Do magnetospheric shear Alfvén waves generate sufficient electron energy flux to power the aurora?

Reply to comment by F. Mottez on "Do magnetospheric shear Alfvén waves generate sufficient electron energy flux to power the aurora?"

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[1] The gauge condition relating the electromagnetic potentials of shear Alfvén waves is one of the key governing equations in simulations of low-frequency waves in the Earth's magnetosphere. *Mottez* [2013] suggests that the gauge condition used in *Watt and Rankin* [2010] is inappropriate for the case where the perpendicular wave number of the shear Alfvén wave varies as a function of distance along the field line. We present an alternate derivation of this gauge equation and argue that the assumptions used are appropriate for low-frequency shear Alfvén waves provided that the variation of the perpendicular wave number parallel to the geomagnetic field is much smaller than the corresponding variation of the field amplitude.

[2] For a shear Alfvén wave in the magnetosphere, it is assumed that the perpendicular wavelengths are much smaller than the parallel wavelengths and that the frequency of the wave is much smaller than the local ion gyrofrequency. These assumptions allow us to represent the wave in terms of the scalar potential ϕ and the parallel component of the vector potential $A_\parallel$ only [*Tikhonchuk and Rankin*, 2000], delivering a considerable reduction in the spatial dimensions required for a numerical treatment.

[3] The shear Alfvén wave is assumed to operate in a plasma which is quasi-neutral, hence $\nabla \cdot \mathbf{J} = 0$, where $\mathbf{J}$ is the current density. For low-frequency waves such as the shear Alfvén wave, this assumption is appropriate. If z is the direction along the field line and x is one of the directions perpendicular to the field, then we could also write

$$\frac{\partial J_\parallel}{\partial z} + \frac{\partial J_\perp}{\partial x} = 0. \tag{1}$$

We use the same ansatz as described in equation (1) of *Mottez* [2013] to describe the variation of scalar and vector potentials in the perpendicular direction.

[4] In the parallel direction, the current is assumed to be carried by the electrons since they are closely tied to the geomagnetic field. We use the reduced Maxwell-Ampère equation to approximate the parallel current:

$$J_\parallel = -\frac{1}{\mu_0} k_\perp^2 A_\parallel, \tag{2}$$

where the perpendicular derivatives have been replaced by $ik_\perp$. In the perpendicular direction, the current is assumed to be carried by cold ions and can be described as

$$J_\perp = \frac{\rho}{B^2} ik_\perp \frac{\partial \phi}{\partial t}, \tag{3}$$

where ρ is the mass density and B is the local magnetic field strength.

[5] In a situation where we allow the perpendicular wave number to vary along the field line, the parallel derivative of $J_\parallel$ leads us to consider the parallel derivative of the product $k_\perp^2 A_\parallel$. The perpendicular derivative of the $J_\perp$ is obtained by assuming that the perpendicular variation of ρ and B is small in comparison to the perpendicular variation of ϕ. This assumption allows us to use the simplified one-dimensional description to describe the physics of the shear Alfvén wave all along the field line.

[6] Our evaluation of equation (1) then becomes

$$\frac{\partial \phi}{\partial t} + \frac{v_A^2}{k_\perp^2}\frac{\partial}{\partial z}(k_\perp^2 A_\parallel) = 0, \tag{4}$$

as used in all DK1D simulations with a nonuniform magnetic field [*Watt et al.*, 2006; *Watt and Rankin*, 2009, 2010, 2012]. Equation (4) is valid provided $A_\parallel \partial k_\perp^2/\partial z$ is much smaller than $k_\perp^2 \partial A_\parallel/\partial z$. The gauge condition that Mottez describes in his comment agrees with this result in the same limit. To summarize, this approximate gauge condition relies upon the assumptions of quasi-neutrality, small variations in plasma, and magnetic field strength in the direction perpendicular to the field, a strict separation of parallel and perpendicular motion in order to supply electrical currets, and a negligibly small displacement term. However, these conditions form a reasonable approximation of the plasma environment and the length and temporal scales in which magnetospheric shear Alfvén waves operate. To verify the validity of the approximation implied by equation (4), we have compared the magnitude of the neglected term by calculating it during one of the simulations presented in *Watt and Rankin* [2010]. The result is shown in Figure 1, which shows that in this particular case, the $A_\parallel \partial k_\perp^2/\partial z$ term is approximately 2% of the magnitude of the $k_\perp^2 \partial A_\parallel/\partial z$ term.

[7] Superficially, a one-dimensional model such as DK1D seems less realistic than higher-dimension models. The

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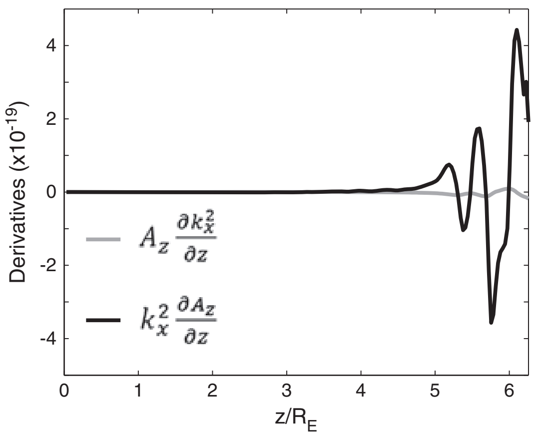

Figure 1. Estimate, using central differences, of the two gradients indicated during simulation C documented in *Watt and Rankin* [2010]. The gradients are shown as a function of distance along the simulation domain at the time when the wave envelope applied to the upper boundary has reached its midpoint (i.e., the time of maximum amplitude in $A_z = A_{\|}$). The approximate scaling of the perpendicular wavelength is appropriate when the values indicated in grey are small in comparison to the values indicated in black. In this case, the term involving the derivative of $k_x^2 = k_\perp^2$ is approximately 2% of the size of the term involving the derivative of A_z.

two-dimensional kinetic simulations of *Swift* [2007] and *Damiano and Johnson* [2012] independently show that the perpendicular wave number of shear Alfvén waves is not fixed in time. Two and a half dimension particle-in-cell studies [*Génot et al.*, 2001a, 2001b, 2004] demonstrate how perpendicular plasma gradients can contribute to the evolution of shear Alfvén waves and the development of the important narrow scales. However, all of these higher-dimensional numerical models must incorporate different assumptions in turn, either by using a significantly reduced simulation domain, or by enforcing a less realistic periodic boundary condition, or by using a relatively small amount of macroparticles to simulate the entire warm plasma population, resulting in noisy electromagnetic fields that require significant smoothing and averaging. The insight gained from each different model complements the insight from other idealized models with different assumptions. It is important to realize that the full physics of the problem would require a five-dimensional gyrokinetic simulation that could cover vast distances along the field, as well as describe the details of the small-scale variations across the field, and that these simulations should be able to reproduce physical times of tens of seconds in order to study full wave periods of the low-frequency Alfvén waves. These types of numerical simulations are not tractable, even given the considerable recent advances in computational power.

[8] Although the DK1D simulations are idealizations of the shear Alfvén wave interaction with electrons, they have provided new insight into the location of the electron acceleration along auroral field lines and have provided an upper estimate for the amount of energy flux that can be extracted from the waves. The differential energy flux predicted by the idealized simulations displays the same characteristics as the observable electron energy flux associated with shear Alfvén waves at both low and high altitudes [*Watt and Rankin*, 2012]. For example, the energization of electrons in the parallel and antiparallel directions observed by the Polar spacecraft at $r > 4\,R_E$ [*Wygant et al.*, 2002; *Janhunen et al.*, 2006] is seen clearly in the DK1D simulation at high altitudes [*Watt and Rankin*, 2012]. The rapid decrease of earthward wave Poynting flux at $r \sim 4\,R_E$, highlighted in statistical studies of Polar data [*Janhunen et al.*, 2004], is also reproduced by the simulations [*Watt and Rankin*, 2010]. And finally, the low-altitude signatures of Alfvén wave acceleration are reproduced in minute detail [*Watt et al.*, 2006] by simulations that use the same approximate gauge condition as given in equation (4). At the current time, researchers must often choose between a multidimensional simulation of the kinetic physics over a small section of the field line [e.g., *Génot et al.*, 2001a] and a one-dimensional simulation of electron kinetic physics over a large distance along the field [e.g., *Watt and Rankin*, 2010]. The hybrid two-dimensional particle-in-cell models of *Swift* [2007] and *Damiano and Johnson* [2012] provide a compromise, although large amplitude waves must be simulated in order to reliably extract information from the noisy wavefields. It is encouraging that even simplified simulation models such as DK1D can shed light on the complicated nonlinear plasma physics that governs some aspects of auroral acceleration, especially when these processes are only fleetingly observed with in situ spacecraft.

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