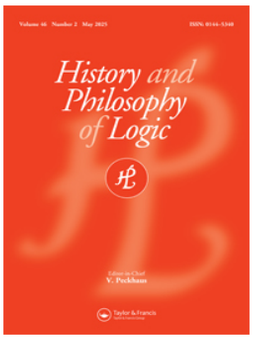
Tractarian functions are not propositional functions



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Tractarian Functions are not Propositional Functions

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ABSTRACT

The central claim is that in the *Tractatus* Wittgenstein refuses Russell's notion of propositional function, and Russell's notions of range of significance and types for variables. Drawing on Russell's notion of well-defined function, for which all values are given in advance, Wittgenstein takes a different approach, that propositions realise a common characteristic, the general form, this being definitional of the propositional. This common characteristic is a formal concept, an *Urbild* (prototype), demonstrable by definition by abstraction. What is novel is the delimitation of the notion of a function *fx* in 5.501, and the argument showing that the general form, which Wittgenstein regards as a variable, is immanent to its realisations.

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1. The Absence of 'Propositional Function'

When Wittgenstein talks about functions in the *Tractatus*, and in particular, the 'function *fx*' in 5.501, what he has in mind is not, I suggest, Russell's notion of propositional function. Conflating Wittgenstein's notion of a function with Russell's notion of propositional function is so widespread that, in quoting 5.501, Rogers and Wehmeier insert the word as if Wittgenstein had, perhaps carelessly, omitted it: 'a [propositional] function *fx*, whose values for all values of *x* are the propositions to be described (5.501)' (*Rogers and Wehmeier 2012*, p. 563).

Reading the familiar Russellian notion of propositional function into the *Tractatus* is a barrier to understanding what Wittgenstein is doing, not least because despite outward similarities with the notation of *Principia Mathematica*, it isn't there to be found. In fact, the expression 'propositional function' doesn't appear in *Wittgenstein 1984a*, *Wittgenstein 1984b*, *Wittgenstein 1984c*, *Wittgenstein 1971*, the *Tractatus*, or the correspondence (*Wittgenstein 2012*, *Wittgenstein 1973*). Wittgenstein's earliest use of the expression is at *Wittgenstein 1974*, p. 125.

Rogers and Wehmeier note that Wittgenstein could have used Russell and Whitehead's $f\hat{x}$ notation (*Rogers and Wehmeier 2012*, p. 564, n. 48):

> That Wittgenstein was familiar and comfortable with representing propositional functions by means of the circumflex notation is evidenced by *Wittgenstein 1971*, 5.3321:

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> '[T]he proposition "*Only* one x satisfies $F(\hat{x})$", will read "$(\exists x).\, Fx :\sim (\exists x, y).Fx.Fy$" '. Compare [*Tractatus*] 5.5321, where $F(\hat{x})$ is replaced by $f(\ )$.

But one can equally read this as Wittgenstein deliberately suppressing the earlier use of the circumflex notation, in preparing his work for publication. Rogers and Wehmeier also quote from *Ramsey 2006*, p. 15, where Ramsey imposes Russellian notation on Wittgenstein; '$\bar{\xi}$ is the set of values of $f\hat{x}$'.[1] They then say, 'Although the review [*Ramsey 2006*] was written before Ramsey's consultations with Wittgenstein in the summer of 1923, the passage demonstrates how natural it was for someone writing in the *Principia* tradition to assume the availability of circumflex notation' (*Rogers and Wehmeier 2012*, p. 565). But this is a comment about *Ramsey*, not about *Wittgenstein*. The only use of circumflex notation in *Wittgenstein 1971* is in 5.3321. It appears in *Wittgenstein 1984c* at pp. 11, 16, 19, 28 and 32, at *Wittgenstein 1984a*, p. 101 (cf. *Potter 2009*, p. 278), and once in the correspondence (L.W. to B.R. November 1913, *Wittgenstein 1984c*, pp. 128/9, *Wittgenstein 2012*, pp. 57/59). It is not used in the *Tractatus* at all. It is true that absence of evidence is not evidence of absence, but it is reasonable to think that such absence may be indicative of theoretical distinctions. This is the moral that I seek to draw.

It is noticeable that the only appearance of the expression 'propositional function' in some editions of the *Tractatus* is in Russell's introduction, where Russell attributes to Wittgenstein an account of generality based on 'all propositions of the form *fx* where *fx* is *a given propositional function*' (*Russell 1961*, p. xxi, emphasis added). Wittgenstein initially refused to have Russell's introduction printed, because on translation into German 'what remained was superficiality and misunderstanding' (L.W. to B.R. 6 May 1920, *Wittgenstein 2012*, p. 119). The topic of the present paper is, I suggest, an illustration of such divergence.

It may seem that I have singled out Rogers and Wehmeier, but in general their paper is excellent and has been justly influential. Nevertheless the presumption that Wittgenstein was operating with the Russellian notion of propositional function in the *Tractatus* is a largely unquestioned orthodoxy.[2] Fisher and McCarty are, as far as I am aware, the only authors to note the absence of the phrase 'propositional function' in the text (*Fisher and McCarty 2016*, p. 305), but far from questioning this omission, they make extensive use of the notion in their reconstruction of the logic of the *Tractatus*.

Rather than address individual attributions of the notion of propositional function to the early Wittgenstein, my contention is that Wittgenstein's approach is not directly based on this notion at all. The rest of this section consists of preliminary remarks, for orientation.

The notion of propositional function involves, essentially, a mapping from the values of a variable to a truth-value; from, in the present context, the members of a domain of individuals to the truth-values *true* and *false*. In the Tractarian context there are two immediate difficulties with this. First, there is no straightforward notion in the *Tractatus* of a domain of individuals; the Tractarian notion of *object* is notoriously obscure. The second, related difficulty concerns the *context principle*, that it is only in the context of a proposition that

[1] The notation $\bar{\xi}$ first appears at *Wittgenstein 1961*, 5.501; from hereon naked numbers refer to propositions in the *Tractatus*.

[2] The notion of propositional function is specifically attributed at, for example, *Ishiguro 1981*, pp. 43, 49, *Miller 1995*, § 3, pp. 210ff, *Hylton 1997*, p. 141, *Potter 2009*, p. 44, *Ricketts 2013*, p. 129, n. 4, *Halimi 2016*, p. 289, *Weiss 2017*, p. 5, *Connelly 2021*, p. 174. But one can find tacit attribution in pretty much the entirety of the secondary literature.

a name has a meaning (3.3). Given this principle it does not follow, without further argument, that a name has the same meaning in different propositional contexts. In fact these difficulties stand, or fall, together. For the moment, these are just framing remarks, to motivate the discussion. What I want to suggest is that we may come to a better understanding of the logic of the *Tractatus* if we set aside familiar textbook conceptions of first-order logic and quantification; that is, textbook presentations in the tradition that originates in *Hilbert and Ackermann 1928* that, critically, assume a domain of well-defined, discrete individuals. So when Hintikka, for instance, says that 'a full understanding of the most central concept of contemporary logic, the notion of quantifier [...] eluded Wittgenstein' (*Hintikka 1996*, p. 261), and von Plato discusses 'Wittgenstein's failure to understand quantificational logic' (*von Plato 2017*, p. 184; cf. p. 24), these reflect, I think, an assumption that Wittgenstein should be doing first-order logic with quantification, and a consequent failure to appreciate that this is not what he is doing.

It is not enough, though, merely to point out that Wittgenstein does not use the terminology. What I seek to do in the following is to present an alternative; that Wittgenstein begins his logic with elementary propositions, where these are *functions of names* (3.202, 3.318). Tractarian functions are then *abstractions* from elementary propositions, such abstractions called by Wittgenstein *Urbild* (generally translated as *prototype*). In taking this approach Wittgenstein develops one of the two approaches Whitehead and Russell take to propositions in their book and, to propositional functions, one that involves beginning with a totality of propositions as all the values of a propositional function. Wittgenstein departs from Russell and Whitehead by working wholly from the top down—abstracting functions from propositions, and not in any sense from the bottom up—assembling propositions from constituents, where these include a propositional function.

On the top-down approach it is argued that what Wittgenstein seeks is what is common to all and only propositions, this being what will be called the *realisation* of the general form of the proposition, that is, the realisation of an *Urbild*. For an elementary proposition, this is primarily a matter of being a *function of names*. For all propositions, elementary, complex, and everyday, it is a matter of being a *truth-function of elementary propositions*, expressible by means of the *N* operator. This is stated at 5, with the parenthetical remark that 'An elementary proposition is a truth-function of itself', to cover the case of an elementary proposition considered by itself. Paradoxical though it might sound, when Wittgenstein says that 'The general propositional form is a variable' (4.53), what he means is what is *variable* is what is *realised in all cases*.[3]

The plan of the paper is to begin with significance, taken together with Russell's and Whitehead's two approaches to propositions and propositional functions, and Russell's multiple-relations theory of judgement, in Sections 2 and 3. Wittgenstein's response to Russell's theory of judgement and to Whitehead's and Russell's approach to the variable is addressed in Section 4. The top-down/bottom-up strategies are outlined in Section 5, and the critical notion of *Urbild* (*prototype*) in Section 6. The general form of the proposition is addressed in Section 7, and the concomitant notion of variability in Section 8. Abstraction and bracket expressions as the relevant mechanisms are addressed in Section 9. The hard case, the talk of functions in 5.501, is addressed in Section 10.

[3] The same line of thought can be seen in 6.022, with reference to number.

2. Russell's Notion of Significance

In *Whitehead and Russell 1910–13*, Russell (on the basis that Russell was largely responsible for this aspect of the text), says (*Whitehead and Russell 1910–13*, Vol. I, pp. 17–18):

> Let ϕx be a statement containing a variable x and such that it becomes a proposition when x is given any fixed determined meaning. Then ϕx is called a 'propositional function' […] 'x is hurt' is an ambiguous 'value' of a propositional function. When we wish to speak of the propositional function corresponding to 'x is hurt', we shall write '$\hat{x}$ is hurt'. Corresponding to any propositional function $\phi\hat{x}$, there is a range, or collection of values, consisting of all the propositions (true or false) which can be obtained by giving every possible determination to x in ϕx […] Any value 'ϕx' of the function $\phi\hat{x}$ can be asserted. Such an assertion of an ambiguous member of the values of $\phi\hat{x}$ is symbolised by '$\vdash \phi x$' […] This […] embodies the use of the [real] variable.

The critical point is an implied differentiation between a propositional function, and determinations of the variable that result in a proposition. Since Russell was engineering a way of evading paradoxes, given such separation he needed a way of specifying the range of determination (the 'total variation', *Whitehead and Russell 1910–13*, Vol. I, p. 16) of x. This is addressed at *Russell 1956a*, p. 72, n. *, where Russell sets out his notion of significance:

> A function is said to be significant for the argument x if it has a value for this argument. Thus we may say shortly 'ϕx is significant', meaning 'the function ϕ has a value for the argument x'. The range of significance of a function consists of all the arguments for which the function is true, together with all the arguments for which it is false.

He then makes a rather odd remark (*Russell 1956a*, p. 73, emphasis added):

> [I]f the function ceases to be significant when the variable goes outside a certain range, then the variable is *ipso facto* confined to that range, *without the need of any explicit statement to that effect*.

How is such confinement supposed to come about? Now, a range of significance is equated with a type; 'A *type* is defined as the range of significance of a propositional function, i.e. as the collection of arguments for which the said function has values' (*Russell 1956a*, p. 75). But how is a type to be specified? Here we get the critical move, to evade vicious circles (*Whitehead and Russell 1910–13*, Vol. I, p. 41, emphasis added):

> When we say that 'ϕx' ambiguously denotes ϕa, ϕb, ϕc, etc., we mean that 'ϕx' means one of the objects ϕa, ϕb, ϕc, etc., though not a definite one, but an undetermined one. It follows that 'ϕx' only has a well-defined meaning […] if the objects ϕa, ϕb, ϕc, etc., are well defined. That is to say, *a function is not a well-defined function unless all its values are already well-defined*.

Russell simply assumes that the range of significance of a propositional function is already present and correct, given in advance.[4] On this basis he can 'say [that] "ϕx is a proposition", but "$\phi\hat{x}$" is a propositional function' (*Whitehead and Russell 1910–13*, Vol. I, p. 42). As Quine notes, the x in ϕx (not $\phi\hat{x}$) only looks like a free variable; 'because the free variable is in this way unhampered, Russell likes to suppress a universal quantifier when it has the whole of a theorem as its scope' (*Quine 1967*, p. 151). The critical point is that Russell's ϕx is propositional, appearances notwithstanding. This is, I think, what Wittgenstein is going to rely on, implicitly, in the *Tractatus*, that the *fx* of 5.501 can be regarded as propositional, that is, as akin to Russell's ϕx and not his $\phi\hat{x}$, because it is well-defined, and hence that all the relevant propositions are effectively already present.[5]

In developing Russell's approach Wittgenstein aims to resolve Russell's difficulties over propositional unity, and with judging whether a given proposition is true, or false. Wittgenstein states his position at *Wittgenstein 1984a*, p. 98:

> (1) Symbols are not what they seem to be. In 'aRb', 'R' looks like a substantive, but is not one [...] Similarly in 'ϕx', 'ϕ' looks like a substantive but is not one.

This is aimed at Russell, because Wittgenstein thinks that the former's approach involves treating signs for properties and relations (the universals of *Russell 1912*, Chap. 9) as names, and thus what they name, as substantives. Hence a Russellian proposition becomes a list of names for substantives such that there isn't a proposition at all, resulting in ever more sophisticated versions of the multiple-relations theory of judgement in, so Wittgenstein holds, a vain effort to reassemble what Russell's theoretical commitments have hopelessly sundered. But this is not the Russell we have seen so far. What Wittgenstein is doing is not only developing one aspect of Russell's approach, but also bringing out the fact that Russell has two different and unreconciled treatments of the notion of proposition.

3. Technical and Philosophical Approaches to Propositions

The consequence of this is nevertheless a profound tension between assuming that the values of a propositional function (a class of propositions) are given in advance ('already well defined'), and the idea that a proposition 'is not a single entity at all' (*Whitehead and Russell 1910–13*, Vol. I, p. 46), that its disparate elements—an n-adic propositional function and n determinations of its variable(s)—are only assembled into a proposition in a mental act by a judging subject. Russell ostensibly rides two horses, treating propositions both as somehow given in advance, and thus as suitably unified entities, *and* as somehow constructed in judgement. These will be called respectively the 'technical approach' (that propositional

[4] The significance of significance can be seen in its ubiquity, at, *inter alia*, *Russell 1973b*, pp. 201–202, 205–206, *Russell 1973c*, p. 177, *Russell 1956a*, pp. 72–75, *Whitehead and Russell 1910–13* Vol. I, pp. 43–44, 48, 50, ∗1.11, ∗1.72, ∗3 summary, ∗3.03, ∗9.14, ∗10.121, ∗10.14, ∗10.4f, ∗11.14, ∗13.22, ∗13.3, ∗20.631f, ∗21.631, *Russell 1919*, pp. 159, 162, *Russell 1956b*, pp. 337–338.

[5] Of 5.501(2) Halimi claims, treating Wittgenstein's *fx* as a propositional function, that this can only work if 'a magical "range of significance" is supposed to be attached to any function and given along with it: This is actually Russell's resort in *Russell 1956a*' (*Halimi 2016*, p. 288). Halimi doesn't appreciate that Wittgenstein's notions of Tractarian function, propositional variability, and prototype in 3.313ff assume exactly such attachment.

functions are well-defined, that propositions can be taken for granted) and the 'philosophical approach' (multiple-relations theory). This bifurcation is clearest in *Whitehead and Russell 1910–13*, Vol. I, Chap. II, §§ II–III.[6]

In his technical work Russell's preference is for the first. He was fully aware of Wittgenstein's objection, that you can't assemble a proposition out of disparate constituents (the philosophical approach) unless you already know that they are such that they can be so assembled, and one can manufacture different responses depending which text one chooses to concentrate on. *Russell 1984* represents the apotheosis of the disparate end of the assembly problem (the multiple-relations theory), with the approach assuming that all values are given in advance (the technical approach) in *Russell 1956a* and, arguably, in *Whitehead and Russell 1910–13*, constituting the opposite extreme.

That Wittgenstein presumes the technical approach in the *Tractatus* is, I think, anticipated at *Wittgenstein 1984c*, p. 54; 'In these investigations I always seem to be unconsciously taking the elementary proposition as my starting point'. Ramsey notes this presupposition of elementary propositions in the *Tractatus* in a record of his 1923 conversations with Wittgenstein.[7] To be clear, though, what Wittgenstein takes over is a strategy of beginning with elementary propositions. What he does not take over is the whole Russellian apparatus, and in particular, the notion of propositional function.

With reference to Russell's multiple-relations theory of judgement, concentrating on Wittgenstein's mature views in the *Tractatus*, beginning with elementary propositions enables him to sidestep both major problems; first, the 'wide direction' problem:

> WD is the problem of ensuring that a judgement contains at least one predicate or relation, that the relation occurs in the correct position and thus that the judgement has sense.

Second, the 'narrow direction' problem:

> ND is simply the problem of distinguishing, for example, Othello's belief that Desdemona loves Cassio from Othello's belief that Cassio loves Desdemona.[8]

Assuming elementary propositions from the start sidesteps the wide direction problem. The narrow direction problem is sidestepped by assuming that elementary propositions are bipolar, that they are either true, or false; the issue of the direction of the relation becomes a matter of the actual truth-value of the propositions concerned, ascertained by comparison with the obtaining facts.

The assumption of elementary propositions enables Wittgenstein to reject any approach that treats a Russellian propositional function as an entity in its own right, containing one or more free variables, such that a determination of its variables results in a proposition; and any associated strictures needed to ensure that the determinations are drawn from the

[6] Cf. discussion at *Linsky 1999*, pp. 23–25.

[7] Cf. Ramsey papers #002-27-01 p. 24, quoted at *Klagge 2022*, p. 55.

[8] The wide/narrow distinction originates in *Griffin 1985*, the formulations here are taken from *Connelly 2021* p.35 and p.33 respectively. Wittgenstein's views are thoroughly discussed in *Connelly 2021*; the successive phases of the multiple-relations theory are well set out in *Candlish 1996*. The impact of Wittgenstein's criticisms on Russell is controversial (cf. *MacBride 2018*, pp. 156–161), however as this is tangential to the concerns of the present paper I have stripped the discussion to the bone.

range of significance of the propositional function in question. The Russellian, by contrast, needs a statement of the range of significance of a propositional function in addition to the function itself.

4. Variables Real and Apparent, Individual and Propositional

In discussing Tractarian functions, Soames says 'The function mentioned […] is not really a function, but a formula containing a (free) variable. Such formulas have often been accorded the misleading title "propositional function" ' (*Soames 1983*, p. 578, n. 14). Soames does not explain what he has in mind by 'misleading', referring his reader to the 'clear explanation' to be found at *Ramsey 1990*, p. 171:

> A propositional function is an expression of the form $f\hat{x}$, which is such that it expresses a proposition when any symbol (of a certain appropriate logical type depending on f) is substituted for $\hat{x}$. Thus '$\hat{x}$ is a man' is a propositional function. We can use propositional functions to collect together the range of propositions which are all the values of the function for all possible values of x. Thus '$\hat{x}$ is a man' collects together all the propositions 'a is a man', 'b is a man', etc.

Ramsey was clearly familiar with Whitehead and Russell's notation, and its need for a specification of appropriate logical type. With this in mind, we can turn back to *Rogers and Wehmeier 2012*, p. 556:

> By a *propositional variable*, Wittgenstein understands any variable whose values are propositions. A *variable proposition* is an open sentence, that is, the result of replacing, in a proposition, a constituent by a variable. Finally, a *propositional function* is a function that sends names to propositions in such a way that a name a is mapped to the result $\phi(a)$ of replacing the variable x in a variable proposition $\phi(x)$ with a.[9]

The notion of a *propositional variable* is certainly to be found in the *Tractatus* (cf. 3.313), as is that of a *variable proposition* (3.315):

> If we turn a constituent of a proposition into a variable, there is a class of propositions all of which are values of the resulting variable proposition.

The 'function *fx*' of 5.501 is thus a *variable proposition*, 'whose values for all values of x are the propositions to be described'. 3.315 is premonitory of 5.501, and what Wittgenstein is doing is, I suggest, exploiting Russell's technical approach, that all the instances are given in advance; this is, I think, the approach adumbrated in *Whitehead and Russell 1910–13*, that 'when we say "ϕx is a proposition", we mean to state something which is true for every possible value of x, though we do not decide what value x is to have' (*Whitehead and Russell 1910–13*, Vol. I, p. 42). The salient phrasing here is 'true for every possible value of x', where what is presumed is all the resulting propositions, and not a working through (a

[9] The notions of propositional variable and prototype are discussed at *Ishiguro 1981*, p. 56, *Glock 1996*, p. 146, *Marion 1998*, pp. 35, 37, *Connelly 2021*, p. 174, *Klagge 2022*, p. 91.

ranging over) every element in the domain to see whether or not the result is a proposition. Significance ensures that the 'possible values' are already present and correct.

Variable propositions certainly look like open sentences, containing free variables, but as we have already seen noted by Quine, this appearance has to be treated with care. Whitehead and Russell distinguish *real* and *apparent* variables (*Whitehead and Russell 1910–13*, Vol. I, pp. 18, 17):

> When we are considering or asserting ϕx, the variable x is called a 'real variable' [...] The x which occurs in '$(x).\phi x$' or '$(\exists x).\phi x$' is called (following Peano) an 'apparent variable'.

This certainly looks like the familiar distinction between free and bound variables, but one should be wary. Wittgenstein recurs to this in his correspondence with Russell in June 1912 and in July 1913 (*Wittgenstein 2012*, pp. 30, 42):

> The prop[ositions] of logic contain only APPARENT variables.All my progress comes out of the idea that the *indefinables* of logic are of the general kind [...] and this again comes from the abolition of the real variable.

In rejecting real variables Wittgenstein is also, I think, rejecting the notion of propositional function, and this is, I think, constant from the early correspondence right through to the published text of the *Tractatus*. What looks like an open sentence with a real or free variable is not what it might appear to be, because Wittgenstein develops and adapts Russell's technical approach, that we are dealing only with well-defined functions all of whose values are given in advance, on the basis that we begin with *propositions*, and then look for variability *within* propositions. This is the force of 3.314:

> An expression has meaning only in a proposition. All variables can be construed as propositional variables. (Even variable names.)

It is significant that 3.314 is a comment on the context principle at 3.3, that 'Only propositions have sense; only in the nexus of a proposition does a name have meaning'. The key point is that what looks like an open sentence, or a propositional function, in the *Tractatus*, is not an open sentence or a propositional function in the now familiar sense, and to construe the *fx* of 5.501 as a Russellian propositional function is to go astray from the beginning.[10]

[10] Diamond does not attribute the Russellian notion of propositional function to Wittgenstein in *Diamond 2014*, but I will not discuss her paper because of a significant misinterpretation, I think, of Russell's views. Diamond asserts that 'according to Russell in *Whitehead and Russell 1910–13*, the number of values of a propositional function is necessarily infinite' (*Diamond 2014*, p. 23). Diamond refers to *Whitehead and Russell 1910–13*, Vol. I, pp. 39–40, and p. 51, where Russell discusses the vicious-circle principle. What he says is that if we have a proposition $(\phi).f(\phi\hat{z}, x)$, 'it follows that the totality of values of $\phi\hat{z}$ concerned in $(\phi).f(\phi\hat{z}, x)$ is not the totality of all functions in which x can occur as argument, and that there is no such totality as that of all functions in which x can occur as argument' (*Whitehead and Russell 1910–13*, Vol. I, p. 51). For Diamond's claim to go through, either all propositional functions would have to be impredicative, and impredicativity would necessarily have to start off an infinite process, or there would have to be, necessarily, an infinite domain to be considered. It is not the case that all propositional functions are impredicative, and Russell does not hold that the domain is *necessarily* infinite (the axiom of infinity is an hypothesis, as per remarks to *Whitehead and Russell 1910–13*, Vol. II, ∗120.03). Diamond does not discuss the differing approaches in *Principia Mathematica*, reducibility, or significance,

5. Top-Down and Bottom-Up

As a closing speculative remark in a 2014 paper, Hanks suggests that Wittgenstein 'held a bottom-up view from *Wittgenstein 1984a* through to the composition of *Wittgenstein 1984c*, his view was in transition as he compiled *Wittgenstein 1971*, and that he took a decisive turn in favour of the top-down view in the final composition of the *Tractatus*' (*Hanks 2014*, p. 12). The bottom-up approach correlates names with objects and then assembles propositions therefrom, whereas the top-down approach reverses this, invoking the context principle, that 'a name can *only* have a meaning in the context of a proposition' (*Hanks 2014*, p. 10). Hanks also notes Kremer's observation, that the context principle is elevated in the numbering hierarchy, from 3.202 in *Wittgenstein 1971* to 3.3 in the *Tractatus* (*Kremer 1997*, p. 90):

> The [context principle], as *Wittgenstein 1971* 3.202, appears as simply one point among many about propositions and propositional signs—which are often not clearly distinguished. In *Tractatus*, the [context principle] heads up the largest group of remarks in the 3's, numbering thirty-three in all, almost half of the seventy-four remarks in the 3's, and more than twice as many as any other major group.

It is noticeable that of the propositions in the *Tractatus* setting up the notion of propositional variable, as well as being grouped as remarks on the context principle, four of them—3.311–3.314—have no counterpart in *Wittgenstein 1971*. The counterpart to 3.315 is 4.102274 in *Wittgenstein 1971*, with 3.316 and 3.317 spread out over 5.004–5.005 in the earlier work. This indicates the shift Hanks notes, a decisive rejection of Russell's philosophical approach to propositions, in favour of one more akin to his technical approach.

With respect to 3.31–3.313 Zalabardo argues, with reference to Palmer and Morris[11] , that 'these passages put forward a picture in which propositions are not regarded as complex entities, produced by the combination of more simple items. They are treated instead as basic units' (*Zalabardo 2015*, p. 112). On this basis, 'the fundamental semantic relation [is] a mapping of (true) propositions onto facts' (*Zalabardo 2015*, p. 139), and is not a matter of correlating names and objects. This is a top-down view in spades, and there is much to recommend it. It does, though, overshoot the mark because one can have too much unity, such that one cannot make sufficient sense of the idea that an elementary proposition is a nexus, a concatenation, of names (4.22). Zalabardo attempts to account for this in terms of objects as 'features that states of affairs share with one another' (*Zalabardo 2015*, p. 117), but whether or not this works, for present purposes it will be assumed that an account can be given of whatever is needed at the level of objects/names.[12] The point in this section is to educe evidence for the argument that in the *Tractatus* Wittgenstein leans towards an

and makes reference only to the passages in *Principia* noted above, and to *Principles of Mathematics*, Chap. VII, and the summary thereof (pp. 106–107). Of the secondary literature she refers only to *Hylton 1990*, *Hylton 1994*, and *Hylton 1997*.

[11] *Palmer 1996*, *Morris 2008*.

[12] In his review of *Zalabardo 2015*, Proops says, 'as if in defiance of 2.03, Zalabardo claims that the *Tractatus*'s approach to the problem of factual unity takes an altogether different form. Wittgenstein, he maintains, is dissolving rather than solving the problem […] The stubborn truth […] is that the reading's textual fit is poor' (*Proops 2017*, p. 534). There is a difficult balance to be struck between propositional unity and generalisation over names as propositional constituents, but this will not be pursued further here.

approach that regards propositions as unities in which distinctions are to be drawn, rather than as assemblages of previously given constituents.

6. Tractarian Prototypes

In getting beneath the level of the propositional, that is, within elementary propositions, Wittgenstein's key notion is that of *prototype* (*Urbild*). This can be seen at 3.315, where he talks of variable propositions as resulting from turning a constituent of a proposition into a variable; substituting a variable name for a name, yielding a Tractarian function *fx* out of a proposition *fa*.[13] For present purposes the long-standing argument over whether objects include properties and relations can be set aside, on the basis that an elementary proposition is a concatenation of names. For logical purposes *name* is a single category. In concluding 3.315 Wittgenstein says, 'It [i.e. a variable proposition] corresponds to a logical form—a logical prototype [*einem logischen Urbild*]'. So what is a prototype?

The first difficulty is etymological, as Black observes (*Black 1964*, p. 126):

> *Urbild* is perhaps an unfortunate neologism. For the *Urbild* is not a picture of the form in the sense of Wittgenstein's 'picture theory', but rather embodies the form; it shows what every proposition that is an instance of the *Urbild* has in common with its sense.

Black is right that a prototype is a *common characteristic*, something that all instances have in common. Wittgenstein calls such characteristics *formal properties* (4.126), a formal property being an instance of a formal concept. This ties in with the notion of *variable* in the *Tractatus*, because 'Every variable is the sign for a formal concept' (4.1271). We have a tight circle, of formal concept, (propositional) variable, prototype, all instanced by propositions. The crux is that a variable standing for a formal concept does not *range over* a domain, looking for entities that can be substituted for it to produce a value (in the most salient context here, ranging over individuals such that a propositional function becomes a proposition). A variable shows a common characteristic, so in the Tractarian function *fx* the formal concept *x* shows that the instances *fa*, *fb*, and so on, are propositional. A variable for a formal concept shows a common characteristic, it is not a gap in need of satisfaction.

Where matters go awry is in assimilating the notion of *Urbild* with the picture theory of the proposition (*Bildtheorie*). Anscombe's and von Wright's usual translation of *Urbild* in *Wittgenstein 1984c* as *proto-picture* is consequently, I think, unfortunate.[14] As Black points out, with reference to *Anscombe 1971*, p. 67, Anscombe's brief mention of *Urbild* occurs in an extended discussion of the picture theory; 'Anscombe identifies the proto-picture with the "picture", without individual correlations' (*Black 1964*, p. 126). At this point in her text Anscombe is discussing how a proposition can function as a picture of a situation, for example, of two men fencing. Anscombe is fully alert to the top-down nature of the *Tractatus* (*Anscombe 1971*, p. 67):

[13] Given that a Tractarian elementary proposition is a concatenation of names, for example *cdkrrpsa*, it is clearly more convenient to treat *cdkrrps* as *f*, that is, a Tractarian function, for the sake of the argument. This is a merely heuristic simplification.

[14] Not in all cases; it is translated as *prototype* at *Wittgenstein 1984c*, p. 29.

> The picture of two men fencing was intelligible as a picture, without our making any correlations of the figures with individual men. We might compare to this picture, without individual correlations, what Wittgenstein at 3.24 calls 'the proto-picture' [*Urbild*] occurring in the generality notion: the '*xRy*', for example, in '$(Ex)(y)xRy$'.

By usually translating *Urbild* as *proto-picture*, and associating it almost entirely with the picture theory of the proposition Anscombe succeeded, I think, in largely overlooking the significance of this notion, because what Wittgenstein is after is what is common to the propositional *per se*, and not just what is bound up with the notion of the propositional as pictorial. If there is a general approach to this in the literature it is to see the notion of *prototype* as a Russellian hangover, to be acknowledged, at best, before moving swiftly on. This is typified by Klagge, who asserts that 'What Wittgenstein calls a "logical form" or a "logical prototype" Russell calls "the form of the proposition" ' (*Klagge 2022*, p. 92).[15]

Quite simply, if one thinks that a Tractarian prototype (*Urbild*) is a Russellian logical form, then one has already gone wrong. A Russellian logical form is 'the way in which the constituents are combined in a complex' (*Russell 1984*, p. 98) but, as the schema $U(S, x, R, y, \gamma)$ set out therein shows, it (i.e. γ) is in some sense a constituent of the proposition as understood (as assembled in thought and in judgement) by the subject *S*. Wittgenstein vehemently rejected this whole approach. In short, however much a Tractarian function might look like a Russellian propositional function—whatever use Wittgenstein might make of such notation as *xRy* (at, e.g. 4.1252, 4.1273), and however tempting it might be to assimilate a Tractarian *Urbild* to a Russellian propositional form—Wittgenstein is not following Russell. The temptation to find Russell in the *Tractatus* is strong, though: 'In contrast to [Russell] Wittgenstein offers no theory whatsoever about the kind of entity which a propositional function is' (*Ishiguro 1981*, p. 51). This is hardly surprising, given that Wittgenstein eschews the notion altogether. A Tractarian prototype is not a Russellian logical form, but given the notational similarities it is reasonable to ask how they differ.

The critical difference is this. A Tractarian prototype is a *formal concept*, characteristic of, or immanent to, a class of instances. It is not something external to or independent of a proposition, such that pre-given constituents are combined by means thereof into a proposition. So Wittgenstein owes his reader an account of how the formal concept that characterises propositions is a *prototype*, and thus how we get the notion of the *propositional variable*.

7. General Form of the Proposition

The way into this is to consider the general form of the proposition, stated at 6, bearing in mind Wittgenstein's curious remark at 4.53, that 'The general propositional form is a variable'. The aim is to show what is meant by this, and by the closing remarks of 4.126:

[15] Russell uses the term 'prototype' once, in an early discussion of no-class theory; 'in respect to the different values of $p(x/a)$, we may call *p* the prototype and *a* the *origin* or *initial subject*' (*Russell 1973a*, p. 155). In *Russell 1973c* '*p/a*' is a *matrix*, with *substitution* rather than *determination* for a variable, where 'this shadowy symbol *p/a* represents a *class*' (*Russell 1973c*, pp. 169, 170). There is clearly an influence on Wittgenstein's approach in the *Tractatus*, as if there are no classes then there is no obvious sense in which a variable can be said to *range over*.

> [T]he sign for the characteristics of a formal concept is a distinctive feature of all symbols whose meanings fall under the concept. So the expression for a formal concept is a propositional variable in which this distinctive feature alone is constant.

That 'proposition' is a formal concept is shown by the fact that one cannot sensibly say '*p* is a proposition'. A proposition can only show that it is such by being used to express a sense, that is, a way in which things—objects—can stand to one another, as per the concluding remark of 4.5, that 'The general form of a proposition is: This is how things stand'.

Of 4.5, Black says 'the form of words offered is cryptic to the point of unintelligibility', and that 'We might try saying, more ponderously: The essence of a proposition is its capacity to say something about how matters are in reality' (*Black 1964*, pp. 236, 237). But this is, I think, to over-egg the pudding. The remark at 4.5 is not intended as some profound insight, it is, rather, quite the opposite. Wittgenstein's aim is to identify a characteristic that is common to and distinctive of the propositional, so what is on offer is minimalistic to the point almost of banality. Propositions express sense; what is, or is not, but might have been, the case. One can say, *this* is how things stand, where the proposition deployed to say *this* is true, if they do so stand, and false, if they don't. And that's all there is to it.

The notion of *propositional variable* is effectively presented in 6:

> The general form of a truth-function is $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$.This is the general form of a proposition.

In other words, as per 4.53—'The general propositional form is a variable'—$[\bar{p}, \bar{\xi}, N(\bar{\xi})]$ is a variable. This, Wittgenstein tells us, is the common characteristic of the propositional.

It is striking, first, that Wittgenstein implies an equivalence between *proposition* and *truth-function*. The reason is Wittgenstein's idiosyncratic terminology. Here, as elsewhere, he takes over terms from Frege, or from Whitehead and Russell, and gives them a twist. So a Tractarian function is not a mapping or a correlation that sends names to propositions, and a Tractarian truth-function is not a mapping from propositions to more (or less) complex propositions. For the latter, at least, Wittgenstein has the notion of *operation*. Broadly speaking in the *Tractatus* an operation is what you *do*, a function is what you *get*. Sundholm asserts that 'Several *ion*-words show similar ambiguities in the *Tractatus*. *Funktion* is ambiguous between, on the one hand, "function of" and, on the other, the Fregean unsaturated notion' (*Sundholm 1992*, p. 60, n. 7). An elementary proposition is the primitive or degenerate case (a truth-function of itself, as 5). Complex propositions are truth-functions, the result of the application of a truth-operation.

Wittgenstein expresses this notion of function at 3.318, that 'Like Frege and Russell I construe a proposition as a function of the expressions contained in it'. A proposition is a function of its constituent expressions, whether it be names combining to form elementary propositions (a function of names), or whether it be (at a first application) elementary propositions combining by means of truth-operations to form complex propositions (a truth-function of elementary propositions). This process can be iterated without limit, resulting in more and more complex propositions, because an operation can take its own results as its base, whereas a function cannot be its own argument (5.251) (so after a first application *N* can take complex propositions generated at earlier stages as input).

Wittgenstein offers little explanation of the symbols used in 6, a deficit rectified by Russell in his introduction (*Russell 1961*, p. xv). $\bar{p}$ is the totality of all elementary propositions (5.5561), $\bar{\xi}$ a selection therefrom, N the joint negation truth-operator, so $N(\bar{\xi})$ is the result of applying N to the members of $\bar{\xi}$. According to Russell, 'The symbol [i.e. the whole expression $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$] is intended to describe a process by the help of which, given the elementary propositions, all others can be manufactured' (*Russell 1961*, p. xv). What Russell says is correct. The [_ _ _] expression in 6 does show how to manufacture complex propositions as truth-functions of less complex/elementary propositions. N is *the* truth-operation, and can be applied iteratively without limit.[16] But this is only part of the story and not, for Wittgenstein, I think, the most important part. Because what Russell does not explain is how the expression in 6 is a *variable*.

8. What is and What is not Variable

This is difficult to grasp because, confronted with the notion of a variable, we naturally look for a range of values for the variable (or a range of substitution instances). But this is not how Wittgenstein thinks of the notion. For Wittgenstein the notion of variability is bound up with a range of instances of a common characteristic, of a formal concept. What can be thought of as variable is the range of instances that realise a common characteristic. In the case of propositions, a proposition is an instance of the common characteristic—the general form—of the proposition, in that it (the proposition) says, this is how things stand. Hence 'To give values for a propositional variable is *to give the propositions* whose common characteristic the variable is' (3.317, emphasis in the original).

What Wittgenstein has in mind is the notion that all propositions are given in advance. It isn't difficult to trace this back through the claims that all objects are given in advance ('objects are what is unalterable and subsistent', 2.0271) and hence 'at the same time all *possible* states of affairs are also given' (2.0124). The totalities of names and elementary propositions follow immediately (4.52, 5.5262, 5.5561) and given 6 we get all possible truth-functional combinations (complex/everyday propositions). Since all of this is given, any concerns over type theory, impredicativity, and reducibility take care of themselves (and this is reflected in a correct notation). These are not small presumptions but they are all to be found readily in the text.

It follows that what varies, for Wittgenstein, is the ways in which a common characteristic, a prototype, is realised. The ostensible blandness of 4.5 is not because it is a triviality but because it is the statement of a common characteristic, the lowest common denominator. 6 can be thought of, as it was by Russell, as a way of manufacturing complex propositions, but what Wittgenstein really intends 6 to serve as is a statement of what is fundamental to the propositional, this being truth-functionality. What one might choose to call the 'range' of a variable is the class of instances of a formal concept; the variable—the prototype—is immanent, realised by the instances.

We can now see the force of 3.312, 'in this form the expression will be *constant* and everything else *variable*'. This can be rephrased as 'the expression of the general form of

[16] Griffiths and Paseau say that Wittgenstein 'leaves it an open question [in the *Tractatus*] whether *N* could apply to infinitely many propositions. As Wittgenstein's exposition of these matters is less than lucid, we leave it to Wittgenstein scholars to interpret him better' (*Griffiths and Paseau 2022*, p. 74, n. 20). Beyond ruling out the nondenumerable, as the *Tractatus* deals in totalities, it isn't clear that there is a definitive answer here.

the common characteristic, the prototype, is *constant*, and everything else *variable*'. When Wittgenstein says that 'The general propositional form is a variable' (4.53) what he means is, however paradoxical it may sound, that the variable is what stays fixed, variation arising from the range of instances that realise the common characteristic. And to understand the *Tractatus* one has to appreciate not just the promotion of the context principle as compared to *Wittgenstein 1971*, and the rearrangement of the 3.1s between the former and the latter, but also that the critically significant 3.311–3.314 in the *Tractatus* have no counterpart in *Wittgenstein 1971*. Wittgenstein moves a long way in a short period of time.

9. Definition by Abstraction, and [_ _ _] Expressions

It might seem that Wittgenstein's approach, deploying familiar notation with unfamiliar intent, is aberrant. What lies in the background is, I think, the notion of *definition by abstraction* (*Mancosu 2016*, p. 14):

> [W]e can say that a definition by abstraction takes its start from an equivalence relation $\sim$ over a class of entities [...] The equivalence relation can be thought of as capturing a common feature, a similarity, among the entities standing in the $\sim$ relation. One then ignores any individual feature of the objects except whether they stand in the $\sim$ relation thereby 'abstracting' from any other feature distinguishing the elements so related.

Wittgenstein would have been familiar with this approach through Frege's use of Hume's principle in his account of number in *Frege 1884*, and through Russell's definition of cardinal number in *Russell 1937*, § 111:

> [T]his method [...] [which] applies to all the cases in which Peano employs definition by abstraction [...] is, to define as the number of the class the class of all classes similar to the given class.

The approach in the *Tractatus* is to show what, exactly, all the instances of a formal concept have in common. If pressed as to how, I think the answer would have been by analysis, whatever process that might be. Analysis is intended to show the 'real' logical form of a proposition (cf. 4.0031), and in the end, the general form, the common characteristic of all propositions. Once analysis has stripped away 'the outward form of the clothing' (4.002) we are left with a common characteristic, propositions forming, on this basis, an equivalence class. This is why there is no need for the Russellian prefix '$p \supset p$' (5.5351), because what is propositional shows itself to be so, by realising the general form.

Looking at Wittgenstein's other uses of [_ _ _] expressions helps to make this clearer. Such expressions can be found at 5.2522, 6.01, and 6.03, as well as at 6, quoted earlier:

$[a, x, 0'x]$ (general term of a series of forms) (5.2522)
$[\bar{\xi}, N(\bar{\xi})]'(\bar{\eta})$ $(= [\bar{\eta}, \bar{\xi}, N(\bar{\xi})])$ (general form of an operation) (6.01)
$[0, \xi, \xi + 1]$ (general form of an integer) (6.03)

Sundholm criticises Wittgenstein, at *Sundholm 1992*, p. 66, on the basis that whereas:

> Wittgenstein is at pains [in 5.2522] to emphasise the homogeneity of arguments [...] in 6, on the other hand, homogeneity is violated in the most blatant way: the third argument-place is taken by a result of an application of the joint negation operator N to a certain range of *Sätze*, that is, by a *Satz*, whereas the other two places are taken by ranges of *Sätze*, whether elementary or not. It is incumbent on any *Tractatus* interpretation to try to make sense of this deviant use of the square brackets.

But this is not, I think, an objection that would have concerned Wittgenstein. What he is trying to do is identify common characteristics of instances of formal concepts. Uses of [___] expressions are intended as elucidatory, and not as comprehensive, as while the 'list' of formal concepts at 4.1272 mentions *complex*, *fact*, *function*, and *number*, and clearly applies to *object* and to *proposition*, in most cases not only are we not given [___] expressions, it is hard to see what expression there could be. For elementary propositions we might have $[\bar{x}, \bar{y}, C'(\bar{y})]$, where $\bar{x}$ is the totality of names for objects (cf. 5.556), $\bar{y}$ is a selection from these, and C the concatenation relation of 4.22. But this won't work in any useful sense because whereas N can be applied iteratively in unrestricted fashion to propositions, C cannot be so applied to objects, because objects have forms and as such unrestricted combinatorialism does not apply (2.0123) (i.e. not any list of Tractarian names is propositional).

What is more difficult in the propositional context is, I think, the talk in 5.2522 of a *series* of forms (*Formenreihe*); 'I call a series that is ordered by an *internal* relation a series of forms (*Formenreihen*)' (4.1252). This is clearly Wittgenstein's technique to get out numbers, as a series (4.1252, 4.1273), although the consequence is, as Russell points out of 6.03, a limitation to the finite.[17] But what is programmatically unclear is what analogue there is for *successor* or *ancestral* when it comes to propositions; in what sense does the successive application of N yield a formal series, where the terms of the series are internally related to one another? What does Wittgenstein mean when he says in 6.001 that 'every proposition is a result of successive applications' of $N'(\bar{\xi})$?[18]

That every proposition can be so generated is true, assuming expressive completeness, provided one accepts that *successive* means that there is *some* point in a process of repeated application to selections of propositions as input at which one will get a particular proposition as output. There is no significance to the order in which propositions appear, and no obvious sense in which propositions can be ordered in terms of 'ancestor of' or 'successor of'.[19] One could order propositions into disjoint classes by complexity (number of constituent elementary propositions, number of applications of N), or by arithmetisation, but it is hard to see why one would do so. What is consequential is Wittgenstein's implicit assumption that the bracket expression $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$ encapsulates what can be called the *constructional history* of a proposition. Because it is in this sense that the [___] expression in 6 can be thought of as an *Urbild* realised by a proposition; it is in this sense that the general propositional form $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$ is a variable (4.53).

[17] 'You only get finite ordinals. You deny classes, so cardinals collapse. What happens to $\aleph_0$?' (B.R. to L.W. 13 August 1919, *Wittgenstein 2012*, pp. 96–97). Cf. *Russell 1996*, p. 323.

[18] The ' is present in *Wittgenstein 1981*, missing in *Wittgenstein 1961*, and reinstated in *Wittgenstein 2023*.

[19] Cf. *Marion 1998*, p. 26, and *Cheung 2000*, p. 253, the latter in opposition to Fogelin and Anscombe.

10. Functions in 5.501

In discussing an argument put forward in *Diamond 1991*, and with respect to 5.501, Sullivan claims that the argument he (Sullivan) has given 'considers only one possible style for a variable ranging over propositions, when the *Tractatus* recognises three' (*Sullivan 2004*, p. 51). It is, I hope, by now clear that there is no such thing in the *Tractatus* as a *variable ranging over propositions*. The general form is a variable that is *realised by* propositions, there is no *ranging over*. With this in mind, we can turn to Sullivan's discussion of the relevant part of 5.501 (*Sullivan 2004*, p. 51):

> We may distinguish three kinds of description: 1. Direct enumeration [...] 2. Giving a function *fx*, whose values for all values of *x* are the propositions to be described. 3. Giving a formal law, according to which those propositions are constructed. In this case the [values] are all the terms of a formal series (5.501).

This is the Ogden translation, the word 'values' replacing the phrase 'terms of the expression in brackets'. But what is going on in (1)–(3) is not, I think, three different ways in which a variable ranges over propositions. Rather we get three different ways in which a class of propositions can be described such that they form a class $\bar{\xi}$ that can be input to *N*. The wording is difficult, but it becomes a great deal more difficult if one reads it à la Russell rather than à la Wittgenstein.

Since we are dealing with propositions, all of them are covered by 6; they all realise the general form of the proposition. It is a matter now of choosing a class out of the totality of propositions, elementary and complex, as input to *N*, assuming for the sake of the argument that this totality is of manageable cardinality. If we get the members of the class by (1), then there is no difficulty. If we can give the members of the class by means of a Tractarian function *fx* (2), all of whose values—propositions—are given in advance, then we get all propositions that realise the prototype *fx*. We don't have a propositional function that ranges over values, rather we have a disjoint subclass of all propositions, whose members realise a prototype. If it is objected that we can't do this, the answer is that we can, in principle, at least, if we can just find a way of emulating the logical god of 5.123. In the case of (3) we have a [_ _ _] expression, where this expression is the variable; it is a specification of the common characteristic, the prototype, realised by a selection from the totality of propositions.

The point is that we are not looking for three different styles of variable that range over, rather we are looking for three different ways in which a variable, a common characteristic that is more discriminating than the general form, is realised. This is potentially awkward for (1), because the only common characteristic may be *appearing in this direct enumeration*. Stipulation may be enough to carry the day. This is, perhaps, what Wittgenstein has in mind in his talk of 'stipulat[ing the] values for a propositional variable' at 3.316 and 3.317. It is noticeable that in *Wittgenstein 1971* such stipulation is discussed at 5.004ff, this series of propositions including the analogue of 5.501 (5.00531 in *Wittgenstein 1971*). The implied link is lost in rearrangement.

In the case of (2), in principle at least, we need to know what prototypes there are. However, 'we are [...] unable to give the composition of elementary propositions' (5.55), but all that follows from this is that we do not know *a priori* how many different prototypes there

are. We have however 'some concept of elementary propositions quite apart from their particular logical forms' (5.555), because we possess 'a system by which we can create symbols' (5.555), that is, to create whatever symbols we need for however many prototypes there are (fx, ϕxy, and so on). The members of (2) are given by realising a particular prototype, thus picking out a disjoint subclass of the class of elementary propositions. In the case of (3) we have of course a [_ _ _] expression.

The argument of Diamond's that Sullivan addresses is a strategy of replacement, that we will achieve philosophical insight by replacing philosophical vocabulary 'by a notation designed to make logical similarities and differences clear' (*Diamond 1991*, p. 183). But in the case of, at least, *proposition* this is not, I think, what Wittgenstein is doing. Rather in saying that a proposition is a truth-function, that the common characteristic of the propositional is a matter of *This is how things stand* (4.5), what is on offer for this common characteristic is a definition by abstraction. Once we see that this is so, we no longer need to ask whether or not some candidate expression is a proposition; rather we ask, do things stand thus and so, or don't they? We are past the point of sense, because what we are dealing with expresses a sense, and have already advanced to the question of truth-value. If we are unable to make sense then we can only conclude that what we are dealing with is not propositional; appearances notwithstanding, it is senseless, or nonsense.

11. Conclusions

In summary, in the *Tractatus* Wittgenstein has a top-down approach that assumes that all objects and thus all propositions, elementary, complex, and everyday, are all given in advance. What he then sets out to show is what it is that these have in common, what their common characteristic is. This is the general form of the proposition, and it is a variable in terms of definition by abstraction. What is propositional is what realises the general form. There is no Russellian apparatus of propositional functions, types, significance, or a bottom-up strategy generally to bring together disparate elements into propositions. Superficial similarities between Wittgenstein's and Russell's work conceal deep differences.

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References

Anscombe, G. E. M. 1971. *An Introduction to Wittgenstein's Tractatus*, 1st ed. 1959, new ed. 1971, Reprinted Bristol: Thoemmes Press [1996].

Black, M. 1964. *A Companion to Wittgenstein's Tractatus*, Ithaca, NY: Cornell University Press.

Candlish, S. 1996. 'The unity of the proposition and Russell's theories of judgement', in R. Monk and A. Palmer (eds.), *Bertrand Russell and the Origins of Analytical Philosophy*. Bristol: Thoemmes Press, pp. 103–133.

Cheung, L. K. C. 2000. 'The Tractarian operation *N* and expressive completeness', *Synthèse*, **123**, 247–261.

Connelly, J. R. 2021. *Wittgenstein's Critique of Russell's Multiple Relation Theory of Judgement*, London: Anthem Press.
Diamond, C. 1991. 'Throwing away the ladder: how to read the *Tractatus*', in C. Diamond (ed.), *The Realistic Spirit*, Cambridge, MA: MIT Press, pp. 179–204.
Diamond, C. 2014. 'Addressing Russell resolutely?', *Philosophical Topics*, **42** (2), 13–43.
Fisher, D., and McCarty, C. 2016. 'Reconstructing a logic from *Tractatus*: Wittgenstein's variables and formulae', in S. Costreie (ed.), *Early Analytic Philosophy—New Perspectives on the Tradition*, Cham: Springer, pp. 301–324.
Frege, G. 1884. *Die Grundlagen der Arithmetik*, Reprinted Stuttgart: Reclam [1987].
Glock, H-J. 1996. *A Wittgenstein Dictionary*, Oxford: Blackwell.
Griffin, N. 1985. 'Russell's multiple relation theory of judgement', *Philosophical Studies*, **47**, 213–247.
Griffiths, O., and Paseau, A. C. 2022. *One True Logic, A Monist Manifesto*, Oxford: Oxford University Press.
Halimi, B. 2016. 'The concept of "essential" general validity in Wittgenstein's *Tractatus*', in S. Costreie (ed.), *Early Analytic Philosophy—New Perspectives on the Tradition*, Cham: Springer, pp. 283–300.
Hanks, P. 2014. 'Bipolarity and sense in the *Tractatus*', *Journal for the History of Analytical Philosophy*, **2** (9), 1–14.
Hilbert, D., and Ackermann, W. 1928. *Grundzüge der theoretischen Logik*, Berlin: Springer.
Hintikka, J. 1996. 'Wittgenstein on being and time', In J. Hintikka (ed.), *Ludwig Wittgenstein, Half-Truths and One-and-a-Half Truths*, Dordrecht: Kluwer Academic Publishers, pp. 241–274.
Hylton, P. 1990. *Russell, Idealism, and the Emergence of Analytical Philosophy*, Oxford: Clarendon Press.
Hylton, P. 1994. 'Functions and propositional functions in *Principia Mathematica*', in A. D. Irvine and G.A. Wedeking (eds.), *Russell and Analytic Philosophy*, Toronto: University of Toronto Press, pp. 342–360.
Hylton, P. 1997. 'Functions, operations, and sense in Wittgenstein's *Tractatus*', in W. W. Tait (ed.), *Early Analytic Philosophy, Frege, Russell, Wittgenstein*, Chicago, IL: Open Court, pp. 91–106.
Ishiguro, H. 1981. 'Wittgenstein and the theory of types', in I. Block (ed.), *Perspectives on the Philosophy of Wittgenstein*, Oxford: Blackwell, pp. 43–59.
Klagge, J. C. 2022. *Tractatus in Context*, Abingdon: Routledge.
Kremer, M. 1997. 'Contextualism and holism in the early Wittgenstein: from *Prototractatus* to *Tractatus*', *Philosophical Topics*, **25** (2), 87–120.
Linsky, B. 1999. *Russell's Metaphysical Logic*, Stanford, CA: CSLI Publications.
MacBride, F. 2018. *On the Genealogy of Universals, the Metaphysical Origins of Analytical Philosophy*, Oxford: Oxford University Press.
Mancosu, P. 2016. *Abstraction and Infinity*, Oxford: Oxford University Press.
Marion, M. 1998. *Wittgenstein, Finitism, and the Foundations of Mathematics*, Oxford: Clarendon Press.
Miller, H. 1995. 'Tractarian semantics for predicate logic', *History and Philosophy of Logic*, **16** (2), 197–215.
Morris, M. 2008. *Routledge Philosophy Guidebook to Wittgenstein and the Tractatus*, Abingdon: Routledge.
Palmer, A. 1996. 'The complex problem and the theory of symbolism', in R. Monk and A. Palmer (eds.), *Bertrand Russell and the Origins of Analytical Philosophy*, Bristol: Thoemmes Press, pp. 155–182.
Potter, M. 2009. *Wittgenstein's Notes on Logic*, Oxford: Oxford University Press.
Proops, I. 2017. 'Review of José Zalabardo, *Representation and reality in Wittgenstein's Tractatus*', *Philosophical Review*, **126** (4), 532–535.
Quine, W. v. O. 1967. 'Introduction to Russell's "Mathematical logic as based on the theory of types" ', in J. van Heijenoort (ed.), *From Frege to Gödel, A Source Book in Mathematical Logic, 1879–1931*, Cambridge, MA: Harvard, pp. 150–152.
Ramsey, F. P. 1990. 'The foundations of mathematics', in D. H. Mellor (ed.), *Philosophical Papers*, Cambridge: Cambridge University Press, pp. 164–224.
Ramsey, F. P. 2006. Review of *Tractatus*. in I. M. Copi and R. W. Beard (eds.), *Essays on Wittgenstein's Tractatus*, Abingdon: Routledge, pp. 9–23.

Ricketts, T. 2013. 'Logical segmentation and generality in Wittgenstein's *Tractatus*', in P. Sullivan and M. Potter (eds.), *Wittgenstein's Tractatus, History & Interpretation*, Oxford: Oxford University Press, pp. 125–142.
Rogers, B., and Wehmeier, K. F. 2012. 'Tractarian first-order logic: identity and the N-operator', *Review of Symbolic Logic*, **5** (4), 538–573.
Russell, B. 1912. *The Problems of Philosophy*, Reprinted Oxford: Oxford University Press [1980].
Russell, B. 1919. *An Introduction to Mathematical Philosophy*, Reprinted London: Routledge [1993].
Russell, B. 1937. *The Principles of Mathematics*, 2nd ed., London: Routledge.
Russell, B. 1956a. 'Mathematical logic as based on the theory of types', in R. C. Marsh (ed.), *Logic and Knowledge*, London: Routledge, pp. 59–102.
Russell, B. 1956b. 'Logical atomism', In R. C. Marsh (ed.), *Logic and Knowledge*, London: Routledge, pp. 323–343.
Russell, B. 1961. 'Introduction', in L. Wittgenstein (ed.), *Tractatus Logico-Philosophicus*, London: Routledge, pp. ix–xxii.
Russell, B. 1973a. 'On some difficulties in the theory of transfinite numbers and order types', in D. Lackey (ed.), *Essays in Analysis*, London: George Allen & Unwin, pp. 135–164.
Russell, B. 1973b. 'On "insolubilia" and their solution by symbolic logic', in D. Lackey (ed.), *Essays in Analysis*, London: George Allen & Unwin, pp. 190–214.
Russell, B. 1973c. 'On the substitutional theory of classes and relations', in D. Lackey (ed.), *Essays in Analysis*, London: George Allen & Unwin, pp. 165–189.
Russell, B. 1984. 'Theory of knowledge, the 1913 manuscript', in E. R. Eames and K. Blackwell (eds.), *The Collected Papers of Bertrand Russell*, Volume 7, London: George Allen & Unwin, pp. 5–178.
Russell, B. 1996. 'The limits of empiricism', in J. G. Slater (ed.), *The Collected Papers of Bertrand Russell*, Volume 10, London: Routledge, pp. 314–328.
Soames, S. 1983. 'Generality, truth functions, and expressive capacity in the *Tractatus*', *Philosophical Review*, **92** (4), 573–589.
Sullivan, P. M. 2004. '"The general propositional form is a variable" (*Tractatus* 4.53)', *Mind*, **113**, 43–56.
Sundholm, G. 1992. 'The general form of the operation in Wittgenstein's *Tractatus*', *Grazer Philosophische Studien*, **42**, 57–76.
von Plato, J. 2017. *The Great Formal Machinery Works, Theories of Deduction and Computation at the Origins of the Digital Age*, Princeton, NJ: Princeton University Press.
Weiss, M. 2017. 'Logic in the *Tractatus*', *The Review of Symbolic Logic*, **10** (1), 1–50.
Whitehead, A. N., and Russell, B. 1910–13. *Principia Mathematica*, Volumes I–III, 1st ed., Cambridge: Cambridge University Press.
Wittgenstein, L. 1961. *Tractatus Logico-Philosophicus*, transl. D. Pears and B.F. McGuinness, London: Routledge.
Wittgenstein, L. 1971. *Prototractatus: An Early Version of the Tractatus Logico-Philosophicus*, London: Routledge Kegan Paul.
Wittgenstein, L. 1973. *Letters to C.K. Ogden*, Oxford and London: Blackwell and Routledge Kegan Paul.
Wittgenstein, L. 1974. *Philosophical Grammar*, Oxford: Blackwell.
Wittgenstein, L. 1981. *Tractatus Logico-Philosophicus*, transl. C. K. Ogden, London: Routledge.
Wittgenstein, L. 1984a. 'Notes on logic', in L. Wittgenstein (ed.), *Notebooks 1914–16*, 2nd ed., Chicago, IL: University of Chicago Press, pp. 93–107.
Wittgenstein, L. 1984b. 'Notes dictated to G.E. Moore in Norway', in L. Wittgenstein, *Notebooks 1914–16*, 2nd ed., Chicago, IL: University of Chicago Press, pp. 108–119.
Wittgenstein, L. 1984c. *Notebooks 1914–16*, 2nd ed., Chicago, IL: University of Chicago Press.
Wittgenstein, L. 2012. 'Correspondence', in B. F. McGuinness (ed.), *Wittgenstein in Cambridge, Letters and Documents 1911–1951*, 4th ed., Chichester: Wiley-Blackwell.
Wittgenstein, L. 2023. *Tractatus Logico-Philosophicus*, transl. M. Beaney, Oxford: Oxford University Press.
Zalabardo, J. L. 2015. *Representation and Reality in Wittgenstein's Tractatus*, Oxford: Oxford University Press.