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# Transition from an antiphase error-correction mode to a synchronization mode in mutual hand tracking

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Proactive motion in hand tracking and in finger bending, in which the body motion occurs prior to the reference signal, has been previously reported when a periodic target signal was shown to the subjects at relatively high frequencies. These phenomena indicate that the human sensory-motor system tends to choose an anticipatory mode rather than a reactive mode, when the target motion is relatively fast. The present research was undertaken to study what kind of mode appears in the sensory-motor system when two persons were asked to track the hand position of the partner at various mean tracking frequency. The experimental results showed that a transition from a mutual error-correction mode to a synchronization mode occurred in the same region of the transition frequency with the one from a reactive error-correction mode to a proactive anticipatory mode reported previously in the target tracking experiments of the single subjects. Present research indicated that synchronization of body motion occurred only when both of the pair subjects operated in a proactive anticipatory mode. We also presented mathematical models to explain the behavior of the error-correction mode and the synchronization mode.

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## I. INTRODUCTION

Reactive motion with some delay to the observed information and predictive anticipatory motion prior to the environmental change are vitally important functions for survival of higher animals including human beings [1]. The condition for the appearance of the two functions is an interesting subject for physical science as well as for brain science. Quantitative research on the reaction-anticipation transition in human perception and action was reported by Engström *et al.* [1]. The transition from a reactive to anticipatory mode in the finger bending experiments to periodic stimuli occurred at a stimulus frequency from 0.5 Hz to 0.75 Hz. They asserted that reaction and anticipation were the two modes of a single dynamical human perceptomotor system. The anticipation tendency in the finger tapping paradigm, called negative asynchrony, has been studied by a number of researchers [2,3]. The mechanism, however, has not yet been completely clarified to the knowledge of the present authors.

In contrast to the finger bending which provides discrete timing information, the hand tracking experiment can provide us continuous analog data of the positional difference between the target and subject motions. Utilizing this advantage, Ishida *et al.* [4,5] showed that the amount of proactive motion corresponds to the value for which the error created by the sudden change of the target motion was minimized, that is, anticipation creates proaction to avoid danger in motion caused by an unexpected environmental change. Subsequently, utilizing intermittent visual information in the hand tracking experiment, Hayashi *et al.* [6] showed that the proactive motion appeared either when the visual information was artificially masked or when the amount of visual information exceeded the human capacity of information processing for motor control.

Anticipation of a subject so far has been studied for the motion of a visual target or a series of external discrete stimuli

to which the subject interacts unidirectionally. Therefore, experimental and theoretical studies of mutually coupled visual hand tracking of two subjects would be a natural extension of these reported series of research.

The present work is undertaken to study how the proactive anticipatory hand motion in the unidirectional coupling hand tracking changes, when the programmed target motion is replaced by the motion of the other subject who simultaneously tries to track the motion of the other. Using a similar experimental setup modified for the present purpose, we studied the following in this paper: what will be the attractor of a coupled dynamical system of the two mutually anticipative subjects?

## II. EXPERIMENTAL DESIGN

### A. Procedures

The hand tracking experiment is a paradigm to trace a moving object to reveal a mechanism of the visual-motor control based on the perception of position of the object. In a mutual tracking experimental system, a cross-feedback system was set up using two computers connected through a cross cable as shown in Fig. 1. The time delay of transmission was regulated less than 5 ms to achieve reasonable accuracy for mutual tracking of a human, with a comparable time scale with a refresh rate of the liquid crystal display.

A subject was seated in front of a computer screen, and was asked to trace a moving visual target (a red closed circle of 6 mm diameter) as accurately as possible by the motion of a cursor (a blue closed circle of 6 mm diameter) in the screen. A pair of subjects sat side by side, maintaining a distance so that they felt free from each other.

A subject traced the moving target by the cursor which was synchronized with the motion of the computer mouse. The linear dependence of the motion of the mouse and the motion of the tracer on display was checked fine. The tracer in display of each subject was set to move on a two-dimensional plane of the screen, but the target motion was set to be confined along

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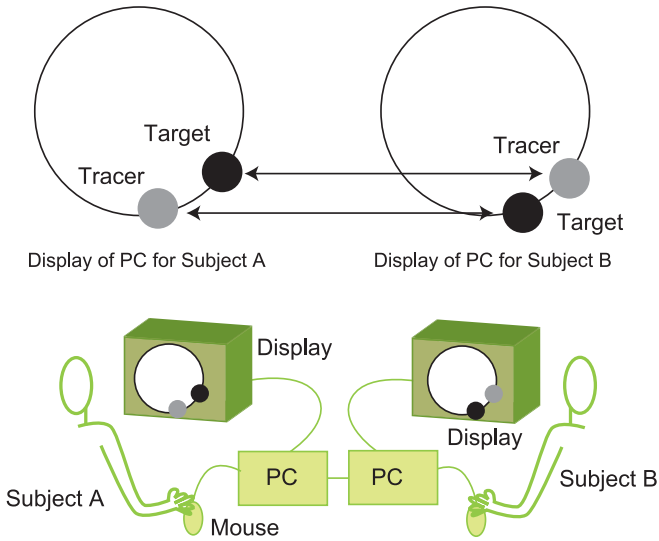


FIG. 1. (Color online) Experimental setup for mutual tracking experiments. A cross feedback system is constructed, i.e., the motion of the target of subject A is synchronized with the motion of the tracer of subject B and vice versa.

the guideline circle of 13 cm in diameter, i.e., the phase angle of the tracer was calculated, transmitted to the other computer, and was presented along the guideline circle on the display of the other person. The initial position of target and tracer prior to tracking was located on a positive  $x$  axis, and the target started to move along the circle in a counterclockwise sense.

We employed three types of tracking conditions.

(1) Fully visible tracking experiments: target was fully visible along the entire guideline circle.

(2) Intermittent-visible tracking experiments: circular orbit was constructed by two target-hidden regions (each 30% at the top and the bottom) and two target-visible regions (each 20% in horizontal location), as shown in Fig. 2.

(3) Readin tracking experiments: a subject was asked to track the target motion which was the recording of the tracer motion of his partner subject in the fully visible experiment.

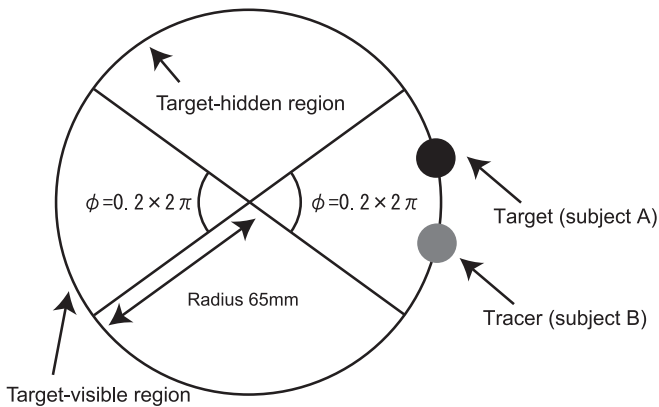


FIG. 2. Display of the intermittent-visible tracking experiments. The circular orbit was constructed by two target-hidden regions (each 30% at top and bottom) and two target-visible regions (each 20% in horizontal location). The target is represented by a black circle and the tracer is represented by a gray circle.

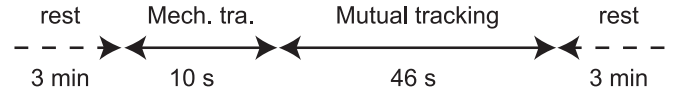


FIG. 3. Experimental protocol of mutual tracking experiments. In a mechanical tracking period, the subject was asked to track the computer-programed target at constant velocity in order to regulate the tracer velocity in the mutual tracking period.

Prior to the mutual tracking experiments, the subject was asked to track the computer-programed target at constant velocity in order to regulate the tracer velocity in the mutual tracking period (Fig. 3). This is called a mechanical tracking period. After 10 s of tracking a programed target, the target image was switched to the tracer motion of the other subject without notifying a pair of subjects. This mutual tracking period was set to last for 46 s, and the subject was asked to keep the initial velocity shown by the programed target and to trace the target as accurately as possible. The programed target frequency was set to 0.1 Hz, 0.3 Hz, 0.5 Hz, and 0.7 Hz in the mechanical tracking period. The detail experimental protocol was given as follows.

(1) One trial in the mutual tracking experiments was set to last for 56 s in total.

(2) After finishing each trial, subjects took a rest for 3 min.

(3) Initial target frequency was progressively increased from 0.1 Hz to 0.7 Hz in steps of 0.2 Hz.

(4) A subject performed 10 trials at each frequency; these 10 trials in total were referred as one set.

(5) After finishing each set at each frequency, a subject took a rest for about 5 min.

(6) It was regarded that one clue was finished when a pair finished all the tracking experiments at each frequency (total 40 trials).

A pair of subjects began by performing three clues as learning trials to become familiar with the experimental procedure and the motion of the target produced by the partner.

In the time duration of mutual tracking, the first and the last 3 s of tracking data was omitted, and the phase angle of target and tracer was calculated every 5 ms. One set (10 trials) was used to calculate an average and a standard deviation in each initial frequency of the target. The characteristics of hand motion were mainly discussed in the power spectrum of tracer velocity and the cross-correlation functions of the tracer velocities between the pair subjects.

## B. Subject information

In all the experimental conditions, the subjects who were well trained in target tracking experiments participated in the mutual tracking experiments. All of them were right-handed persons. The subjects consisted of four male and one female in their twenties. The training level was measured as follows: the subject was asked to trace a target at constant frequency of 0.1 Hz and 0.3 Hz. The distribution of relative phase errors between target and tracer was calculated and was fitted by a Gaussian distribution. In the course of training, if the standard deviation of the obtained Gaussian distribution became less than 0.3 rad, the person was selected to participate in the mutual tracking experiments. The subjects were randomly chosen to make five pairs. The institutional ethics committee at Tohoku

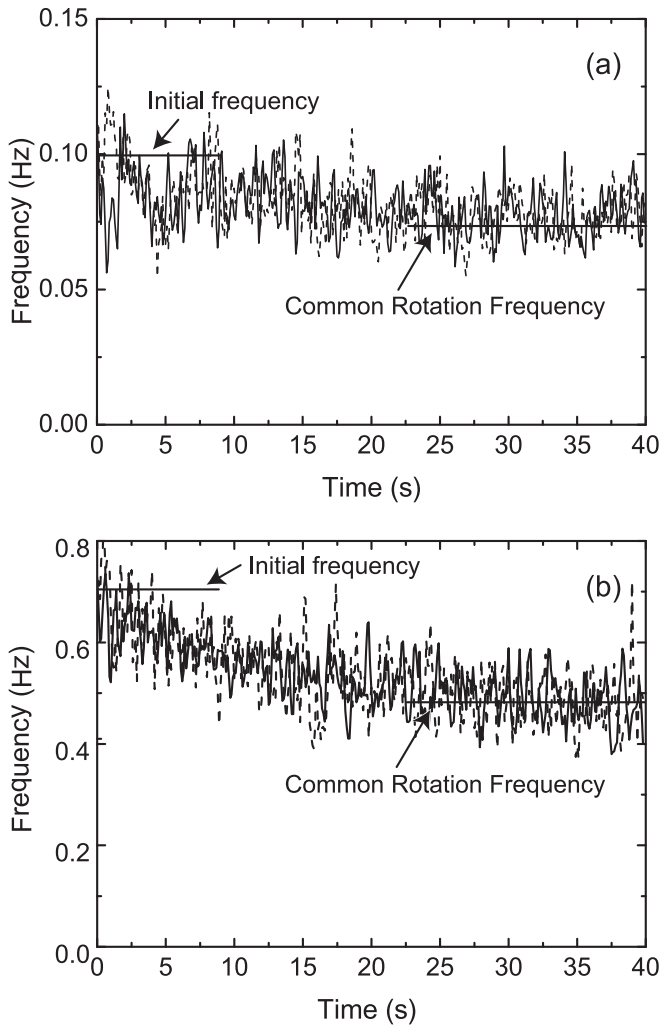


FIG. 4. Temporal development of the local speed in the unit of the rotation frequency of the hand motion during the mutual tracking period. The solid line and the broken line represent the speed of the two mutually tracking hand motions. (a) Initial target frequency is 0.1 Hz and (b) initial target frequency is 0.7 Hz.

Gakuin University approved the procedure of the experiments, and the subjects gave informed consent before participation.

### III. EXPERIMENTAL RESULTS ON MUTUAL TRACKING

Figure 4 shows a temporal development of the local rotation speed in the unit of frequency. Even though the subjects tried to keep the initial speed, their tracking speed relaxed to a certain value (hereafter we refer to it as the common rotation frequency,  $f_{cr}$ ) which was determined by the mutual interactions as the two subjects perform the mutual tracking. We examined  $f_{cr}$  as a function of the initial frequency, and Fig. 5 shows that  $f_{cr}$  is always smaller than the initial frequency.

Figure 6 shows a power spectrum of the tracer velocity when  $f_{cr}$  is relatively low. In the previous target tracking experiment [6], a broad Gaussian-like distribution whose peak was located around 1.0 Hz appeared as shown in Fig. 6 by the broken line. It was explained that the hand motion reflected some intrinsic time scale of 1.0 s for positional error correction, when the target moved at a low frequency. However, in the

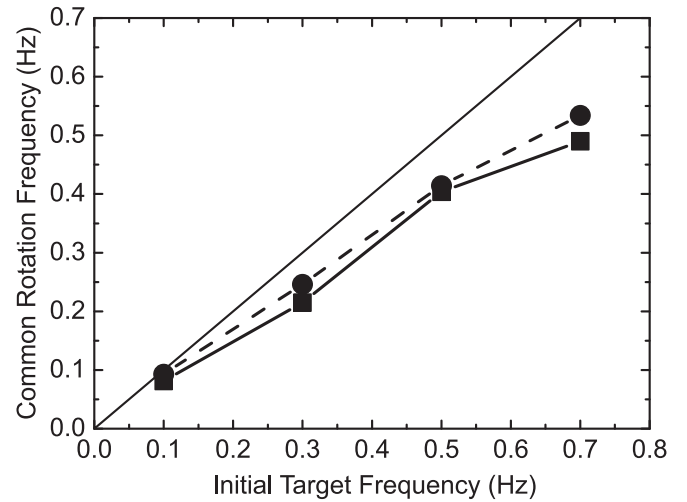


FIG. 5. Common rotation frequency as a function of initial target frequency. The solid line represents the common rotation frequency in the fully visible tracking experiments, and the dashed line represents the common rotation frequency in the intermittent-visible tracking experiments. The data was averaged over 10 trials of one pair.

present mutual tracking experiments, the spectrum profile showed no broad peak around 1.0 Hz, and extended to a higher-frequency region as shown in Fig. 6 by the solid line. This interesting fact suggests that the mutual error correction weakens the fluctuation of the intrinsic time scale of motion.

Figure 7 shows a cross-correlation function between the pair subjects. A deep valley at zero time lag and a weak oscillation whose amplitude decreased with increase of the time lag were observed. These features in the cross-correlation function would suggest that the time series of antiphase oscillation in the hand positions of the pair subjects should be triggered

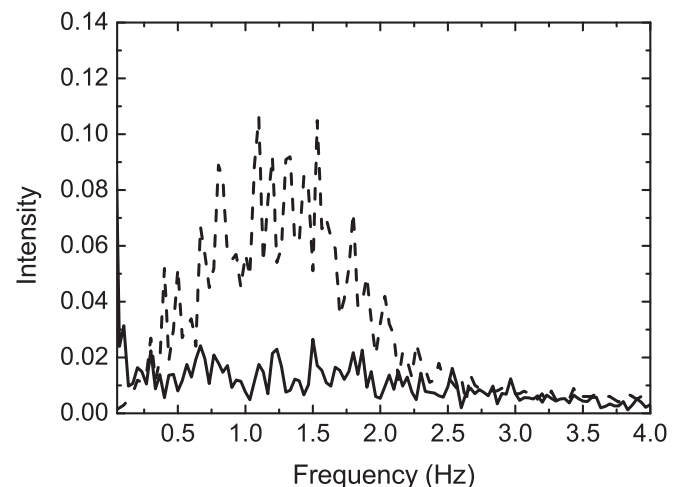


FIG. 6. Power spectrum of tracer velocity in the mutual tracking experiments (fully visible condition). The solid line represents the power spectrum of tracer velocity in the mutual tracking experiments. The initial target frequency was 0.1 Hz. The data was averaged over 10 trials. The standard deviation was 0.017 (averaged over frequency). The dashed line represents the power spectrum of tracer velocity in the target tracking experiments. The initial target frequency was 0.1 Hz.

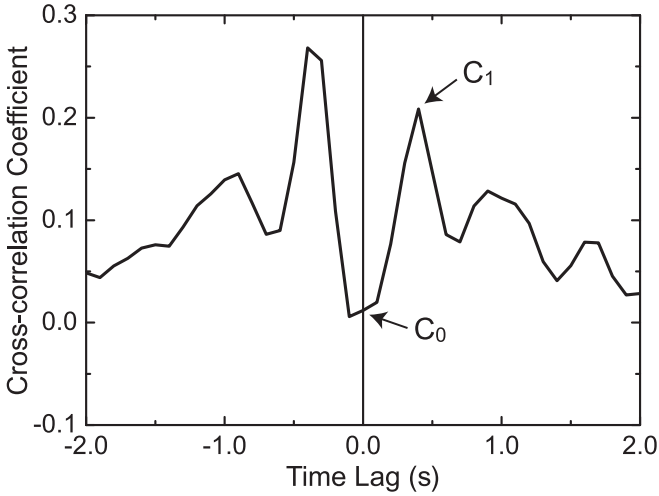


FIG. 7. Cross-correlation function of velocity in the fully visible tracking experiment. The initial target frequency was 0.1 Hz. The data was averaged over 10 trials. The standard deviation was 0.13 (averaged over time lag).

from time to time by various percept-motor noise. From the distances between the peaks, the period of the oscillation is estimated to be approximately 0.6 s and, from the decreasing rate of the amplitude, the decay time is estimated to be close to 1.0 s.

To understand the nature of the antiphase damped oscillations found in the cross-correlation function of two subjects, we carried out a readin tracking experiment in which the time development of a tracer frequency was recorded beforehand from a mutual tracking experiment, and then the target trajectory (readin data) was replayed on the screen. The other subject was asked to follow this replayed trajectory of the target. Therefore, in the readin tracking experiment, the movement of the target is predetermined, independent of the tracer, even though it has a characteristic of the mutual motion.

Figure 8 shows a cross-correlation function between the velocities of the subject's hand and of the readin target trajectory. Interestingly, the difference of the curve profile between Figs. 7 and 8 is clear, i.e., the positive peak of the cross-correlation on the positive time delay of 0.3 s in the mutual tracking (Fig. 7) disappeared in the readin tracking (Fig. 8), whereas the positive peak with a negative time delay of  $-0.3$  s was enhanced. This is a natural result, considering that the subject who tries to track the target moving rather chaotically can only follow the trajectory of the target with time delay in the sensory-motor system, but cannot lead the complex motion of the readin target trajectory.

Figure 9 shows a power spectrum of the tracer velocity in the intermittent-visible mutual tracking experiment with the initial rotation frequency of 0.5 Hz. When the common rotation frequency,  $f_{cr}$ , is greater than 0.3 Hz, a rhythmic component,  $2f_{cr}$ , appears in the power spectrum. The hand motion becomes rhythmic with double frequency of the common rotation frequency, because the visual intermittent tracking display has twofold symmetry as shown in Fig. 2.

Figure 10 shows a cross-correlation function of the velocities of the hand motion of the two subjects when the initial target velocity is set to 0.7 Hz. The graph suggests that the

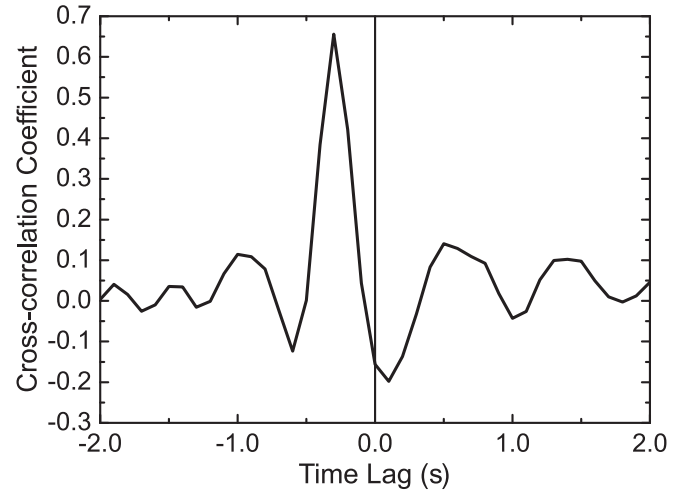


FIG. 8. Cross-correlation function of tracer velocity in the readin tracking experiments (solid line). The readin data was recorded from the mutual tracking; the initial target frequency was 0.1 Hz. The average was calculated from 10 trials. The standard deviation was 0.038 (averaged over time lag).

tracer velocities include an in-phase oscillation component with noise. In the target tracking experiments, it was shown [6] that, even when the rotation frequency was lower than 0.3 Hz, a rhythmic component was created locally in the target-hidden region in the intermittent-visible experiments and that, when the common rotation frequency increased greater than 0.3 Hz, this rhythmic component in the rotation frequency expanded over the entire orbit at a rotational frequency,  $2f_{cr}$ . We discuss in the latter part of this section why the two subjects can synchronize without phase delay.

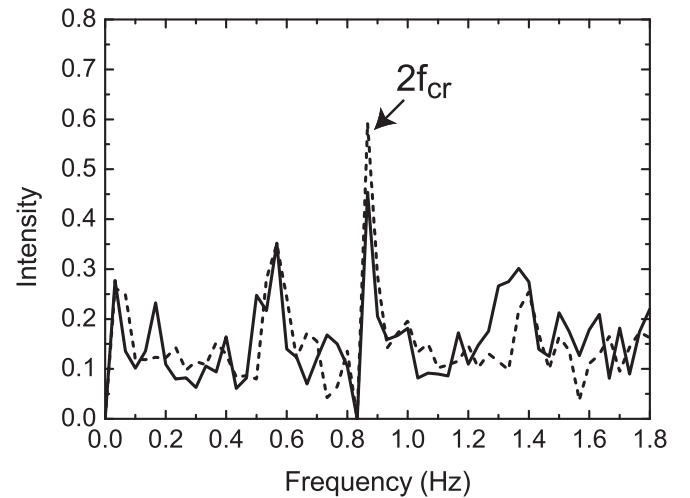


FIG. 9. Power spectrum of tracer velocity in the intermittent-visible tracking experiments. The initial target frequency was 0.5 Hz. The solid line represents the power spectrum of the tracer frequency of subject A and the dashed line represents the power spectrum of the tracer frequency of subject B. The standard deviation was 0.017 for subject A and 0.019 for subject B (averaged over frequency). The data was averaged over 10 trials.



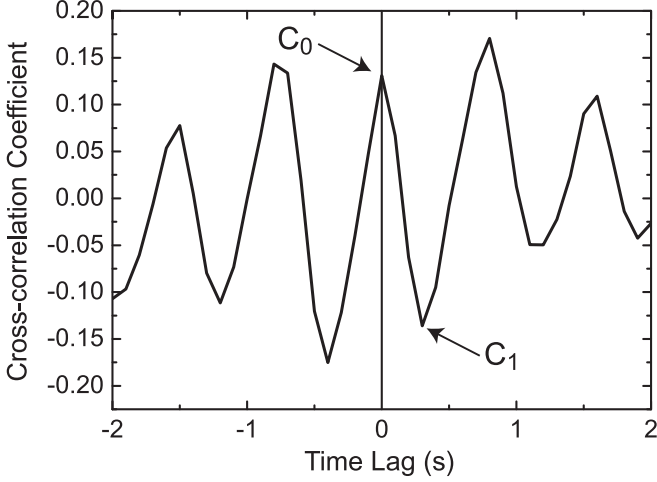


FIG. 10. Cross-correlation function of tracer velocity in the intermittent-visible tracking experiments. The initial target frequency was 0.7 Hz. The data was averaged over 10 trials. The standard deviation was 0.081 (averaged over time lag).

To measure the degree of synchronization, we introduce a strength of synchronization (SS) as

$$\text{Strength of synchronization} = C_0 - C_1, \quad (1)$$

where  $C_0$  denotes the cross-correlation coefficient at zero time lag and  $C_1$  denotes the cross-correlation coefficient at the peak position (Fig. 7) or at the valley position (Fig. 10) nearest to zero time lag.

The SS value is shown in Fig. 11(a) as a function of  $f_c$ , showing a transition from a damped antiphase oscillation mode to a synchronization mode. We also measured the strength of

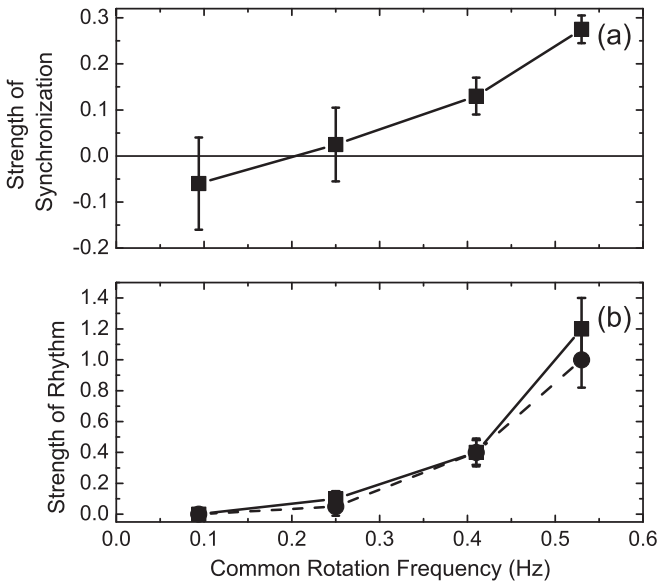


FIG. 11. (a) Strength of synchronization,  $C_0 - C_1$  as a function of  $f_{cr}$ . (b) Strength of the rhythmic component as a function of  $f_{cr}$ . The solid line represents the strength of rhythm of subject A and the dashed line represents the strength of rhythm of subject B. The average and standard deviations were calculated from 10 trials. The results of one pair were shown. Two of the 20% visible belts were set in the intermittent-visible tracking experiments.

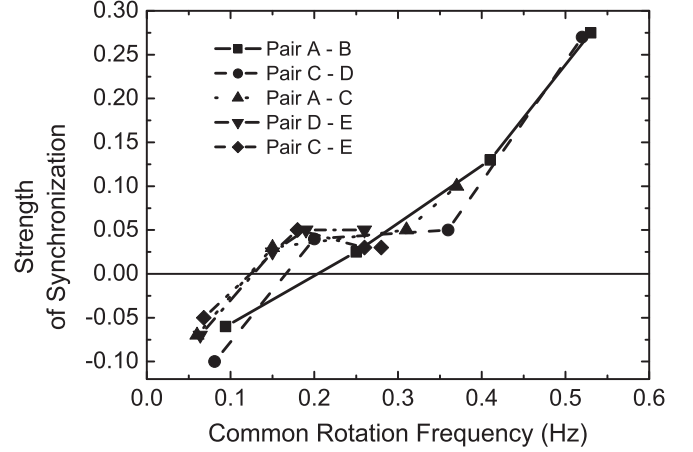


FIG. 12. Strength of synchronization,  $C_0 - C_1$  in the mutual tracking experiments as a function of  $f_{cr}$ . The results of five pairs were presented. The subject *id* was described in the figure for each pair. The data points were averaged over 10 trials. Two of the 20% visible belts were set in the intermittent-visible tracking experiments.

the rhythmic component in the hand motion from the height of the peak in the power spectrum of tracer velocity, and found that the rhythmic component was strengthened as  $f_c$  increased [Fig. 11(b)]. Thus it seems reasonable to assume that the transition between two modes is caused by the generation of a rhythmic component. We shall discuss this point later in Sec. IV B. Figure 12 shows a strength of synchronization for five pairs. All the pairs showed a transition of mode from error correction to synchronization.

#### IV. MATHEMATICAL DESCRIPTION OF THE MUTUAL HAND TRACKING FOR LOW AND HIGH COMMON ROTATION FREQUENCY REGIONS

It was experimentally shown in the previous section that the hand motion of the two subjects in the mutual hand tracking experiment showed a damped antiphase oscillation at a low common frequency region ( $f_{cr} < 0.3$  Hz). In a higher common rotation frequency region, a synchronized rhythmic component was generated in their hand motion. The strength of the synchronization increased with increasing common rotation frequency,  $f_{cr}$ .

##### A. Low common rotation frequency

In this section we show that a model of the coupled feedback equations with delay exhibits a damped antiphase oscillation, which may be related to the experimental observation (Fig. 7) at a low common rotation frequency ( $f_{cr} < 0.3$  Hz). Coupled delayed error-correction dynamics for the common rotation frequency,  $f_{cr}$ , is written as

$$\frac{d\phi_1}{dt} = \frac{1}{\tau} [\phi_2(t - \delta) - \phi_1(t - \delta)] + 2\pi f_{cr}, \quad (2)$$

$$\frac{d\phi_2}{dt} = \frac{1}{\tau} [\phi_1(t - \delta) - \phi_2(t - \delta)] + 2\pi f_{cr}, \quad (3)$$

where  $\phi_1$  and  $\phi_2$  are phase positions,  $\delta$  is the delay time of the visual-motor system, and  $\tau$  is the time constant for the error correction. The first term describes the positional

error-correction motion and the second term is given to keep the constant velocity. The solution is

$$\phi_1(t) = V_0 t + C f(t), \quad (4)$$

$$\phi_2(t) = V_0 t - C f(t), \quad (5)$$

where  $C$  is an arbitrary constant, and

$$f = \exp(-\gamma t) \exp(i\omega t), \quad (6)$$

where  $\gamma$  and  $\omega$  satisfy the following relation:

$$-\tau\gamma = 2 \exp(\gamma\delta) \cos(\omega\delta), \quad (7)$$

$$\tau\omega = -2 \exp(\gamma\delta) \sin(\omega\delta). \quad (8)$$

It was shown in the previous section (Fig. 7) that the hand motion of two subjects in the mutual tracking experiments showed a damped oscillation with the values for  $\gamma = 1 \text{ s}^{-1}$  and  $\omega = 10 \text{ s}^{-1}$ . Substituting these values for Eq. (7) and Eq. (8), we obtain rough estimates:  $\delta = 0.45 \text{ s}$  and  $\tau = 0.3 \text{ s}$ . They are rather close to the value of the time delay of the visual-motor system and the value of the error-correction time. In the real experimental systems, the damped oscillations would be triggered from time to time by various percept-motor noise and decayed by the factor,  $\gamma$ , given by Eq. (6). We believe that a delayed mutual error-correction model explains qualitatively and quantitatively the observed experimental data of the damped oscillation.

### B. High common rotation frequency

At a higher common rotation frequency region, a rhythmic component appears in addition to the common rotation frequency. The coupling between the two subjects is more macroscopic in this region. The coupling is no more through the difference of the positions of the two, but through the phase difference of the rhythmic components of the two. Therefore, one can describe the phase velocities of the two subjects as

$$\frac{d\phi_1}{dt} = 2\pi f_{\text{cr}} + a_1 \sin[2\pi f_{\text{rh}} t + \Psi_1(t)], \quad (9)$$

$$\frac{d\phi_2}{dt} = 2\pi f_{\text{cr}} + a_2 \sin[2\pi f_{\text{rh}} t + \Psi_2(t)]. \quad (10)$$

The first term is the constant velocity term and the second term is the rhythmic term, where  $f_{\text{cr}}$  is the common rotation frequency (Fig. 4) and  $f_{\text{rh}} = 2f_{\text{cr}}$  is the frequency of the rhythmic component (Fig. 9). The present purpose is to understand why the rhythmic component of the two subjects synchronizes, i.e.,  $\Psi_1(t) - \Psi_2(t)$  goes to zero, when  $f_{\text{cr}}$  is greater than 0.3.

At a higher value of  $f_{\text{cr}}$  the amplitude of the rhythmic component is prominent (Figs. 9 and 10) and, therefore, the frequency of the rhythmic component is established and the remaining phase variables are slow variables compared to  $f_{\text{cr}}$ . Since the phase variables  $\Psi_1(t)$  and  $\Psi_2(t)$  are the slow variables, it is reasonable to treat them as the discrete functions  $\Psi_1(n)$  and  $\Psi_2(n)$  of the  $n$ th rotation. Also, one may write the mutual error-correction dynamics as

$$\{\Psi_1(n+1) - \Psi_1(n)\} / T = k_1 \sin[\Psi_2(n) - \Psi_1(n)], \quad (11)$$

$$\{\Psi_2(n+1) - \Psi_2(n)\} / T = k_2 \sin[\Psi_1(n) - \Psi_2(n)], \quad (12)$$

where  $k_1$  and  $k_2$  are the small coupling constants of positive value and  $T$  is the period of oscillation equal to  $1/f_{\text{cr}}$ . An important assumption in this dynamics is that the time delay,  $\delta = 0.45 \text{ s}$ , is short enough compared to the period, and neglected. Therefore, the dynamics is instantaneous as far as we deal with the discrete dynamics. The dynamics of the phase difference,  $\Delta\Psi(n) = \Psi_2(n) - \Psi_1(n)$ , is derived from Eqs. (11) and (12) as

$$\{\Delta\Psi(n+1) - \Delta\Psi(n)\} / T = -(k_1 + k_2) \sin \Delta\Psi(n). \quad (13)$$

For a small initial value of the phase difference,

$$\Delta\Psi(n) = \{1 - (k_1 + k_2)T\}^n \Delta\Psi(0). \quad (14)$$

From Eq. (14) one can easily see that the phase difference will disappear as  $n$  increases. One can conclude that the appearance of a well established rhythm creates synchronization of the rhythmic component, which, in turn, synchronizes the hand motion  $\phi_1$  and  $\phi_2$ .

In summary, we obtained the following interesting observation: in the low-frequency region, the difference of the positional information of the two subjects is taken into the brain with delay, resulting in a damped antiphase oscillation, while in the higher-frequency region, the difference of the phase information of the rhythmic component is taken into the brain without time delay, resulting in the synchronization.

### C. Relation between amplitude and synchronization strength

In order to understand the strength of synchronization [Fig. 11(a)] and its relation to the strength of rhythm [Fig. 11(b)] as a function of  $f_{\text{cr}}$ , we first delete  $\Psi_1(t)$  and  $\Psi_2(t)$  from Eqs. (9) and (10), because we are now considering the synchronization mode. A dependence of the strength of synchronization on percept-motor noise should be also discussed; thus the noise term is added,

$$\frac{d\phi_1}{dt} = 2\pi f_{\text{cr}} + a_1 \sin(2\pi f_{\text{rh}} t) + \Gamma_1(t), \quad (15)$$

$$\frac{d\phi_2}{dt} = 2\pi f_{\text{cr}} + a_2 \sin(2\pi f_{\text{rh}} t) + \Gamma_2(t), \quad (16)$$

where  $\Gamma_1(t)$  and  $\Gamma_2(t)$  are the random noise in the muscles of each subject. One obtains the cross-correlation function,  $C(\tau)$ , between the tracer velocities of the two subjects as follows:

$$\begin{aligned} C(\tau) &= \langle d\phi_1(t)/dt, d\phi_2(t+\tau)/dt \rangle \\ &= \frac{a_1 a_2 (\cos 2\pi f_{\text{rh}} \tau) + 2\langle \Gamma_1(t), \Gamma_2(t+\tau) \rangle}{(a_1^2 + 2\Gamma_1^2)^{\frac{1}{2}} (a_2^2 + 2\Gamma_2^2)^{\frac{1}{2}}}, \end{aligned} \quad (17)$$

where  $\langle \Gamma_1(t), \Gamma_2(t+\tau) \rangle$  is the cross-correlation function of the noise terms of tracer velocities; furthermore,  $\Gamma_1$  and  $\Gamma_2$  are the root mean square of  $\Gamma_1(t)$  and  $\Gamma_2(t)$ . Regarding the strength of the synchronization, we theoretically assume that  $C_0 - C_1$  is twice the amplitude of the cross-correlation function at  $\tau = 0$ . If we further assume, to make the discussion simpler, that the two subjects behave statistically equivalent,  $a_1 = a_2 = a$  and  $\Gamma_1 = \Gamma_2 = \Gamma$ , but with no cross correlation in noise terms,  $\langle \Gamma_1(t), \Gamma_2(t+\tau) \rangle = 0$ , we obtain a simple form from Eq. (17) as

$$C_0 - C_1 = 2C(0) = \frac{a^2}{a^2 + 2\Gamma^2}. \quad (18)$$



In the case  $a^2 \gg \Gamma^2$  for the experimental rotation frequency region of Fig. 11,  $C(0)$  should be near unity, and therefore the strength of synchronization,  $2C(0)$ , would be as large as 2. However, this is not the case in the present experimental situation as seen in Fig. 11(a). Contrarily, Fig. 11(a) suggests  $a^2 \ll \Gamma^2$ , and in this case Eq. (18) could be written as

$$\text{Strength of synchronization} = \frac{1}{2}(a/\Gamma)^2 \quad (19)$$

by neglecting the  $a^2$  term in the denominator of Eq. (18). This analysis suggests the reasonable fact that the amplitude of the rhythmic component must increase more rapidly than the noise part with increasing common rotation frequency in order to observe the synchronization between the two subjects at high rotation frequency.

## V. DISCUSSION

It was demonstrated in the section of the experimental results of this paper that the hand motion of the two subjects in the mutual tracking experiments on a circular orbit showed a damped antiphase oscillation in a mutual error-correction mode at a low rotation frequency region ( $f_{cr} < 0.3$  Hz), and that, in a higher rotation frequency region ( $f_{cr} > 0.3$  Hz), it showed synchronized motion with no phase difference. The transition from a mutual error-correction mode to a synchronization mode was, thereby, observed when the common rotation frequency was in the vicinity of 0.3 Hz. This frequency was found close to that of the transition from a reactive error-correction mode to an anticipatory mode in the previously reported research in the single subject tracking experiments of the PC programed target on a circular orbit [6,7].

The main features of the periodic hand tracking at a higher rotation frequency region are the proactive anticipatory motion in the case of the single subject tracking of a PC programed target and synchronization motion in the case of mutual tracking of the two subjects. Both of these characteristic features appear at the rotation frequency higher than 0.3 Hz. At the same time in this region, a spontaneous rhythmic component was observed on top of the constant tracking speed. The amplitude was weak in the frequency region where the motions of the pair subjects were not synchronized,

and it became stronger with increasing rotation frequency [Fig. 11(b)]. A similar behavior of a rhythmic component was observed in the single subject tracking experiments of a PC programed target [7]. These experimental results suggested that the existence of the rhythmic component of a finite amplitude should be responsible for the proactive hand motion of the single subject tracking a PC programed target, and for the synchronized hand motions of the two subjects tracking each other. Also, these results showed experimentally that the interpersonal synchronization is only possible when the two subjects are operating in the anticipatory mode.

The physiological asymmetry [8] in the eye motion between the horizontal and vertical directions may be a cause for the birth of a rhythmic component in the hand motion, which may be enhanced due to the anisotropy of eye motion when the speed of the target is high. Preliminary data showed that the motion of the fovea of the subject in the vertical direction was suppressed compared to the motion in the horizontal direction for the circular motion of the target, and the motion of the fovea deviated from a circle to an ellipsoid when the target speed was high [7]. Also, the asymmetry of the muscle-skeletal system of the subjects in the vertical and horizontal motions may also play a role for the rhythmic component [9]. The origin, the location, and the rotation frequency dependence of the relevant rhythmic component might be identified in future research, including the neuronal correlation of the two visual motor systems under a cooperative task [10].

Mathematical description of the dynamics of the amplitude equation of the rhythmic component, as the order parameter of the system, based on this research would be indispensable for further understanding of visual tracking phenomena. We hope that the combination of the dynamical equations of the order parameters of the pair subjects with Eqs. (11) and (12) of the present paper should give us a closed picture of the mode transition in the visual hand tracking.

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