Quantifying observation error correlations in remotely sensed data

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Quantifying observation error correlations in remotely sensed data

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Talk structure

★ Introduction and theory
  - Motivation
  - Variational data assimilation
  - Observation error covariance matrices

★ Quantifying observation error correlations
  - Desrozières’ method of statistical approximation
  - Application to IASI data
  - Results

★ Modelling observation error correlation structure
  - Approximate structures for $R$
What are observation error correlations?

Every observation $y$ of a atmospheric variable $x$ has an associated error $\epsilon$: $y = Hx + \epsilon$

→ observation error correlations are present when components of the error vector $\epsilon$ are related
→ measurement errors are attributed to 3 sources: instrument noise, forward model error and representativity error
Introduction and Theory

Quantifying observation error correlations

Modelling error correlation structure

Error sources

- **Instrument noise**
  - temperature converted neΔt value
  - regular calibrations ensure noise is uncorrelated between channels

- **Forward model error**
  - errors in discretisation of radiative transfer equation
  - errors in mis-representation of gaseous contributors
  - errors from undetected cloud

- **Representativeness error**
  - contrasting model and observation resolutions
  - observations resolve spatial scales or features that the model cannot
  - contributes to cross channel observation error correlations

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Why are correlations important?

**Problems**

- **-ve** magnitude and behaviour relatively unknown
- **-ve** reduce weighting of observations in analysis
- **-ve** for an observation vector of size $10^6$, difficult to store and invert observation error matrix if correlations are included

**Benefits**

- **+ve** increase accuracy of gradients of the observed field represented in the analysis
- **+ve** works with the prior error covariance to specify how observation features should be smoothed
- **+ve** more information available from observations
Observation error correlation and Shannon Information Content

Figure: The SIC under different approximations of $R$

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Variational data assimilation

Assimilation objective

Model forecast + Observation data $\rightarrow$ State of atmosphere

Assimilation method Minimise a cost function which measures distance of a solution state $x$ from the observations $y^o \in \mathbb{R}^m$ and the background field $x^b \in \mathbb{R}^n$

Cost Function

$$J(x) = \frac{1}{2}(x - x^b)^T B^{-1} (x - x^b) + \frac{1}{2}(y^o - H(x))^T R^{-1} (y^o - H(x))$$

where $B$ and $R$ are the background and observation error covariance matrices respectively
An error covariance matrix structure

The observation error covariance matrix takes the form:

\[ R = D^{1/2} C D^{1/2} \]

where \( C \) is the error correlation matrix

\[
C = \begin{pmatrix}
1 & \rho_{12} & \ldots & \rho_{1m} \\
\rho_{12} & 1 & \ldots & \rho_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{1m} & \rho_{2m} & \ldots & 1
\end{pmatrix}
\]

and \( D \) is the error variance matrix

\[
D = \begin{pmatrix}
\sigma^2_1 & 0 & \ldots & 0 \\
0 & \sigma^2_2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \sigma^2_m
\end{pmatrix}
\]
A desirable error covariance matrix

Main issue in observation error correlation modelling

★ need to calculate matrix-vector product $R^{-1}(y^o - Hx)$ every time we calculate cost function $J$
★ relatively easy if $R = D \equiv m$ scalar multiplications
★ \textbf{BUT} $y \in \mathbb{R}^{10^6}$ and so $R \in \mathbb{R}^{10^6 \times 10^6}$ which, if dense, is impossible to store and invert

The perfect partner: what do we want from $R \neq D$?

❤ structure resulting in an $R^{-1}$ suitable for storage / can be used cheaply in a matrix-vector product
❤ representative of the true error correlation structure
❤ greater access to information from the observations and improved analysis accuracy

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Quantifying cross-channel correlations: a study

Objective

Generate the true observation error correlation structure for a sample set of remotely sensed data typical of NWP

Data type

- IASI (infrared atmospheric sounding interferometer) observations
- measurements of the infrared radiation emitted by the earth’s surface and atmosphere at different wavelengths

Method

We use a post analysis diagnostic derived from variational data assimilation theory [Desroziers, 2005]
Desroziers’ method of statistical approximation

Recall the background state, $x_b$, and observation vector, $y$, are approximations to the true state of the atmosphere, $x_t$, where $\epsilon^o$ and $\epsilon^b$ are the observation and background errors respectively.

The best linear unbiased estimate of the true state, $x_a$, is given by

\[
\begin{align*}
y &= Hx_t + \epsilon^o \\
x_t &= x_b + \epsilon^b
\end{align*}
\]

\[
x_a = x_b + K(y - Hx_b) = x_b + K\epsilon^o
\]

\[
K = BH^T(HBH^T + R)^{-1}
\]

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Desroziers’ method of statistical approximation

### Innovation vector

\[ d^o_b = y - Hx_b = Hx_t + \epsilon^o - Hx_b \]
\[ \approx \epsilon^o + H\epsilon^b \]

### Analysis innovation vector

\[ d^o_a = y - Hx_a = y - H(x_b + Kd^o_b) \]
\[ \approx (I - HK)d^o_b \]
\[ \approx R(HBH^T + R)^{-1}d^o_b \]
Desroziers’ method of statistical approximation

Taking the expectation of the cross product of $d_a^o$ and $d_b^o$, and assuming

$$\mathbb{E}[\epsilon^o(\epsilon^b)^T] = \mathbb{E}[\epsilon^b(\epsilon^o)^T] = 0,$$

we find a statistical approximation for the observation error covariances

$$\mathbb{E} \left[ d_a^o(d_b^o)^T \right] \approx \mathbb{E} \left[ R(HBH^T + R)^{-1} d_b^o(d_b^o)^T \right]$$

$$\approx R(HBH^T + R)^{-1} \mathbb{E} \left[ (\epsilon^o + H\epsilon^b)(\epsilon^o + H\epsilon^b)^T \right]$$

$$\approx R(HBH^T + R)^{-1}(R + HBH^T)$$

$$\approx R$$
Application to IASI data

Figure: Assimilation process
Methodology

- aim to identify correlations between 139 IASI channels used in 4D-Var assimilation
- only use clear sky, sea surface observations from night and day
- $\mathbf{R}$ matrix is calculated using $\mathbb{E}[d_a^o(d_b^o)^T]$
Observation error correlation matrix

Figure: Error correlation matrix for 139 channels used in Var
**Introduction and Theory**

**Quantifying observation error correlations**

**Modelling error correlation structure**

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**Observation error correlation matrix**

*Figure:* Error correlation matrix for (a) temperature sounding channels; (b) water vapour channels
Operational and diagnosed error variances

**Figure:** Operational error variances (black line), diagnosed error variances (red line), and first off-diagonal error covariance (green line).
Diagnosed error variances: comparison with Hollingsworth-Lonnberg (H-L) method

Figure: Diagnosed error variances (red line), H-L diagnosed error variances for 84.8km (blue and black line) and 61.9km (green line) separation. Plot provided by James Cameron, UK Met Office.
Quantifying cross-channel correlations: a summary

- Strong off-diagonal correlations are present between channels with similar spectral properties
- Channels highly sensitive to water vapour have large observation error variances and covariances
- The observation error variance is being overestimated in current assimilation algorithms
- Diagnosed error variances are comparable with those using the H-L diagnostic
- Non-symmetric matrices! → future work
Modelling error correlation structure

What next?

Investigate how to approximate the true error correlation structure within operational assimilation methods...

Current approaches

- a diagonal matrix approximation
- diagonal variance inflation

Alternative approaches

- a Markov error covariance approximation
- a truncated eigendecomposition approximation [Fisher, 2005]
- a Toeplitz to circulant matrix approximation [Healy, 2005]
A Markov error covariance approximation

Consider a Markov covariance matrix of the form

\[ R_{ij} = \sigma^2 \rho^{|i-j|}, \quad \rho = \exp\left(-\frac{\delta z}{h}\right) \]

where \( \sigma^2 \) is the error variance, \( \delta z \) is the level spacing, and \( h \) is the length scale.

This is equivalent to a correlation matrix of the form

\[ C = \begin{pmatrix}
1 & \rho & \rho^2 & \ldots & \rho^n \\
\rho & 1 & \rho & \ldots & \rho^{n-1} \\
\rho^2 & \rho & 1 & \ldots & \rho^{n-2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\rho^n & \cdots & \rho^2 & \rho & 1
\end{pmatrix} \]
The benefit of this choice is that $C$ has a tri-diagonal inverse

$$C^{-1} = \frac{1}{1 - \rho^2} \begin{pmatrix} 1 & -\rho & 0 & \ldots & 0 \\ -\rho & 1 + \rho^2 & -\rho & \ldots & 0 \\ 0 & -\rho & 1 + \rho^2 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & -\rho & 1 + \rho^2 & -\rho \\ 0 & \ldots & 0 & -\rho & 1 \end{pmatrix}$$

and therefore as does $R$: $R^{-1} = \frac{1}{\sigma^2} I \times C^{-1}$

No need to store and invert $R$!
An eigendecomposition approximation

Describe $C$ by a truncated eigendecomposition using its leading eigenpairs

$$\tilde{R} = D^{1/2} \tilde{C} D^{1/2} = D^{1/2} \left( \alpha I + \sum_{k=1}^{K} (\lambda_k - \alpha) v_k v_k^T \right) D^{1/2}$$

where $(\lambda_k, v_k)$ is an eigenvalue, eigenvector pair of $C$, $K$ is the number of eigenpairs used, and $\alpha$ is chosen such that $\text{trace}(\tilde{R}) = \text{trace}(D)$ [Fisher, 2005]

This matrix also has an easily attainable inverse

$$\tilde{R}^{-1} = D^{-1/2} \left( \alpha^{-1} I + \sum_{k=1}^{K} (\lambda_k^{-1} - \alpha^{-1}) v_k v_k^T \right) D^{-1/2}$$

No need to store and invert $R$!
Summary

★ Observation error correlations are often created because of contrasting model and observation resolutions
★ Including observation error correlation structure can increase analysis accuracy and information content
★ In IASI data, observation error correlations are strongest between channels with similar spectral properties
★ In IASI data, the largest observation error covariances are between channels highly sensitive to water vapour
★ In order to include observation error correlation structure in data assimilation algorithms, the $\mathbf{R}$ matrix must be suitably structured
Future work

- Working with a symmetric matrix, e.g. fitting a correlation function to the data, taking the symmetric part
- Investigation using the diagnostic update in a identical twin 1D shallow water model experiment
