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Adapting the International System of Units to the 21st century

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Abstract

We review the proposal of the International Committee for Weights and Measures (CIPM), currently being considered by the General Conference on Weights and Measures (CGPM), to revise the International System of Units, the SI. The proposal includes new definitions for four of the seven base units of the SI, and a new form of words to present the definitions of all the units. The objective of the proposed changes is to adopt definitions referenced to constants of nature, taken in the widest sense, so that the definitions may be based on what are believed to be true invariants. In particular, whereas in the current SI the kilogram, ampere, kelvin and mole are linked to exact numerical values of the mass of the international prototype of the kilogram, the magnetic constant (permeability of vacuum), triple-point temperature of water, and molar mass of carbon 12, respectively, in the new SI these units are linked to exact numerical values of the Planck constant, elementary charge, Boltzmann constant and Avogadro constant, respectively. The new wording used expresses the definitions in a simple and unambiguous manner without the need for the distinction between base and derived units. The importance of relations among the fundamental constants to the definitions, and the importance of establishing a *mise en pratique* for the realisation of each definition, is also discussed.

Key words: International System of Units, SI; definition of the new SI; fundamental constant; SI reference constant; explicit-constant definition; explicit-unit definition.

1. Introduction

The International System of Units, the SI (from the French: *Le Système international d'unités*), is generally presented as being founded on the seven base units metre, kilogram, second, ampere, kelvin, mole and candela. These seven base units are the SI units for the corresponding base quantities, length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity. The SI units for all other quantities in the physical sciences, known as derived units, are then obtained as products of powers of these seven base units by using equations based on the known relations between the corresponding quantities. It follows that the definitions of the seven base units are of central importance to the whole system.

The definition of each base unit is specified by relating the unit to a quantity of the same kind which is believed to be a suitable invariant to use as a reference. The definition is made explicit by specifying the numerical value, either directly or indirectly, of the ratio of the unit to the specified reference quantity. The present definition of the kilogram is referenced to the mass of the international prototype of the kilogram (IPK), but for other base units the property of a simple atom or a fundamental constant is used as a reference, as in the case of the second which is referenced to the period of a specified transition frequency in the caesium atom, or the metre which is referenced to the distance travelled by light in vacuum in a specified time interval. The desirable qualities for a good definition are that the reference quantity should be a true invariant, should be available to anyone at any time, should be realisable as accurately as the best measurements require, and should preferably be as simple as possible both to comprehend and to realise.

Definitions and changes in the definitions of the base units are made by the General Conference on Weights and Measures (CGPM) on the advice of the International Committee for Weights and Measures (CIPM), which itself takes advice from its various consultative committees (CCs), and particularly from the Consultative Committee for Units (CCU). The CGPM and the CIPM act with the authority given to them by the Metre Convention of 1875 which created the Bureau International des Poids et Mesures (BIPM), located at Sèvres close to Paris. The definition of the SI is published by the BIPM in the form of the SI Brochure [1], which is revised in new editions published at intervals of approximately 5 or more years, which record any changes that are made to the system.

Many of the definitions have been changed over the last 120 years since the BIPM was established and the first definitions were made, and particularly since the SI was formally adopted by the 11th General Conference in 1960. These changes generally reflect improvements in the accuracy and reproducibility with which the definitions can be realised by adopting a revised choice for one of the reference quantities. The kilogram is an exception: it has remained unchanged since the current definition was adopted in 1889. The kilogram is defined as the mass of the IPK, international prototype of the kilogram, denoted $\mathcal{K}$, kept in a vault at the BIPM. However it has been known for more than fifty years that the kilogram is particularly in need of an improved definition, because it fails on all of the first three desirable qualities listed above. In particular it is believed to be slowly changing in mass compared with other supposedly identical prototypes machined and polished to have the same mass at the time of manufacture. The changes are of the order of 50 μg (5 parts in 10^8), and may perhaps be as much as 100 μg over the last 100 years, although the precise magnitude is not known. This may be contrasted with our ability to compare the masses of two prototype kilograms which is at least as good as 1 μg, or 1 part in 10^9. These changes may be owing to wear and tear, changing surface properties due to adsorbed molecules, or gas that was originally trapped in the prototype when it was cast in 1879 and may now be leaching out. The kilogram is also involved in the definitions of several of the other base units, so that the uncertain stability of the definition of the kilogram becomes spread among the definitions of a number of other units of our system.

This paper reviews the proposal of the CIPM which will be considered by the CGPM at its next meeting to revise the definition of the International System. It is proposed to revise the definitions of the kilogram, ampere, kelvin and mole, leaving the present definitions of the second, metre, and candela unchanged. It is also proposed to make significant changes to the way that the definition of the entire system is presented. These proposals together will constitute a major change to the system, and for convenience we refer to the revised system in this paper as the new SI to distinguish it from the present system which we refer to as the current SI. The proposed changes to the SI are also presented in Draft Resolution A for the CGPM [2], and Draft Chapter 2 for the new SI [3] which are available on the BIPM website.

The changes to the SI proposed here were suggested by Mills *et al.* [4,5], and were developed by the CCU at its meetings in 2009 and 2010. However they have been under discussion for more than ten years, and have been the subject of many published papers (see [6 – 11]). Even assuming these proposals are approved by the CGPM in 2011 they are still unlikely to be adopted for a further two to four years because we await the results of further experiments still in progress, which will improve our knowledge of the numerical values for the fundamental constants to be used in the definitions. Nonetheless, it is desirable that the proposals should be disseminated and widely discussed in the meantime.

It should be emphasised that when the proposed changes are finally adopted the values of the constants used in the new definitions will be chosen to maintain continuity, so that the magnitudes of the four re-defined units in the new SI will be essentially identical to their magnitudes in the current SI at the moment of adoption. For almost all practical applications of the SI by scientific and technical users, and for every-day commerce in the market place, the changes in the definitions of these four units will be of no consequence. Only for the most precise experimental measurements will the changes matter and we believe that in these cases the new definitions will be more robust, more fundamental, and better suited to new scientific developments. The adjective “new” will of course no longer be needed once the change has been made.

This paper is the opening paper presented at the Royal Society Discussion meeting on “The new SI: units of measurement based on fundamental constants” held at the Royal Society of London on 24 and 25 January 2011. It is intended to set the stage for later papers presented at this meeting, also included in this issue of the *Philosophical Transactions of the Royal Society A*, in which the choices made in the new SI will be discussed in greater detail.

In this paper we begin by presenting the proposals for the new SI in Section 2. The choices that have been made to arrive at these proposals, and further details of the changes involved in the new SI, are discussed in later sections of the paper.

2. The new SI

The definition of the new SI is presented below in Section 2.1 in two parts. In the first part the entire system is defined in a single statement by specifying the exact numerical values of seven chosen fundamental constants when they are expressed in terms of their new SI units. This has the effect of scaling the magnitudes of all the units of the new SI. In this statement no distinction is made between base units and derived units, nor is such a distinction needed. This approach is viewed as the simplest and most fundamental way of defining the International System.

The second part of the definition contains the individual definitions of the familiar seven base units second, metre, kilogram, ampere, kelvin, mole and candela, which follow from the first part. This second part is analogous to previous presentations of the definition of the SI, except that the order of the base units has been changed to s, m, kg, A, K, mol, cd, so that although several of the definitions depend on others in the list, no definition depends in a significant way on that of a unit that comes later in the list.

Although the scaling statement is itself sufficient to define the entire system, in recognition of the historical structure of the SI and the utility of the concepts of base and derived units the new SI retains these concepts. In particular the names and symbols of the 7 base units, and the 22 derived units with their relations to the base units, are retained unchanged in the new SI; these are listed in table 4 in the appendix in order to keep the main text of this paper concise. Table 5 in the appendix also lists the names and symbols for the multiple and sub-multiple prefixes that may be used to define units that are much larger or smaller than the coherent SI units; these are also unchanged from the current SI.

All the definitions are presented here by specifying the exact numerical values of seven fundamental constants when expressed in the new SI units. We describe this format as the use of "explicit-constant" definitions, and we shall call the seven constants "the SI reference constants". However each explicit-constant definition of a base unit in Part 2 of Section 2.1 is followed by the corresponding "explicit-unit" definition, in which the magnitude of the unit to be defined is expressed in terms of the magnitude of a quantity of the same kind as the unit which is related to the values of the SI reference constants.

It should be recognized that the choice of the seven units called "base units", from which all other "derived units" are obtained by multiplying together powers of the base units, is actually somewhat arbitrary. There is nothing new in this; the choice of the seven base units has always involved an arbitrary element, although this is not often stated.

2.1 The definition of the new SI

Part 1

The International System of Units, the SI, is the system of units in which

- the ground state hyperfine splitting frequency of the caesium 133 atom $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$ is exactly 9 192 631 770 hertz,
- the speed of light in vacuum c is exactly 299 792 458 metre per second,

- the Planck constant h is exactly 6.626 069... $\times10^{-34}$ joule second,
- the elementary charge e is exactly 1.602 176... $\times10^{-19}$ coulomb,
- the Boltzmann constant k is exactly 1.380 648... $\times10^{-23}$ joule per kelvin,
- the Avogadro constant N_A is exactly 6.022 141... $\times10^{23}$ reciprocal mole,
- the luminous efficacy K_{cd} of monochromatic radiation of frequency 540 $\times10^{12}$ Hz is exactly 683 lumen per watt,

where (see also table 4 in the appendix)

(i) the second is the unit of time, the metre the unit of length, the kilogram the unit mass, the ampere the unit of electric current, the kelvin the unit of thermodynamic temperature, the mole the unit of amount of substance and the candela is the unit of luminous intensity, having symbols, s, m, kg, A, K, mol, and cd respectively,

(ii) the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, and W, respectively, are related to the units second, metre, kilogram, ampere, kelvin, mole, and candela, according to $\text{Hz} = \text{s}^{-1}$, $\text{J} = \text{s}^{-2}\ \text{m}^2\ \text{kg}$, $\text{C} = \text{A s}$, $\text{lm} = \text{cd m}^2\ \text{m}^{-2} = \text{cd sr}$, and $\text{W} = \text{s}^{-3}\ \text{m}^2\ \text{kg}$, and

(iii) the numerical values that appear in these statements for the Planck constant h, the elementary charge e, the Boltzmann constant k and the Avogadro constant N_A should include additional digits that are not yet decided, to be taken from the most recent CODATA adjustment of the values of the fundamental constants [12] at the time the new SI is adopted, to preserve continuity so that the magnitudes of the new units agree with their current values to the precision that they are known in the current SI.

Part 2

It follows from Part 1 that the definition of each of the seven base units can be presented individually as shown below. For each base unit the formal definition is presented first in a bold sans-serif type in an explicit-constant form, followed by further words in a serif type to provide background and further understanding of the effect of each formal definition, and also to include the fully equivalent explicit-unit form for each definition.

second, unit of time

The second, s, is the SI unit of time; its magnitude is set by fixing the numerical value of the ground state hyperfine splitting frequency of the caesium 133 atom, at rest and at zero thermodynamic temperature, to be equal to exactly 9 192 631 770 when it is expressed in the unit s^{-1}, which is equal to Hz.

Thus we have the exact relation $\Delta\nu(^{133}\text{Cs})_{\text{hfs}} = 9\ 192\ 631\ 770$ Hz,

or $$\text{Hz} = \frac{\Delta\nu\left(^{133}\text{Cs}\right)_{\text{hfs}}}{9\ 192\ 631\ 770}$$

The effect of this definition is that the second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.

metre, unit of length

> **The metre, m, is the SI unit of length; its magnitude is set by fixing the numerical value of the speed of light in vacuum to be equal to exactly 299 792 458 when it is expressed in the unit $\mathbf{m\ s^{-1}}$.**

Thus we have the exact relation $c = 299\,792\,458$ m/s, or $\text{m/s} = \dfrac{c}{299\,792\,458}$

The effect of this definition is that the metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.

kilogram, unit of mass

> **The kilogram, kg, is the SI unit of mass; its magnitude is set by fixing the numerical value of the Planck constant to be equal to exactly $\mathbf{6.626\,069\ ... \times10^{-34}}$ when it is expressed in the unit $\mathbf{s^{-1}\ m^2\ kg}$, which is equal to J s.**

Thus we have the exact relation $h = 6.626\,069\ ... \times10^{-34}$ J s,

or $\text{J s} = \dfrac{h}{6.626\,069... \times10^{-34}}$

The value of the Planck constant is a constant of nature, which may be expressed as the product of a number and the unit joule second, the unit of action, where $\text{J s} = \text{s}^{-1}\ \text{m}^2\ \text{kg}$.

The effect of this definition, together with those for the second and the metre, is to express the unit of mass in terms of the unit of frequency through the fundamental equations $E = mc^2$ and $E = h\nu$.

ampere, unit of electric current

> **The ampere, A, is the SI unit of electric current; its magnitude is set by fixing the numerical value of the elementary charge to be equal to exactly $\mathbf{1.602\,176\ ... \times10^{-19}}$ when it is expressed in the unit s A, which is equal to the coulomb C.**

Thus we have the exact relation $e = 1.602\,176\ ... \times10^{-19}$ C, or $\text{C} = \dfrac{e}{1.602\,176... \times10^{-19}}$

The effect of this definition is that the ampere is the electric current corresponding to the flow of $1/(1.602\,176... \times10^{-19})$ elementary charges per second.

kelvin, unit of thermodynamic temperature

> **The kelvin, K, is the SI unit of thermodynamic temperature; its magnitude is set by fixing the numerical value of the Boltzmann constant to be equal to exactly $\mathbf{1.380\,648\ ... \times10^{-23}}$ when it is expressed in the unit $\mathbf{s^{-2}\ m^2\ kg\ K^{-1}}$, which is equal to $\mathbf{J\ K^{-1}}$.**

Thus we have the exact relation $k = 1.380\,648\ ... \times10^{-23}$ J/K, or $\quad \mathrm{J/K} = \dfrac{k}{1.380\ 648\ldots\times 10^{-23}}$

The effect of this definition is that the kelvin is equal to the change of thermodynamic temperature T that results in a change of thermal energy kT by 1.380 648 ... $\times 10^{-23}$ J.

mole, unit of amount of substance

> **The mole, mol, is the SI unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles; its magnitude is set by fixing the numerical value of the Avogadro constant to be equal to exactly 6.022 141 ... $\times 10^{23}$ when it is expressed in the unit mol^{-1}.**

Thus we have the exact relation $N_\mathrm{A} = 6.022\ 141\ldots \times 10^{23}\ \mathrm{mol}^{-1}$,

or $\quad \mathrm{mol}^{-1} = \dfrac{N_\mathrm{A}}{6.022\,141\ldots\times 10^{23}}$

The effect of this definition is that the mole is the amount of substance of a system that contains 6.022 141 ... $\times 10^{23}$ specified elementary entities.

candela, unit of luminous intensity

> **The candela, cd, is the unit of luminous intensity in a given direction; its magnitude is set by fixing the numerical value of the luminous efficacy of monochromatic radiation of frequency 540 $\times 10^{12}$ Hz to be equal to exactly 683 when it is expressed in the unit $\mathrm{s^3\ m^{-2}\ kg^{-1}\ cd\ sr}$, or $\mathrm{cd\ sr\ W^{-1}}$, which is equal to $\mathrm{lm\ W^{-1}}$.**

Thus we have the exact relation $K_\mathrm{cd} = 683\ \mathrm{lm/W}$, or $\mathrm{lm/W} = \dfrac{K_\mathrm{cd}}{683}$,

where K_cd is the luminous efficacy of monochromatic radiation of frequency $\nu = 540 \times 10^{12}$ Hz. The effect of this definition is that the candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 $\times 10^{12}$ Hz and that has a radiant intensity in that direction of 1/683 W/sr.

2.2 Relations among the individual definitions of the base units

In Part 2 of Section 2.1 the individual definitions of the seven base units of the new SI are presented. Of these seven definitions only the first, the definition of the second, and the sixth, the definition of the mole, are independent of the other definitions. Fixing the numerical value of the speed of light in vacuum actually defines the unit of speed, m/s, so that the definition of the second is required to complete the definition of the metre. Fixing the numerical value of the Planck constant actually defines the unit of action, $\mathrm{J\ s = s^{-1}\ m^2\ kg}$, so that the definitions of the second and the metre are required to complete the definition of the kilogram. Fixing the numerical value of the elementary charge actually defines the unit of charge, A s, or coulomb, C, so that the definition of the second is required to complete the definition of the ampere. Fixing the numerical value of the Boltzmann constant actually fixes the value of the unit of energy per temperature interval, J K^{-1}, so that the definitions of the second, kilogram, and metre are required to complete the definition of the kelvin. And finally, fixing the numerical value of the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz actually defines the unit of luminous efficacy, lm/W = cd sr W^{-1}, or $s^3\ m^{-2}\ kg^{-1}$ cd sr, so that the definitions of the second, kilogram, and metre are required to complete the definition of the candela.

It follows that the definitions of the base units in Part 2 must be taken together as a coherent group for the definitions of the base units of the new SI, and should not be regarded as independent definitions of the individual base units. The same was true in all previous presentations of the SI.

3 Explicit constant and explicit unit definitions

In Section 2.1 the definitions of the units have been presented in each case by specifying the exact numerical value for each of the seven chosen fundamental constants, the SI reference constants, used in the definitions. We call these "explicit-constant" definitions. It has been customary in the past (for example in all editions of the SI Brochure) to define each unit in terms of the value of a chosen reference quantity of the same kind as the unit. We call these "explicit-unit" definitions. Although these two alternatives are equivalent, we believe that explicit-constant definitions are fundamentally simpler and thus to be preferred. In particular it is the use of the explicit-constant format which allows us to present the definition of the entire SI in a single statement, as in Part 1 of Section 2.1, without introducing the distinction between base and derived units – which indeed is not really required. In Part 2, where we present the individual definitions of the seven traditional base units, we have followed each formal explicit-constant definition (in bold sans-serif type) with the fully equivalent explicit-unit definition (in serif type), for comparison and ease of interpretation.

Explicit-constant definitions may be understood as follows. To express the value of a fundamental constant, we write it as the product of a number $\{Q\}$ and a unit $[Q]$,

$$Q = \{Q\}\ [Q]$$

The value of the constant Q is assumed to be a true invariant. It is for this reason that we describe it as a constant of nature. However we may choose the unit $[Q]$ in different ways, giving different numerical values $\{Q\}$. If we define the unit $[Q]$ independently, then we must determine the corresponding number $\{Q\}$ by experiment, and it will have an experimental uncertainty. Alternatively we may take the number $\{Q\}$ to have an exact value chosen to suit our convenience, but then the value of the unit $[Q]$ will be defined by our choice. This is an explicit-constant definition.

As an example, consider the speed of light in vacuum c, which may be expressed by the equation

$$c = \{c\}\ [c] = 299\ 792\ 458 \text{ m/s}$$

Before 1983 the metre was defined in terms of the wavelength of a specified krypton atomic line, and the second was defined in terms of the frequency of the caesium hyperfine transition; we then had to determine the numerical value $\{c\}$ by experiment, and it had an experimental uncertainty. However in 1983 the CGPM decided to fix the numerical value to the exact number 299 792 458, so that it had no uncertainty; the effect of this was then to define the value of the unit m/s. Since the definition of the second was retained unchanged in terms of the caesium transition, the effect was (indirectly) to define the value of the metre.

The formal definition of the entire SI in Part 1 of Section 2.1 is expressed by defining the exact numerical values of the seven SI reference constants, following the explicit-constant format. Each of the formal definitions of the seven base units in Part 2 is presented first as an explicit-constant definition, each then being followed by the equivalent explicit-unit definition to provide further understanding and show the relationship to the format of earlier presentations. Drafting an appropriate explicit-unit definition of the kilogram, in Part 2 of Section 2.1, poses difficulties in finding simple words to relate the unit of mass to the unit of action. The words we have chosen use the two equations $E = mc^2$ and $E = h\nu$ (see Bordé, reference [13]).

4 Choosing the fundamental constants to be used in the definitions

The seven reference constants used to define units of the SI, which we are calling the *SI reference constants*, are summarized in table 1, both for the current SI and for the new SI. In the traditional representation of the SI the seven SI reference constants are taken to define the seven base units individually (as in Part 2 of Section 2.1); in the alternative presentation of the entire system in a single statement (as in Part 1) the seven SI reference constants are not necessarily associated with base units.

Table 1

Reference constants used to define the seven base units in the current SI and the new SI

Unit		*Reference constant used in* the *current SI*		*Reference constant used in the new SI*	
second,	s	$\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$		$\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$	Cs hyperfine splitting
metre,	m	c		c	speed of light in vacuum
kilogram,	kg	$m(\mathcal{K})$	mass of the IPK	h	Planck constant
ampere,	A	μ_0	magnetic constant	e	elementary charge
kelvin,	K	T_{TPW}	temperature of the triple point of water	k	Boltzmann constant
mole,	mol	$M(^{12}\mathrm{C})$	molar mass of ^{12}C	N_{A}	Avogadro constant
candela,	cd	K_{cd}		K_{cd}	luminous efficacy of a 540 THz source

In the current SI the mass of the international prototype of the kilogram (IPK), $m(\mathcal{K})$, the magnetic constant μ_0 (permeability of vacuum), the thermodynamic temperature of the triple point of water T_{TPW}, and the molar mass of the carbon 12 atom $M(^{12}\mathrm{C})$, are used to define the kilogram, ampere, kelvin and mole; in the new SI these four constants are replaced by the Planck constant h, the elementary charge e, the Boltzmann constant k and the Avogadro constant N_{A} respectively. The hyperfine splitting frequency of the caesium 133 atom $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$ and the speed of light in vacuum c are retained to define the second and the metre. The candela is a special case, and there is no proposal to change the definition at present. The present definition of the candela is referenced to the constant K_{cd} which may be regarded as a constant of nature.

The four new reference constants listed above, h, e, k and N_{A}, which follow the recommendation of the CCU and the CIPM, were chosen for the following reasons. The values of h and e, together with $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$ and c, define the metre, kilogram and ampere, as in Part 2 of Section 2.1. Having exactly defined values for h and c, which are the fundamental constants of quantum mechanics and relativity theory, will be a significant advantage for fundamental physics. Similarly having exactly defined values of both h and e will be a significant advantage for electromagnetic measurements, because precise measurements in this field depend on a knowledge of the quantities $2e/h$ and h/e^2, which are generally regarded as the theoretical expressions for the Josephson constant K_{J} and the von Klitzing constant R_{K} involved in Josephson effect measurements to determine electromotive force and quantum Hall effect measurements to determine electrical resistance. In the new SI there will no longer be any need to use the conventional values $K_{\mathrm{J-90}}$ and $R_{\mathrm{K-90}}$, which were adopted in 1990 when uncertainties in the experimental values of $2e/h$, and h/e^2 were greater than the reproducibility of Josephson and quantum Hall effect experiments. The conventional values lead to electromagnetic units that are not actually part of the current SI; thus in the new SI electromagnetic measurements and units will be brought into the system by fixing the values of h and e. The elementary charge e is also a conceptually simpler constant to use as a reference than the magnetic constant μ_0, and will be easier to teach. Although defining the kilogram by fixing the value of the Planck constant is conceptually more complex than, for example, defining the kilogram in terms of the mass of the electron or a specified atom, the other advantages of fixing the value of the Planck constant make this the preferable definition. See also the paper by Bordé [13], as well as those of Becker [14], Stock [15] and Davis [16] in this issue.

The Boltzmann constant k will have a precisely defined value in the new SI, in place of the thermodynamic triple point temperature of water T_{TPW} in the current SI, to provide the definition of the kelvin. The realization of the definition in terms of T_{TPW} requires that a particular state of matter, the triple point of pure water of a specified isotopic composition, be set up and that the temperatures of other states of matter be related to it by primary methods of thermometry. The new definition will allow the kelvin to be realized by a wide variety of experiments, over a wide range of different temperatures, by direct measurement of the thermodynamic temperature of any state of matter (or radiation) at equilibrium. See also the paper by Pitre [17] in this issue.

The Avogadro constant N_{A} will have a precisely defined value in the new SI to define the mole, in place of the molar mass of carbon 12, $M(^{12}\mathrm{C})$, in the current SI. The new definition will have the advantage of being a conceptually simpler way of defining the mole, which emphasizes that the quantity amount of substance, of which the mole is the unit, is concerned with counting entities, to be distinguished from the quantity mass.

Amount of substance n and mass m are both quantities that may be used to quantify a sample, and for a pure sample s of molar mass M_s they are related by the equation

$$n_s = m_s/M_s$$

However they are fundamentally different quantities; it is the relation of amount of substance to counting entities which is the reason for its importance to stoichiometric relations in chemistry. See also the paper and by Milton [18] in this issue, and that by Milton and Mills [19].

Tables 2 and 3 illustrate the effect of the new definitions on the uncertainties with which the fundamental constants will be known in the new SI in comparison with the current SI. The uncertainties in these tables are based on the CODATA 2010 recommended values of the constants [12] but are expected to be smaller by the time the new SI is adopted.

Table 2 shows the relative standard uncertainty u_r in the constants used to define the kilogram, ampere, kelvin and mole in both the current SI and the new SI. The table shows that in each case the value of u_r is switched between the two constants that are interchanged on introducing the new SI. Table 3 shows the uncertainty u_r in a selection of fundamental constants in both the current SI and the new SI. The table shows that the change from the current SI to the new SI would lead to the value of u_r, in almost every case, being either reduced by more than an order of magnitude, or being reduced to zero for those constants that become exactly determined, on introducing the new SI.

Table 2

The effect of changing from the current SI to the new SI on the relative standard uncertainties of the constants used to define the kilogram, ampere, kelvin and mole.

unit	*constant used to define the unit*	*symbol*	*uncertainty in the current SI*		*uncertainty in the new SI*	
kg	mass of the IPK	$m(\mathcal{K})$	exact	0.0	expt	4.4×10^{-8}
	Planck constant	h	expt	4.4×10^{-8}	exact	0.0
A	magnetic constant	μ_0	exact	0.0	expt	3.2×10^{-10}
	elementary charge	e	expt	2.2×10^{-8}	exact	0.0
K	temperature of TPW	T_{TPW}	exact	0.0	expt	9.1×10^{-7}
	Boltzmann constant	k	expt	9.1×10^{-7}	exact	0.0
mol	molar mass of ^{12}C	$M(^{12}C)$	exact	0.0	expt	7.0×10^{-10}
	Avogadro constant	N_A	expt	4.4×10^{-8}	exact	0.0

Table 3

Relative standard uncertainties for a selection of fundamental constants in the current SI and the new SI, multiplied by 10^8 (i.e. in parts per hundred million)

constant	*current SI*	*new SI*	*constant*	*current SI*	*new SI*
$m(\mathcal{K})$	0	4.4	α	0.032	0.032
h	4.4	0	K_J	2.2	0
e	2.2	0	R_K	0.032	0
k	91	0	μ_0	0	0.032
N_A	4.4	0	ε_0	0	0.032
R	91	0	Z_0	0	0.032
F	2.2	0	$N_A h$	0.070	0
σ	360	0	J↔kg	0	0
m_e	4.4	0.064	J↔m^{-1}	4.4	0
m_u	4.4	0.070	J↔Hz	4.4	0
$m(^{12}C)$	4.4	0.070	J↔K	91	0
$M(^{12}C)$	0	0.070	J↔eV	2.2	0

Note: in this table the symbols denote the following constants. R: the molar gas constant; F: the Faraday constant; σ: the Stefan-Boltzmann constant; m_e: the electron mass; m_u: the unified atomic mass constant; $m(^{12}C)$: the mass of a carbon 12 atom; $M(^{12}C)$: the molar mass of carbon 12; α : the fine structure constant; K_J and R_K: the Josephson and von Klitzing constants; μ_0 and ε_0: the magnetic and electric constants; Z_0: the impedance of vacuum; $N_A h$: the molar Planck constant; and U_a↔U_b: the conversion factor between unit a and unit b.

5 Choosing the numerical values of the constants to be adopted in the definitions

In the new SI the numerical values to be chosen for the four new constants to be used in the definition, namely h, e, k, and N_A, must be taken from the most recent CODATA least squares review of the values of the fundamental constants at the time the new SI is adopted, in order to preserve continuity with the magnitudes of the units in the current SI. These CODATA reviews appear at four-yearly intervals; the most recent published at present is from Mohr *et al.* [12]. There are a number of relations between the constants used in the definitions which follow from the laws of physics, and for this reason it is essential that the values used for the constants are obtained from a single least-squares calculation of all the available data, in order to ensure the internal consistency of the values adopted. Because new experimental results are expected to be published in the next few years, the actual adoption of the proposals for the new SI presented here should await these results, and the new CODATA review of the values of the fundamental constants which will follow from the new data.

A few of the relevant quantity equations relating the fundamental constants used in the definitions, in both the current SI and the new SI, are given below. All these equations are relations between physical quantities; they remain true whatever units are used to specify the values of the quantities involved. In particular they remain true whether units of the current SI or units of the new SI are used. However any self consistent set of values for the physical constants must satisfy all of these equations, which also yield information on the relative standard uncertainties u_r in the fundamental constants as shown in tables 2 and 3.

$$M(^{12}\mathrm{C}) = N_A m(^{12}\mathrm{C}) \tag{5.1}$$

$$M_u = N_A m_u \tag{5.2}$$

In the current SI the molar mass of carbon 12, $M(^{12}\mathrm{C})$, is exactly defined as 12 g/mol, and $M_u = M(^{12}\mathrm{C})/12$, the molar mass constant [10], is exactly defined as 1 g/mol, whereas the two factors on the right hand side of (5.1) and (5.2) have a relative standard uncertainty u_r which is the same for both factors: $u_r(N_A) = u_r(m_u) = u_r(m(^{12}\mathrm{C}))$. In the new SI N_A will be exactly defined, and both $M(^{12}\mathrm{C})$ and M_u will have an uncertainty, which will be the same as the uncertainty in $m(^{12}\mathrm{C})$ and m_u.

$$\frac{h}{m_e} = \frac{\alpha^2 c}{2R_\infty} \tag{5.3}$$

$$\frac{h}{m_u} = \frac{\alpha^2 c\, A_r(\mathrm{e})}{2R_\infty} = \frac{\alpha^2 c}{2R_\infty}\left(\frac{m_e}{m_u}\right) \tag{5.4}$$

$$\frac{h}{m(^{12}\mathrm{C})} = \frac{\alpha^2 c}{2R_\infty}\left(\frac{m_e}{m(^{12}\mathrm{C})}\right) \tag{5.5}$$

$$\frac{h}{m(^{28}\mathrm{Si})} = \frac{\alpha^2 c}{2R_\infty}\left(\frac{m_e}{m(^{28}\mathrm{Si})}\right) \tag{5.6}$$

$$h N_A = \frac{\alpha^2 c\, A_r(\mathrm{e}) M_u}{2R_\infty} \tag{5.7}$$

Equation (5.3) is the equation for the Rydberg constant originally derived by Bohr, and (5.4) through (5.7) are essentially different ways of writing (5.3). The symbol $A_r(\mathrm{e}) = 12 m_e/m(^{12}\mathrm{C}) = m_e/m_u$ denotes the relative atomic mass of the electron. In all these equations α^2, c, $A_r(\mathrm{e})$, and R_∞ are either known exactly or are known to better than a part in 10^9. The same applies to $m_e/m(^{12}\mathrm{C})$ and to $m_e/m(^{28}\mathrm{Si})$. Thus equations (5.3) through (5.6) provide essentially exact relations between the two quantities on the left hand side of each equation.

In the current SI, equation (5.7) provides a similar relation between h and N_A. However in the new SI the values of h and N_A will both be exactly known, but M_u will be an experimentally determined quantity with an uncertainty. Thus equation (5.7) can provide information on the value and uncertainty of M_u in terms of the values and uncertainties of the other factors on the right hand side, as shown in table 3.

It is customary to describe the x-ray crystal density (XRCD) experiment as one experiment to determine the value of the Avogadro constant, but this will be misleading in the new SI where both h and N_A will be exactly known. Instead the XRCD experiment will become an alternative means comparable in accuracy to the watt balance to realize the unit of mass in the new SI.

Another important relation between the physical constants is that between μ_0 and e,

$$\frac{h}{e^2} = \frac{\mu_0 c}{2\alpha} \tag{5.8}$$

This relation shows that in the new SI, where c, h and e will all be known exactly, the relative uncertainty $u_r(\mu_0)$ will be the same as $u_r(\alpha)$, as shown in tables 2 and 3.

6 Realising the definitions

An essential aspect of the definitions of the units in the SI is the practical need to realize the definitions experimentally. This is particularly important for the seven base units, since the realization of a derived unit, in principle, follows from expressing it in terms of the base units and then realizing the base units. The definitions of the base units proposed in the new SI have been made with the need for their experimental realization in mind, but the definitions themselves should not, and do not, imply any particular experiment to realize the unit. As new experiments are developed new methods of realizing the units may be proposed. Thus it is an advantage of the explicit-constant format for the definitions that it does not imply any particular method of experimental realization. For this reason the *mises en pratique* are not discussed in detail in this paper. Nonetheless helpful advice on realizing the definitions is provided in the form of *mises en pratique* by the Consultative Committees of the CIPM. These are not published in the SI Brochure [1], but are available on the BIPM website for each of the base units. See Appendix 2 of Ref. [1], entitled "Practical realization of the definitions of some important units", which is published in electronic form only and is available on the BIPM website at **http://www.bipm.org/en/si/si_brochure/appendix2/**. These *mises en pratique* will be adapted to the new SI when it is adopted, and will be revised from time to time as new experiments are devised, probably more frequently than new editions of the Brochure are published. For example, advice for the realization of the kilogram in the new SI, defined by reference to the Planck constant, will clearly refer to both the watt-balance experiment, and the x-ray crystal density experiment using a single crystal sphere of nearly pure silicon 28.

7 Summary

This paper presents the proposal which is currently being considered to revise and modernize the international system of units, the SI, to meet the challenges of the 21st century. Although the current SI has served the needs of the scientific community well since it was established in 1960, there are now compelling reasons for modernizing the system. In brief, the new SI as presented here will meet the growing needs of science, technology, and commerce in the 21st century by providing

- a definition of the kilogram based on an invariant of nature, the Planck constant, rather than a material artefact, so as to ensure the long-term stability of the SI unit of mass and other mechanical units of the SI, as well as to enable the SI unit of mass to be realized at any place, at any time, by anyone;
- exactly known values of h and e, thereby providing exactly known values of the Josephson constant $K_J = 2e/h$ and the von Klitzing constant $R_K = h/e^2$ and hence a means, via the Josephson and quantum Hall effects, to realize the ampere, volt, and ohm as well as other electric units with unprecedented accuracy resulting in the elimination of the present system of conventional non-SI electric units;
- a definition of the kelvin based on a true invariant of nature, the Boltzmann constant, which can be realized by a wide variety of experiments at a wide range of temperatures, rather than a property of water that can be realized at only one temperature and depends on impurity content, isotopic composition, etc.;
- a definition of the mole that will help eliminate the present poor understanding of the SI base quantity amount of substance, which is independent of mass, and its unit mole, which is a unit to count the number of entities;
- definitions of all seven SI base units written in a common explicit-constant form, thereby bringing to the SI base-unit definitions simplicity, uniformity, and ease of understanding; and
- significant improvements in our knowledge of the values of the fundamental constants, an advance that will be of major benefit to the many scientists, engineers, and students who regularly use values of the constants in their work – not only will h, e, k, and N_A be exactly known, but many other constants and energy equivalency factors will also be exactly known or will have significantly reduced uncertainties, implying that the changes in the recommended values of the constants and factors from one CODATA least-squares adjustment to the next will be significantly reduced.

Above all, the changes in the SI proposed here will strengthen the philosophical foundation of our system of units in relation to our present understanding of theoretical and quantum physics.

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Appendix

Table 4

Relations between the 7 base units and the 22 derived units with special names in the SI. The same relations hold without change in the new SI. Note that the first seven entries are the traditional base quantities and units.

quantity	*name of unit*	*symbol for unit*	*expression in terms of other units*
time	Second	s	
length	Metre	m	
mass	Kilogram	kg	
electric current	Ampere	A	
thermodynamic temperature	Kelvin	K	
amount of substance	Mole	mol	
luminous intensity	Candela	cd	
plane angle	Radian	rad	= m/m = 1
solid angle	Steradian	sr	= m^2/m^2 = 1
frequency	Hertz	Hz	= s^{-1}
force	Newton	N	= s^{-2} m kg
pressure, stress	Pascal	Pa	= N/m^2 = $s^{-2}\,m^{-1}$ kg
energy, work, amount of heat	Joule	J	= N m = $s^{-2}\,m^2$ kg
power, radiant flux	Watt	W	= J/s = $s^{-3}\,m^2$ kg
electric charge, amount of electricity	Coulomb	C	= s A
electric potential difference	Volt	V	= W/A = J/C = $s^{-3}\,m^2\,kg\,A^{-1}$
capacitance	Farad	F	= C/V = $s^4\,m^{-2}\,kg^{-1}\,A^2$
electric resistance	Ohm	Ω	= V/A = $s^{-3}\,m^2\,kg\,A^{-2}$
electric conductance	Siemens	S	= A/V = $s^3\,m^{-2}\,kg^{-1}\,A^2$
magnetic flux	Weber	Wb	= V s = $s^{-2}\,m^2\,kg\,A^{-1}$
magnetic flux density	Tesla	T	= Wb/m^2 = $s^{-2}\,kg\,A^{-1}$
inductance	Henry	H	= Wb/A = $s^{-2}\,m^2\,kg\,A^{-2}$
Celsius temperature	degree Celsius	^{o}C	= K
luminous flux	Lumen	lm	= cd sr = cd
illuminence	Lux	lx	= lm/m^2 = m^{-2} cd
activity referred to a radionuclide	Becquerel	Bq	= s^{-1}
absorbed dose, specific energy (imparted), kerma	Gray	Gy	= J/kg = $s^{-2}\,m^2$

dose equivalent, ambient dose equivalent	Sievert	Sv	= J/kg = $s^{-2}\,m^2$
catalyticactivity	Katal	kat	= s^{-1} mol

Table 5

The SI multiple and sub-multiple prefixes. When prefixes are used, the prefix name and the unit name are combined to form a single word, and the prefix symbol and the unit symbol are written without any space to form a single symbol, which may itself be raised to any power.

factor	*name*	*symbol*	*factor*	*name*	*symbol*
10	deca	da	10^{-1}	deci	d
10^2	hecto	h	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^6	mega	M	10^{-6}	micro	μ
10^9	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a
10^{21}	zeta	Z	10^{-21}	zepto	z
10^{24}	yotta	Y	10^{-24}	yocto	y