Variational data assimilation for morphodynamic model parameter estimation

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Outline

- Background/ motivation
  - what is morphodynamic modelling?
  - why do we need morphodynamic models?
- A simple 1D morphodynamic model
- Data assimilation and parameter estimation
  - how can we use data assimilation to estimate uncertain model parameters?
  - how do we model the background error covariances?
- Results
- Summary

Terminology

- Bathymetry - the underwater equivalent to topography
  - coastal bathymetry is dynamic and evolves with time
  - water action erodes, transports, and deposits sediment, which changes the bathymetry, which alters the water action, and so on
- Morphodynamics - the study of the evolution of the bathymetry in response to the flow induced sediment transport
- Morphodynamic prediction
  - why?
  - how?

Picture courtesy of Nigel Cross, Lancaster City Council

18km

Kent channel

1960s

1970s

1997

- Channel movement
  - impacts on habitats in the bay
  - affects access to ports
  - has implications for flooding during storm events

Morphodynamic modelling

- Operational coastal flood forecasting is limited near-shore by lack of knowledge of evolving bathymetry
  - but it is impractical to continually monitor large coastal areas
- Modelling is difficult
  - longer term changes are driven by shorter term processes
  - uncertainty in initial conditions and parameters
- An alternative approach is to use data assimilation

Parameter estimation

- Model equations depend on parameters
  - exact values are unknown
  - inaccurate parameter values can lead to growth of model error
  - affects predictive ability of the model
- How do we estimate these values *a priori*?
  - theoretical values
  - calibration

or ...

- data assimilation
  - choose parameters based on observations
  - state augmentation: model parameters are estimated alongside the model state

Simple 1D model

Based on the sediment conservation equation

$$\frac{\partial z}{\partial t} = -\left(\frac{1}{1-\varepsilon}\right)\frac{\partial q}{\partial x}$$

where $z(x,t)$ is the bathymetry, t is time, q is the sediment transport rate in the x direction and ε is the sediment porosity.

For the sediment transport rate we use the power law

$$q = Au^n$$

where $u(x,t)$ is the depth averaged current and A **and** n **are parameters whose values need to be set**

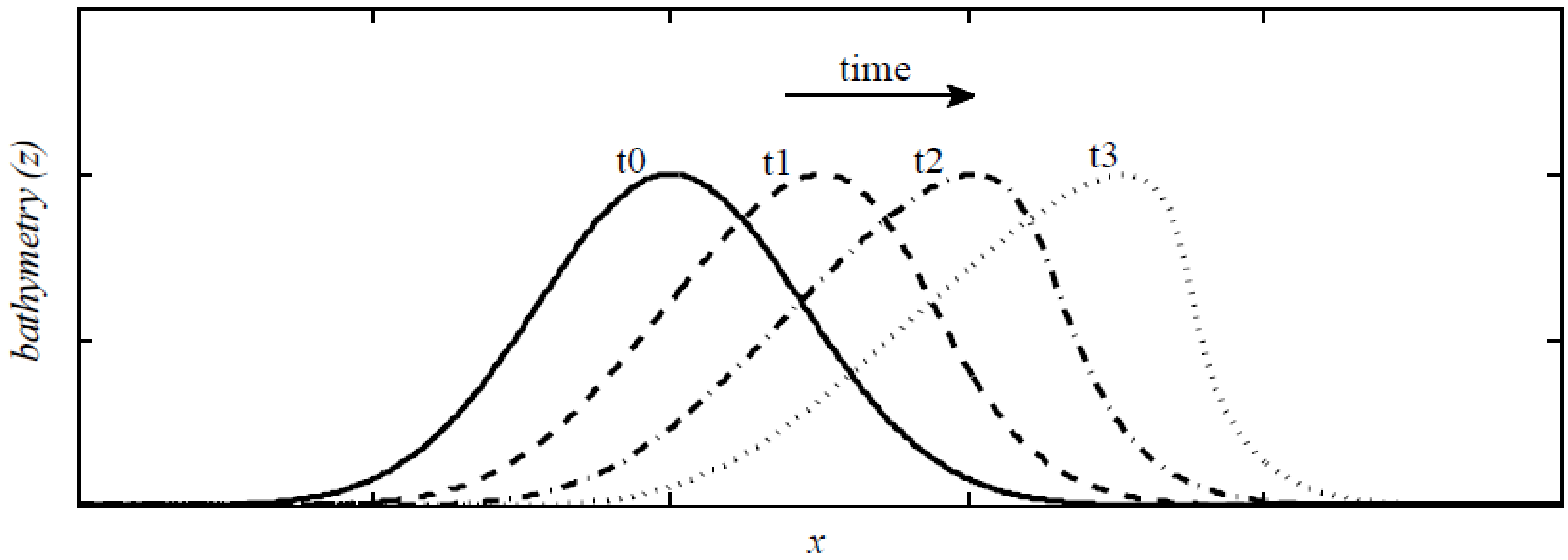
If we assume that water flux (F) and height (H) are constant

$$F = u(H - z)$$

we can rewrite the sediment conservation equation as

$$\frac{\partial z}{\partial t} + a(z, H, F, \varepsilon, A, n)\frac{\partial z}{\partial x} = 0$$

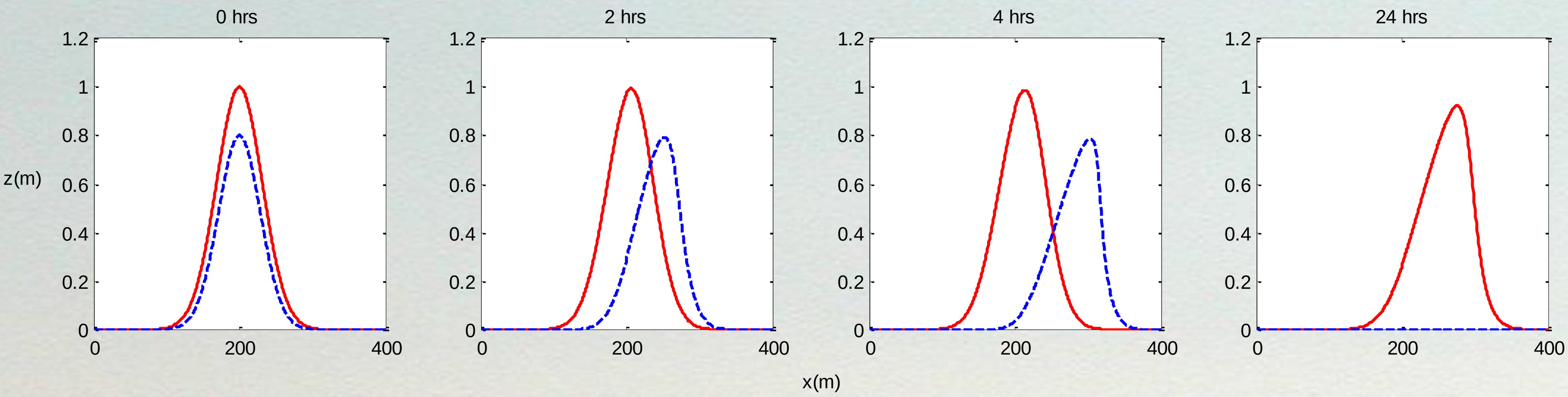
where $a(z, H, F, \varepsilon, A, n)$ is the advection velocity or bed celerity.



time
t0
t1
t2
t3
bathymetry (z)
x

Can we use data assimilation to estimate the parameters A and n?

model run with incorrect parameters & without data assimilation



red line = correct parameters

blue line = incorrect parameters (A over estimated, n under estimated)

State augmentation

- Dynamical system model

$$\mathbf{z}_{k+1} = \mathbf{f}(\mathbf{z}_k, \mathbf{p}_k)$$

(discrete, non-linear, time invariant)

- Parameter evolution

$$\mathbf{p}_{k+1} = \mathbf{p}_k.$$

- Augmented system model

$$\mathbf{w}_{k+1} = \begin{pmatrix} \mathbf{z}_{k+1} \\ \mathbf{p}_{k+1} \end{pmatrix} = \begin{pmatrix} \mathbf{f}(\mathbf{z}_k, \mathbf{p}_k) \\ \mathbf{p}_k \end{pmatrix} = \tilde{\mathbf{f}}(\mathbf{w}_k)$$

- Observations

$$\mathbf{y}_k = \mathbf{h}(\mathbf{z}_k)$$

in terms of the augmented system ...

$$\mathbf{y}_k = \tilde{\mathbf{h}}(\mathbf{w}_k)$$

where

$$\tilde{\mathbf{h}}(\mathbf{w}) = \tilde{\mathbf{h}}\begin{pmatrix} \mathbf{z} \\ \mathbf{p} \end{pmatrix} = \mathbf{h}(\mathbf{z}).$$

3D Var

Cost function:

$$\tilde{J}(\mathbf{w}) = (\mathbf{w} - \mathbf{w}^b)^{\mathrm{T}}\tilde{\mathbf{B}}^{-1}(\mathbf{w} - \mathbf{w}^b) + (\mathbf{y} - \tilde{\mathbf{h}}(\mathbf{w}))^{\mathrm{T}}\mathbf{R}^{-1}(\mathbf{y} - \tilde{\mathbf{h}}(\mathbf{w}))$$

$\tilde{\mathbf{B}}$ and $\mathbf{R}$ are the covariance matrices of the background and observation errors.

$$\tilde{\mathbf{B}} = \begin{pmatrix} \mathbf{B}_{\mathbf{zz}} & \mathbf{B}_{\mathbf{zp}} \\ (\mathbf{B}_{\mathbf{zp}})^{\mathrm{T}} & \mathbf{B}_{\mathbf{pp}} \end{pmatrix}.$$

$\mathbf{B}_{\mathbf{zz}}$ state background error covariance

$\mathbf{B}_{\mathbf{pp}}$ parameter background error covariance

$\mathbf{B}_{\mathbf{zp}}$ state parameter error cross covariance augmented gain matrix:

$$\begin{aligned}
\tilde{\mathbf{K}} &= \tilde{\mathbf{B}}\tilde{\mathbf{H}}^{\mathbf{T}}\left[\tilde{\mathbf{H}}\tilde{\mathbf{B}}\tilde{\mathbf{H}}^{\mathbf{T}} + \mathbf{R}\right]^{-1} \\
&= \begin{pmatrix} \mathbf{B_{zz}H^T} \\ \mathbf{B_{zp}^T H^T} \end{pmatrix} \left[\mathbf{H B_{zz} H^T} + \mathbf{R}\right]^{-1} \\
&\stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{K_z} \\ \mathbf{K_p} \end{pmatrix}
\end{aligned}$$

state & parameter updates:

$$\begin{aligned}
\mathbf{z}^{a} &= \mathbf{z}^{b} + \mathbf{K_z}(\mathbf{y} - \mathbf{h}(\mathbf{z}^{b})) \\
\mathbf{p}^{a} &= \mathbf{p}^{b} + \mathbf{K_p}(\mathbf{y} - \mathbf{h}(\mathbf{z}^{b}))
\end{aligned}$$

State-parameter cross covariances

The Extended Kalman filter (EKF)

State forecast:

$$\mathbf{w}_{k+1}^{f} = \tilde{\mathbf{f}}_k(\mathbf{w}_k^a)$$

Error covariance forecast:

$$\mathbf{P}_{k+1}^{f} = \mathbf{F}_k \mathbf{P}_k^a \mathbf{F}_k^T$$

where

$$\mathbf{F}_k = \left.\frac{\partial \tilde{\mathbf{f}}}{\partial \mathbf{w}}\right|_{\mathbf{w}_k^a} = \left.\begin{pmatrix} \frac{\partial \mathbf{f}(\mathbf{z},\mathbf{p})}{\partial \mathbf{z}} & \frac{\partial \mathbf{f}(\mathbf{z},\mathbf{p})}{\partial \mathbf{p}} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}\right|_{\mathbf{z}_k^a,\mathbf{p}_k^a} = \begin{pmatrix} \mathbf{M}_k & \mathbf{N}_k \\ \mathbf{0} & \mathbf{I} \end{pmatrix}$$

Error covariance forecast:

$$\mathbf{P}^f_{k+1} = \begin{pmatrix} \mathbf{M}_k \mathbf{P}^a_{\mathbf{zz}_k} \mathbf{M}_k^T + \mathbf{N}_k \mathbf{P}^a_{\mathbf{pp}_k} \mathbf{N}_k^T & \mathbf{N}_k \mathbf{P}^a_{\mathbf{pp}_k} \\ \mathbf{P}^a_{\mathbf{pp}_k} \mathbf{N}_k^T & \mathbf{P}^a_{\mathbf{pp}_k} \end{pmatrix}$$

a new hybrid approach ...

$$\tilde{\mathbf{B}}_k = \begin{pmatrix} \mathbf{B}_{\mathbf{zz}} & \mathbf{N}_k \mathbf{B}_{\mathbf{pp}} \\ \mathbf{B}_{\mathbf{pp}} \mathbf{N}_k^T & \mathbf{B}_{\mathbf{pp}} \end{pmatrix}$$

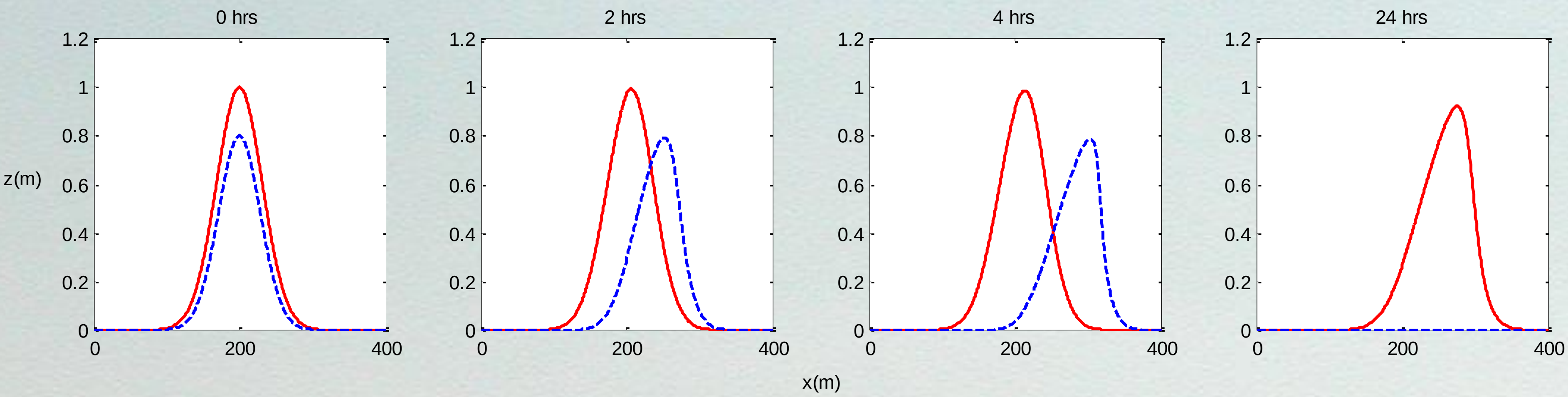
for our simple 2 parameter model

$$
\begin{aligned}
\mathbf{B}_{\mathbf{zp}_k} &= \mathbf{N}_k \mathbf{B}_{\mathbf{pp}} \\
&= \begin{pmatrix} \frac{\partial \mathbf{f}_k}{\partial A} & \frac{\partial \mathbf{f}_k}{\partial n} \end{pmatrix} \begin{pmatrix} \sigma_A^2 & \sigma_{An} \\ \sigma_{An} & \sigma_n^2 \end{pmatrix} \\
&= \begin{pmatrix} \sigma_A^2 \frac{\partial \mathbf{f}_k}{\partial A} + \sigma_{An} \frac{\partial \mathbf{f}_k}{\partial n} & \sigma_n^2 \frac{\partial \mathbf{f}_k}{\partial n} + \sigma_{An} \frac{\partial \mathbf{f}_k}{\partial A} \end{pmatrix}
\end{aligned}
$$

Model setup

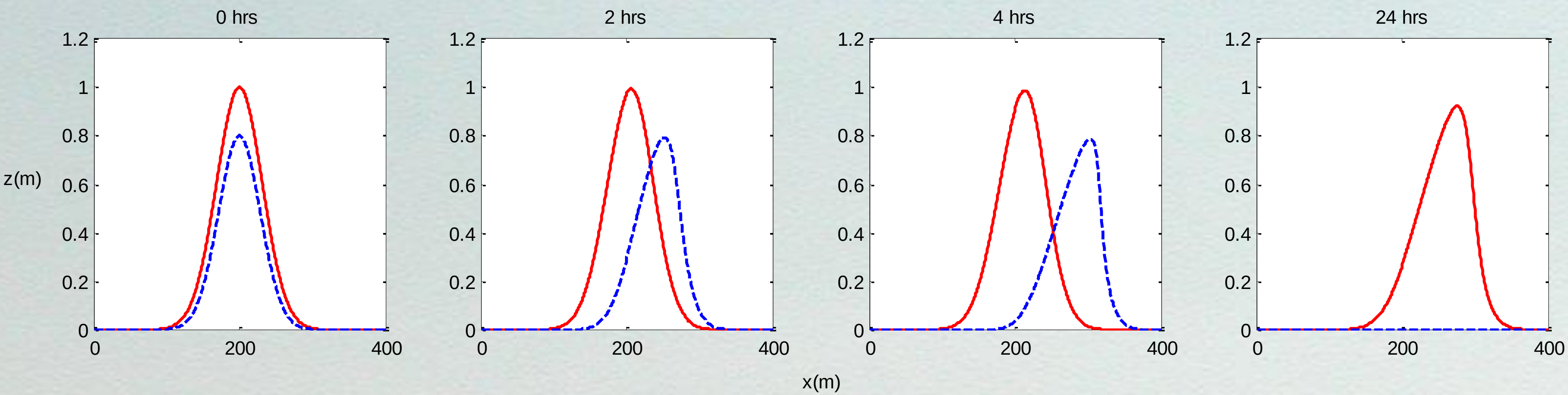
- Assume perfect model and observations
- Identical twin experiments
  - reference solution generated using Gaussian initial data and parameter values $A = 0.002\ ms^{-1}$ and $n = 3.4$
- Use incorrect model inputs
  - inaccurate initial bathymetry
  - inaccurate parameter estimates
- 3D Var algorithm is applied sequentially
  - observations taken at fixed grid points & assimilated every hour
  - the cost function is minimized iteratively using a quasi-Newton descent algorithm
- Covariances
  - $\mathbf{B}_{zz}$ fixed
  - $\mathbf{B}_{zp}$ time varying

without data assimilation

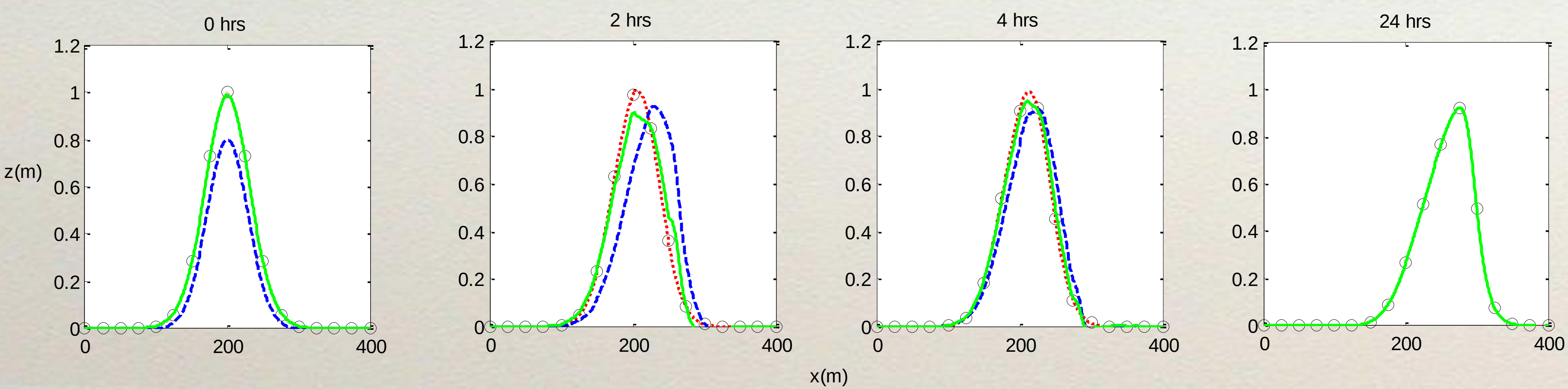


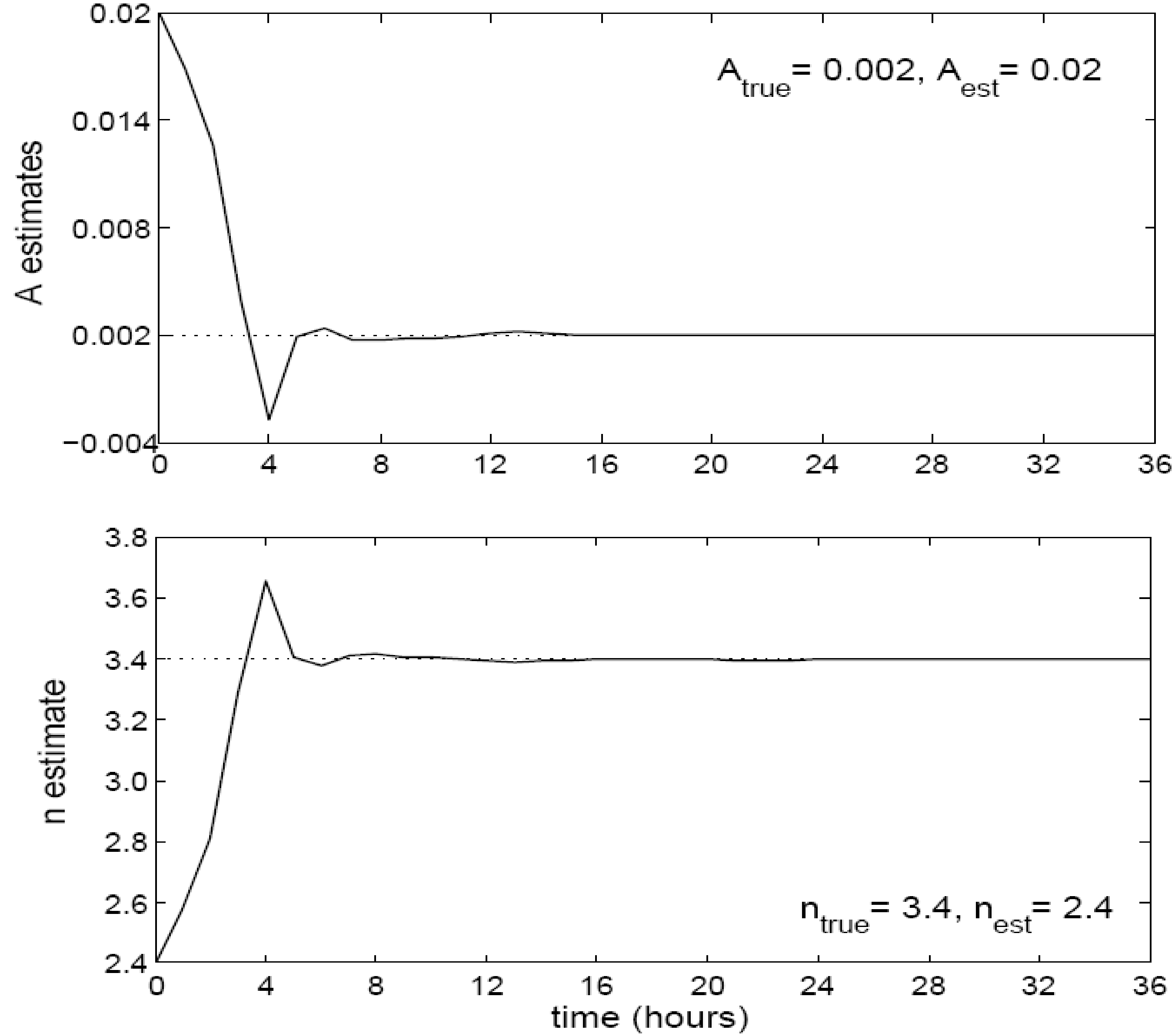
with data assimilation …

without data assimilation

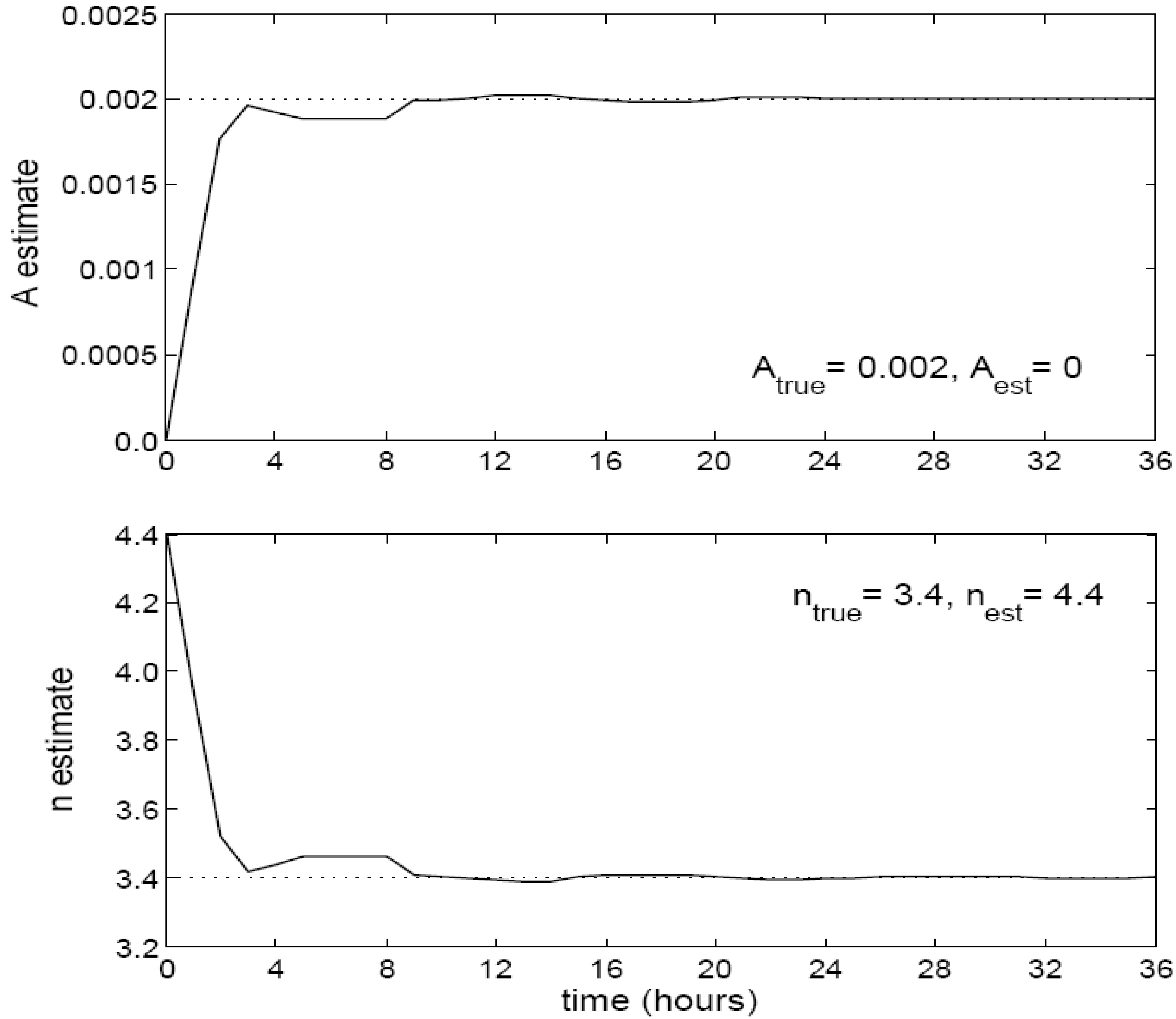


with data assimilation …

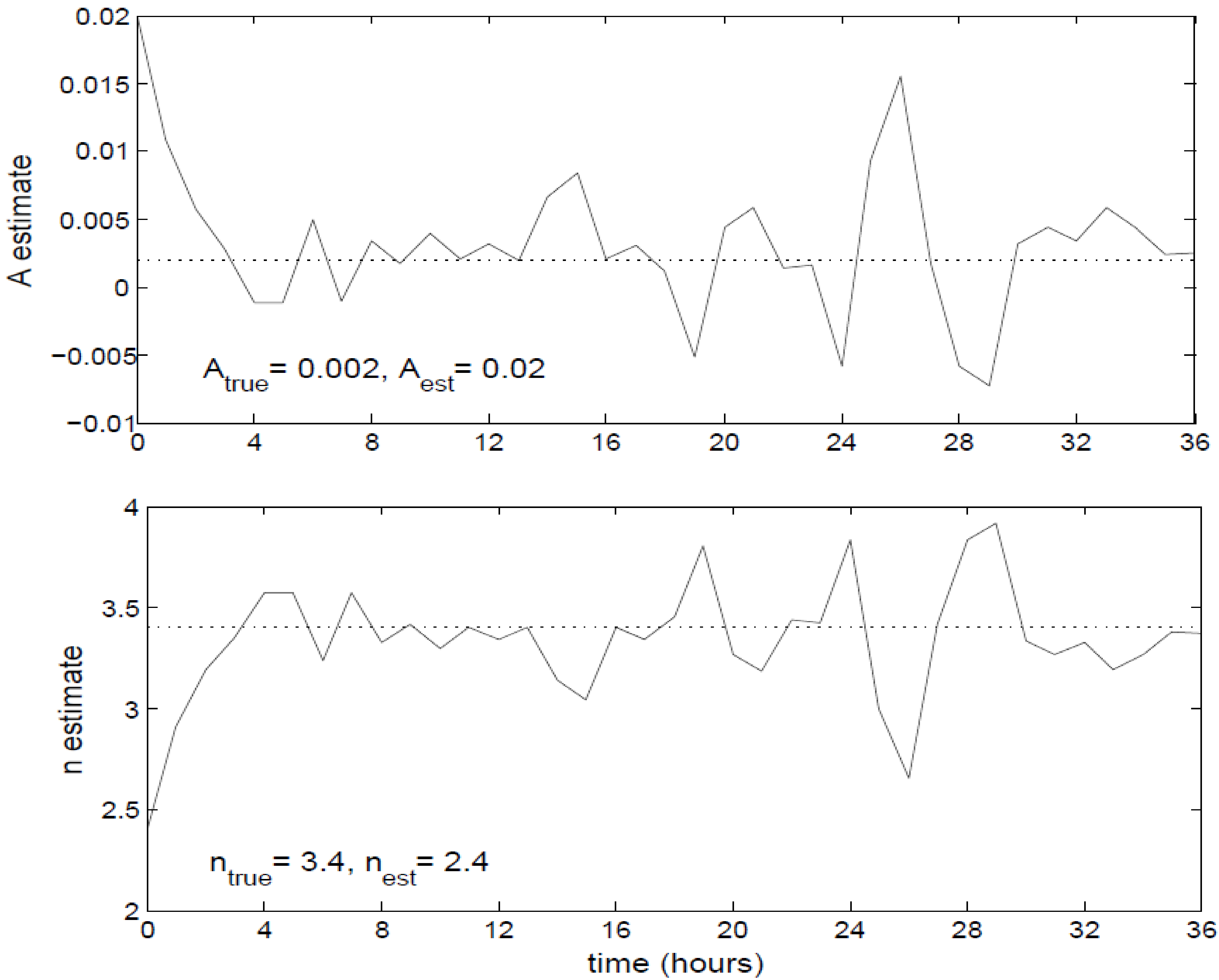




A estimates
A_{true}= 0.002, A_{est}= 0.02
0.02
0.014
0.008
0.002
−0.004
0
4
8
12
16
20
24
28
32
36
n estimate
n_{true}= 3.4, n_{est}= 2.4
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
time (hours)



A estimate
0.0025
0.002
0.0015
0.001
0.0005
0.0
0
4
8
12
16
20
24
28
32
36
A_{true}= 0.002, A_{est}= 0
n estimate
4.4
4.2
4.0
3.8
3.6
3.4
3.2
n_{true}= 3.4, n_{est}= 4.4
time (hours)



A estimate
0.02
0.015
0.01
0.005
0
−0.005
−0.01
A_{true}= 0.002, A_{est}= 0.02
0
4
8
12
16
20
24
28
32
36
n estimate
4
3.5
3
2.5
2
n_{true}= 3.4, n_{est}= 2.4
0
4
8
12
16
20
24
28
32
36
time (hours)

Summary

- Presented a novel approach to model parameter estimation using data assimilation
  - demonstrated the technique using a simple morphodynamic model
- Results are very encouraging
  - scheme is capable of recovering near-perfect parameter values
  - improves model performance
- What next …?
  - can our scheme be successfully applied to more complex models?
  - can we say anything about the convergence of the system?

Questions?

Simple Models of Changing Bathymetry with Data Assimilation

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Numerical Analysis Report 10/2007*

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Mathematics Report 2/2008*

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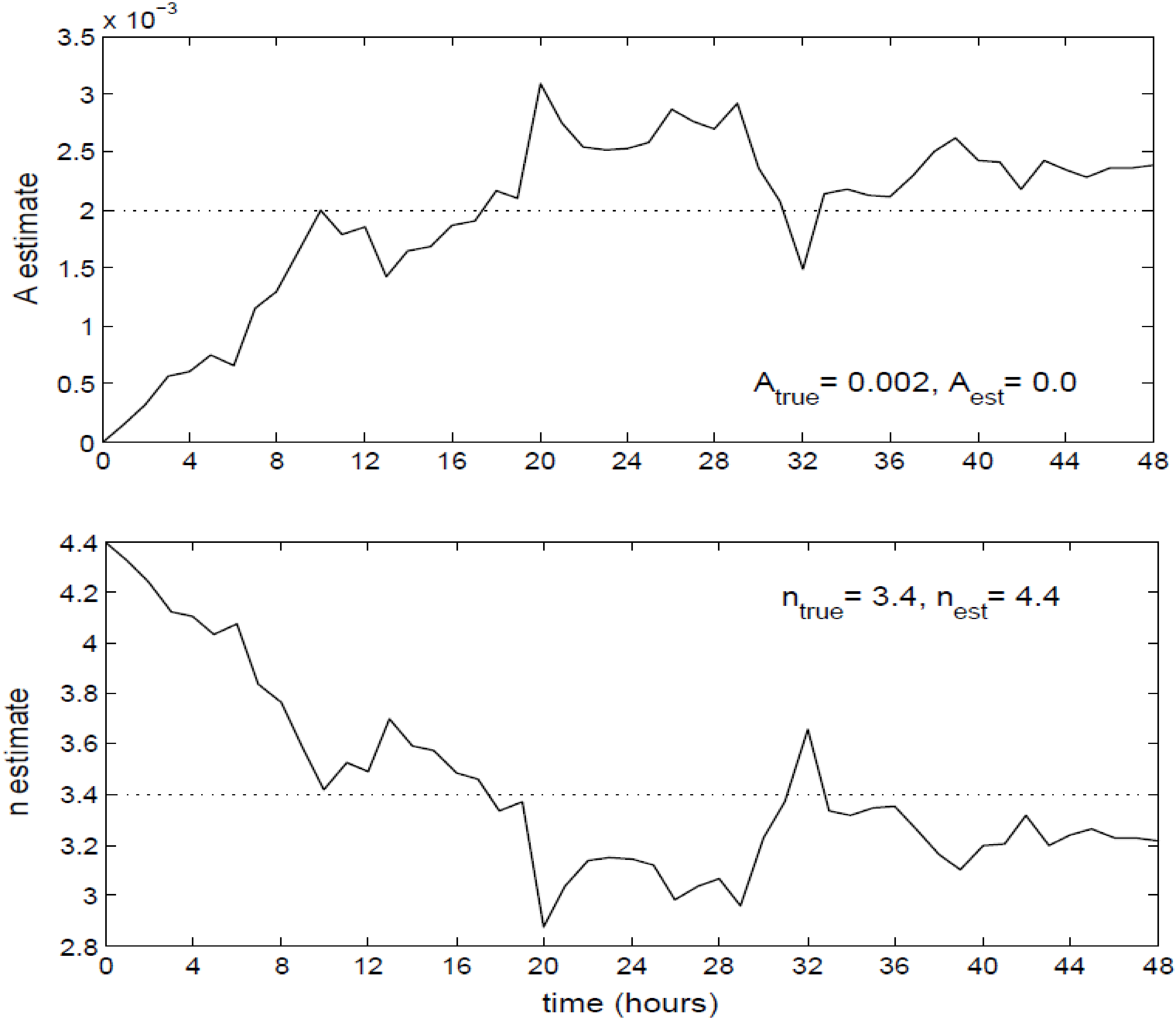
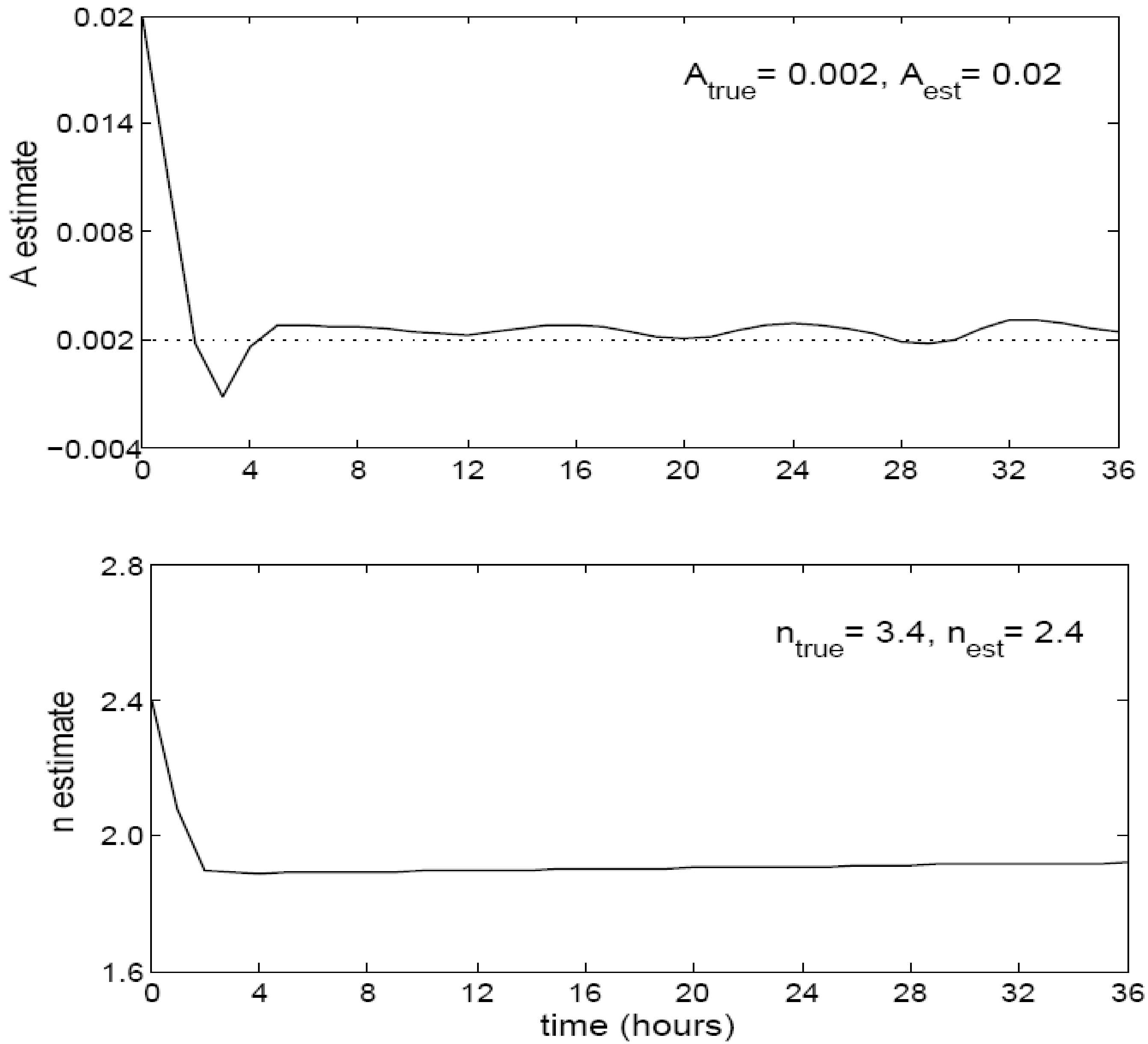


x 10^-3
A estimate
A_true= 0.002, A_est= 0.0
n estimate
n_true= 3.4, n_est= 4.4
time (hours)



A_{true}= 0.002, A_{est}= 0.02
A estimate
0.02
0.014
0.008
0.002
−0.004
0
4
8
12
16
20
24
28
32
36
n_{true}= 3.4, n_{est}= 2.4
n estimate
2.8
2.4
2.0
1.6
time (hours)