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Reply to "Comments on 'On the Steadiness of Separating Meandering Currents'"

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ABSTRACT

The authors thank Nof et al. for their comments on the authors' paper "On the steadiness of separating meandering currents." The authors' paper was motivated by a series of papers by Nof et al. Under a certain set of conditions (reduced gravity, steady state, no meridional velocity at outflow, and parallel outflow), Nof et al. showed that a separating and retroflecting frictionless current cannot be steady because of a momentum imbalance. The main conclusion of the authors' paper was that they agree with the Nof et al. result that a momentum imbalance exists and extended the proof to all possible configurations of retroflecting currents, even including friction. The authors' results point to a new mechanism for the generation of variability in the ocean that is not related to dynamical instability of the flow.

The main claim in the comments is that the authors incorrectly argued in the appendix that the steady-state solutions presented by Nof et al. in several papers fulfill the extra constraint $u^2 = g'h$. In the original paper, the authors showed that it follows from the geostrophic assumption stated implicitly in all these Nof et al. papers, because the flow is assumed to be parallel. Nof et al. now argue that the flow is only approximately geostrophic in all Nof et al. papers. The authors show in this reply that for steady weakly meandering outflows approximate geostrophy does lead to a momentum imbalance paradox as Nof et al. claim. However, for a steady strongly meandering outflow, approximate geostrophy is not enough and one has to use the method explored by van Leeuwen and De Ruijter to derive a momentum imbalance paradox.

1. Introduction

van Leeuwen and De Ruijter (2009, hereafter VL-DR) try to find general criteria for the steadiness of separating meandering flows. It is mentioned here up front that VL-DR only treat flow-configuration cases in which the outflow is along a fixed zonal boundary in the appendix.

The starting point is the momentum imbalance paradox derived by Nof and Pichevin in several papers (Nof and Pichevin 1996, 1999; Nof et al. 2004; Pichevin and Nof 1996, 1997; Pichevin et al. 1999). VL-DR generalize the derivations by Nof and Pichevin, showing that any retroflecting current has to be unstable and also that any separating current is most likely unstable, pointing to a new mechanism for the generation of time-varying flow and/or eddies in the world oceans, which is completely unrelated to a dynamic instability of the flow.

The comments by Nof et al. (2012) on which this paper is a reply can be split into two parts. The first part contains comments on the appendix in VL-DR, in which VL-DR treat the derivation of the momentum imbalance paradox by Nof and Pichevin and argue that the derivation of the momentum balance paradox by Nof and Pichevin is related to too-strong assumptions on the outflow at the eastern boundary of the domain for the meandering outflow. We still believe that to be the case for certain flow configurations, as discussed in section 2.

The second part of the comment by Nof et al. (2012), their discussion, deals with the apparent contradiction in the literature on steady separating flows before VL-DR. Nof et al. (2012) argue that there was no apparent

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contradiction, whereas we argued that there were papers presenting steady separation solutions, if only partial solutions, and Nof and Pichevin had shown that to be impossible.

We will comment on both issues below. However, before we do that, to take away any misunderstanding by a "typical reader," we did not argue against any of the unsteady solutions presented by Nof and Pichevin.

2. The steady-state momentum imbalance paradox

Our starting point is the equation obtained by integrating the zonal momentum equation over any area bounded by contour ϕ,

$$\int_\phi huv\,dx - \int_\phi \left(hu^2 - f\psi + \frac{1}{2}g'h^2\right)dy \qquad (1)$$

In this equation, u is the zonal velocity; v is the meridional velocity; ψ is the mass streamfunction; and h is the interface displacement, assumed zero along ϕ. Note that we are discussing the steady-state balance, so time derivatives are zero by definition. A steady state is essential to be able to define the mass streamfunction from the continuity equation, which reads as

$$(hu)_x + (hv)_y = 0, \qquad (2)$$

so that $\psi_y = -hu$ and $\psi_x = hv$.

a. The case with outflow along a zonal boundary

For the configuration depicted in Fig. 1 of Nof et al. (2012), one has $v = 0$ on the coastal boundary by continuity, and the momentum integral becomes

$$\int_0^L \left(hu^2 - f\psi + \frac{1}{2}g'h^2\right)dy = 0, \qquad (3)$$

in which L is the current width along the boundary. To proceed, Nof and Pichevin and implicitly Nof et al. (2012) use geostrophy in the meridional direction to show that the last two terms in this balance tend to cancel, leading to

$$\int_0^L hu^2\,dy = 0. \qquad (4)$$

This condition cannot be fulfilled by any flow and is called the momentum imbalance paradox by Nof and Pichevin.

All basic papers on the momentum imbalance paradox (e.g., Nof and Pichevin 1996; Pichevin and Nof 1996, 1997) mention that the outflow is parallel and (so) geostrophic. Nof et al. (2012) argue that the outflow is actually only approximately geostrophic and point to VL-DR for using this too-strong relation, leading to $u^2 = g'h$. Below, we investigate if a momentum imbalance paradox can be derived using only approximate geostrophy.

The meridional momentum equation reads as

$$(huv)_x + (hv^2)_y + fhu + \frac{1}{2}g'hh_y = 0. \qquad (5)$$

Integrating this equation from position y to the most northern extent of the current at $y = L$ leads to

$$\int_y^L (huv)_x\,dy' - hv^2 + f\psi + \int_y^L \beta\psi\,dy' - \frac{1}{2}g'h^2 = 0, \qquad (6)$$

where we have taken $\psi(L) = 0$, so that $\psi > 0$ inside the flow. Combining this with the integrated zonal momentum equation by eliminating the $f\psi - 1/2g'h^2$ term gives

$$\int_0^L \left(hu^2 - hv^2 + \int_y^L (huv)_x\,dy' + \int_y^L \beta\psi\,dy'\right)dy = 0. \qquad (7)$$

The order of magnitude of the terms is

$$HU^2L_y,\quad HV^2L_y,\quad HUVL_y\frac{L_y}{L_x},\quad \text{and} \quad \beta HUL_y^3 \qquad (8)$$

or, dividing by the magnitude of the first term,

$$1,\quad \frac{V^2}{U^2},\quad \frac{V}{U}\frac{L_y}{L_x},\quad \text{and} \quad \frac{\beta L_y^2}{U}. \qquad (9)$$

The assumption made in Nof et al. (2012) is that $U \gg V$ and $O(\beta R_d/f_0) = 0.01$, in which R_d is the Rossby deformation radius $R_d = \sqrt{g'H}/f_0$. Furthermore, it is also assumed that the zonal length scales L_x are much longer than meridional length scales L_y. Clearly, in that case, the second and third terms are smaller in absolute value than the first, and the last ratio can be written as

$$\frac{\beta R_d}{f_0}\frac{f_0 R_d}{U} = \frac{\beta R_d}{f_0}\frac{\sqrt{g'H}}{U}. \qquad (10)$$

By assumption, the first factor is very small. The second will be larger than 1, but for reasonable velocities the whole term is expected to be smaller than 1. No matter what its magnitude is, both terms 1 and 4 are positive. This, then, is the rationale for the momentum imbalance paradox in Nof and Pichevin, in which at dominant order two positive terms add up to zero.

Concluding, we can say that, although explicitly stated in Nof and Pichevin, pure geostrophy or purely parallel flow is not needed for the momentum imbalance paradox for outflow along a zonal boundary. VL-DR have interpreted the wording in these papers literally, and as such that part of the VL-DR appendix is perhaps misleading. In the next section, we treat the free meandering outflow, also explored by Nof and Pichevin (in Pichevin et al. 1999).

b. Strong meandering outflow

This case is relevant for the Agulhas system treated in Pichevin et al. (1999). The area integrated momentum balance for such a case is

$$\int_\phi huv\,dx - \int_\phi \left(hu^2 - f\psi + \frac{1}{2}g'h^2\right)dy, \qquad (11)$$

which follows from integrating the zonal momentum equation over an area bounded by contour ϕ. Exploring again the meridional momentum equation to eliminate $f\psi - 1/2g'h^2$ we find

$$\int_\phi huv\,dx - \int_\phi \left[hu^2 - hv^2 + \int_y^L (huv)_x\,dy' + \int_y^L \beta\psi\,dy'\right]dy. \qquad (12)$$

An order of magnitude estimate of the different terms in the equation is more elaborate in this case because of the curvature of the flow. VL-DR follow Nof and Pichevin in choosing the eastward boundary of the integration domain along a meridional section where $v = 0$, assuming that is possible. Furthermore, we now have to assume $U \approx V$ and $L_x \approx L_y$. So, it should be realized that, because of the meandering structure of the flow, v_x is maximal and, as u_y, approximately equal to the vorticity of the jet. An order of magnitude estimate for the different terms is now

$$HUVL_x, \quad HU^2L_y, \quad HV^2L_y, \quad HU\zeta L_y^2, \quad \text{and} \quad \beta HUL_y^3 \qquad (13)$$

or, dividing by the magnitude of the second term,

$$\frac{V}{U}\frac{L_x}{L_y}, \quad 1, \quad \frac{V^2}{U^2}, \quad 1, \quad \text{and} \quad \frac{\beta L_y^2}{U}, \qquad (14)$$

where we used $\zeta = V/L_x = U/L_y$. Because $v = 0$ at the section, we find

$$\int_\phi huv\,dx - \int_\phi \left(hu^2 + \int_y^L (huv)_x\,dy' + \int_y^L \beta\psi\,dy'\right)dy, \qquad (15)$$

in which, specifically, terms 2 and 3 have similar magnitude. However, Nof et al. (2012) ignore term 3 and write for this case

$$\int_\phi huv\,dx - \int_\phi \left(hu^2 + \int_y^L \beta\psi\,dy'\right)dy. \qquad (16)$$

This is only consistent if they also assume v_x is much smaller than u_y at the eastward section. This fact is the starting point of the further derivations in the appendix of VL-DR.

To conclude, in our view, Nof and Pichevin assume both v and v_x are small along the outflow boundary of the separating flow, and these assumptions force the flow to be unstable. This part of the appendix by VL-DR stands as is.

3. The state of the field before VL-DR

Nof et al. (2012) argue in their discussion section that VL-DR mention in their introduction that "although the idea of [Nof and Pichevin (1996)] is appealing, it seems to be contradicted by other studies" and argue that we back this up by references to Dijkstra and De Ruijter (2001) and Ou and De Ruijter (1986). Then it is argued by Nof et al. (2012) that VL-DR never spelled out clearly what the actual contradiction is. We would like to make three comments here. First, we do list Moore and Niiler (1974) and not Dijkstra and De Ruijter (2001) in this context. Second, Moore and Niiler (1974) do provide a full-equation steady-state separating meandering solution, in direct contradiction with Nof and Pichevin. However, VL-DR prove that the solution by Moore and Niiler (1974) is in error. Finally, Ou and De Ruijter (1986) is a steady approximate local solution for separation due to nonlinear outcropping and subsequent meandering or retroflection. Indeed, it is not a solution to the full equations of motion, and in that sense the contradiction is seemingly (i.e., at first sight) not actual. Nof et al. (2012) subsequently argue that VL-DR "have not resolved any clearly identifiable problem or contradiction." We disagree for the arguments given above.

4. Conclusions

The comments by Nof et al. (2012) seem to be largely related to a misunderstanding. VL-DR took the Nof and Pichevin article literally when they assumed that the flow was geostrophic and parallel and derived the condition $u^2 = g'h$ for that case. Nof et al. (2012) now argue that Nof and Pichevin actually meant that the flows are only approximately parallel. So the outflow is now argued to be only approximately geostrophic, and the condition $u^2 = g'h$ does not appear. We have shown in this reply that for steady weakly meandering outflows approximate geostrophy leads to a momentum imbalance paradox. However, for a steady strongly meandering outflow, as treated in Pichevin et al. (1999), approximate geostrophy is not enough and one has to use the method explored by VL-DR to derive a momentum imbalance paradox in this case.

The main new ingredient of VL-DR is that they improved on the derivation of the momentum imbalance paradox for steady retroflecting free outflow currents. They showed that this momentum imbalance is actually much more general that previously thought, showing that the new mechanism for eddy generation that is not related to instability of the flow found by Nof and Pichevin is perhaps a major player in ocean eddy generation.

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