Electron acceleration and parallel electric fields due to kinetic Alfvén waves in plasma with similar thermal and Alfvén speeds

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Abstract: We investigate electron acceleration due to shear Alfvén waves in a collisionless plasma for plasma parameters typical of 4-5R_E radial distance from the Earth along auroral field lines. Recent observational work has motivated this study, which explores the plasma regime where the thermal velocity of the electrons is similar to the Alfvén speed of the plasma, encouraging Landau resonance for electrons in the wave fields. We use a self-consistent kinetic simulation model to follow the evolution of the electrons as they interact with the wave, which allows us to determine the parallel electric field of the shear Alfvén wave due to both electron inertia and electron pressure effects. The simulation demonstrates that electrons can be accelerated to keV energies in a modest amplitude wave. We compare the parallel electric field obtained from the simulation with those provided by fluid approximations.
Electron acceleration and parallel electric fields due to kinetic Alfvén waves in plasma with similar thermal and Alfvén speeds

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Abstract

We investigate electron acceleration due to shear Alfvén waves in a collisionless plasma for plasma parameters typical of $4 - 5R_E$ radial distance from the Earth along auroral field lines. Recent observational work has motivated this study, which explores the plasma regime where the thermal velocity of the electrons is similar to the Alfvén speed of the plasma, encouraging Landau resonance for electrons in the wave fields. We use a self-consistent kinetic simulation model to follow the evolution of the electrons as they interact with the wave, which allows us to determine the parallel electric field of the shear Alfvén wave due to both electron inertia and electron pressure effects. The simulation demonstrates that electrons can be accelerated to keV energies in a modest amplitude wave. We compare the parallel electric field obtained from the simulation with those provided by fluid approximations.

Key words: kinetic simulation, self-consistent, Landau damping

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Evidence from high-latitude in-situ observations of Earth’s magnetosphere indicates that shear Alfvén waves measured near the plasma sheet possess sufficient parallel Poynting flux which could, if converted to parallel electron energy flux, be responsible for some instances of auroral brightening (Wygant et al., 2000, 2002; Keiling et al., 2002, 2003; Chaston et al., 2005; Dombeck et al., 2005). However, the details of this conversion process are still not well understood.

For example, there is still much discussion regarding the location of the auroral acceleration region which is governed by waves. For those electrons which are accelerated through a quasi-static potential drop to form discrete auroral arcs, the evidence indicates that this acceleration occurs in a region at $2−3R_E$ radial distance. However, it has not yet been determined whether wave-mediated auroral acceleration only occurs at the same location as the potential drop, or whether it occurs higher up, nearer $4−5R_E$ radial distance (Janhunen et al., 2004, 2006), or indeed whether both cases are possible.

In order for shear Alfvén waves to accelerate electrons in the field-aligned direction, there must exist a component of the wave electric field in the parallel direction. Shear Alfvén waves can support a parallel electric field when the perpendicular scale length is comparable to the electron inertial length $\lambda_e = c/\omega_{pe}$ (Goertz and Boswell, 1979), or the ion acoustic gyroradius $\rho_{ia} = C_s/\Omega_i$ (Hasegawa, 1976). Here, $c$ is the speed of light, $\omega_{pe} = (n_e q_e^2/(m_e \epsilon_0))^{1/2}$ is the electron plasma frequency, $C_s = (2k_B T_e/m_e)^{1/2}$ is the ion acoustic speed, $\Omega_i = q_i B_0/m_i$ is the ion gyrofrequency, $n_\alpha$ is the number density, $T_\alpha$ is the
temperature, $m_\alpha$ is the mass, and $q_\alpha$ is the charge of plasma species $\alpha$. The inertial regime ($k_\perp \lambda_e \sim 1$, where $k_\perp$ is the perpendicular wavenumber) is more suitable for $2 - 3R_E$ radial distance, where it is expected that $v_{th,e} << v_A$ ($v_{th,e} = (2k_B T_e/m_e)^{1/2}$ is the electron thermal speed and $v_A = B_0 (\mu_0 n_i m_i)^{-1/2}$ is the Alfvén speed). However, at $4 - 5R_E$ radial distance, the ambient electron population is more energetic (see e.g. Wygant et al., 2000), and it is more likely that $v_{th,e} \sim v_A$. Although this plasma regime is between the inertial and the kinetic ($k_\perp \rho_m \sim 1$) limits, it may be important for shear Alfvén wave acceleration, since a significant number of electrons will be in Landau resonance with a shear Alfvén wave. If the wave can also support a parallel electric field at this location, then conditions are optimal for parallel electron acceleration.

Chaston et al. (2003) studied the behaviour of test-particle electrons along geomagnetic field lines between 100km and $5R_E$ altitude, using both kinetic and inertial corrections to the two-fluid wave solution. The results from this study predicted that the parallel electric field carried by the shear Alfvén wave would be reduced from that predicted by only using the inertial approximation, since the kinetic approximation for the parallel electric field has the opposite sign to that determined by the inertial approximation. However, a test-particle simulation is not able to clarify how the electron acceleration is affected by both a reduction in parallel electric field due to the kinetic correction, and an increase in the number of resonant particles due to the higher temperature electrons at $4 - 5R_E$ radial distance. To study these effects completely, it is necessary to take the whole distribution function into account. In this paper, we use a self-consistent kinetic simulation code to investigate the acceleration of electrons by shear Alfvén waves in plasma with $v_{th,e} \sim v_A$. Section 2 describes
the simulation code and physical model used to study this phenomenon, and
Section 3 details the results from a case study which demonstrates the sim-
ilarities and differences between the $v_{th,e} \sim v_A$ intermediate regime and the
$v_{th,e} \gg v_A$ inertial regime, which has been reported previously. We present
our conclusions in Section 4.

2 Simulation Model

The simulation code used to obtain the results in this paper is the self-
consistent drift-kinetic simulation code developed by Watt et al. (2004), which
has previously been shown to compare favourably with in-situ FAST satellite
observations (Watt et al., 2005, 2006). The model follows the evolution of
three variables: the scalar potential $\phi$, the parallel component of the vector
potential $A_\parallel$ and the electron distribution function $f_e$. By using the potential
description of the electromagnetic shear Alfvén waves, we can describe the
physical system in one dimension. It is assumed that electrons carry the par-
allel current and that ions carry the perpendicular current. Parallel ion motion
is neglected. The electron distribution function is allowed to evolve in time on
a fixed grid in phase space according to the gyro-averaged Vlasov equation:

$$
\frac{\partial f}{\partial t} + \left( p_\parallel - \frac{q_e}{m_e} A_\parallel \right) \frac{\partial f}{\partial z} + \left[ \frac{q_e}{m_e} \left( p_\parallel - \frac{q_e}{m_e} A_\parallel \right) \frac{\partial A_\parallel}{\partial z} - \frac{\partial \phi}{\partial z} \right] \frac{\partial f}{\partial p_\parallel} = 0, \quad (1)
$$

where $p_\parallel = v_\parallel + (q_e/m_e) A_\parallel$ is the parallel canonical momentum per unit mass,
$v_\parallel$ is the parallel velocity coordinate, $z$ is the parallel spatial coordinate and
$t$ is time. Note that in this paper, we consider a spatially uniform ambient
magnetic field, and so the magnetic mirror term in the Vlasov equation is not
required.
The potential variables are defined on fixed grid-points in the spatial \((z)\) domain. Using the first moment of the distribution function as the source term for the parallel current, we consider the parallel component of Ampère’s Law \((∇ \times B)_{\parallel} = \mu_0 J_{\parallel}\), to obtain an expression for the parallel component of the vector potential:

\[
A_{\parallel} = \frac{\mu_0 q e}{k_{\perp}^2 + \mu_0 \frac{q^2}{m_e}} \int_{-\infty}^{\infty} p_{\parallel} f dp_{\parallel},
\]  

(2)

Here, we neglect the displacement current and assume that all perpendicular variations can be expressed in the form \(\exp(-ik_{\perp} x)\), where \(x\) is a perpendicular coordinate. Under the latter assumption, all the perpendicular gradients can be reduced to factors of \(ik_{\perp}\), and in this fashion, we can reduce the simulation model to only one spatial dimension.

The system of equations is closed through the polarization current equation, which is combined with the perpendicular component of Ampere’s Law under the same assumptions as above to obtain:

\[
\frac{\partial \phi}{\partial t} = -v_A^2 \frac{\partial A_{\parallel}}{\partial z}.
\]

(3)

The simulation domain length is \(L_z = 4.7R_E\) and we assume uniform ambient magnetic field strength, and uniform initial plasma number density and temperature. In the magnetosphere, these three quantities vary along the field line with scale lengths that are much smaller than \(1R_E\). However, we ignore these variations in the present study.

We are interested in studying the behaviour of shear Alfvén waves along auroral field lines at geocentric distances of \(4 - 5R_E\). Hence, we choose magnetic field and plasma values which are representative (Wygant et al., 2000; Chaston
et al., 2003): $n_e = 10^5$ m$^{-3}$, $B_0 = 8.7 \times 10^{-7}$ nT, $T_e = 500$ eV. Note that the observations indicate a slightly higher electron temperature ($T_e \sim 1 - 2$ keV) than is used here, but we limit our study to $T_e = 500$ eV in order to ensure that the velocity grid does not have to cover large velocities which would require a relativistic treatment. A temperature of $T_e = 500$ eV is sufficient to demonstrate the behaviour we want to study. In this plasma regime we have $v_{th}/v_A = 0.22$, $\beta_e = 2.7 \times 10^{-5} \ll m_e/m_i$ and the important scale lengths are $\lambda_e = 16.8$ km and $\rho_{ia} = 3.71$ km.

The electron distribution function in the simulation is initialized with a Maxwellian distribution function. The potentials are initially set to zero at all points in the simulation domain, and a pulse potential of the form $\phi(t) = (1/2)\phi_0(1 - \cos[2\pi(t/t_1)])$ is added to the scalar potential at the top of the simulation domain ($z = 4.7 R_E$) for $0 < t < t_1$, where $t_1 = 0.25$ s and $\phi_0 = 600 V$. The initial perpendicular electric field strength corresponds to $E_\perp = 60$ mV/m, and changes self-consistently as the wave interacts with the plasma. The pulse travels in the $-z$ direction until it reaches the lower boundary, where the boundary conditions for the potentials are such that the wave is partially reflected (see Watt et al., [2004]): $A_\parallel = -\mu_0 \Sigma_P \phi$, where $\Sigma_P$ is the height-integrated Pedersen conductivity. We are interested only in the behaviour of the plasma before the pulse reaches the lower boundary, and therefore the boundary condition is not particularly important for the calculations presented here.

The perpendicular scale length of the wave for this case study is chosen to be $\lambda_\perp = 6.3 \times 10^4$ m ($k_\perp = 10^{-4}$ m$^{-1}$), which, when mapped to ionospheric altitudes, corresponds to a scale length of 8.2 km. Observational studies of auroral arc widths (Knudsen et al., 2001) indicate that this is a reasonable choice of perpendicular scale. The key wave parameters are therefore $k_\perp \lambda_e = 1.68$.
and $k_{\perp} \rho_{ia}(= k_{\perp} \lambda_{e} v_{th,e}/v_A) = 0.37$. Note that $\rho_{ia}$ is often used as a convenient shorthand in place of $\lambda_{e} v_{th,e}/v_A$, which can lead to the impression that it is ion effects which generate the parallel electric field in the kinetic limit. However, the parallel electric field in the kinetic limit arises from finite electron pressure, and so when $k_{\perp} \lambda_{e} v_{th,e}/v_A \sim 1$, there will be a finite parallel electric field even if $T_i = 0$ (Nakamura, 2000). In this intermediate plasma regime of $v_{th} \sim v_A$, both electron inertia and electron pressure are important for the formulation of parallel electric fields. Neither the kinetic nor inertial limits are appropriate for $v_{th,e} \sim v_A$, and so a fully kinetic code is necessary to study the nonlinear shear Alfvén wave-particle interactions.

3 Simulation Results and Discussion

Figure 1 shows some selected plasma and field diagnostics from a simulation with the initial plasma parameters given in the previous section. Each quantity is displayed as a function of time at $z = 1R_E$, i.e. the pulse has travelled through $\sim 3.7R_E$ of plasma before reaching this point. Figure 1(a) shows the differential electron energy flux of the downward moving electrons, (b) shows the absolute value of the parallel electron energy flux ($Q_\parallel = \int v^2 v_\parallel dv$), (c) shows the parallel current ($J_\parallel = q_e \int v_\parallel f dv$), (d) shows the perpendicular electric field ($E_\perp = -\nabla_\perp \phi$), (e) shows the parallel electric field ($E_\parallel = -\partial A_\parallel/\partial t - \partial \phi/\partial z$), and (f) shows the perpendicular magnetic field perturbation ($B_\perp = (\nabla \times A_\parallel)_\perp$). The pulse shape exhibits a slight change from its original sinusoidal form, with modest steepening of the leading edge, but this steepening is not as pronounced as in the inertial cases reported in Watt et al. (2004, 2005). In those cases $q\phi \gg k_B T_e$, but here we have $q\phi \sim k_B T_e$ because
the electron temperature is much higher. Note that the electromagnetic field perturbation observed for $1.02 < t < 1.25$ s is a signature of the reflected pulse. This upward travelling pulse has a smaller amplitude because there is only partial reflection at the lower boundary.

Previous studies (Watt et al., 2005; Watt and Rankin, 2006) of electron acceleration due to shear Alfvén waves have shown that the parallel electron energy flux $Q_\parallel$ will be enhanced due to this acceleration. However, it is important to distinguish an increase in $Q_\parallel$ which is due to the resonantly accelerated beam electrons, and an enhancement which corresponds to the parallel current of the wave that is carried by the electrons. The vertical dashed line in Figure 1 shows the approximate time when the beam electrons have almost all passed $z = 1R_E$ and the signature of the parallel current begins. For the inertial cases reported in previous publications, the steepened wave profile made it easy to identify which $Q_\parallel$ signature was due to the beam, and which signature was due to the wave parallel current. This was because the wave profile followed the same steepened characteristics. In this case, the individual signatures of the accelerated beam electrons and the parallel current are not as easy to distinguish, but it is clear that the parallel electron energy flux is enhanced before the parallel current starts to increase, and so at least some of the parallel electron energy flux shown in Figure 1(b) is due to the resonantly accelerated beam electrons which arrive before the shear Alfvén wave pulse.

Figure 1(a) shows evidence of resonantly accelerated electrons in the form of an energy-dispersed beam for $0.53 < t < 0.65$ s. These electrons have energies between 3.3keV and 9.0keV. Non-MHD effects reduce the phase speed of the wave below the Alfvén speed $v_A = 6.0 \times 10^7$ m/s. In this simulation, the pulse moves down the simulation domain with a measured speed of $v_{ph} \sim 3.4 \times$
10^7 m/s. Hence the resonant electron energy is 3.19 keV. From the differential
electron energy flux in Figure 1(a), it can be seen that electrons are accelerated
above this resonant energy, and form a high-energy beam.

Even though the maximum parallel electric field amplitude in the interme-
diate regime (\(v_{th,e} \sim v_A\)) has the same magnitude as that reported previ-
ously in inertial regime studies (Watt et al., 2004; Watt and Rankin, 2006),
\(E_\parallel \sim 0.2 \) mV/m, it can be seen that electrons are accelerated to much higher
energies, keV instead of hundreds of eV. This is only possible because the
resonant phase velocity is high, and the hot electron distribution function
provides sufficient electrons with matching velocities.

There has been much discussion regarding the calculation of the magnitude
of the parallel electric field in the intermediate plasma regime (\(v_{th,e} \sim v_A\))
(Chaston et al., 2003; Shukla and Stenflo, 2004; Chaston, 2004). For interest,
we have calculated the parallel electric field in the inertial and kinetic limits,
even though neither accurately apply to this situation. In the inertial limit we
have (e.g. Lysak, 1990):

\[
E_{\parallel,i} = -\frac{\lambda_e^2 e}{1 + \lambda_e^2 k^2 \parallel} \frac{\partial}{\partial z} \nabla \cdot E_{\perp},
\]

and in the kinetic limit:

\[
E_{\parallel,k} = \frac{\lambda_e^2 v_{th,e}}{v_A^2} \frac{\partial}{\partial z} \nabla \cdot E_{\perp}.
\]

Figure 2 shows the parallel electric field as determined from the self-consistent
simulation potentials \(E_{\parallel,s} = -\nabla \phi - (\partial A_\parallel / \partial t)\) (solid line), \(E_{\parallel,i}\) (dashed line),
\(E_{\parallel,k}\) (dotted line), and finally from the sum of the electric field approximations
\(E_{\parallel,i} + E_{\parallel,k}\) (dot-dashed line). The simulation parallel electric field is indeed
reduced from the inertial approximation. However, what is most surprising is that it is reduced by exactly the amount predicted from the kinetic approximation (note that the dot-dashed line is difficult to make out in Figure 2 because it lies almost exactly on top of the $E_\parallel$ from the simulation). Hence, the approximations used by Chaston et al. (2003) appear to yield the correct parallel electric field. Note that the parallel electric field in the studies presented here is a diagnostic of the simulation code and not an intrinsic simulation variable [i.e., it does not appear in equations (1)-(3)]. It is also important to note that equations (4) and (5) relate the size of the parallel electric field to the gradient of the perpendicular electric field, $E_\perp$. In this simulation, $E_\perp$ varies in response to the plasma evolution. This evolution of $E_\perp$ can be a change in profile (e.g. nonlinear steepening) or a change in amplitude due to the wave particle interactions. Analysis of wave and plasma energy changes in the simulation presented here shows that by the time the wave reaches $z = 1 R_E$, it has converted 37% of its Poynting flux to accelerated electron energy flux (energy flux contained in the beam electrons which arrive before the pulse).

Although a combination of equations (4) and (5) yields a very good approximation for $E_\parallel$, it is also necessary to have the correct form of $E_\perp$. Therefore, in order to obtain the correct amplitude and profile for the parallel electric field, and therefore the correct numbers and energies of accelerated electrons, a self-consistent simulation code is essential.

Watt et al. (2006) showed using a self-consistent simulation code with a non-uniform magnetic field that as a pulse travels through regions of increasing Alfvén speed (e.g. travels along a magnetic field line from the plasma sheet towards the ionosphere) it can catch up to previously accelerated electrons and accelerate them further, to even higher energies, through the same resonant
process. The results of Watt et al. (2006) and the results presented here suggest that electrons resonantly accelerated to keV by shear Alfvén waves at $4 - 5R_E$ radial distance may experience further acceleration by the same wave as the wave progresses to regions of higher Alfvén speed closer to the Earth. In order to test this prediction, we plan to extend our simulation code to follow the evolution of a pulse and its interaction with the ambient electron population from radial distances of $5R_E$ to 200km altitude in order to investigate the conditions for excitation of high-energy electron beams when one takes into consideration the changing temperature, number density and magnetic field profiles in this region.

4 Conclusions

We have investigated, using self-consistent kinetic simulations, the similarities and differences between shear Alfvén waves in an intermediate ($v_{th,e} \sim v_A$) regime of parallel electron acceleration for plasma parameters that are appropriate to radial distances of $4 - 5R_E$. Previous studies of electron acceleration in this plasma regime have been performed using a test-particle approach, which required the use of assumptions regarding the form of the parallel electric field. Using our self-consistent code, we can be confident that the parallel electric field we obtain is correct, and that the parallel electron acceleration seen is quantitatively more realistic.

We have shown that the expression for the parallel electric field as a function of the gradient of the perpendicular electric field as used by Chaston et al. (2003) provides a reasonable approximation to the parallel electric field obtained from a fully nonlinear self-consistent simulation that includes the necessary electron
inertia and electron pressure effects.

A definitive answer as to whether electrons are accelerated by shear Alfvén waves at $2 - 3R_E$ or $4 - 5R_E$ radial distance will require the marriage of a large number of in-situ observations and sophisticated nonlinear kinetic models which include the effects of plasma and magnetic field inhomogeneities. However, the self-consistent simulation results shown in this paper indicate that for plasma conditions typical of $4 - 5R_E$ radial distance, shear Alfvén waves can accelerate electrons in the parallel direction to keV energies for modest amplitude waves and hence our results motivate further study of electron acceleration in this plasma regime.

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Fig. 1. The time evolution of plasma and wave diagnostics from the simulation run at \( z = 1R_E \): (a) the differential electron energy flux of the downward moving electrons, (b) the absolute value of the parallel electron energy flux, (c) the parallel current, (d) the perpendicular electric field, (e) the parallel electric field, and (f) the perpendicular magnetic field perturbation.

Fig. 2. The parallel electric field determined from simulation parameters: due to simulation potentials \( E_{||,s} = -\nabla ||\phi - (\partial A||/\partial t) \) (solid line); approximation due to inertial effect \( E_{||,i} \) (dashed line); approximation due to kinetic effect \( E_{||,k} \) (dotted line), and the sum of the electric field approximations \( E_{||,i} + E_{||,k} \) (dot-dashed line).
Figure 1

(a) Energy (eV) vs. time (s)
(b) $|Q_\parallel|$ (µW/m²) vs. time (s)
(c) $J_\parallel$ (µA/m²) vs. time (s)
(d) $E_\perp$ (mV/m) vs. time (s)
(e) $E_\parallel$ (mV/m) vs. time (s)
(f) $B_\perp$ (nT) vs. time (s)

(differential electron energy flux (m⁻²s⁻¹))