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2011 J. Phys.: Conf. Ser. 307 012012

(http://iopscience.iop.org/1742-6596/307/1/012012)

Extending Murty interferometry to the Terahertz part of the spectrum

S. Hadjiloucas, G.C. Walker and J.W. Bowen

Cybernetics, School of Systems Engineering, The University of Reading, RG6 6AY, UK

Correspondance email: s.hadjiloucas@reading.ac.uk

Abstract. We discuss some novel technologies that enable the implementation of shearing interferometry at the terahertz part of the spectrum. Possible applications include the direct measurement of lens parameters, the measurement of refractive index of materials that are transparent to terahertz frequencies, determination of homogeneity of samples, measurement of optical distortions and the non-contact evaluation of thermal expansion coefficient of materials buried inside media that are opaque to optical or infrared frequencies but transparent to THz frequencies. The introduction of a shear to a Gaussian free-space propagating terahertz beam in a controlled manner also makes possible a range of new encoding and optical signal processing modalities.

1. Introduction

Shearing interferometry is a well established measurement modality which has been extensively implemented using various optical topologies at the optical or infrared parts of the spectrum [1-21]. Advances in monochromatic terahertz sources as well as electro-optic detector technologies now enable us to directly translate some of these systems to longer wavelengths. Advantages from the proposed approach are the longer wavelength implementation which results in better fringe stability and visibility due to low phase noise, the ability to accurately control the source wavelength through heterodyne frequency locking techniques, the use of very efficient broadband polarizing beam splitting components (wire grids), a naturally diffractive spreading of the beam, reduced scattering, greater penetration length of terahertz radiation through fog or ash clouds, etc. The parallel or slightly wedged plate that is used in most shearing interferometers can be made out of high density polyethylene, TPX or other polymer material that happens to be transparent at the terahertz part of the spectrum. Of particular interest is the possibility to use pyrolytic Boron Nitride which is transparent at THz frequencies [22], and has a high refractive index. Detection of the sheared wavefronts should be possible either directly with bolometer arrays which are currently under development [23-27] or indirectly through an electro-optic imaging detection system, where the THz radiation is spatially diplexed with a polarized infrared beam incident on an electro-optic crystal so as to induce a change of infrared polarization which is then monitored using a CCD camera. Alternatively, a Wollaston prism can be used to spatially separate the two polarization states so that a differential imaging photodetection imaging scheme (pixel by pixel) may provide the required fringe visibility. In the following section we discuss some interesting measurement modalities that can be explored.

In any shear interferometer, the original wavefront $W(x,y)$ interferes with a copy of itself laterally displaced by an amount S in the x direction so the new wavefront is $W(x-S,y)$. By expanding in a taylor series, the optical path difference between the two wavefronts may be written as:

$$W(x,y) - W(x-S,y) = \left(\frac{\partial W}{\partial x}\right) S - \left(\frac{\partial^2 W}{\partial x^2}\right)\frac{S^2}{2} + \dots \tag{1}$$

For very small displacement S, only the first terms needs to be considered i.e., $S << 2\left(\frac{\partial W}{\partial x}\right) / \left(\frac{\partial^2 W}{\partial x^2}\right)$.

To a first order approximation, the shape of the fringes is given by: $(\partial W / \partial x) S = n\lambda$. A lateral shear interferometer, therefore, measures the transverse aberration in the direction of the shear. The lateral shear for a plate of thickness T and refractive index n assuming an incidence angle θ, is given by:

$$S = \frac{T \sin 2\theta}{\sqrt{n^2 - \sin^2\theta}} \tag{2}$$

2. Direct measurement of refractive index and birefringence of materials,

In Figure 1, a THz shearing interferometer for the measurement of the refractive index of a liquid is presented.

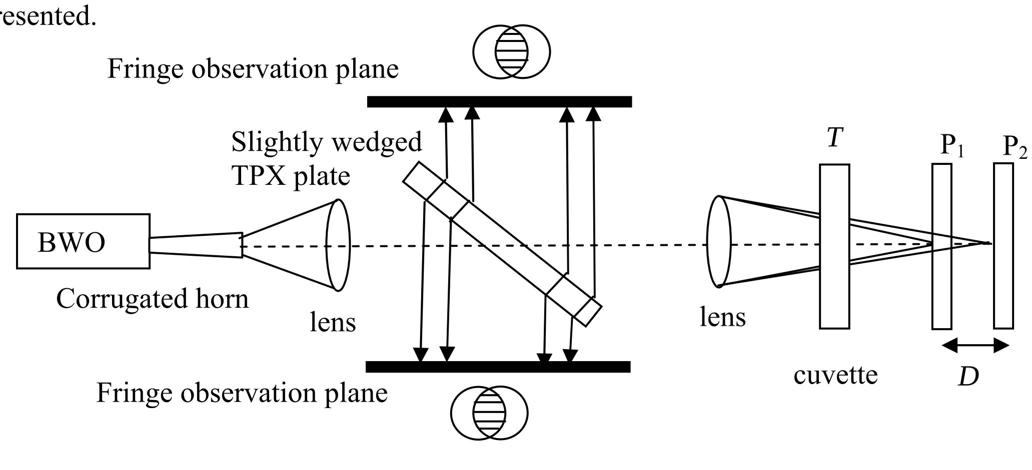

Figure 1. parallel plate shearing interferometer for the measurement of refractive index of liquids.

Initially a measurement is performed with the empty cuvette in place and horizontal fringes are observed for a plane mirror positiononed at P_1. The introduction of liquid in the cuvette, leads to a new position P_2 for which the fringes are parallel in the observation plane. The refractive index of the liquid introduced in the cuvette is given from:

$$n_l = \frac{T}{T-D} \tag{3}$$

For the measurement of birefringence, three measurements need to be performed, by moving the lens in three consecutive positions and the cuvette is replaced by a parallel plate sample and similar expressions for the refractive index of the ordinary and extraordinary rays as shown in equation 3 are performed. Changing the polarization of the source from horizontal to vertical is necessary in this arrangement, this can be conveniently accomplished by introducing a Martin-Puplett interferometer between the source and the wedged plate. The birefringence can then be obtained from:

$$n_e - n_o = \frac{1}{2}\left[\left(n_1^* - n_1\right) + \left(n_2^* - n_2\right) + \left(n_3^* - n_3\right)\right] \tag{4}$$

where subscripts indicate the refractive index of the ordinary and extraordinary rays measurements and that measurements are performed along the three directions of the sample. Similar arrangements can be used to measure the refractive index of materials that are transparent to terahertz frequencies, determine the homogeneity of samples, measure optical distortions and to perform the non-contact evaluation of thermal expansion coefficient of materials buried inside media that are opaque to optical or infrared frequencies but transparent to THz frequencies.

3. Shearing interferometry with phase-conjugate surfaces

It is well known that phase-conjugate surfaces developed for the terahertz part of the spectrum, when placed at regularly spaced intervals, can be used to efficiently eliminate wavefront distortion (Figure 2). For an incident wave-front $e^{i(\omega t+\phi)}$ on a non-linear surface with a phase correction $e^{i2\omega t}$ the reflected waves will have a phase $e^{i(\omega t+\phi)}e^{i2\omega t}=e^{i(2\omega t-\omega t-\phi)}=e^{i(\omega t-\phi)}$. The concept has already been demonstrated at mm-wave frequencies using an electronic approach to conjugate the signal at specific antenna elements [28]. Sampling at sufficiently dense intervals it is possible to conjugate an entire wave-front thus eliminating any distortions. This information can be used for improving the coupling between THz waveforms.

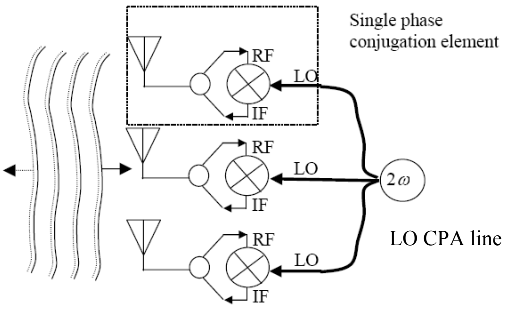

Figure 2. Array of phase conjugation elements sampling an entire THz wavefront with a local oscillator (LO) signal distributed by a network of CPA lines.

The proposed system can be used to introduce a programmable shear to a free space propagating beam or used in tandem with a shearing interferometer to perform accurate measurements by nulling the observed change in the fringes.

4. The application of shearing interferometry for information processing

It is well known that a thin lens can produce a 2-dimensional Fourier transform of an image on the back of its plane. Introduction of a shear in a controlled manner to a free space propagating Gaussian beam, provides a new degree of freedom for encoding and decoding information. The shear performs a transformation of fractional order, which can be used for phase retrieval, signal analysis, and filtering [29-54]. The relevance of fractional order transforms to understand the free space propagation of Gaussian beams described by Gauss-Laguerre or Gauss Hermite polynomials will be discussed in more detail at the conference.

5. Conclusion

A range of new wavefront shearing measurement modalities that can be implemented in the THz part of the spectrum have been suggested. Terahertz shearing interferometry is still at its infancy but because of the possible applications, it is likely to become an emergent research area once fringe imaging technologies become more widely available.

References

[1] Bates W.J., A wavefront shearing interferometer *Proc. Phys. Soc.* (*Lond.*), **59**, (1947), 940.
[2] Drew R.L., A simplified shearing interferometer *Proc. Phys. Soc.* (*Lond.*), **864**, (1951), 1005.
[3] Francon M., *Progress in microscopy.* Pergamon Press, New York, 1951 p. 150.
[4] Brown D. S. The application of shearing interferometry to routine optical testing *J. Sci. Instrum.* **32** (1955) 137.

[5] Baker J., An interferometer for measuring the spatial frequency response of a lens system. *Proc. Phys. Soc.* (*Lond.*), **868**, (1955), 871.
[6] Gates J.W., Reverse shearing interferometry *Nature* **176** (1955) 259.
[7] Smith F.H., In *Modern Methods of Microscopy.* Butterworths, London, 1956, p. 76.
[8] Hariharan, P., and Sen, D., Cyclic shering interferometer. *J. Sci. Instrum.* **37** (1960) 340.
[9] Saunders, J.B., Wavefront shearing prism interferometer. *J. Res. Nat. Bur. Stand.* **68C** (1964) 155.
[10] Murty, M.V.R.K., Some modifications of Jamin interferometer useful in optical testing. *Appl. Optics*, **3** (1964) 535.
[11] Francon, M., Polarization apparatus for interference microscopy of isotropic transparent objects. *J. Opt.Soc. Am.,* **47** (1967) 528.
[12] Saunders, J.B., A simple inexpensive wavefront shearing interferometer. *Appl. Optics*, **6** (1967) 1581.
[13] Murty, M.V.R.K., Interferometry applied to the testing of optics. *Bull. Opt. Soc. India*, **1** (1967) 33.
[14] Lohmann, A., and Bryndahl O.,A., A lateral wavefront shearing interferometer with variable shear. *Appl. Optics*, **6** (1967) 1934.
[15] Van Rooyen, E., Design for a variable shear prism interferometer *Appl. Optics*, **7** (1968) 2423.
[16] Briers J.D., Prism shearing interferometer. *Opt. Technol*., **1** (1969) 196.
[17] Van Rooyen, E., and Van Houten A.G., Design of a wavefront shearing interferometer useful for testing large aperture optical systems. *Appl. Optics*, **8** (1969) 19.
[18] Murty, M.V.R.K.,A compact lateral shearing interferometer based on the Michelson interferometer. *Appl. Optics*, **9** (1970) 1146.
[19] Kelly J.C. and Hargreaves R.A., A rugged inexpensive shearing interferometer. *Appl. Optics*, **9** (1970) 948.
[20] Nyssonen, D. and Jerke J.M., Lens testing with a simple wavefront shearing interferometer. *Appl. Optics*, **12** (1973) 2061.
[21] Hariharan P. Simple laser interferometer with variable shear and tilt. *Appl. Optics*, **14** (1976) 1056.
[22] M. Naftaly, R.Dudley and J. Leist, 'Terahertz spectroscopy of Boron Nitride,' *J. Phys. Conf. Ser.,* Dielectrics 2011 (*in press*).
[23] R Qi Li, Sheng-Hui Ding, Rui Yao, and Qi Wang Real-time terahertz scanning imaging by use of a pyroelectric array camera and image denoising, *J. Opt Soc. Am. A*, **27**, (2010) 2381-2386.
[24] Miller A.J., Luukanen A., Grossman E.N., 'Micromachined antenna-coupled uncooled microbolometers for terahertz imaging arrays, terahertz for military and security applications II,' ed. R.J. Hwu, D.L. Woolard, *Proc. SPIE* **5411**, (Bellingham WA, 2004) 18-24
[25] Wei Lee A. and Hu Q., 'Real-time, continuous-wave terahertz imaging by use of a microbolometer focal-plane array,' *Optics Lett.*, **30**, (2005) 2563-2565.
[26] Wu, Q. Hewitt, T. D. and Zhang, X.-C., 'Two-dimensional electro-optic imaging of THz beams,' *Appl. Phys. Lett.* **69** (1996) 1026-1028.
[27] Mickan S., Abbott, D. Munch, J. Zhang X.-C., van Doorn T., Analysis of system trade-offs for terahertz imaging,' *Microelectron. Journal* **31** (2000) 503–514.
[28] Chang, Y., Fetterman, H., Newberg, I. and Panaretos, S. "Millimeter-wave phase conjugation using artificial nonlinear surfaces," *Appl. Phys. Lett.* **72**, (1998) 745-747.
[29] Flannery D.L. and Horner J.L., Fourier optical signal processors, *Proc. IEEE,* **77**, (1989), 1511-1527.
[30] Walther A., Radiometry and Coherence, *J. Opt Soc. Am.*, **58** (1968), 1256-1259
[31] Walther A., Propagation of the generalized radiance through lenses, *J. Opt Soc. Am.*, **68** (1978), 1606-1610.
[32] Mendlovic, D. and Ozaktas, H.M., Fractional Fourier transforms and their optical implementation I, *J. Opt Soc. Am. A.*, **10** (1993), 1875-1881.
[33] Mendlovic, D. and Ozaktas, H.M., Fractional Fourier transforms and their optical implementation II, *J. Opt. Soc. Am. A.*, **10** (1993), 2324-2329.
[34] Ozaktas, H.M., and Mendlovic, D., Fourier transforms of fractional order and their optical interpretation, *Opt. Commun.,* **101**, (1993), 163-169.

[35] Lohmann A.W., Image rotation, Wigner rotation, and the fractional Fourier transform, *J. Opt. Soc. Am. A.*,10, (1993) 2181-2186.
[36] Lohmann A.W., Mendlovic, D., Zalevsky Z., and Dorch R.G., Some important fractional transformations for signal processing, *Opt. Commun.,* **125**, (1996), 18-20.
[37] Ozaktas, H.M., Zalevsky Z. and Kutay M.A., *The fractional Fourier transform with applications in Optics and Signal Processing,* Wiley New York, 2001.
[38] Torre A., The fractional Fourier transform and some of its applications in optics, in *Progress in Optics,* **43**, ed. E. Wolf, Nonrth-Holland, Amsterdam, (2002), 531-596.
[39] Almeida L.B., The fractional Fourier transform and time-frequency representations, *IEEE Trans. Signal Process.,* **42**, (1994), 3084-3091.
[40] Alieva, T. and Bastiaans, M.J., On fractional Fourier transform moments, *IEEE Signal Process. Lett.*, **7**, (2000), 320-323.
[41] Alieva, T. and Bastiaans, M.J., Phase-space distributions in quasi-polar coordinates and the fractinal Fourier transform *J. Opt Soc. Am. A.*, **17** (2000), 2324-2329.
[42] Teague R.M., Deterministic phase retrieval: a Green function solution, *J. Opt Soc. Am.*, **73** (1983), 1434-1441.
[43] Alieva, T. Stancović, L.J. and Bastiaans, M.J., Signal reconstructin from from two close fractional Fourier power spectra, *IEEE Trans. Signal Process.,* **51**, (2003), 112-123.
[44] Bastiaans, M.J., and Wolf K.B., 'Phase reconstruction from intensity measurements in linear systems,' *J. Opt. Soc. Am., A*, **20** (2003), 1046-1049.
[45] Dong B.Z., Zhang Y. Gu B.Y. and Yang G.Z., Numerical investigation of phase retrieval in a fractional Fourier transform, *J. Opt. Soc. Am., A*, **14** (1997), 2709-2714.
[46] Alieva T. and Agullo-Lopez F., Diffraction analysis of random fractal fields, *J. Opt. Soc. Am., A*, **15** (1998), 669-674.
[47] Merlo D.R., Romo J.A.RM., Alieva T. and Calvo M.L., Fresnel diffraction by deterministic fractal gratings, an experimental study, *Opt. Spectrosc.,* **95**, (2003) 131-133.
[48] Alieva T. and Calvo M.L., Paraxial diffraction on structures generated by multiplicative iterative procedures, *J. Opt. A., Pure Appl. Opt.,* **5**, (2003) S324-S328.
[49] Ozaktas, H.M., Barshan, B., Mendlovic, D. and Onural, L., 'Convolution, filtering and multiplexing in fractional Fourier domainsand their relationships to chirp and wavelet transforms,' *J. Opt. Soc. Am., A*, **11** (1994), 547-559.
[50] Almeida L.B., 'Product and convolution theorems for the fractional Fourier transform,' *IEEE Signal Process. Lett.,* **4**, (1997) 15-17.
[51] Mendlovic, D., Ozaktas, H.M. and Lohmann A.W., Fractional correlation, *Appl. Opt.,* **34**, (1995) 303-309.
[52] Lohmann A.W., Zalevsky Z. and Mendlovic, D., 'Synthesis of pattern recognition filters for fractional Fourier processing,' *Opt. Commun.,* **128**, (1996) 199-204.
[53] Alieva, T. and Calvo M.L., 'Generalized fractional convolution,' in *Perspectives in Modern Optics and Optical Instrumentation*, eds. J. Joseph, A. Sharma, and V.K. Rastogi, Anita Publications New Delhi, (2002), 163-173.
[54] Akay O., and Boudreaux-Bartels G.F., 'Fractional convolution and correlationvia operator methods and an application to detection of linear FM signals,' *IEEE Trans. Signal Process.,* **49** (2001) 979-993.