

Variational data assimilation for morphodynamic model parameter estimation

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Presentation

Presentation slides

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Variational data assimilation for morphodynamic model parameter estimation

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Outline

- Background
 - what is morphodynamic modelling?
 - why do we need morphodynamic models?
- Data assimilation and parameter estimation
 - how can data assimilation provide us with better estimates of uncertain model parameters?
 - what is state augmentation?
 - how do we model the error covariances?
 - Application to a simple 1D morphodynamic model
 - does it work?
- Summary

Terminology

- **Bathymetry** - the underwater equivalent to topography
 - coastal bathymetry is dynamic and evolves with time
 - water action erodes, transports, and deposits sediment, which changes the bathymetry, which alters the water action, and so on
- **Morphodynamics** - the study of the evolution of the bathymetry in response to the flow induced sediment transport
- **Morphodynamic prediction**
 - why?
 - how?



Kent channel

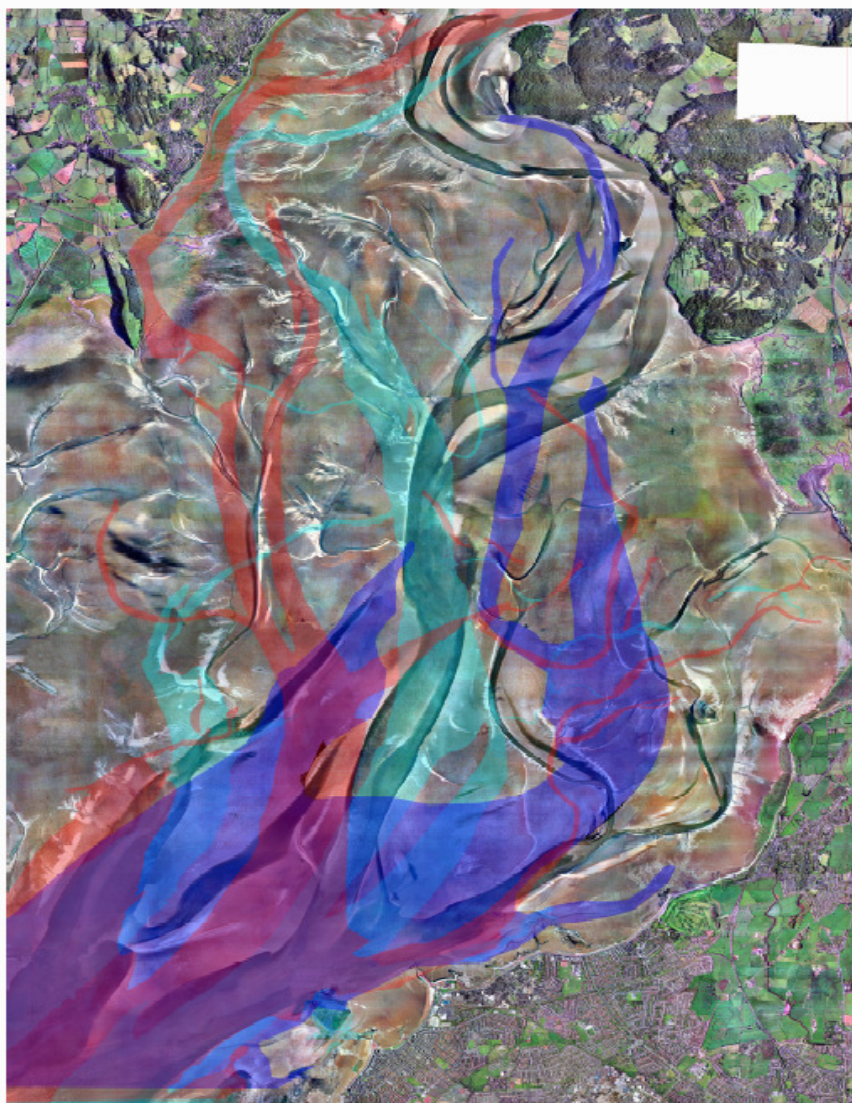
1960s

1970s

1997

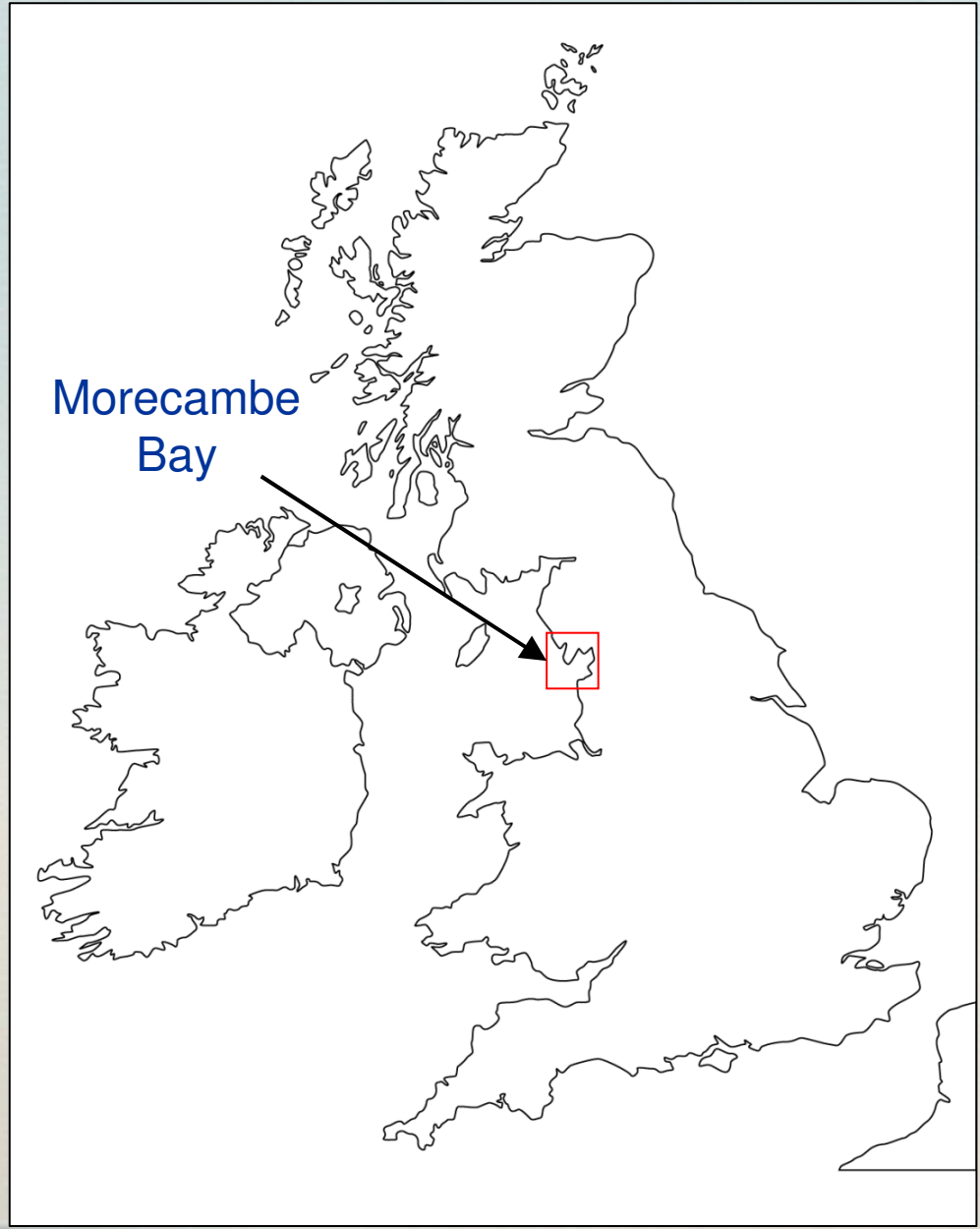
18km

- Channel movement
 - impacts on habitats in the bay
 - affects access to ports
 - has implications for flooding during storm events



Scale
0 1 2 3
kilometres

Picture courtesy of Nigel Cross,
Lancaster City Council



Morecambe
Bay

Kent channel

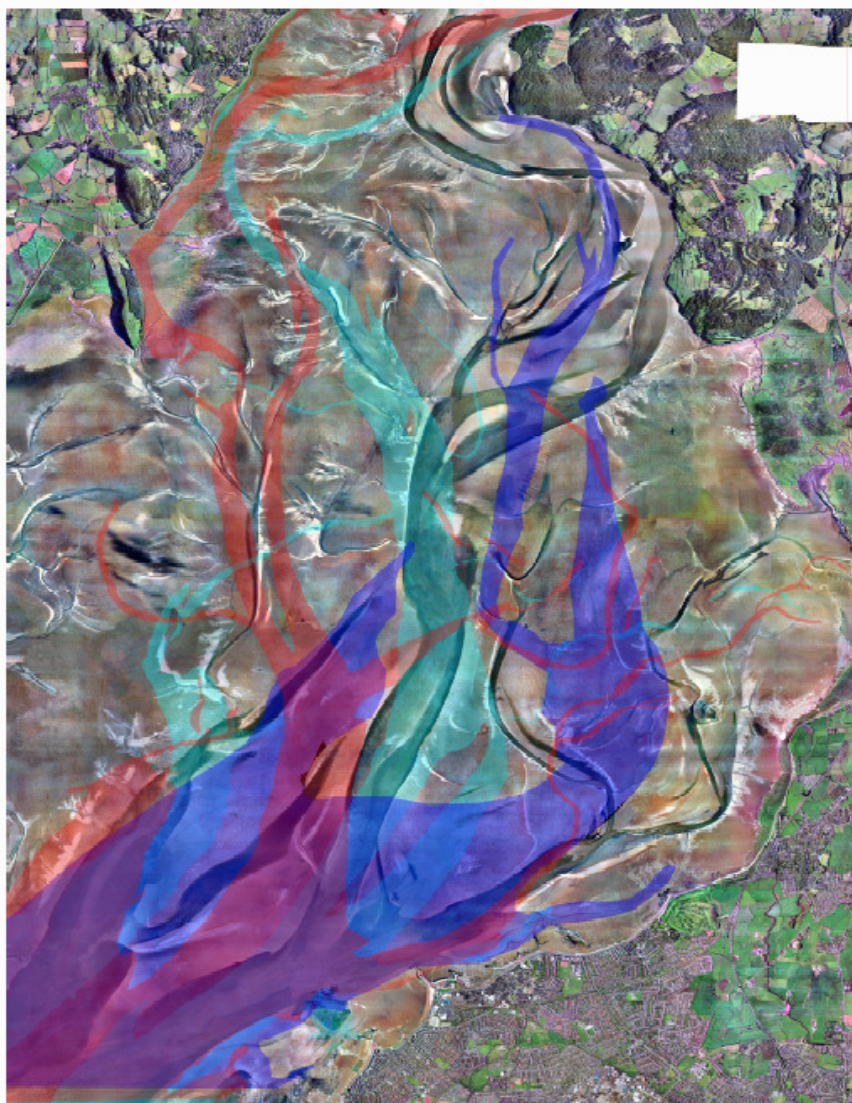
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Morphodynamic modelling

- Operational coastal flood forecasting is limited near-shore by lack of knowledge of evolving bathymetry
 - but it is impractical to continually monitor large coastal areas
- Modelling is difficult
 - longer term changes are driven by shorter term processes
 - uncertainty in initial conditions and parameters
- An alternative approach is to use data assimilation
 - improved estimates of current model bathymetry
 - improved predictions of future bathymetry
 - better value for money for investments in observations & model development



Parameter estimation

- Model equations depend on parameters
 - exact values are unknown
 - inaccurate parameter values can lead to growth of model error
 - affects predictive ability of the model
- How do we estimate these values *a priori*?
 - theoretical values
 - calibration

or ...

- data assimilation
 - choose parameters based on observations
 - state augmentation: model parameters are estimated alongside the model state

3D Var cost function

- Measures the distance of the solution from the background and observations weighted by the inverse of their errors

$$\tilde{J}(\mathbf{w}) = (\mathbf{w} - \mathbf{w}^b)^T \tilde{\mathbf{B}}^{-1} (\mathbf{w} - \mathbf{w}^b) + (\mathbf{y} - \tilde{\mathbf{h}}(\mathbf{w}))^T \mathbf{R}^{-1} (\mathbf{y} - \tilde{\mathbf{h}}(\mathbf{w}))$$

background term

observation term

$\mathbf{w} = \begin{pmatrix} \mathbf{z} \\ \mathbf{p} \end{pmatrix}$ is the augmented model state vector and $\tilde{\mathbf{B}}$ and \mathbf{R} are the covariance matrices of the background and observation errors.

Notation:

- \mathbf{z} model state vector
- \mathbf{p} vector of parameters
- \mathbf{w}^b background state
- \mathbf{y} vector of observations
- $\tilde{\mathbf{h}}$ augmented observation operator

Background error covariance

Augmented B matrix:

$$\tilde{\mathbf{B}} = \begin{pmatrix} \mathbf{B}_{zz} & \mathbf{B}_{zp} \\ (\mathbf{B}_{zp})^T & \mathbf{B}_{pp} \end{pmatrix}.$$

If we assume errors are unbiased

$\mathbf{B}_{zz} = E(\boldsymbol{\varepsilon}_b \boldsymbol{\varepsilon}_b^T)$ state covariance

$\mathbf{B}_{pp} = E(\boldsymbol{\varepsilon}_p \boldsymbol{\varepsilon}_p^T)$ parameter covariance

$\mathbf{B}_{zp} = E(\boldsymbol{\varepsilon}_b \boldsymbol{\varepsilon}_p^T)$ state parameter cross covariances

where $\boldsymbol{\varepsilon}_b = \mathbf{z}^b - \mathbf{z}^t$ and $\boldsymbol{\varepsilon}_p = \mathbf{p}^b - \mathbf{p}^t$

Augmented gain matrix

$$\begin{aligned}\tilde{\mathbf{K}} &= \tilde{\mathbf{B}}\tilde{\mathbf{H}}^T \left[\tilde{\mathbf{H}}\tilde{\mathbf{B}}\tilde{\mathbf{H}}^T + \mathbf{R} \right]^{-1} \\ &= \begin{pmatrix} \mathbf{B}_{zz}\mathbf{H}^T \\ \mathbf{B}_{zp}^T\mathbf{H}^T \end{pmatrix} \left[\mathbf{H}\mathbf{B}_{zz}\mathbf{H}^T + \mathbf{R} \right]^{-1} \\ &\stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{K}_z \\ \mathbf{K}_p \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\mathbf{z}^a &= \mathbf{z}^b + \mathbf{K}_z(\mathbf{y} - \mathbf{h}(\mathbf{z}^b)) \\ \mathbf{p}^a &= \mathbf{p}^b + \mathbf{K}_p(\mathbf{y} - \mathbf{h}(\mathbf{z}^b))\end{aligned}$$

Simple 1D model

Based on the sediment conservation equation

$$\frac{\partial z}{\partial t} = - \left(\frac{1}{1 - \varepsilon} \right) \frac{\partial q}{\partial x}$$

where $z(x,t)$ is the bathymetry, t is time, q is the sediment transport rate in the x direction and ε is the sediment porosity.

For the sediment transport rate we use the power law

$$q = Au^n$$

where $u(x,t)$ is the depth averaged current and **A and n are parameters whose values need to be set**

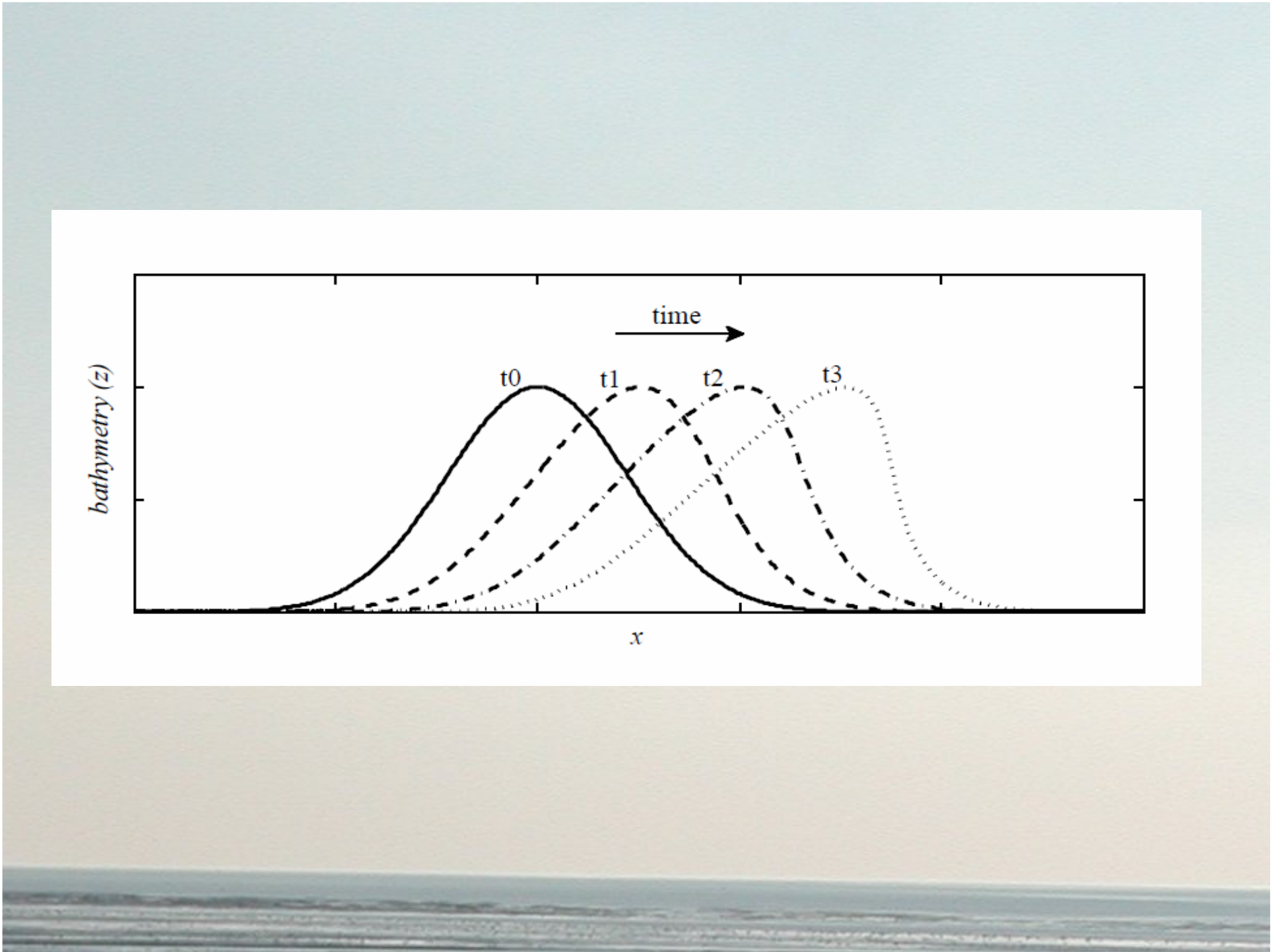
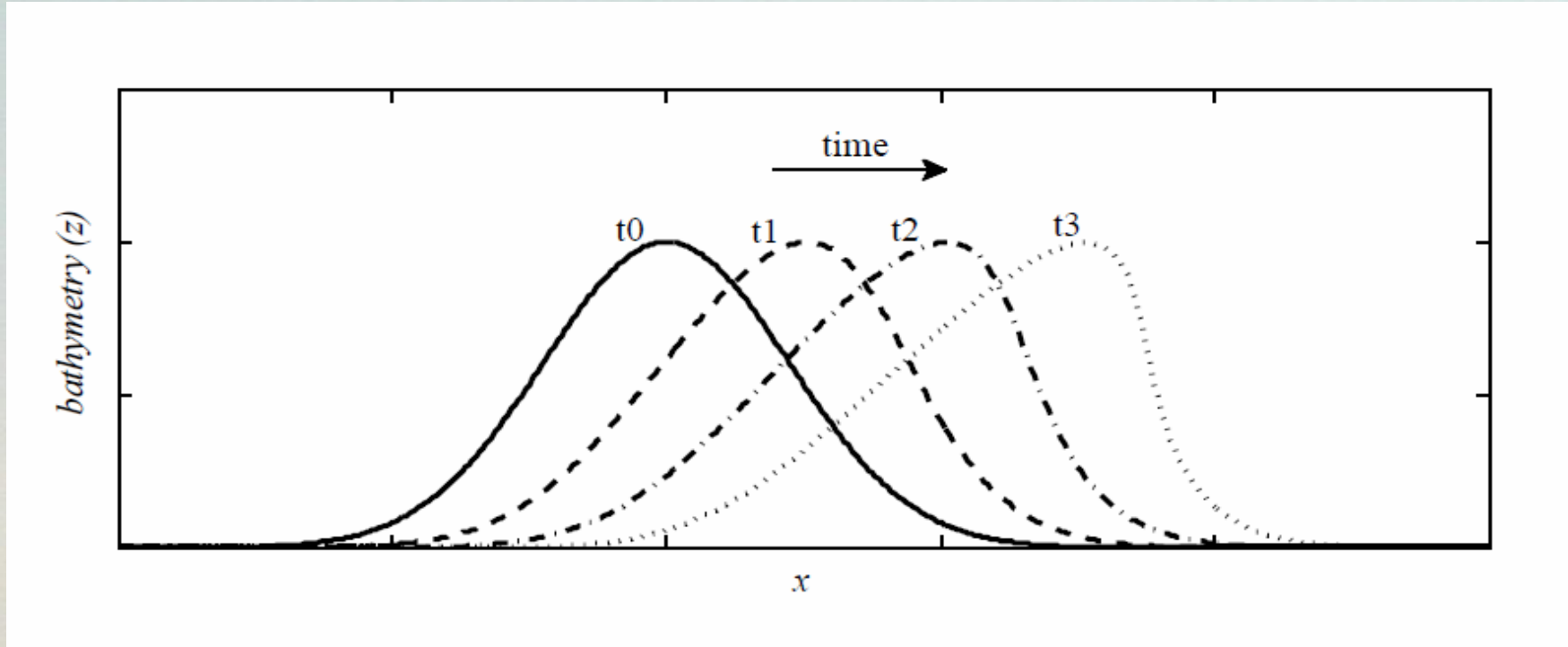
If we assume that water flux (F) and height (H) are constant

$$F = u(H - z)$$

we can rewrite the sediment conservation equation as

$$\frac{\partial z}{\partial t} + a(z, H, F, \varepsilon, A, n) \frac{\partial z}{\partial x} = 0$$

where $a(z, H, F, \varepsilon, A, n)$ is the advection velocity or bed celerity.





Can we use data assimilation to estimate the parameters A and n ?

State-parameter cross covariances

The Extended Kalman filter

Augmented system model:

$$\mathbf{w}_{k+1} = \begin{pmatrix} \mathbf{z}_{k+1} \\ \mathbf{p}_{k+1} \end{pmatrix} = \begin{pmatrix} \mathbf{f}(\mathbf{z}_k, \mathbf{p}_k) \\ \mathbf{p}_k \end{pmatrix} = \tilde{\mathbf{f}}(\mathbf{w}_k)$$

State forecast:

$$\mathbf{w}_{k+1}^f = \tilde{\mathbf{f}}_k(\mathbf{w}_k^a)$$

Error covariance forecast:

$$\mathbf{P}_{k+1}^f = \mathbf{F}_k \mathbf{P}_k^a \mathbf{F}_k^T$$

Tangent linear model:

$$\mathbf{F}_k = \left. \frac{\partial \tilde{\mathbf{f}}}{\partial \mathbf{w}} \right|_{\mathbf{w}_k^a} = \left(\begin{array}{cc} \frac{\partial \mathbf{f}(\mathbf{z}, \mathbf{p})}{\partial \mathbf{z}} & \frac{\partial \mathbf{f}(\mathbf{z}, \mathbf{p})}{\partial \mathbf{p}} \\ \mathbf{0} & \mathbf{I} \end{array} \right) \bigg|_{\mathbf{z}_k^a, \mathbf{p}_k^a} = \left(\begin{array}{cc} \mathbf{M}_k & \mathbf{N}_k \\ \mathbf{0} & \mathbf{I} \end{array} \right)$$

Error covariance forecast:

$$\mathbf{P}_{k+1}^f = \left(\begin{array}{cc} \mathbf{M}_k \mathbf{P}_{\mathbf{z}\mathbf{z}_k}^a \mathbf{M}_k^T + \mathbf{N}_k \mathbf{P}_{\mathbf{p}\mathbf{p}_k}^a \mathbf{N}_k^T & \mathbf{N}_k \mathbf{P}_{\mathbf{p}\mathbf{p}_k}^a \\ \mathbf{P}_{\mathbf{p}\mathbf{p}_k}^a \mathbf{N}_k^T & \mathbf{P}_{\mathbf{p}\mathbf{p}_k}^a \end{array} \right)$$

Hybrid approach

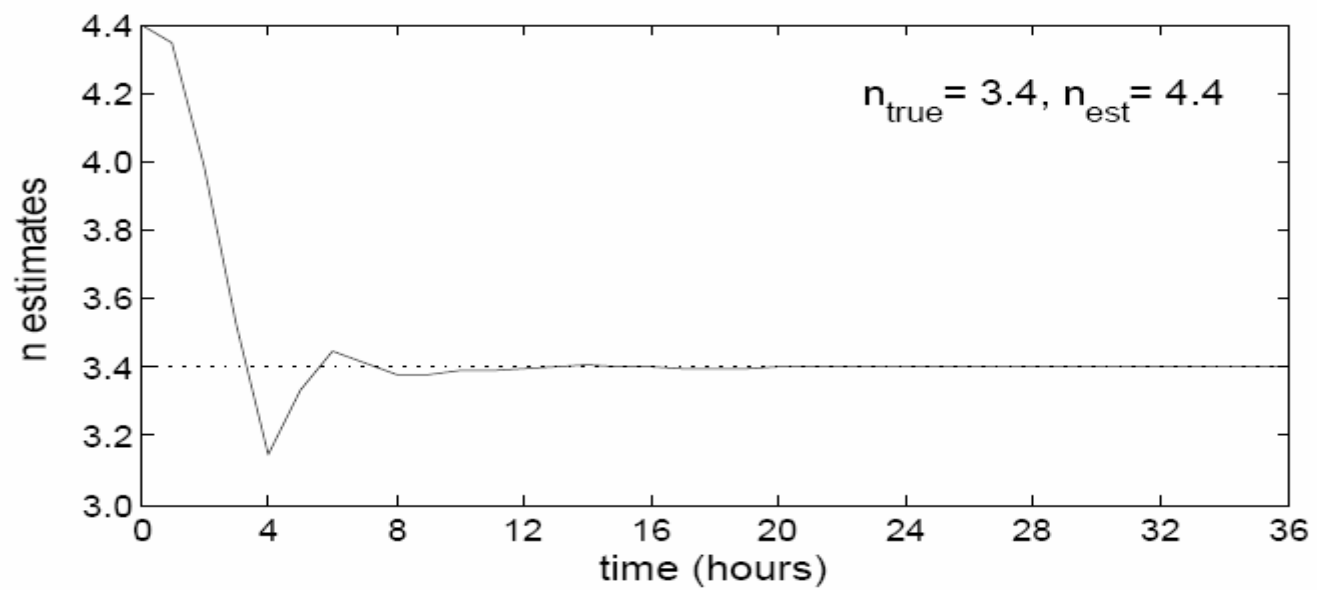
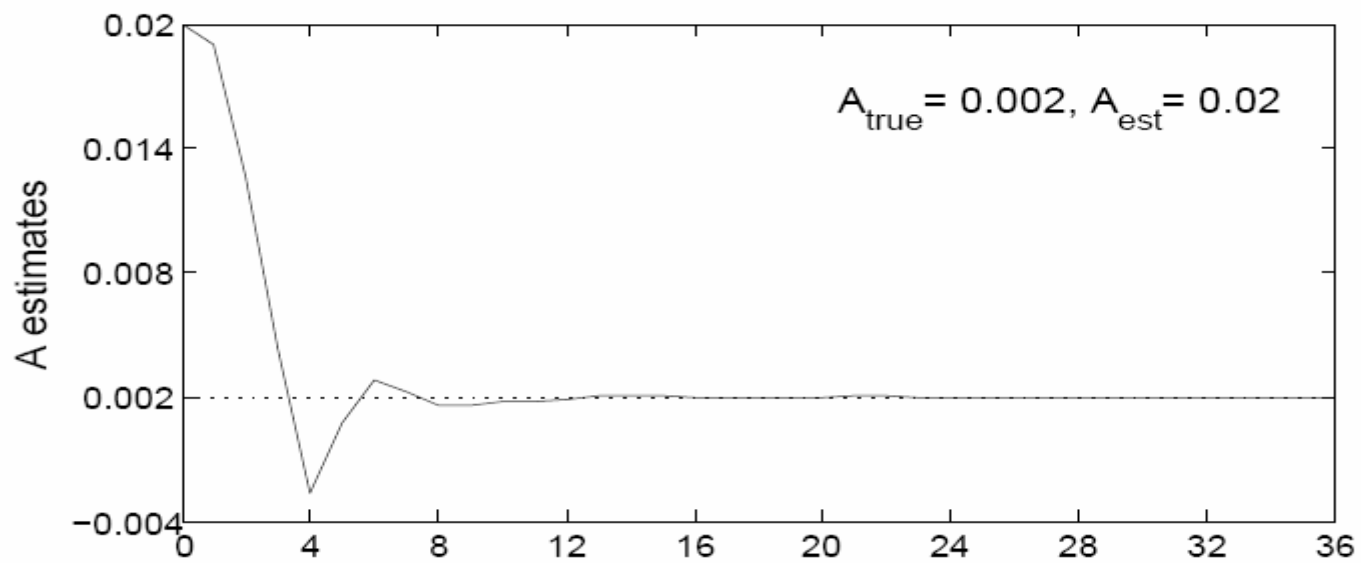
$$\tilde{\mathbf{B}}_k = \begin{pmatrix} \mathbf{B}_{zz} & \mathbf{N}_k \mathbf{B}_{pp} \\ \mathbf{B}_{pp} \mathbf{N}_k^T & \mathbf{B}_{pp} \end{pmatrix}$$

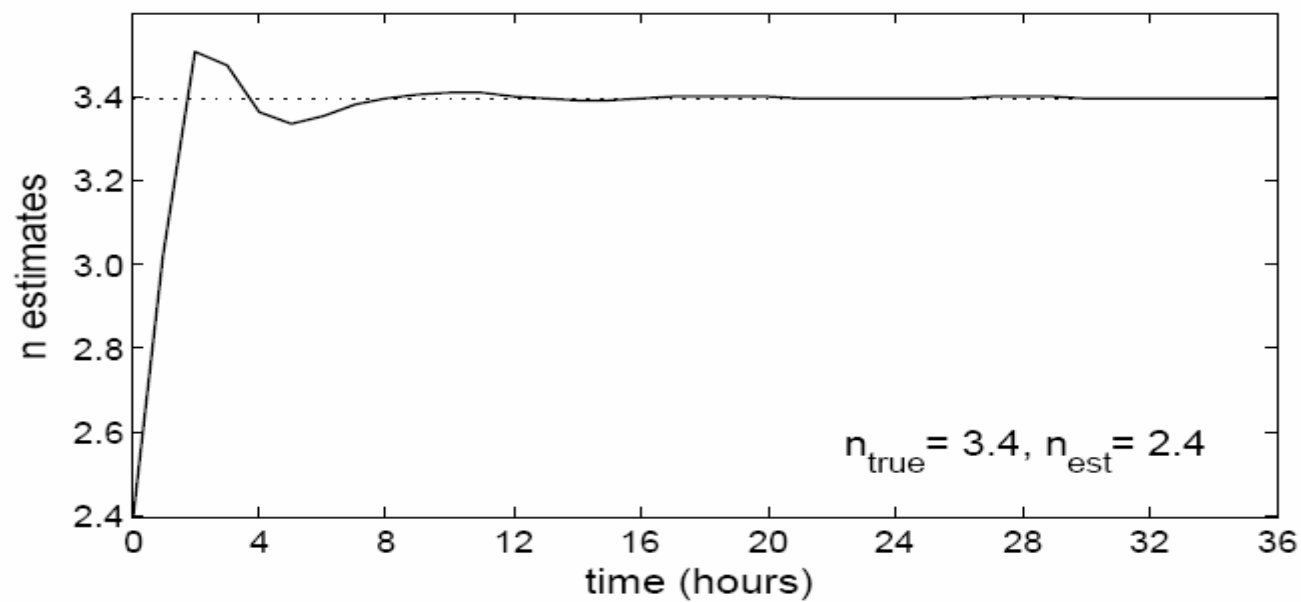
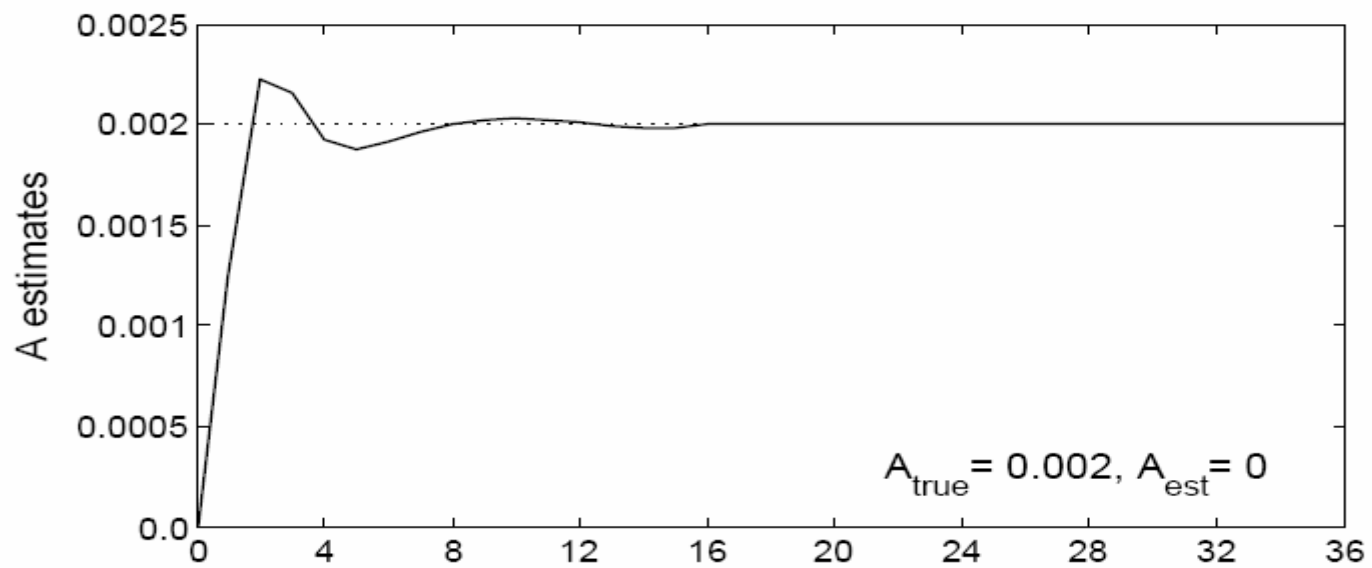
State-parameter cross covariances for our simple model are given by

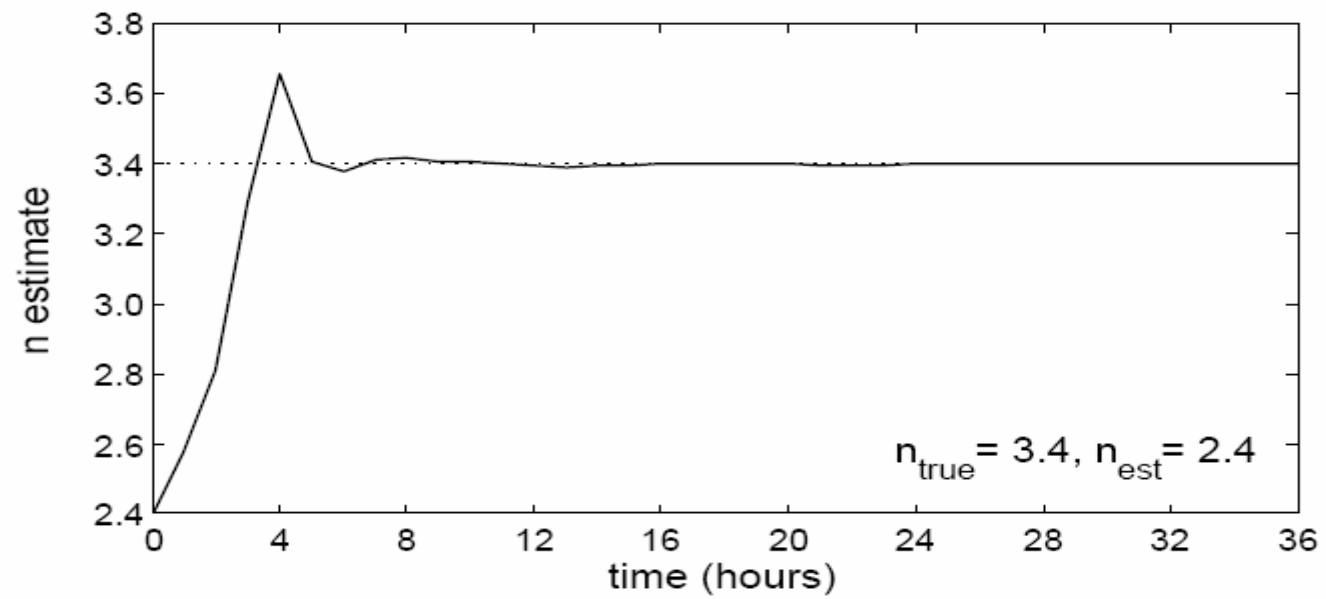
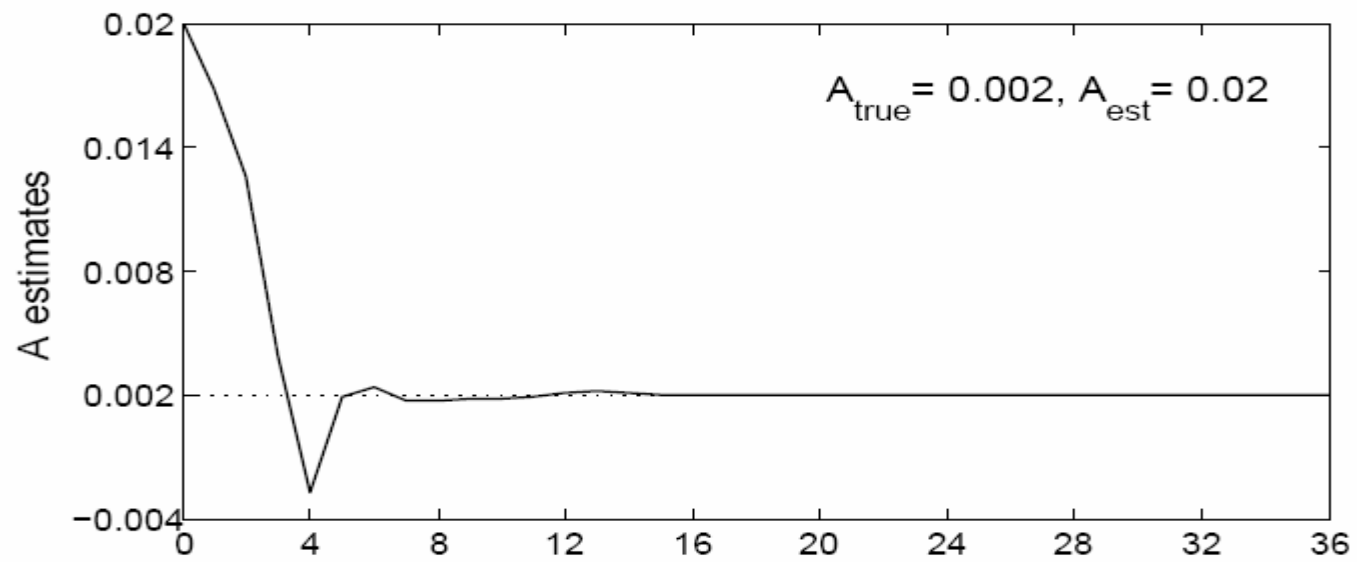
$$\begin{aligned} \mathbf{B}_{zpk} &= \mathbf{N}_k \mathbf{B}_{pp} \\ &= \begin{pmatrix} \frac{\partial \mathbf{f}_k}{\partial A} & \frac{\partial \mathbf{f}_k}{\partial n} \end{pmatrix} \begin{pmatrix} \sigma_A^2 & \sigma_{An} \\ \sigma_{An} & \sigma_n^2 \end{pmatrix} \\ &= \begin{pmatrix} \sigma_A^2 \frac{\partial \mathbf{f}_k}{\partial A} + \sigma_{An} \frac{\partial \mathbf{f}_k}{\partial n} & \sigma_n^2 \frac{\partial \mathbf{f}_k}{\partial n} + \sigma_{An} \frac{\partial \mathbf{f}_k}{\partial A} \end{pmatrix} \end{aligned}$$

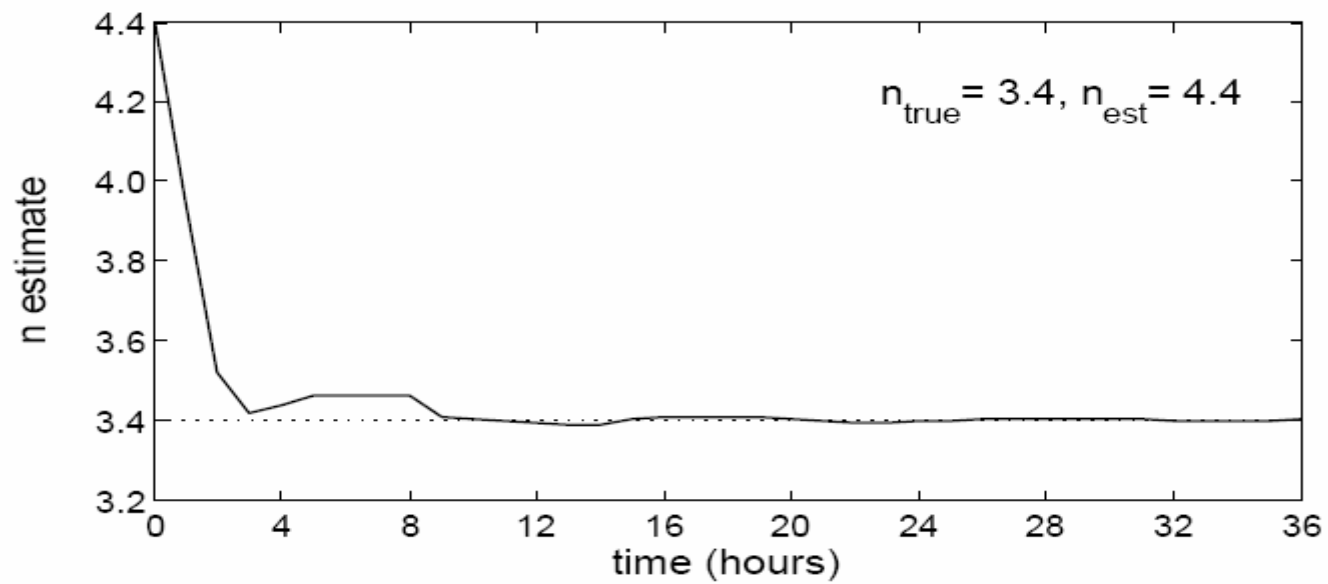
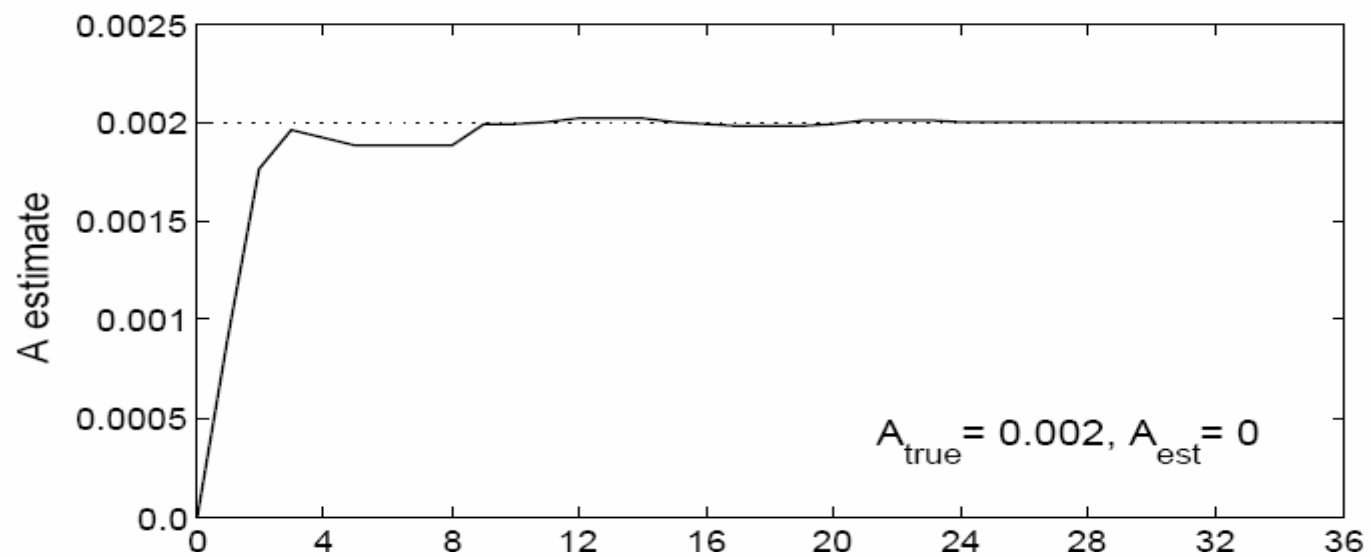
Model setup

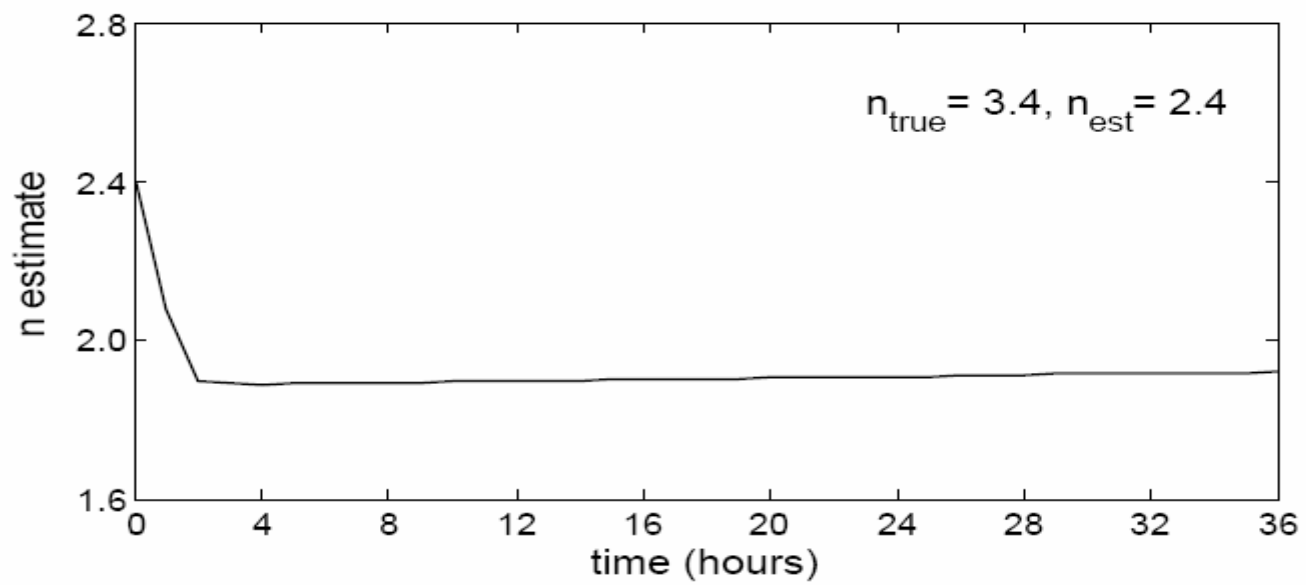
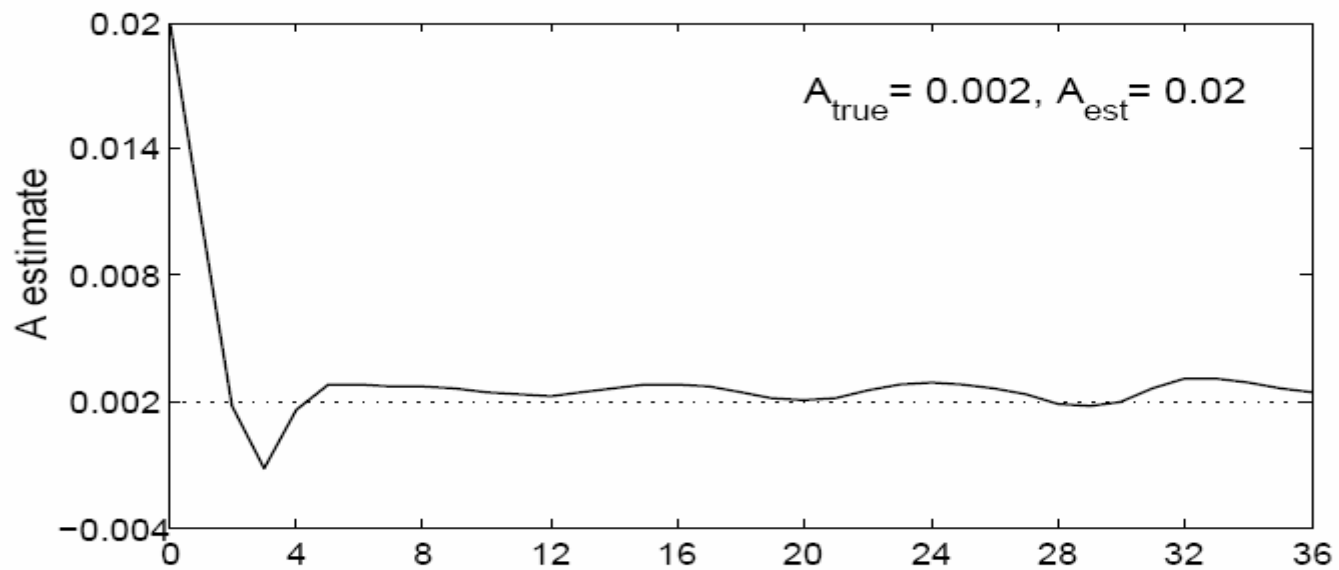
- Assume perfect model and observations
- Identical twin experiments
 - reference solution generated using Gaussian initial data and parameter values $A = 0.002 \text{ ms}^{-1}$ and $n = 3.4$
- Use incorrect model inputs
 - inaccurate initial bathymetry
 - inaccurate parameter estimates
- 3D Var algorithm is applied sequentially
 - observations taken at fixed grid points & assimilated every hour
 - the cost function is minimized iteratively using a quasi-Newton descent algorithm
- Covariances
 - \mathbf{B}_{zz} fixed
 - \mathbf{B}_{zp} time varying











Summary

- Presented a novel approach to model parameter estimation using data assimilation
 - demonstrated the technique using a simple morphodynamic model
- Results are very encouraging
 - scheme is capable of recovering near-perfect parameter values
 - improves model performance
- What next ...?
 - can our scheme be successfully applied to more complex models?
 - can we determine an optimal observing network?

Morecambe Bay at sunset



Simple Models of Changing Bathymetry with Data Assimilation

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Department of Mathematics, University of Reading

Numerical Analysis Report 10/2007*

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Mathematics Report 2/2008*

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