Chess Endgame News

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NOTE

CHESS ENDGAME NEWS

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This note begins with an apology for a recent error in the Journal and then examines some issues and two new types of EGT associated with the ply-counting rules of play over the board. It also reviews reaction to the published 'Haworth's Law' conjecture and provides evidence to support supplementary conjectures.

The advent of the chess endgame table (EGT) has changed the activity of chess composition. It has become a key tool for creating correct sub-8-man compositions, so much so that the dates of their availability (Haworth, 2014a) should be better known. Extra kudos should go to those who created such studies before helpful EGTs were available. It is unfortunate that the Codex for Chess Composition (WFCC, 2014b) does not encourage or even provide for a description of how the study was composed: effort and achievement are both, separately, worthy of acclaim. In connection with Müller and Haworth (2013), the authors would like to apologise to Noam Elkies for the fact that an incorrect reference to the use of EGTs was not removed from the published text as intended. Noam's 1993 study composition (Van der Heijden, 2010) preceded the KRPKBP EGT by a decade.

id	m	w-b	Endgame	FEN	Value 1-0?	5-way	ply-depth	in metric	Notes
m1	5	2-3	KQKRB	K4b2/8/8/8/6Q1/8/k3r3/8 b - - 0 1	1-0	-2	-140	DTM_{50}	a maxDTM_{50} 2-3m_P-less pos for $pc = 0$
m2	5	2-3	KQKRB	K4b2/8/8/8/6Q1/8/k3r3/8 b - - 11 1	1-0	-2	-144	DTM_{50}	a maxDTM_{50} 2-3m_P-less pos for $pc \geq 0$
m3	5	2-3	KQKRP	8/8/1pk5/K7/8/8/1r6/4Q3 w - - 0 1	1-0	2	193	DTM_{50}	the maxDTM_{50} 2-3m pos for $pc = 0$
m4	5	2-3	KQKRP	8/1p4Q1/3k4/8/r7/8/8/2K5 b - - 33 1	1-0	-2	-214	DTM_{50}	a maxDTM_{50} 2-3m pos for $pc \geq 0$
m5	5	3-2	KBNKN	8/8/1N6/8/6B1/1K3n2/8/k7 b - - 0 1	1-0	-2	-160	DTM_{50}	a maxDTM_{50} s6m_P-less pos for $pc = 0$ or $pc \geq 0$
m6	5	3-2	KPPKP	8/8/8/1p3K2/3P4/3P4/7k/8 b - - 0 1	1-0	-2	-282	DTM_{50}	1 of 4 maxDTM_{50} s6m pos for $pc = 0$ or $pc \geq 0$
m7	5	3-2	KNNKP	6k1/p7/8/8/7N/7K/2N5/8 w - - 0 1	1-0	2	223	DTM_{50}	a maxDTM_{50} KNNKP pos for $pc = 0$
m8	5	3-2	KNNKP	8/8/5N2/p7/8/k1K5/8/1N6 b - - 4 1	1-0	-2	-256	DTM_{50}	a maxDTM_{50} KNNKP pos for $pc \geq 0$
m9	5	3-2	KQPKQ	3Q4/8/8/5K2/3P4/7k/1q6 b - - 0 1	1-0	-2	-274	DTM_{50}	a maxDTM_{50} KQPKQ pos for $pc = 0$
m10	5	3-2	KQPKQ	3Q4/8/8/5K2/8/3P3k/1q6 w - - 88 1	1-0	2	275	DTM_{50}	a maxDTM_{50} KQPKQ pos for $pc \geq 0$
m11	6	3-3	KBBKNN	7k/7B/2B5/3K4/2n5/8/5n2 w - - 0 1	1-0	2	179	DTM_{50}	a maxDTM_{50} KBBKNN pos for $pc = 0$
m12	6	3-3	KBBKNN	2n5/8/3B4/8/3K4/1B6/6n1/2k5 w - - 43 1	1-0	2	181	DTM_{50}	a maxDTM_{50} KBBKNN pos for $pc \geq 0$
z1	5	3-2	KQPKQ	8/8/1P5Q/1K6/3q4/8/5k2/8 w	1-0	2	99	DTZ_{50}'	the maximum known |dtz_{50} - dtz|. dtz = 1p; 1. b7?? but 1. Qg5''''
z2	5	3-2	KRPKP	6R1/P7/1k6/8/8/8/p2K4/8 b	'1-0'	-1	-1	DTZ_{50}'	dtz = -2p; 1. … a1=Q'''' 2. a8=Q'''' (dtz = -106p)
z3	6	3-3	KBBKQN	1n2K3/7q/6B1/8/8/B7/3k4/8 w	1-0	2	7	DTZ_{50}'	dtz = 1p; 1. Bb4+'''' Ke3'' 2. Bc5+'''' Kf4'' 3. Bd6+'''' K~ 4. Bxh7''
z4	6	2-4	KQKBBN	8/bb6/5Q2/8/8/3k4/8/1K1n4 w	1-0	1	89	DTZ_{50}'	dtz = 101p, so dtz > 100p but, even so, $dtz_{50}' <$ dtz
z5	6	4-2	KRRPKQ	7q/7k/8/6R1/8/K1P2R2/8 b	'1-0'	-1	-387	DTZ_{50}'	|dtz| ≤ 385 < |dtz_{50}'|; a maxDTZ_{50}' s7m_P-ful position
pr1	4	2-2	KPKQ	8/8/8/k7/8/q7/1K4P1/8 w	1-0	2	57	DTM	1 ply precursor to a maxDTM(3) pos: 1. Kxa3''''
pr2	6	3-3	KRPKQN	8/8/1k6/R7/8/nq6/1K4P1/8 w	1-0	2	59	DTM	3 ply precursor to a maxDTM(3) pos: 1. Kxb3'''' Kxa5'' 2. Kxa3''''
pr3	5	3-2	KQPKR	8/5k2/2PK4/5Q2/5r2/8/8/8 b	1-0	-2	-86	DTM	1 ply precursor to a maxDTM(4) pos: 1. … Rxf5''
pr4	7	4-1	KQPPKBP	8/8/1p6/4P3/k1b1P3/Q1K5/8 b	'1-0'	-2	-256	DTM	2 ply precursor to a maxDTM(5) pos: 1. … Kxa2'' 2. Kxc3'''' (Ka3''')
pr5	8	4-3	KRNNKQNN	6qk/5n2/5N2/8/8/5n2/1RK5/1N6 w	'1-0'	1	525	DTM	2 ply precursor to a maxDTM(6) pos: 1. Nxg8'''' 2. Kxg8''
pr6	9	4-5	KQRPKQRBN	1n1k4/6Q1/5KP1/8/6Rq/1r4b1/8/8 w	'1-0'	1	1,099	DTM	2 ply precursor to a maxDTM(7) pos: 1. Rxh4'''' Bxh4''
c1	3	2-1	KQK	7K/6Q1/8/8/8/8/2k5/8 b	1-0	-2	-20	DTC/Z	1 of 8 maxDTC/Z KQK positions
c2	5	3-2	KQBKB	1BKQ4/8/8/8/8/8/8/1k3b2 b	1-0	-2	-16	DTC/Z	1 of 998 maxDTC/Z KQBKB positions
c3	5	3-2	KQNKN	7N/8/8/8/8/7n/2k1K3/4Q3 b	1-0	-2	-18	DTC/Z	1 of 39 maxDTC/Z KQNKN positions
c4	5	3-2	KQPKP	8/2p5/8/8/k7/2P5/8/K1Q5 b	1-0	-2	-12	DTC	1 of 39,633 maxDTC KQPKP positions

Table 1. Cited positions including DTM_{50} records, DTZ_{50}' examples and precursors of DTM records.
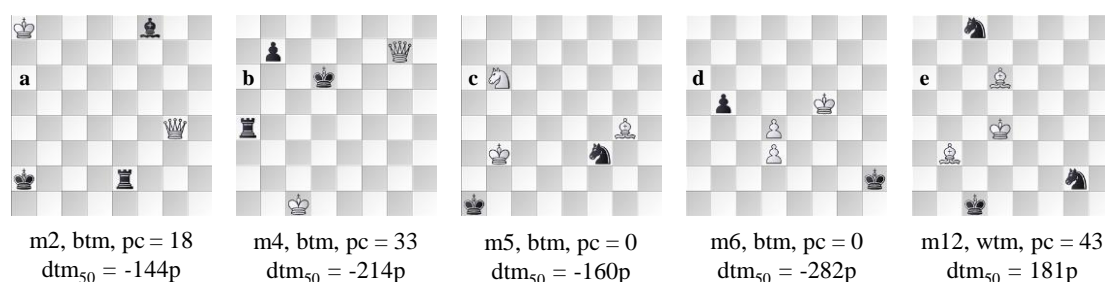
A number of developments suggest a focus on the FIDE *k*-move rules of play (*k*mr) involving the ply-count, despite the fact that such rules are mutable and have been changed several times in the past.

First, FIDE (2014) have drawn up Rule 9.6b, effective 1st July 2014, which defines a position with a ply-count of 150 as drawn, at which point the arbiter should intervene in the game. The logistical difficulties of this 75mr will no doubt be discussed (Gijssen, 2014): perhaps chess technology should include a facility to indicate the ply-count. The ICGA will consider the applicability of the 75mr in its Computer Chess tournaments. The World Federation for Chess Composition (WFCC, 2014a), previously the FIDE PCCC, will also need to comment on the relevance of the 75mr to Retrograde Problems, the only type of composition for which the 50mr is relevant. There are many endgames, e.g. KRRPKQ (maxDTZ ≥ 383 ply) and KRNPKRB (519 ply), where both sides would reasonably hope to win over the board even after 150 ply of optimal play.

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Secondly, the International Correspondence Chess Federation (ICCF, 2014a/b) voted at its 2013 congress in Kraków to award the theoretical result of games to anyone making a correct claim based on a 6-man position as evaluated by the Lomonosov 6-man DTM EGTs. The author is assured that this rule, effective 1st January 2014, also applies to positions with less than six men. Epistemologists should note that the s7m EGTs have finally been given the imprimatur of correctness from an international body. They have indeed been checked, position for position, against the more widely used Nalimov EGTs (Bleicher, 2014) which, by implication, have earned the same mark of approval.

The ICCF's decision implicitly suspends both the 50mr and 75mr for sub-7-man correspondence chess. Convecta (2014) will provide ICCF tournament directors with free access to s7m EGTs. The Lomonosov 7-man DTM EGTs have not been independently and fully verified yet, a task scheduled for this August, but we may look forward to the ICCF adjudicating at the 7-man point.

| m2, btm, pc = 18 | m4, btm, pc = 33 | m5, btm, pc = 0 | m6, btm, pc = 0 | m12, wtm, pc = 43 |
| dtm_{50} = -144p | dtm_{50} = -214p | dtm_{50} = -160p | dtm_{50} = -282p | dtm_{50} = 181p |

Figure 1. maxDTM$_{50}$ records in KQKRB, KQKRP, KBNKN, KPPKP and KBBKNN.

Thirdly, there have been two excellent initiatives in the creation of EGTs which recognise the 50-move rule. Huntington (2013) has created DTM$_{50}$ EGTs which give the Depth to Mate value in the context of the 50-move rule and the ply-count *pc*. De Man has created his 'syzygy' WDL′ and DTZ$_{50}$′ EGTs (CPW, 2013a/b; de Man, 2013a/b/c) which give, respectively, a 5-way indication of 'value'[2] and a depth to the zeroing of the ply-count, again in the context of the 50-move rule.

As *pc* increases and less ply are available, the winning side becomes more constrained in the current phase of the game. DTM$_{50}$ may therefore increase as the number of available ply decreases, usually to the point where the win slips away to a draw. Thus DTM$_{50}$, for a given position and side-to-move, has a number of values, each corresponding to some range of *pc* values. The main Huntington innovation is that these are all stored in his DTM$_{50}$ EGTs. Thus, the following questions may be answered for a specific endgame:

- Is maxDTM$_{50}$ greater, equal to or less than maxDTM?
- What is maxDTM$_{50}$ for *pc* = 0, for *pc* = n, or taken across all values of *pc*?
- What positions have the most different values of *dtm$_{50}$* across all values of *pc*?[3]
- What positions carry the largest 50mr-penalty, *dtm$_{50}$*−*dtm*?[4]

Extreme DTM$_{50}$ values, positions and lines (Haworth, 2014a/b) for 5-man *n-m* 1-0 wins, P-less and P-ful, are to be found in Table 1's m1-m6, Figure 1 and the appendix. The Huntington EGTs also answer long-standing questions about the maximal 50mr-mates in, e.g., KNNKP (Dekker, van den Herik and Herschberg, 1989), KQPKQ (Tamplin and Haworth, 2004) and KBBKNN (Huntington, 2013), q.v., Table 1's m7-m12.

Some DTM$_{50}$ lines show the loser playing moves which are not DTZ-optimal, most notably, ending a phase by Pawn-push and/or capture. The winner also departs from DTZ-optimality as in Table 1 z1. Both may depart from DTZ$_{50}$- or DTM-optimality, either ignoring or being constrained by *pc*. Clearly the SM$_{(50)}$Z and SZM$_{(50)}$ strategies, successively filtering moves on one metric then the other, are not identical to the SM$_{50}$ strategy.

Huntington is notable as only the second EGT author to use a functional programming language, in this case, HASKELL. Hurd previously used HOL to generate chess EGTs in order to show that functional languages could manage large volumes of data (Hurd and Haworth, 2010).

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[2] '2' ≡ 50mr-win, '1' ≡ other wins, '0' ≡ draw, '-1' ≡ loss but not 50mr-loss and '-2' ≡ 50mr-loss.
[3] 8/p7/2N3K1/8/8/2N1k3/8 w has 26 DTM$_{50}$ *pc*-dependent values (61-112m), the most for KNNKP.
[4] k7/p4K2/4N3/8/2N5/8/8/8 w has DTM$_{50}$ = 9m (*pc* = 0-90) but 104m (*pc* = 91-92) with *pc*-cost 95m.
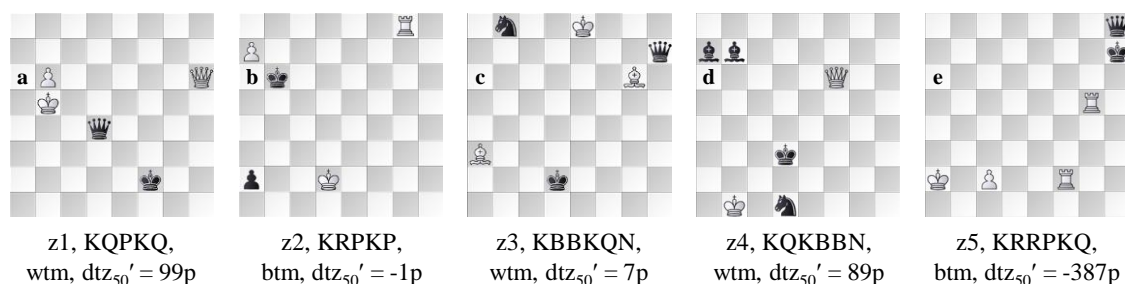
Figure 2. Some maxDTZ$_{50}'$ records as in Table 1's z1-z5.

Turning now to de Man's 'syzygy' DTZ$_{50}'$ EGTs, it should first be explained that the notation DTZ$_{50}'$ is the author's rather than de Man's. It is intended to emphasise the fact that the metric of the syzygy EGTs, unlike the DTZ$_{50}$ metric, also ascribes depths to type-1 decisive positions.[5] After identifying the 50mr-wins and losses, de Man has a DTZ$_{50}$ EGT: remaining positions are unconditional draws or type-1 wins drawn under the 50mr.

The type-1 wins include two subsets of positions, one being the immediate precursors of type-2 wins of depth 100 ply, the other being immediate precursors of type-1 positions in the next phase. De Man continues his retrograde analysis by 'unmoves' from positions in these two subsets which are given a nominal DTZ$_{50}$ depth of 101 ply, even when they convert in one ply to the next phase. This idea clearly differentiates the type-1 and type-2 wins in the statistics but creates an ambiguity in that '$dtz_{50}' = 101$' can mean '$dtz_{50}' = 101$' or '$dtz_{50}' = 1$' as in Table 1's z2.[6] This note reports actual dtz_{50}' values, particularly as the dtz/dtz_{50}' comparison is of interest.

The actual dtz_{50}' may be less than, equal to or greater than dtz. $dtz_{50}' < dtz$ for some type-1 results with $dtz \leq 100$p and even for $dtz > 100$p, q.v., Table 1's z4.[7] $dtz_{50}' = dtz$ for the majority of positions where the transition to the next phase is not to a position affected by the 50mr. $dtz_{50}' > dtz$ where the 50mr complicates the win as in, e.g.,Table 1's z1 and z3. This may also be the case therefore when $dtz > 100$ ply: Table 1's z5 is a max DTZ$_{50}'$ s6m_P-ful position with $dtz_{50}' = 387$ ply but $dtz \leq 385$ ply (Bourzutschky and Konoval, 2012).

The notation DTZ$_k'$ also recognises the fact that there are other ways to supplement a '$k$mr' EGT: these could be indicated by DTZ$_k''$ etc. One could for example simply enter DTZ or DTR (Haworth, 2000, 2001) values for the type-1 wins/losses. It is timely here to acknowledge that de Man was the first to point out that the proposed DTR EGTs would not necessarily indicate the optimal move over the board if the DTR-optimal move required, as indicated by the DTZ$_R$ metric, more ply than were available. This is analogous to the lowest nominal DTM$_{50}$ not being achievable if too few ply are available, as is evidenced by Huntington's DTM$_{50}$ EGTs. The required on-demand, runtime solution to the 'DTR problem' is to search forward in the EGT and/or recompute a DTR EGT, performing 'unmoves' for only n cycles if there are only n ply available in the phase. This could be made more efficient by using prior knowledge of position values and DTZ depths, given that $dtr \geq dtz$.

De Man's implementation approach included the following innovations, several being original to him:

- 5-way position-evaluation: '± 2' $\equiv$ 50mr-wins/losses, '± 1' $\equiv$ 'win/loss' 50mr-draws, '0' $\equiv$ draw,
- Separate EGTs for 'value' and 'depth': easier depth notation, improved EGT compression,
- Retaining only the smaller of the 'wtm' and 'btm' sides of the EGT,
- Depths computed and quoted in ply rather than in moves,[8]
- Using compression-optimising values for 'broken' positions, and positions at $dtz_{50}' = 1$.

DTZ$_{50}'$ data is available, to some extent, via HOUDINI 4, the 'SYZYGY' STOCKFISH engine and the DEEPFRITZ14 GUI, this last used by the author. Currently, the DEEPFRITZ14 GUI does not report the 5-way value of the position. It also incorrectly decrements DTZ$_{50}'$ by '1' per 4 plies for DTZ$_{50}' > 100$ ply. This means that moves that appear to be DTZ$_{50}'$-equi-optimal may not be: examples have been encountered of the winner 'covertly' stalling by one move[9] and the loser conceding one move. Chessbase are however fixing this bug.

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[5] Type-1 wins/losses are those which are decisive in the absence of the 50mr but drawn under the 50mr.

[6] An alternative way to differentiate the depths of type-1 and type-2 positions is to add 100 to all type-1 depths.
For a small compression cost, this is equivalent to reporting (*value*, dtz_{50}') rather than just dtz_{50}'.

[7] For Table 1's z4, $SZ_{(50)}^-/SZ_{(50)}^+$: 1. Qf1+'''' Kd2'' and 'priority' is forcing the 50mr draw rather than delaying conversion.

[8] e.g., a DTZ$_{50}'$ line from Table 1's z5 ends with 1R6/8/8/k7/4q3/1R6/K1P5/8 w and 194. Ra8+'' Qxa8° 1-0.

[9] e.g., 8/8/5q2/8/2k5/3R4/1KP5/2R5 w: the DEEPFRITZ14 GUI reports 1. Ka2' Qa6' 2. Kb2' Qf6' {position 1w!}

These 6-man DTZ_{50}' and the Lomonosov team's 7-man DTZ_{50} EGTs indicate that the computation and publication of 6-man DTZ EGTs need not be far away.

Still on the subject of the rules of chess, Magnus Carlsen has joined Nigel Short in suggesting that if a player is stalemated, they should be considered to have lost rather than drawn as was determined in the 19th century. This would of course change the nature of the game and invalidate the content of almost all EGTs. An alternative is that the stalemated side is deemed to play a *null move*: double stalemate would presumably remain a draw!

Victor Zakharov provides first information about the team who computed the 7-man DTM EGTs. Victor himself provided ideas, management and promotion while Vladimir Makhnychev (now with Google) was the main author of the generator code. Vladislav Schukin coded many utilities, a new EGT-compressor and a DTZ generator. Alongside these three leaders, Artem Mostyaev created the first WDL ternary tables, and worked on compression and AQUARIUM, Peter Luferenko (now with Microsoft) created the first version of the online probe server, and Vladimir Ivanov and Kirill Smorodin worked on the Chessok 7m-server site. Ekaterina Kortunova is working to interface mobile devices with the Lomonosov EGTs. Truly, this is a team effort to be congratulated.

There has been some response to the publication of the 'Haworth's Law' conjecture which suggests that the observed linear trend in $\log(\text{maxDTM}(k))$ will continue for some time (Haworth, 2013). Online discussion, at times passionately flawed, even considered the longest '50m' game and whether the original position is decisive! The relevant notation here and some supplementary conjectures are:

$E \equiv WB$, an endgame with White force W and Black force B,
$Em \equiv WmBm$, endgame E with man m added to both sides,
$\text{maxDTM}(E) \equiv$ the maximum DTM in plies of the White wins in E ('0' if there are no wins), and
$\text{maxDTM}(k) \equiv \max\{\text{maxDTM}(E) \mid E$ is a k-man endgame$\}$

C1) if $k \geq 2$, $\text{maxDTM}(k+1) > \text{maxDTM}(k)$,
C2) if $k \geq 2$, a maxDTM k-man position p_k may be modified to a DTM-deeper, prior position pr_{k+1}:
    the side which does not have the move may often be imagined to have just captured a man,
C3) if $k \geq 2$, there is a k-man endgame E and man m such that $\text{maxDTM}(Em) \geq \text{maxDTM}(E)$,

C1 is most likely to be true and C1/2 are certainly true[10] for $k = 2\text{-}6$, see Table 1's pr1-pr6. C3 has been verified for $k = 2\text{-}5$ for all endgames E, not just 'at least one'. It soon becomes clear that C3 is not true if the metric DTM is replaced by DT(C/Z)_k with $k > 9$: maxDTC/Z(KQK) = 10m while maxDTC = 8m (KQBKB), 9m (KQNKN) and 6m (KQPKP), q.v., Table 1's c1-c4 (Tamplin, 2014).

It is certainly difficult to imagine the linear trend in $\log(\text{maxDTM}(k))$ continuing all the way to 32-man chess. If it did, there would be, without $k$mr drawing rules, 32-man wins in 298 billion moves and 32-man phases of 161 billion moves. How would a crowded 64-square board accommodate so many moves? Ken Regan suggests that we are seeing the start of an asymptote-seeking ogive, a curve seen shedding load in 13th century Gothic arches or minimising resistance in the noses of 21st century projectiles. A more extreme suggestion is:

C4) for some k, $\text{maxDTM}(k+2) - \text{maxDTM}(k+1) < \text{maxDTM}(k+1) - \text{maxDTM}(k)$.

Again, if this were not so, there would be 32-man wins in 7,724 moves, but these may exist anyway if the predicted linear trend persists to 11-man chess.

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[10] Here, with no decisive KK positions but KBKB losses in 0, *null* is considered to be 'less than' zero.

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Appendix

One of four 'any *pc*' $maxDTM_{50}$ sub-6-man (KPPKP) losses and wins, 8/8/8/1p3K2/3P4/3P4/7k/8 b - - 0 1:

$\{SV^+M_{50}^-/SV^+M_{50}^+$: moves are also DTZ_{50}-optimal if possible. **dtc/m/m$_{50}$/z/z$_{50}$ = -4/-124/-141/-1/-1m}**
1. ... b4″ **{SZ$_{(50)}^+$: K~″}** {a **maxDTM$_{50, 0/pc}$ s6m wtm win: dtc/m/m$_{50}$/z/z$_{50}$ = 4/124/141/1/1m}** 2. d5″″ b3″ 3. d6″″ b2″ 4. d7″″ b1=Q″ 5. d8=Q″″ **{KQP(d3)KQ: dtc/m/m$_{50}$/z$_{(50)}$ = -110/-120/-137/-50m}** Qf1+″ 6. Ke5′ Qe2+″ 7. Kd4″ Qd2″ 8. Kc4″″ Qc2+′ 9. Kb5″″ Qb2+′ 10. Kc6″″ Qc2+″ 11. Kb7″″ Qg2+″ 12. Ka7″″ Qg7+″ 13. Ka6″″ Qg6+″ 14. Kb5″″ Kg3″ 15. Kc5″″ Qf5+″ 16. Kc6″″ Kf2″ 17. Qd4+″″ Kf1″ 18. Kc7″″ Qa5+″ 19. Kd7″″ Qf5+″ 20. Ke7″″ Qh7+″ 21. Kd8″″ Qg8+″ 22. Kc7″″ Qf7+″ 23. Qd7″″ Qf4+″ 24. Qd6″″ Qf7+″ 25. Kc6″″ Qf3+″ 26. Qd5″″ Qf6+′ 27. Kd7″″ Qg7+″ 28. Ke8″″ Qh8+″ 29. Ke7″″ Qh4+″ 30. Kd7″″ Qg4+′ 31. Kd6″″ Qf4+″ 32. Qe5″″ Qf8+″ 33. Kd7′ Qf7+″ 34. Kc6″″ Qf3+″ 35. Qe4″″ Qf8″ 36. Kc7′ Qf7+′ 37. Kb6″″ Qf6+″ 38. Kb5′ Qb2+″ 39. Kc5′ Qc1+″ 40. Kd4″″ Qb2+″ 41. Kd5″″ Qb5+″ 42. Kd6″″ Qb8+″ 43. Ke7″″ Qc7+″ 44. Ke8′ Qc8+′ 45. Kf7″″ Qc7+″ 46. Kg6″″ Qg3+″ 47. Kf6″″ Qd6+″ 48. Kf5″″ Qf8+″ 49. Kg5′ **{SM$^-$: Kg4″}** Qg7+″ 50. Kf4″″ Qf6+″ 51. Qf5″″ Qd4+″ 52. Kg5+″″ Ke2″ 53. Qe4+″″ Qe3+″ 54. Kf5″″ Kd2″ 55. d4″″ **{KQP(d4)KQ: dtc/m/m$_{50}$/z$_{(50)}$ = -74/-84/-87/-44m}** Qh3+″ 56. Kf6″ Qh6+″ 57. Kf7′ Qh5+″ 58. Ke7″ Qg5+″ 59. Kd7″ Qg7+″ 60. Kc6″ Qf6+″ 61. Kb5″ Qf1+″ 62. Ka5″ Qc4″ 63. Kb6″″ Qb4+″ 64. Kc6″ Qa4+″ 65. Kd6″ Qb4+″ 66. Ke6″ Qc4+″ 67. Kf6″ Kc3″ 68. Qe3+″″ Kb4″ 69. Qe7+′ Ka4″ 70. Qe5″ Qc6+′ 71. Ke7″″ Qb7+″ 72. Kd8′ Qb6+′ 73. Kd7″ Qa7+″ 74. Ke6″ Qa8″ 75. Qd6″ Qe4+″ 76. Kd7″ Qb7+″ 77. Kd8″ Qa8+″ 78. Kc7″ Qa7+″ 79. Kc6″ Qa6+″ 80. Kd5″ Qb5+″ 81. Ke6″ Qe8+″ 82. Kf6″ Qe4″ 83. Qa6+″ Kb4″ 84. Qb6+″ Ka4″ 85. Qa7+″ Kb3″ 86. Qc5″ Qh4+″ 87. Ke6″ Qg4+′ 88. Kd6″ Qg6+″ 89. Kc7″ Qf7+″ 90. Kb6″ Qf6+″ 91. Kb5″ Qf1+″ 92. Ka5″ **{SM$^-$: Kc6″}** Qc4″ **{SZ$_{(50)}^+$: Qe1+″}** 93. Qb5+′ Kc3″ 94. d5″″ **{KQP(d5)KQ: dtc/m/m$_{(50)}$/z$_{(50)}$ = -37/-48/-12m}** Qc7+″ 1-0