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Panel regression models for bilateral exchange rates: The numéraire effect

Michael Kunkler

Department of Economics, University of Reading, Whiteknights, PO Box 217, Reading, RG6 6AH, Berkshire, UK

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ABSTRACT

The objective of this paper is to measure and assess the statistical significance of the numéraire effect. This effect arises from the adoption of a common single-currency numéraire for bilateral exchange rates in bilateral-panel regression models. To achieve this objective, exchange rates and explanatory variables are divided into two hierarchical levels of aggregation: a disaggregated level, which consists of multilateral exchange rates and multilateral explanatory variables, and an aggregated level, which consists of bilateral exchange rates and bilateral explanatory variables. The statistical significance of the numéraire effect is examined by testing whether the slope coefficient in a bilateral-panel regression—at the aggregated level—suffers from aggregation bias. If the slope coefficient in a bilateral-panel regression suffers from aggregation bias, it is recommended that the analysis be performed with a multilateral-panel regression—at the disaggregate level—where the slope coefficient is free from aggregation bias. The numéraire effect is measured as the difference between the slope coefficient in a bilateral-panel regression—based on the common single-currency numéraire—and the slope coefficient in a multilateral-panel regression. To illustrate two applications—the uncovered interest rate parity (UIP) condition, and the uncovered equity parity (UEP) condition—we use a monthly dataset covering 45 years (from 1980 to 2024), comprising six bilateral exchange rates expressed in US dollars. The results indicate that a substantial share of the slope coefficients estimated from bilateral panel regressions exhibit significant aggregation bias. These findings provide strong support for the use of multilateral panel regression models.

1. Introduction

Panel regression models are widely employed in the analysis of bilateral exchange rate dynamics. They allow for the simultaneous examination of cross-sectional and temporal variation, which enables them to account for heterogeneity that cannot be captured by cross-sectional or time-series analyses alone (Baltagi, 2021). However, before applying panel regression models to bilateral exchange rates, a common single-currency numéraire must be selected. The motivation of this paper is to shed light on the choice of a common single-currency numéraire in bilateral-panel regression models.

Bilateral-panel regression models refer to panel regression models that use a system of bilateral exchange rates expressed in terms of a common single-currency numéraire. The key output of a bilateral-panel regression model is the slope coefficient, which shows the expected average change in the bilateral exchange rates when their corresponding bilateral explanatory variables increase by one unit. The common single-currency numéraire must be chosen and can significantly influence the slope coefficient of bilateral-panel

E-mail address: michael.kunkler@reading.ac.uk.

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regression models.

Similarly, multilateral-panel regression models refer to panel regression models that use a system of multilateral exchange rates expressed in terms of a common multicurrency numéraire (weighted currency basket). The key output of a multilateral-panel regression model is the slope coefficient, which shows the expected average change in the multilateral exchange rates when their corresponding multilateral explanatory variables increase by one unit. Even though a common multicurrency numéraire must also be specified, this paper adopts multilateral exchange rates expressed in terms of an equally-weighted currency basket. While alternative weighted currency baskets are available, equally-weighted currency baskets are consistent and yield numéraire-invariant results—essential properties not reliably preserved under other weighting schemes (see Kunkler, 2024).

The objective of this paper is to measure and examine the statistical significance of the numéraire effect, which arises when a particular currency is designated as the common single-currency numéraire for bilateral exchange rates in bilateral-panel regressions. This objective is fulfilled by expressing each bilateral exchange rate as the difference—an aggregation—between two multilateral exchange rates (Mahieu and Schotman, 1994). Similarly, bilateral explanatory variables can be expressed as the difference—another aggregation—between two multilateral explanatory variables. As a consequence, exchange rates and their associated explanatory variables can be divided into two hierarchical levels of aggregation: a disaggregated level, which consists of multilateral exchange rates and multilateral explanatory variables, and an aggregated level, which consists of bilateral exchange rates and bilateral explanatory variables (Kunkler and MacDonald, 2015). The two hierarchical levels of aggregation can be used to examine whether the slope coefficient from a bilateral-panel regression model suffers from aggregation bias.

This paper contributes to the literature by introducing a measure of the numéraire effect and examines the statistical significance of the numéraire effect. The numéraire effect is measured by the difference between the slope coefficient from a bilateral-panel regression—based on the common single-currency numéraire—and the slope coefficient from a multilateral-panel regression. The statistical significance of the numéraire effect is examined by testing whether the slope coefficient from a bilateral-panel regression suffers from aggregation bias.

To illustrate this paper's contributions to two applications—uncovered interest rate parity (UIP), and uncovered equity parity (UEP)—a monthly data sample spanning 45 years (from 1980 to 2024) is used, comprising a system of six bilateral exchange rates expressed in terms of the US dollar. As a result, the system consists of seven unique currencies. For each application, seven separate bilateral-panel regression models are estimated, each using a different common single-currency numéraire. The analysis reveals that a substantial number of the observed bilateral-panel slope coefficients exhibit significant aggregation bias. The results show that when aggregation bias is not corrected, the bilateral-panel slope coefficients can either overstate or understate the extent to which the underlying economic conditions are accepted or rejected.

The organisation of this paper is as follows: Section 2 provides a selective literature review; Section 3 presents the methodology; Section 4 reports some empirical results of two applications: UIP, and UEP; and Section 5 concludes.

2. Selective literature review

Panel regression models provide methodological advantages over conventional single cross-sectional and single time-series regression approaches. For example, panel regression models account for unobserved heterogeneity that cannot be captured by either single cross-sectional regression models or single time-series analyses regression models (Baltagi, 2021). They account for unobserved heterogeneity with two types of fixed effects: individual and time (Baltagi, 2021). In this paper, the individual fixed effects are associated with exchange rates (bilateral or multilateral) and the time fixed effects are associated with the cross-section of the system of exchange rates (bilateral or multilateral) at each point in time.

Floating bilateral exchange rates arose in the 1970s after the collapse of the Bretton Woods system, which had previously maintained fixed but adjustable bilateral exchange rates. Frankel (1986) raised the issue of the low statistical power of unit root tests for bilateral real exchange rates in the post Bretton Woods era. A natural approach to addressing this issue involved extending the time series (see Froot and Rogoff, 1995, and the references within). However, Frankel and Rose (1996) argued that extending the time-series to include the data from the pre Bretton Woods era could be inappropriate due to regime changes between fixed and floating bilateral exchange rates. In contrast, panel regression models address the issue of low statistical power by providing a larger number of observations than single time-series models (Manzur, 2018). Consequently, panel regression models have been widely adopted across various areas of research on exchange rate dynamics, such as: unit root tests (Levin et al., 2002; Im et al., 1995); uncovered interest rate parity (UIP) (Chinn, 2006; Herger, 2016); the monetary model (Mark, 2001; Groen, 2005; Cerra and Saxena, 2010); purchasing power parity (PPP) (Frankel and Rose, 1996; MacDonald, 1996; Taylor and Sarno, 1998; Papell and Theodoridis, 2001; Pedroni, 2004; Banerjee et al., 2005); uncovered equity parity (UEP) (Jung and Jung, 2021); and predictive models (Engel et al., 2007; Rogoff and Stavrakeva, 2008; Mark and Sul, 2012).

Before using bilateral exchange rates in a bilateral-panel regression model, a common single-currency numéraire must be chosen. The US dollar is typically adopted as the common single-currency numéraire. However, choosing a common single-currency numéraire raises several questions: What criteria should guide its selection? Is there an optimal choice? And should all possible options be considered? Similar common numéraire issues occur when modelling currencies in the Frankel-Wei regression framework, where all currencies are expressed in terms of a common numéraire: either a common single-currency numéraire or a common multicurrency numéraire (Frankel and Wei, 1994, 2007; Frankel, 2009).

Selecting different common single-currency numéraires can lead to varying and often inconsistent outcomes. Papell and Theodoridis (2001) showed that the choice of common single-currency numéraire matters in panel tests of purchasing power parity (PPP). O'Connell (1998) mentions that empirical studies were more likely to reject the random walk hypothesis for real bilateral exchange rates when the German mark was used as the common single-currency numéraire, compared to when the US dollar was used. For example, Jorion and Sweeney (1996) reported that European currencies exhibited faster mean reversion when the German mark served as the common single-currency numéraire, compared to when the US dollar served that role. Similarly, using a system of 19 currencies within a bilateral-panel regression, Coakley and Fuertes (2000) demonstrated the presence of a numéraire effect: significant mean reversion was observed when the German mark served as the common single-currency numéraire, whereas no significant mean reversion occurred when the US dollar served that role. Significant fluctuations from trend were identified by Mark and Sul (2001) as unit root processes when the German mark served as the common single-currency numéraire, whereas no such significance was reported when the US dollar was served that role.

Multilateral-panel regression models use a system of multilateral exchange rates expressed in terms of a weighted currency basket (a multicurrency numéraire). Compared to bilateral-panel regression models, their application has been relatively limited within the exchange rate literature. A weighted currency basket must be chosen when modelling a system of multilateral exchange rates, where the weights assigned to each currency in the basket must be positive and sum to one. Alternative specifications of weighted currency baskets are available for modelling multilateral exchange rates, such as: trade-weighted baskets, risk-weighted baskets, and GDP-weighted baskets. Kunkler (2024) showed that multicurrency numéraires can be categorized using two key variables: currency selection (idiosyncratic vs. common) and currency weights (unequal vs. equal). These dimensions create four separate types of multicurrency numéraires: idiosyncratic-unequal, idiosyncratic-equal, common-unequal, and common-equal. Kunkler (2024) showed that idiosyncratic multicurrency numéraires are inconsistent and recommended using common equally-weighted multicurrency numéraires. An example of an idiosyncratic equally-weighted multicurrency numéraire would be one that does not encompass all relevant currencies (see Lustig et al., 2011; Verdelhan, 2018; Aloosh and Bekaert, 2022). Trade-weighted baskets are examples of idiosyncratic unequally-weighted multicurrency numéraires, where each currency is expressed in terms of its own weighted currency basket, weighted according to the country's major trading partners. An example of a common unequally-weighted multicurrency numéraire is the International Monetary Fund's Special Drawing Right (SDR), where each currency is expressed in terms of the same SDR currency basket. Examples of common equally-weighted baskets include Hovanov et al. (2004) and Kunkler and MacDonald (2015). Equally-weighted currency indices have been shown to remain invariant to the choice of common single-currency numéraire for a system of bilateral exchange rates (Hovanov et al., 2004, 2007). In addition, Kunkler (2023) presents a multivariate regression framework, which models a system of multilateral exchange rates expressed in terms of an equally-weighted currency basket.

Mahieu and Schotman (1994) showed that a system of bilateral exchange rates can be decomposed into two multilateral exchange rates, where each bilateral exchange rate can be expressed as the difference—an aggregation—between two multilateral exchange rates. Kunkler and MacDonald (2015) imposed an arbitrage condition to create a unique system of multilateral exchange rates and classified exchange rates into two hierarchical levels of aggregation: a disaggregated level, which consisted of multilateral exchange rates and multilateral explanatory variables, and an aggregated level, which consisted of bilateral exchange rates and bilateral explanatory variables. At the aggregated level, the slope coefficients from bilateral regression models can be tested to make sure that they do not suffer from aggregation bias (Lee et al., 1990; Kunkler and MacDonald, 2015; Zellner, 1962). Finally, this paper has similarities with the multivariate regression framework presented in Kunkler (2023), which also modelled a system of multilateral exchange rates expressed in terms of an equally-weighted currency basket.

3. Methods

The regression models in this section are contemporaneous, but are adaptable to predictive regression models by using lagged explanatory variables. For example, the application to uncovered interest rate parity (UIP) condition later in the paper uses lagged interest rate levels as explanatory variables.

3.1. Bilateral exchange rate returns

A bilateral exchange rate is the price of a currency expressed in terms of a numéraire currency (a single-currency numéraire), represented by η. In log terms, for a system of N currencies, let $s_{i/\eta}(t)$ represent the ith/ηth bilateral exchange rate at time t, which is the price of the ith currency expressed in terms of a common single-currency (ηth) numéraire, where $t = 0, \ldots, T$. Any currency from a system of N currencies can be chosen as the common single-currency numéraire. The bilateral exchange rate is zero when the common single-currency numéraire is expressed in terms of itself ($i = \eta$), with $s_{\eta/\eta}(t) = 0$.

In log terms, the return of a bilateral exchange rate can be written as:

$$\Delta s_{i/\eta}(t) = s_{i/\eta}(t) - s_{i/\eta}(t-1) \quad (1)$$

where $i = 1, \ldots, N$; $t = 1, \ldots, T$; $\Delta s_{i/\eta}(t)$ is the ith/ηth bilateral exchange rate return at time t; $s_{i/\eta}(t)$ is the ith/ηth bilateral exchange rate at time t; and $s_{i/\eta}(t-1)$ is the ith/ηth bilateral exchange rate at time $t-1$ (see Tsay, 2005). The bilateral exchange rate return is zero when the common single-currency numéraire is expressed in terms of itself ($i = \eta$), with $\Delta s_{\eta/\eta}(t) = 0$.

3.2. Bilateral regression models

Bilateral regression models refer to regression models that use the returns of a bilateral exchange rate expressed in terms of a common single-currency numéraire against a corresponding bilateral explanatory variable. The form of a bilateral regression model can be written as:

$$\Delta \boldsymbol{s}_{i/\eta} = \alpha_{i/\eta} + \boldsymbol{x}_{i/\eta}\beta_{i/\eta} + \boldsymbol{u}_{i/\eta} \tag{2}$$

where $i = 1,\ldots,N$; $i \neq \eta$; $\Delta \boldsymbol{s}_{i/\eta}$ is a $T \times 1$ vector of the ith/ηth bilateral exchange rate returns; $\boldsymbol{x}_{i/\eta}$ is a $T \times 1$ vector of the ith/ ηth bilateral explanatory variable; $\alpha_{i/\eta}$ is a bilateral intercept term; $\beta_{i/\eta}$ is a bilateral slope coefficient; and $\boldsymbol{u}_{i/\eta}$ is a $T \times 1$ vector of the ith/ ηth bilateral disturbances (see Sarno and Taylor, 2002).

The slope coefficient from a bilateral regression model quantifies both the strength and direction of the relationship between the log returns of a bilateral exchange rate (the dependent variable) and a bilateral explanatory variable. Economically, it indicates how much an individual bilateral exchange rate is expected to move when the bilateral explanatory variable increases by one unit. A positive slope coefficient suggests that, on average, an increase (or decrease) in the bilateral explanatory variable leads to a corresponding increase (or decrease) in the bilateral exchange rate. Conversely, a negative slope coefficient implies that an increase (or decrease) in the bilateral explanatory variable is associated with a decrease (or increase) in the corresponding bilateral exchange rate.

The ordinary least squares (OLS) estimator of the bilateral slope coefficient is:

$$\widehat{\beta}_{i/\eta} = \frac{cov\left(\Delta \boldsymbol{s}_{i/\eta}, \boldsymbol{x}_{i/\eta}\right)}{var\left(\boldsymbol{x}_{i/\eta}\right)} \tag{3}$$

where $i = 1,\ldots,N$; $i \neq \eta$; $\widehat{\beta}_{i/\eta}$ is the OLS estimator of the bilateral slope coefficient; $cov\left(\Delta \boldsymbol{s}_{i/\eta}, \boldsymbol{x}_{i/\eta}\right)$ is the covariance between the ith/ ηth bilateral exchange rate returns and the ith/ηth bilateral explanatory variable; and $var\left(\boldsymbol{x}_{i/\eta}\right)$ is the variance of the ith/ ηth bilateral explanatory variable.

3.3. Bilateral-panel regression models

Bilateral-panel regression models refer to panel regression models that use a system of bilateral exchange rates expressed in terms of a common single-currency numéraire. A balanced structure is assumed so that all regression variables have the same number of observations: with no missing data. In this situation, the ordinary least squares (OLS) estimator of the bilateral-slope coefficient using the demeaned regression variables is the same as the OLS estimator of the bilateral-slope coefficient using the raw regression variables where the individual fixed-effect coefficients are included (Stock and Watson, 2020). Although the raw regression variables could be demeaned for both individual and time fixed effects, they are only adjusted for individual fixed effects. Adjusting for time fixed effects is beyond the scope of this paper and left for future research.

Both the bilateral exchange rate returns and the bilateral explanatory variables are demeaned for the individual fixed effects. The demeaned bilateral exchange rate return is:

$$\Delta \widetilde{s}_{i/\eta}(t) = \Delta s_{i/\eta}(t) - \Delta \overline{s}_{i/\eta} \tag{4}$$

where $\Delta \widetilde{s}_{i/\eta}(t)$ is the demeaned ith/ηth bilateral exchange rate return; and $\Delta \overline{s}_{i/\eta} = \frac{1}{T}\sum_{t=1}^{T} \Delta s_{i/\eta}(t)$ is the average of the ith/ ηth bilateral exchange rate returns. Similarly, the demeaned bilateral explanatory variable is:

$$\widetilde{x}_{i/\eta}(t) = x_{i/\eta}(t) - \overline{x}_{i/\eta} \tag{5}$$

where $\widetilde{x}_{i/\eta}(t)$ is the demeaned ith/ηth bilateral explanatory variable; and $\overline{x}_{i/\eta} = \frac{1}{T}\sum_{t=1}^{T} x_{i/\eta}(t)$ is the average of the ith/ ηth bilateral explanatory variable.

The form of a bilateral-panel regression model for the $N-1$ panel of demeaned bilateral exchange rate returns against the corresponding $N-1$ panel of demeaned bilateral explanatory variables can be written as:

$$\Delta \widetilde{\boldsymbol{s}}_{:/\eta} = \widetilde{\boldsymbol{x}}_{:/\eta}\beta_{:/\eta} + \widetilde{\boldsymbol{u}}_{:/\eta} \tag{6}$$

where $\Delta \widetilde{\boldsymbol{s}}_{:/\eta} = \left[\Delta \widetilde{\boldsymbol{s}}_{1/\eta}; \cdots; \Delta \widetilde{\boldsymbol{s}}_{N/\eta}\right]$ is a $(N-1)T \times 1$ stacked vector of demeaned bilateral exchange rate returns; $\widetilde{\boldsymbol{x}}_{:/\eta} = \left[\widetilde{\boldsymbol{x}}_{1/\eta}; \cdots; \widetilde{\boldsymbol{x}}_{N/\eta}\right]$ is a $(N-1)T \times 1$ stacked vector of demeaned bilateral explanatory variables; $\beta_{:/\eta}$ is a bilateral-panel slope coefficient; and $\widetilde{\boldsymbol{u}}_{:/\eta} = \left[\widetilde{\boldsymbol{u}}_{1/\eta}; \cdots; \widetilde{\boldsymbol{u}}_{N/\eta}\right]$ is a $(N-1)T \times 1$ stacked vector of disturbances. Note that there is a system of N currencies; however, the bilateral-panel regression model does not incorporate the regression variables linked to the common single-currency numéraire. For example, the stacked vector of demeaned bilateral exchange rate returns $\Delta \widetilde{\boldsymbol{s}}_{:/\eta}$ excludes the demeaned bilateral exchange rate returns of the common single-currency numéraire $\Delta \widetilde{s}_{\eta/\eta}$. Similarly, the stacked vector of demeaned bilateral explanatory variables $\widetilde{\boldsymbol{x}}_{:/\eta}$ excludes the demeaned bilateral explanatory variable of the common single-currency numéraire $\widetilde{\boldsymbol{x}}_{\eta/\eta}$.

The slope coefficient from the bilateral-panel regression model quantifies both the strength and direction of the relationship between the panel of bilateral exchange rate returns (the dependent variable) and the panel of explanatory variables. Economically, it indicates how much a bilateral exchange rate is expected to move when its corresponding bilateral explanatory variable increases by one unit. A positive slope coefficient suggests that, on average, an increase (or decrease) in a bilateral explanatory variable is associated with an increase (or decrease) in the corresponding bilateral exchange rate. Conversely, a negative slope coefficient implies that an increase (or decrease) in a bilateral explanatory variable is associated with a decrease (or increase) in the corresponding bilateral exchange rates.

The OLS estimator of the bilateral-panel slope coefficient is:

$$\widehat{\beta}_{:/\eta} = \frac{cov\left(\Delta\widetilde{\boldsymbol{s}}_{:/\eta}, \widetilde{\boldsymbol{x}}_{:/\eta}\right)}{var\left(\widetilde{\boldsymbol{x}}_{:/\eta}\right)} \tag{7}$$

where $cov\left(\Delta\widetilde{\boldsymbol{s}}_{:/\eta}, \widetilde{\boldsymbol{x}}_{:/\eta}\right)$ is the covariance between the panel of demeaned bilateral exchange rate returns and the panel of demeaned bilateral explanatory variables; and $var\left(\widetilde{\boldsymbol{x}}_{:/\eta}\right)$ is the variance of the panel of demeaned bilateral explanatory variables. The covariance term in Eq. (7) can be expanded to give:

$$\begin{aligned} cov\left(\Delta\widetilde{\boldsymbol{s}}_{:/\eta}, \widetilde{\boldsymbol{x}}_{:/\eta}\right) &= \frac{1}{N-1}\sum_{i=1}^{N} I_{i\neq\eta}\frac{1}{T}\sum_{t=1}^{T} cov\left(\Delta\widetilde{s}_{i/\eta}(t)\widetilde{x}_{i/\eta}(t)\right) \\ &= \frac{1}{N-1}\sum_{i=1}^{N} I_{i\neq\eta} cov\left(\Delta\widetilde{\boldsymbol{s}}_{i/\eta}, \widetilde{\boldsymbol{x}}_{i/\eta}\right) \\ &= \frac{1}{N-1}\sum_{i=1}^{N} I_{i\neq\eta} cov\left(\Delta\boldsymbol{s}_{i/\eta}, \boldsymbol{x}_{i/\eta}\right) \end{aligned} \tag{8}$$

where $I_{i\neq\eta}$ is an indicator function which is one when $i \neq \eta$ and zero when $i = \eta$; and $cov\left(\Delta\widetilde{\boldsymbol{s}}_{i/\eta}, \widetilde{\boldsymbol{x}}_{i/\eta}\right) = cov\left(\Delta\boldsymbol{s}_{i/\eta}, \boldsymbol{x}_{i/\eta}\right)$ is the covariance between the demeaned ith/ηth bilateral exchange rate returns and the demeaned ith/ηth bilateral explanatory variable. The variance term in Eq. (7) can also be expanded to give:

$$\begin{aligned} var\left(\widetilde{\boldsymbol{x}}_{:/\eta}\right) &= \frac{1}{N-1}\sum_{i=1}^{N} I_{i\neq\eta}\frac{1}{T}\sum_{t=1}^{T} var\left(\widetilde{x}_{i/\eta}(t)\right)^2 \\ &= \frac{1}{N-1}\sum_{i=1}^{N} I_{i\neq\eta} var\left(\widetilde{\boldsymbol{x}}_{i/\eta}\right) \\ &= \frac{1}{N-1}\sum_{i=1}^{N} I_{i\neq\eta} var\left(\boldsymbol{x}_{i/\eta}\right) \end{aligned} \tag{9}$$

where $var\left(\widetilde{\boldsymbol{x}}_{i/\eta}\right) = var\left(\boldsymbol{x}_{i/\eta}\right)$ is the variance of the demeaned ith/ηth bilateral explanatory variable.

Both Eq. (8) and Eq. (9) can be substituted into Eq. (7) to write the OLS estimator of the bilateral-panel slope coefficient as:

$$\begin{aligned} \widehat{\beta}_{:/\eta} &= \frac{\sum_{i=1}^{N} I_{i\neq\eta} cov\left(\Delta\widetilde{\boldsymbol{s}}_{i/\eta}, \widetilde{\boldsymbol{x}}_{i/\eta}\right)}{\sum_{i=1}^{N} I_{i\neq\eta} var\left(\widetilde{\boldsymbol{x}}_{i/\eta}\right)} \\ &= \sum_{i=1}^{N} I_{i\neq\eta}\left(\frac{cov\left(\Delta\boldsymbol{s}_{i/\eta}, \boldsymbol{x}_{i/\eta}\right)}{\sum_{j=1}^{N} I_{j\neq\eta} var\left(\boldsymbol{x}_{j/\eta}\right)}\right) \\ &= \sum_{i=1}^{N} I_{i\neq\eta}\left(\frac{var\left(\boldsymbol{x}_{i/\eta}\right)}{\sum_{j=1}^{N} I_{j\neq\eta} var\left(\boldsymbol{x}_{j/\eta}\right)}\right)\left(\frac{cov\left(\Delta\boldsymbol{s}_{i/\eta}, \boldsymbol{x}_{i/\eta}\right)}{var\left(\boldsymbol{x}_{i/\eta}\right)}\right) \\ &= \sum_{i=1}^{N} I_{i\neq\eta} w_{i/\eta}\widehat{\beta}_{i/\eta} \end{aligned} \tag{10}$$

where $\widehat{\beta}_{i/\eta} = cov\left(\Delta\boldsymbol{s}_{i/\eta}, \boldsymbol{x}_{i/\eta}\right) / var\left(\boldsymbol{x}_{i/\eta}\right)$ from Eq. (3); and:

$$w_{i/\eta} = \frac{var\left(\boldsymbol{x}_{i/\eta}\right)}{\sum_{j=1}^{N} I_{j\neq\eta} var\left(\boldsymbol{x}_{j/\eta}\right)} \tag{11}$$

is the ith/ηth bilateral-panel weight. All of the bilateral-panel weights in Eq. (11) are positive ($w_{i/\eta} > 0$) and sum to one:

$$\sum_{i=1}^{N} I_{i\neq\eta} w_{i/\eta} = 1 \tag{12}$$

In summary, the slope coefficient from a bilateral-panel regression is a weighted average of the $N-1$ slope coefficients from separate bilateral (non-panel) regressions, where the weights are determined by the variance of the corresponding bilateral explanatory variable relative to the sum of all the variances of the bilateral explanatory variables.

3.4. Multilateral exchange rate returns

A multilateral exchange rate represents the price of a currency expressed in terms of a weighted currency basket (a multicurrency numéraire), represented by $\mathscr{M}$. It is assumed that all of the basket weights are positive and sum to one. For a system of N currencies at time t, Mahieu and Schotman (1994) showed that a system of bilateral exchange rates can be decomposed into two multilateral exchange rates:

$$s_{i/\eta}(t) = s_{i/\mathscr{M}}(t) - s_{\eta/\mathscr{M}}(t) \tag{13}$$

where $i = 1, \ldots, N$; $i \neq \eta$; $t = 0, \ldots, T$; $s_{i/\eta}(t)$ is the ith/ηth bilateral exchange rate; $s_{i/\mathscr{M}}(t)$ is the multilateral exchange rate for the ith currency; and $s_{\eta/\mathscr{M}}(t)$ is the multilateral exchange rate for the common single-currency (ηth) numéraire. Note that when $i = \eta$, the ηth/ηth bilateral exchange rate is not included is the system of $N - 1$ bilateral exchange rates. Thus, bilateral exchange rates are the difference—an aggregation—between two multilateral exchange rates.

Kunkler and MacDonald (2015) introduced a no-arbitrage (equilibrium) condition on the system of N multilateral exchange rates:

$$\sum_{i=1}^{N} s_{i/\mathscr{M}}(t) = 0 \tag{14}$$

where $t = 0, \ldots, T$. Subtracting the multilateral exchange rate for the common single-currency (ηth) numéraire $s_{\eta/\mathscr{M}}(t)$ from both sides of Eq. (14) gives:

$$s_{\eta/\mathscr{M}}(t) = -\sum_{i=1}^{N} I_{i \neq \eta} s_{i/\mathscr{M}}(t) \tag{15}$$

where $t = 0, \ldots, T$.

The no-arbitrage condition in Eq. (14) can be included into the decomposition of the system of $N - 1$ bilateral exchange rates in Eq. (13), which creates a combined system of N equations ($N - 1$ bilateral exchange rates and one no-arbitrage condition) that is the same number as the number of N unknowns (multilateral exchange rates). In this context, Kunkler and MacDonald (2015) showed that the unique solution is a system of N multilateral exchange rates expressed in terms of an equally-weighted multicurrency numéraire:

$$s_{i/\mathscr{M}}(t) = \frac{1}{N}\sum_{j=1}^{N} s_{i/j}(t) \tag{16}$$

where $t = 0, \ldots, T$; $s_{i/\mathscr{M}}(t)$ is the multilateral exchange rate for the ith currency; and $s_{i/j}(t)$ is the ith/jth bilateral exchange rate.

In log terms, by using Eq. (16), the return of a multilateral exchange rate for the ith currency at time t can be written as:

$$\Delta s_{i/\mathscr{M}}(t) = s_{i/\mathscr{M}}(t) - s_{i/\mathscr{M}}(t-1)$$

$$= \frac{1}{N}\sum_{j=1}^{N} \Delta s_{i/j}(t) \tag{17}$$

where $i = 1, \ldots, N$; $t = 1, \ldots, T$; $\Delta s_{i/j}(t) = s_{i/j}(t) - s_{i/j}(t-1)$ is the ith/jth bilateral exchange rate return at time t. In addition, the returns of the system of N multilateral exchange rates in Eq. (17) are related by:

$$\sum_{i=1}^{N} \Delta s_{i/\mathscr{M}}(t) = \sum_{i=1}^{N} s_{i/\mathscr{M}}(t) - \sum_{i=1}^{N} s_{i/\mathscr{M}}(t-1) = 0 \tag{18}$$

where $t = 1, \ldots, T$; $\sum_{i=1}^{N} s_{i/\mathscr{M}}(t) = 0$ from Eq. (14); and $\sum_{i=1}^{N} s_{i/\mathscr{M}}(t-1) = 0$ from Eq. (14). Thus, the no-arbitrage (equilibrium) condition in Eq. (14) also applies to the system of N multilateral exchange rate returns. Subtracting the multilateral exchange rate return for the ηth currency from both sides of Eq. (18) gives:

$$\Delta s_{\eta/\mathscr{M}}(t) = -\sum_{i=1}^{N} I_{i \neq \eta} \Delta s_{i/\mathscr{M}}(t) \tag{19}$$

where $t = 0, \ldots, T$; $\Delta s_{\eta/\mathscr{M}}(t)$ is the multilateral exchange rate return for the common single-currency (ηth) numéraire.

3.5. *Multilateral explanatory variables*

Fundamental explanatory variables such as growth rates, inflation rates, interest rates, monetary variables, and industrial production indexes, are usually associated with countries, or regions. However, once fundamental explanatory variables enter bilateral regression models, they are typically modelled in bilateral (differential) form, such as bilateral growth rates, bilateral inflation rates, bilateral interest rates, bilateral monetary variables, and bilateral industrial production indexes. The methodology used for calculating multilateral exchange rates can be applied to transform bilateral explanatory variables into multilateral ones.

For a system of N currencies at time t, a bilateral explanatory variable can be decomposed into two multilateral explanatory variables by:

$$x_{i/\eta}(t) = x_{i/\mathscr{M}}(t) - x_{\eta/\mathscr{M}}(t) \tag{20}$$

where $i = 1, \ldots, N$; $i \neq \eta$; $t = 1, \ldots, T$; $x_{i/\eta}(t)$ is the ith/ηth bilateral explanatory variable; $x_{i/\mathscr{M}}(t)$ is the multilateral explanatory variable for the ith currency; and $x_{\eta/\mathscr{M}}(t)$ is the multilateral explanatory variable for the numéraire (ηth) currency. Note that when $i = \eta$, the ηth/ηth bilateral explanatory variable is not included is the system of $N - 1$ bilateral explanatory variable. Thus, bilateral explanatory variables are the difference—an aggregation—between two multilateral explanatory variables.

Similar to the no-arbitrage condition for the system of N multilateral exchange rates in Eq. (14), there is a no-arbitrage condition for the system of N multilateral explanatory variables:

$$\sum_{i=1}^{N} x_{i/\mathscr{M}}(t) = 0 \tag{21}$$

where $t = 1, \ldots, T$. Subtracting the multilateral explanatory variable for the ηth currency $x_{\eta/\mathscr{M}}(t)$ from both side of Eq. (21) gives:

$$x_{\eta/\mathscr{M}}(t) = -\sum_{i=1}^{N} I_{i\neq\eta} x_{i/\mathscr{M}}(t) \tag{22}$$

where $t = 1, \ldots, T$.

The no-arbitrage condition in Eq. (21) can be included into the decomposition of the system of $N-1$ bilateral explanatory variables in Eq. (20), which creates a combined system of N equations ($N-1$ bilateral explanatory variables and one no-arbitrage condition) that is the same number as the number of N unknowns (multilateral explanatory variables). In this situation, there is a unique solution to the system of N equations given by:

$$x_{i/\mathscr{M}}(t) = \frac{1}{N}\sum_{j=1}^{N} x_{i/j}(t) \tag{23}$$

where $t = 1, \ldots, T$; and $x_{i/j}(t)$ is the ith/jth bilateral explanatory variable. Note that $\mathscr{M}$ is associated with an equally-weighted basket of N bilateral explanatory variables.

3.6. Multilateral regressions

Multilateral regression models refer to regression models that use the log returns of a multilateral exchange rate expressed in terms of a common equally-weighted multicurrency numéraire against a corresponding multilateral explanatory variable. The form of a multilateral regression model can be written as:

$$\Delta \boldsymbol{s}_{i/\mathscr{M}} = \alpha_{i/\mathscr{M}} + \boldsymbol{x}_{i/\mathscr{M}}\beta_{i/\mathscr{M}} + \boldsymbol{u}_{i/\mathscr{M}} \tag{24}$$

where $i = 1, \ldots, N$; $\Delta \boldsymbol{s}_{i/\mathscr{M}}$ is a $T \times 1$ vector of the multilateral exchange rate returns of the ith currency; $\boldsymbol{x}_{i/\mathscr{M}}$ is a $T\times 1$ vector of the multilateral explanatory variable of the ith currency; $\alpha_{i/\mathscr{M}}$ is an multilateral intercept term; $\beta_{i/\mathscr{M}}$ is a multilateral slope coefficient; and $\boldsymbol{u}_{i/\mathscr{M}}$ is a $T \times 1$ vector of multilateral disturbances.

The slope coefficient from the multilateral regression model quantifies both the strength and direction of the relationship between an individual multilateral exchange rate returns (the dependent variable) and the multilateral explanatory variable. Economically, it indicates how much an individual multilateral exchange rate is expected to move when the multilateral explanatory variable increases by one unit. A positive slope coefficient suggests that, on average, an increase (or decrease) in the multilateral explanatory variable leads to a corresponding increase (or decrease) in the multilateral exchange rate. Conversely, a negative slope coefficient implies that an increase (or decrease) in the multilateral explanatory variable is associated with a decrease (or increase) in the multilateral exchange rate.

The OLS estimator of the multilateral slope coefficient is:

$$\widehat{\beta}_{i/\mathscr{M}} = \frac{cov\left(\Delta \boldsymbol{s}_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}}\right)}{var\left(\boldsymbol{x}_{i/\mathscr{M}}\right)} \tag{25}$$

where $i = 1, \ldots, N$; $\widehat{\beta}_{i/\mathscr{M}}$ is the OLS estimator of the multilateral slope coefficient; $cov\left(\Delta \boldsymbol{s}_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}}\right)$ is the covariance between the multilateral exchange rate returns of the ith currency and the multilateral explanatory variable of the ith currency; and $var\left(\boldsymbol{x}_{i/\mathscr{M}}\right)$ is the variance of the multilateral explanatory variable of the ith currency.

3.7. Cross-currency multilateral regressions

Cross-currency multilateral regression models refer to regression models that use the log returns of a multilateral exchange rate expressed in terms of a common equally-weighted multicurrency numéraire against any multilateral explanatory variable. The form of a cross-currency regression model can be written as:

$$\Delta \boldsymbol{s}_{i/\mathscr{M}} = \alpha_{i,j/\mathscr{M}} + \boldsymbol{x}_{j/\mathscr{M}}\beta_{i,j/\mathscr{M}} + \boldsymbol{u}_{i,j/\mathscr{M}} \tag{26}$$

where $i, j = 1, \ldots, N$; $\Delta \boldsymbol{s}_{i/\mathscr{M}}$ is a $T \times 1$ vector of multilateral exchange rate returns of the ith currency; $\boldsymbol{x}_{j/\mathscr{M}}$ is a $T \times 1$ vector of the multilateral explanatory variable of the jth currency; $\alpha_{i,j/\mathscr{M}}$ is a cross-currency multilateral intercept term; $\beta_{i,j/\mathscr{M}}$ is a cross-currency multilateral slope coefficient; and $\boldsymbol{u}_{i,j/\mathscr{M}}$ is a $T \times 1$ vector of multilateral disturbances.

The OLS estimator of the cross-currency multilateral slope coefficient is:

$$\widehat{\beta}_{i,j/\mathscr{M}} = \frac{cov(\Delta s_{i/\mathscr{M}}, \boldsymbol{x}_{j/\mathscr{M}})}{var(\boldsymbol{x}_{j/\mathscr{M}})} \tag{27}$$

where $i,j = 1,\ldots,N$; $\widehat{\beta}_{i,j/\mathscr{M}}$ is the OLS estimator of the cross-currency multilateral slope coefficient; $cov(\Delta s_{i/\mathscr{M}}, \boldsymbol{x}_{j/\mathscr{M}})$ is the covariance between the multilateral exchange rate returns of the ith currency and the multilateral explanatory variable of the jth currency; and $var(\boldsymbol{x}_{j/\mathscr{M}})$ is the variance of the multilateral explanatory variable of the jth currency.

Note that when $i = j$, the OLS estimator of the cross-currency multilateral slope coefficient $\widehat{\beta}_{i,i/\mathscr{M}}$ in Eq. (27) is equivalent to the OLS estimator of the multilateral slope coefficient $\widehat{\beta}_{i/\mathscr{M}}$ in Eq. (25):

$$\widehat{\beta}_{i,i/\mathscr{M}} = \frac{cov(\Delta s_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}})}{var(\boldsymbol{x}_{i/\mathscr{M}})} = \widehat{\beta}_{i/\mathscr{M}} \tag{28}$$

where $i = 1,\ldots,N$.

3.8. Revisiting bilateral regression models

The OLS estimator of the slope coefficient $\widehat{\beta}_{i/\eta}$ in a bilateral regression in Eq. (3) can be expressed as a combination of four slope coefficients from four separate multilateral regressions:

$$\begin{aligned}
\widehat{\beta}_{i/\eta} &= \frac{cov(\Delta s_{i/\eta}, \boldsymbol{x}_{i/\eta})}{var(\boldsymbol{x}_{i/\eta})} \\
&= \frac{cov(s_{i/\mathscr{M}} - s_{\eta/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}} - \boldsymbol{x}_{\eta/\mathscr{M}})}{var(\boldsymbol{x}_{i/\mathscr{M}} - \boldsymbol{x}_{\eta/\mathscr{M}})} \\
&= \frac{cov(s_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}})}{var(\boldsymbol{x}_{i/\mathscr{M}} - \boldsymbol{x}_{\eta/\mathscr{M}})} - \frac{cov(s_{\eta/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}})}{var(\boldsymbol{x}_{i/\mathscr{M}} - \boldsymbol{x}_{\eta/\mathscr{M}})} - \frac{cov(s_{i/\mathscr{M}}, \boldsymbol{x}_{\eta/\mathscr{M}})}{var(\boldsymbol{x}_{i/\mathscr{M}} - \boldsymbol{x}_{\eta/\mathscr{M}})} + \frac{cov(s_{\eta/\mathscr{M}}, \boldsymbol{x}_{\eta/\mathscr{M}})}{var(\boldsymbol{x}_{i/\mathscr{M}} - \boldsymbol{x}_{\eta/\mathscr{M}})} \\
&= \delta_i \left(\frac{cov(s_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}})}{var(\boldsymbol{x}_{i/\mathscr{M}})} - \frac{cov(s_{\eta/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}})}{var(\boldsymbol{x}_{i/\mathscr{M}})} \right) - \delta_\eta \left(\frac{cov(s_{i/\mathscr{M}}, \boldsymbol{x}_{\eta/\mathscr{M}})}{var(\boldsymbol{x}_{\eta/\mathscr{M}})} - \frac{cov(s_{\eta/\mathscr{M}}, \boldsymbol{x}_{\eta/\mathscr{M}})}{var(\boldsymbol{x}_{\eta/\mathscr{M}})} \right) \\
&= \delta_i \left(\widehat{\beta}_{i/\mathscr{M}} - \widehat{\beta}_{\eta,i/\mathscr{M}} \right) - \delta_\eta \left(\widehat{\beta}_{i,\eta/\mathscr{M}} - \widehat{\beta}_{\eta/\mathscr{M}} \right)
\end{aligned} \tag{29}$$

where $i = 1,\ldots,N$; $i \neq \eta$;

$$\delta_i = \frac{var(\boldsymbol{x}_{i/\mathscr{M}})}{var(\boldsymbol{x}_{i/\mathscr{M}} - \boldsymbol{x}_{\eta/\mathscr{M}})} \tag{30}$$

is the delta term for the ith currency;

$$\delta_\eta = \frac{var(\boldsymbol{x}_{\eta/\mathscr{M}})}{var(\boldsymbol{x}_{i/\mathscr{M}} - \boldsymbol{x}_{\eta/\mathscr{M}})} \tag{31}$$

is the delta term for the ηth currency; $\widehat{\beta}_{i/\mathscr{M}} = cov(s_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}})/var(\boldsymbol{x}_{i/\mathscr{M}})$ from Eq. (25); $\widehat{\beta}_{\eta,i/\mathscr{M}} = cov(s_{\eta/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}})/var(\boldsymbol{x}_{i/\mathscr{M}})$ from Eq. (27); $\widehat{\beta}_{i,\eta/\mathscr{M}} = cov(s_{i/\mathscr{M}}, \boldsymbol{x}_{\eta/\mathscr{M}})/var(\boldsymbol{x}_{\eta/\mathscr{M}})$ from Eq. (27); $\widehat{\beta}_{\eta/\mathscr{M}} = cov(s_{\eta/\mathscr{M}}, \boldsymbol{x}_{\eta/\mathscr{M}})/var(\boldsymbol{x}_{\eta/\mathscr{M}})$ from Eq. (25).

In summary, the slope coefficient from a bilateral regression can be expressed as a weighted average of four multilateral components: two slope coefficients from two separate multilateral regressions, and two slope coefficients from two separate cross-currency multilateral regressions. Although the decomposition outlined in this section does not represent the paper's principal contribution, it nevertheless highlights the capacity of slope coefficients derived from separate multilateral regressions to account for the slope coefficient observed in a bilateral regression.

3.9. Aggregation bias

Bilateral exchange rates were shown to be the difference—an aggregation—between two multilateral exchange rates in Eq. (13). Similarly, bilateral explanatory variables were shown to be the difference—another aggregation—between two multilateral explanatory variables in Eq. (20). This breakdown creates two hierarchical levels of aggregation: a disaggregated level, which consists of multilateral exchange rates and multilateral explanatory variables, and an aggregated level, which consists of bilateral exchange rates and bilateral explanatory variables. The two hierarchical levels of aggregation can be used to formulate a hypothesis to assess whether the slope coefficient from a bilateral regression in Eq. (2) suffers from aggregation bias (see Kunkler and MacDonald, 2015).

At this point, it is helpful to rename the bilateral regression models in Eq. (2) as homogeneous bilateral regression models. The bilateral explanatory variable used in the homogeneous bilateral regression model is the difference between two multilateral explanatory variables (see Eq. (20)). By substituting this difference and introducing separate slope coefficients for each multilateral explanatory variable, the homogeneous bilateral regression model can be reformulated into a heterogeneous bilateral regression model:

$$\Delta \boldsymbol{s}_{i/\eta} = \alpha_{i/\eta} + \boldsymbol{x}_{i/\mathscr{M}}\gamma_{i/\mathscr{M}} - \boldsymbol{x}_{\eta/\mathscr{M}}\gamma_{\eta/\mathscr{M}} + \boldsymbol{u}_{i/\eta} \tag{32}$$

where $i = 1,\ldots,N$; $i \neq \eta$; $\gamma_{i/\mathscr{M}}$ is a multilateral slope coefficient of the ith currency; $\boldsymbol{x}_{i/\mathscr{M}}$ is a $T \times 1$ vector of the multilateral explanatory variable of the ith currency; $\gamma_{\eta/\mathscr{M}}$ is a multilateral slope coefficient of the common single-currency (ηth) numéraire; $\boldsymbol{x}_{\eta/\mathscr{M}}$ is a $T\times 1$ vector of the multilateral explanatory variable of the common single-currency (ηth) numéraire; and the other terms are discussed in Eq. (2).

Although the heterogeneous bilateral regression model analyses a bilateral exchange rate, it allows the two multilateral slope coefficients to differ from each other. The heterogeneous bilateral regression model is equivalent to the homogeneous bilateral regression model when the two multilateral slope coefficients are equal ($\gamma_{i/\mathscr{M}} = \gamma_{\eta/\mathscr{M}}$). Thus, the homogeneous bilateral regression model implicitly assumes that the two multilateral slope coefficients from the heterogeneous bilateral regression model are equal. If this assumption does not hold, the slope coefficient from the homogeneous bilateral regression suffers from aggregation bias.

The two hierarchical levels of aggregation enable the formulation of a hypothesis to test whether the slope coefficient from the homogeneous bilateral-panel regression suffers from aggregation bias. The null hypothesis posits that the two multilateral slope coefficients from the heterogeneous bilateral regression model are equal ($H_0 : \gamma_{i/\mathscr{M}} = \gamma_{\eta/\mathscr{M}}$), while the alternative hypothesis asserts that they differ ($H_A : \gamma_{i/\mathscr{M}} \neq \gamma_{\eta/\mathscr{M}}$). If there is insufficient evidence to reject the null hypothesis, the bilateral slope coefficient does not suffer from aggregation bias and it is appropriate to use the homogeneous bilateral regression model. In contrast, if there is sufficient evidence to reject the null hypothesis, the bilateral slope coefficient suffers from aggregation bias and it is inappropriate to use the homogeneous bilateral regression model when compared to the heterogeneous bilateral regression model. When the slope coefficient from a homogeneous bilateral regression suffers from aggregation bias, it is recommended that the analysis be performed at the disaggregate level, since the slope coefficient from a multilateral regression is free from aggregation bias.

3.10. *Multilateral-panel regressions*

Multilateral-panel regression models refer to panel regression models that use a system of multilateral exchange rates expressed in terms of a common equally-weighted currency basket. The multilateral-panel regression model for a system of N multilateral exchange rate returns is assumed to be balanced, where all of the multilateral exchange rate returns have the same number of observations with no missing data. In this situation, the OLS-estimated slope coefficient using the demeaned regression variables is the same as the OLS-estimated slope coefficient using the raw variables and including the individual fixed-effect coefficients (see Stock and Watson, 2020).

Although the regression variables could be demeaned for both individual and time fixed effects, they are only adjusted for individual fixed effects due to the absence of time fixed effects in multilateral-panel regressions. The absence of time fixed effects in multilateral-panel regressions arises from the fact that the cross-sectional average is zero—at each time t—for both the N multilateral exchange rate returns and the N multilateral explanatory variables. The cross-sectional average of the system of N multilateral exchange rate returns is:

$$\frac{1}{N}\sum_{i=1}^{N} \Delta s_{i/\mathscr{M}}(t) = 0 \tag{33}$$

where $t = 1,\ldots,T$; and $\sum_{i=1}^{N} s_{i/\mathscr{M}}(t) = 0$ from Eq. (14). Similarly, the cross-sectional average of the system of N multilateral explanatory variables is:

$$\frac{1}{N}\sum_{i=1}^{N} x_{i/\mathscr{M}}(t) = 0 \tag{34}$$

where $t = 1,\ldots,T$; and $\sum_{i=1}^{N} x_{i/\mathscr{M}}(t) = 0$ from Eq. (21).

Thus, the regression variables are only demeaned for the individual fixed effects. The demeaned multilateral exchange rate return for the individual fixed effect is:

$$\Delta \widetilde{s}_{i/\mathscr{M}}(t) = \Delta s_{i/\mathscr{M}}(t) - \Delta \overline{s}_{i/\mathscr{M}} \tag{35}$$

where $\Delta \widetilde{s}_{i/\mathscr{M}}(t)$ is the demeaned multilateral exchange rate return of the ith currency; and $\Delta \overline{s}_{i/\mathscr{M}} = \frac{1}{T}\sum_{t=1}^{T} \Delta s_{i/\mathscr{M}}(t)$ is the average of the multilateral exchange rate returns of the ith currency. Similarly, the demeaned multilateral explanatory variable for the individual fixed effect is:

$$\widetilde{x}_{i/\mathscr{M}}(t) = x_{i/\mathscr{M}}(t) - \overline{x}_{i/\mathscr{M}} \tag{36}$$

where $\widetilde{x}_{i/\mathscr{M}}(t)$ is the demeaned multilateral explanatory variable of the ith currency; and $\overline{x}_{i/\mathscr{M}} = \frac{1}{T}\sum_{t=1}^{T} x_{i/\mathscr{M}}(t)$ is the average of the multilateral explanatory variable of the ith currency.

The form of a multilateral-panel regression model for the panel of demeaned multilateral exchange rate returns against the corresponding panel of demeaned multilateral explanatory variables can be written as:

$$\Delta\widetilde{\boldsymbol{s}}_{:/\mathscr{M}} = \widetilde{\boldsymbol{x}}_{:/\mathscr{M}}\beta_{:/\mathscr{M}} + \widetilde{\boldsymbol{u}}_{:/\mathscr{M}} \tag{37}$$

where $\Delta\widetilde{\boldsymbol{s}}_{:/\mathscr{M}} = [\Delta\widetilde{\boldsymbol{s}}_{1/\mathscr{M}}; \cdots; \Delta\widetilde{\boldsymbol{s}}_{N/\mathscr{M}}]$ is an $NT \times 1$ stacked vector of demeaned multilateral exchange rate returns; $\widetilde{\boldsymbol{x}}_{:/\mathscr{M}} = [\widetilde{\boldsymbol{x}}_{1/\mathscr{M}}; \cdots; \widetilde{\boldsymbol{x}}_{N/\mathscr{M}}]$ is an $NT \times 1$ stacked vector of demeaned multilateral explanatory variables; $\beta_{:/\mathscr{M}}$ is a multilateral-panel slope coefficient; and $\widetilde{\boldsymbol{u}}_{:/\mathscr{M}} = [\widetilde{\boldsymbol{u}}_{1/\mathscr{M}}; \cdots; \widetilde{\boldsymbol{u}}_{N/\mathscr{M}}]$ is an $NT \times 1$ stacked vector of disturbances. Note that there is a system of N currencies; however, unlike the bilateral-panel regression model in Eq. (6), the multilateral-panel regression model incorporates the regression variables linked to the common single-currency numéraire.

The slope coefficient from the multilateral-panel regression model quantifies both the strength and direction of the relationship between the panel of multilateral exchange rate returns (the dependent variable) and the panel of multilateral explanatory variables. Economically, it indicates how much a multilateral exchange rate is expected to move when the corresponding multilateral explanatory variable increases by one unit. A positive slope coefficient suggests that, on average, an increase (or decrease) in a multilateral explanatory variable leads to an increase (or decrease) in the corresponding multilateral exchange rate. Conversely, a negative slope coefficient implies that an increase (or decrease) in a multilateral explanatory variable leads to a decrease (or increase) in the corresponding multilateral exchange rate.

The OLS estimator of the multilateral-panel slope coefficient is:

$$\widehat{\beta}_{:/\mathscr{M}} = \frac{cov(\Delta\widetilde{\boldsymbol{s}}_{:/\mathscr{M}}, \widetilde{\boldsymbol{x}}_{:/\mathscr{M}})}{var(\widetilde{\boldsymbol{x}}_{:/\mathscr{M}})} \tag{38}$$

where $\widehat{\beta}_{:/\mathscr{M}}$ is the OLS estimator of the multilateral-panel slope coefficient; $cov(\Delta\widetilde{\boldsymbol{s}}_{:/\mathscr{M}}, \widetilde{\boldsymbol{x}}_{:/\mathscr{M}})$ is the covariance between the panel of demeaned multilateral exchange rate returns and the panel of demeaned multilateral explanatory variables; and $var(\widetilde{\boldsymbol{x}}_{:/\mathscr{M}})$ is the variance of the panel of multilateral explanatory variables. The covariance term in Eq. (38) can be expanded to give:

$$\begin{aligned} cov(\Delta\widetilde{\boldsymbol{s}}_{:/\mathscr{M}}, \widetilde{\boldsymbol{x}}_{:/\mathscr{M}}) &= \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\Delta\widetilde{s}_{i/\mathscr{M}}(t)\widetilde{x}_{i/\mathscr{M}}(t)\right) \\ &= \frac{1}{N}\sum_{i=1}^{N} cov(\Delta\widetilde{\boldsymbol{s}}_{i/\mathscr{M}}, \widetilde{\boldsymbol{x}}_{i/\mathscr{M}}) \\ &= \frac{1}{N}\sum_{i=1}^{N} cov(\Delta\boldsymbol{s}_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}}) \end{aligned} \tag{39}$$

where $cov(\Delta\widetilde{\boldsymbol{s}}_{i/\mathscr{M}}, \widetilde{\boldsymbol{x}}_{i/\mathscr{M}}) = cov(\Delta\boldsymbol{s}_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}})$ is the covariance between the demeaned multilateral exchange rate returns of the ith currency and the multilateral explanatory variables of the ith currency. The variance term in Eq. (38) can also be expanded to give:

$$\begin{aligned} var(\widetilde{\boldsymbol{x}}_{:/\mathscr{M}}) &= \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\widetilde{x}_{i/\mathscr{M}}(t)\right)^2 \\ &= \frac{1}{N}\sum_{i=1}^{N} var(\widetilde{\boldsymbol{x}}_{i/\mathscr{M}}) \\ &= \frac{1}{N}\sum_{i=1}^{N} var(\boldsymbol{x}_{i/\mathscr{M}}) \end{aligned} \tag{40}$$

where $var(\widetilde{\boldsymbol{x}}_{i/\mathscr{M}}) = var(\boldsymbol{x}_{i/\mathscr{M}})$ is the variance of the demeaned multilateral explanatory variable of the ith currency.

Substituting both Eq. (39) and Eq. (40) into the equation for the OLS estimator of the multilateral-panel slope coefficient in Eq. (38) gives:

$$\begin{aligned} \widehat{\beta}_{:/\mathscr{M}} &= \frac{\sum_{i=1}^{N} cov(\Delta\boldsymbol{s}_{i/\mathscr{M}}\boldsymbol{x}_{i/\mathscr{M}})}{\sum_{i=1}^{N} var(\boldsymbol{x}_{i/\mathscr{M}})} \\ &= \sum_{i=1}^{N}\frac{cov(\Delta\boldsymbol{s}_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}})}{\sum_{j=1}^{N} var(\boldsymbol{x}_{i/\mathscr{M}})} \\ &= \sum_{i=1}^{N}\left(\frac{var(\boldsymbol{x}_{i/\mathscr{M}})}{\sum_{j=1}^{N} var(\boldsymbol{x}_{i/\mathscr{M}})}\right)\left(\frac{cov(\Delta\boldsymbol{s}_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}})}{var(\boldsymbol{x}_{i/\mathscr{M}})}\right) \\ &= \sum_{i=1}^{N} w_{i/\mathscr{M}}\widehat{\beta}_{i/\mathscr{M}} \end{aligned} \tag{41}$$

where:

$$w_{i/\mathscr{M}} = \frac{var(\boldsymbol{x}_{i/\mathscr{M}})}{\sum_{j=1}^{N} var(\boldsymbol{x}_{j/\mathscr{M}})} \tag{42}$$

is the multilateral-panel weight of the ith currency. The multilateral-panel weights are positive ($w_{i/\mathscr{M}} > 0$) and sum to one:

$$\sum_{i=1}^{N} w_{i/\mathscr{M}} = 1 \tag{43}$$

In summary, the slope coefficient from a multilateral-panel regression is a weighted average of the slope coefficients from the N separate multilateral regression models. The weights are determined by the variance of the corresponding multilateral explanatory variable relative to the sum of all of the variances of the multilateral explanatory variables.

3.11. Revisiting bilateral-panel regression models

This section shows that the slope coefficient obtained in a bilateral-panel regression can be decomposed into a weighted average of two multilateral components. It is helpful to restate the OLS estimator of the bilateral-panel slope coefficient in Eq. (7):

$$\widehat{\beta}_{:/\eta} = \frac{cov(\Delta\widetilde{\boldsymbol{s}}_{:/\eta}, \widetilde{\boldsymbol{x}}_{:/\eta})}{var(\widetilde{\boldsymbol{x}}_{:/\eta})} \tag{44}$$

The covariance term in Eq. (44) can be expanded to give:

$$\begin{aligned}
cov(\Delta\widetilde{\boldsymbol{s}}_{:/\eta}, \widetilde{\boldsymbol{x}}_{:/\eta}) &= \sum_{i=1}^{N} I_{i\neq\eta} cov(\Delta\boldsymbol{s}_{i/\eta}, \boldsymbol{x}_{i/\eta}) \\
&= \frac{1}{N-1}\sum_{i=1}^{N} I_{i\neq\eta} \mathrm{cov}(\Delta\boldsymbol{s}_{i/\mathscr{M}} - \Delta\boldsymbol{s}_{\eta/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}} - \boldsymbol{x}_{\eta/\mathscr{M}}) \\
&= \frac{1}{N-1}\left[\sum_{i=1}^{N} I_{i\neq\eta} cov(\Delta\boldsymbol{s}_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}}) - cov\left(\sum_{i=1}^{N} I_{i\neq\eta}\Delta\boldsymbol{s}_{i/\mathscr{M}}, \boldsymbol{x}_{\eta/\mathscr{M}}\right)\right] \\
&+ \frac{1}{N-1}\left[-cov\left(\Delta\boldsymbol{s}_{\eta/\mathscr{M}}, \sum_{i=1}^{N} I_{i\neq\eta}\boldsymbol{x}_{i/\mathscr{M}}\right) + (N-1)cov(\Delta\boldsymbol{s}_{\eta/\mathscr{M}}, \boldsymbol{x}_{\eta/\mathscr{M}})\right] \\
&= \frac{1}{N-1}\left[N cov(\Delta\boldsymbol{s}_{\eta/\mathscr{M}}, \boldsymbol{x}_{\eta/\mathscr{M}}) + \sum_{i=1}^{N} cov(\Delta\boldsymbol{s}_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}})\right] \\
&= \frac{N}{N-1}\left[cov(\Delta\boldsymbol{s}_{\eta/\mathscr{M}}, \boldsymbol{x}_{\eta/\mathscr{M}}) + \frac{1}{N}\sum_{i=1}^{N} cov(\Delta\boldsymbol{s}_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}})\right]
\end{aligned} \tag{45}$$

where $\Delta\widetilde{\boldsymbol{s}}_{\eta/\mathscr{M}} = -\sum_{i=1}^{N} I_{i\neq\eta}\Delta\widetilde{\boldsymbol{s}}_{i/\mathscr{M}}$ from Eq. (19) so that $cov\left(\sum_{i=1}^{N} I_{i\neq\eta}\Delta\widetilde{\boldsymbol{s}}_{i/\mathscr{M}}, \widetilde{\boldsymbol{x}}_{\eta/\mathscr{M}}\right) = -cov(\Delta\widetilde{\boldsymbol{s}}_{\eta/\mathscr{M}}, \widetilde{\boldsymbol{x}}_{\eta/\mathscr{M}})$; and $\widetilde{\boldsymbol{x}}_{\eta/\mathscr{M}} = -\sum_{i=1}^{N} I_{i\neq\eta}\widetilde{\boldsymbol{x}}_{i/\mathscr{M}}$ from Eq. (22) so that $cov\left(\widetilde{\boldsymbol{x}}_{\eta/\mathscr{M}}, \sum_{i=1}^{N} I_{i\neq\eta}\widetilde{\boldsymbol{x}}_{i/\mathscr{M}}\right) = -var(\widetilde{\boldsymbol{x}}_{\eta/\mathscr{M}})$. The variance term in Eq. (44) can also be expanded to give:

$$\begin{aligned}
var(\widetilde{\boldsymbol{x}}_{:/\eta}) &= \frac{1}{N-1}\sum_{i=1}^{N} I_{i\neq\eta} var(\boldsymbol{x}_{i/\eta}) \\
&= \frac{1}{N-1}\sum_{i=1}^{N} I_{i\neq\eta} var(\boldsymbol{x}_{i/\mathscr{M}} - \boldsymbol{x}_{\eta/\mathscr{M}}) \\
&= \frac{1}{N-1}\left[\sum_{i=1}^{N} I_{i\neq\eta} var(\boldsymbol{x}_{\eta/\mathscr{M}}) - 2cov\left(\boldsymbol{x}_{\eta/\mathscr{M}}, \sum_{i=1}^{N} I_{i\neq\eta}\boldsymbol{x}_{i/\mathscr{M}}\right) + (N-1)var(\boldsymbol{x}_{\eta/\mathscr{M}})\right] \\
&= \frac{1}{N-1}\left[N var(\boldsymbol{x}_{\eta/\mathscr{M}}) + \sum_{i=1}^{N} var(\boldsymbol{x}_{i/\mathscr{M}})\right] \\
&= \frac{N}{N-1}\left[var(\boldsymbol{x}_{\eta/\mathscr{M}}) + \frac{1}{N}\sum_{i=1}^{N} var(\boldsymbol{x}_{i/\mathscr{M}})\right]
\end{aligned} \tag{46}$$

where $var(\boldsymbol{x}_{i/\eta}) = var(\boldsymbol{x}_{i/\mathscr{M}} - \boldsymbol{x}_{\eta/\mathscr{M}})$ is the variance of the bilateral explanatory variable of the ith currency; and $\boldsymbol{x}_{\eta/\mathscr{M}} = -\sum_{i=1}^{N} I_{i\neq\eta}\boldsymbol{x}_{i/\mathscr{M}}$ from Eq. (22) so that $cov\left(\boldsymbol{x}_{\eta/\mathscr{M}}, \sum_{i=1}^{N} I_{i\neq\eta}\boldsymbol{x}_{i/\mathscr{M}}\right) = -var(\boldsymbol{x}_{\eta/\mathscr{M}})$.

Substituting both Eq. (45) and Eq. (46) into the equation for the OLS estimator of the bilateral-panel slope coefficient in Eq. (44) gives:

$$\begin{aligned}
\widehat{\beta}_{:/\eta} &= \frac{cov\left(\Delta \boldsymbol{s}_{\eta/\mathscr{M}}, \boldsymbol{x}_{\eta/\mathscr{M}}\right) + \frac{1}{N}\sum_{i=1}^{N} cov\left(\Delta \boldsymbol{s}_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}}\right)}{var\left(\boldsymbol{x}_{\eta/\mathscr{M}}\right) + \frac{1}{N}\sum_{i=1}^{N} var\left(\boldsymbol{x}_{i/\mathscr{M}}\right)} \\
&= \delta_{\eta}\frac{cov\left(\Delta \boldsymbol{s}_{\eta/\mathscr{M}}, \boldsymbol{x}_{\eta/\mathscr{M}}\right)}{var\left(\boldsymbol{x}_{\eta/\mathscr{M}}\right)} + (1-\delta_{\eta})\sum_{i=1}^{N}\left(\frac{var\left(\boldsymbol{x}_{i/\mathscr{M}}\right)}{\sum_{j=1}^{N} var\left(\boldsymbol{x}_{j/\mathscr{M}}\right)}\right)\left(\frac{cov\left(\Delta \boldsymbol{s}_{i/\mathscr{M}}, \boldsymbol{x}_{i/\mathscr{M}}\right)}{var\left(\boldsymbol{x}_{i/\mathscr{M}}\right)}\right) \\
&= \delta_{\eta}\frac{cov\left(\Delta \boldsymbol{s}_{\eta/\mathscr{M}}, \boldsymbol{x}_{\eta/\mathscr{M}}\right)}{var\left(\boldsymbol{x}_{\eta/\mathscr{M}}\right)} + (1-\delta_{\eta})\sum_{i=1}^{N} w_{i/\mathscr{M}}\widehat{\beta}_{i/\mathscr{M}} \\
&= \delta_{\eta}\widehat{\beta}_{\eta/\mathscr{M}} + (1-\delta_{\eta})\widehat{\beta}_{:/\mathscr{M}}
\end{aligned} \tag{47}$$

where $\widehat{\beta}_{\eta/\mathscr{M}} = cov\left(\Delta \boldsymbol{s}_{\eta/\mathscr{M}}, \boldsymbol{x}_{\eta/\mathscr{M}}\right)/var\left(\boldsymbol{x}_{\eta/\mathscr{M}}\right)$ from Eq. (25); $\widehat{\beta}_{:/\mathscr{M}} = \sum_{i=1}^{N} w_{i/\mathscr{M}}\widehat{\beta}_{i/\mathscr{M}}$ from Eq. (41); $w_{i/\mathscr{M}} = var\left(\boldsymbol{x}_{i/\mathscr{M}}\right)/\sum_{j=1}^{N} var\left(\boldsymbol{x}_{j/\mathscr{M}}\right)$ from Eq. (42); and:

$$\delta_{\eta} = \frac{var\left(\boldsymbol{x}_{\eta/\mathscr{M}}\right)}{var\left(\boldsymbol{x}_{\eta/\mathscr{M}}\right) + \frac{1}{N}\sum_{i=1}^{N} var\left(\boldsymbol{x}_{i/\mathscr{M}}\right)} \tag{48}$$

is the multilateral delta of the common single-currency numéraire.

In summary, the slope coefficient from a bilateral-panel regression can be decomposed into a weighted average of two multilateral components: (1) the slope coefficient from a multilateral-panel regression, and (2) the slope coefficient from a multilateral (non-panel) regression—based on a common single-currency numéraire.

3.12. *Numéraire effect*

The numéraire effect arises when a particular currency is designated as the common single-currency numéraire for bilateral exchange rates in bilateral-panel regressions. Selecting different common single-currency numéraires can lead to varying and often inconsistent outcomes. Therefore, it is important to examine the statistical significance and quantify the impact of the numéraire effect. Any currency within the system of N currencies can serve as the common single-currency (ηth) numéraire.

Exchange rates and explanatory variables can be divided into two hierarchical levels of aggregation: a disaggregated level, which consists of multilateral exchange rates and multilateral explanatory variables, and an aggregated level, which consists of bilateral exchange rates and bilateral explanatory variables. The $(N-1)T \times 1$ stacked bilateral explanatory variable used in the bilateral-panel regression model in Eq. (6) can be decomposed into two stacked multilateral explanatory variables:

$$\begin{aligned}
\widetilde{\boldsymbol{x}}_{:/\eta} &= \left[\widetilde{\boldsymbol{x}}_{1/\eta}; \cdots; \widetilde{\boldsymbol{x}}_{N/\eta}\right] \\
&= \left[\widetilde{\boldsymbol{x}}_{1/\mathscr{M}} - \widetilde{\boldsymbol{x}}_{\eta/\mathscr{M}}; \cdots; \widetilde{\boldsymbol{x}}_{N/\mathscr{M}} - \widetilde{\boldsymbol{x}}_{\eta/\mathscr{M}}\right] \\
&= \left[\widetilde{\boldsymbol{x}}_{1/\mathscr{M}}; \cdots; \widetilde{\boldsymbol{x}}_{N/\mathscr{M}}\right] - \left[\widetilde{\boldsymbol{x}}_{\eta/\mathscr{M}}; \cdots; \widetilde{\boldsymbol{x}}_{\eta/\mathscr{M}}\right] \\
&= \widetilde{\boldsymbol{z}}_{:/\mathscr{M}} - \widetilde{\boldsymbol{z}}_{\eta/\mathscr{M}}
\end{aligned} \tag{49}$$

where $\widetilde{\boldsymbol{z}}_{:/\mathscr{M}} = \left[\widetilde{\boldsymbol{x}}_{1/\mathscr{M}}; \cdots; \widetilde{\boldsymbol{x}}_{N/\mathscr{M}}\right]$ is an $(N-1)T \times 1$ stacked vector of demeaned multilateral explanatory variables, which excludes the demeaned multilateral explanatory variable of the common single-currency (ηth) numéraire ($\widetilde{\boldsymbol{x}}_{\eta/\mathscr{M}}$); and $\widetilde{\boldsymbol{z}}_{\eta/\mathscr{M}} = \left[\widetilde{\boldsymbol{x}}_{\eta/\mathscr{M}}; \cdots; \widetilde{\boldsymbol{x}}_{\eta/\mathscr{M}}\right]$ is an $(N-1)T \times 1$ stacked and repeated vector of the demeaned multilateral explanatory variable corresponding to the common single-currency (ηth) numéraire. Thus, panels of bilateral explanatory variables are the difference—an aggregation—between two panels of multilateral explanatory variables.

At this point, it is helpful to rename the bilateral-panel regression models in Eq. (6) as homogeneous bilateral-panel regression models. By substituting Eq. (49) and introducing separate slope coefficients for each of the stacked vectors, the homogeneous bilateral-panel regression model can be reformulated into a heterogeneous bilateral-panel regression model:

$$\Delta \widetilde{\boldsymbol{s}}_{:/\eta} = \widetilde{\boldsymbol{z}}_{:/\mathscr{M}}\gamma_{:/\mathscr{M}} - \widetilde{\boldsymbol{z}}_{\eta/\mathscr{M}}\gamma_{\eta/\mathscr{M}} + \widetilde{\boldsymbol{u}}_{:/\eta} \tag{50}$$

where $\gamma_{:/\mathscr{M}}$ is a multilateral-panel slope coefficient; $\gamma_{\eta/\mathscr{M}}$ is a multilateral-panel slope coefficient of the common single-currency (ηth) numéraire; and the other terms are discussed in both Eq. (6) and Eq. (49).

While the heterogeneous bilateral-panel regression model analyses a panel of bilateral exchange rates, it does allow the two multilateral-panel slope coefficients to differ from each other. The heterogeneous bilateral-panel regression model is equivalent to the homogeneous bilateral-panel regression model when the two multilateral-panel slope coefficients are equal ($\gamma_{:/\mathscr{M}} = \gamma_{\eta/\mathscr{M}}$). Thus, the homogeneous bilateral-panel regression model implicitly assumes that the two multilateral-panel slope coefficients are equal. If this assumption does not hold, the slope coefficient from the homogeneous bilateral-panel regression suffers from aggregation bias.

The two hierarchical levels of aggregation enable the formulation of a hypothesis to test whether the slope coefficient from the homogeneous bilateral-panel regression suffers from aggregation bias. The null hypothesis posits that the two multilateral-panel slope coefficients within the heterogeneous bilateral-panel regression are equal ($H_0 : \gamma_{:/\mathscr{M}} = \gamma_{\eta/\mathscr{M}}$), while the alternative hypothesis asserts that they differ ($H_A : \gamma_{:/\mathscr{M}} \neq \gamma_{\eta/\mathscr{M}}$). If there is insufficient evidence to reject the null hypothesis, the bilateral-panel slope coefficient does not suffer from aggregation bias and it is appropriate to use the homogeneous bilateral-panel regression model. In contrast, if there is sufficient evidence to reject the null hypothesis, the bilateral-panel slope coefficient suffers from aggregation bias and it is inappropriate to use the homogeneous bilateral-panel regression model when compared to the heterogeneous bilateral-panel regression model. When the slope coefficient from a homogeneous bilateral-panel regression suffers from aggregation bias, it is recommended that the analysis be performed at the disaggregate level, since the slope coefficient from a multilateral-panel regression is free from aggregation bias.

A formula can be derived to measure the numéraire effect—using any chosen common single-currency numéraire—by computing the difference between the observed of the bilateral-panel slope coefficient and that of the multilateral-panel slope coefficient:

$$\widehat{\varphi}_{:/\eta} = \widehat{\beta}_{:/\eta} - \widehat{\beta}_{:/\mathscr{M}} \tag{51}$$

where $\widehat{\varphi}_{:/\eta}$ is the numéraire effect of choosing the ηth currency as the common single-currency numéraire; $\widehat{\beta}_{:/\eta}$ is the OLS-estimated slope coefficient from a homogeneous bilateral-panel regression; and $\widehat{\beta}_{:/\mathscr{M}}$ is the OLS-estimated multilateral-panel slope coefficient. An alternative representation of the numéraire effect can be computed by substituting Eq. (47) into Eq. (51):

$$\begin{aligned}\widehat{\varphi}_{:/\eta} &= \delta_\eta \widehat{\beta}_{\eta/\mathscr{M}} + (1-\delta_\eta)\widehat{\beta}_{:/\mathscr{M}} - \widehat{\beta}_{:/\mathscr{M}} \\ &= \delta_\eta \left(\widehat{\beta}_{\eta/\mathscr{M}} - \widehat{\beta}_{:/\mathscr{M}}\right)\end{aligned} \tag{52}$$

where $\widehat{\varphi}_{:/\eta}$ is the numéraire effect of choosing the ηth currency as the common single-currency numéraire; δ_η is from Eq. (48); $\widehat{\beta}_{\eta/\mathscr{M}}$ is a multilateral slope coefficient from a homogeneous bilateral-panel regression; and $\widehat{\beta}_{:/\mathscr{M}}$ is a multilateral-panel slope coefficient. The alternative representation shows that the numéraire effect represents a scaled difference between the slope coefficient from a multilateral (non-panel) regression—based on a common single-currency numéraire—and the slope coefficient from a multilateral-panel regression.

This section examined the statistical significance and measured the magnitude of the numéraire effect resulting from the selection of a specific currency as the common single-currency numéraire in homogeneous bilateral-panel regressions. The two hierarchical levels of aggregation provided a basis for a hypothesis test on whether the slope coefficient from the homogeneous bilateral-panel regression suffers from aggregation bias. When such bias is present, it is advisable to conduct the analysis at the disaggregated level, since the slope coefficient from a multilateral-panel regression is not subject to aggregation bias. The numéraire effect can be estimated by the difference between the bilateral-panel slope coefficient—based on the common single-currency numéraire—and that of the multilateral-panel slope coefficient.

4. Results

This section provides two applications, namely, the uncovered interest rate parity (UIP) condition and the uncovered equity parity

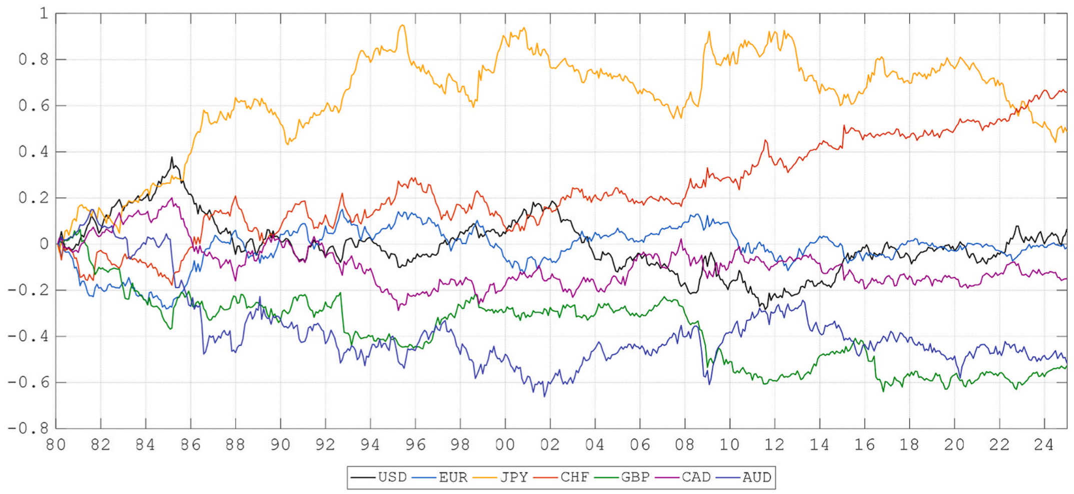

Fig. 1. Multilateral exchange rates. *Notes:* Fig. 1 displays the cumulative log returns of the seven multilateral exchange rates.

(UEP) condition, using a monthly data sample spanning 45 years containing a system of six US dollar bilateral exchange rates.

4.1. Data sample

The data sample is monthly data from 1st January 1980 to 31st December 2024, and consists of a system of six bilateral US dollar exchange rates (sourced from Bloomberg), seven MSCI total return equity market indexes (sourced from Bloomberg), and seven 3-month interbank rates (sourced from the OECD). The system of seven currencies consists of: the US dollar (USD), the euro (EUR), the Japanese yen (JPY), the Swiss franc (CHF), the British pound (GBP), the Canadian dollar (CAD), and the Australian dollar (AUD).

4.2. Exchange rates

The cumulative log returns of the seven multilateral exchange rates are displayed in Fig. 1. According to the no-arbitrage condition in Eq. (14), the system of these multilateral exchange rates, when expressed in log terms, are centred around zero. A significant period followed the signing of the Plaza Accord agreement in 1985, where depreciations occurred for the US dollar (USD), the Canadian dollar (CAD), and the Australian dollar (AUD), and appreciations occurred for the Japanese yen (JPY), the euro (EUR), and the Swiss franc (CHF). A further significant period was marked by the global financial crisis in 2008, where sharp depreciations occurred for the British pound (GBP) and the Australian dollar (AUD), and sharp appreciations occurred for the Japanese yen (JPY) and the US dollar (USD).

The descriptive statistics for the log returns of the seven multilateral exchange rates and the log returns of the six bilateral US dollar exchange rates are reported in Table 1. The decomposition of a bilateral exchange rate into two multilateral exchange rates in Eq. (13) is equally applicable to determining the average annualised return of that bilateral exchange rate:

$$\Delta \bar{s}_{i/USD} = \Delta \bar{s}_{i/\mathscr{M}} - \Delta \bar{s}_{USD/\mathscr{M}} \tag{53}$$

where $i = 1, \ldots, 7$; $i \neq USD$; $t = 1, \ldots, T$; $\Delta \bar{s}_{i/USD}$ is from Eq. (4); $\Delta \bar{s}_{i/\mathscr{M}}$ and $\Delta \bar{s}_{USD/\mathscr{M}}$ are from Eq. (35). As a result, the ranking of the average annualised returns for the multilateral exchange rates matches that of the bilateral exchange rates. For example, the highest average annualised return for the multilateral exchange rates is 1.46% for the Swiss franc (CHF) and for the bilateral exchange rates is 1.32% for the Swiss franc/US dollar (CHF/USD).

The highest annualised volatility is 8.61% for the Japanese yen (JPY) multilateral exchange rate and 11.25% for the Australian dollar/US dollar (AUD/USD) bilateral exchange rate. For the multilateral exchange rates, the observed skewness values are significantly positive with 0.856 for the Japanese yen (JPY) and 0.502 for the Swiss franc (CHF), and is significantly negative with −0.888 for the Australian dollar (AUD), −0.723 for the British pound (GBP), and −0.239 for the Canadian dollar (CAD). In contrast, for the bilateral exchange rates, the currencies with significant observed skewness values are similar, except for the Swiss franc/US dollar (CHF/USD), which is insignificant. The observed kurtosis values are significant for all multilateral exchange rates and all bilateral exchange rates. The significant results of the Augmented Dickey-Fuller (ADF) unit root test suggest that multilateral exchange rate movements and bilateral exchange rate movements are stationary.

Table 1
Descriptive statistics.

Panel A: Multilateral exchange rates					
	Average	Volatility	Skewness	Kurtosis	ADF
USD	0.15%	6.37%	0.021	3.51**	−22.0**
EUR	−0.03%	5.69%	−0.144	3.76**	−22.8**
JPY	1.08%	8.61%	0.856**	6.15**	−22.8**
CHF	1.46%	6.65%	0.502**	5.09**	−24.1**
GBP	−1.17%	6.38%	−0.723**	7.23**	−22.0**
CAD	−0.33%	6.21%	−0.239*	3.93**	−24.6**
AUD	−1.15%	8.42%	−0.888**	5.63**	−22.2**
Panel B: Bilateral exchange rates					
	Average	Volatility	Skewness	Kurtosis	ADF
EUR/USD	−0.18%	10.41%	−0.113	3.77**	−22.3**
JPY/USD	0.93%	11.00%	0.460**	4.48**	−22.2**
CHF/USD	1.32%	11.07%	0.155	3.92**	−22.8**
GBP/USD	−1.32%	9.84%	−0.213*	5.17**	−21.8**
CAD/USD	−0.48%	7.07%	−0.490**	7.43**	−24.6**
AUD/USD	−1.29%	11.25%	−0.576**	5.31**	−22.3**

Notes: Table 1 reports the descriptive statistics for the log returns of the seven multilateral exchange rates (Panel A) and the log returns of the six bilateral US dollar exchange rates (Panel B). The columns are the annualised average return (Average); the annualised standard deviation (Volatility); the observed skewness (Skewness); the observed kurtosis (Kurtosis); and the Augmented Dickey-Fuller (ADF) unit root test displaying the test statistic. Significance at the 10% level, the 5% level, and 1% level is denoted by *, **, and ***, respectively.

4.3. Uncovered interest rate parity

Uncovered interest rate parity (UIP) is a fundamental principle used to explain how exchange rates are determined. The bilateral UIP condition states that the expected return in a bilateral exchange rate should be equal to the negative of the differential in the respective interest rate levels (see Sarno and Taylor, 2002; Engel, 2014). The form of a bilateral regression model to test the bilateral UIP condition can be written as:

$$\Delta s_{i/\eta}(t) = \alpha_{i/\eta} - r_{i/\eta}(t-1)\beta_{i/\eta} + u_{i/\eta}(t) \tag{54}$$

where $i = 1,\ldots,N$; $i \neq \eta$; $t = 1,\ldots,T$; $\Delta s_{i/\eta}(t)$ is the ith/ηth bilateral exchange rate returns; $r_{i/\eta}(t-1) = r_i(t-1) - r_\eta(t-1)$ is the ith/ ηth bilateral interest rate level (the differential in the respective interest rate levels); $r_i(t-1)$ is the interest rate level for the ith currency; $r_\eta(t-1)$ is the interest rate level for the ηth currency; $\alpha_{i/\eta}$ is a bilateral intercept term; $\beta_{i/\eta}$ is a bilateral slope coefficient; and $u_{i/\eta}(t)$ is the ith/ηth bilateral disturbance (see Sarno and Taylor, 2002). The bilateral explanatory variables for the homogeneous bilateral regressions are the negative of the lagged bilateral interest rate levels. According to economic theory, the bilateral UIP condition implies a bilateral intercept of zero ($\alpha_{i/\eta} = 0$) and a bilateral slope coefficient of one ($\beta_{i/\eta} = 1$). Empirically, the bilateral UIP condition is strongly rejected, with studies commonly finding negative bilateral slope coefficients (MacDonald, 2007).

The multilateral UIP condition states that the expected return in a multilateral exchange rate should be equal to the negative of the multilateral interest rate level. The form of a multilateral regression model to test the multilateral UIP condition can be written as:

$$\Delta s_{i/\mathscr{M}}(t) = \alpha_{i/\mathscr{M}} - r_{i/\mathscr{M}}(t-1)\beta_{i/\mathscr{M}} + u_{i/\mathscr{M}}(t) \tag{55}$$

where $i = 1,\ldots,N$; $t = 1,\ldots,T$; $\Delta s_{i/\mathscr{M}}(t)$ is the multilateral exchange rate returns of the ith currency; $r_{i/\mathscr{M}}(t-1)$ is the multilateral interest rate level for the ith currency; $\alpha_{i/\mathscr{M}}$ is an multilateral intercept term; $\beta_{i/\mathscr{M}}$ is a multilateral slope coefficient; and $u_{i/\mathscr{M}}(t)$ is the multilateral disturbance. The multilateral explanatory variables for the multilateral regressions are the negative of the lagged multilateral interest rate levels. According to economic theory, the multilateral UIP condition implies a multilateral intercept of zero ($\alpha_{i/\mathscr{M}} = 0$) and a multilateral slope coefficient of one ($\beta_{i/\mathscr{M}} = 1$).

4.3.1. Interest rate levels

The seven annualised multilateral interest rate levels are displayed in Fig. 2. According to the no-arbitrage condition in Eq. (21), the system of the multilateral interest rate levels, when expressed in log terms, are centred around zero. The levels of the multilateral interest rates were considerably more volatile from 1980 to the mid-1990s compared to the subsequent years. Furthermore, following the global financial crisis in 2008, the levels of multilateral interest rates began to converge, except for the Australian dollar (AUD). The levels of the multilateral interest rates converge further and remain flat for two years during the Covid years of 2020-21.

The descriptive statistics for the seven multilateral interest rate levels and the six bilateral interest rate levels expressed in terms of the US interest rate level are reported in Table 2. The decomposition of a bilateral interest rate level (explanatory variable) into two multilateral interest rate levels (explanatory variables) using Eq. (20) is equally applicable to determining the average of that bilateral interest rate level:

$$\bar{r}_{i/USD} = \Delta\bar{r}_{i/\mathscr{M}} - \Delta\bar{r}_{USD/\mathscr{M}} \tag{56}$$

where $i = 1,\ldots,7$; $i \neq USD$; $t = 1,\ldots,T$; $\bar{r}_{i/USD}$ is from Eq. (5); $\bar{r}_{i/\mathscr{M}}$ and $\bar{r}_{USD/\mathscr{M}}$ are from Eq. (36). As a result, the ranking of the average for

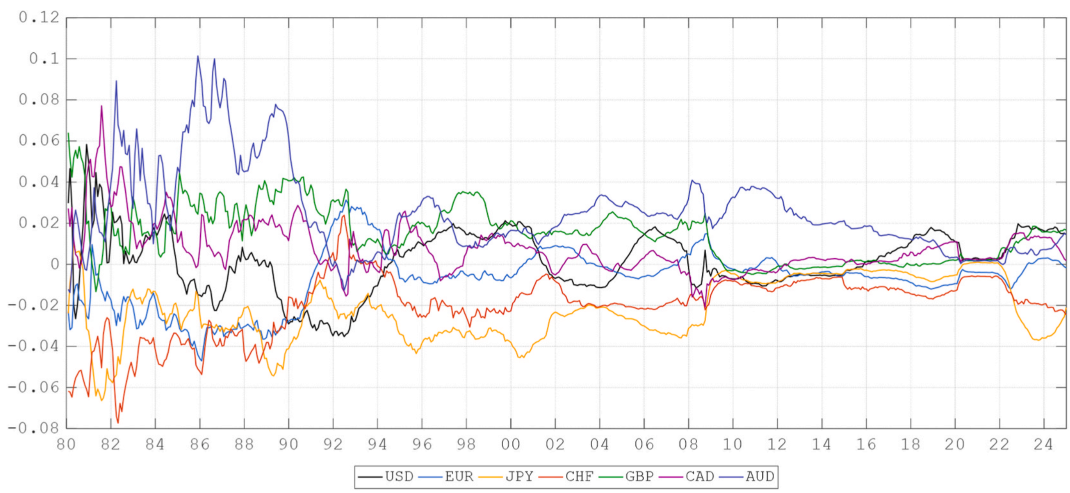

Fig. 2. Multilateral interest rate levels. *Notes:* Fig. 2 displays the seven annualised multilateral interest rate levels.

Table 2
Descriptive statistics.

Panel A: Multilateral interest rate levels					
	Average	Volatility	Skewness	Kurtosis	ADF
USD	0.16%	0.43%	−0.132	3.33**	−3.4**
EUR	−0.70%	0.40%	−0.282**	3.41**	−2.0*
JPY	−2.19%	0.43%	−0.216*	2.45**	−1.1
CHF	−2.00%	0.43%	−0.925**	4.33**	−2.3*
GBP	1.49%	0.38%	0.530**	2.89**	−3.2**
CAD	0.74%	0.35%	1.616**	7.69**	−3.2**
AUD	2.50%	0.60%	1.260**	4.53**	−1.6
Panel B: Bilateral interest rate levels					
	Average	Volatility	Skewness	Kurtosis	ADF
EUR/USD	−0.86%	0.64%	0.578**	4.34**	−3.1**
JPY/USD	−2.36%	0.68%	−0.495**	2.89**	−2.1*
CHF/USD	−2.17%	0.72%	−0.275**	4.69**	−2.7**
GBP/USD	1.33%	0.62%	0.828**	3.73**	−2.8**
CAD/USD	0.58%	0.43%	0.788**	3.56**	−3.2**
AUD/USD	2.33%	0.82%	0.763**	3.95**	−2.4*

Notes: Table 2 reports the descriptive statistics for the seven multilateral interest rate levels (Panel A) and the six bilateral interest rate levels relative to the US interest rate levels (Panel B). The columns are the annualised average return (Average); the annualised standard deviation (Volatility); the observed skewness (Skewness); the observed kurtosis (Kurtosis); and the Augmented Dickey-Fuller (ADF) unit root test displaying the test statistic. Significance at the 10% level, the 5% level, and 1% level is denoted by *, **, and ***, respectively.

the bilateral interest rate levels matches that of the multilateral interest rate levels. For example, the highest average annualised return for the multilateral interest rate levels is 2.50% for the Australian dollar (AUD) and for the bilateral interest rate levels is 2.33% for the Australian dollar/US dollar (AUD/USD).

The highest annualised volatility for the multilateral interest rate levels is 0.60% for the Australian dollar (AUD) and for the bilateral interest rate levels is 0.82% for the Australian dollar/US dollar (AUD/USD). For the multilateral interest rate levels, the observed skewness values are significantly negative with −0.925 for the Swiss franc (CHF), −0.282 for the euro (EUR), and −0.216 for the Japanese yen (JPY) and, and is significantly positive with 1.616 for the Canadian dollar (CAD), 1.260 for the Australian dollar (AUD) and 0.530 for the British pound (GBP). In contrast, for the bilateral interest rate levels, all of the observed skewness values are significant. The observed kurtosis is significant for all multilateral interest rate levels and all bilateral interest rate levels. Although most of the Augmented Dickey-Fuller (ADF) unit root tests are significant, which suggest that the corresponding multilateral interest rate levels and bilateral interest rate levels are stationary. However, the ADF unit root tests are insignificant for both the Japanese yen (JPY) multilateral interest rate level and the Austrian dollar (AUD) multilateral interest rate level.

4.3.2. *Multilateral-panel regressions*

The observed slope coefficient from a multilateral-panel regression of the seven multilateral exchange rate returns against the corresponding seven multilateral interest rate levels is reported in Table 3, together with the constituents of the weighted average in Eq. (41). The multilateral-panel slope coefficient quantifies both the strength and direction of the relationship between the panel of seven multilateral exchange rate returns and the panel of seven multilateral interest rate levels. Economically, the multilateral-panel

Table 3
Weighted average of the slope coefficient from a multilateral-panel regression.

i	$\widehat{\beta}_{i/\mathscr{M}}$	$var(\boldsymbol{x}_{i/\mathscr{M}})$	$w_{i/\mathscr{M}}$	$w_{i/\mathscr{M}}\widehat{\beta}_{i/\mathscr{M}}$
Panel	**−0.5991*** **	**0.00001113**	**1.0000**	**−0.5991**
USD	−2.3391***	0.00000153	0.1372	−0.3209
EUR	0.3760	0.00000135	0.1213	0.0456
JPY	−0.8269**	0.00000153	0.1379	−0.1140
CHF	−1.3768***	0.00000153	0.1375	−0.1893
GBP	−1.1531***	0.00000120	0.1080	−0.1246
CAD	−1.1612***	0.00000100	0.0899	−0.1044
AUD	0.7773	0.00000298	0.2682	0.2085

Notes: Table 3 reports the observed slope coefficient from the multilateral-panel regression, together with the variables used in the weighted average in Eq. (41). The values are: $\widehat{\beta}_{i/\mathscr{M}}$ is the observed slope coefficient from a multilateral (non-panel) regression of the ith currency; $var(\boldsymbol{x}_{i/\mathscr{M}})$ is the monthly variance of the multilateral interest rate levels of the ith currency; and $w_{i/\mathscr{M}}$ is the multilateral weight for the ith currency. The multilateral slope coefficients showing sufficient evidence to reject the null hypothesis of one are denoted by *, **, and *** at the 10% level, the 5% level, and 1% level, respectively.

slope coefficient measures how much each multilateral exchange rate is expected to change when its corresponding negative multilateral interest rate changes by one unit, since the multilateral interest rates enter the multilateral-panel regression equation with a negative sign.

The observed multilateral-panel slope coefficient of −0.5991 is significantly different from the theoretical value of one at the 1% level, implying that the multilateral UIP condition does not hold for the panel of seven multilateral exchange rate returns. It was shown in Eq. (41) that the observed multilateral-panel slope coefficient is a weighted average of the multilateral slope coefficients from separate multilateral (non-panel) regressions:

$$\widehat{\beta}_{:/\mathscr{M}} = \sum_{i=1}^{7} w_{i/\mathscr{M}} \widehat{\beta}_{i/\mathscr{M}} = -0.5991 \tag{57}$$

This breakdown makes clear how each individual currency contributes to the overall estimate of the multilateral-panel slope coefficient. The multilateral slope coefficients themselves differ across the separate multilateral (non-panel) regressions. For example, both the Australian dollar (AUD) and the euro (EUR) yield positive multilateral slope coefficients—0.7773 and 0.3760, respectively—that are not statistically different from the theoretical value of one, suggesting that the multilateral UIP condition holds for these currencies. In contrast, the other five currencies display multilateral slope coefficients that differ significantly from the theoretical value of one, indicating a failure of the multilateral UIP condition in those cases. The US dollar (USD) multilateral exchange rate accounts for the largest share of the weighted average, with a weighted slope coefficient ($w_{USD/\mathscr{M}} \widehat{\beta}_{USD/\mathscr{M}}$) of −0.3209. The associated multilateral weight is:

$$w_{USD/\mathscr{M}} = \frac{var(\widetilde{x}_{USD/\mathscr{M}})}{\sum_{j=1}^{7} var(\widetilde{x}_{j/\mathscr{M}})} = \frac{0.00000153}{0.00001113} = 0.1372 \tag{58}$$

where $\sum_{j=1}^{7} var(\widetilde{x}_{j/\mathscr{M}}) = 0.00001113$.

The significant multilateral-panel slope coefficient of −0.5991 shows that the panel of multilateral exchange rates move in the same direction as the corresponding panel of interest rate levels, contrary to the economic predictions of the multilateral UIP condition. The observed multilateral-panel slope coefficient reflects a weighted average of the seven multilateral (non-panel) slope coefficients. Even though some of these individual multilateral slope coefficients do not differ significantly from the theoretical value of one, the aggregate estimate of the multilateral-panel slope coefficient is statistically different from one.

4.3.3. *Bilateral-panel regressions*

The slope coefficients from seven separate homogeneous bilateral-panel regression models are estimated, each using a different single-currency numéraire, as reported in the $\widehat{\beta}_{:/\eta}$ column of Table 4. According to the bilateral uncovered interest rate parity (UIP) condition, the bilateral-panel slope coefficients should equal one. However, contrary to this theoretical prediction, the estimated slope coefficients from all seven bilateral-panel regressions deviate significantly from one. The largest negative bilateral-panel slope coefficient of −1.4515 occurs when the US dollar (USD) serves as the common single-currency numéraire. In contrast, the largest positive bilateral-panel slope coefficient of 0.2989 occurs when the Australian dollar (USD) serves as the common single-currency numéraire.

Each bilateral-panel slope coefficient can be decomposed into two multilateral terms using in Eq. (47), namely, the slope coefficient from a multilateral-panel regression ($\widehat{\beta}_{\eta/\mathscr{M}}$), and the slope coefficient from a multilateral (non-panel) regression ($\widehat{\beta}_{:/\mathscr{M}}$)—based on a common single-currency numéraire (see Table 4). For example, bilateral-panel slope coefficient when the US dollar serves as the

Table 4
Numéraire effect and slope coefficients.

η	$\widehat{\beta}_{:/\eta}$	$\widehat{\varphi}_{:/\eta}$	$\widehat{\gamma}_{\eta/\mathscr{M}}$	$\widehat{\gamma}_{:/\mathscr{M}}$	$\widehat{\gamma}_{\eta/\mathscr{M}} - \widehat{\gamma}_{:/\mathscr{M}}$	δ_η	$\widehat{\beta}_{\eta/\mathscr{M}}$	$1 - \delta_\eta$	$\widehat{\beta}_{:/\mathscr{M}}$
USD	−1.4515***	−0.8524	−2.6843***	−0.2676***	−2.4167***	0.4899	−2.3391***	0.5101	−0.5991**
EUR	−0.1514***	0.4477	0.5653	−0.7598***	1.3251**	0.4591	0.3760	0.5409	−0.5991**
JPY	−0.7110***	−0.1119	−0.8722***	−0.5555***	−0.3167	0.4912	−0.8269**	0.5088	−0.5991**
CHF	−0.9806***	−0.3814	−1.5312***	−0.4505***	−1.0807*	0.4905	−1.3768***	0.5095	−0.5991**
GBP	−0.8377***	−0.2386	−1.2588***	−0.5192***	−0.7396	0.4306	−1.1531***	0.5694	−0.5991**
CAD	−0.8162***	−0.2170	−1.2658***	−0.5333***	−0.7325	0.3862	−1.1612***	0.6138	−0.5991**
AUD	0.2989***	0.8981	1.1112	−1.2260***	2.3372***	0.6525	0.7773	0.3475	−0.5991**

Notes: Table 4 reports the observed slope coefficients from the seven separate homogeneous bilateral-panel regressions, together with the values in the decomposition of the observed bilateral-panel slope coefficient. The values are: $\widehat{\beta}_{:/\eta}$ is a bilateral-panel slope coefficient; $\widehat{\varphi}_{:/\eta}$ is the numéraire effect; $\widehat{\gamma}_{\eta/\mathscr{M}}$ is a multilateral-panel slope coefficient of the common single-currency (ηth) numéraire; $\widehat{\gamma}_{:/\mathscr{M}}$ is a multilateral-panel slope coefficient of all the currencies except the common single-currency (ηth) numéraire a multilateral slope coefficient of the numéraire currency; $\widehat{\beta}_{\eta/\mathscr{M}}$ is a multilateral slope coefficient of the numéraire currency; and $\widehat{\beta}_{:/\mathscr{M}}$ is a multilateral-panel slope coefficient; and δ_η is the multilateral delta for the common single-currency numéraire. The slope coefficients showing sufficient evidence to reject the null hypothesis of one are denoted by *, **, and *** at the 10% level, the 5% level, and 1% level, respectively. In contrast, the difference slope coefficients ($\widehat{\gamma}_{\eta/\mathscr{M}} - \widehat{\gamma}_{:/\mathscr{M}}$) showing sufficient evidence to reject the null hypothesis of zero are denoted by *, **, and *** at the 10% level, the 5% level, and 1% level, respectively.

common single-currency numéraire can be decomposed by:

$$\widehat{\beta}_{:/USD} = \delta_{USD}\widehat{\beta}_{USD/\mathscr{M}} + (1-\delta_{USD})\widehat{\beta}_{:/\mathscr{M}}$$
$$= (0.4899)(-2.3391) + (0.5101)(-0.5991)$$
$$= -1.4515 \tag{59}$$

where $\widehat{\beta}_{USD/\mathscr{M}} = -2.3391$ is the multilateral slope coefficient of the US dollar (USD); and $\widehat{\beta}_{:/\mathscr{M}} = -0.5991$ is the multilateral-panel slope coefficient; and $\delta_{USD} = 0.4899$ is the multilateral delta for the US dollar (USD). More generally, the multilateral-panel slope coefficient $\widehat{\beta}_{:/\mathscr{M}}$ of -0.5991 has an average weight $(1-\delta_\eta)$ of approximately 0.5 over the seven separate homogeneous bilateral-panel regressions, showing that the decomposition assigns—on average—roughly equal importance to both the multilateral-panel slope coefficient and the multilateral slope coefficients associated with the different common single-currency numéraires.

The slope coefficients $(\beta_{:/\eta})$ for the seven separate homogeneous bilateral-panel regressions are tested for aggregation bias. The tests are carried out by estimating the two multilateral-panel slope parameters from each of the seven heterogeneous bilateral-panel regressions in Eq. (50), where each regression employs a distinct single-currency numéraire, as reported in the $\widehat{\gamma}_{\eta/\mathscr{M}}$ and $\widehat{\gamma}_{:/\mathscr{M}}$ columns of Table 4. The null hypothesis posits that the two multilateral-panel slope coefficients from each heterogeneous bilateral-panel regression are equal ($\widehat{\gamma}_{\eta/\mathscr{M}} = \widehat{\gamma}_{:/\mathscr{M}}$), where $\eta = 1, \ldots, 7$. The null hypothesis is tested—for each heterogeneous bilateral-panel regression—by comparing the difference between the two multilateral-panel slope coefficients with zero, as reported in the $\widehat{\gamma}_{\eta/\mathscr{M}} - \widehat{\gamma}_{:/\mathscr{M}}$ column of Table 4. For example, when the US dollar (USD) serves as the common single-currency numéraire, the difference between the two multilateral-panel slope coefficients is $\widehat{\gamma}_{USD/\mathscr{M}} - \widehat{\gamma}_{:/\mathscr{M}} = -2.4167$, which is significantly different from zero at the 1% level. In this case, there is sufficient evidence to reject the null hypothesis, and conclude that the bilateral-panel slope coefficient suffers from aggregation bias. More generally, four of the seven bilateral-panel slope coefficients suffer from aggregation bias. The aggregation bias is significant at the 1% level for both the Australian dollar (AUD) and the US dollar (USD), at the 5% level for the euro (EUR), and at the 10% level for the Swiss franc (CHF). Thus, many of the bilateral-panel slope coefficients suffer from aggregation bias and it is inappropriate to use homogeneous bilateral-panel regressions when compared to the corresponding heterogeneous bilateral-panel regressions.

The numéraire effect in each of the seven homogeneous bilateral-panel regressions can be expressed in two ways, namely those outlined in Eq. (51) and Eq. (52). Fig. 3 presents the numéraire effects derived from the seven homogeneous bilateral-panel regressions, together with the slope-coefficient terms appearing in the decomposition of the numéraire effect in Eq. (51). The most intuitive representation appears in Eq. (51), which expresses the numéraire effect as the difference between the bilateral-panel slope coefficient and the multilateral-panel slope coefficient. This formulation highlights the role of the multilateral-panel slope coefficient, -0.5991, as an anchor from which the bilateral-panel slope coefficients diverge by an amount equal to their respective numéraire effects. For instance, the Australian dollar (AUD) exhibits the largest numéraire effect, at 0.8981, which shifts its bilateral-panel slope coefficient away from the multilateral-panel anchor. This relationship becomes clear upon rearranging Eq. (51):

$$\widehat{\beta}_{:/AUD} = \widehat{\beta}_{:/\mathscr{M}} + \widehat{\varphi}_{:/AUD}$$
$$= -0.5991 + 0.8981$$

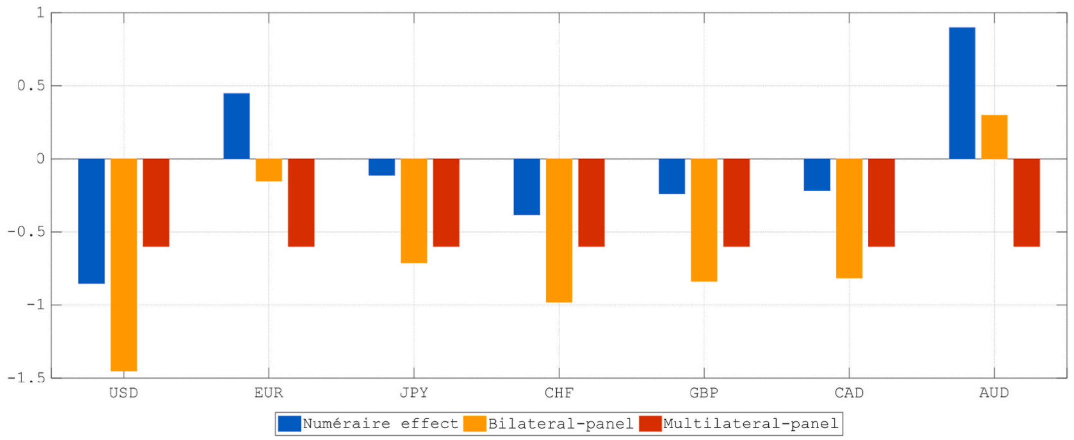

Fig. 3. Numéraire effect and slope coefficients. *Notes:* Fig. 3 displays the observed numéraire effect $\widehat{\varphi}_{:/\eta}$ (Numéraire effect); the observed slope coefficients $\widehat{\beta}_{:/\eta}$ in the seven separate bilateral-panel regressions (Bilateral-panel); and the observed slope coefficient $\widehat{\beta}_{:/\mathscr{M}}$ in the multilateral-panel regression, which is repeated seven times for comparison (Multilateral-panel).

$$= 0.2989 \tag{60}$$

In addition, the US dollar (USD) exhibits the second largest observed numéraire effect, at −0.8524, which shifts its bilateral-panel slope coefficient away from the multilateral-panel anchor. This relationship once again becomes clear upon rearranging Eq. (51):

$$\widehat{\beta}_{:/USD} = \widehat{\beta}_{:/\mathscr{M}} + \widehat{\varphi}_{:/USD}$$

$$= -0.5991 - 0.8524$$

$$= -1.4515 \tag{61}$$

Although all seven bilateral-panel slope coefficients differ significantly from the theoretical value of one, the conclusions reach depend on the choice of the common single-currency numéraire. For instance, the bilateral-panel slope coefficients reported in the literature are generally negative, yet using the Australian dollar (AUD) as the common numéraire yields a positive coefficient. With an average numéraire effect of −0.0651, the results suggest that the bilateral-panel slope coefficients generally overstate the degree to which the uncovered interest rate parity (UIP) condition is rejected.

In summary, the slope coefficients from the seven separate homogeneous bilateral-panel regressions were tested for aggregation bias by using their corresponding heterogeneous bilateral-panel regressions. The evidence showed that over half of the slope coefficients from the homogeneous bilateral-panel regressions exhibited significant aggregation bias. If aggregation bias is left uncorrected, the resulting bilateral-panel slope coefficients may either overstate or understate the degree to which the uncovered interest rate parity (UIP) condition is rejected. As a consequence, the strength of the empirical findings can vary with the particular single-currency numéraire employed.

4.3.4. Rolling regressions

A visual examination is conducted using three-year rolling homogeneous bilateral-panel regressions to capture the temporal evolution of the slope coefficients and the numéraire effect values. This approach reveals the dynamic nature of the slope coefficients and numéraire effect values across seven separate homogeneous bilateral-panel regressions, each employing a separate common single-currency numéraire. For comparison, the visual examination also includes the slope coefficients from three-year rolling multilateral-panel regressions.

The observed slope coefficients from the three-year rolling homogeneous bilateral-panel regressions, together with the observed slope coefficients from three-year rolling multilateral-panel regressions, are displayed in Fig. 4. There are many periods where the observed multilateral-panel slope coefficients are below zero, which is expected due to the observed multilateral-panel slope coefficient over the entire period being −0.5991 (see Table 3). There was significant volatility during the European Sovereign Debt Crisis, which began in 2009 and extended into the 2010s. For instance, major market movements occurred in January 2015 when the Swiss franc (CHF) was unpegged from the euro (EUR).

The observed numéraire effect values of the corresponding observed bilateral-panel slope coefficients are displayed in Fig. 5. As anticipated, the increase in variability observed in the bilateral-panel slope coefficients is also evident in the numéraire effect values.

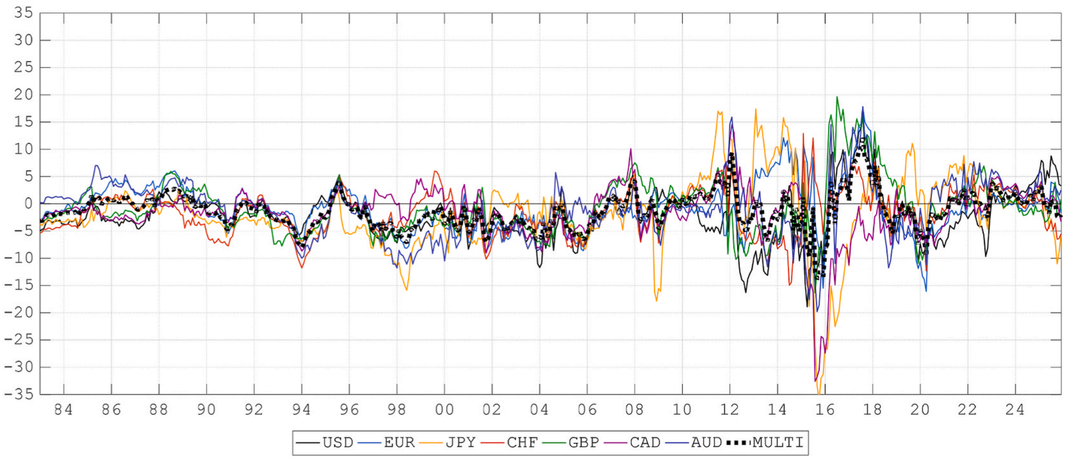

Fig. 4. Slope coefficients over time. *Notes:* Fig. 4 displays the observed slope coefficients from three-year rolling multilateral-panel regressions (*MULTI*), together with the observed slope coefficients from seven separate three-year rolling homogeneous bilateral-panel regressions, each employing a different common single-currency numéraire.

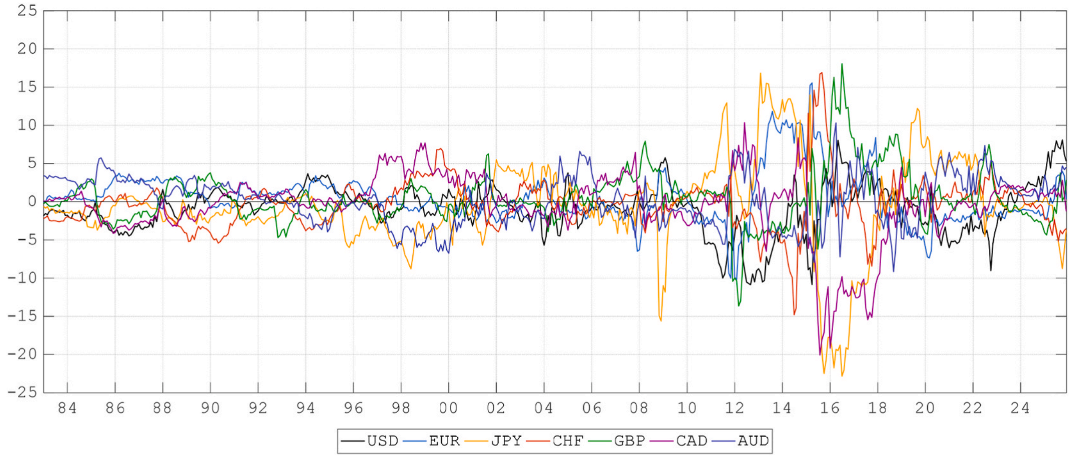

Fig. 5. Numéraire effect over time. *Notes:* Fig. 5 displays the numéraire effect of the seven separate three-year rolling homogeneous bilateral-panel regressions, each employing a different common single-currency numéraire.

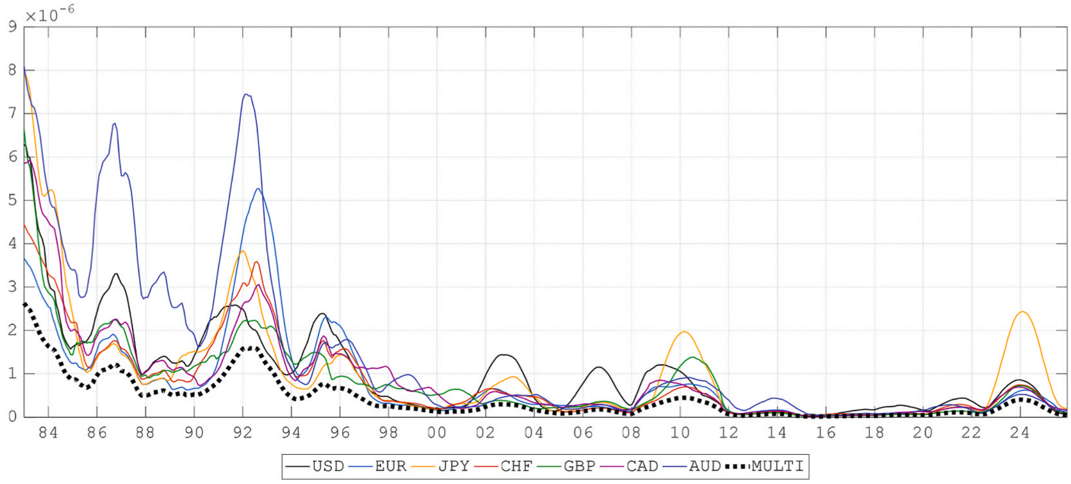

Fig. 6. Variances of the interest rate levels over time. *Notes:* Fig. 6 displays the three-year rolling monthly variances of the panel of demeaned multilateral interest rate levels (*MULTI*), together with the three-year rolling monthly variances of the seven panels of demeaned bilateral interest rate levels, each corresponding to a different common single-currency numéraire.

For both the bilateral and multilateral panel regressions, the slope coefficients are calculated as the ratio of a covariance term to a variance term. Fig. 6 displays the three-year rolling variances of the seven panels of bilateral interest rate levels—each constructed corresponding to different a common single-currency numéraire—alongside the three-year rolling variances of the panel of multilateral interest rate levels. Notably, the variances of all interest rate levels reach their lowest points over the same period in which slope coefficient variability increases. Consequently, the heightened variation in slope coefficients between 2012 and 2020 can be attributed to the reduction in interest rate level variances.

4.4. Uncovered equity parity

Similar to uncovered interest rate parity (UIP), uncovered equity parity (UEP) is used to explain how exchange rates are determined, differing in that it employs equity indices instead of interest rates. The bilateral uncovered equity parity condition refers to a negative relationship between bilateral exchange rate returns and the corresponding bilateral equity index returns (Hau and Rey, 2006; Kim, 2011). The form of a bilateral regression model for the bilateral UEP condition can be written as:

$$\Delta s_{i/\eta}(t) = \alpha_{i/\eta} + e_{i/\eta}(t)\beta_{i/\eta} + u_{i/\eta}(t) \tag{62}$$

where $i = 1,\ldots,N$; $t = 1,\ldots,T$; $i \neq \eta$; $\Delta s_{i/\eta}(t)$ is the ith/ηth bilateral exchange rate returns; $e_{i/\eta}(t) = e_i(t) - e_\eta(t)$ is the ith/ ηth bilateral equity index return; $e_i(t)$ is the equity index return for the ith currency; $e_\eta(t)$ is the equity index return for the ηth currency; $\alpha_{i/\eta}$ is a bilateral intercept term; $\beta_{i/\eta}$ is a bilateral slope coefficient; and $u_{i/\eta}(t)$ is the ith/ηth bilateral disturbance. The bilateral explanatory variables for the bilateral regressions for the bilateral UEP condition are the contemporaneous bilateral equity index returns. According to economic theory, the bilateral UEP condition implies a negative bilateral slope coefficient ($\beta_{i/\eta} < 0$). While empirical work has identified a negative UEP relationship (Hau and Rey, 2006), the support for such a relationship across all market pairs is not uniformly strong.

The multilateral uncovered equity parity condition refers to a negative relationship between multilateral exchange rate returns and the multilateral equity index returns. The form of a multilateral regression model for the multilateral UEP condition can be written as:

$$\Delta s_{i/\mathscr{M}}(t) = \alpha_{i/\mathscr{M}} + e_{i/\mathscr{M}}(t)\beta_{i/\mathscr{M}} + u_{i/\mathscr{M}}(t) \tag{63}$$

where $i = 1,\ldots,N$; $t = 1,\ldots,T$; $\Delta s_{i/\mathscr{M}}(t)$ is the multilateral exchange rate returns of the ith currency; $e_{i/\mathscr{M}}(t)$ is the multilateral equity index return for the ith currency; $\alpha_{i/\mathscr{M}}$ is an multilateral intercept term; $\beta_{i/\mathscr{M}}$ is a multilateral slope coefficient; and $u_{i/\mathscr{M}}(t)$ is the multilateral disturbance. The multilateral explanatory variables for the multilateral UEP condition are the contemporaneous multilateral equity index returns. According to economic theory, the multilateral UEP condition implies a negative multilateral slope coefficient ($\beta_{i/\mathscr{M}} < 0$).

4.4.1. *Equity index returns*

The cumulative log returns of the seven multilateral equity indexes are displayed in Fig. 7. According to the no-arbitrage condition in Eq. (21), the system of the multilateral equity indexes, when expressed in log terms, are centred around zero. The Japanese multilateral equity index (JPY) depreciates from 1990 until late 2012.

The descriptive statistics for the log returns of the seven multilateral equity indexes and the six bilateral equity indexes expressed in terms of the US equity index are reported in Table 5. The decomposition of the log return of a bilateral equity index (explanatory variable) into the log returns of two multilateral equity indexes (explanatory variables) using Eq. (20) is equally applicable to determining the average annualised return of that bilateral equity index:

$$\overline{e}_{i/USD} = \Delta\overline{e}_{i/\mathscr{M}} - \Delta\overline{e}_{USD/\mathscr{M}} \tag{64}$$

where $i = 1,\ldots,7$; $i \neq USD$; $t = 1,\ldots,T$; $\overline{e}_{i/USD}$ is from Eq. (4); $\overline{e}_{i/\mathscr{M}}$ and $\overline{e}_{USD/\mathscr{M}}$ are from Eq. (36). As a result, the ranking of the average annualised returns for the bilateral equity indexes matches that of the multilateral equity indexes. For example, the highest average annualised return for the multilateral equity index is 2.44% for the Swiss franc (CHF) and for the bilateral equity index is 2.66% for the Swiss franc/US dollar (CHF/USD).

The highest average annualised volatility is 13.48% for the Japanese yen (JPY) multilateral equity index and 17.04% for the Japanese yen/US dollar (JPY/USD) bilateral equity index. For the multilateral equity indexes, there is significant positive skewness of 0.428 for the US dollar (USD) and 0.399 for the British pound (GBP) and significant negative skewness of −0.652 for the Australian dollar (AUD) and the −0.507 for the euro (EUR). Similar observed skewness values occurs for bilateral equity indexes, except for the Swiss franc/US dollar (CHF/USD) which has significant negative skewness and the British pound/US dollar (GBP/USD) which has insignificant skewness. The observed kurtosis is significant for all multilateral equity indexes and all equity indexes. The significant results of the Augmented Dickey-Fuller (ADF) unit root test suggest that multilateral equity index movements and bilateral equity index movements are stationary.

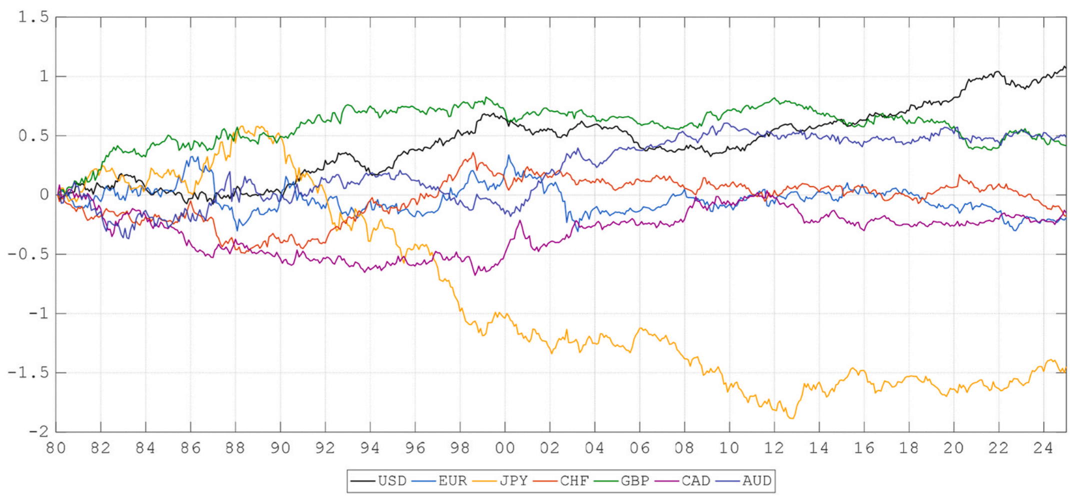

Fig. 7. Multilateral equity indexes. *Notes:* Fig. 7 displays the cumulative log returns of the seven multilateral equity indexes.

Table 5
Descriptive statistics.

Panel A: Multilateral equity indexes					
	Average	Volatility	Skewness	Kurtosis	ADF
USD	2.37%	7.50%	0.428**	4.36**	−26.3**
EUR	−0.43%	11.63%	−0.507**	6.26**	−23.6**
JPY	−3.22%	13.48%	0.107	4.17**	−22.9**
CHF	−0.40%	8.57%	0.063	3.95**	−22.4**
GBP	0.92%	8.25%	0.399**	4.15**	−25.5**
CAD	−0.34%	8.87%	0.029	3.69**	−24.2**
AUD	1.08%	11.01%	−0.652**	10.52**	−26.6**

Panel B: Bilateral equity indexes					
	Average	Volatility	Skewness	Kurtosis	ADF
EUR/USD	−2.80%	14.85%	−0.664**	6.29**	−24.1**
JPY/USD	−5.59%	17.04%	−0.121	4.10**	−23.4**
CHF/USD	−2.77%	12.09%	−0.377**	5.33**	−24.4**
GBP/USD	−1.45%	11.50%	−0.004	3.86**	−26.9**
CAD/USD	−2.71%	10.37%	−0.014	4.00**	−25.0**
AUD/USD	−1.29%	14.57%	−0.392**	7.66**	−27.4**

Notes: Table 5 reports the descriptive statistics for the log returns of both the seven multilateral equity indexes (Panel A) and the six bilateral equity indexes (Panel B). The columns are the annualised average return (Average); the annualised standard deviation (Volatility); the observed skewness (Skewness); the observed kurtosis (Kurtosis); and the Augmented Dickey-Fuller unit root test (ADF) displaying the test statistic. Significance at the 10% level, the 5% level, and 1% level is denoted by *, **, and ***, respectively.

Table 6
Weighted average of the slope coefficient from a multilateral-panel regression.

i	$\widehat{\beta}_{i/\mathscr{M}}$	$var(\boldsymbol{x}_{i/\mathscr{M}})$	$w_{i/\mathscr{M}}$	$w_{i/\mathscr{M}}\widehat{\beta}_{i/\mathscr{M}}$
Panel	**−0.0672*** **	**0.005967**	**1.0000**	**−0.0672**
USD	−0.1736***	0.000470	0.0788	−0.0137
EUR	−0.0183	0.001129	0.1892	−0.0035
JPY	−0.0714***	0.001517	0.2543	−0.0182
CHF	−0.1217***	0.000613	0.1027	−0.0125
GBP	−0.2225***	0.000569	0.0953	−0.0212
CAD	−0.0231	0.000657	0.1101	−0.0025
AUD	0.0256	0.001012	0.1696	0.0043

Notes: Table 6 reports the observed slope coefficient from the multilateral-panel regression, together with the variables used in the weighted average in Eq. (41). The values are: $\widehat{\beta}_{i/\mathscr{M}}$ is the observed slope coefficients from individual multilateral (non-panel) regressions; $var(\boldsymbol{x}_{i/\mathscr{M}})$ is the monthly variance of the multilateral equity index returns of the ith currency; and $w_{i/\mathscr{M}}$ is the multilateral weight for the ith currency. The multilateral slope coefficients showing sufficient evidence to reject the null hypothesis of zero are denoted by *, **, and *** at the 10% level, the 5% level, and 1% level, respectively.

4.4.2. Multilateral-panel regressions

The observed slope coefficient from a multilateral-panel regression of the seven multilateral exchange rate returns against the corresponding seven multilateral equity index returns is reported in Table 6, together with the constituents of the weighted average in Eq. (41). The multilateral-panel slope coefficient quantifies both the strength and direction of the relationship between the panel of seven multilateral exchange rate returns and the panel of seven multilateral equity index returns. Economically, the multilateral-panel slope coefficient measures how much each multilateral exchange rate is expected to change when its corresponding multilateral equity index changes by one unit.

The observed multilateral-panel slope coefficient of −0.0672 is significantly different from the theoretical value of zero at the 1% level, implying that the UEP condition holds for hold for the panel of seven multilateral exchange rate returns. It was shown in Eq. (41) that the observed multilateral-panel slope coefficient is a weighted average of the multilateral slope coefficients obtained from separate multilateral (non-panel) regressions:

$$\widehat{\beta}_{:/\mathscr{M}} = \sum_{i=1}^{7} w_{i/\mathscr{M}}\widehat{\beta}_{i/\mathscr{M}} = -0.0672 \tag{65}$$

This breakdown makes clear how each individual currency contributes to the overall estimate of the multilateral-panel slope coefficient. The multilateral slope coefficients themselves differ across the separate multilateral (non-panel) regressions. For example, the Australian dollar (AUD), the euro (EUR), and the Canadian dollar (CAD) all display multilateral slope coefficients—0.0256, −0.0183, and −0.0231, respectively—that are not statistically different from the theoretical value of zero, suggesting a failure of the multilateral UEP condition for these currencies. In contrast, the other four currencies exhibit negative multilateral slope coefficients that differ significantly from the theoretical value of zero, suggesting that the multilateral UEP condition holds for those currencies. The British pound (GBP) multilateral exchange rate accounts for the largest share of the weighted average, with a weighted slope coefficient ($w_{GBP/\mathscr{M}}\widehat{\beta}_{GBP/\mathscr{M}}$) of −0.0212. The associated multilateral weight is:

$$w_{GBP/\mathscr{M}} = \frac{var(\widetilde{\boldsymbol{x}}_{GBP/\mathscr{M}})}{\sum_{j=1}^{7} var(\widetilde{\boldsymbol{x}}_{j/\mathscr{M}})} = \frac{0.000569}{0.005967} = 0.0953 \tag{66}$$

where $\sum_{j=1}^{7} var(\widetilde{\boldsymbol{x}}_{j/\mathscr{M}}) = 0.005967$.

The significant multilateral-panel slope coefficient of −0.0672 shows that the panel of multilateral exchange rate returns move against the corresponding panel of multilateral equity index returns, consistent with the economic predictions of the multilateral UEP condition. The observed multilateral-panel slope coefficient reflects a weighted average of the seven multilateral (non-panel) slope coefficients. Even though some of these individual multilateral slope coefficients do not differ significantly from the theoretical value of zero, the aggregate estimate of the multilateral-panel slope coefficient is statistically different from zero.

4.4.3. Bilateral-panel regressions

The slope coefficients from seven separate homogeneous bilateral-panel regression models are estimated, each using a different single-currency numéraire, as reported in the $\widehat{\beta}_{:/\eta}$ column of Table 7. According to the bilateral uncovered equity parity (UEP) condition, the bilateral-panel slope coefficients should be negative. Consistent with theoretical expectations, slope coefficients from all seven bilateral-panel regressions are negative, with six deviating significantly from zero. The insignificant bilateral-panel slope coefficient of −0.0168 occurs when the Australian dollar (USD) serves as the common single-currency numéraire. In contrast, the largest negative bilateral-panel slope coefficient of −0.1293 occurs when the British pound (GBP) serves as the common single-currency numéraire for the bilateral-panel regression.

Each bilateral-panel slope coefficient can be decomposed into two multilateral terms using in Eq. (47), namely, the slope coefficient from a multilateral-panel regression ($\widehat{\beta}_{\eta/\mathscr{M}}$), and the slope coefficient from a multilateral (non-panel) regression ($\widehat{\beta}_{:/\mathscr{M}}$)—based on a common single-currency numéraire (see Table 7). For example, bilateral-panel slope coefficient when the British pound (GBP) serves as the common single-currency numéraire can be decomposed by:

$$\begin{aligned}\widehat{\beta}_{:/GBP} &= \delta_{GBP}\widehat{\beta}_{GBP/\mathscr{M}} + (1-\delta_{GBP})\widehat{\beta}_{:/\mathscr{M}} \\ &= (0.4001)(-0.2225) + (0.5999)(-0.0672) \\ &= -0.1293\end{aligned} \tag{67}$$

where $\widehat{\beta}_{GBP/\mathscr{M}} = -0.2225$ is the multilateral slope coefficient of the British pound (GBP); and $\widehat{\beta}_{:/\mathscr{M}} = -0.0672$ is the multilateral-panel slope coefficient; and $\delta_{GBP} = 0.4001$ is the multilateral delta for the British pound (GBP). More generally, the multilateral-panel slope coefficient $\widehat{\beta}_{:/\mathscr{M}}$ of −0.0672 has an average weight $(1-\delta_\eta)$ of approximately 0.5 over the seven separate homogeneous bilateral-panel regression models, showing that the decomposition assigns—on average—roughly equal importance to both the multilateral-panel slope coefficient and the multilateral slope coefficients associated with the different common single-currency numéraires.

The slope coefficients ($\beta_{:/\eta}$) for the seven separate homogeneous bilateral-panel regressions are tested for aggregation bias. The tests are carried out by estimating the two multilateral-panel slope parameters from each of the seven heterogeneous bilateral-panel re-

Table 7
Numéraire effect and slope coefficients.

η	$\widehat{\beta}_{:/\eta}$	$\widehat{\varphi}_{:/\eta}$	$\widehat{\gamma}_{\eta/\mathscr{M}}$	$\widehat{\gamma}_{:/\mathscr{M}}$	$\widehat{\gamma}_{\eta/\mathscr{M}} - \widehat{\gamma}_{:/\mathscr{M}}$	δ_η	$\widehat{\beta}_{\eta/\mathscr{M}}$	$1-\delta_\eta$	$\widehat{\beta}_{:/\mathscr{M}}$
USD	−0.1050***	−0.0378	−0.1931***	−0.0564***	−0.1367***	0.3555	−0.1736***	0.6445	−0.0672***
EUR	−0.0394***	0.0278	−0.0079	−0.0811***	0.0732***	0.5698	−0.0183	0.4302	−0.0672***
JPY	−0.0699***	−0.0027	−0.0724***	−0.0654***	−0.0069	0.6403	−0.0714***	0.3597	−0.0672***
CHF	−0.0900***	−0.0228	−0.1320***	−0.0598***	−0.0722**	0.4183	−0.1217***	0.5817	−0.0672***
GBP	−0.1293***	−0.0621	−0.2516***	−0.0478***	−0.2038***	0.4001	−0.2225***	0.5999	−0.0672***
CAD	−0.0480***	0.0192	−0.0147	−0.0737***	0.0590**	0.4353	−0.0231	0.5647	−0.0672***
AUD	−0.0168	0.0504	0.0449**	−0.0901***	0.1350***	0.5427	0.0256	0.4573	−0.0672***

Notes: Table 7 reports the observed slope coefficients from the seven separate homogeneous bilateral-panel regressions, together with the values in the decomposition of the observed bilateral-panel slope coefficient. The values are: $\widehat{\beta}_{:/\eta}$ is a bilateral-panel slope coefficient; $\widehat{\varphi}_{:/\eta}$ is the numéraire effect; $\widehat{\gamma}_{\eta/\mathscr{M}}$ is a multilateral slope coefficient of the common single-currency (ηth) numéraire; $\widehat{\gamma}_{:/\mathscr{M}}$ is a multilateral slope coefficient of all the currencies except the common single-currency (ηth) numéraire; $\widehat{\beta}_{\eta/\mathscr{M}}$ is a multilateral slope coefficient of the numéraire currency; and $\widehat{\beta}_{:/\mathscr{M}}$ is a multilateral-panel slope coefficient; and δ_η is the multilateral delta for the common single-currency numéraire. Slope coefficients showing sufficient evidence to reject the null hypothesis of zero are denoted by *, **, and *** at the 10% level, the 5% level, and 1% level, respectively.

gressions in Eq. (50), where each regression employs a distinct single-currency numéraire, as reported in the $\widehat{\gamma}_{\eta/\mathscr{M}}$ and $\widehat{\gamma}_{:/\mathscr{M}}$ columns of Table 7. The null hypothesis posits that the two multilateral-panel slope coefficients from each heterogeneous bilateral-panel regression are equal ($\widehat{\gamma}_{\eta/\mathscr{M}} = \widehat{\gamma}_{:/\mathscr{M}}$), where $\eta = 1, \ldots, 7$. The null hypothesis is tested—for each heterogeneous bilateral-panel regression—by comparing the difference between the two multilateral-panel slope coefficients with zero, as reported in the $\widehat{\gamma}_{\eta/\mathscr{M}} - \widehat{\gamma}_{:/\mathscr{M}}$ column of Table 7. For example, when the British pound (GBP) serves as the common single-currency numéraire, the difference between the two multilateral-panel slope coefficients is $\gamma_{GBP/\mathscr{M}} - \gamma_{:/\mathscr{M}} = -0.2038$, which is significantly different from zero at the 1% level. In this case, there is sufficient evidence to reject the null hypothesis, and conclude that the bilateral-panel slope coefficient suffers from aggregation bias. More generally, six out of the seven bilateral-panel slope coefficients suffer from aggregation bias. The aggregation bias is significant at the 1% level for the US dollar (USD), the Australian dollar (AUD), the euro (EUR), and the British pound (GBP), and at the 5% level for the Swiss franc (CHF) and the Canadian dollar (CAD). Thus, most of the bilateral-panel slope coefficients suffer from aggregation bias and it is inappropriate to use homogeneous bilateral-panel regressions when compared to the corresponding heterogeneous bilateral-panel regressions.

The numéraire effect in each of the seven homogeneous bilateral-panel regressions can be expressed in two ways, namely those outlined in Eq. (51) and Eq. (52). Fig. 8 presents the numéraire effects derived from the seven homogeneous bilateral-panel regressions, together with the slope-coefficient terms appearing in the decomposition of the numéraire effect in Eq. (51). The most intuitive representation appears in Eq. (51), which expresses the numéraire effect as the difference between the bilateral-panel slope coefficient and the multilateral-panel slope coefficient. This formulation highlights the role of the multilateral-panel slope coefficient, −0.0672, as an anchor from which the bilateral-panel slope coefficients diverge by an amount equal to their respective numéraire effects. For instance, the British pound (GBP) exhibits the largest numéraire effect, at −0.0621, which shifts its bilateral-panel slope coefficient away from the multilateral anchor. This relationship becomes clear upon rearranging Eq. (51):

$$\begin{aligned}\widehat{\beta}_{:/GBP} &= \widehat{\beta}_{:/\mathscr{M}} + \widehat{\varphi}_{:/GBP} \\ &= -0.0672 - 0.0621 \\ &= -0.1293 \end{aligned} \tag{68}$$

In addition, the Australian dollar (AUD) shows the second-largest numéraire effect, at 0.0504, which likewise shifts its bilateral-panel slope coefficient away from the multilateral anchor. This relationship once again becomes clear upon rearranging Eq. (51):

$$\begin{aligned}\widehat{\beta}_{:/AUD} &= \widehat{\beta}_{:/\mathscr{M}} + \widehat{\varphi}_{:/AUD} \\ &= -0.0672 + 0.0504 \\ &= -0.0168 \end{aligned} \tag{69}$$

Although seven out of six bilateral-panel slope coefficients differ significantly from the theoretical value of zero, the conclusions reach depend on the choice of the common single-currency numéraire. For instance, when the Australian dollar (AUD) serves as the common single-currency numéraire, the observed bilateral-panel slope coefficient not significantly from the theoretical value of zero, suggesting that the uncovered equity parity (UEP) condition does not hold. With an average numéraire effect of −0.0040, the results suggest that the bilateral-panel slope coefficients generally overstate the degree to which the uncovered equity parity (UEP) condition is accepted.

In summary, the slope coefficients from the seven separate homogeneous bilateral-panel regressions were tested for aggregation bias by using their corresponding heterogeneous bilateral-panel regressions. The evidence showed that most of the slope coefficients from the homogeneous bilateral-panel regressions exhibited significant aggregation bias. If aggregation bias is left uncorrected, the resulting bilateral-panel slope coefficients may either overstate or understate the degree to which the uncovered equity parity (UEP) condition is accepted. As a consequence, the strength of the empirical findings can vary with the particular single-currency numéraire employed.

4.4.4. *Rolling regressions*

A visual examination is conducted using three-year rolling homogeneous bilateral-panel regressions to capture the temporal evolution of the slope coefficients and the numéraire effect values. This approach reveals the dynamic nature of the slope coefficients and numéraire effect values across seven separate homogeneous bilateral-panel regressions, each employing a separate common single-currency numéraire. For comparison, the visual examination also includes the slope coefficients from three-year rolling multilateral-panel regressions.

The observed slope coefficients from three-year rolling homogeneous bilateral-panel regressions, together with the observed slope coefficients from three-year rolling multilateral-panel regressions, are displayed in Fig. 9. Overall, the data suggests two separate periods with opposite signs: a positive phase from the beginning of the data sample until 1993, followed by a negative phase thereafter. During the negative phase, the multilateral-panel slope coefficient experiences a sharp decline amid the 2008 financial crisis. This drop appears to persist only for the three years covered by the rolling regressions, suggesting the presence of outliers related to the crisis. A similar pattern emerges during the COVID-19 period; however, instead of a decline, there is an increase.

The observed numéraire effect values of the corresponding observed bilateral-panel slope coefficients are displayed in Fig. 10. As

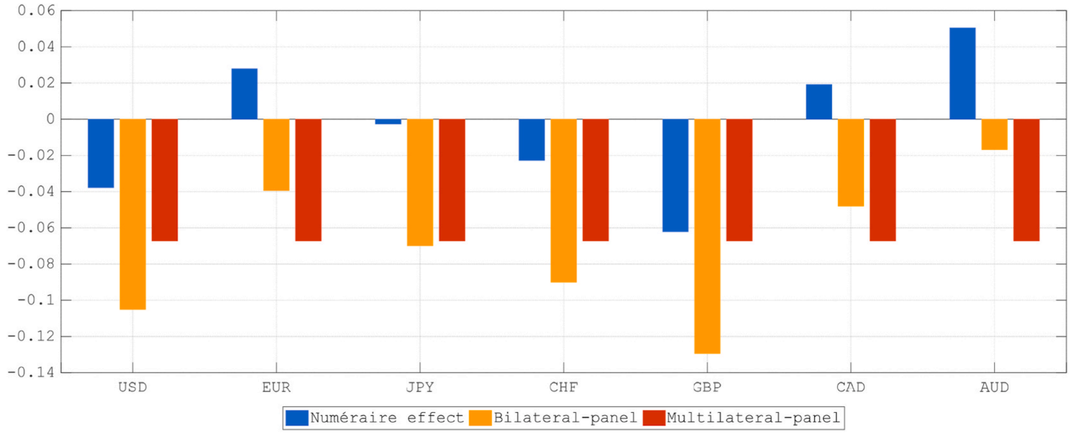

Fig. 8. Numéraire effect and slope coefficients. *Notes:* Fig. 8 displays the observed numéraire effect $\widehat{\varphi}_{:/\eta}$ (Numéraire Effect); the observed slope coefficients $\widehat{\beta}_{:/\eta}$ in the seven separate bilateral-panel regressions (Bilateral-panel); and the observed slope coefficient $\widehat{\beta}_{:/\mathscr{M}}$ in a multilateral-panel regression, which is repeated seven times for comparison (Multilateral-panel).

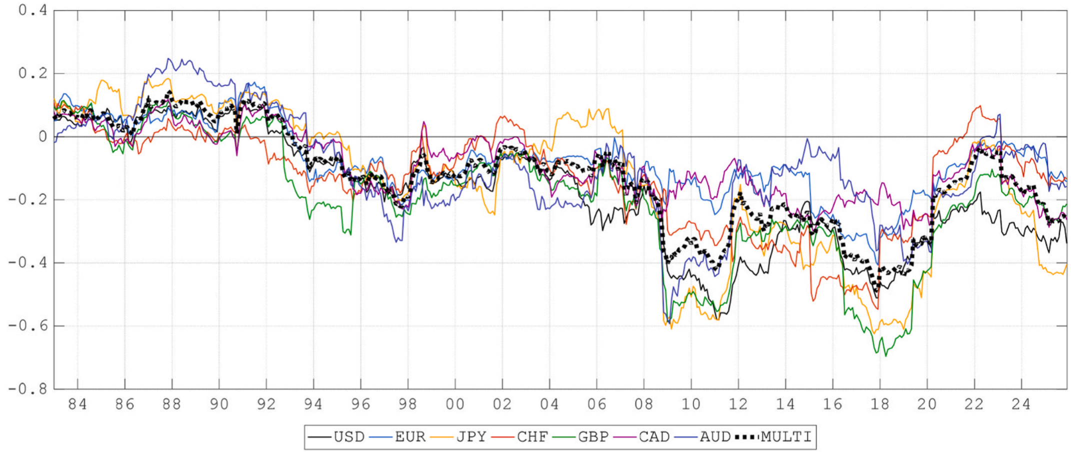

Fig. 9. Slope coefficients over time. *Notes:* Fig. 9 displays the observed slope coefficients from three-year rolling multilateral-panel regressions (*MULTI*), together with the observed slope coefficients from seven separate three-year rolling homogeneous bilateral-panel regressions, each employing a different common single-currency numéraire.

anticipated, the increase in variability observed in the bilateral-panel slope coefficients is also evident in the numéraire effect values. For both the bilateral and multilateral panel regressions, the slope coefficients are calculated as the ratio of a covariance term to a variance term. Fig. 11 displays the three-year rolling variances of the seven panels of bilateral equity index returns—each constructed corresponding to different a common single-currency numéraire—alongside the three-year rolling variances of the panel of multilateral equity index returns. Notably, the variances of all equity index returns reach their lowest levels over the same period in which slope coefficient variability increases. Consequently, the heightened variation in slope coefficients post-2008 financial crisis can be attributed to the reduction in the variances of the equity index returns.

5. Conclusion

The numéraire effect arises from the adoption of a common single-currency numéraire for bilateral exchange rates in bilateral-panel regression models. This paper measured and examined the statistical significance of the numéraire effect. The numéraire effect was measured as the difference between the slope coefficient from a bilateral-panel regression—based on a common single-currency numéraire—and the slope coefficient from a multilateral-panel regression. The statistical significance of the numéraire effect was examined by testing whether the slope coefficient from a homogeneous bilateral-panel regression suffers from aggregation bias. In cases where such bias is present, it is recommended that the analysis be performed at the disaggregate level, since the slope coefficient from a multilateral-panel regression is not affected by aggregation bias.

An analysis of two applications—uncovered interest rate parity (UIP), and uncovered equity parity (UEP)—revealed that a

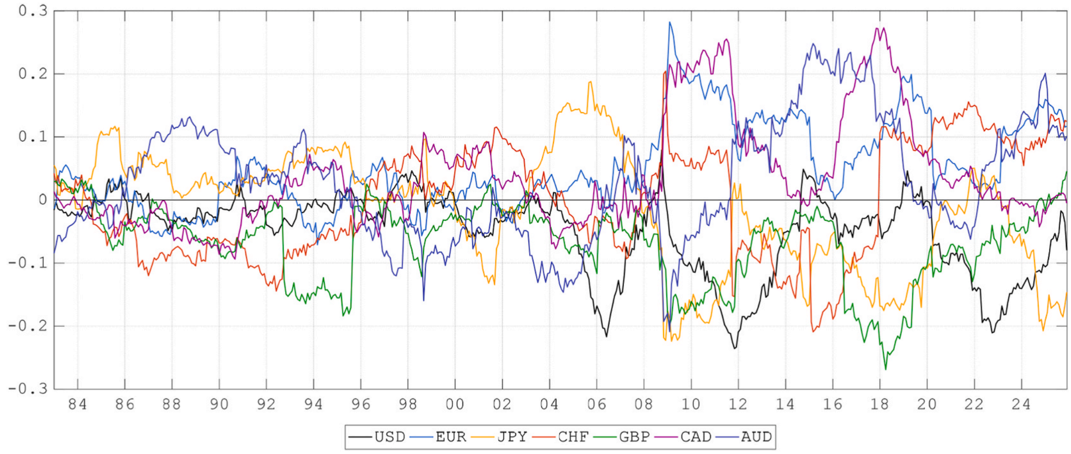

Fig. 10. Numéraire effect over time. *Notes:* Fig. 10 displays the observed numéraire effect of the seven separate three-year rolling homogeneous bilateral-panel regressions, each employing a different common single-currency numéraire.

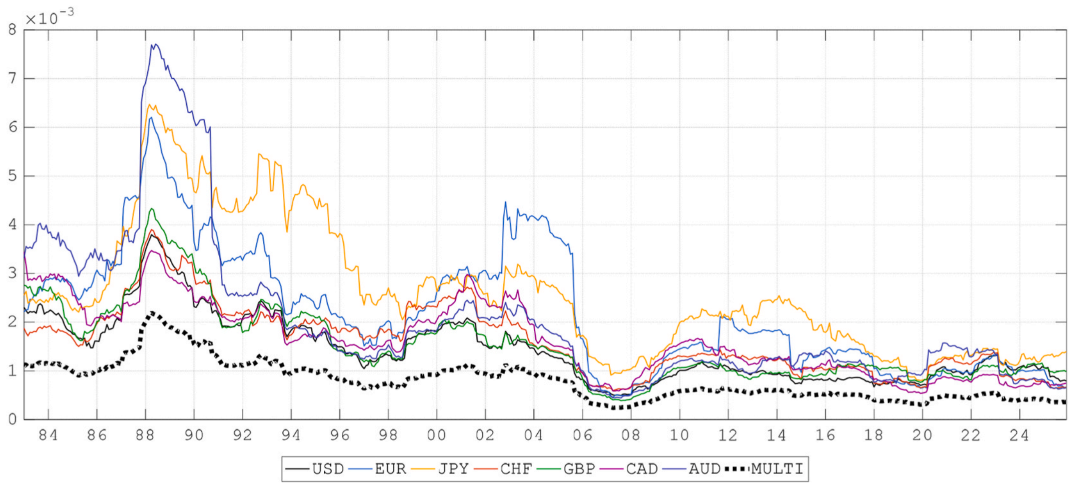

Fig. 11. Variances of the equity index returns over time. *Notes:* Fig. 11 displays the three-year rolling monthly variances of the panel of demeaned multilateral equity index returns (*MULTI*), together with the three-year rolling monthly variances of the seven panels of demeaned bilateral equity index returns, each corresponding to a different common single-currency numéraire.

substantial number of the observed slope coefficients from separate homogeneous bilateral-panel regressions exhibited significant aggregation bias. If aggregation bias is left uncorrected, the resulting bilateral-panel slope coefficients may either overstate or understate the degree to which the underlying economic conditions are accepted or rejected. For instance, the analysis overstated both the rejection of the UIP condition and the acceptance of the UEP condition. As a consequence, the strength of the empirical findings can vary with the particular single-currency numéraire employed.

Selecting different common single-currency numéraires can lead to varying and often inconsistent outcomes. Rather than selecting a common single-currency numéraire for the bilateral exchange rates in a homogeneous bilateral-panel regression, this study advocates using a multilateral-panel regression with multilateral exchange rates expressed in terms a common equally weighted multi-currency numéraire. Under this approach, the estimated slope coefficient becomes invariant to the choice of common single-currency numéraire and free from aggregation bias.

Disclosure statement

The author reports that there are no competing interests to declare.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing

interests: Michael Kunkler reports was provided by University of Reading. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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