, though, there is an aspect of Zalabardo's approach to discuss, his "existential claim" (B):

> There is a set Δ of [elementary] propositions and a function f pairing each truth-value assignment on Δ with a truth-value assignment on [the set] ED [of everyday propositions], such that for every $p \in ED$ and every $\Gamma \subseteq ED$, $\Gamma \models p$ just in case for every truth-value assignment s on Δ, if f(s) pairs every element of Γ with the value True, then f(s) also pairs p with the value True. (Zalabardo 2015, p.205)

In the terms used here, the set Δ is the totality of elementary propositions $\bar{p}$, as 6, and the truth-value assignments are distributions of truth-values across this totality. Everyday propositions as truth-functions of elementary propositions simply supervene on this distribution. My concerns are not over this, but with Zalabardo's "function f".

Given unrestricted truth-functional combinatorialism, as this is required for full truth-tables, there is only a "function f" in the nugatory sense that a truth-value distribution is a function that maps each elementary proposition

[19] This is I think broadly the line taken by Pilch, that "Time evolution in state space is given by a series of possible worlds, ordered by a time-parameter ... The time evolution of reality can be seen as a path in state space consisting of just those time-ordered transitions, from one possible world to the next, that in fact become actualised." (Pilch 2017, p.37) A "state space" for Pilch is a possible world, so a path in state space is what is called here a trajectory in logical space.

p in $\bar{p}$ to one of the truth-values, $\top$, or $\bot$. In information-theoretic terms, there is no redundancy or compressibility. Given a function $\forall x \in \mathbb{R}.fx \mapsto x^2$, there is a rule to follow, and one can survey the result in graph form, so there is a high degree of compressibility. For the function $\forall p \in \bar{p}.fp \mapsto \{\top, \bot\}$ there is only an algorithm that generates recursively all permutations of ordered pairs $< p_i, X >$, where X is a variable ranging over $\{\top, \bot\}$, with a non-surveyable result (the GTT). There are no shortcuts here.

The thing is, though, that Zalabardo's "function f" is introduced for programmatic reasons, because Zalabardo uses it to operationalise his notion of "our inferential inclinations" (cf. pp.64, 130). There is no problem with this "function f" as long as it is construed as a mapping, but if constraints on possible worlds are to be imposed by restricting the range of "function f" in accord with our inferential inclinations then this is, I contend, to lapse into psychologism.

5.10 The extent of language

Concerning relations between logical space and language, a way into this is to entertain haecceitism, that each object is not only numerically but also qualitatively different from any other, and see how things go astray. For this to be the case each object has to have a different form, and hence a different name, from any other (i.e. if haecceitism then there are only names, and not variable names, contra 4.1272; cf. §2.5). It is immediately obvious that we would find ourselves saddled with a language of an unmanageable and unlearnable extent. We would not be able to "understand the sense of a propositional sign without its having been explained to us" (4.02), because every proposition would be different from any other.

On the present account learnability is addressed by compositionality, consequent on the totality of objects being partitioned according to form, with objects of the same form being numerically and not formally different. This totality of objects is delimited as a totality at 5.5561; "Empirical reality is limited by the totality of objects." It is, I think, noteworthy that it is reality as *empirical* that is limited by—is determined by—the totality of objects. As we have seen (p.86), reality, the world, and the objects that there are, are one and the same. A world is simply a point in the logical space of worlds, *a* way reality can be, *a* combination of the objects that there are according

to their forms.

The distinction between reality and the world is implicit in 5.5262, that "The truth or falsity of every proposition does make some alteration in the general construction of the world." Varying the distribution of truth-values across the totality of elementary propositions moves us from one possible world to another, from one point in the logical space of worlds to another, from one row of the GTT to another.

Given the totality of elementary propositions, provided we make the imaginative leap into logical space, it can be seen that we have all possible complex propositions, as they arise from distributions of truth-values across the totality of elementary propositions. This is stated at 4.52, that "Propositions comprise all that follows from the totality of all elementary propositions." This is rendered clear by mentally inserting "Complex/everyday" at the beginning of 4.52.

Finally, in this backwards progression (skipping 4.11 for the moment) we come to 4.001, that "The totality of propositions is language." But what does not follow is that there is a 1:1 correlation between, in a possible world, the true elementary propositions and the obtaining states of affairs. Elementary propositions are types and not tokens. There may be many token cats on token mats, but there is only one Tractarian proposition. It follows that there is a discontinuity between the neat picture of the GTT, and how things are on the ground. The neat picture can be maintained by adopting haecceitism but the price is, I suggest, too high; because the result is incompatible with language as compositional and learnable.

There are ways round this. We could demand a full specification of time and place, or sufficient detail; that he buttered the toast slowly, in the kitchen, with a silver-mounted hunting knife made for the 7th Duke of Monmouth. But it isn't clear that determinacy of sense makes such demands. If, to use Pears' phrase, we talk of the "total demand" that the truth of a proposition makes on what there is (Pears 1987, pp.70–1), then this may outstrip what we can be reasonably held to know, and thus be able to express, even though such total demand is in principle what we say and think.[20] What does not obviously follow from determinacy of sense is that everyday propositions have to be tokens and not types; and it follows from this that elementary propositions may be types and not tokens, that is, in a

[20] Cf. Schroeder 2006, p.47.

possible world there may be obtaining states of affairs that are numerically and not qualitatively different. There may be many Emilys buttering toast in kitchens in the morning, just as there are many qualitatively indistinguishable nuts, bolts, rivets, and pipe clips in an aircraft.

The *logical* point is, I think, that in any given possible world there will be as many true elementary propositions as are required such that every object that there is, is a constituent of a state of affairs. It may be the case that there are states of affairs that differ only numerically, and thus elementary propositions that are true with respect to multiple instances. The GTT for a *language* has one column for each elementary proposition. But each row may not fully capture a possible *world*, because there may be multiple instances for any true (T) elementary proposition. This is a further reason for distinguishing $\top$ (and $\bot$) from T (and F). It follows, also, that the lamp array model will break down as a model unless it is extended to take account of the totality of objects. At bottom, though, one has to accept that the shift to logical space is an imaginative leap, that it is, essentially, an abstract mathematical construct that one can only grasp via an abstract mathematical mode of thinking.

5.11 How many objects?

As noted, the objects that there are exist, as a totality. This totality is partitioned by form. Objects are held to be necessary existents, having necessarily the forms that they have.

Objects of the same form are qualitatively identical, and numerically different. But how many objects there are, and how many objects there are of any particular form, is far from evident. When Wittgenstein says, "it is just as impossible to say, 'There are 100 objects', or, 'There are $\aleph_0$ objects' " (4.1272), this is not because there are not 100, or $\aleph_0$, objects. It is because the strictures of the *Tractatus* rule out any such statement ("it is nonsensical to *speak of the total number of objects*" (4.1272)). But in the background is a need to deal with Russell's not obviously logical axiom of infinity.[21]

[21] Cf. *Principia Mathematica*, $*120.03$; "[I]f the axiom of infinity is false, then, in any assigned type, all the cardinals after a certain one are $\bigwedge$." (*Principia Mathematica*, ii p.203) $\bigwedge$ is the null class, defined in $*24$. Grattan-Guinness points out that if there is a reliance on the empirical for a large enough number of individuals, it follows "that logic was *a posteriori*—surely a mistake, requesting the physicists to decide about a basic

It isn't completely clear what the issue is here. Williamson says that "Necessitists who defend their view by appeal to contingently non-concrete things rather than recombinations of necessary atoms are likely to be forced into postulating very large numbers of possibly concrete things." (Williamson 2013, p.8) Given that Wittgenstein's view is based on recombination of necessarily existing objects, and given 5.5561—"Empirical reality is limited by the totality of objects"—why not accept that there is a number, even if it is not up to the logician to say what that number is? According to Russell:

> When I was discussing the *Tractatus* with him at The Hague in 1919, I had before me a sheet of white paper and I made on it three blobs of ink. I besought him to admit that, since there were these three blobs, there must be at least three things in the world; but he refused, resolutely. He would admit that there were three blobs on the page, because that was a finite assertion, but he would not admit that anything at all could be said about the world as a whole. (Russell 1959, p.86)

Of course to say how many objects there are might be viewed as an admission that one knows exactly what an object is, that it is an entity that is individually identifiable and countable. This Wittgenstein wanted to resist. And if one treats number as an exponent, in terms of the notion of *operation* (6.021), why not just say that operations are infinitely repeatable, just as one can pile up $\sim$s in front of p; and that this is all that needs to be said for logical/Tractarian purposes about number? But this limits one to a potential and not an actual infinity, so the price is Cantorian set theory and much of higher mathematics.

I do not want to deny Floyd's claim that "one cannot appreciate the full philosophical force of Wittgenstein's treatment of logic without taking his remarks on mathematics into account." (Floyd 2002, p.309) However Floyd's remark that, contra Frege and Russell, Wittgenstein "revitalises the standard algebraic talk of 'operations', hurling the old-fashioned language back at Frege and Russell" (Floyd 2002, p.317; cf. *op. cit.* p.314) rather suggests that Wittgenstein was going backwards.

In terms of such revitalising, Floyd makes the substantive point concerning the notion of an operation that Wittgenstein reinstates the notion

feature of logicism." (Grattan-Guinness 2000, p.300)

"and so on", that "The concept of successive applications of an operation is equivalent to the concept 'and so on'." (5.2523)[22] That is, in constructing a formal series (*Formenreihen*) "and so on" is shown by dots of continuation and not by dots of laziness (4.1252, 4.1273).[23] According to Floyd, "The answer 'and so on' was the very notion Frege and Russell wished to eliminate from the foundations of logic (and mathematics) by means of their ancestral construction." (Floyd 2002, p.331) At face value, the implication is that Wittgenstein is prepared to countenance only a potential and not an actual infinity. But this requires a closer look at what Wittgenstein says about infinity, and about totalities.

There are five salient remarks:

> 2.0131(1) A spatial object must be situated in infinite space.
> 4.2211 Even if the world is infinitely complex, so that every fact consists of infinitely many states of affairs and every state of affairs is composed of infinitely many objects, there would still have to be objects and states of affairs.
> 4.463(3) A tautology leaves open to reality the whole—the infinite whole—of logical space: a contradiction fills the whole of logical space leaving no point of it for reality.
> 5.43 Even at first sight it seems scarcely credible that there should follow from one fact p infinitely many *others*, namely $\sim\sim p$, $\sim\sim\sim\sim p$, etc. And it is no less remarkable that the infinite number of propositions of logic (mathematics) follows from half a dozen 'primitive propositions'.
> 5.535 All the problems that Russell's 'axiom of infinity' brings with it can be solved at this point. What the axiom of infinity is intended to say would express itself in language through the existence of infinitely many names with different meanings.

On the basis of these remarks Marion, following unpublished work by Michael Wrigley, claims that 4.2211 in particular, concerning the number of objects, "opens the door to an actual infinity . . . since 'the whole logical space must be given' with a particular proposition (3.42) it is implicit in Wittgenstein's doctrines that the totality of propositions be infinite and determinate, that is in a sense actual." (Marion 1998, p.34)

[22] Cf. *Notebooks 1914–16*, pp.89–90.
[23] Cf. Floyd 2002, Appendix, pp.341–5.

Marion also paraphrases a remark made by Wittgenstein to Kreisel, "that he had put down his system in a finite setting without bothering about the infinite case, assuming that if a problem was to be found in the infinite case, then there would have already been a problem in the finite case." (Marion 1998, p.34) If the totalities of the *Tractatus* are finite then there are no particular difficulties to address, and since they are totalities, they can't be nondenumerable. It is the denumerable case that needs to be considered.

If Wittgenstein treats a denumerable totality, an actual infinity, as if it is finite, then he is in good company. Hallett gives the following as one of three principles underpinning Cantor's treatment of infinite numbers:

> *Cantor's principle of finitism.* The transfinite is on a par with the finite and mathematically is to be treated as far as possible like the finite. (Hallett 1984, p.7)[24]

The point is, I think, that a denumerably infinite set can be thought of as finite by contrast with a nondenumerably infinite set, because the latter cannot be put in a one-to-one correlation with the former. In principle, then, one can treat a denumerably infinite totality *as if* it has a finite number associated with it. As Wittgenstein later remarked, one of his "erroneous ideas" in the *Tractatus* was of "Treating infinity as a number, and supposing that there can be an infinite number of propositions." (*Lee Lecture Notes*, p.119) Leaving aside questions of how sophisticated Wittgenstein's approach is, it is, I think, plausible to hold that he could have countenanced denumerably infinite totalities on the basis that they could be treated as on a par with very large finite totalities.

The one thing that can I think be stated *a priori* is that if the number of objects is infinite, then it will be denumerably so. Given that the totality of objects is partitioned according to form, and thus given that unrestricted combinatorialism does not obtain, it follows that given $\aleph_0$ objects there cannot be more than $\aleph_0$ states of affairs. And given that we are dealing with the discrete and not the continuous it is reasonable to assume that the extent of logical space is at most denumerably infinite. At bottom from a logical point of view it does not obviously matter whether the cardinality of the totality of objects is very large, or denumerably infinite. It is what

[24] Cf. Hallett 1984, pp.32–40, Clark 1986, p.525, Mayberry 1986, p.431, Maddy 1997, pp.51–3.

it is, and given that objects are in principle nameable it is shown by the number of different names needed to name them (noting that the totality of objects is of greater cardinality than that of the number of Tractarian names *as variable names*, as there are objects of the same form, substitutable for the same variable name; hence the uses made of indices). At bottom it seems reasonable to demand *a priori* that one's logical system be able to cope whether the number of objects that there are is finite, or denumerably infinite. Mathematics must then take care of itself.

Chapter 6

Probability

6.1 A neglected aspect

Very little has been written about Wittgenstein's views on probability in the *Tractatus*. A rare exception is White, but in his brief discussion he says "It is difficult to see why Wittgenstein devotes so much space to what must count as a relative side issue." (White 2006, p.86) The aim in this chapter is to show that probability is much more significant in the *Tractatus* than has been realised. White may share in the underestimation of the role of probability, but he is astute in affording it any comment at all. The remarks on probability are ostensibly contained in 5.15–5.156, but in fact probability is mentioned at the start of the discussion of truth-grounds (5.1), and is thus bound up with inference. Beyond this there is a brief anticipation at *Notebooks 1914–16*, pp.27–8.

In the literature on the *Tractatus*, reference is generally made to Waismann's 'A Logical Analysis of the Concept of Probability' (Waismann 1930). The paper begins with the stated aim of discussing "the logical clarification of the concept of probability", but in so doing he states that he is "using Wittgenstein's ideas", and, "I do not know how far my views agree in detail with Wittgenstein's. Wittgenstein is preparing a major work which will also deal with the concept of probability." (Waismann 1930, pp.4, 21n.1) Considering the content of the paper, it cannot be regarded straightforwardly as an exposition of Wittgenstein's views in the *Tractatus*. As Carnap makes clear, concerning his further development of the logical concept of probability after Waismann's paper, this is on "the basis of an idea of Wittgenstein's," stated

in 5.15. (Carnap 1962, p.299) Carnap later dropped mention of Wittgenstein and attributed the logical approach to Keynes (Carnap 1966, pp.28ff). If one wants to understand probability in the *Tractatus*, neither Waismann nor Carnap is a good place to start.

However, perhaps because of difficulties in understanding the *Tractatus*, Waismann's paper has been influential. It is discussed by von Wright in his seminal paper 'Wittgenstein's Views on Probability' (von Wright 1969, reprinted with changes to §5 in von Wright 1982), and thence by McGuinness (McGuinness 1982, written in the main as a review of von Wright), and by Cuffaro (Cuffaro 2010, pagination cited for version downloaded from www.michaelcuffaro.com). Probability is discussed by Black (Black 1964, pp.247–58), but his exposition draws heavily on Carnap, and on Waismann. Frascolla's extended discussion (Frascolla 2007, pp.198–203) concentrates on the *Tractatus*, but he treats probability as a side issue, isolated from the context it appears in, as one of a number of topics that can't be meaningfully (propositionally) spoken of. In the notes to his discussion he refers to Waismann, Carnap, and von Wright (Frascolla 2007, p.234 n.22–26). Fogelin's discussion is similarly self-contained, although he does note that Wittgenstein's discussion of probability "is not only consistent with the main themes of the *Tractatus*, but develops naturally from them." (Fogelin 1987, p.53) There is a brief and inconclusive discussion at Anscombe 1971, pp.156–8, in part directed at equiprobability/independence for elementary propositions. Glock treats probability in an entry for induction, probability not warranting its own entry (Glock 1996, pp.170–4).

As to Wittgenstein's sources, the most obvious candidates are von Kries and Keynes. Boltzmann refers to the former in *Populäre Schriften*, known to have been read by Wittgenstein. Von Kries's "Prinzip der Spielräume" looks to appear twice in the *Tractatus*, at 4.463 and 5.5262. Von Wright says that Wittgenstein may have been familiar with von Kries' book, but this is disputed by McGuinness (cf. von Wright 1969, p.266, McGuinness 1982, p.206n7, also Cuffaro 2010, p.7n4). McGuinness may be misled by assuming that probability is confined to 5.15–5.156, whereas the *Spielraum* concept appears outside these bounds. Keynes 1921 was published too late to influence the *Tractatus*, but the ideas therein were developed before the war and discussed extensively with Russell.[1] In his preface Keynes says,

[1] Cf. headnote to Russell 1922c.

"I have been influenced by Johnson, Moore, and Russell, that is to say by Cambridge" (Keynes 1921, p.v) More specifically, in a letter to Keynes dated 12 June 1919 Wittgenstein says, "Have you done any more work on probability? My M-S. [i.e., the *Tractatus*] contains a few lines about it which, I believe,—solve the essential question." (*Wittgenstein in Cambridge*, p.91)[2]

Von Wright also mentions Bolzano, suggesting that the truth-functional approach outlined in §6.2 is sufficiently similar to Bolzano that it might be called the *Bolzano-Wittgenstein definition* (von Wright 1969, p.264).[3] However von Wright expresses doubts that Wittgenstein had ever read Bolzano.

Here one circles back to Wittgenstein's remark in the Preface that he "makes no claim to novelty in detail, and the reason why I give no sources is that it is a matter of indifference to me whether the thoughts that I have had have been anticipated by someone else." Wittgenstein's views on probability are not novel in themselves. What is of interest is the use he makes of these ideas in the context of the *Tractatus*.

A second difficulty concerns the overlap between Wittgenstein's remarks on probability and those concerning induction and laws of nature. The focus in this chapter is on probability, with Wittgenstein's views on induction, laws of nature, and on science generally held over to Chapter 7.

At 5, Wittgenstein turns from elementary propositions to complex propositions. The first substantive mention of probability occurs shortly afterwards, in 5.1; "Truth-functions can be arranged in series. That is the foundation of the theory of probability."[4] So von Wright's statement that Wittgenstein's views on probability are to be found in 5.15–5.156, fully $2\frac{1}{2}$ pages, or twenty-three numbered remarks, after 5.1, is surprising. Similarly Black remarks, of 5.1, "This will be elaborated in sections 5.15ff." (Black 1964, p.241) Black echoes Stenius here, the latter quoting 5.1 and then stating, "what immediately follows after 5.1 refers to its first paragraph only" (Stenius 1960, p.13), that is, the remark concerning the arrangement of truth-functions into series. Stenius does not otherwise discuss probability.[5] This is characteristic, I think, of readings that miss the significance of probability in

[2]Cf. Goldstein 2004, pp.155–6.

[3]Bolzano's views on probability are addressed in Lapointe 2011, Chapter 4. See also *op. cit.* p.158n7 and Goldstein 2004, p.155 for a possible indirect influence of Bolzano on Wittgenstein.

[4]There is an earlier parenthetical mention at 4.464, addressed at p.184.

[5]In similar fashion McGinn jumps from 5.143 to 5.2, omitting the discussion of probability completely (McGinn 2006, p.220).

the *Tractatus*. What is lost is a continuity of theoretical development, that truth-functionality for propositions (and inference) discussed in 5.1–5.143 leads seamlessly into the treatment of probability. Treating 5.15–5.156 as a separable account of probability turns it, erroneously, into a "relative side issue". To understand what Wittgenstein has to say one must first see the discussion of probability in the broader context of the 5s generally.

Central to the concept of probability is some notion of a gap or separation, bound up with time, or with knowledge. Talk of events or happenings or outcomes implies a separation in time. In 'Truth and Probability' Ramsey takes as an example "the probability of recovery from smallpox" (Ramsey 1926, p.54), so here probability is bound up with a process that occurs over time. If one bets on who or what is behind a series of closed doors then there is a knowledge gap, one satisfied by a process that unfolds over time. It is hard to see how one can have any conception of a static notion of probability. To understand what Wittgenstein has to say about probability one has to have in mind a dynamic and not a static conception, specifically, in the language of §5.9, a trajectory through the fourth stage of logical space, a temporal succession of possible worlds.

6.2 A logical theory

A logical theory of probability is based on truth-functional relations between propositions, postulating a parallel between conditional probability claims of the form:

$$\text{Prob}(p/E) = x,$$

where E is a body of true propositions, and statements of the form:

$$E \rightarrow p.^{6}$$

If $E \rightarrow p$ is regarded as expressing a logical entailment between E and p, that it cannot be the case that E is true and p false, then this can be shown in a truth-table. If $\text{Prob}(p/E) = 1$, then wherever there is a $\top$ in the p column, there is a $\top$ in the E column.

The central idea is that if $\text{Prob}(p/E) = x$ where $0 \leq x \leq 1$, this is shown by distribution of truth-values in the truth-table. If we consider the rows in

[6] The presentation borrows from Horwich 1982, pp.34–6, in turn based on Carnap 1962.

which E is $\top$, and add up the number of such rows in which p is $\top$, then $\text{Prob}(p/E)$ is the number of rows in which p is $\top$ divided by the number of rows in which E is $\top$.

Turning to 5.15–5.151 and taking, for the sake of the argument, the case of $p \,.\, q$:

p	q	$p \,.\, q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\bot$

In a world with two possible states of affairs and one logical connective $.$, waiving all other considerations for the sake of the argument, $\text{P}(p \,.\, q)= \frac{1}{4}$; the cardinality of the range of possibilities L_n (the *Spielraum*) is $2^2 = 4$, and in only one possible world is the proposition true. $\top$he *Spielraum* for an elementary proposition is $\top$ or $\bot$ (its degree of freedom, as p.163), but the truth-value of any elementary proposition so to speak ripples across logical space because it determines the truth-values (is the truth-ground) of a range of complex propositions. This is stated in 4.463 and 5.5262.

Following the notation of 5.151, if:

r	s
$\top$	$\top$
$\top$	$\bot$
$\bot$	$\top$
$\bot$	$\bot$

then $\frac{\top_{rs}}{\top_r} = \frac{1}{2}$; there is one row in which r and s are both $\top$ out of the two rows in which r is $\top$ so r gives to s the probability $\frac{1}{2}$.[7] Probability is a matter of correlating truth-values, so it is a *logical* theory of probability. Since $\{\sim, .\}$ is an adequate set of connectives, this can be extended to the other logical constants by definition.

Probability is simply $\frac{|\top|}{L_n}$, the number of $\top$s in the range divided by L_n. For $p.q$ this is $\frac{1}{4}$, for $p \vee q$ $\frac{3}{4}$, and so on. Given that we have no logical reason to prefer one of these possible worlds to any other, each possible world has a probability of $\frac{1}{16}$ (the range is 2^{2^2}, each being logically equipossible).

[7] Cf. von Wright 1969, pp.262–3, Fogelin 1987, pp.51–2, Cuffaro 2010, pp.6–8.

It may be objected that this isn't really what we mean by probability. Truth-tables, for example, are no help for betting on horse races. In criticising Keynes's version of the logical theory Ramsey says, "There really do not seem to be any such things as the probability relations he describes." (Ramsey 1926, p.57) But what we have here is a bifurcation. A logical theory of probability is really about induction and generalisation, about laws of nature expressed as hypotheses, in the form *If . . . , then . . .* If one thinks of probability in terms of prediction and explanation, the logical theory has something to be said for it. If one thinks of betting behaviour and games of chance, then it doesn't. Hence Keynes draws a distinction between probability as it relates to events and happenings, and our expectations thereof, and probability as treated mathematically (Keynes 1921, Chapter VII). Wittgenstein's account is directed at relations between propositions and not at the mathematical. Consequently my strategy is to get clear about Wittgenstein's views on probability as a preamble to investigating his views on laws of nature.

The first requirement for applying a logical theory is the Independence Thesis, that any given elementary proposition is, at any given moment/in any given possible world, true or false; not both, and not neither. This is the gist of 4.023, "A proposition must restrict reality to two alternatives, yes or no." For the truth-table method to work *a priori* it has to be the case that the truth-value of any given elementary proposition is as it is independently of any other. This is effectively definitional, as 4.211; "It is a sign of a proposition's being elementary that there can be no elementary proposition contradicting it." This condition must be met for the mechanical generation of the columns of the truth-table.

The logical constants then fall out, by correlation. Given (p,q), we get ($\top\bot\bot\bot$), ($\top\top\top\bot$), ($\top\bot\bot\top$), and so on. Wittgenstein calls truth-tables or sequences like ($\top\top\bot\top$) (p,q) at 4.442 *propositional signs*, stating the truth-conditions for a complex proposition. The signs for the logical constants are a convenient shorthand. If we consider the prefixes (XXX . . .), with X a schematic letter that can take $\top$ and $\bot$ as values, then a logical constant can be regarded as an equivalence class of similar patterns of $\top$s and $\bot$s. The point is that there is nothing substantive to the logical constants, this line of thinking standing behind Wittgenstein's "fundamental idea," that the logical constants "are not representatives." (4.0312) It doesn't follow,

though, from not being representative that the logical constants don't do anything. What they do is the subject of the present and the following chapter.

6.3 Lamp array models

In the first sentence of 5.1 Wittgenstein says, "Truth-functions can be arranged in series (*Reihen*)." At 4.1252 and 4.1273 he talks of series, generated by a process that can be called *functional abstraction*[8], and generalisation; aRb, $(\exists x) : aRx.xRb$, $(\exists x, y) : aRx.xRy.yRb$, etc. 4.1252 terms this a *formal series* (*Formalreihen*) because it is ordered by an internal relation. We can think of an old-fashioned computer hooked up to a metronome, such that each time the metronome ticks it applies the relevant operations, taking the output from the previous operation as its input.

In the case of 5.1 we can think of a similar machine as §5.7 with an array of lamps displaying the output, such that each time the metronome ticks it applies the Sheffer stroke according to a suitable algorithm (this algorithm is, in Tractarian terms, a formal series, or at least, the mechanism to generate a formal series).[9] The illumination pattern across the lamp array shows the distribution of truth-values after each application. In this way all possible combinations for any two elementary propositions are generated, albeit not in any particular order, from $\top\top\top\top$ to $\bot\bot\bot\bot$ (as 5.101).

The truth grounds of a proposition are "those truth-possibilities of its truth-arguments that make it true." (5.101) Only complex propositions can have truth-grounds, where these are simply the distribution(s) of truth-values over its constituent elementary propositions that render the complex proposition true. So the truth-grounds of $p.q$ are the truth of p, and of q. And so on. An elementary proposition cannot have a truth-ground because it is a truth-ground.

To say that a formal series is ordered by an internal relation is to say that there is nothing to the relation over and above a correlation (noting that with the exception of number, and the functional abstraction of 4.1252 and 4.1273, there is no sound notion of order here; cf. §4.8). The relation is not in any way substantive. In Tractarian terms such a relation is an *operation*, as

[8] "Functional abstraction" because what is abstracted is a Tractarian function and not a propositional function, as Chapter 4.

[9] Cf. Proops' remarks on the N-operator (5.502) at Proops 2000, p.16.

carried out by our machine; "We can determine the general term of a series of forms by giving its first term in the general form of the operation that produces the next term out of the proposition that precedes it." (4.1273) Given $(\top\bot\bot\bot)(p,q)$ we already have $.$, just as given $(\top\top\top\bot)(p,q)$ we already have $\vee$. This is why they are internal relations because, so to speak, they are already present and correct.

From here Wittgenstein turns to deductive inference, at 5.12–5.143; the remarks on tautologies and contradictions at 5.142–5.143 introduce nothing new, as they deal with these purely in terms of inference. Deductive inference is essentially truth-functional, shown by the propositions themselves, as discussed in §3.2. This is expressed near enough *in toto* in 5.1241; " '$p.q$' is one of the propositions that affirm 'p' and at the same time one of the propositions that affirm 'q'." For much of this section of the *Tractatus* one could simply substitute a set of natural deduction rules (cf. §3.2). Inference is simply a matter of a mechanical application of natural deduction rules.

Given a lamp array with a lamp for every elementary proposition, a possible world is a distribution of truth-values across this array, with some lamps lit ($\top$) and others dark ($\bot$). From such a distribution the logical god of 5.123 can see at once all true complex propositions, and all inferences therefrom.[10] That is, a logical god can move immediately from the second stage of logical space, of elementary propositions representing states of affairs (§5.7) to the third stage, of complex propositions representing situations (§5.8). From here, for our logical god, it is an immediate step to the fourth stage of logical space (§5.9) because this is given by the distribution of truth-values over the totality of elementary propositions. But this is an essentially *static* picture. Inference occurs *within* a possible world, that is, within a distribution of truth-values across the totality of elementary propositions. Statically, there is no meaningful sense of probability, because there is no gap, over and above ignorance. And ignorance is not *logically* significant.

6.4 From the static to the dynamic I

When Wittgenstein says that "the only necessity that exists is *logical* necessity" (6.37(2)), this is because the only necessity there is, is a static

[10] Akin to seeing the world *sub specie aeternitatis*; "The usual way of looking at things sees objects as it were from the midst of them, and the view *sub specie aeternitatis* from outside." (*Notebooks 1914–16*, p.83). Cf. Hacker 2021, p.98.

conception of inference *within* a given possible world, on the basis of the truth-functional structuring of complex propositions. That there is something else going on is shown by the first sentence of 6.37; "There is no compulsion making one thing happen *because another has happened*." (6.37, emphasis added) Now we have temporality, a *dynamic* conception, relating to changes in distributions of truth-values across the totality of elementary propositions. We now have worlds that are *temporally* distinct from this world. This is the fourth stage of logical space for reality, the range of possible worlds. At any given moment the world will occupy a location in this space. This can be represented as follows, at t_i, t_j, t_k, and so on:

	t_i				t_j				t_k			
Totality of	$\top$	$\bot$	$\top$	.	$\top$	$\top$	$\bot$	.	$\bot$	$\bot$	$\top$	.
Elementary	$\top$	$\top$	$\bot$	.	$\bot$	$\bot$	$\top$	.	$\top$	$\bot$	$\top$	.
Propositions	$\bot$	$\top$	$\top$	.	$\top$	$\top$	$\top$	.	$\bot$	$\bot$	$\bot$	.
	.	.	.	.	.	.	.	.	.	.	.	.
Logical	.	.	.	.	.	.	.	.	.	.	w_k	.
Space	.	.	w_i	.	.	.	.	.	.	.	.	.
for reality	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	w_j	.	.	.	.	.	.

There is a correlation between these, with the distribution of truth-values across the totality of elementary propositions correlated with the location of the world in logical space for reality. What is of interest is how the truth-value distribution changes over time and, correlatively, the trajectory of the world through logical space.

As we have seen, the objects that there are, and their forms, are the same in all possible worlds, with each possible world a more or less extensive recombination of objects into states of affairs. It is contingent for any given elementary proposition whether it is true or false; in some worlds it will be true, in others, false. The truth-values of elementary propositions must change over worlds or we would get a frozen world, and this is evidently not the case. *A priori*, this is all that can be said.

Moving into the part of the *Tractatus* that most obviously addresses probability (5.15–5.156), Wittgenstein says, "Two elementary propositions give one another the probability $\frac{1}{2}$." (5.152) This is on one level an imme-

diate corollary of the Independence Thesis, but on another level it states a relation—a probabilistic relation—between any two elementary propositions. The thing to note is that inference is the limiting case, where the relations between propositions have probability 1, or 0. What we get now is the general case, where the relation of *following from* can take any rational value between 1 and 0.[11] As Frascolla says, "In an elegant way, metalogical [inferential] relations between propositions turn out to be mere particular cases of probability relations between propositions." (Frascolla 2007, p.199) This is prefigured in 4.464, the first mention of probability in the *Tractatus*; "A tautology's truth is certain, a proposition's possible, a contradiction's impossible. (Certain, possible, impossible: here we have the first indication of the scale that we need in the theory of probability.)" (4.464) What this shows is that probability is not a side issue, it is integrated into Wittgenstein's views on propositions from the outset.

However the presumption that any two elementary propositions give to one another the probability $\frac{1}{2}$ is not immediate. Why would one think that the alternative to the absence of an inferential relation (of probability 1 or 0) is a probabilistic relation, value $\frac{1}{2}$? The reason is Wittgenstein's reliance on a familiar notion of equiprobability, stated by Keynes as the *Principle of Indifference*:

> The Principle of Indifference asserts that if there is no *known* reason for predicating of our subject one rather than another of several alternatives, then relatively to such knowledge the assertions of each of these alternatives have an *equal* probability. (Keynes 1921, p.45)[12]

Keynes considers several objections, discovering a consistent pattern, with the proposed alternatives found to be further divisible:

[11] Rational, not real, because countenancing a non-denumerably infinite number of values leads to contradiction (Keynes 1921, pp.51–2). Kamlah terms this the *von Kries-Bertrand paradox* (Kamlah 1987, p.104). This goes together with the point at p.165 that the cardinality of L_n is $\mathbb{N}^{\mathbb{N}}$ and not $2^{\aleph_0}$.

[12] Anscombe describes this as an "arbitrary dogma" (Anscombe 1971, p.156), but this is misplaced. It is reasonable to assume equiprobability as a convention, subject to revision as the evidence comes in. This is proposed by Borel and Poincaré (Keynes 1921, pp.52–3). Fogelin's objection that Wittgenstein's account can't be readily extended to a theory of confirmation because he has nothing to say about sampling, and that we have no guide as to applying a logical theory of probability in everyday life (Fogelin 1987, p.53) can be similarly addressed. See also Glock 1996, p.172.

> The paradoxes and contradictions arose, in each case, when the alternatives, which the Principle of Indifference treated as equivalent, actually contained or might contain a different or an indefinite number of more elementary units. (Keynes 1921, p.65)

This is tacitly resolved in the *Tractatus*, because elementary propositions are not further divisible in the relevant sense; in short, bipolar entails truth-values as discrete and not continuous. The conditions for indifference are thus met.

In terms of the lamp array, a possible world is represented by an illumination pattern, showing the totality of elementary propositions that are true in this world (the facts, the obtaining states of affairs that constitutes this world). Whether or not any particular lamp is lit is contingent, with a probability $\frac{1}{2}$. This is of course not changed by its being lit in this world, because this probability is *a priori*. A logical god can see at a glance all the complex propositions that are true and all the inferences that can be made therefrom.

Now the metronome ticks, there is a more or less extensive recombination of objects, and the distribution of truth-values change; the illumination pattern changes. Our logical god immediately takes in the changes in the complex propositions that are true, and what can be inferred therefrom. But why the transition from that world w_i to this world w_j? Why this particular ballet of objects with this particular outcome?

Wittgenstein says, shortly before the remarks directly on probability (5.15–5.156), that "We *cannot* infer the events of the future from those of the present ... There is no causal nexus to justify such an inference." (5.1361, 5.136) Later, in discussing laws of nature, he says, "It is an hypothesis that the sun will rise tomorrow: and this means that we do not *know* whether it will rise." (6.36311) Here we get the rather implausible presumption that there can only be *knowledge* in the domain of the logical.

This unsettled an early reviewer:

> The immediate consequence of this ... is sufficiently startling, though Mr Wittgenstein does not hesitate to embrace it. It is the essential invalidity of all attempts at prediction ... It is to be noted that the grounds alleged by Mr Wittgenstein apply to probable, as well as to certain, reasoning—though he himself does not make this clear. If facts

> consist of absolutely independent elements, we have not the slightest reason for supposing that the sun will rise upon another day. (De Laguna 1924, p.27)

This is an over-statement, but here we are trenching on laws of nature and science generally, of which Tejedor notes that "Little sustained attention has been paid to what, precisely, Wittgenstein understands the role of the natural sciences to be in the *Tractatus* and how this understanding affects his approach to causation." (Tejedor 2015, p.113n.2) The reason for this is, I think, in large measure a failure to get clear first about probability.

Going back to the lamp array model, the question becomes, what operation is applied when the metronome ticks? Because this is the crux of the problem. Whereas a distinction between deductive inference and induction is, I think, generally drawn in the literature, this is not tied to the static/dynamic conception presented here. One cannot, I suggest, begin to conceptualise the issues until one starts to think dynamically.

Tejedor sees Wittgenstein as rejecting a causal necessity view, "that causation involves necessary connections between causes and effects." (Tejedor 2015, p.91) Whatever else one might say about laws and causation, it is nigh on impossible to do so without talking about ordering in time. Wittgenstein's views are relevantly close to those expressed in Russell's 'On the Notion of Cause', in which he rejects science as a quest for exceptionless causal laws, on the basis that any mooted sequence may be interrupted. He offers instead that "Certain differential equations can be found, which hold at every instant for every particle of the system, and which, given the configuration and velocities at one instant, or the configurations at two instants, render the configuration at any other earlier or later instant theoretically calculable." (Russell 1912b, p.186) "Theoretically calculable" states what comes next *ceteris paribus*, and this is not necessitating.

If there were exceptionless causal laws, then when the metronome ticks we (or at least our logical god) would be able to infer what comes next. We would *know* what comes next, at least where such causal laws apply. This is *all* that Wittgenstein denies. But this is a narrowly logical point. That we do not *know* whether the sun will rise tomorrow (6.36311), that "There is no compulsion making one thing happen because another has happened" (6.37), leaves untouched the fact that it is more likely than not that the worlds following the one that obtains now are overwhelmingly ones

in which the sun rises tomorrow. But to see this, why there are laws of nature expressed in the forms of hypotheses (6.36311) concerning the motions of the heavenly bodies, relies on first understanding how probability is involved in what happens when the metronome ticks.

The basic idea is this; laws of nature arise from the trajectory of the world through logical space—from temporal sequences of changing distributions of truth-values across the totality of elementary propositions. This is why one cannot make progress with laws of nature expressed as hypotheses unless one has first grasped the transition from the static to the dynamic.

Turning back to Keynes, he says that the theory he proposes is a logical theory because it is a matter of "cognis[ing] correctly a logical connection between one set of propositions which we call our evidence and which we suppose ourselves to know, and another set which we call our conclusions, and to which we attach more or less weight according to the grounds supplied by the first." (Keynes 1921, p.5) If our evidence is the set of true elementary propositions of w_i, then our conclusions are those of w_j. The point about a logical theory of probability is that it "emphasise[s] the existence of *a logical relation between two sets of propositions* in cases where it is not possible to argue demonstratively [i.e. inferentially] from one to the other." (*op. cit.* p.8, emphasis in the original) This is indeed the situation we find ourselves in.

This is also where Wittgenstein's interest stops, so one must look elsewhere to fill out the account. When Waismann takes up Wittgenstein's ideas, he says that the logical relation can "be called the degree of 'logical proximity' between statements." (Waismann 1930, p.9) Intuitively we can think of "logical proximity" in terms of closeness between possible worlds, along lines suggested by Lewis; "We may say that one world is *closer to actuality* [the world as now] than another if the first resembles our actual world more than the second does, taking account of all the respects of similarity and difference and balancing them off against one another." (Lewis 1973a, p.196)[13]

[13]Intuitively, but there are differences because Lewis's accounts in Lewis 1973a, Lewis 1973b, and Lewis 1979 involve determinism, miracles, and convergence/divergence between possible worlds (indeterminism is discussed at Lewis 1986, pp.58–65), whereas the reconstruction given here is probabilistic, without miracles and, since the totality of objects is given, there is no divergence or convergence. Counterfactuals are supported, but only probabilistically, so Lewis's "If it were the case that A, then it would be the case that C" becomes, *If it were the case that* A*, then it is more or less likely that it will be* C. On the present account laws are consequent on observed sequences, so closeness is *shown*.

In terms of the trajectory of the world through logical space, it is reasonable to think that this will be smooth, that is, taking in worlds that are pretty close. It is because of this that we can generate laws of nature inductively, as hypotheses. The critical point is that on the basis of experience we revise the probabilities of propositions, and take as laws of nature those whose measure of probability approaches 1. What remains to be shown is how the logical constants function in the complex propositions that state laws of nature as placeholders for relations between possible worlds, where these relations can be treated as statistical regularities and not as necessities.

Chapter 7

Laws of Nature

7.1 Totalities

The aim is first, to set out a view on science and laws of nature, that can be extracted from the *Tractatus*, and second, to consider whether the view so extracted is plausibly what Wittgenstein had in mind.

The first parameter is 6.375, that "Just as the only necessity that exists is *logical* necessity, so too the only impossibility that exists is *logical* impossibility." It follows that necessity belongs only to tautologies and contradictions, where these are shown by the propositional sign, this having a solid row of $\top$s in the left-hand parenthesis in the case of a tautology, and a solid row of $\bot$s in the case of a contradiction. We also know (4.462) that tautologies and contradictions do not represent possible situations, that they are not in any sense representative. So if we think that science in general and laws of nature in particular deal with what there is, then it follows that the propositions of science generally and those stating laws of nature in particular are not tautologies and contradictions. And they cannot be treated as bridge principles, as per the equations of mathematics, because as Wittgenstein makes clear, while the latter are not tautologies or contradictions, they, like the "propositions" of logic, *show* the logic of the world. Indeed if a propositional sign shows that it is a tautology or a contradiction then we can say immediately that it is not a proposition of any science except, of course, of logic, if we regard logic as being itself a science. This is the force of 4.11:

> The totality of true propositions is the whole of natural science (or the whole corpus of the natural sciences.) (4.11)

This, I suggest, says a great deal more than is at first apparent. At any given moment, the totality of true propositions is the world, represented by a maximally consistent totality of elementary propositions. Its complement is a maximal totality of elementary propositions, where this totality is inconsistent. That we cannot fully specify or list these totalities is not a logical objection. These totalities taken together comprise reality.

The totality of elementary propositions, together with the N operator, to generate complex/everyday propositions out of elementary propositions, represents reality. However the totality mentioned in 4.11 is neither the world, nor reality. It is the totality of true propositions *irrespective of time*. And this is, I think, consequential.

7.2 Necessities and accidental generalisations

To grasp Wittgenstein's views on science and laws of nature one must distinguish carefully between propositions that are necessary, and propositions that happen to be true at all times and in all places, the latter being *accidental generalisations* (cf. p85):

> The mark of a logical proposition is *not* general validity. To be general means no more than to be accidentally valid for all things. (6.1231)
> The general validity of logic might be called essential, in contrast with the accidental general validity of such propositions as "All men are mortal". (6.1232)

Necessity—logical necessity—applies *only* to propositions that are tautologies and contradictions, shown by their propositional signs. Propositions that happen to be true at all times and at all places are not necessary, they are merely accidentally universally true. Formally, validity is limited to the propositions of logic, i.e. tautologies and contradictions, and does not apply to what is accidentally general.

Two consequences follow. First, Wittgenstein rejects any mooted reductive account of necessity in terms of truth in all possible worlds, because this collapses the necessary into the accidentally general. The second concerns the precise wording of 4.11. When Wittgenstein says, "The totality of *true* propositions is the whole of natural science" (emphasis added) what I think he intends is the totality of propositions that are, contingently, true at some

moment in time and somewhere in space. It is logically possible for any one or more of them to be false at present, or to be false at some time in the past or in the future. What has not as yet been addressed is the question of how we can identify such propositions. Wittgenstein does not pursue the question in this form, because this is a matter for empirical science and not for logic. The issue is nevertheless involved in the *forms* of the laws of nature, discussed in the 6.3s.

7.3 The forms of laws

The obvious form of a law is, roughly, whenever something X happens, something Y follows. Or we might say that Xs cause Ys. In early work Armstrong argues that we can think of laws of nature as of the form $N(F, G)$, where N asserts a physical necessitation between Fs and Gs, these being instances of what he calls "quasi-universals"; they "resemble universals in permitting a multitude of instances, but unlike universals proper they involve temporal or spatial restrictions." (Armstrong 1983, p.80) Now in the case of the *Tractatus* what is of interest is a change in the distribution of truth-values over the totality of elementary propositions. So if we have a world w_t now, represented by a totality $\bar{\xi}_t$ of true elementary propositions, at w_{t+1}, whatever temporal increment is involved, there will be a different distribution of truth-values and thus a different totality of true elementary propositions, say, $\bar{\xi}_{t+1}$. If we think in terms of states of affairs instead of Armstrong's quasi-universals, and instead of temporal or spatial restrictions, that a state of affairs may come into being at a time t_1 if at that moment an appropriate collection of objects so combines, and of it ceasing to obtain at a later time t_n when the state of affairs disintegrates and the objects concerned recombine with other objects as required to form different state(s) of affairs, then Armstrong's N relates to, I think, some measure of the likelihood of the states of affairs obtaining at w_t being followed by the states of affairs at w_{t+1}. This, I want to assert, is not a random process, that it is not the case that any possible world can be followed by any other possible world.

In later work avowedly influenced by the *Tractatus* Armstrong put forward an account of laws of nature with $N(F, G)$ construed as a causal relation obtaining between states of affairs types, where a state of affairs is a non-mereological composition of suitable constituents (Armstrong 1997,

p.122). Without getting too deeply into Armstrong, the point that he raises concerns the connection N between states of affairs types; where for the sake of the argument his states of affairs types are sufficiently close to Tractarian states of affairs as combinations of objects according to the forms of the objects concerned to make his account of interest, not least because Tractarian states of affairs can be thought of as types (because there can be more than one object of any given form). The question is, what is the causal relation Armstrong requires? And how can thinking about this assist in understanding the *Tractatus*?

The reason for asserting some such relation between states of affairs types as following in a temporal sequence (ignoring the spatial, beyond assuming that one is within the light cone) is that the world as we experience it is not some random chaos of one damn thing after another. And here I think we should not be misled by the remarks leading up to 6.37:

> This procedure [induction], however, has no logical justification but only a psychological one. It is clear that there are no grounds for believing that the simplest eventuality will in fact be realised. (6.3631)
> It is an hypothesis that the sun will rise tomorrow: and this means that we do not *know* whether it will rise. (6.36311)
> There is no compulsion making one thing happen because another has happened. The only necessity that exists is *logical* necessity. (6.37)

As we have seen (p.185), reading too much into this led De Laguna to ascribe to Wittgenstein a commitment to one damn thing after another. What he missed is Wittgenstein's very tight demarcation of the logical and, concomitantly, of knowledge. What is empirical is not logical, so of course there is no *logical* justification for induction. But to say that there is only a psychological justification need not be read pejoratively. All it means is that one's grounds for deploying induction are not logical, not that such grounds are irrational. We do not know that the sun will rise tomorrow, because we can only have knowledge in the domain of the logical; "The connexion between knowledge and what is known is that of logical necessity" (5.1362). But this is so only if one demands, as Wittgenstein seems to, that knowledge is restricted to the logical. This is of a piece with his earlier assertion that "Theory of knowledge is the philosophy of psychology" (4.1121). *Prima facie* this is absurdly strong, implying that outside of logic there is only super-

stition (*Aberglaube*, as 5.1361). It is consistent, though, with Wittgenstein's disregard for what is not logical, for the empirical; for what is somebody else's job. Where I think De Laguna goes wrong, though, is in asserting that Wittgenstein's approach rules out probable reasoning because, as will be argued, it is the notion of the probable that is central to his approach.

In going back over his earlier arguments Armstrong says, in treating laws of nature as relations between universals (between states of affairs types), "The connection was represented as $N(F, G)$ and the entailment of a universally quantified truth, a regularity, seemed a mystery. For N I am now substituting C for cause ... *The fundamental causal relation is a nomic one, holding between states of affairs types.*" (Armstrong 1997, pp.228, 227) What I want to show is how, without invoking causes or compulsions or physical or nomic necessity, one can find an account of laws of nature within the *Tractatus* such that it is reasonable to think that the sun is overwhelmingly likely to rise tomorrow.

7.4 From the static to the dynamic II

The account to be given assumes states of affairs as combinations of objects according to their forms, represented by elementary propositions, and of complex propositions as truth-functions of elementary propositions. Complex propositions are the results of applying truth-operations, specifically, N (the Tractarian N, of course, and not Armstrong's), to elementary propositions.

It also draws on the earlier account of logical space, at the level of elementary propositions, in terms of the lamp array, with a lamp for each elementary proposition (cf. §5.7, §6.3). A possible world is represented by a (maximally consistent) totality of true elementary propositions, shown by an illumination pattern to the lamp array, with each illuminated bulb standing for a true elementary proposition, and each dark bulb, a false elementary proposition.

Each illumination pattern can be represented by complex propositions. These can take the form of conjunctions, disjunctions, or material conditionals, with negation as required. Or one can simply use N. Given the interdefinability of the logical constants it is a matter of convenience which one uses. The point (as p.164) is that complex propositions represent no

addition of ontology over and above the elementary propositions that are, and equally are not, the case. This is of course the thrust of 4.0312, Wittgenstein's "fundamental idea ... that the 'logical constants' are not representative". A possible world as a way in which objects can stand to one another is represented exhaustively by a list of the elementary propositions that are true in that world. Overlaying the complex propositions that can be formed—most obviously, a conjunction of all the true ones—is no addition whatsoever. But this being so, one might ask, what do the logical constants do? And here I think one has to shift from a static to a dynamic conception, as §6.4, this being where the notion of time comes in.

7.5 Time and trajectory

To make Wittgenstein's views work it has to be the case, I think, that he holds a relational and not an absolute conception of space and time.[1] This is stated, for time, at 6.3611:

> We cannot compare a process with 'the passage of time'—there is no such thing—but only with another process (such as the working of a chronometer). Hence we can describe the passage of time only by relying on some other process. (6.3611)[2]

The issue is the transition from one possible world to another, as a more or less extensive disintegration of states of affairs into objects followed by a more or less extensive recombination into a different totality of states of affairs. We can perhaps start up a metronome and think, we have a possible world as actual now—tick, and now a different possible world as actual now—tick, and now a different—and so on. The metronome is of course itself an aspect of the world and subject to the same strictures. But given that our concern is with transitions between actualised possible worlds, the trajectory of the world through logical space (§6.4) can be regarded as a temporal process (perhaps even *the* temporal process). There is no external or absolute perspective on this process. Whatever accounting procedures we use (mechanical, electrical, frequencies of the vibrations of crystals) are

[1] I note that Russell advocated dispensing with points and instants on the basis of Occam's razor, at *Our Knowledge of the External World*, pp.146–7.

[2] Cf. L.W. to B.R. Jan 1914, *Wittgenstein in Cambridge*, pp.64–6.

simply those that happen to be available to us. The available temporal accounting procedures can be thought of as more or less accurate and more or less coarse-grained, along the same lines as the net metaphor for mechanics at 6.342. We can choose whatever temporal accounting procedure is most useful or convenient.

There is an upper bound on recombination, involving the totality of objects, but no obvious lower bound except that there can be, I think, no stationary state. Recurrence, eternal or otherwise, is also I think ruled out, but this will not be argued for.

What is of interest is the worlds that are accessible from the world at present. Both von Wright (von Wright 1982, p.200) and Bradley (Bradley 1992, pp.xix, 47ff, 54) attribute S5 to the *Tractatus*, Bradley proposing S5A with an actuality operator, as he is prepared to countenance changes in the number of objects. This goes against the text which clearly states totalities of, in particular, objects (5.5561). The correct modal logic for the *Tractatus* is, I think, S5B, with Barcan. But while this is fine from a purely logical point of view it is not entirely satisfactory, because with S5B any possible world is accessible from any other. While there is no obvious logical objection to this, the result would be unpredictable chaos; there would be no way of predicting, much less knowing, what comes next. In this respect De Laguna's point is salient, and this shows where logic runs out, and other considerations come into play. In other words logic *per se* has nothing to say about laws of nature. All logic can do is, as Wittgenstein points out, furnish "*a priori* insights about the forms in which the propositions of science can be cast." (6.34) Specifying the modal logic of the *Tractatus* as S5B addresses only the domain of the *formal*. The question of what constraints need to be added to S5B such that we get a more or less smooth trajectory through logical space such that the world is as we experience it, more or less systematic, intelligible, and understandable, is an empirical affair.

7.6 A probabilistic account

Reverting to the lamp array model, each possible world is represented by an illumination pattern. Each time the metronome ticks the illumination pattern changes. Concerning laws of nature, what is of interest is how this illumination pattern changes, because this is an indicator of the "closeness

of possible worlds" (p.187). We also have to allow for the claim that, as the metronome ticks, the sequence of possible worlds is not determined. If we take Armstrong's formulations, $N(F, G)$ (nomic necessity), or $C(F, G)$ (cause), and think of N, or C, as the impulse that takes a world to the next temporal world, and F, G as the world at t and at $t + 1$, then our account cannot come out as postulating that only one world, w_{t+1}, can be accessed from W_t. Whatever the impulse involved in the transition from one world to the next, it cannot be necessitating, even if *in fact* there is only one such world (this would be an accidental generality, hence not logically consequential). In practice there is likely to be a range of worlds sufficiently close to any other, such that we can think in terms of a causal or law-like relation between worlds that is not logically necessitating. What is needed is, I suggest, an account of what is most likely to happen next; that is, a *probabilistic* account.

The account of probability given in the *Tractatus* is a logical account, as Chapter 6, based on counting the number of $\top$s in propositional signs. Given the independence of elementary propositions, in any possible world the probability of an elementary proposition $(\top\bot)(p)$ being true is $\frac{1}{2}$, of it being false, $\frac{1}{2}$. Beyond that, the probability of $(\top\bot\bot\bot)(p \wedge q)$ being true is $\frac{1}{4}$, of it being false $\frac{3}{4}$, shown by the propositional sign. For $(\top\bot\bot\bot)(p \vee q)$ the probabilities are $\frac{3}{4}$ and $\frac{1}{4}$ respectively. The account extends straightforwardly from here.

These measures are of course *a priori*. If we consider the world as it is now, then we have no need to think probabilistically; for any elementary or complex proposition, it is either true, or false (ignoring tautologies and contradictions), so its probability has crystallised as 1, or as 0. But the point about probability is that we are interested in the transition from one state to another, that is, a process that unfolds over time. We play cards, perhaps, or draw balls from urns, as 5.54. What we are interested in is the likelihood of what happens next, that "The circumstances—of which I have no further knowledge—gives such and such a degree of probability to the occurrence of a particular event." (5.155)

What we are interested in, then, is the likelihoods involved in the changes in truth-value distribution occurring as the world follows a trajectory in logical space; the likelihood of the truth or falsity of an elementary or complex proposition being followed in the time-period of interest by the truth or

falsity of the same, or different, elementary or complex propositions. This likelihood is, I suggest, a way of getting out laws of nature not as necessitating, but as probabilistic. The closer the measure gets to 1 the more law-like a proposition becomes, where such a law is, of course, an accidental generalisation and not a necessity. Given that we are unable to survey all of space and time probabilistic laws are the best we can attain.

If a law is expressed as a conditional of the form $\forall x.fx \supset gx$, and if we think of fx and gx as states of affairs types in temporally adjacent worlds, then what we are interested in is successive pairs of worlds where fx in w_t is followed by gx in w_{t+1}. The form of a law is then a generalisation over couples of states of affairs types; so if F and G are schematic letters for states of affairs types, the law takes the form, perhaps, $\forall_{FG}(F;G)(F \supset G)$, to be read, for all couples $(F;G)$, where F is in w_t and G is in w_{t+1}, F in w_t is followed by G in W_{t+1}. For this to be of interest there would need to be localisation, that is, there would need to be a distinguishing between F occurring in w_t and G occurring in w_{t+1} where all or some, or where none, of the objects (considered as tokens and not as types) involved in F, are comprised in G. This would be a part of a doctrine of closeness of worlds.

In principle what is sought is a measure of similarity and difference between illumination patterns—truth-distributions—in temporally successive worlds. Empirical data based on observations of change, that is, a comparison of Ts and Fs, would be used to generate the probability of sequences of states of affairs and thus to formulate laws that are essentially probabilistic. So rather than $\forall_{FG}(F;G)(F \supset G)$, laws might take the form $P(F;G)(F \supset G)$, where P is a measure of probability taking a value strictly between 0 and 1.

The account also furnishes an explication of the logical constants, beyond asserting that they are not representative (4.0312), that all they do is show the combinatorial possibilities of elementary propositions in terms of $\top$ and $\bot$. The logical constants are allotted a role as marking probabilistic relations between possible worlds; that counting the $\top$s and $\bot$s in sequences of possible worlds enables prediction of what is more or less likely to happen next.

7.7 Boltzmann and Hertz

This does not, though, account for Wittgenstein's 1931 remark listing Hertz and Boltzmann among the principal influences on his work. Hertz is mentioned twice, briefly, with respect to dynamical models and multiplicity at 4.04, and "that only connexions that are *subject to law* are *thinkable*" at 6.361, and Boltzmann, not at all. So wherein lies their influence?

It is true that Wittgenstein appropriates the work of others to inspire his own thinking, generally without acknowledgement. We know that he read at least the introduction to Hertz's *Principles of Mechanics*, and that he read Boltzmann's *Populäre Schriften*. But one would hardly take Wittgenstein as a reliable guide to anybody else's work, and it is not clear how much of anybody else's work Wittgenstein either read, or understood.

Rather than approach the issues through a consideration of Hertz, or of Boltzmann, or by looking for parallels in the text, it might be better to start by looking at the difficulties Wittgenstein wrestles with. Wittgenstein is keen to dissociate logic and the natural sciences exhaustively by assigning the status of truth-functional tautologyhood/contradictoriness to the propositions of the former, and contingency/bipolarity to the propositions of the latter. Empirical considerations arise only with the latter. In drawing this distinction accidental generalisations are unequivocally categorised as contingent/bipolar. So, one might ask, how does the work of Hertz, and Boltzmann, assist in making this distinction?

To take Boltzmann first, it is plausible to hold that his treatments of phase space, the evolution of thermodynamic systems over time, entropy, and laws as probabilistic, exercised some influence, reflected in the probabilistic account given earlier. This is familiar and, in principle at least, I do not think it is contentious. Wittgenstein's definitive statement that "The only necessity that exists is *logical* necessity" (6.37) is entirely consistent with this. The account given is, I think, a fair recapitulation of at least *a* Wittgensteinian appropriation, although I would seek to make the point more definite than this (i.e., I think the account given is plausibly *the* Wittgensteinian appropriation).

Second, the issue of models and picturing (*Bildtheorie*). One can attribute influence here to either Boltzmann, or to Hertz. The notion of modelling is so ubiquitous in nineteenth century science that to try and make

much of it is perhaps otiose, although one could argue that Boltzmann and Wittgenstein exhibit a certain originality in extending the notion beyond physics, to all our thinking. Nevertheless what Wittgenstein has to say about models and multiplicity (isomorphism), and about making pictures (*Bilde*) to oneself is not, I think, sufficiently out of the usual run to excite particular attention.

Third, though, and this is where Hertz is particularly interesting, is the divide between the logical and the empirical, between the *a priori* and the *a posteriori*. Because it is here that the distinctive Wittgensteinian separation comes into play.

7.8 A hard distinction

In the *Tractatus* there is a hard distinction between the logical, and the contingent. This relates to the formal properties of complex propositions, as to whether they are tautologous/contradictory, or meaningful/bipolar. In particular the latter are, whereas the former are not, representative of how things may, or may not be, in the world.

As noted, accidental generalisations are not logical propositions, because they are not truth-functional tautologies. Laws of nature are accidental generalisations, stated in contingent propositions. The account given in probabilistic terms fits with this. It also leaves space for a notion of *form* for the propositions in which such laws can be cast. Laws of nature are empirical generalisations, whereas laws of logic are tautologies; there is consequently a hard distinction between them, consistent with the hard distinction drawn between philosophy and the natural sciences in 4.111ff, and with the claim at 4.11 that "The totality of true propositions is the whole of natural science"; where this should be construed, I think, to exclude the propositions of logic because, as §2.9, they are not true *in the same way*.

While this is consistent with the opening propositions of the 6.3s, when Wittgenstein gets to Newtonian mechanics at 6.341 things become more difficult. Because what we seem to get is a view that places aspects of mechanics on the *a priori*/necessary side of this hard distinction. There are two immediate points to make. The first is that Wittgenstein's precipitate recourse to the net metaphor in 6.341 is intensely frustrating, because rather than set out a position in terms that the reader can engage with straightforwardly he

puts up a metaphor, or perhaps an analogy, that leaves one near-hopelessly entangled in interpretation. The second point is that the hard distinction Wittgenstein draws at least parallels that drawn by Hertz between Books I and II of his *Principles of Mechanics*, leading one to expect some adherence to Hertz's views. It rapidly becomes unclear, though, how much Wittgenstein draws on Hertz, given that of the three images of mechanics that Hertz sets out in his introduction; (1) a Newtonian image, based on space, time, mass, and force, (2) an energeticist image, based on space, time, mass, and energy, and (3) Hertz's preferred image, based on space, time, and mass, what Wittgenstein says does not make it clear whether he adheres to (1) or to (3). Since he clearly envisages different systems of mechanics (particularly in 6.342) he may have held that the choice is a matter for the physicist, and not for the logician.[3]

The first difficulty is of getting a grip on Hertz's project. It is not self-evident why mechanics, which looks *prima facie* to be an empirical affair, should trade in the *a priori*; that is, in what is independent of experience. Sklar observes that "there is, of course, no pretence on Newton's part that his first principles could themselves be established without reference to empirical experiment", adding, parenthetically, "It was left to Kant to fall into the trap of trying to establish Newtonian dynamics as a fully *a priori* science." (Sklar 2013, p.214) If this is a fair description, then the establishing was primarily effected by Kant in the *Metaphysical Foundations of Natural Science*, a work that Hertz records reading in February 1884 (Hertz 1977, p.191). In the preface Kant offers a hard distinction between the *a priori* and the empirical:

> All *proper* natural science therefore requires a *pure* [i.e., *a priori*] part, on which the apodictic certainty that reason seeks therein can be based. And because this pure part is wholly different, in regard to its principles, from those that are merely empirical, it is also of the greatest utility to expound this part as far as possible, separated and

[3] Eisenthal suggests as a parallel between Wittgenstein and Hertz that "the three different nets [of 6.341]—square, triangular, and hexagonal—can be thought of in comparison with the three different formulations [in Hertz]—traditional, energetics, and Hertzian." (Eisenthal 2024, p.178) But this doesn't obviously square with 6.342, where Wittgenstein suggests that "The net might also consist of more than one kind of mesh: e.g. we could have both triangles and hexagons."

> wholly unmixed with the other part. (*Metaphysical Foundations of Natural Science*, Ak.4 p.469)

Although, as Lützen points out, there is no mention of Kant by name in the introduction to the *Principles of Mechanics* (Lützen 2005, p.124), the reader of the introduction will hardly miss Hertz's use of the word "pure" at p.33. The Kantian aspects of Hertz's work are emphasised by Hyder (Hyder 2003) and comparatively downplayed by Lützen (Lützen 2005, pp.123–4), but there is little evidence of sustained interest in philosophy in Hertz's writings. The only philosophers he read were Plato, and Kant, both the *Metaphysical Foundations of Natural Science* and the *Critique of Pure Reason*.[4] He also read Mach's *Science of Mechanics* (Hertz 1977, p.191), and acknowledges him in the Author's Preface to the *Principles of Mechanics*.

In discussing Hertz, Saunders asserts that "Of course [he] was neither a philosopher nor a historian" (Saunders 1997, p.145), while Hyder states that if one approaches the *Principles of Mechanics* "as an attempt to eliminate the troublesome category of force from our ontology and replace it with 'intuitively imaginable' rigid connections between mass-points in spatio-temporal intuition", this "imports far too much philosophy into Hertz's undertaking, which was after all a work in theoretical physics." (Hyder 2013, p.272) Nordmann says that in the *Principles of Mechanics* Hertz "appeal[ed] to a vague but widely held neo-Kantianism which was a defining feature of the scientific community in Germany." (Nordmann 1997, p.162) What this seems to come down to is not a commitment to any specific set of neo-Kantian views, whether of the Marburg or the Southwest school, but rather of working within a broadly Kantian framework with a distinction between the synthetic *a priori*, and the *a posteriori*. What matters, though, is the significance of this for Wittgenstein, and because it is familiar ground, I will not discuss the influence of Hertz's remarks on conceptual clarification at *Principles of Mechanics*, pp.7–8, 33.

In the middle of the discussion of the net analogy, Wittgenstein says this:

> Mechanics determines one form of description of the world by saying that all propositions used in the description of the world must be obtained in a given way from a given set of propositions—the axioms of mechanics. It thus supplies the bricks for building the edifice of

[4] Cf. Hertz 1977, pp.29, 31, 51, 119, 125, 191.

> science, and it says, "Any building that you want to erect, whatever it may be, must somehow be constructed with these bricks, and with these alone." (6.341)

Wittgenstein's continued resort to metaphor does not fill the reader with confidence. His point seems to be that the axioms of mechanics are wholly general because they are involved in any scientific theory or conception of what there is. But this does not make it clear whether the propositions in which the axioms are stated are *a priori* in the sense of being truth-functionally tautological—or even obvious (non) impossibilities (cf. §2.9)—or whether they are merely accidental generalisations. This point will be returned to. For the moment one might well ask, what does he have in mind by the indispensable bricks, and by the assertion that *we* do the constructing?

7.9 The human stain

The difficulty lies, I think, in getting ourselves out of the picture. A way in is perhaps via a remark of Burge's, in a discussion of Kant's influence on Frege concerning the *a priori*, with respect to geometry and intuition:

> If one strips [Kant's] view of its idealist elements, one can regard pure sensible intuition as a faculty for intuiting the pure structure (not of the faculty itself but) of mind-independent space and time. (Burge 2000, p.31)

Now it is reasonable to say that we have some grasp, which we can call "sensible intuition", of ourselves, and of things, as being in space and time. Evans puts forward a series of compelling arguments to the effect that "our self-conscious thoughts about ourselves also rest upon various ways we have of gaining knowledge of ourselves as physical things" (Evans 1982, p.213), and that location matters to our capacity to keep track and thus to make sense of our world (Evans 1982, pp.278ff). Without getting too deeply into this, the point is that we do seem to have *some* grasp of presence or location in space and time, and of displacement in space and time that, while not exactly independent of space and time, is fundamentally constitutive of our thinking. What Burge has in mind, I think, is that if we strip this approach

of transcendental overtones and think in terms of a mind-independent space and time, and not in terms of a faculty of pure sensible intuition, then we have the beginnings of a kinematics, to which we can reasonably accord the status of apriority. It is such stripping that, I think, lies behind Hertz's and Wittgenstein's views.

The difficulty remains, though, of pinning down the hard distinctions drawn by Hertz and by Wittgenstein, and of clarifying what lies on which side of these distinctions. For Hertz, what is *a priori* in Book I of the *Principles of Mechanics* is space and time in terms of "internal intuitions" (§§1–2), and a conception of mass, given by definition (§§3–8). We do not experience space, time, and mass directly; "only definite times, space-quantities, and masses ... may form the result of a definite experience" (§297). The alignment between theory and realisation is ensured by the *correctness* of a theory, that "the necessary consequents of the images in thought [physical theory] are always the images of [representations of] the necessary consequents in nature of the things pictured." (*Principles of Mechanics*, p.1). What is surprising, though, and not easy to understand, is that Hertz places his Fundamental Law, his encapsulation of Newton's three laws—"Every free system persists in its state of rest or of uniform motion in a straightest path" (§309)—in the *a posteriori* Book II and not in the *a priori* Book I.

At this point, despite earlier reservations (§1.14), it is hard to see how we can make progress without countenancing what is, for us, conceivable. In talking about the "indispensable bricks" one cannot readily make any sense of this other than by talking in terms of more or less impenetrable and persistent material bodies, located in and moveable in a mind-independent space and time. To say that they have mass, and thus inertia, is to say that we cannot conceive of material bodies other than in terms of certain interactions that involve pressure and impact, because this is how mass (inertia) is experienced (interacts with ourselves and with our instruments). We do not *have* to think of mass in terms of force, a line of thought deployed by Hertz in order to dispense with the latter in his preferred image ((3), p.200) in favour of the actions of hidden masses. What can be substantiated along these lines is a *conception* of Newton's laws, restated, perhaps, and more or less adapted as needed, but nevertheless *a priori* and *not* necessary.[5]

[5]Perhaps along the lines of the dynamic or relativised *a priori*, as suggested by Cassirer; cf. Stump 2015, pp.90–4. See also Friedman 2001, pp.64, 66, 71ff.

The laws so arrived at are *a priori* in the sense that while we may think that we can conceive of them as being otherwise, this is an empty possibility because we can't give any *content* to the bare claim that we "can conceive of them being otherwise". They are not necessary because they are not truth-functional tautologies (and are not obviously conceptual truths either).

This gives us perhaps the best available way of making sense of the precise distinction that Wittgenstein draws when he says, "And now we can see the relative position of logic and mechanics." (6.342) The indispensable bricks are space, time, and mass, as mind-independent. Beyond this it is all a matter of how we arrange or axiomatise our thinking. So "the possibility of describing the world by means of Newtonian mechanics tells us nothing about the world: but what does tell us something about it is the precise *way* in which it is possible to describe it by these means" (6.342); that is, the ways in which *we* can describe what happens, and what happens next.

Now the *possible* relates, as we have seen (§1.9), to the forms of objects; the notion of objectuality, that all that can be experienced is a manifestation of the combinatory possibilities and thus the forms of the objects that there are. Space and time are "forms of objects" (2.0251), not because objects are spatial or temporal, but because space and time as we experience them "are produced . . . by the configuration of objects." (2.0231) Similar remarks, I think, apply to the notion of mass, this also being a material property consequent on configurations of objects. Noting that none of this precludes individual objects being locatable relative to one another in space and time, or of having mass.

Paying attention to what, exactly, Wittgenstein says, that "the *possibility* of describing the world by means of Newtonian mechanics tells us *nothing* about the world" (6.342, emphasis added), shows that what is possible lies within the domain of logic, that is, of what is *shown* in tautologies and contradictions, because these do not say anything about the world (they are not representative). At this juncture Wittgenstein is operating at a very high level of abstraction. What he wants to do is set out what is *purely* formal, where at bottom this comes down to the forms of the objects that there are.

As we have seen, that objects have forms falls within the domain of logic. But the content of these forms—the range of combinatorial possibilities for an object into states of affairs—falls within the domain of empirical enquiry. Hence *that* the axioms of mechanics, the geometry of the mesh, *can* describe

the world completely, falls within the domain of logic. But the geometry of the mesh is an empirical affair, a matter of pragmatic choice (what is simplest for the purposes to hand, as 6.363). It is a mistake, though, to think that what is optional extends from *there being a net independently of the choice of mesh geometry*, to it being optional *that there is a net at all*. Tejedor interprets 6.341, that the form of *a* system of mechanics is "optional" or "arbitrary" (*beliebig*), as rendering *all* such forms optional:

> [T]hese forms are optional in that none of them are essential requirements of language or representation. The implication here is that it is possible to construct a description of the world without adopting any of these forms—that is, without adopting any of these sets of principles from the natural sciences. (Tejedor 2015, p.104)

Sluga takes a similar line, arguing that this arbitrariness should extend to *all* propositions:

> The devastating implications of . . . this for Wittgenstein's official 'mirror' account of meaning have generally gone unnoticed . . . A sentence and the state of affairs it is about must have the same logical form. But this obviously does not hold for the propositions of Newtonian mechanics. Their form is said to be arbitrary. Why then should this not also be true of all other kinds of propositions? (Sluga 2012, p.102)

But it does not follow from saying that *any* form is arbitrary, that the notion of form is itself somehow arbitrary. One only gets Sluga's "devastating implications" if one falls into the error of sliding from *any* to *all*. That there is a net is necessary, it is its configuration that is optional; in other words, *form* is necessary, shown in the logic that we have. It is also necessary that objects have *content* (the content of 2.025), but the ascertaining of content is an empirical affair. It is not arbitrary that objects have content. The point here is the same as that made earlier, concerning independence (p.81). It is again a matter of precise location of distinctions. But what, one might ask, does this say of the axioms of mechanics? With reference to the hard distinction, how exactly are these located?

7.10 The axioms of mechanics

At this point it is difficult to extract a completely lucid doctrine from the text. 6.341 states that there are axioms of mechanics, and from these one can, it would seem, construct a particular system of mechanics. So perhaps behind Hertz's three images (p.200) there is "a given set of propositions" (6.341) of which these are, presumably, intertranslatable presentations, each capable of describing the world completely, differing only in terms of simplicity (6.342). But while this looks to cohere with 6.341 there are, I think, two fairly immediate difficulties.

The first is that while Hertz's three images are ostensibly interchangeable, because they address the same phenomena, given the difference in conceptual resources it is not clear that they are intertranslatable and thus that the choice between them is wholly a matter of simplicity. It could be, as McGuinness says, that "Wittgenstein does not say how a system should be chosen, but clearly his view is a variant of the conventionalism not uncommon in his day." (McGuinness 1988, p.127) But to say that the choice is a matter of convention is not, I think, synonymous with saying that it is a matter of simplicity. Because to say that it is conventional implies some degree of more or less arbitrary postulation, whereas simplicity is more obviously a matter of mathematical economy. It isn't clear where the trade-offs lie. This, though, will not be pursued further.

The second concerns the axioms of mechanics themselves. Are they *a priori* or *a posteriori*, and how much sense can be made of such a question? Hertz clearly locates his single Fundamental Law in the *a posteriori* Book II of *Principles of Mechanics* (§309, p.203), stating that it is "inferred from experience" (§308). Planck remarked, in 1915, that mechanics "was founded by Galileo and Newton, and brought to its final form by Euler and Lagrange. This branch of physics forms a complete picture, leaving nothing to be desired in rounding off and filling in, and it can emulate a mathematical theory in strictness." (Planck 1925, p.83) But to be complete and finished is not to be *a priori*.

It would take the wisdom of Solomon, perhaps, to decide exactly where the distinction between the *a priori*/necessary and the *a posteriori*/contingent lies in this domain. McGuinness talks of "general Principles of Nature", these being the principles of induction and least effort and the law of causal-

ity, before saying that:

> These Principles of Nature ... are clearly not tautologies or contradictions but there are two ways detectable in the *Tractatus* in which they can be viewed indirectly as *a priori*. The first is this; it is obvious *a priori* that there can be particular laws connecting phenomena or physical events or measurements, rules that conform to these Principles ... The second and more complicated way in which the Principles can be viewed as *a priori* is this: we may in fact adopt them and make them part of our network for describing the world. (McGuinness 1969, p.118)

As well as blurring Wittgenstein's hard distinctions, this fails to address the status of the axioms of mechanics. The "indirectly *a priori*" status of the high-level Principles and the "obvious *a priori*" fact that there can be particular laws drags us into the domain of what is conceivable by us.

While Wittgenstein has, I think, a very clear image in his mind and knows what he wants to say, he fails to carry out the programme sufficiently that the difficulties of execution come to light. If there are aspects of mechanics that are *a priori* and necessary, that is, are wholly concerned with form, then it should be possible to state these as truth-functional tautologies. But of course Wittgenstein couldn't do this, which is why in between the talk of form we get the remark in 6.3431, that "The laws of physics, with all their logical apparatus, still speak [*sprechen*], however indirectly, about the objects of the world." This should remind the reader of 6.124, that "It is clear that something about the world must be indicated [*anzeigen*] by the fact that certain combinations of symbols—whose essence involves the possession of a determinate character—are tautologies." How is the hard distinction to be bridged? What is being gestured at—form, that is, objectuality, how the forms of objects manifest themselves—is clear enough. But the detail of how this is to be substantiated is sorely lacking.

7.11 Conceptual truths

At bottom there just is no obvious or even non-obvious way of assessing whether or not the laws at stake are *a priori*. Wittgenstein clearly holds

that neither the laws of nature nor the axioms of mechanics can be truth-functional tautologies. There is, consequently, no ready way of making the hard distinction that he wants to make, leading to the recurrent suggestion that we have now seen in von Wright, Baker, and Lampert, in §2.9, that in Tractarian terms there are truths that are somehow necessary but are not truth-functional tautologies.

In the light of this we can perhaps reconsider the net/mesh metaphor. This has not received a great deal of commentary and is usually glossed over.[6] For example, with particular reference to 6.35, Morris says:

> These higher-order laws [causation, least action, induction] are the laws which it is natural to regard as necessary: we can see here that Wittgenstein holds that they do not really tell us anything about how the *world* is—rather, they 'treat of the network'. (Morris 2008, p.317)

But what does this *mean*; why the resort to quote marks? Morris is, I think, aware of the tensions noted by Baker, Hyder, and McGuinness, but this only manifests itself rhetorically:

> Where do such things as higher-order laws fit here? On Wittgenstein's account they seem to be neither tautologies nor empirical statements. Does he, then, after all, assign them an intermediate status, like that of Kant's synthetic *a priori* judgments? (Morris 2008, p.319)[7]

Morris concludes that these higher-order laws "are not really laws at all: in fact, they are not even meaningful, not even sentences. They are attempts to say what cannot be said." (Morris 2008, p.320) Now of course the whole book is nonsense, on its own terms, but Newtonian mechanics is not. If Wittgenstein thought that physics is wholly *a posteriori* and empirical then he could have said so in a single sentence. So what is it to "treat of the network"?

Of the net/mesh metaphor, with respect to Newton and Einstein, Griffin says this:

[6] Eisenthal notes discussions in Black 1964, Griffin 1964, Barker 1979, Hyder 2002, and Tejedor 2015 (Eisenthal 2024, p.178n.21).

[7] That there is an issue here is acknowledged by Diamond; in a 2017 note added to Diamond 2004 she says, "Some time after I wrote this essay, I recognised the significance for the *Tractatus* of propositions that are not bipolar, not tautologies or contradictions, and not nonsensical." (Diamond 2004, p.95n.1)

> Wittgenstein's point is that many scientific laws are like the mesh in this illustration [the black and white surface of 6.341]. Their function is not to make reports . . . but to supply representational techniques by which reports can be made. Thus, when this kind of law is superseded, it is not, for it cannot be, falsified. What happens is that a better method of representation is found . . . what Einstein did was to discover a new representational form, to present a finer grid. (Griffin 1964, p.103)

If we consider two inertial frames in one dimension moving with different velocities then the distance between them is given by the Galilean transformation:

$$x_0 = x_1 - vt$$

where v is the difference in velocity between the frames x_0 and x_1.[8] In special relativity this becomes the Lorentz transformation:

$$x_0 = \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where c is the speed of light. This adjustment is relevant in the case of, for example, GPS satellites in geostationary orbit, where x_0 is the Earth and x_1 the satellite. One could assert, perhaps, that the relativistic version is more accurate—"a finer grid"—but this just looks to extend the metaphor, while side-stepping the question of truth; whether the later theory falsifies the earlier.[9] But at bottom it is not clear that there is any more sense than this to be made of the mesh/net metaphor.

Standing back somewhat, what we see in the historical development of mechanics is a reformulation of principles, these becoming increasingly general in application and in mathematical sophistication.[10] Wittgenstein's talk

[8] Cf. Einstein 1954, Chapter 11.

[9] Watson says, "It is not a matter of Newton's laws being wrong and Einstein's being right . . . in physics we choose the particular method of representation adequate to the purpose in mind" (Watson 1938, p.44). Ladyman and Ross talk of "a partial continuity of mathematical structure" (Ladyman and Ross 2007, p.95), but this leads into very deep waters.

[10] A story well told in Sklar 2013.

of the indispensable bricks and reformulations thereof, and the use of different mesh geometries, perhaps captures the spirit of this. The point that the higher-order laws, of causation, of least action, and induction are not falsifiable coheres with his view that while they don't say anything about the world directly, they "still speak, however indirectly, about the objects of the world." (6.3431)[11]

Wittgenstein has, I think, a plausible story to tell, but it is hamstrung by his hard-edged distinction. If one restricts knowledge to the logical then it is reasonable to say that the deliverances of induction, of empirical enquiry generally, are warranted only on pragmatic grounds, where the pragmatic is construed as psychological; and the psychological is in turn construed as whatever is not logical. But what Wittgenstein is unable to do more than gesture at metaphorically—the net—is a hard distinction between *a priori*/necessary principles in physics, and what is straightforwardly empirical. His difficulties may flow from nailing his colours to the Hertzian mast, because although Hertz's work "did influence the later development [of physics] in several ways, it never became the fundamental basis of a new program" (Lützen 2005, p.287). It may, at bottom, be a mistake to think that there is a viable Hertzian/Tractarian hard distinction to be found; perhaps captivated by the Hertzian picture, Wittgenstein backed the wrong horse. The underlying point is, though, that the failure to establish a hard distinction fails to separate logic definitively from the empirical; and thus fails to establish logic as wholly *sui generis*.

[11] Black says that the phrase "however indirectly" should be deleted from the Pears and McGuinness translation, because it is not in the original German (Black 1964, p.361). Klagge however argues with respect to *Letters to C.K. Ogden*, p.50 that Pears and McGuinness's phrasing is justifiable (Klagge 2022, p.258).

Chapter 8

Objects

There are two remaining topics to be addressed. First, an account of objects is required, consistent with the demands on objects and naming set out in §2.4. Second, an account is needed of colour exclusion, showing that Wittgenstein had a plausible account of this—of the Independence Thesis—at the time of composing the text.

8.1 Objects as substance

A central thesis of the present work is the doctrine of *objectuality* (§1.9), that the world as we experience it is a function of the objects that there are and their combinatorial possibilities, that is, their *forms*; it is "only by the configuration of objects that [material properties] are produced" (2.0231). We experience the world in terms of these material properties. We experience objects, certainly, but in thinking and in making pictures to ourselves we do not experience objects individually.

Further, the remark that "Objects make up the substance of the world" (2.021) is taken entirely seriously. Substance is, I suggest, the basic or elemental stuff things are made of, this elemental stuff furnishing the content of 2.025; that "It [substance] is form and content." But in saying that substance is the elemental stuff of the world, one has to be very clear about what is dependent and what is independent, that is, the distinction between *world* and *reality*; between what is the case now, and what can be the case, whether it is or is not at present. At any given moment substance, the objects that there are, is/are combined into states of affairs. Substance equates

to reality, as what endures or persists (2.027), with the world (any world) a configuration thereof, as 2.024; "Substance is what subsists [*besteht*] independently of what is the case." What matters is that the stuff of the world is such that it can be reconfigured, because the trajectory of the world through logical space arises from more or less extensive recombination of the objects that there are, in accordance with their forms. And as already noted (§1.14) one has to be very clear what is meant by *independence* (*Unabhängig*). Substance can be thought of independently of what is the case, but at any given moment it is wholly combined into states of affairs.

8.2 The logical and the empirical

What is needed next is a clear distinction between the logical and the empirical. In saying that substance "is form and content" (2.025), for objects, there is a clear distinction between what is within the purview of logic (form) and what falls to the natural sciences (content). All that can be said, formally, of objects is that they have forms, and that the totality of objects is fixed, given in advance. There is no effective difference between thinking of an object's form as a range of combinatorial possibilities, or of thinking of it in terms of the totality of states of affairs it can be a constituent of. These are different ways of expressing the same thing.

It might be objected that objects are *logical* objects and consequently it is a mistake—a category error, perhaps—to treat them as physical entities. My reply is that this leaves logic as an uninterpreted calculus dangling in mid-air, failing to connect with its application[1]; whereas Wittgenstein is clear at 5.557 that "Logic has to be in contact with its application." The proponent of an wholly top-down, *sui generis* reading cannot, I think, say how the "propositions of logic" are such that "*something about the world* must be indicated by the fact that certain combinations of symbols ... are tautologies" (6.124, emphasis added), or do justice to 6.3431, that "The laws of physics ... still speak ... about the objects [*Gegenstände*] of the world." Such a reading cannot, I think, accommodate the determinacy of sense, or the substance argument (that there is something and not nothing). If however the proponent of an wholly *sui generis* logical approach manages

[1] As Proops puts it, referring to 5.5521, this would be "a conception of logic as somehow entirely without existential preconceptions, a free-floating entity having its nature quite independently of whether or not there is a world." (Proops 2000, p.90n.207)

to stay in the saddle despite my best efforts to get the facts to buck, then so be it; there is no meeting of minds. The point about application is that what we think and speak of is what there is, and this, I think, is critical.

In distinguishing the logical from the empirical, as §5.11, from a logical perspective it is enough that there are objects, that they have forms, and there is at most $\aleph_0$ of them.

8.3 Space time and colour as formal

In accord with the distinction in Chapter 7 between the *a priori*/necessary and the *a posteriori*/empirical with respect to mechanics and natural science, what there is to say about space, time, and mass lies on the side of the *a priori*/necessary. That is, space, time, and mass are formal properties of objects; this is the point of 2.0251, "Space, time, and colour (being coloured) are forms of objects."[2] It would help the argument run here if Wittgenstein had said mass rather than colour, but I don't think this particularly undermines it. For objects to produce space and time as we experience it by their presence and configuration suggests that some objects at least have mass, but it need not be the case that all objects have mass, any more than that all objects are coloured. 2.0251 says that colour is *a* form of objects, it doesn't say that *all* objects are coloured. I think it has to be the case, though, that all objects are spatial and temporal and that, on Wittgenstein's Newtonian/Hertzian approach, all objects have mass. There is an implicit commitment to physicalism here, because in a pre-relativity/quantum mechanical world having mass is, I think, essential to existing.[3]

The reason why Wittgenstein treats colour in terms of the form of objects is, I think, because he is preparing the ground for a formal/logical solution of colour exclusion (6.3751). The colour of something is a material property produced by a configuration of objects, as 2.0231. That something is coloured is a function of the forms of the objects that are so combined.

[2]It may be significant that 2.0251 arises from a contraction of *Prototractatus* 2.0251, "Space and time are forms of objects", and *Prototractatus* 2.0252, "In the same way colour (being coloured) is a form of visual objects." Omitting the word "visual" suggests that Wittgenstein may have distinguished form for colour from form as it relates to space and time; if so, any such distinction is erased in the *Tractatus*.

[3]If this is so, it accounts for Wittgenstein's later accusation of plagiarism by Carnap, on the basis of "the claim that physicalism is in the *Tractatus*" (Stern 2007, p.322).

8.4 As if this were science

In the quest for objects an early consideration is whether one's candidates are physical entities. In a revealing comment on remarks in *Ludwig Wittgenstein and the Vienna Circle*, Pears says that Wittgenstein "relied on his *a priori* argument for his extreme version of logical atomism and expected the details to be discovered later" (Pears 1987, p.84), footnoted, "*As if this were science*" (Pears 1987, p.84n.72, emphasis added). This, I think, is exactly what Wittgenstein had in mind, that the specification of objects in the *Tractatus* was complete *so far as the logical aspects go*. Objects have forms, that is, restricted combinatorial possibilities into states of affairs. They constitute space, time, and mass, their combination producing the physical world of material properties as we experience it. But their content, how this comes about, is a matter of empirical inquiry. The *Tractatus* constitutes, I think, a promissory note, with Wittgenstein expecting empirical science to come up with suitable fundamental entities.

Two consequences follow. First, one can rule out as candidates entities proposed within wholly logical or metaphysical or conceptual enquiries that are disconnected from empirical enquiries. This is in part because such entities as Moore's and Russell's sense-data were not in need of later discovery, and in part because, as Wittgenstein emphasises, there can be no surprises in logic (6.1251); the logical is fully specified, it is the empirical (the content) that is lacking. If one thinks that the *Tractatus* fails because of empirical discoveries—that there is nothing that meets the specifications laid down for objects—then such failure cannot arise, I suggest, from logical, or metaphysical, or conceptual inquiries. Failure on this basis would be internal and not external and could not be surprising (although it might be a matter of consternation).

Second, if the proposed reading is at all on the right track, in concerning himself with application Wittgenstein must have had *some* view on what he thought empirical science would come up with. Pears' remark that Wittgenstein "was studiously agnostic about any features of objects that would make no difference to the occurrence of their names in logically independent elementary propositions" (Pears 1987, p.91) addresses, though, only part of the story.

To be agnostic suggests that there are things that cannot be known. This

is not, I think, Wittgenstein's position. He thought, rather, that what is *a priori*/logical about objects is already fully known; they have forms, and there are a lot of them. It is the *content* of objects, as 2.025, that needs to be catered for. Now to say what I think fits the bill for objects, the answer is the elements or atoms of late nineteenth century science. Wittgenstein would have been familiar with contemporary controversies over atomism, from his reading of Boltzmann's *Populäre Schriften* (cf. p.67).

By 1905, following Einstein's paper on Brownian motion, the reality of atoms was generally accepted (Kox 2014, p.110). But by this date physicists were already going further; by 1899 Thompson "had produced convincing evidence that electrons—or at least negatively charged ones—were real, measurable components of the physical world" (Hunt 2003, p.325), and by 1911 Rutherford's experimental work, conducted at Manchester coincident with Wittgenstein's studies there, showed that "the atom contains a central charge distributed through a very small volume" (Rutherford 1911, p.623).

Exactly what science would turn up as fundamental was, in terms of the content aspect of objects, in flux. Wittgenstein I think needed only the *a priori* assumption that whatever fundamental particles physics turned up would be classically well-behaved, as p.77. To say that in principle classically well-behaved particles can be numbered and tracked is to say that *in principle* they can be named. Given the significance of naming in the *Tractatus* it seems to me that it is incumbent on any interpretation of the text that it can do justice to this, specifically 3.203, that "A name means an object. The object is its meaning."

At this point the Ishiguro/McGuinness anti-realist, or the resolutist, might say yes, this is all very well, but now you have objects—fundamental particles—that can be named and identified prior to and independently of combination, so you are committed to a "Fido"/Fido model of naming along with all the familiar problems of somehow assembling propositions out of these, where such propositions can only ever be lists of names and not unities. In short the account given fails to respect 3.3, the context principle.

I accept that there is some force to this. On the other hand, though, the aim is to produce a reconstruction that addresses all the points made in the text. The objectors noted in the previous paragraph do not, I think, have an adequate account of naming beyond a semantical category of names, and

this presumes application without explication. This, I suggest, steers too close to psychologism. In terms of the context principle, whatever account is given has to respect the fact that the world changes, that the states of affairs that obtain at present can disintegrate and reform, similarly or otherwise. It is not enough to offer a static account, one must be able to address the dynamics, the trajectory of the world through logical space. Respecting the context principle and the unity of the proposition should not be taken so far that one cannot account for other considerations (cf. Zalabardo's account, as p.63). But first, I will address Grasshoff's assimilation of Tractarian objects to Hertz's *materielle Punkte*. The question to be borne in mind is whether or not Hertz's notions serve as a key to the puzzling aspects of Tractarian objects.

8.5 Hertzian candidate objects

The section in *Ludwig Wittgenstein and the Vienna Circle* headed "On Dogmatism" referred to by Pears, at p.214, continues:

> There is another mistake, which ... pervades my whole book, and that is the conception that there are questions the answers to which will be found at a later date. It is held that, although a result is not known, there is a way of finding it ... In my book I still proceeded dogmatically. Such a procedure is legitimate only if it is a matter of capturing the features of the physiognomy, as it were, of what is only just discernible—and that is my excuse. I saw something from far away and in a very indefinite manner, and I wanted to elicit from it as much as possible. (Wittgenstein 1979a, pp.182, 184)

The context is a discussion of elementary propositions, but as these represent concatenations of objects it is a small step to regarding this as directed towards objects. It is I think reasonable to postulate that Wittgenstein had some idea of what would count as an object, but the details *as to content* depended on the outcomes of non-philosophical investigations. If so it is, I think, implausible to ascribe to Wittgenstein any candidate for the role of object that was *wholly* available to him at the time, whether philosophers' entities (sense-data) or, specifically, the scientific entities to be found in

Hertz's *Principles of Mechanics*. If he had had the latter in mind he could have said so.

The first thing to note is Hertz's clear distinction between the aspects of mechanics that are *a priori*, in Book I, and those that are empirical, in Book II (cf. §7.9). Space, time and mass are addressed in Book I Chapter One. Hertz distinguishes between *materielle Punkte*, and *Massenteilchen*:

> A material point [*materielle Punkte*] ... consists of any number of material particles [*Massenteilchen*] connected with each other. This number is always to be infinitely great: this we attain by supposing the material particles to be of a higher order of infinitesimals than those material points which are regarded as being of infinitely small mass. (*Principles of Mechanics*, §5)[4]

Grasshoff translates *Massenteilchen* as *mass particle* (Grasshoff 1997a, p.105), to avoid difficulties I will stick with Hertz's original German expressions (and bracket them in quoted passages alongside translations).

Lützen offers a thorough account of Hertz's evolving views, from the Kiel lectures of 1884, through three successive drafts of the *Principles of Mechanics*, to the published work. This is a conscious development, Lützen noting that whereas in 1884 "when talking about the smallest parts of matter, he usually had atoms in mind" (Lützen 2005, p.138), by the time of the finished work particles had shrunk to the infinitely small:

> In the second draft of the book (Ms12) Hertz made the *Massenteilchen* infinitely small so that material particles could have finite masses with a continuum of values. Only in the third draft (Ms15) did he make the *Massenteilchen* infinitely small of the second order. It is in this connection that he introduced material particles [i.e., *Massenteilchen*] with infinitely small masses. (Lützen 2005, p.157)

Lützen says, in a footnote, "Hertz's image of matter being built up from indefinitely small 'Massenteilchen' might be a thorn in the flesh of the positivist, but as I shall argue ... this inessential part of the image was introduced for mathematical reasons." (Lützen 2005, p.105n.3) The reason is, I think,

[4] A "higher order of infinitesimals" might lead one into non-standard analysis, but even if I had the mathematical sophistication for this I'm not sure it would make things more intelligible.

to ensure the applicability of differential equations. *Massenteilchen* are not, clearly, the sort of entity that one can encounter, indeed, that any sort of physical process could bring to light. In discussing Hertz's dispensing with the notion of force in his preferred image, van Fraassen says:

> When the kinetic energy of a macroscopic system apparently changes, in a way normally accounted for by postulating forces, the effect is instead accounted for within mechanics by reference to the motion of hidden masses. But these hidden masses consist of [*Massenteilchen*], entities with zero extension, and so the theory does not appear to afford a literal realistic construal. (van Fraassen 2008, p.202)

These postulated hidden masses are, as Hertz states, "Invisible things ... beyond the limits of our senses." (*Principles of Mechanics*, p.25) It does not follow, though, that hidden masses are ontologically different from perceived masses; "We are free to assume that this hidden something is not else than motion and mass again—motion and mass which differ from the visible ones *not in themselves* but in relation to us and to our usual means of perception" (*Principles of Mechanics*, p.25, emphasis added). "Hidden masses" are more of the same.

Eisenthal proposes a "logically oriented interpretation" of Hertz, arguing that the latter was not offering an ontology. Hertz's *a priori* apparatus, rather, "captur[es] the number and the type of a system's degrees of freedom." (Eisenthal 2022, p.290)[5] It is not clear to me what work is being done by degrees of freedom here, but Eisenthal is, I think, right to see *Massenteilchen* as, in a Tractarian sense, logical, because what Hertz tells us about them is purely formal, or *a priori*. It becomes difficult, though, to grasp how such hidden masses are supposed to be beyond our perception. In discussing Hertz, Van Fraassen points out, of the magnitudes in question considered as theoretical parameters, that "We must not understand this to mean that the theoretical parameters are allowed to be empirically ungrounded! They need to be connected to measurement, relative to the theory, in a way that allows for determination of their values in principle." (van Fraassen 2008, p.201) *Massenteilchen* presumably have *some* empirical manifestation if the phenomena that the Newtonian postulates forces for, and the Hertzian hidden

[5] Bizarro also argues that Hertz is not offering an ontology; "Hertz might have misled his reader to thinking that these objects have to be interpreted as physical entities." (Bizarro 2010, p.157)

masses for, are to be addressed, even if they really are intended as wholly mathematical entities with a purely instrumental function.

There is, then, I think, good reason to think that Tractarian objects are not *Massenteilchen*; because if so, Wittgenstein could have easily said as much, and because it is not clear how a transition is to be made from the *a priori* presuppositions to the *a posteriori*, the application. Although it would be helpful if Wittgenstein had made his views clear on continuity (6.34)—the net conception of 6.341–6.342 is *prima facie* discrete and finitistic—and on the infinitesimal, and the application of differential equations.

In proposing an Hertzian construal of Tractarian objects Grasshoff is aware of some, at least, of the problems with *Massenteilchen*, so he asserts rather that Tractarian objects can be thought of as Hertz's *materielle Punkte*:

> Can we find in the *Tractatus* a direct reference to Hertzian material points as examples for simple objects? Many interpreters deny that Wittgenstein ever succeeded to give such an example. In the *Prototractatus* we find a surprise. There *Tractatus* 2.013 is supplemented by a thesis elaborating on the thing as simple object: [Grasshoff's translation]
>
> [*Prototractatus*] 2.0141 Let the thing be the material point [*der materielle Punkt*] surrounded by infinite space. It is obvious that the material point [*der materielle Punkt*] cannot be imagined without indefinite space ...
>
> One cannot imagine a more unambiguous statement about simple objects! Wittgenstein uses the Hertzian notion of material points [*materielle Punkte*] and defines them as things—the simple objects of the external world. (Grasshoff 1997a, pp.115–6)[6]

This is, I think, a case of seeing what one wants to see. *Prototractatus* 2.0141 does indeed use the words „materielle Punkt“, but this is not a *supplement* to the *Tractatus*. Rather the words „materielle Punkt“ were *suppressed* in the rewriting of *Prototractatus* 2.0141 as *Tractatus* 2.013, just

[6]This is repeated near-verbatim at Grasshoff 1997b, p.262, Grasshoff noting (*op. cit.* p.267) that this is a later version of Grasshoff 1997a. Cf. Grasshoff 2006, pp.19–26.

as he suppressed the ˆ notation in rewriting *Prototractatus*, 5.3321 as 5.5321 (cf. p.120). One might well ask why.

Grasshoff is particularly critical of Pears and McGuinness: "The biggest flaw in the translation by Pears and McGuinness is the inconceivable rendering of 'materielle Punkte' as 'point-masses'." (Grasshoff 1997a, p.108) This is with respect to 6.3432, where Ogden translates „materiellen Punkten" as "material points". This is a narrow criticism, applying only to monoglot readers of Pears and McGuinness. Grasshoff says, "Wittgenstein himself gave his translator Ogden a hint how to translate the technical terms of these passages: 'To get the right expression please look up the English translation of Hertz's *Principles of Mechanics*' " (Grasshoff 1997a, p.109), referring to *Letters to C.K. Ogden*, p.35. But there is some misdirection here because Wittgenstein specifically comments on the translation of „gesetzmässige Zusammenhänge" in 6.361, and as the indexing to *Letters to C.K. Ogden* shows, there is no mention in the correspondence of 6.3432 at all. Ogden and Ramsey's translation of „materielle Punkte" as "material point" is of course entirely literal, irrespective of Hertz. Pears and McGuinness arguably stumble on this, but the reader of Grasshoff expecting an indication in *Letters to C.K. Ogden* that the translation of 6.3432 should accord with the English translation of *Principles of Mechanics* will find themselves short-changed.

The crux, then, of Grasshoff's strategy is to interpret Tractarian objects as Hertzian material points („materielle Punkte"). So 2.01, „Der Sachverhalt ist eine Verbindung von Gëgenstanden. (Sachen, Dingen.)" becomes "A state of affairs is a combination of material points." (Grasshoff 1997a, p.114) But this isn't obviously compatible with 2.02, that "Objects are simple", because "A material point . . . consists of any number of material particles connected with each other" (*Principles of Mechanics*, §5), these "material particles" being the *Massenteilchen*. It is hard not to conclude that on Grasshoff's reading of the *Tractatus Massenteilchen* silently disappear, along, presumably, with the problematic notions of being infinitely small and being of infinitely small mass, in favour of material points which on the basis of this disappearance can be treated as simple. This is understandable, but it is a stretch to get it to fit the text.

At bottom Hertz's entities are, I think, wholly mathematical, based on synthetic *a priori* principles bound up with continuity and the infinitesimal. I agree with Eisenthal that Hertz's *materielle Punkte* and *Massenteilchen* are

theoretical/logical entities, but this I think precludes their being candidates for Tractarian objects, because the former are specified wholly logically in a way that leaves no room for an account of content, or of restrictions on combinatorialism.

8.6 The chemical analogy

To return to the main line of argumentation, an interpretation of the *Tractatus* has, I think, to do justice to the remarks concerning naming, that objects can be, in some sense at least, thought of as individual; individuals that combine and recombine with other objects to form states of affairs. One has to have an account of how states of affairs can disintegrate and different ones come into being, how the world can follow a trajectory through logical space, of how the distribution of truth-values over the totality of elementary propositions can change.

This is not met, I think, by essentially static accounts, ones that put heavy emphasis on the context principle, on objects as logical entities that are features of elementary propositions considered as unities. Similar considerations apply to accounts that treat Tractarian propositions as Fregean or Quinean, as Goldfarb (p.46). Such accounts effectively treat elementary propositions as unipolar and not as bipolar. Any account that results in a static world where propositions do not or cannot change their truth-values is, I think, misguided. On the other hand I am not sure that the demands on naming can be met in their entirety; all that can be done is, I think, the proposing of an account that goes some way to meeting the strictures laid down.

Faced with these difficulties commentators often draw an analogy with the chemical theory of valency, of atomic bonding, whereby atoms combine to form molecules (and molecules to form more complex molecules). This theory came to maturity in the second half of the nineteenth century alongside the rise of atomistic theories in physics, specifically in gas theory and statistical mechanics (fields in which Boltzmann made notable advances).

The analogy is not novel, Wittgenstein later remarking:

> My notion in the *Tractatus Logico-Philosophicus* was wrong . . . because I . . . thought that logical analysis had to bring to light what was hid-

> den (as chemical and physical analysis does). (*Philosophical Grammar*, p.210)[7]

The presumption is of a logical analysis yielding elementary propositions and objects in terms of form, that parallels or mirrors a physical or chemical analysis yielding classically well-behaved fundamental particles. What we have on both fronts is a promissory note, outcomes seen through a glass darkly.

The chemical analogy can be readily found in the literature. In a 1959 paper Proctor says, "Objects have formal properties in virtue of which they can combine with one another to make facts—analogous to the sense in which physical atoms are said to have valencies." (Proctor 1959, p.207) In the same year Anscombe wrote, "It is useful here to adopt the metaphor of structural formulae in chemistry for the structure of sentences." (Anscombe 1971, p.38) Stenius talks of "Wittgenstein's chemical vocabulary" in *Philosophical Investigations*, §22, relating this to 4.022 (Stenius 1960, pp.159–61). Ishiguro talks of "The tiny fleck of snow on my hand ... made of H_2O" (Ishiguro 1969, p.47). Hacker says, "Objects have an essential form, consisting of their internal properties—that is, their combinatorial possibilities with other objects (they are, in this respect, akin to chemical ions with a given valency)." (Hacker 1996, p.30) Candlish and Damnjanovic suggest that "Names can therefore come in different forms which, like the valencies of chemical elements, determine which other names they can combine with." (Candlish and Damnjanovic 2012, p.90)

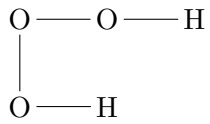
In *Philosophical Investigations*, Wittgenstein remarks:

> Compare 'logically possible' with 'chemically possible'. One might call a combination chemically possible if a formula with the right valencies existed (e.g. H–O–O–O–H). (Wittgenstein 1958, §521)[8]

Strikingly when the same analogy appears in *Philosophical Grammar* (at p.128) the presentation is, in Fregean fashion, two-dimensional:

[7] This is a recurring theme in the transition works; c.f. *Lee Lecture Notes*, pp.35, 90, 109, *The Big Typescript*, pp.66, 82, 385, *Moore Lecture Notes*, p.217, *Wittgenstein-Skinner Manuscripts*, pp.340ff.

[8] The formula is nonsense, because oxygen doesn't form chain molecules with itself. See also discussion dated 13 March 1935 at *Wittgenstein-Skinner Manuscripts*, pp.340–2. This discussion of the "chemically possible" does not appear in Ambrose's notes (lecture XVI ends at *Ambrose Lecture Notes*, p.163) so it may not have formed part of the lecture as delivered.



The comparison with a Tractarian doctrine of elementary propositions as concatenations of names, standing for states of affairs as concatenations of objects, is, I suggest, close at hand. That Frege was perhaps thinking along these lines can be seen in an unpublished 1881 paper:

> [I]n the concept script ... designations never occur on their own, but always in combinations which express contents of possible judgement. I could compare this with the behaviour of the atom: we suppose an atom never to be found on its own, but only combined with others, moving out of one combination only in order to enter immediately into another. (Frege 1880/81, p.17)[9]

Frege appends a footnote, "Wundt makes a similar use of this image in his *Logik*." But that various ideas were in the air is one matter, although it is striking that Frege alludes to the *dynamics* of chemical combination and recombination. The question is whether such analogies can be put to use. The answer in the present context, of a quest for a physical candidate for a formal specification, something that could possibly have been seen through a glass darkly, as if from afar, in the first two decades of the twentieth century, and for the discovery of which there were enquiries afoot is, I think, yes.

8.7 Structural chemistry

The conditions for a model for Tractarian objects are as stated in §2.4, in particular, that candidate objects are not individually identifiable and nameable in "Fido"/Fido fashion, that they do not exhibit material properties individually, that they have a capacity to combine to produce material properties, that they exhibit a range of combinatorial forms such that combination yields instances of material properties, and that they can be thought of independently of such combination. Further, given a dynamic and not a static world, combinations have to be able to disintegrate and their constituents recombine in the same or in different combinations, subject to form.

[9]For the chemical analogy in the Fregean context see also Dummett 1981, pp.32, 62.

Lastly, the model has to be one that Wittgenstein could plausibly have had in mind in composing the text (so for the sake of the argument I assume that Wittgenstein's views at the time of composing the text pre-date relativity and quantum mechanics).

Structural chemistry came of age circa 1860/1900, following Kekulé's work in the late 1850s on the self-linking of carbon atoms, and it was his work on the "combining power" of carbon that led to the concept of *valency*; "The valence of an element is its capacity to combine with a certain number of hydrogen atoms or of atoms with the same combining power as hydrogen. Carbon can combine with four hydrogen atoms, so it has a valence of 4; it is *tetravalent*." (Levere 2001, p.139) Chemistry more broadly also, arguably, came of age during this period, with the presentation of Mendeleev's periodic table in 1869 (Levere 2001, pp.117–20).

My suggestion is, then, that the conditions laid down are plausibly met by the atoms of elements as conceived of in the late nineteenth century.[10] Atoms of elements do not exhibit *material* properties individually. One might object that they have mass but, as §8.3, mass can be regarded as a *formal* property. Whatever comprises the substance of the world has to be something and not nothing, so now it becomes a matter of distinguishing what is formal from what is material.

For present purposes I will not distinguish between *atoms*, *elements*, and *substances*. What is at stake is a model for Tractarian objects, and a detailed study would, I think, rapidly go beyond what can plausibly be ascribed to Wittgenstein, on the basis of his education. For the latter I will refer to two elementary text books, Ostwald's *The Fundamental Principles of Chemistry*, and Nernst's *Theoretical Chemistry from the Standpoint of Avogardo's Rule and Thermodynamics*. These are two of the three textbooks chosen by DeLanda to establish "what ... had become part of consensus practice [in chemistry] by the year 1900" (DeLanda 2015, p.128).

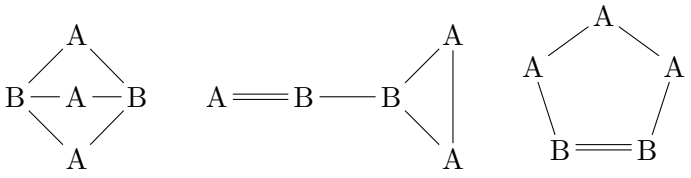
The first point is that chemistry at this level is, in principle, remarkably simple, and doesn't in fact require a grasp of or commitment to atomism. It can be presented as a simple combinatorial theory, based on combining weights. So even the anti-atomist Ostwald can talk of "the so-called structural theory, in which a distinct difference is assumed between ele-

[10] Russell discusses the "new atomism theory ... invented for the needs of chemistry" at *Our Knowledge of the External World*, p.103.

ments which are directly bound to each other and those which are indirectly bound, that is, by means of other elements" (Ostwald 1909, p.325), and also of valency, by which "The elements combine not only in the ratio of single combining weights, but also in very manifold proportions for which no complete and regular principles have yet been found." (Ostwald 1909, p.326)[11]

On the notion of *element*, Ostwald follows Lavoisier; chemical processes "can be carried on until we have substances which cannot be transformed under any conditions into mixtures or solutions. Such substances cannot be decomposed or analysed: they are called *elements*, or simple substances." (Ostwald 1909, p.169)[12] Nernst gives a similar Lavoisier-style account at Nernst 1904, p.6, but then gives a more Mendeleevian account at *op. cit.* p.33, in terms of atomic weights.

Ostwald can thus present as possible structures for a bivalent element A and a trivalent element B the following (Ostwald 1909, p.327):



Nernst discusses this in Book II Chapter IV of his book, noting difficulties in speculation over the arrangements of atoms into molecules, that in so doing "We go into a region of a purely hypothetical nature" (Nernst 1904, p.275). Nevertheless he goes on to say:

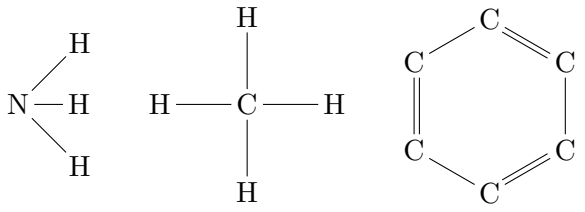
> The usual requirement customary in the careful study of nature, of abstaining from such an attempt, does not appear to be justified ... for such a requirement, on the one hand, would amount to a refusal to obtain many obvious conceptions regarding many important phenomena, which neither the experimental nor the theoretical student can explain in any other way; and, on the other hand, it would not harmonise with the fundamental principle of the method of natural science which commands us to follow out, to the ultimate, such a practical and fruitful

[11] They since have, of course; for textbook presentations see Gray 1973, Atkins and Jones 2008, Chapter 2.

[12] For Lavoisier's definition see Banchetti-Robino 2020, p.97. Lavoisier left it open that the elements of his time might prove further analysable in the future.

> hypothesis as the atomic hypothesis is well known to be. (Nernst 1904, p.275)

He goes on to consider NH_3 and CH_4 molecules (*op. cit.* p.278), and Kekulé's famous benzene ring:



So when Wittgenstein says, twice, that "A proposition is articulate" (3.141, 3.251), that "In a state of affairs objects fit into one another like the links of a chain [*Kette*]" (2.03), that "An elementary proposition . . . is . . . a concatenation [*Verkettung*] of names" (4.22), structural chemistry—bonding relations between atoms—exhibits the salient characteristic, "*that there isn't anything third* that connects the links but that the links *themselves* make connexion with one another" (*Letters to C.K. Ogden*, p.23). It is striking that in 1858 "Kekulé referred to the atomicities or basicities of the elements themselves . . . and so arrived at the fundamental notion of *catenation*, or the linking of carbon atoms into a chain." (Brock 1992, p.250, emphasis added)

In this context one can, I think, invoke Ishiguro's "dummy names" argument (Ishiguro 1969, p.45) (cf. p.72). What is required is entities that can combine and recombine with one another in consistent ways, yielding a dynamic conception of the world as we know it. Atoms are distinguished by their atomic weights, and we can think of these as the formal concepts that partition the totality of objects. So a carbon atom instantiates the formal concept *atomic weight 12*.[13] Not only can we think of C, H, O, N, Na, Cl, and so on, as dummy names for any carbon, hydrogen, and so on, atom in a molecule; we can, I suggest, think of these as names for forms and thus as Tractarian *variable names*. This yields a model that respects the context principle, while allowing for a dynamical approach. We have symbols that

[13] Cf. Wittgenstein 2013, p.385: "One awaits an ultimate logical analysis of facts, as one waits for a chemical analysis of compounds. An analysis that actually enables one to find, say, a 7-place relation, like an element that actually has the specific number 7." Strictly the atomic weight of carbon is 12.01, its atomic number—the number of protons in the nucleus—is 12. But atomic number was not adopted as definitional until 1923 (Earley 2020, pp.109, 113).

combine and recombine according to form, where the ensuing material properties arise from combination. As Banchetti-Robino notes, "Even before the time of Lavoisier, it was realised that, while the properties of substances had to be related in some way to the *natures* of their constituents, they did not match the *properties* that their individual constituents displayed when not combined with other substances" (Banchetti-Robino 2020, p.90); the properties of sulphuric acid are not those of hydrogen, sulphur, and oxygen taken individually or as an aggregate.

My suggestion, in short, is that the basic entities of late nineteenth century chemistry function as a model for Tractarian objects. On this conception atoms are above all classically well-behaved and come in a range of forms, capable of combination and recombination. They are persistent, and capable, on combination and recombination, of producing the material properties of the world as we experience it. We can think of the atoms as individuals, and hold that in principle they are nameable, even if only by means of dummy or variable names; just as in principle there is determinacy of sense as the terminus of an analysis we cannot carry out.

I accept that this is conjectural. But Wittgenstein must, I think, have had *something* in mind to serve as a model for his notion of object, something that works both formally and contentually (2.025). Otherwise there is inconsistency (Hacker), and this diminishes Wittgenstein's achievement, or there is logic as wholly *sui generis*, and this results in a loss of contact with the world (of application, as 5.557).

8.8 Analysis

Given the proposed model it is reasonable to ask, what might an analysis look like? Here, I think, Wittgenstein gives a pretty good account, in later reflections in *Philosophical Investigations*:

> "A *name* signifies only what is an *element* of reality. What cannot be destroyed; what remains the same in all changes."—But what is that? ... We see *component parts* of something composite (of a chair, for instance). We say that the back is part of the chair, but is in itself composed of several bits of wood; while a leg is a simple component part. We also see a whole which changes (is destroyed) while its com-

> ponent parts remain unchanged. These are the materials from which we construct that picture of reality. (*Philosophical Investigations*, §59)

My suggestion is that a piece of homogeneous stuff—a piece of wood, for example—constitutes a state of affairs. A broom is then an arrangement or structure of its component parts, the handle, the brush block, the screw that holds these together, and the bristles, taken individually. Now the immediate objection is that "The broom is in the corner" treats the truth-functional structuring of the elementary propositions (the handle, the brush block, the screw, and so on) as a spatial structuring of these constituents; constituents that are made for each other but nevertheless require assembly. This looks to go against 5.42, "It is self-evident that $\vee$, $\supset$, etc. are not relations in the sense in which right and left etc. are relations."

This would be fine, if Wittgenstein had an answer to the *Satzverband* problem. But he doesn't and, as argued in §5.4, he posits an account of complex/everyday propositions that just assumes structuring; that brooms are available for sweeping and that motor car accidents occur. If we choose to give him this, then what we can say is that there is a sequence of sufficiently close possible worlds in which the components of this broom are fabricated and assembled, and in which the broom is employed until it wears out or disintegrates, its substance becoming available for recombination. The critical point is that if we think of space as a function of the forms of objects—of space as relational, a matter of the relations between objects—and this is discussed further in the remainder of this chapter—then we can see how logical/truth-functional relations between elementary/complex propositions can in some sense go proxy for spatial relations between everyday things. This line of thought can I think be seen in the following:

> Suppose that, instead of saying "Bring me the broom", you said "Bring me the broomstick *and* the brush which is fitted onto it." !—Isn't the answer: "Do you want the broom? Why do you put it so oddly?" (*Philosophical Investigations*, §60, emphasis added)

The point being, I take it, that while the account given in the *Tractatus* is manifestly absurd as an account of our mastery of language, it may be useful in other contexts, if, for example, one is asked to set up a production line manufacturing brooms.

8.9 Problems with generalisations

The starting point was Hacker's assertion that "if one searches for an example of something that will satisfy Wittgenstein's specifications, one will search in vain. As Wittgenstein himself realised in 1929, the specifications are inconsistent, and there can be no such thing as a simple object as conceived in the *Tractatus*." (Hacker 2021, p.70) Now it is true that quantum mechanics at least undermines the account given, because what physics came up with is fundamental particles that are not classically well-behaved. But what has been argued is that the specification for Tractarian objects is not formally inconsistent and that it was plausibly met at the time of composition and publication. And one can resist the quantum mechanical slide by sticking to the level of atoms and not going sub-atomic. Ostensibly, though, the account given in the *Tractatus* is put under pressure on two fronts; the problem of exclusion, specifically colour exclusion, and a loss of faith in treating quantifiers in terms of conjunctions and disjunctions.

The latter is, I think, the lesser difficulty, not least because it arises from concerns external to the text. There are two reasons for losing faith in the treatment offered of generalised propositions, the first concerning finiteness, the second, that this way of thinking about generalisations is a poor fit with natural language.

Of the first, Wittgenstein was not, I think, concerned with the possibility of a nondenumerable domain. Rather he rejected the conjunction/disjunction analysis because of a mistake:

> Now let's talk of following, not as here from molecular propositions, but from or to general propositions. Take $(x)fx$ entails fa E.G. if anybody is here he has a hat entails if Smith is here he has a hat. Or take fa entails $(\exists x)fx$.
> Now there's a temptation, to which I yielded in *Tractatus*, to say that
> $(x)fx$ = logical product $fa\,.\,fb\,.\,fc\ldots$
> $(\exists x)fx$ = logical sum, $fa \vee fb \vee fc\ldots$
> This is wrong but not as absurd as it looks.
> Suppose we say that: Everybody in this room has a hat = Ursell has a hat . Richards has a hat etc ...
> This is obviously false, because you have to add & a, b, c ... are the only people in the room.

> This I knew and said in *Tractatus*.
> But now, suppose we talk of "individuals" in Russell's sense, e.g. atoms or colours; & give these names, then there would be no proposition analogous to "And a, b, c ... are the only people in the room". (*Moore Lecture Notes*, pp.214–5)[14]

But this is not obviously absurd. If there is a finite number of cases, one can work through them until there are no further cases (bearing in mind that we are dealing with realisations, with instances of a prototype, and not with objectual quantifiers ranging over a domain of objects which have to be individually identified as instantiating some property). In terms of 5.501 we can simply treat this as a case of a function fx, all of whose values are realised (cf. p.140).

Wittgenstein nevertheless rejects the Tractarian approach as mistaken, and distinguishes between generalisations that can, and those that cannot, be treated as conjunctions/disjunctions. In cases where they can, the dots in $\forall x.fx \supset (fa \wedge fb \wedge fc \wedge \dots)$ are termed "the & so on of laziness", because in such cases these dots "*could* be replaced by an enumeration" (*Moore Lecture Notes*, p.217). In the *Tractatus* there are only dots of laziness, because we are dealing with totalities. But there are, Wittgenstein says, circa 1930/33, cases where what looks like a logical product is not a sum (a conjunction) but a limit, and in these latter cases the conjunction/disjunction analysis fails. Here he draws some sort of parallel between series in mathematics and the conjunction/disjunction analysis:

> [I]s a fallacy of thinking $1 + 1 + 1 + \dots$ is a sum.
> It is a muddling up a sum with a limit of a sum.
> $\frac{dx}{dy}$ is not a quotient, but the limit of a quotient. It doesn't obey all the rules that $\frac{x^2}{x}$ obeys. (*Moore Lecture Notes*, p.217)

The discussion peters out without, I think, offering any substantive account of what is held to be wrong or mistaken with the conjunction/disjunction analysis. This can, though, be taken together with the section on generality in *The Big Typescript*. The discussion here is again obscure, with this remark occurring midway:

[14] Cf. Moore 1954/55, p.297.

> Indeed it's clear that I do not recognise any logical sum as a definition of the proposition "The cross is between the lines". And really that says everything that is to be said. (*The Big Typescript*, p.253e)

The case discussed earlier (*The Big Typescript*, p.241e) is of positioning a cross somewhere on a line between a pair of vertical lines. The point is, I think, that we can understand the particular case as a particular albeit indeterminate case without needing to understand first all the possibilities, with its position *here* being one of the possibilities. This is what he seems to have in mind in the following:

> [E]ven that still doesn't seem to me to hit upon the most important point of this matter. I believe that what matters isn't really the infinity of possibilities, but a kind of indeterminacy. Indeed, if I were asked how many possibilities a circle in my visual field had of being within a particular square, I could neither name a finite number, nor say that there were infinitely many ... Rather, although we never come to an end here, the series isn't endless in the sense in which $|1, \xi, \xi + 1|$ [the general form of the integer, 6.03] is. (*The Big Typescript*, p.255e)

This sheds some light on the mathematical parallel, but Wittgenstein's point at bottom is, I think, that the shift in his position is not logical but, in Tractarian terms, psychological. It is not that the conjunction/disjunction analysis is wrong. It is rather that if one is concerned with how we think and speak it is absurd. This I suggest is not a rejection of the *Tractatus* on its own terms but rather a rotating of our interests away from the logical towards what we actually say and do (cf. *Philosophical Investigations*, §108).

This also allows Wittgenstein to sidestep difficulties over the transfinite, that is, the mathematical parallel noted above; "an expression ending with the words 'and so on' does not point towards a possibility waiting to be realised (an empirical possibility) but shows a possibility of the symbolism" (Marion 1998, p.181), with reference to a passage in *Remarks on the Foundations of Mathematics* that such cases may lack "the institution of the end, that it is not finished off" (*Remarks on the Foundations of Mathematics*, ii §45). But again this represents a change in position, a rejection of the earlier presumption of fixed logically tractable totalities.

8.10 Space as relational I

This brings us to the genuinely problematic, that a consequence of the independence of elementary propositions is that *prima facie* one cannot do justice to the problem of exclusion; that "this surface is red and this surface is blue" represents a clearly impossible situation and is thus *prima facie* contradictory. Except it is not truth-functionally contradictory in the sense that one cannot say that it is a contradiction by inspection of the proposition itself (6.112); the stated proposition is a conjunction so however absurd it may seem its propositional sign is $(\top\bot\bot\bot)(p,q)$, and is not of the form $(\top\top\top\ldots)(p,q,r,\ldots)$, with the contents of the left-hand parentheses a solid block of $\top$s. So how, one might ask, could Wittgenstein have coherently thought that he had a solution to the colour exclusion problem at the time he wrote the *Tractatus*, on the basis only of what is said in the text itself?

An immediate observation is that if one holds to an absolute view of space and time, of points and instants as locations to be occupied, then exclusion is inescapable. So the first move is to attribute a relational account of space and time to Wittgenstein.[15]

The account to be given relies on regarding space and time as we experience them as functions of the forms of the objects that there are. That is, all objects are spatially constitutive, and all objects are persistent in time. This is in part at least, I suggest, what is meant by 4.441, that "There are no 'logical objects' "; the only objects that there are, by my reckoning, are physical objects.

There may be an issue over temporal exclusion but given that it is hard to see how to make any sense of this, and given the presumption that objects are persistent I will set this aside. It is spatial exclusion that is salient.

If we think that space as we experience it is a function of the forms of objects then there is nothing to space, and motion in space, over and above the spatial relations in which objects stand in to one another. So what is needed is a full-blooded relational account, to show that there is nothing to space over and above form—the sum of the relative positions of all the

[15] Grasshoff suggests that Wittgenstein was aware of the relational/absolutist debate, as the position stated in L.W. to B.R. January 1914 (*Wittgenstein in Cambridge*, pp.64–5) on space and time as relative corresponds closely to Petzold's views presented in a course of lectures on the foundations of mechanics given at the Charlottenberg Hochschule in Summer 1907 (Grasshoff 2006, p.14).

objects that there are—such that exclusion can be defused as an objection.

The account to be given draws on Earman 1989, Chapter 2. Earman presents a range of classical space-time structures, from Mach to Aristotle. Machian space-time "consists only of an absolute simultaneity and a Euclidean metric structure for the instantaneous spaces" (Earman 1989, p.28), that is, E^3, described by Torretti as "infinitely extended Euclidean space, with its infinitely articulated net of depthless planes and widthless lines, meeting at dimensionless points" (Torretti 1996, p.21).[16] At an opposite extreme is Aristotelian space time, with the full structure of Newtonian absolute space-time with a preferred location as the centre of the universe (Earman 1989, p.34).

Earman's approach is, for present purposes, useful, because what he points out is that what questions are meaningful—i.e., propositional—depends on how rich a space-time structure is postulated; "As the space-time structure becomes richer, the symmetries become narrower, the list of absolute quantities increases, and more and more questions about motion become meaningful." (Earman 1989, p.36) The underlying idea, in the Tractarian context, is to present a model in which questions about exclusion can be excluded as meaningless.

Concerning Earman's classification, Machian space-time has so little structure that questions about motion are meaningless, because it lacks a time metric. So for present purposes we can adopt Earman's Leibnizian space-time, consisting of Machian space-time plus a time metric.[17] In Liebnizian space-time:

> It is now meaningful to ask of two particles: How fast is the distance between them changing? How fast is the relative speed changing? etc. In general, the only questions about motion that are meaningful in this setting are questions about the relative motions of particles, which would seem to make it the ideal setting for implementing the slogan that all motion is the relative motions of bodies. (Earman 1989, p.31)

To begin with a static picture, we can consider reality as it is at any given

[16] Cf. Maudlin 2012; E^3 is homogeneous and isotropic (p.35) and one can attach a noninertial coordinate system, provided if what there is rotates, then the coordinate system rotates along with it (p.65).

[17] Earman's Leibnizian space-time is, I think, Maudlin's Galilean space-time (Maudlin 2012, p.54), provided any attached coordinate system is noninertial.

moment—a possible world—comprising the totality of objects, combined in states of affairs in accord with their forms. There is, I suggest, nothing more to space than the relations that the objects that there are stand in to one another; that is, the *situations* that obtain. Objects don't move *in* space, rather they move *relative to one another*. We can think of space in terms of E^3 by a process of abstraction and idealisation (cf. Torretti 1996, p.21). We can choose a location as the origin of a coordinate system, and from this point map where objects are relative to one another. What we call *space* arises from the fact that objects are such that they can stand in spatial relations. If we adopt methodological solipsism and take the origin as an extensionless point to which reality is coordinated (5.64), then this location is the origin of one's visual field. This is why I think Wittgenstein talks not so much of *space* as of the *visual field*, with the visual field equated with the metaphysical subject as the limit of the world (5.632–5.634). Everything that we can sense must be sensible (2.0131) but there is no sense, I think, whereby we can perceive either space, or time. We can only compare what occurs in space or in time with some other process (cf. 6.3611, the working of a chronometer). This parallels 2.171–2.172, that although a spatial picture can depict what is spatially arrayed, it cannot depict space itself, because space is a matter of form. And this can only be shown. This is all rather abstract, so I want to propose here what will be called *Rubik's cube world* as a model, to bring out particular points.

Each face of a Rubik's cube has nine cubies[18] with, in the distinguished or original configuration, each face made up of nine cubies of the same colour. The cube has an internal mechanism so certain moves are allowed; the basic move is that any face, consisting of nine cubies, can be rotated through 90° relative to the remainder of the cube. This clearly changes the relative configurations of the rows of three cubies to the adjacent faces. Relatively speaking the six face centre cubies do not move, however the faces are rotated. These cubies can only be said to move from a position external to the cube, just as one has to move into higher dimensions to make certain figures congruent (6.36111). From the perspective of the cube itself it makes no sense to ask if a face centre cubie can move.

To move from a static to a dynamic conception, we can imagine a cube

[18]The term used in Provenza 2012. There is an extensive literature on the cube, see for example Zeng et al. 2019.

with an internal mechanism connected to a clock such that every time the clock ticks a face is rotated 90°. However disorganised the cube is it can by a directed series of moves be returned to the original state in at most twenty moves (Rokicki et al. 2014). But if the moves are randomly generated then given the number of possible configurations (4.3×10^{19}, Rokicki et al. 2014, p.645) such a return is unlikely.

A complete description of the cube, at any moment, takes the form of a conjunction of six propositions, specifying the colours of the eight edge cubies associated with a particular face centre cubie. So if we take for example the face with the face centre cubie yellow, the proposition will specify the colours of the eight other cubies. There isn't complete independence because, for example, each of the eight corner cubies presents three different colours. This complicates matters in detail but not in principle. The question to be asked is this; in Rubik's cube world, what sense does it make to talk of colour exclusion?

8.11 Colour exclusion

The key proposition here is 6.3751:

> For example, the simultaneous presence of two colours at the same time in the visual field is impossible, in fact logically impossible, since it is ruled out by the logical structure of colour. (1)
> Let us think how this contradiction appears in physics: more or less as follows—a particle cannot have two velocities at the same time; that is to say, it cannot be in two places at the same time; that is to say, particles that are in different places at the same time cannot be identical. (2)
> (It is clear that the logical product of two elementary propositions can neither be a tautology nor a contradiction. The statement that a point in the visual field has two different colours at the same time is a contradiction.) (3) (6.3751)

Hacker uses Johnson's terminology of *determinate* values of *determinables* in his discussion, a distinction Wittgenstein was doubtless familiar with.[19]

[19] Johnson taught Wittgenstein logic in 1911/1912 (Klagge 2022, p.148). The distinction is set out in Johnson 1921, Chaps.I and II.

Colour is a pre-eminent case, as Johnson notes; "We may illustrate the relation of incompatibility among adjectives by *red* and *green* as characterising the same patch" (Johnson 1921, p.15):

> One relational characteristic holds in all cases; namely that, if any determinate adjective characterises a given substantive, then it is impossible that any other determinate under the same determinable should characterise the same substantive: e.g. the proposition that 'this surface is red' is incompatible with the proposition that 'this (same) surface is blue.' (Johnson 1921, p.181)

Whatever the physical aspects of this, Johnson addresses it in a treatise on logic, so he presumably regards it as a matter for logical and not for empirical determination. He also goes some way to sketching the logical structure of colour, stating that in many cases determinates can be ordered; "the several determinates are to be conceived as necessarily assuming a certain serial order, which develops from the idea of what may be called 'adjectival betweenness'. The term 'between' is used here in a familiar metaphorical sense derived from spatial relations." (Johnson 1921, pp.181–2) This I take it forms part of the background to Wittgenstein's views in the *Tractatus*.

Given 6.3751, one might expect to see the determinates/determinables distinction deployed elsewhere in the *Tractatus*. But this is not what one finds. Rather we get a diametrically different conception, the Independence Thesis, that states of affairs obtain or do not obtain, and the elementary propositions that represent them are true, or are false, *independently* of one another.

Since Hacker is my stalking-horse, I will quote at some length from *Insight & Illusion*:

> It will be objected correctly that determinates under a determinable are mutually exclusive. So if 'A' names a spatio-temporal point, 'A is red' cannot be an elementary proposition, since it is *not* logically independent. It entails 'A is not green (yellow, orange, etc.)'. Wittgenstein was, to be sure, aware of this. It showed, he thought, that such a proposition was further analysable, and would show itself to *contain* the exclusion of the incompatible properties, i.e. colour exclusion must be a matter of *tautology* (and 'A is red. A is green' must be a logical

> contradiction). If 'p' contradicts 'q', then 'p' must be analysable into, say, a conjunction e.g. '$r.s.t$', and 'q' into '$v.w. \sim t$', so that their incompatibility is transparent. Wittgenstein seems not to have realised that analysing colours into fine shades would get him no closer to the desired atomicity requirement (this is evident *ex post facto* from *Philosophical Remarks*, Ch.viii; see Hacker 2021, pp.110f). And analysing colour in terms of wavelength of reflected light (as seems intimated by 6.3751) is itself an instance of determinate exclusion, since if a certain lightwave is 621 nanometres long, it follows that it is not also 521 or 421 or 620 nanometres. Undoubtedly he had not thought the matter through. (Hacker 2021, pp.71–2)[20]

By Tractarian standards, then, "A is red" cannot be an elementary proposition. As we are not given an account of analysis, and given that "This surface is here, now, red and blue all over" (I will call this the "mooted proposition") is self-evidently an impossibility—this being, I take it, undeniable—it is *prima facie* reasonable to postulate that analysis will show a contradiction (ignoring, for the moment, that 6.3751 clearly presents the mooted proposition as a contradiction in advance of *any* analysis). But within the strict terms of the *Tractatus*, the mooted proposition is not a contradiction.

At this point Wittgenstein's wavering between truth-functional tautologies and obvious impossibilities, set out in §2.9, comes to the fore. There simply aren't resources available in the *Tractatus* to *show* that the mooted proposition is a truth-functional contradiction. But given that Wittgenstein was aware of the colour exclusion problem at the time he wrote the *Tractatus*, and given that he was well aware of what moves one can, and cannot make, within the strictures laid down in the text, and given his confident pronouncement in the Preface that he had stated the "unassailable and definitive truth", how exactly did he think he had solved colour exclusion? Assuming, for the sake of the argument, that he had thought things through.

If Wittgenstein thought he had a purely formal, or logical, response, then 6.3751(2) is superfluous. Any plausible account should have something to say about this invoking of physics (and perhaps address the change in ter-

[20] In asking why Wittgenstein had assumed that analysis would reveal a contradiction, Pears draws a similar to conclusion to Hacker's, phrased as a rhetorical question; "But why had he made such an implausible assumption?" (Pears 1987, p.84) Cf. the notion of "hidden contradiction" at *Lee Lecture Notes*, p.106, *Moore Lecture Notes*, pp.193, 204.

minology, from *Kontradiktion* to *Widerspruch* and from *Gegenstand*, *Ding*, and *Sache*, to *Teilchen*). Hacker mentions this, but does not explain how 6.3751(2) says something more or different from 6.3751(1) and 6.3751(3). In his review of the *Tractatus*, Ramsey considers Wittgenstein's analysis, in terms of vibrations:

> [E]ven supposing that the physicist thus provides an analysis of what we mean by 'red', Mr Wittgenstein is only reducing the difficulty to that of the *necessary* properties of space, time, and matter or the ether. He explicitly makes it depend on the *impossibility* of a particle being in two places at the same time. These necessary properties of space and time are hardly capable of a further reduction of this kind. For example, considering between in point of time as regards my experiences; if B is between A and D, and C between B and D, then C must be between A and D; but it is hard to see how this can be a formal tautology. (Ramsey 1923, p.18)

So Ramsey makes the point that 6.3751(2) looks to offer an explanation not in terms of the logical doctrines of the *Tractatus* but in terms of what is physically necessary. This is at odds, though, with 6.375, that "Just as the only necessity that exists is *logical* necessity, so too the only impossibility that exists is *logical* impossibility." (6.375)

How is reference to physics supposed to illuminate the logical? The Hintikkas claim that 6.3751(2) is "not an explanation of the physical basis" (i.e., not a reduction to physics) but "Instead, it presents (Wittgenstein thinks) a solvable *analogue* to the problem from the field of particle mechanics." (Hintikka and Hintikka 1986, p.125)[21] But an analogy is supposed to work as a comparison, by means of agreement or similarity; "a form of reasoning in which a similarity between two or more things is inferred from a known similarity between them in other respects" (*Collins English Dictionary*). But by 6.375 there cannot be impossibilities *in the world*; there are only *logical* impossibilities. So there is no basis for an analogy. Any "solution" in particle physics could relate only to what is *physically* impossible.

On the basis of Hacker's rather damning conclusion, that "undoubtedly he had not thought the matter through", if Moore and Russell had pressed

[21] Jacquette argues similarly, referring to Ramsey's review, that 6.3751(2) is not a reduction but an analogy (Jacquette 1990, p.357).

Wittgenstein on 6.375 and 6.3751 at his PhD viva, how might the discussion have gone? Taking 6.3751(1) and 6.3751(3), as presented by Hacker, Wittgenstein has no come-back whatsoever. In the absence of an analysis all he can say is that colour exclusion has to be an impossibility, and thus a logical impossibility, because that is the only sort of impossibility there is—but this is not an argument.

Of 6.3751(2), if this is an invocation of physics, then it falls foul of 4.111, that "Philosophy is not one of the natural sciences." The natural sciences deal in propositions with sense, and not with tautologies and contradictions, that is, the senseless. If one takes the point that "Tautologies and contradictions ... do not represent any possible situations" (4.462), then there is none of the empirical data that natural science gets to grips with. If one has to look at the world then one is not doing logic, because "if we get into a position where we have to look at the world for an answer to such a problem [a question that can be decided by logic], that shows that we are on a completely wrong track." (5.551) A fallback on physics is inconsistent with the text. So what will be argued in the next section is, rather, that Wittgenstein must have been thinking in terms of a relational treatment of space.

Surveying the literature shows that Hacker's criticism has been hugely influential. Austin quotes his remark, that "Wittgenstein's first philosophy collapsed over its inability to solve one problem—colour exclusion" (Austin 1980, p.143). The same remark is quoted in the motto to Sievert 1989. But while it appears in the first edition of *Insight and Illusion* (at p.86), it is not to be found in later editions, where the relevant passage is more muted; "the weakness [of the structure of the *Tractatus*] became exposed at what might appear a matter of detail ... The colour exclusion problem" (Hacker 1986, p.108; the pagination of the revised, 1986 and the third, 2021 editions is the same). It is hard to see how one can get past Hacker's *coup de force*.

This difficulty manifests itself in articles on colour exclusion in a tendency to ignore the *Tractatus* and concentrate on the transition period, as in Austin 1980, Pears 1981, Sievert 1989, and Jacquette 1990. In particular, there is a tendency to assimilate the views in the *Tractatus* to those in 'Some Remarks on Logical Form'. This can be seen in Moss 2012, where she argues that her proposed solution is feasible in the Tractarian context because "Wittgenstein suggests that even logical constants can appear in our representations of elementary propositions" (Moss 2012, p.849). But this runs wholly counter

to central doctrines of the *Tractatus*, and the only defence Moss offers is to quote from 'Some Remarks on Logical Form'. Blue retorts that Moss's "strained reading is not supported by the rest of 'Some Remarks on Logical Form' or any other contemporary writing of Wittgenstein's, and Moss gives no reason for extending it to the *Tractatus*" (Blue 2021, p.2n.7).

In his paper Blue sets out five stages in the evolution of Wittgenstein's thinking on colour statements, based on *The Big Typescript*, pp.340–1. His (S1) relates to the *Tractatus*, his (S2) to Wittgenstein's work in early 1929. The timescale is short, with his (S4) and (S5) all featuring in 'Some Remarks on Logical Form'. Blue claims that on Wittgenstein's initial return to philosophy he still held the views in the *Tractatus*, so (S1) and (S2) are the same. This may be so, but since 'Some Remarks on Logical Form' contains a direct rejection of views in the *Tractatus*—"The mutual exclusion of unanalysable statements of degree contradicts an opinion which was published by me several years ago and which necessitated that [elementary] propositions could not exclude one another" ('Some Remarks on Logical Form', p.35)—it isn't clear how Blue's analysis advances the argument. Further, it is hard to see how Wittgenstein could have made it any clearer that his views in 'Some Remarks on Logical Form' differ from those in the *Tractatus*. There is nothing in the *Tractatus* to indicate analysis in terms of shade or units of colour.[22] Indeed the whole point of the self-evidence criterion is that there is no obvious need to rely on analysis at all (cf. p.237); but there is no getting round the fact that the mooted proposition expresses a self-evident *impossibility*, and not a self-evident *contradiction* (shown by the symbol itself). The orthodoxy, after Hacker, is that analysis will show that the mooted proposition is a truth-functional contradiction. The question is not, I suggest, about the finer points of analysis, but *whether or not this whole approach is mistaken*. Hacker is right, I think, to postulate that the mooted proposition is senseless, but not in effecting this by decreeing that it is a truth-functional tautology (to be shown on analysis). What needs to be shown is, I suggest, that the *possibility* of colour exclusion cannot be

[22] Blue deploys Frascolla's suggestion, that "forms of linguistic expressions [in the *Tractatus*] have an arithmetical structure" (quoted at Blue 2021, p.17), but there is no obvious basis for this in the text. When Wittgenstein talks of shades of blue, at 4.123, he talks of internal relations and not an analysis involving number. Gandon notes that "It is only in 1929 that Wittgenstein relates the two problems [colour exclusion and degree attribution] ... Nothing shows that the author of the *Tractatus* ever endorsed this thesis [attribution of degrees of colour]." (Gandon 2016, p.264)

sensibly stated.

8.12 Space as relational II

The task, then, is to show how the mooted proposition—"This surface is here, now, red and blue all over"—is senseless, otherwise than by a postulated analysis uncovering a contradiction. My suggestion is, in terms of Rubik's cube world, to say that a cubie being red excludes a cubie being blue is senseless. It is not nonsense because, in one's visual field (6.3751(1),(3)), it is *possible* for one to be seeing, here and now, any of six colours. But it being red, say, does not *exclude* it being blue, white, green, yellow, or orange, because there isn't *a location* independent of a particular cubie where a different cubie of similar form (corner, edge, face centre) could be located. At any given moment everything that there is, is where it is relative to every other thing, in E^3, and the *truth* of something being where it is relative to other things is thereby crystallised. If things were relatively speaking otherwise then a different set of elementary propositions would be true, a different distribution of truth-values across the totality of elementary propositions would obtain. The possibilities remain as they are, at the level of sense; what changes as the world changes is what is *actually* true, or *actually* false. The notion of exclusion is senseless because, I suggest, it cannot make sense unless one can take up a position external to the world.

To see how this is so, from such an external position there is the trivial solution of peeling off the coloured stickers on the faces as required and resticking them. But this is not a possibility within the world itself. What, though, of a noninertial coordinate system attached to the cube? We can now express cubie positions in terms of (x_i, y_i, z_i), list off the coordinates for the corners and edges (the face centres are fixed) and then say, if *this* configuration obtains, then *these* possible configurations are excluded? This is fair enough but the relationist can still stick to their guns and say that space as E^3 is an abstraction from the relations in which things stand to one another. If one abstracts such a space and attaches it as a coordinate system and then claims that possibilities are thereby excluded what one has actually done is postulated a fixed, absolute space. And in this way one reaches an impasse.

The point is not, though, to produce a knock-down argument, rather it

is to produce a model that Wittgenstein could plausibly have had in mind at the time of composing the text. If Wittgenstein had thought of space in terms of Earman's Leibnizian space-time, or Maudlin's Galilean space-time, perhaps with a noninertial coordinate system attached, then exclusion can be refused as senseless.

This is, I suggest, not only sufficient for present purposes, it can also account for 6.3751(2), the invocation of physics. If space is an abstraction in terms of E^3 then it is conceptually impossible for a particle to have two velocities at the same time, or to be in two different relative places at the same time, or for particles that are relatively distinct to be identified. Consequently what is on offer is not an analogy, it functions as a view on the structure of space.

Further, it serves as an argument for determinacy of sense. If space is abstracted from relations then the position of a watch on a desk cannot be indeterminate. Consequently we have an underpinning for Wittgenstein's talk of determinacy at 3.23–3.251.

8.13 Going empirical

So why, one might ask, did Wittgenstein come to reject his earlier views? The usual reasons given are reservations about colour exclusion, and over the conjunction/disjunction treatment of generalisations. This is familiar, thoroughly addressed in 'Some Remarks on Logical Form', the *Lee Lecture Notes*, the *Ambrose Lecture Notes*, and the *Moore Lecture Notes*. None of this is in dispute. But these are *internal* to Wittgenstein's philosophical project. There are, I suggest, equally interesting *external* reasons for rejection. In brief, in the *Tractatus* Wittgenstein flew too close to the empirical sun.

The models proposed to try and make *System Tractatus* work are scientific models, drawing on chemistry and spatial relationalism. However the critical Tractarian innovation is held to be that logic is *independent* of what there is, that it is divorced from the empirical:

> Since the propositions of logic, unlike those in physics, are not descriptions of the properties and relations of objects in a certain domain, since they are senseless, they cannot constitute a genuine and

> anankastic foundation for prescriptive norms of thinking. This conception of the nature of logic and logical truth was a colossal advance over nineteenth-century thought and over the philosophical ideas of Frege and Russell. It was still far from the whole truth; nor was it wholly true, but it had taken great strides in the right direction, liberating the philosophy of logic from its antecedent failure to differentiate adequately the truths of logic from empirical, psychological or putative Platonist generalisations. (Hacker 1996, p.34)

I accept that this is Wittgenstein's intention, of construing logic as a *sui generis* discipline that is in contact with but does not overlap its application (5.557), that mirrors the structuring or ordering of what there is such that the "propositions of logic" are connected with the world, "that something about the world must be indicated by the fact that certain combinations of symbols ... are tautologies." (6.124) Further, I accept that this would have worked, if only all the *obvious impossibilities* could be cast in the form of *truth-functional tautologies*, such that their tautologyhood is shown by their propositional signs (6.113). But this is a programme that, I think, cannot be effected without having a view on the nature of what there is, and in particular, on the structure and nature of space. Because only so can one get full truth-tables, and it is only with full truth-tables that logic can plausibly claim to be a *sui generis* discipline.

So either, I think, one accepts something along the lines of the reconstruction on offer, in which case one accepts that Wittgenstein contracted into not metaphysical but empirical commitments. Or one accepts Hacker's contention that the *Tractatus* was flawed from the outset, because the very notion of object is inconsistent. But if one accepts the latter then I think one has also to regard the early Wittgenstein as a gravely diminished figure.

If, though, one accepts the reconstruction, then the questions about Wittgenstein's later rejection open up. In terms of internal reasons, that it is an absurd account of our mastery of language is fair enough. That we cannot possibly survey the totalities of objects and of elementary propositions is also fair enough. But perhaps it is more interesting to look at reasons external to the *Tractatus*, at developments in science.

Beginning with relativity, the only way, I think, to get the Independence Thesis to work and thus to get full truth-tables is to hold a relational view of space, as an abstraction in terms of E^3. But this is a pre-relativistic

conception of space as nonsubstantive. Introducing c as a constant at least undermines wholesale relationalism (cf. Maudlin 2012, p.67), and Earman states that "Relativity theory, in either its special or general form, is more inimical to a relational conception of motion than is classical physics." (Earman 1989, p.101) This is beyond my capacity to assess, so I merely note that the proposed way out of exclusion problems becomes at least insecure.

Concerning quantum mechanics, the issue here concerns the quest for what is ultimately simple. To be even in principle nameable what is ultimately simple has to be substantial, that is, persistent and, I think, classically well-behaved. But quantum mechanics turned up ultimate particles that are not classically well-behaved and are not even in principle nameable.

By the mid/late 1920s the classical nineteenth century particle had evaporated in favour of the wave/particle duality at the centre of the new quantum mechanics. But this isn't by itself fatal to an account that relies on ultimately pinning names to elementary particles. Neither is Heisenberg's uncertainty principle, that you cannot simultaneously measure the position and the momentum of an elementary particle. Given a wave there is, to speak loosely, a particle in there somewhere. So long as there is some way of counting them, in principle at least one can still "name" objects. The "in principle" may be very stretched, but then again, it always was.

However, by late 1927/early 1928 it was clear that quantum mechanics confounded expectations, because possible solutions included those with "negative energy" (Hanson 1963, p.145). Consequently Dirac's wave equation, published in paper form in 1928 and book form in 1930, proposed a new particle, the *positron*, the positively charged electron. And now it turns out that two particles of opposite energy, an electron and a positron, can come into being and then disappear against a background of a "sea of negative energy."[23] Whatever else one might or might not be able to say about particles, they can no longer be determinately counted. Ultimately it is energy that is conserved, and not objects; we might say that energy is the substance of the world. The positron was, as expected, duly "discovered" by C.D. Anderson, on 2 August 1932 (Hanson 1963, p.135). According to Heisenberg, this discovery of antimatter was "of utmost importance, because it changed our whole picture of matter" (Heisenberg 1973, pp.31–2):

[23] Cf. Polkinghorne 2002, pp.70–2, Close 2009, p.43, Hobson 2017, pp.91–3.

> In nonrelativistic physics the number of particles of any kind was a constant of motion like energy or momentum. In relativistic physics this number was not a good quantum number any more. A hydrogen atom, for example, did not necessarily consist of proton and electron, it may be taken as consisting of proton, two electrons and one positron, even if this latter configuration would only amount to a small relativistic correction of the complete wave function of hydrogen. One of the consequences of this situation was the conjecture that in a very energetic collision of two particles a larger number of new particles may be created, and these possibilities should be limited only by the laws of conservation of energy, momentum, isospin etc. ... What actually happens in a very energetic collision of two particles is the creation of new particles out of the kinetic energy. Energy becomes matter by assuming the form of elementary particles. (Heisenberg 1976, pp.58, 59)

Any philosophical theory that relies on scientific discoveries risks empirical refutation, and if Tractarian objects are identified with whatever simple particles science comes up with, that is arguably what happened. As noted (p.229) Wittgenstein could have stopped short at the level of atoms, but if one is going to stop analysis short by fiat, one might as well do so at the level of the everyday propositions of 5.5563 (and this is arguably what Wittgenstein does in *Philosophical Investigations*). What is striking is that in the transition period Wittgenstein didn't revert to treating objects as philosopher's entities. The emphasis becomes use, on what we do with our language, on our forms of life. McManus notes, of the transition works, that "One does not find there the working out of a new metaphysical outlook, criticism of the idea of an *ineffable* metaphysics, or any substantial indication that the criticisms of con-formism that one does find there are criticisms directed at his earlier self" (McManus 2006, p.235), where "con-formism" is "something like a fit, an isomorphism between the 'form of thought' and the 'form of the world' " (McManus 2006, p.5). The reason is, I suggest, that Wittgenstein was not doing metaphysics at all, rather he was beating the bounds of science. But modern particle physics wrecks any idea of accounting for the meaningfulness of the propositions of everyday language by means of an analysis terminating in names for well-behaved objects. This is perhaps why complicating the logical system of the *Tractatus* proved such

a short-lived stage in the development of Wittgenstein's thinking. It also throws into sharp relief his strictures against philosophical views on the "queer connexion" of naming, at *Philosophical Investigations*, §§37–47:

> But what are the simple constituent parts of which reality is composed?—What are the simple constituent parts of a chair?—The bits of wood of which it is made? *Or the molecules, or the atoms?*—'Simple' means: not composite. And here the point is: in what sense 'composite'? It makes no sense at all to speak absolutely of the 'simple parts of a chair'. (*Philosophical Investigations*, §47, emphasis added.)

Instead of the nonsensical, we get the senseless:

> Eddington says that whenever you turn a light-ray on an electron it vanishes: I might also say that there is a white rabbit on my sofa which cannot be seen because whenever anyone looks at it it vanishes. These two propositions are on exactly the same level: both are merely senseless. (*Lee Lecture Notes*, p.111)[24]

However, when it comes to the Heisenberg/Bohr debate over the Copenhagen interpretation of quantum mechanics, Wittgenstein appears unsure what to say. Camilleri quotes Heisenberg, from a December 1930 lecture:

> Heisenberg insisted that "the indeterminacy relations hence should not simply be conceived of as the impossibility of precisely *knowing* or *measuring* the position and velocity [of an electron]; the indeterminacy relations signify that an *application of the words* 'position, velocity' loses any reasonable meaning beyond specified limits." (Camilleri 2009, p.106)

This line of thinking is perhaps alluded to by Wittgenstein in a 1933 lecture:

> It's absurd to say I can't see an electron because it's too small. It's not just <u>false</u> to say "I see an electron vibrating (schwingen)", but it's <u>nonsense</u> to say either that or that I don't. (*Moore Lecture Notes*, p.363)

[24] In a lecture on laws of nature and freedom of the will Wittgenstein is quoted as saying, "In the case of electrons one simply gives up. 'No. There are no laws here.' " (Wittgenstein 1939, p.432)

Absurd, false, or nonsense? Here a descriptive approach, regarding our propositions in perfectly good order just as they stand, hits the buffers. Consideration of such matters is hardly a part of everyday language, but it is telling that Wittgenstein's next sentence reverts to the familiar world of classical mechanics; "Suppose we observed heavenly bodies moving in epicycloids . . . " As to ruling out tautology and contradiction, Wittgenstein's later responses veer towards the polemical:

> Think of the case of the Liar. It is very queer in a way that this should have puzzled anyone—much more extraordinary than you might think . . . if a man says "I am lying" we say that it follows that he is not lying, from which it follows that he is lying and so on. Well, so what? You can go on like that until you are black in the face. Why not? It doesn't matter . . . Now suppose a man says "I am lying" and I say "Therefore you are not, therefore you are, therefore you are not . . . "—What is wrong? Nothing. Except that it is of no use; it is just a useless language-game, and why should anybody be excited? (*Lectures on the Foundations of Mathematics*, pp.206–7)[25]

At this point I will draw the discussion to a close. The critical lesson is, I think, that one cannot keep logic out of at least one of psychology, metaphysics, or science. An wholly *sui generis* conception of logic is possible, but this is the study of abstract structures, that may or may not be applicable; a discipline that trades in wholly formal properties and not in truth, other than by accident (the accident that a system so conceived happens to be applicable). This is not, I think, what Wittgenstein had in mind. What he intended is a logical system that is applicable (5.557) and whose propositions somehow show something about the world (6.124). What has been regarded as failure shows, I think, that if logic is going to trade in truth and application then it cannot be wholly purged of extra-logical commitments.

[25]Cf. *Remarks on the Foundations of Mathematics*, Pt.I Appendix III §§12–13.

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