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Extending Murty interferometry to the Terahertz part of the spectrum

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Abstract. We discuss some novel technologies that enable the implementation of shearing interferometry at the terahertz part of the spectrum. Possible applications include the direct measurement of lens parameters, the measurement of refractive index of materials that are transparent to terahertz frequencies, determination of homogeneity of samples, measurement of optical distortions and the non-contact evaluation of thermal expansion coefficient of materials buried inside media that are opaque to optical or infrared frequencies but transparent to THz frequencies. The introduction of a shear to a Gaussian free-space propagating terahertz beam in a controlled manner also makes possible a range of new encoding and optical signal processing modalities.

1. Introduction

Shearing interferometry is a well established measurement modality which has been extensively implemented using various optical topologies at the optical or infrared parts of the spectrum [1-21]. Advances in monochromatic terahertz sources as well as electro-optic detector technologies now enable us to directly translate some of these systems to longer wavelengths. Advantages from the proposed approach are the longer wavelength implementation which results in better fringe stability and visibility due to low phase noise, the ability to accurately control the source wavelength through heterodyne frequency locking techniques, the use of very efficient broadband polarizing beam splitting components (wire grids), a naturally diffractive spreading of the beam, reduced scattering, greater penetration length of terahertz radiation through fog or ash clouds, etc. The parallel or slightly wedged plate that is used in most shearing interferometers can be made out of high density polyethylene, TPX or other polymer material that happens to be transparent at the terahertz part of the spectrum. Of particular interest is the possibility to use pyrolytic Boron Nitride which is transparent at THz frequencies [22], and has a high refractive index. Detection of the sheared wavefronts should be possible either directly with bolometer arrays which are currently under development [23-27] or indirectly through an electro-optic imaging detection system, where the THz radiation is spatially diplexed with a polarized infrared beam incident on an electro-optic crystal so as to induce a change of infrared polarization which is then monitored using a CCD camera. Alternatively, a Wollaston prism can be used to spatially separate the two polarization states so that a differential imaging photodetection imaging scheme (pixel by pixel) may provide the required fringe visibility. In the following section we discuss some interesting measurement modalities that can be explored.

In any shear interferometer, the original wavefront $W(x,y)$ interferes with a copy of itself laterally displaced by an amount S in the x direction so the new wavefront is $W(x-S,y)$. By expanding in a taylor series, the optical path difference between the two wavefronts may be written as:

$$W(x,y) - W(x-S,y) = \left(\frac{\partial W}{\partial x}\right)S - \left(\frac{\partial^2 W}{\partial x^2}\right)\frac{S^2}{2} + \ldots \tag{1}$$

For very small displacement S, only the first terms needs to be considered i.e., $S << 2\left(\frac{\partial W}{\partial x}\right) / \left(\frac{\partial^2 W}{\partial x^2}\right)$.

To a first order approximation, the shape of the fringes is given by: $(\partial W / \partial x) S = n\lambda$. A lateral shear interferometer, therefore, measures the transverse aberration in the direction of the shear. The lateral shear for a plate of thickness T and refractive index n assuming an incidence angle θ, is given by:

$$S = \frac{T \sin 2\theta}{\sqrt{n^2 - \sin^2\theta}} \tag{2}$$

2. Direct measurement of refractive index and birefringence of materials,

In Figure 1, a THz shearing interferometer for the measurement of the refractive index of a liquid is presented.

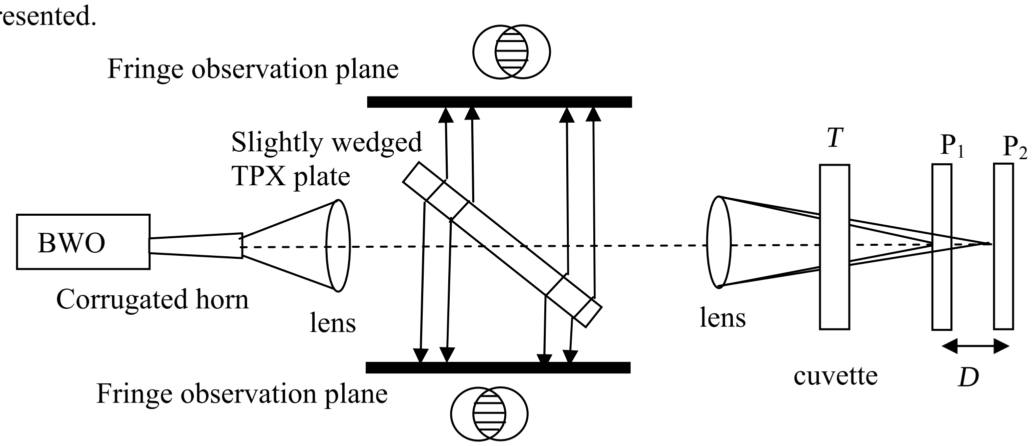

Figure 1. parallel plate shearing interferometer for the measurement of refractive index of liquids.

Initially a measurement is performed with the empty cuvette in place and horizontal fringes are observed for a plane mirror positiononed at P_1. The introduction of liquid in the cuvette, leads to a new position P_2 for which the fringes are parallel in the observation plane. The refractive index of the liquid introduced in the cuvette is given from:

$$n_l = \frac{T}{T-D} \tag{3}$$

For the measurement of birefringence, three measurements need to be performed, by moving the lens in three consecutive positions and the cuvette is replaced by a parallel plate sample and similar expressions for the refractive index of the ordinary and extraordinary rays as shown in equation 3 are performed. Changing the polarization of the source from horizontal to vertical is necessary in this arrangement, this can be conveniently accomplished by introducing a Martin-Puplett interferometer between the source and the wedged plate. The birefringence can then be obtained from:

$$n_e - n_o = \frac{1}{2}\left[\left(n_1^* - n_1\right) + \left(n_2^* - n_2\right) + \left(n_3^* - n_3\right)\right] \tag{4}$$

where subscripts indicate the refractive index of the ordinary and extraordinary rays measurements and that measurements are performed along the three directions of the sample. Similar arrangements can be used to measure the refractive index of materials that are transparent to terahertz frequencies, determine the homogeneity of samples, measure optical distortions and to perform the non-contact evaluation of thermal expansion coefficient of materials buried inside media that are opaque to optical or infrared frequencies but transparent to THz frequencies.

3. Shearing interferometry with phase-conjugate surfaces

It is well known that phase-conjugate surfaces developed for the terahertz part of the spectrum, when placed at regularly spaced intervals, can be used to efficiently eliminate wavefront distortion (Figure 2). For an incident wave-front $e^{i(\omega t+\phi)}$ on a non-linear surface with a phase correction $e^{i2\omega t}$ the reflected waves will have a phase $e^{i(\omega t+\phi)}e^{i2\omega t} = e^{i(2\omega t-\omega t-\phi)} = e^{i(\omega t-\phi)}$. The concept has already been demonstrated at mm-wave frequencies using an electronic approach to conjugate the signal at specific antenna elements [28]. Sampling at sufficiently dense intervals it is possible to conjugate an entire wave-front thus eliminating any distortions. This information can be used for improving the coupling between THz waveforms.

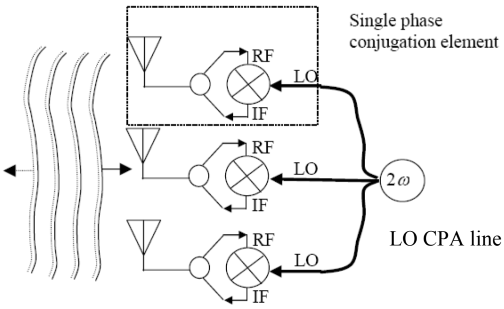

Figure 2. Array of phase conjugation elements sampling an entire THz wavefront with a local oscillator (LO) signal distributed by a network of CPA lines.

The proposed system can be used to introduce a programmable shear to a free space propagating beam or used in tandem with a shearing interferometer to perform accurate measurements by nulling the observed change in the fringes.

4. The application of shearing interferometry for information processing

It is well known that a thin lens can produce a 2-dimensional Fourier transform of an image on the back of its plane. Introduction of a shear in a controlled manner to a free space propagating Gaussian beam, provides a new degree of freedom for encoding and decoding information. The shear performs a transformation of fractional order, which can be used for phase retrieval, signal analysis, and filtering [29-54]. The relevance of fractional order transforms to understand the free space propagation of Gaussian beams described by Gauss-Laguerre or Gauss Hermite polynomials will be discussed in more detail at the conference.

5. Conclusion

A range of new wavefront shearing measurement modalities that can be implemented in the THz part of the spectrum have been suggested. Terahertz shearing interferometry is still at its infancy but because of the possible applications, it is likely to become an emergent research area once fringe imaging technologies become more widely available.

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