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Abstract

While Nalimov's endgame tables for Western Chess are the most used today, their Depth-to-Mate metric is not the most efficient or effective in use. The authors have developed and used new programs to create tables to alternative metrics and recommend better strategies for endgame play.

Key words: chess: conversion, data, depth, endgame, goal, move count, statistics, strategy

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1 Introduction

Chess endgames tables (EGTs) to the 'DTM' Depth to Mate metric are the most commonly used, thanks to codes and production work by Nalimov [10,7]. DTM data is of interest in itself, even if *conversion*, i.e., change of force, is more often adopted as an interim objective in human play. However, more effective endgame strategies using different metrics can be adopted, particularly by computers [3,4]. A further practical disadvantage of the DTM metric is that, as maxDTM increases, the EGTs take longer to generate and are less compressible.

Here, we focus on metrics DTC, DTZ[1] and DTZ_{50}[2]; the first two were effectively used by Thompson [19], Stiller [14], and Wirth [20]. New programs by Tamplin [15] and Bourzutschky [2] have already enabled a complete suite of 3-to-5-man DTC/Z/Z_{50} EGTs to be produced [18]. This note is an update, focusing solely on Tamplin's continuing work, assisted by Bourzutschky, with the latter code on 6-man, pawnless endgames for which DTC ≡ DTZ and DTC_{50} ≡ DTZ_{50}. Section 2 outlines the algorithm used. Sections 3 and 4 review the new DTZ and DTZ_{50} data tabled in the Appendix. In section 5, *endgame strategy* is defined and improved strategies are recommended for the 50-move and *k*-move contexts.

2 The NBT code

Here, we review the algorithm and the 'NBT' code developed in turn by Nalimov, Bourzutschky and Tamplin. The first author extended Nalimov's DTM-code to enable it to generate EGTs to metrics $DTC_{(k)}$, DTM_k and $DTZ_{(k)}$[3]. This involved generalising some DTM-specific aspects of the algorithm, as well as making the obvious changes to the iterative formula for deriving depth. For $DTC_{(k)}$, the code retains the efficiencies of the DTM-code while requiring maxDTC rather than maxDTM cycles[4]. Because EGT

[1] DTC ≡ *Depth to Conversion*, i.e., to force change and/or mate. DTZ ≡ *Depth to (Move-Count) Zeroing (Move)*, i.e., to Pawn-push, force change and/or mate – when a move-counter is set to zero again.
[2] dtz_k = *dtz* unless a *k*-move rule allowing a draw-claim sets a value of *draw*.
[3] The board-size, piece-type and rule generalizations also effected are not covered here.
[4] An advantage, as, e.g., KQBNKN has maxDTM = 107 but maxDTC = 6.

generation to the DTZ metric has not been implemented generically as a sequence of 'fixed pawn structure' sub-EGT generations, this is not so for $DTZ_{(k)}$ computations. The second author ran the code on single- and multi-processor UNIX systems, and evolved the code to:

a) increase portability as Nalimov's C++ is non-standard and Windows-oriented,
b) manage virtual stores and files greater than 2GBytes,
c) accumulate integer counts greater than 2^{31}-1,
d) pursue EGT depths > 126, requiring 16-bit database entries, and
e) synchronise multiple processes more rigorously.

Experience confirms the observation [13] that manual file-management can be a source of error. This suggests that the Nalimov file-format should include a file-header to help prevent such errors with details, e.g., of author, code version, metric, degree and date of completion, and compression algorithm.

Table 1. Examples of extreme, atypical maxDTC wins and losses.

Endgame	Result	Position	maxDTC	avgeDTC	maxDTC/avgeDTC
KRPKN	0-1	K1k5/8/Pn6/8/R7/8/8/8 w	1	0.01	98.00
KRBNKQ	1-0	1k4q1/8/N2K4/8/8/8/8/R3B3 b	98	1.31	74.96
KRBNKQ	1-0	1k4q1/8/3K4/8/1N6/8/8/R3B3 w	99	1.33	74.45
KQRKQR	1-0	4q3/7R/7Q/4r3/4k3/8/8/2K5 w	92	1.92	48.03
KQPKN	0-1	K1k5/8/Pn6/8/Q7/8/8/8 w	1	0.02	47.00
KRBKR	1-0	8/3B4/8/1R6/5r2/8/3K4/5k2 w	59	1.4	42.13

Table 2. Chess EGTs: comparative file sizes.

		DTM MB	DTC %	DTZ %	DTZ_{50} %	$\delta(DTZ_{50}, DTZ)$ %	DTZ + 'δ' %
3-5-man	pawnless	1,822	71.29	71.29	0.00	0.00	71.29
3-5-man	with Ps	5,579	59.14	43.36	15.36	0.70	44.06
3-5-man	all	7,401	62.13	50.24	11.58	0.53	50.77
3-3/4-2	pawnless	220,623	56.37	56.37	20.56	1.12	57.50
3-6-man	to date	228,024	56.56	56.17	20.27	1.11	57.28

3 The DTC and DTZ metrics

DTC EGTs are interesting, not only for completeness, but because *conversion* is an intuitively obvious objective and the DTC EGTs document precisely the phase of play when the material nominated is on the board. The DTZ metric is more important than DTC, being necessary if the length of the current phase of play is to be *guarded* in the context of chess' *k*-move rule, *k* currently being 50. Where no Pawns are involved, as here, DTZ ≡ DTC.

The NBT-code measures depth consistently in *winner's moves* and does not assume that conversion is effected by the winner. Also, it does not allow the loser to make a voluntary, 'natural' if unavailing capture, e.g., {wKe1Qf1Rb1/bKa1 b: 1. ... Kxb1}. The ICGA web-site (2004) provides the latest data, including %-wins and average win-length. Because there are many wins in 1, the *% of positions won* does not characterise well the presence of wins in an endgame. Similarly, maxDT*x* is not a good indicator of typical DT*x* and Table 1 gives some maxDTC positions for endgames with extreme maxDTC/averageDTC. We therefore calculate a new characteristic,

$$x\text{-}Presence \equiv \text{\%\_of\_decisive\_positions} \times (\text{Average DT}x)$$

x-Presence may be compared with *maxDTx* and *%-wins* [8]. It is not unduly affected by the wins in 1 or by the long tail of deep wins, and is the number of moves for which a win is expected to be on the board when DT*x* ≡ DTC.

3.1 A Review of the DTZ data

The results are in the Appendix, Table 3. These agree with the earlier results of Stiller [14] and Thompson [17] with two exceptions.[5, 6] Note that legal but unreachable positions can affect the statistics.[7]

KBNK wtm wins had the largest C-presence (2455.76) of 3-5-man endgames with density 99.51% and average DTC 24.68. Only KRBKNN btm losses exceed this (4068.54) with density 57.52% and average DTC 70.73.

Table 2 summarises the absolute and comparative sizes of the various EGTs.

4 The DTZ_{50} metric

The DTZ_{50} metric rates as wins only those positions winnable against best play given the 50-move rule. Figure 1 shows those 5-man endgames for which some DTZ and DTZ_{50} depths differ[8], thereby affecting the value or depth of some 6-man positions. Let *KwKb*, e.g., KBBKN, be an endgame with wtm and btm 1-0 wins impacted by the 50-move rule. Then the DTZ_{50} EGTs for *KwxKb* and *KwKby*, e.g., KQBBKN and KBBKNN, must be computed and are likely to differ from their DTZ equivalents.

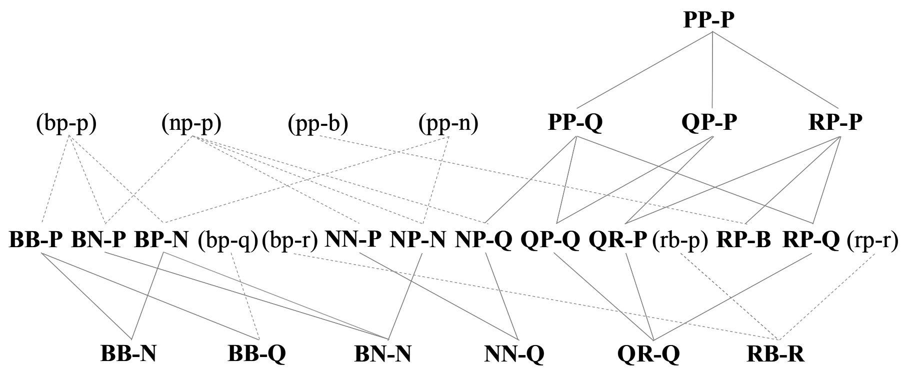

Figure 1. 5-man endgames with $EZ_{50} \neq EZ$.

Table 4 in the Appendix lists 6-man DTZ_{50} EGT data for endgames where $EZ_{50} \neq EZ$. Table 5 summarises 50-move impact, minimal for KRRKRB (1-0), considerable for KBBKNN. Table 6 gives an example position for each affected endgame. 63 of the 135 6-man pawnless endgames are affected by the 50-move rule. Although $DTZ_{50} \geq DTZ$, $maxDTZ_{50}$ is rarely greater than maxDTZ: KQNKBB, KQQKBB, KBBBKN and KBNNKN are the only examples to date. Wins frustrated by the 50-move rule produce a

[5] Their maxDTC for KQNKRR and KQNNKQ is 1 greater: in both cases, Black is forced to convert.
[6] For KBNNKN [17], ‘27’ should be ‘28’: a foreshortened line went unseen.
[7] e.g., KQQKNN has '1 wtm loss in 1’ in 8/8/8/8/1n6/QQn5/K2k4 w. The double-check is impossible.
[8] Endgames where DTZ and DTZ_{50} might have differed, but did not, are bracketed in lower-case.

$maxDTZ_{50}$ < maxDTZ ≤ 50 for only KBBKBN and KBBKNN so far. KBBKNN has the majority of its wins frustrated, and relatively few wins can be retained by a deeper strategy in the current phase. Here, the 50-move rule bars the now well defined route to many KBBKN wins [12]. There are significant percentages of frustrated wins in KBB*x*KQ (0-1), and of delayed 1-0 wins in KBB*x*KN. Elsewhere, the 50-move impact is sparsely distributed and one might expect that this becomes sparser as the number of men increases.

Note that, as $DTZ_{50} \geq DTZ$ for a decisive position, we may construct an EGT coding, $EdZ_{50}Z$, of $\delta(DTZ_{50}, DTZ)$[9] enabling DTZ_{50} to be derived from DTZ and $EdZ_{50}Z$. The latter notes only DTZ_{50}-DTZ for the delayed wins, and 'new draws' when DTZ ≤ 50: DTZ > 50 already implies 'new draw'. If $EdZ_{50}Z$ is *null*, it is not required. For 3-5-men, these EGTs are only 0.53% the size of the corresponding DTM EGTs. They can in fact be made much smaller by designer-compression techniques more tailored to the data than the established compression method adopted by Nalimov.

5 Endgame strategies

An *endgame strategy*, denoted here by S*s*, is an algorithm for filtering the available moves to a preferred choice. Endgame strategies can be applied in sequence. $Ss_1s_2 \ldots s_n$ denotes a compound endgame strategy using strategies Ss_1, Ss_2,…, Ss_n in turn. Let *dtx* be the depth by metric DT*x*, and E*x* an EGT to metric DT*x*. Let Sx^- be an endgame strategy minimising *dtx*, e.g., 'quickest mate' SM^-, SC^-, SZ^- or SZ_{50}^-. Let Sx^+ be a strategy maximising *dtx*. With some exceptions, q.v. Section 5.2, Sx^- strategies are used by attackers and Sx^+ strategies are used by defenders.

Let SZ^o and SZ_k^o be endgame strategies *guarding* the length of the current phase in the context of a *k*-move rule and a remaining *mleft* moves before a possible draw claim. By definition, if *dtx* > *mleft*, $Sx^o \equiv Sx^-$.

Some elementary observations are worth noting first:

- S*x* must not filter out all available moves, hence the contingency definition of Sx^o,
- S*xy* defines at least as narrow a choice of moves as S*x*,
- if S*xy* fails to safeguard the theoretical value of the position, then S*x* also fails,
- if S*y* has no effect after the use of S*x*, then S*xy* ≡ S*x*,
- SZ_k^o has no effect if the position is a draw under the *k*-move rule
- S*xx* ≡ S*x*, i.e. a strategy 'filter' has no further effect when applied a 2nd time,
- S*xy* is not necessarily identical to S*yx*, e.g., SM^-Z^- and SZ^-M^- are different,
- S*xy* ≡ S*x* ≡ S*yx* if S*x* excludes any move that S*y* excludes,
- $SZ^oZ^- \equiv SZ^-$: SZ^o allows DTZ-optimal moves through its filter in all positions.

A likely set of goals for an attacking endgame strategy is to:

- win from any position that can be won under the prevailing *k*-move rule,
- avoid a draw-claim in the current phase if possible, and
- maximize the probability of finessing a win from a draw against a fallible player.

[9] In fact, intelligent access-code interpreting 'DTZ_{50}' > 50 as "draw" enables this $EdZ_{50}Z$ encoding:
"DTZ > 50 ∨ EZ code = EZ_k code" ⇒ $EdZ_{50}Z$ stores 0 (reducing, e.g., KRNKNN $EdZ_{50}Z$ to *null*).
"DTZ ≤ 50 but new EZ_{50} draw" ⇒ $EdZ_{50}Z$ stores 1. "0 < DTZ50 – DTZ = δ" ⇒ $EdZ_{50}Z$ stores δ+1.

It is already clear from KBBKP, KNNKP, KQPKQ and KRPKP examples [18] that the three strategies SC^-, SM^- and SZ^-, even in combination, are not enough to achieve even the first goal. As conjectured by Haworth [3], and demonstrated by Bourzutschky [2], KBBKNN includes positions where these three strategies all fail, not even including the move which safeguards a win available under the current 50-move rule. Similar positions have been found in KBBKBN, KQNKBB and KBNNKQ by Tamplin and their strategy-driven lines are illustrated in Appendix 1 after Table 6. However, the first objective is in fact relatively easy: SZ_k^- wins any position winnable against best play under a *k*-move rule. As *k* is currently 50, DTZ_{50} EGTs and SZ_{50}^- have a clear role. The strategy SZ_{50}^- provides no help in other situations where finesse and/or the opponent's acquiescence are required: more sophisticated strategies are required.

5.1 Strategies for playing a fallible opponent

By definition, a *fallible* opponent is not certain to achieve a result as good as the theoretical value of the position. They may lose a half or full point, fail to avoid a 50-move draw claim from the opponent or fail to defend a lost position long enough to claim an available draw. Let us suppose that it is possible to avoid a draw-claim in the current phase, if not in a later phase. Clearly, it is critical to achieve this if a win is to follow.

The strategy SZ^- does so but strives for nothing else. The strategy $SZ^oZ_k^-$ does so, and also seizes on any winnable position once offered. The strategy $SZ^oZ_k^-Z^-$ also achieves a third, ancillary goal of achieving both goals in the shortest current phase. $SZ^oZ_k^-Z^-$ is not however the best use of DTZ and DTZ_k data. It does not attempt to minimize the difficulty of finessing the win in the second and subsequent phases of play. In particular, the third goal runs counter to giving the fallible opponent the best opportunity to concede ground in the current phase.

To increase the chance of finessing a win against a fallible opponent, it is helpful to play the opponent as well as the game by exploiting any apparent fallibility [5,6,9]. This is done by having an *opponent model* OM, e.g., R_c [5], and using it in a forward search. As the opponent's fallibility replaces certainty by probabilities, the forward search minimaxes expected depth rather than depth. The OM may be revised by a Bayesian learning process in the light of experience during play.

5.2 Winning under a k-move rule

The underlying difficulty is that the data so far does not help us to answer the question "By how much does the current position fail to be a win under the 50-move rule?". However, the question implicitly defines a new metric:

dtr = the least *k* for which a position is won or lost, given a *k*-move drawing rule,

$0 \leq dtr \leq dtm$ and therefore the integer *dtr* can be determined. *dtr-k* measures the defender's margin for error and the attacker's challenge when there are *k* moves left before a draw-claim in the current phase. Although the 50-move rule seems unlikely to be changed to a different *k*-move rule, the DTR EGT enables an attacker to win any position winnable under any *k*-move rule, regardless of *k*. It obviates the need for specific DTZ_{50} EGTs.

Because a sequence of positions on the winning line can have the same DTR value, the following metric is also necessary [4] while generating and using the DTR EGTs:

dtz_R = the minimaxed depth to a (move-count zeroing) move while minimaxing *dtr*

$SR^-Z_R^-$ is a necessary and sufficient strategy to achieve any win available against best play given a *k*-move rule. $SR^+Z_R^+$ is a necessary and sufficient strategy to defend a *k*-move draw.

Generating the DTR EGTs remains a future challenge, made the more difficult because two metrics are used in parallel, and the process is not as efficient as that for DTC, DTM and potentially DTZ. However, because $dtr \geq dtz_R \geq dtz$, dtz_R and *dtr* may be derived economically from tables EZ, EdZ_RZ and $EdRZ_R$ in the same way[10] as dtz_{50} is derived from tables EZ and $EdZ_{50}Z$.

The $SZ^oR^-Z_R^-$ strategy minimizes DTR, but only within the constraints of completing the current phase in the available moves and without forward search. It might therefore require too many moves to retain a target *dtr* to the end of the phase.

With the addition of the SZ_R^o filter, strategy $SZ^oZ_R^oR^-Z_R^-$ aims to adopt an in-range DTR goal to ameliorate this problem. It:

– guards the length of the current phase in the context of the current *k*-move rule,
– wins any position that is winnable under whatever *k*-move rule is in force,
– aims to minimize *dtr* for the attacking side with pragmatic DTR goals, and
– achieves the first three goals in a current phase of least possible moves.

Similar caveats apply to $SZ^oZ_R^oR^-Z_R^-$ as to $SZ^oZ_k^-Z^-$. The strategy does not necessarily minimize DTR, or Ř = Expected[DTR] against a fallible opponent. It does not even make best use of the moves available to give the opponent more opportunity to err. Within constraints which avoid 3x repetition[11], a more liberal strategy such as $SZ^oZ_R^oR^-Z_R^+$ can be more effective than $SZ^oZ_R^oR^-Z_R^-$. In position NN-P[12], $SZ^oZ_R^oR^-Z_R^-$ makes the optimal move-choice[13] Nb1+: $SZ^oZ_{50}^-$ can, and $S\sigma$ ($\sigma \equiv C^-, M^-, Z^-, Z^oZ_{50}^-Z^-$) do, concede DTR depth with Kc2.

5.3 Strategy effectiveness

The effectiveness of an attacking strategy may be measured in two dimensions:

– % of theoretically won positions in which the strategy retains the win
  i.e. in which the strategy offers no moves which are not offered by SZ_{50}^-
– % of drawn positions in which a win is finessed against a fallible opponent

Different reference defenders are needed for the two dimensions. We suggest here:

– for a lost position, an infallible defender playing strategy $SR^+Z_R^+$, and otherwise,
– a fallible defender R_c [6] playing ‘to’ DTR and DTZ_R.

[10] Because there are no ‘extra’ draws as in $EdZ_{50}Z$, $EdZ_RZ \equiv \{dtz_R - dtz\}$ and $EdRZ_R \equiv \{dtr - dtz_R\}$.
[11] e.g., sufficient but not necessary, no $\{DTR, DTZ_R\}$ combination to be visited three times.
[12] NN-P: 8/8/8/2pN4/8/k1N5/8/2K5 w. *dtm*=115p, *dtr*=102p, *dtz*=42p, dtz_R=60p.
[13] $SZ_R^oR^-Z_R^-$ - $SR^+Z_R^+$: 1. Nb1+' Ka4'. White retains DTR = 102p and converts in 30m.

In the context of the 50-move rule, SZ_{50}^- retains the win in 100% of positions. Although this has not been examined, we expect SZ^-, SC^-, SM^-C^- and SM^- to exhibit increasing rates of failure. SZ^- fails both in the 0.34% of positions where DTZ < DTZ_{50} and in positions with DTZ = DTZ_{50} where it offers moves which SZ_{50}^- rejects.[14]

6 EGT integrity

All EGT files were immediately given MD5sum signatures [11] to guard against subsequent corruption or loss[15]. The EGTs were checked for errors in various ways:

- DT*x* EGTs {E*x*}, *x* = Z and Z_{50}, verified by Nalimov's standard test.
- consistency of the {EM} and {EZ} EGTs confirmed:
  counts of all positions found identical to predicted index-ranges, and
  theoretical values found identical with *dtm* ≥ *dtz*.
- consistency of the {EZ_{50}} and {EZ} EGTs confirmed:
  values identical with $dtz_{50} \geq dtz$, or 'EZ' win/loss an 'EZ_{50}' draw,
- DTZ statistics compared with Stiller's results [14],
- published DTZ-minimaxing lines [14] checked against DTZ EGTs, and
- DTZ statistics compared with Thompson's results [17].

Multi-metric working introduces new risks to the process of EGT generation and we recommend that the EGTs are self-identifying to increase integrity assurance.

7 Summary

This paper is a second snapshot of continuing work on the evolution and use of a multi-metric code 'NBT'. This was created by Nalimov, generalized by Bourzutschky [2] and managed on Unix by Tamplin. Here, we surveyed the newly completed 6-man pawnless DTZ and DTZ_{50} data. The 3-6-man pawnless DTZ EGTs {EZ} to date are 56.17% the size of the equivalent set {EM} and the compressed $EdZ_{50}Z$ EGTs increase this figure to 57.28%. These percentages will reduce as the 6-man P-endgame and 5-1 pawnless EGTs are generated. This is an attractive, practical benefit as the 3-to-6-man EMs will be some 1.45 TB in size.

Clearly, there are more effective and efficient endgame strategies than the commonly used SM^-, and the only constraint is access to EGTs. It is recommended that SC^-M^-, $SZ^oM^-Z^-$, $SZ^oZ_{50}^-Z^-$ and perhaps other strategies are considered, and that the EC, EZ and $EdZ_{50}Z$ EGTs are made available to enable their use. The computation of DTR and DTZ_R EGTs remains a future challenge. Endgame strategies related to $SZ^oZ_R^oR^-Z_R^-$ promise to remove many of the chessic artificialities induced by current metric-based strategies, such as DTZ-motivated sacrifices by the attacker and incorrect choices of defensive goal by the losing side.

[14] e.g., 7K/8/3q4/3B4/5Nk1/8/3B4/8 b: DTZ = DTZ_{50} = 13 but SZ^- allows Qc7 leading to a 50m-draw.

[15] An invaluable guard which enabled the successful recovery of almost all the 0.6TB of EGT data at risk after a RAID crash in the last stages of production work for this paper.

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Appendix: Chess Endgame Data and Examples

Table 3a. Chess Endgames: 6-man, pawnless DTC/Z data.[16]

			DTC Metric							
Endgame			# of maxDTC positions				maxDTC, moves			
			1-0		0-1		1-0		0-1	
Endgame	**GBR**	**w-b**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**
KBBKBB	**0080.00**	3-3	704	224	224	704	6	5	5	6
KBBKBN	**0053.00**	3-3	10	2	26	180	28	27	9	10
KBBKNN	**0026.00**	3-3	11	1	488	1,518	38	38	3	4
KBNKBN	**0044.00**	3-3	29	4	4	29	9	8	8	9
KBNKNN	**0017.00**	3-3	1	1	12	154	13	12	6	7
KNNKNN	**0008.00**	3-3	44	8	8	44	7	6	6	7
KQBKBB	**1070.00**	3-3	3	13	1,317	6,118	13	13	3	4
KQBKBN	**1043.00**	3-3	13	107	944	4,097	16	16	3	4
KQBKNN	**1016.00**	3-3	71	331	28	81	13	13	2	3
KQBKQB	**4040.00**	3-3	2	3	3	2	46	45	45	46
KQBKQN	**4013.00**	3-3	2	1	3	15	36	36	32	32
KQBKRB	**1340.00**	3-3	2	11	15	30	42	41	6	7
KQBKRN	**1313.00**	3-3	1	6	9	34	27	27	7	8
KQBKRR	**1610.00**	3-3	1	79	21	23	85	84	10	11
KQNKBB	**1061.00**	3-3	8	32	1,521	6,573	15	15	3	4
KQNKBN	**1034.00**	3-3	1	7	3	3	17	17	4	5
KQNKNN	**1007.00**	3-3	27	137	74	207	16	16	2	3
KQNKQN	**4004.00**	3-3	6	2	2	6	29	29	29	29
KQNKRB	**1331.00**	3-3	11	26	8	20	26	26	8	9
KQNKRN	**1304.00**	3-3	1	1	2	11	40	40	9	9
KQNKRR	**1601.00**	3-3	7	6	6	7	152	152	11	12
KQQKBB	**2060.00**	3-3	984	5,128	137	714	6	6	3	4
KQQKBN	**2033.00**	3-3	4	28	99	376	8	8	3	4
KQQKNN	**2006.00**	3-3	2	8	1	36,110	7	7	1	1
KQQKQB	**5030.00**	3-3	8	1	1	2	62	62	22	23
KQQKQN	**5003.00**	3-3	4	26	4	20	50	50	18	19
KQQKQQ	**8000.00**	3-3	1	2	2	1	44	44	44	44
KQQKQR	**5300.00**	3-3	4	2	1	12	48	47	56	56
KQQKRB	**2330.00**	3-3	4	22	21	26	14	13	5	6
KQQKRN	**2303.00**	3-3	2	12	14	11	14	14	5	6
KQQKRR	**2600.00**	3-3	3	7	483	575	18	18	5	6
KQRKBB	**1160.00**	3-3	3	13	689	3,514	12	12	3	4
KQRKBN	**1133.00**	3-3	3	14	419	1,645	11	11	3	4
KQRKNN	**1106.00**	3-3	1	243	20	40	11	10	2	3
KQRKQB	**4130.00**	3-3	2	12	5	3	73	73	31	32
KQRKQN	**4103.00**	3-3	3	4	2	6	71	71	26	27
KQRKQR	**4400.00**	3-3	3	1	1	3	92	92	92	92
KQRKRB	**1430.00**	3-3	2	10	75	92	21	21	5	6
KQRKRN	**1403.00**	3-3	1	7	8	16	21	21	6	7
KQRKRR	**1700.00**	3-3	6	4	2	8	34	34	10	11
KRBKBB	**0170.00**	3-3	14	3	97	252	83	83	5	6
KRBKBN	**0143.00**	3-3	1	6	1	9	98	98	5	6
KRBKNN	**0116.00**	3-3	1	2	82	196	223	222	2	3
KRBKRB	**0440.00**	3-3	5	1	1	5	17	16	16	17
KRBKRN	**0413.00**	3-3	78	45	2	25	21	20	13	14
KRNKBB	**0161.00**	3-3	13	14	4	20	140	140	9	10
KRNKBN	**0134.00**	3-3	1	7	12	36	190	189	5	6

[16] The 'GBR' code, created by Guy, Blandford and Roycroft, associates the endgame force with a number of form qrbn.(w)p(b)p, assigning '1' to White's men and '3' to Black's. Thus KQNKRB ≡ 1331.00.

A '9' indicates more than two like pieces of a colour. Thus, KBBBKB ≡ 0090.00/31.

Table 3b. Chess Endgames: 6-man, pawnless DTC/Z data.

Endgame			DTC Metric							
			# of maxDTC positions				maxDTC, moves			
			1-0		0-1		1-0		0-1	
Endgame	**GBR**	**w-b**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**
KRNKNN	**0107.00**	3-3	1	7	29	54	243	242	3	4
KRNKRN	**0404.00**	3-3	6	3	3	6	21	20	20	21
KRRKBB	**0260.00**	3-3	2	16	1	4	37	37	4	5
KRRKBN	**0233.00**	3-3	3	42	6	30	26	25	4	5
KRRKNN	**0206.00**	3-3	2	3	37	77	33	33	2	3
KRRKRB	**0530.00**	3-3	22	13	1	455	54	54	6	6
KRRKRN	**0503.00**	3-3	2	3	37	89	73	73	6	7
KRRKRR	**0800.00**	3-3	2	3	3	2	18	17	17	18
KBBBKB	**0090.00/31**	4-2	19	6	6,150	21,903	20	20	1	2
KBBBKN	**0093.00/30**	4-2	6	6	951	4,838	12	12	0	1
KBBBKQ	**1090.00/30**	4-2	1	9	1	3	10	9	51	51
KBBBKR	**0390.00/30**	4-2	1	23	13	72	69	68	4	5
KBBNKB	**0051.00**	4-2	3	4	10,340	38,254	36	36	1	2
KBBNKN	**0024.00**	4-2	9	54	3,663	18,984	31	31	0	1
KBBNKQ	**3021.00**	4-2	122	16	17	1	12	11	62	63
KBBNKR	**0321.00**	4-2	4	2	10	50	68	68	6	7
KBNNKB	**0042.00**	4-2	6	4	4,779	18,249	38	38	1	2
KBNNKN	**0025.00**	4-2	17	56	4,335	22,890	28	28	0	1
KBNNKQ	**3012.00**	4-2	5	1	1	4	12	11	49	49
KBNNKR	**0312.00**	4-2	12	4	1	398	49	48	7	7
KNNNKB	**0039.00/30**	4-2	1	2	1,275	2,891	92	91	0	1
KNNNKN	**0009.00/31**	4-2	2	2	1,584	8,562	86	86	0	1
KNNNKQ	**1009.00/30**	4-2	1	1	6	11	9	8	35	35
KNNNKR	**0309.00/30**	4-2	2	2	8	31	12	11	6	7
KQBBKB	**1050.00**	4-2	221	1,027	9,168	34,389	8	8	1	2
KQBBKN	**1023.00**	4-2	122	515	1,327	6,813	7	7	0	1
KQBBKQ	**4020.00**	4-2	1	1	2	3	93	93	15	16
KQBBKR	**1320.00**	4-2	2	12	146,288	830,146	20	20	1	2
KQBNKB	**1041.00**	4-2	28	191	7,873	31,019	7	7	1	2
KQBNKN	**1014.00**	4-2	133	708	3,262	17,347	6	6	0	1
KQBNKQ	**4011.00**	4-2	1	1	1	1	65	65	16	17
KQBNKR	**1311.00**	4-2	4	28	408,029	2,319,030	22	22	1	2
KQNNKB	**1032.00**	4-2	3	21	1,457	3,516	11	11	0	1
KQNNKN	**1005.00**	4-2	7	21	1,806	9,962	9	9	0	1
KQNNKQ	**4002.00**	4-2	2	2	5	20	71	71	13	14
KQNNKR	**1302.00**	4-2	2	12	25	163	22	22	2	3
KQQBKB	**2040.00**	4-2	2	10	1,665	7,712	5	5	1	2
KQQBKN	**2013.00**	4-2	23	130	440	2,285	5	5	0	1
KQQBKQ	**5010.00**	4-2	6	30	7	23	29	29	9	10
KQQBKR	**2310.00**	4-2	1	5	75,802	478,709	26	26	1	2
KQQNKB	**2031.00**	4-2	9,757	37,511	383	1,005	4	4	0	1
KQQNKN	**2004.00**	4-2	49	260	477	2,700	5	5	0	1
KQQNKQ	**5001.00**	4-2	1	1	2	13	28	28	8	9
KQQNKR	**1301.00**	4-2	2	12	112,955	720,034	24	24	1	2
KQQQKB	**9030.00/30**	4-2	673,004	2,775,033	0	0	3	3	—	—
KQQQKN	**9003.00/30**	4-2	827	4,016	0	0	4	4	—	—
KQQQKQ	**9000.00/31**	4-2	6	40	1	5	19	19	9	10
KQQQKR	**9300.00/30**	4-2	3	19	11,025	77,175	20	20	1	2
KQQRKB	**2130.00**	4-2	438	1,766	0	0	4	4	—	—
KQQRKN	**2103.00**	4-2	5	29	572	2,459	5	5	0	1
KQQRKQ	**5100.00**	4-2	3	7	3	13	28	28	9	10
KQQRKR	**2400.00**	4-2	1	7	63,979	447,853	24	24	1	2
KQRBKB	**1140.00**	4-2	83	415	5,158	23,146	5	5	1	2

Table 3c. Chess Endgames: 6-man, pawnless DTC/Z data.

			DTC Metric							
Endgame			**# of maxDTC positions**				**maxDTC, moves**			
			1-0		**0-1**		**1-0**		**0-1**	
Endgame	**GBR**	**w-b**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**
KQRBKN	**1113.00**	4-2	977	4,872	3,390	15,732	5	5	0	1
KQRBKQ	**4110.00**	4-2	6	19	4	9	49	49	12	13
KQRBKR	**1410.00**	4-2	1	7	269,633	1,690,187	25	25	1	2
KQRNKB	**1131.00**	4-2	1,358,087	5,054,177	1,150	2,838	4	4	0	1
KQRNKN	**1104.00**	4-2	12	76	3,450	16,495	6	6	0	1
KQRNKQ	**4101.00**	4-2	3	7	1	3	55	55	11	12
KQRNKR	**1401.00**	4-2	2	14	375,359	2,375,039	24	24	1	2
KQRRKB	**1230.00**	4-2	74,085	294,223	0	0	4	4	—	—
KQRRKN	**1203.00**	4-2	299	1,474	1,498	6,333	5	5	0	1
KQRRKQ	**4200.00**	4-2	1	4	1	2	41	41	8	9
KQRRKR	**1500.00**	4-2	12	82	115,042	805,294	23	23	1	2
KRBBKB	**0150.00**	4-2	4	13	12,789	47,143	18	18	1	2
KRBBKN	**0123.00**	4-2	7	57	3,717	17,552	12	11	0	1
KRBBKQ	**3120.00**	4-2	3	1	2	1	44	44	25	26
KRBBKR	**0420.00**	4-2	1	27	104	787	36	35	2	3
KRBNKB	**0141.00**	4-2	9	2	10,985	42,661	13	13	1	2
KRBNKN	**0114.00**	4-2	1	6	8,152	39,422	12	12	0	1
KRBNKQ	**3111.00**	4-2	4	3	3	1	99	98	28	29
KRBNKR	**0411.00**	4-2	1	1	9	55	36	36	3	4
KRNNKB	**0132.00**	4-2	31	44	2,094	4,814	12	12	0	1
KRNNKN	**0105.00**	4-2	154	2,477	4,138	20,608	13	12	0	1
KRNNKQ	**3102.00**	4-2	2	1	2	3	28	27	41	41
KRNNKR	**0402.00**	4-2	1	3	28	114	39	39	3	4
KRRBKB	**0240.00**	4-2	530	1,911	3,931	17,132	7	7	1	2
KRRBKN	**0213.00**	4-2	2,459	12,709	3,664	16,427	6	6	0	1
KRRBKQ	**3210.00**	4-2	3	4	2	5	82	82	16	17
KRRBKR	**0510.00**	4-2	2	10	221,774	1,375,964	31	31	1	2
KRRNKB	**0231.00**	4-2	716	2,439	825	1,937	7	7	0	1
KRRNKN	**0204.00**	4-2	69	333	3,537	16,109	7	7	0	1
KRRNKQ	**3201.00**	4-2	5	2	2	1	101	101	18	19
KRRNKR	**0501.00**	4-2	9	46	289,032	1,811,539	33	33	1	2
KRRRKB	**0930.00/30**	4-2	51,108	219,810	0	0	5	5	—	—
KRRRKN	**0903.00/30**	4-2	6	30	950	3,965	6	6	0	1
KRRRKQ	**3900.00/30**	4-2	3	5	1	2	65	65	13	14
KRRRKR	**0900.00/31**	4-2	3	6	64,686	452,802	21	21	1	2

Table 4a. Chess Endgames: 6-man, pawnless DTZ_{50} data.

			DTZ_{50} Metric							
Endgame			**# of maximal positions**				**max depth, moves**			
			1-0		**0-1**		**1-0**		**0-1**	
	GBR	**w-b**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**
KBBKBN	**0053.00**	3-3	5	1	26	180	21	20	9	10
KBBKNN	**0026.00**	3-3	46	17	488	1,518	29	28	3	4
KBNKBN	**0044.00**	3-3	29	4	4	29	9	8	8	9
KBNKNN	**0017.00**	3-3	1	1	12	154	13	12	6	7
KQBKBB	**1070.00**	3-3	8	30	1,317	6,118	13	13	3	4
KQBKNN	**1016.00**	3-3	71	331	28	81	13	13	2	3
KQBKRR	**1610.00**	3-3	111,887	251,377	21	23	50	50	10	11
KQNKBB	**1061.00**	3-3	15	61	1	6,826	15	15	4	4
KQNKBN	**1034.00**	3-3	1	7	3	3	17	17	4	5
KQNKNN	**1007.00**	3-3	27	137	74	207	16	16	2	3

Table 4b. Chess Endgames: 6-man, pawnless DTZ_{50} data.

Endgame			DTZ_{50} Metric: # of maximal positions				max depth, moves			
			1-0		0-1		1-0		0-1	
	GBR	**w-b**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**
KQNKRR	**1601.00**	3-3	3,007,192	2,814,979	6	7	50	50	11	12
KQQKBB	**2060.00**	3-3	1	5	137	714	8	8	3	4
KQQKNN	**2006.00**	3-3	2	8	1	36,110	7	7	1	1
KQQKQB	**5030.00**	3-3	81	247	1	2	50	50	22	23
KQQKQR	**5300.00**	3-3	4	2	6	26	48	47	50	50
KQRKBB	**1160.00**	3-3	3	13	689	3,514	12	12	3	4
KQRKNN	**1106.00**	3-3	1	243	20	40	11	10	2	3
KQRKQB	**4130.00**	3-3	1,989	1,841	5	3	50	50	31	32
KQRKQN	**4103.00**	3-3	1,953	1,698	2	6	50	50	26	27
KQRKQR	**4400.00**	3-3	1,191	837	837	1,191	50	50	50	50
KQRKRB	**1430.00**	3-3	2	10	75	92	21	21	5	6
KRBKBB	**0170.00**	3-3	69,308	36,223	97	252	50	50	5	6
KRBKBN	**0143.00**	3-3	12,633,808	15,861,502	1	9	50	50	5	6
KRBKNN	**0116.00**	3-3	1,944,494	2,800,448	82	196	50	50	2	3
KRBKRB	**0440.00**	3-3	5	1	1	5	17	16	16	17
KRBKRN	**0413.00**	3-3	78	45	2	25	21	20	13	14
KRNKBB	**0161.00**	3-3	2,037,618	1,042,171	4	20	50	50	9	10
KRNKBN	**0134.00**	3-3	2,488,599	1,948,808	13	38	50	50	5	6
KRNKNN	**0107.00**	3-3	1,202,592	1,198,532	29	54	50	50	3	4
KRRKRB	**0530.00**	3-3	372	107	1	455	50	50	6	6
KRRKRN	**0501.00**	3-3	4,335	3,898	37	89	50	50	6	7
KBBBKN	**0093.00/30**	4-2	3	6	951	4,838	14	14	0	1
KBBBKQ	**3090.00/30**	4-2	1	9	11	15	10	9	50	50
KBBBKR	**0390.00/30**	4-2	685,975	1,619,489	13	72	50	50	4	5
KBBNKN	**0024.00**	4-2	9	54	3,663	18,984	31	31	0	1
KBBNKQ	**3021.00**	4-2	122	16	8,148	4,176	12	11	50	50
KBBNKR	**0321.00**	4-2	139,436	248,016	10	50	50	50	6	7
KBNNKN	**0015.00**	4-2	3	3	4,335	22,890	29	29	0	1
KBNNKQ	**3012.00**	4-2	5	1	1	4	12	11	49	49
KNNNKB	**0039.00/30**	4-2	195,576	232,786	1,275	2,891	50	50	0	1
KNNNKN	**0009.00/31**	4-2	6,272	12,853	1,584	8,562	50	50	0	1
KNNNKQ	**3009.00/30**	4-2	1	1	6	11	9	8	35	35
KQBBKN	**1023.00**	4-2	122	515	1,327	6,813	7	7	0	1
KQBBKQ	**4020.00**	4-2	52,602	136,241	2	3	50	50	15	16
KQBNKN	**1014.00**	4-2	135	719	3,262	17,347	6	6	0	1
KQBNKQ	**4011.00**	4-2	297	885	1	1	50	50	16	17
KQNNKQ	**4002.00**	4-2	10,534	9,796	5	20	50	50	13	14
KQQRKQ	**5100.00**	4-2	3	7	3	13	28	28	9	10
KQRBKQ	**41100.00**	4-2	6	19	4	9	49	49	12	13
KQRBKR	**1410.00**	4-2	1	7	269,633	1,690,187	25	25	1	2
KQRNKQ	**4101.00**	4-2	12	76	1	3	50	50	11	12
KQRRKQ	**4200.00**	4-2	1	4	1	2	41	41	8	9
KRBBKN	**0123.00**	4-2	7	57	3,717	17,552	12	11	0	1
KRBBKQ	**3120.00**	4-2	3	1	2	1	44	44	25	26
KRBBKR	**0420.00**	4-2	1	27	104	787	36	35	2	3
KRBNKN	**0114.00**	4-2	1	6	8,152	39,422	12	12	0	1
KRBNKQ	**3111.00**	4-2	120,325	34,369	3	1	50	50	28	29
KRBNKR	**0411.00**	4-2	1	1	9	55	36	36	3	4
KRNNKQ	**3102.00**	4-2	2	1	2	3	28	27	41	41
KRRBKQ	**3210.00**	4-2	23,857	56,552	2	5	50	50	16	17
KRRBKR	**0510.00**	4-2	2	10	221,774	1,375,964	31	31	1	2
KRRNKQ	**3201.00**	4-2	35,405	45,611	2	1	50	50	18	19
KRRRKQ	**3900.00/30**	4-2	271	1,195	1	2	50	50	13	14

Table 5a. The impact of the 50-move drawing rule on 6-man pawnless endgames.[17]

Endgame		**nominal wins**				**% of nominal wins**			
		# extra draws		**# delayed**		**extra draws**		**delayed**	
	res.	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**
KBBKBN	1-0	128,572,657	16,294,259	884,907	109,678	47.03	66.89	0.32	0.45
KBBKNN	1-0	141,874,223	38,562,549	4,961,624	1,402,773	50.15	70.98	1.75	2.58
KBNKBN	1-0	1,222,632	9,420	5,616	117	2.53	0.92	0.01	0.01
KBNKNN	1-0	1,179,997	14,499	17,361	918	2.81	1.19	0.04	0.08
KQBKBB	1-0	250,935	6,569,025	7,089,297	29,692,117	0.01	0.40	0.40	1.81
KQBKNN	1-0	397	38,516	23,320	38,516	ε	ε	ε	ε
KQBKRR	1-0	586,397	1,305,447	0	0	0.04	0.16	0	0
KQNKBB	1-0	300,774	6,546,430	11,971,950	45,591,146	0.02	0.41	0.64	2.84
	0-1	6,167,236	125,922,828	17,522	259,838	69.89	47.75	0.20	0.10
KQNKBN	0-1	3,703	1,213,657	26	1,328	1.05	2.80	0.01	ε
KQNKNN	1-0	188	36,110	59,575	242,663	ε	ε	ε	0.01
KQNKRR	1-0	72,985,602	79,251,396	0	0	4.87	15.42	0	0
KQQKBB	1-0	23,343	6,776,509	1,244,572	5,432,160	ε	0.58	0.18	0.47
KQQKNN	1-0	130	44,687	4,704	22,000	ε	ε	ε	ε
KQQKQB	1-0	689	2,278	0	0	ε	ε	0	0
KQQKQR	0-1	17,313	41,775	42,552	66,504	0.02	0.01	0.04	0.01
KQRKBB	1-0	125,901	6,357,673	2,948,393	11,781,268	0.01	0.37	0.18	0.69
KQRKNN	1-0	249	39,230	9,116	46,469	ε	ε	ε	ε
KQRKQB	1-0	23,934	17,235	94,650	90,746	ε	ε	ε	0.01
KQRKQN	1-0	12,641	11,010	70,821	86,758	ε	ε	ε	0.01
KQRKQR	1-0	21,395	12,416	48,844	50,736	ε	ε	ε	0.01
KQRKRB	0-1	251	11,459	3	410	0.01	0.02	ε	ε
KRBKBB	1-0	2,561,991	1,304,230	0	0	0.22	0.68	0	0
KRBKBN	1-0	426,514,269	767,645,636	0	0	12.14	41.47	0	0
KRBKNN	1-0	331,894,421	676,322,987	0	0	16.18	45.17	0	0
KRBKRB	1-0	9,084	783	1,605	122	ε	ε	ε	ε
KRBKRN	1-0	9,706	1,202	2,684	359	ε	ε	ε	ε
KRNKBB	1-0	407,078,847	370,216,259	0	0	26.20	66.08	0	0
	0-1	13,836,487	133,053,338	117,223	640,177	65.00	47.50	0.55	0.23
KRNKBN	1-0	139,761,310	107,975,414	0	0	4.98	15.04	0	0
	0-1	9,921	1,225,920	316	6,092	1.12	2.53	0.04	0.01
KRNKNN	1-0	82,794,630	83,586,263	0	0	5.18	14.78	0	0
KRRKRB	1-0	380	145	0	0	ε	ε	0	0
	0-1	396	11,281	30	799	0.02	0.03	ε	ε
KRRKRN	1-0	17,610	16,206	0	0	ε	ε	0	0
KBBBKN	1-0	743,762	37,035,833	55,589,963	161,070,140	0.15	6.16	11.28	26.80
KBBBKQ	0-1	21,650,797	31,223,711	6,004,068	11,096,464	15.04	6.15	4.17	2.19
KBBBKR	1-0	463,105	1,079,492	0	0	0.10	0.35	0	0
KBBNKN	1-0	640,358	36,582,112	136,891,517	318,970,567	0.03	1.74	6.44	15.17
KBBNKQ	0-1	55,226,710	40,880,784	27,763,565	27,296,005	10.16	2.52	5.11	1.68
KBBNKR	1-0	184,213	312,436	0	0	0.01	0.05	0	0
KBNNKN	1-0	96,123	1,016,653	10,322,215	13,062,956	ε	0.05	0.46	0.70
KBNNKQ	0-1	178,774	178,631	179,015	143,015	0.03	0.01	0.03	0.01
KNNNKB	1-0	539,360	648,931	0	0	0.08	0.20	0	0
KNNNKN	1-0	86,880	154,950	0	0	0.01	0.03	0	0
KNNNKQ	0-1	125,488	181,848	91,063	99,907	0.09	0.04	0.07	0.02
KQBBKN	1-0	122,388	45,118,478	24,140,183	88,092,478	0.01	1.72	1.55	3.35
KQBBKQ	1-0	206,322	526,510	0	0	0.01	0.05	0	0
	0-1	413,225	39,206,954	96	4,608	12.21	8.59	ε	ε

[17] ‘ε’ indicates a non-zero value less than 0.005.

Table 5b. The impact of the 50-move drawing rule on 6-man pawnless endgames.

Endgame		nominal wins				% of nominal wins			
		# extra draws		# delayed		extra draws		delayed	
	res.	wtm	btm	wtm	btm	wtm	btm	wtm	btm
KQBNKN	1-0	38,709	1,197,026	852,368	2,263,825	ε	0.02	0.03	0.04
KQBNKQ	1-0	1,347	5,171	0	0	ε	ε	0	0
KQNNKQ	1-0	49,329	38,050	0	0	ε	0.01	0	0
	0-1	1,538	206,733	0	2	0.04	0.05	0	ε
KQQRKQ	1-0	70	3,469	1,646	9,539	ε	ε	ε	ε
KQRBKQ	1-0	153	4,061	4,771	22,119	ε	ε	ε	ε
KQRBKR	1-0	598	31,924	21,765	66,560	ε	ε	ε	ε
KQRNKQ	1-0	654	6,196	4,707	22,857	ε	ε	ε	ε
KQRRKQ	1-0	186	4,325	2,632	14,630	ε	ε	ε	ε
KRBBKN	1-0	237,234	45,273,232	22,875,477	92,309,468	0.01	1.73	1.22	3.52
KRBBKQ	0-1	6,552,902	57,721,197	434,948	2,088,056	12.88	7.91	0.86	0.29
KRBBKR	1-0	4,834	29,950	115,546	131,589	ε	ε	0.01	0.01
KRBNKN	1-0	43,735	1,208,539	2,631,449	6,577,857	ε	0.02	0.07	0.13
KRBNKQ	1-0	1,172,828	314,964	0	0	0.06	0.08	0	0
KRBNKR	1-0	6,661	30,114	190,074	226,929	ε	ε	ε	0.01
KRNNKQ	0-1	33,448	252,183	10,270	30,764	0.04	0.03	0.01	ε
KRRBKQ	1-0	102,282	248,335	0	0	0.01	0.03	0	0
KRRBKR	1-0	918	30,159	76,780	179,899	ε	ε	ε	0.01
KRRNKQ	1-0	225,245	274,440	0	0	0.01	0.03	0	0
KRRRKQ	1-0	1,137	4,225	0	0	ε	ε	0	0

Table 6a. Example Positions showing $EZ_{50} \neq EZ$.[18]

Key		Position	stm	depth in plies				Notes
$EZ_{50} \neq EZ$				dtm	dtr	dtz	dtz_{50}	
BB-BN	1-0	7b/6nB/8/8/3B4/8/2K5/4k3	w	131	?	3	35	1. Bd3?? Ne6" 2. Bxh8 {*dtz* =52m}
BB-NN	1-0	8/8/6n1/8/k3BB2/8/n1K5/8	w	133	?	1	55	1. Bxg6?? {*dtz* =54m}
BN-BN	1-0	5n2/8/8/8/8/2K2b2/3N4/k3B3	w	11	11	1	11	1. Nxf3?? {*dtz* =70m}
BN-NN	1-0	8/8/8/8/2B5/2n2N2/2K4n/k7	w	147	?	1	11	1. Nxh2?? {*dtz* =51m} Nd5"
QB-BB	1-0	8/8/5b2/8/8/Q6b/4k2B/K7	w	39	?	3	23	1. Be5?? Bxe5+ {*dtz* =65m}
QB-NN	1-0	8/7Q/8/8/4n3/Bkn5/8/3K4	w	57	?	7	23	1. Ke1?? Kxa3 {*dtz* =52m}
QB-RR	1-0	8/2Kr4/5k2/8/8/5B2/6Q1/3r4	w	213	169	169	—	a maxDTM/Z pos.
QN-BB1	1-0	8/6bb/5N2/1Q6/5k2/8/8/K7	w	41	?	3	23	1. Qb4+?? Kg5 2. Qg4+ Kxf6
QN-BB2	0-1	1b6/8/8/K6N/8/8/6Q1/3k1b2	b	129	?	1	7	1. … Bxg2?? {*dtz* =52m}
QN-BN	0-1	8/8/8/6Q1/4n3/8/KNk4b	b	5	5	1	5	1. … Nxg4?? {*dtz* =53m}
QN-NN	1-0	8/6Q1/4n3/8/2k2n2/3N4/8/2K5	w	37	?	3	15	1. Qg4?? Kxd3 {*dtz* =52m}
QN-RR	1-0	r5r1/8/k7/8/8/8/3K4/1Q4N1	b	348	305	305	—	a maxDTM/Z pos.
QQ-BB	1-0	8/Q7/8/3bb3/8/8/3k4/K4Q2	w	17	13	3	13	SZ- ×; 1. Qd4+?? Bxd4
QQ-NN	1-0	8/8/8/3n4/Q7/4k3/2K3Q1/4n3	w	69	?	3	7	1. Kd1?? Nxg2 {*dtz* =52m}
QQ-QB	1-0	7Q/4Q3/8/8/6K1/8/2kq4/5b2	b	142	124	124	—	a maxDTM/Z pos.
QQ-QR	0-1	Q2Q4/2K5/8/8/8/8/r7/1k5q	b	91	?	1	71	1. … Qxa8?? {*dtz* =60m}
QR-BB	1-0	8/8/5bb1/8/8/Q7/4k3/K2R4	w	35	?	5	19	1. Rd4?? Bxd4 {*dtz* =66m}
QR-NN	1-0	8/8/8/1Q6/3n4/2k5/8/1RK3n1	w	19	?	1	7	1. Rb3+?? Nxb3º {*dtz* =51m}

[18] *stm* ≡ ‘side to move’. Without a DTR EGT, it is not always possible to determine *dtr* precisely.

Table 6b. Example Positions showing $EZ_{50} \neq EZ$.

Key		Position	stm	depth in plies				Notes
$EZ_{50} \neq EZ$				dtm	dtr	dtz	dtz_{50}	
QR-QB	1-0	8/1Q6/4q3/8/8/6k1/8/1RK4b	w	115	?	1	89	1. Qxh1?? {*dtz* =58m}
QR-QN	1-0	1Q6/8/8/5q2/8/4k3/8/1RK4n	w	101	?	17	75	1. Qb6+??
QR-QR	1-0	8/7R/8/3q4/8/8/1K3k2/Q6r	w	85	?	1	41	1. Qxh1?? {*dtz* =57m}
QR-RB	0-1	8/4R3/5b2/6Q1/8/2k5/6r1/K7	b	7	7	1	7	1. … Rxg5?? {*dtz* =56m}
RB-BB	1-0	7k/4R2B/8/8/8/3K2bb/8/8	w	183	149	149	—	a maxDTM pos.
RB-BN	1-0	Bb6/8/8/8/8/1R6/3kn3/K7	b	224	196	196	—	a maxDTM/Z pos.
RB-NN	1-0	8/8/8/8/2n2k2/2n5/5BR1/1K6	w	475	445	445	—	a maxDTM/Z pos.
RB-RB	1-0	1R6/8/8/1b6/8/B7/k1K5/r7	w	23	?	1	15	1. Rxb5?? {*dtz* =54m}
RB-RN	1-0	8/8/3n4/4B3/3K2r1/8/5R2/k7	w	95	?	9	25	1. Kc3?? Kb1" 2. Bxd6 {*dtz* =52m}
RN-BB1	1-0	2k1b3/7R/8/8/4NK2/8/8/6b1	w	137	103	103	—	
RN-BB2	0-1	8/3b4/8/8/5b2/K6R/8/1k5N	b	51	?	1	13	1. … Bxh3?? {*dtz* =53m}
RN-BN1	1-0	NbR5/8/n7/8/8/8/8/2K2k2	w	417	379	379	—	a maxDTM/Z pos.
RN-BN2	0-1	2N5/5R2/8/7b/8/2k5/8/1K2n3	b	163	?	1	11	1. … Bxf7?? {*dtz* =52m}
RN-NN	1-0	6k1/5n2/8/8/8/5n2/1RK5/1N6	w	523	485	485	—	a maxDTM/Z pos.
RR-RB1	1-0	3R4/8/R7/8/8/8/6r1/k3K2b	b	122	102	102	—	
RR-RB2	0-1	8/8/8/1r6/R4b2/6R1/2k5/K7	b	67	7	1	7	1. … Bxg3?? {*dtz* =55m}
RR-RN	1-0	2K5/k2RR3/8/8/6n1/8/8/r7	b	178	146	146	—	a maxDTM/Z pos.
BBB-N	1-0	8/1B6/8/8/4n3/2BkB3/8/1K6	w	43	?	2	23	1. Ba6+?? Kxe3 {*dtz* =59m}
BBB-Q	0-1	5q2/7K/8/6B1/8/B6B/8/k7	b	91	?	1	59	1. … Qxa3?? {*dtz* =64m}
BBB-R	1-0	6B1/8/8/6r1/8/7k/7B/K5B1	w	149	137	137	—	a maxDTM/Z pos.
BBN-N	1-0	8/8/8/5n2/2K5/1N6/1BkB4	w	79	?	4	41	1. Bf3 Kxb1 {*dtz* =55m}
BBN-Q	0-1	8/4K3/7q/1B6/8/3k4/N7/4B3	b	133	85	13	85	1. … Kd4??
BBN-R	1-0	N7/6B1/8/8/8/7B/1r1k4/K7	b	170	136	136	—	a maxDTM/Z pos.
BNN-N	1-0	8/8/8/8/1k6/N7/2K5/N3n2B	w	77	?	3	47	1. Kd2?? Kxa3 {*dtz* =55m}
BNN-Q	0-1	7N/6q1/8/8/2N5/3K1k2/8/B7	b	125	?	1	71	S(M/Z)σ ×; 1. … Qxa1??
NNN-B	1-0	6bN/8/8/8/8/1N6/2k5/K6N	w	191	183	183	—	a maxDTM/Z pos.
NNN-N	1-0	7N/N7/8/1k6/8/8/2K1n3/1N6	b	180	172	172	—	a maxDTM/Z pos.
NNN-Q	0-1	N7/8/8/q7/5KN1/8/3k3N	b	127	?	1	41	1. … Qxa8?? {*dtz* =57m}
QBB-N	1-0	1Q6/8/8/8/7k/BB6/K3n3	w	11	?	2	8	1. Qg3+ ?? Kxg3° {*dtz* =54m}
QBB-Q1	1-0	8/7K/8/2B5/8/1k2Bq2/7Q	b	192	186	186	—	a maxDTM/Z pos.
QBB-Q2	0-1	8/Q7/8/2B4B/2K5/q7/2k5	b	61	?	1	7	1. … Qxa7?? {*dtz* =52m}
QBN-N	1-0	Q7/1B6/8/2n5/8/5N2/1k1K4	w	9	?	2	7	1. Qa1+?? Kxa1° {*dtz* =51m}
QBN-Q	1-0	8/8/2K5/8/8/1Q1B4/8/2kN2q1	b	168	130	130	—	a maxDTM pos.
QNN-Q1	1-0	7q/1Q6/8/5N2/8/8/8/K1k4N	w	107	101	101	—	1. Ng7" ...
QNN-Q2	0-1	8/2N5/8/2q5/5N2/2k5/8/2K4Q	b	9	7	5	7	SZσ ×; SM ok. 1. … Qa3+??
QQR-Q	1-0	8/7R/8/5q2/7Q/5k2/2K4Q	w	25	?	2	19	1. Qe3+ Qxe3+ {*dtz* =56m}
QRB-Q	1-0	1R5Q/1B6/6k1/5q2/8/8/8/1K6	w	41	?	3	35	1. Be4?? Qxe4+ {*dtz* =51m}
QRB-R	1-0	6B1/8/3r4/8/8/8/3KRQ2/7k	w	63	?	3	18	1. Qd4?? Rxd4 {*dtz* =52m}
QRN-Q	1-0	8/7q/8/7N/6k1/2K5/1R5Q	w	83	?	3	67	1. Nxf5+?? Qxf5+ {*dtz* =54m}
QRR-Q	1-0	2R5/3q4/8/8/8/1k6/8/Q2K2R1	w	39	?	6	29	1. Kc1?? Qxc8+ {*dtz* =54m}
RBB-N	1-0	8/8/8/1k6/2R5/1nB5/3K4/7B	w	19	?	3	13	1. Kc2?? Kxc4 {*dtz* =55m}
RBB-Q	0-1	8/8/q7/5K2/8/1B6/3k1B2/2R5	b	131	?	1	29	1. … kxc1 {*dtz* =55m}
RBB-R	1-0	8/8/8/B7/3K4/8/4R3/2Bk2r1	w	51	?	7	47	1. Kd3?? Kxc1 {*dtz* =55m}
RBN-N	1-0	8/8/8/8/2n4B/8/2N3k1/3K3R	w	25	?	2	13	1. Ne1+?? Kxh1° {*dtz* =76m}
RBN-Q	1-0	1k4q1/8/3K4/8/1N6/8/8/R3B3	w	241	197	197	—	a maxDTM/Z pos.
RBN-R	1-0	8/8/8/3R4/1B4r1/1k1K4/N7/8	w	47	?	7	37	1. Bd6?? Kxa2" {*dtz* =54m}
RNN-Q	0-1	7N/R2q4/8/N7/3k4/8/4K3/8	b	125	?	23	65	1. Qg4+??
RRB-Q	1-0	1RK5/1R6/8/1q6/k7/8/7B/8	b	180	164	164	—	a maxDTM/Z pos.
RRB-R	1-0	8/8/R7/8/6r1/B7/R2K4/1k6	w	13	?	2	12	1. Ra1+?? Kxa1° {*dtz* =55m}
RRN-Q	1-0	2K5/7k/8/8/4q3/7R/8/5R1N	b	216	202	202	—	a maxDTM/Z pos.
RRR-Q	1-0	1R4R1/8/1q6/7R/8/8/5k2/3K4	b	138	130	130	—	a maxDTM/Z pos.

The following lines, starting from selected positions listed in Table 6, show strategy SZ_{50}^- delivering the available win while other strategies fail to retain it. They and others were discovered using the Tamplin (2004) web service, and include an established notation showing the criticality of the moves:

" ≡ *unique value-preserving move*; ' ≡ *strategy's only optimal move*; ° ≡ *only legal move*.

Some themes emerge. The attacker can avoid making an ill-advised sacrifice[19] and we include only QRN-Q here. More interestingly, White can delay a capture[20] or go directly for mate[21]. The defender often avoids capturing where, against a fallible player, it would be in its interests to do so to maximize DTR.

KBBKBN position BB-BN – *dtm* = 66m, *dtz* = 2m, dtz_{50} = 18m:
$S\sigma\tau - SZ_{50}^+$, $\sigma = C^-$, M^- or Z^-: 1. Bd3'?? Ne6" 2. Bxh8" {*dtz* = 52m; Black can 50m-draw} ½-½.
$SZ_{50}^- - SZ_{50}^+$: 1. Kd3" Kf1' 2. Bg8" Kg2' 3. Ke4" Kg3' 4. Ba2" Kg4 5. Bb1" Kg3' 6. Bc2' Kg2' 7. Bd1' Kg3 8. Be5+' Kg2' 9. Bg4' Kf2' 10. Kd3' Kf1' 11. Kd2 Kf2' 12. Bd1 Kg2' 13. Ke2 Kh3' 14. Kf2' Kh4' 15. Bf6+' Kh3° 16. Bf3' Kh2' 17. Bg2' Nh5 18. Bxh8" {*dtm* = 19m} 1-0.

KBBKNN position BB-NN – *dtm* = 67m, *dtz* = 1m, dtz_{50} = 28m:
$S\sigma\tau - S\varphi$, $\sigma = C^-$, M^- or Z^-: 1. Bxg6'?? {*dtz* = 54m; Black can 50m-draw} ½-½.
$SZ_{50}^- - SZ_{50}^+$: 1. Bd6" Nh8' 2. Bc6+" Ka5° 3. Kb3" Nc1+' 4. Kc4" Nf7' 5. Bc7+" Ka6° 6. Bd5" Nh8' 7. Bf3' Ng6' 8. Bd6" Nh4' 9. Be4" Ne2' 10. Bh2" Ka5' 11. Bc7+' Ka6' 12. Kc5' Ka7' 13. Bd3' Ng1' 14. Bg3 Ng2' 15. Kc6' Nh3' 16. Bf1' Nhf4' 17. Bf2+" Kb8' 18. Bb6' Ka8' 19. Ba6' Kb8' 20. Bc4' Nh5' 21. Bc7+' Ka7 22. Be5' Nhf4' 23. Bd6' Nh5 24. Kc7' Nf6' 25. Bc5+' Ka8° 26. Bb5 Nd5+' 27. Kc8" Ne1 28. Bc6#'.

KBNKBN position BN-BN - *dtm* = 6m, *dtz* = 1m, dtz_{50} = 6m:
$SZ^- - SZ_{50}^+$: 1. Nxf3'?? {*dtz* = 70m; Black can 50m-draw} ½-½.
$SZ_{50}^- - SZ_{50}^+$: 1. Kb3" Ne6' 2. Bf2" Bd1+ 3. Ka3" Bc2 4. Bb6 Bd1 5. Ba5' Nd4 6. Bc3#" 1-0.

KBNKNN position BN-NN – *dtm* = 74m, *dtz* = 1m, dtz_{50} = 6m:
$SZ^- - SZ_{50}^+$: 1. Nxh2'?? {*dtz* = 51m; Black can 50m-draw} ½-½.
$SZ_{50}^- - SZ_{50}^+$: 1. Nd4" Nb1' 2. Be6 Na3+' 3. Kc1" Nf1' 4. Nb3+" Ka2° 5. Nd2+' Ka1' 6. Nxf1' {*dtz* = 38m, *dtm* = 68m} 1-0.

KQNKBB position QN-BB2 – *dtm* = 65m, *dtz* = 1m, dtz_{50} = 4m:
$S\varphi - S\sigma\tau$, $\sigma = C^-$, M^- or Z^-: 1. … Bxg2?? {*dtz* = 52m; White can 50m-draw} ½-½.
$SZ_{50}^- - SZ_{50}^+$: 1. ... Bc7+" 2. Kb4' Bd6+" 3. Kc3' Be5+" 4. Kb4 Bxg2' {*dtm* = 18m} 0-1.

KQNKBN position QN-BN - *dtm* = 3m, *dtz* = 1m, dtz_{50} = 3m:
$S\varphi - SZ^-\sigma$: 1. … Nxg4'?? {*dtz* = 53m; White can 50m-draw} ½-½.
$SZ_{50}^+ - SZ_{50}^-$: 1. ... Nc2+" 2. Ka2° Bd5+" 3. Qc4° Bxc4#" 0-1.

KQQKQR position QQ-QR – *dtm* = 46m, *dtz* = 1m, dtz_{50} = 36m:
$S\varphi - SZ^-\sigma$: 1. … Qxa8'?? {*dtz* = 60m; White can 50m-draw} ½-½.
$SZ_{50}^+ - SZ_{50}^-$: 1. ... Qh2+" 2. Kd7' Qh3+" 3. Kc7' Qg3+' 4. Kb6' Qe3+' 5. Kb5' Qb3+" 6. Kc5' Qc3+" 7. Kd6' Qd4+" 8. Ke6' Re2+" 9. Kf7 Rf2+" 10. Ke6' Qg4+' 11. Kd5' Rd2+" 12. Kc5' Rc2+" 13. Kd6' Qf4+' 14. Ke6' Re2+" 15. Kd7' Qf5+' 16. Kc7' Rc2+" 17. Kb8' Qf4+' 18. Ka7' Ra2+' 19. Kb6' Rb2+' 20. Kc6' Rc2+' 21. Kb7' Qf7+' 22. Kb8' Rb2+' 23. Kc8' Qc4+' 24. Qc7' Qg4+' 25. Qd7' Rc2+' 26. Kd8' Qg5+" 27. Qe7' Rd2+" 28. Ke8' Qg8+" 29. Qf8° Re2+' 30. Kd7' Qe6+" 31. Kc7' Rc2+" 32. Kb8' Qe5+" 33. Ka7' Qa5+' 34. Kb7' Rc7+ 35. Kb8° Qb6+" 36. Qb7° Qxb7#' 0-1.

KQRKQB position QR-QB – *dtm* = 58m, *dtz* = 1m, dtz_{50} = 45m:
$SZ^- - S\varphi$: 1. Qxh1'?? {*dtz* = 58m; Black can 50m-draw} ½-½.
$SZ_{50}^- - SZ_{50}^+$: 1. Rb3+" Kf4' 2. Qb4+" Be4' 3. Qd2+" Kg4' 4. Qe2+" Kf5' 5. Qf2+" Ke5' 6. Qg3+' Kd5' 7. Qg5+" Kc6' 8. Rc3+" Kd7' 9. Qg7+" Ke8' 10. Qh8+' Kd7' 11. Rc8' Qg6' 12. Qd8+" Ke6° 13. Qb6+" Ke5' 14. Qb8+' Ke6' 15. Re8+' Kf7' 16. Rf8+" Ke6' 17. Qb6+ Ke7' 18. Qd8+' Ke6° 19. Re8+' Kf5' 20. Qd7+' Kf4' 21. Qd2+' Kf3' 22. Qd1+' Kf4' 23. Qf1+' Ke3' 24. Qe1+" Kd4' 25. Qd2+' Kc4' 26. Qe2+' Kd5' 27. Rd8+' Ke6' 28. Qc4+" Kf5' 29. Rf8+ Ke5' 30. Qc3+' Kd5 31. Rd8+" Ke6' 32. Qc8+' Ke5' 33. Qc7+' Kf5' 34. Rf8+' Ke6' 35. Kb2' Qg2+' 36. Ka3" Bc6' 37. Qf4 Kd7' 38. Qf5+' Kc7 39. Qa5+' Kd6' 40. Rf6+' Kd7' 41. Qa7+' Bb7' 42. Rf7+" Kc8' 43. Qc5+' Kb8 44. Rf8+' Bc8° 45. Rxc8+' {*dtm* = 2m} 1-0.

[19] e.g., positions QB-BB/NN, QN-BB1/NN, QQ-BB/NN, QR-BB/NN, BBB-N, BBN-N, BNN-N, QBB-N, QBN-N, QQR-Q, QRB-Q/R, QRN-Q, QRR-Q, RBB-N/R, RBN-N/R and RRB-R.
[20] e.g., positions BB-BN, BN-NN, QN-BB2, QR-QB/QN/QR, RB-RB/RN, RN-BN2, RR-RB, BBB-Q, BBN-Q, BNN-Q, NNN-Q, QBB-Q and RBB-Q.
[21] e.g., positions BB-NN, BN-BN, QN-BN, QQ-QR, QR-RB, RN-BB2/BN2 and QNN-Q2.

KQRKQN position QR-QN - *dtm* = 51m, *dtz* = 9m, dtz_{50} = 38m:

SZ^- – Sφ: 1. Qb6+'?? Ke2 2. Qa6+' Kf3" 3. Qc6+' Kg3" 4. Qxh1" {*dtz* = 59m; Black can 50m-draw} ½-½.

SZ_{50}^- – SZ_{50}^+: 1. Qb3+" Kf4' 2. Qc3" Qg5' 3. Qd2+" Kg4' 4. Rb4+" Kh5' 5. Rf4" Ng3' 6. Kd1" Kh6 7. Qd6+' Kh5' 8. Qd4' Nf5' 9. Qh8+" Kg6' 10. Qe8+" Kf6' 11. Qc6+' Ke7' 12. Qe4+' Kf6' 13. Kc2' Qh5' 14. Rf2" Qh3' 15. Kb2" Kg5 16. Rg2+" Kf6' 17. Qc6+' Ke5' 18. Qc7+' Kf6' 19. Qd8+' Kf7' 20. Qg8+' Kf6' 21. Rg6+' Ke5' 22. Re6+ Kf4' 23. Qb8+" Kg5' 24. Qd8+ Kf4' 25. Qd2+' Kg4' 26. Rg6+' Kf3' 27. Rg8' Qh7' 28. Qg2+' Kf4' 29. Rg4+' Ke5' 30. Qe4+' Kd6' 31. Qd3+' Ke7 32. Re4+' Kf6' 33. Qd8+' Kg6' 34. Rg4+' Kf7' 35. Qd7+' Ne7' 36. Rf4+' Kg8 37. Qe8+ Kg7° 38. Qxe7+' {*dtm* = 2m} 1-0.

KQRKQR position QR-QR – *dtm* = 43m, *dtz* = 1m, dtz_{50} = 21m:

SZ^- – Sφ: 1. Qxh1'?? {*dtz* = 57m; Black can 50m-draw} ½-½.

SZ_{50}^- – SZ_{50}^+: 1. Qa7+" Kf3' 2. Qa3+" Kg4' 3. Qb4+" Kg5' 4. Qe7+" Kg4' 5. Qg7+' Kf3 6. Qf8+' Ke2 7. Qe8+" Kd3' 8. Qg6+" Kc4' 9. Qg4+ Kb5' 10. Qe2+" Kc5 11. Qe3+' Kc6 12. Qe8+" Kc5 13. Rc7+' Kd4' 14. Rd7' Rh2+' 15. Kc1' Rh1+' 16. Kd2' Rh2+' 17. Ke1' Rh1+' 18. Kf2' Rh2+' 19. Kg3' Rg2+' 20. Kh3' Rg5 21. Rxd5+' {*dtz* = 29m} 1-0.

KQRKRB position QR-RB - *dtm* = 4m, *dtz* = 1m, dtz_{50} = 4m:

Sφ – SZ^-σ: 1. … Rxg5'?? {*dtz* = 56m; White can 50m-draw} ½-½.

SZ_{50}^+ – SZ_{50}^-: 1. … Kb3+" 2. Kb1' Rb2+" 3. Ka1' Ra2+' 4. Kb1° Ra1#'0-1.

KRBKRB position RB-RB – *dtm* = 12m, *dtz* = 1m, dtz_{50} = 8m:

SZ^- – Sφ: 1. Rxb5'?? {*dtz* = 54m; Black can 50m-draw} Rg1" 2. Bd6" Rg2+" 3. Kc3" Rg6" ½-½.

SZ_{50}^- – SZ_{50}^+: 1. Ra8'" Bd3+ 2. Kc3" Be4' 3. Ra4 Kb1 4. Rb4+" Ka2° 5. Bb2' Bc6' 6. Rb6" Rh1 7. Ra6+" Ba4 8. Rxa4+" {*dtm* = 1m} 1-0.

KRBKRN – position RB-RN – *dtm* = 48m, *dtz* = 5m, dtz_{50} = 13m:

SZ^- – Sφ: 1. Kc3'?? {Black can 50m-draw} Kb1" 2. Rf1+' Ka2° 3. Bxd6' {*dtz* = 52m}.

SZ_{50}^- – SZ_{50}^+: 1. Kd3+" Kb1' 2. Rb2+" Kc1' 3. Ra2" Rb4' 4. Bc3" Rb5' 5. Re2 Rd5+' 6. Bd4" Kb1' 7. Rb2+" Kc1' 8. Ra2' Rb5' 9. Re2" Kb1' 10. Re1+" Ka2° 11. Kc2" Ka3' 12. Bc3" Rb2+ 13. Bxb2+" {*dtm* = 15m} 1-0.

KRNKBB position RN-BB2 - *dtm* = 26m, *dtz* = 1m, dtz_{50} = 7m:

Sφ – SZ^-σ: 1. … Bxh3'?? {*dtz* = 53m; White can 50m-draw} ½-½.

SZ_{50}^+ – SZ_{50}^-: 1. … Kc2" 2. Rh5' Bd6+" 3. Ka2' Be6+" 4. Ka1' Bb4" 5. Rh2+' Kc1" 6. Rf2 Bc3+" 7. Rb2° Bxb2#' 0-1.

KRNKBN position RN-BN2 - *dtm* = 82m, *dtz* = 1m, dtz_{50} = 6m:

SZ_{50}^+ – SZ^-: 1. ... Bxf7'?? {*dtz* = 52m; White can 50m-draw} ½-½.

SZ_{50}^+ – SZ_{50}^-: 1. … Bg6+" 2. Ka1' Nc2+" 3. Kb1' Nb4+' 4. Kc1' Nd3+" 5. Kd1' Bh5+" 6. Rf3° Bxf3#".

KRRKRB position RR-RB – *dtm* = 34m, *dtz* = 1m, dtz_{50} = 4m:

Sφ – SZ^-σ: 1. … Bxg3'?? {*dtz* = 55m; White can 50m-draw} ½-½.

SZ_{50}^+ – SZ_{50}^-: 1. … Be5+" 2. Ka2' Rb2+" 3. Ka3 Bd6+" 4. Rb4° Rxb4" {*dtm* = 30m} 0-1.

KBBBKQ position BBB-Q – *dtm* = 46m, *dtz* = 1m, dtz_{50} = 30m:

Sφ – SZ^-σ: 1. ... Qxa3'?? {*dtz* = 64m; White can 50m-draw} ½-½.

SZ_{50}^+ – SZ_{50}^-: 1. ... Qf3" 2. Bc8' Qh5+" 3. Bh6' Qf7+" 4. Bg7+' Ka2" 5. Baf8' Qd5" 6. Kh8' Kb3' 7. Bh6 Ka4' 8. Bh3' Kb5 9. Kg7' Qe5+" 10. Kh7' Qc7+' 11. Bfg7' Qc2+" 12. Kh8' Qe4" 13. Bf8' Kb6' 14. Kg7' Qe5+" 15. Kh7' Qc7+' 16. Bfg7' Qc2+" 17. Kh8' Qe4" 18. Bf8' Kb7' 19. Kg7' Qe5+" 20. Kh7' Qc7+' 21. Bfg7' Qc2+" 22. Kh8' Qg6' 23. Bf8 Kc7' 24. Bf4+ Kd8" 25. B8h6' Ke7' 26. Bf1' Qc2' 27. Kg7 Qb2+" 28. Kg8' Qa2+' 29. Kg7 Qa1+" 30. Kg6 Qxf1' {*dtm* = 17m} 0-1.

KBBNKN position BBN-N - *dtm* = 40m, *dtz* = 2m, dtz_{50} = 21m:

SZ^- – SZ_{50}^+: 1. Bf3'?? Kxb1" {*dtz* = 55m; Black can 50m-draw} ½-½.

SZ_{50}^- – SZ_{50}^+: 1. Bh7' Ne2+' 2. Kb3" Nd4+' 3. Ka2' Kd2' 4. Ka3' Kc3 5. Na4+' Kd2' 6. Bg4' Ke3' 7. Kb4' Nc6+' 8. Kc5' Ne5' 9. Bh3 Nf7 10. Kd5 Ng5' 11. B7f5' Kf4 12. Nc5 Kg3' 13. Bhg4' Nf7' 14. Bh5 Nh6' 15. Ke6 Kf4 16. Bfg6 Ke3 17. Kf6 Kd4 18. Na6 Ng8+ 19. Kf7' Nh6+' 20. Kg7' Ng4 21. Bxg4' {*dtm* = 17m} 1-0**.**

KBBNKQ position BBN-Q – *dtm* = 67m, *dtz* = 7m, dtz_{50} = 43m:

SZ_{50}^+ – SZ^-: 1. ... Kd4'?? 2. Bf2+" Ke5' 3. Bg3+" Kd5 4. Nc3+" Kd4" 5. Bd6" Kxc3' {*dtz* = 51m; White can 50m-draw} ½-½.

SZ_{50}^+ – SZ_{50}^-: 1. ... Ke3" 2. Be8' Qg5+" 3. Kf8' Qc5+' 4. Kg8' Qc8' 5. Kf8' Qa8' 6. Nb4' Qa3" 7. Kg7' Qb2+" 8. Kf7' Qb3+" 9. Kg7' Kf4' 10. Bd2+' Kg4" 11. Bd7+' Kh5' 12. Be8+' Kh4' 13. Be1+' Kg4' 14. Bd7+' Kf3' 15. Bf5' Kf4" 16. Bh7' Qb2+" 17. Kg6' Kg4' 18. Nd3' Qd4 19. Kf7' Qd7+' 20. Kg8' Qe8+ 21. Kg7° Qe7+' 22. Kg6' Qg5+' 23. Kf7° Kh5' 24. Bb4' Qd5+" 25. Kg7' Qd4+ 26. Kg8 Qg4+ 27. Kh8' Qc8+' 28. Bg8' Qc7' 29. Bd2' Qd6" 30. Nf4+' Kg4 31. Bd5' Kf5' 32. Kg8' Qb8+ 33. Kf7' Qc7+' 34. Kg8' Qc2' 35. Be6+' Ke5' 36. Be3' Qe4" 37. Bc1' Kf6' 38. Bb2+' Kg5" 39. Bc1' Qa8+' 40. Kf7' Qb7+" 41. Kf8 Kf6 42. Ba3 Qa8+ 43. Bc8° Qxc8#' 0-1.

KBNNKN position BNN-N - *dtm* = 39m, *dtz* = 2m, dtz_{50} = 24m:

SZ^- – SZ_{50}^+: 1. Kd2'?? Kxa3" {*dtz* = 55m; Black can 50m-draw} ½-½.

SZ_{50}^- – SZ_{50}^+: 1. Kb2" Nd3+' 2. Ka2" Nc1+' 3. Kb1' Nd3' 4. N1c2+ Kc5' 5. Ba8 Kd6 6. Ne3 Kc5' 7. Kc2 Nb4+' 8. Kc3 Na2+' 9. Kd2' Nb4' 10. Nac4 Na6' 11. Kd3 Nb4+ 12. Ke4' Nc6 13. Ne5' Na7' 14. Nd3+' Kd6 15. Nc4+' Kc7' 16. Nb4' Kb8 17. Bd5' Nb5' 18. Bc6' Na7' 19. Ba4 Nc8' 20. Ke5 Ka7' 21. Ke6 Kb8 22. Kd7' Kb7 23. Nd6+' Kb6 24. Nxc8+' {*dtm* = 28m} 1-0.

KBNNKQ position BNN-Q – *dtm* = 63m, *dtz* = 1m, dtz_{50} = 36m:

Sφ – Sσ, σ = C⁻, M⁻ or Z⁻: 1. ... Qxa1'?? {*dtz* = 52m; White can 50m-draw} ½-½.

SZ_{50}^+ – SZ_{50}^-: 1. ... Qh7+" 2. Kd2' Qd7+" 3. Kc3' Ke2' 4. Bb2' Qg4" 5. Kb3' Qe6" 6. Kc3' Qe4" 7. Kb3' Qg4' 8. Kc3' Qf4' 9. Kb3' Qb8+' 10. Kc2' Qb4' 11. Na3' Qe4+" 12. Kb3' Qd5+' 13. Kc3' Qf3+' 14. Kc4' Kd1 15. Kb4' Qb7+" 16. Nb5' Kc2' 17. Bd4' Qe7+' 18. Kc4' Qe6+' 19. Kc5' Qf5+' 20. Kc4' Qc8+' 21. Kb4' Qf8+' 22. Ka4 Qg8' 23. Kb4 Kd3' 24. Bc3' Qd5' 25. Bd4 Qc4+' 26. Ka5' Qg8' 27. Ka4' Qa8+' 28. Kb4' Qf8+' 29. Kb3' Qe7' 30. Bb2' Qe6+' 31. Ka4 Qa2+ 32. Ba3' Qc4+' 33. Ka5 Qd5' 34. Kb4' Qe4+ 35. Ka5 Qa8+' 36. Kb6 Qxh8 {*dtm* = 22m} 0-1.

KNNNKQ position NNN-Q - *dtm* = 64m, *dtz* = 2m, dtz_{50} = 21m:

Sφ – SZ⁻: 1. ... Qxa8'?? {*dtz* = 57m; White can 50m-draw} ½-½.

SZ_{50}^+ – SZ_{50}^-: 1. ... Qa3+" 2. Kf4' Qd6+' 3. Kg4' Qd4+' 4. Kf3' Qf6+' 5. Kg4' Qg7+' 6. Kf3' Kd2' 7. Ne4+' Kd3" 8. Nc5+' Kc4' 9. Nd7' Qf7+" 10. Ke3 Qe6+' 11. Kf2' Qf5+' 12. Kg2' Qd5+' 13. Kh2 Qd2+' 14. Kg3 Kd3' 15. Nab6' Ke2" 16. Kg2' Qb2' 17. Kg3' Qd4' 18. Kg2' Qe4+' 19. Kh2 Kf3' 20. Nf8 Qg4 21. Ng3 Qxg3+' {*dtm* = 1m} 0-1.

KQBBKQ position QBB-Q2 – *dtm* = 31m, *dtz* = 1m, dtz_{50} = 4m:

Sφ – SZ⁻: 1. ... Qxa7'?? {*dtz* = 52m; White can 50m-draw} ½-½.

SZ_{50}^+ – SZ_{50}^-: 1... Qd2+" 2. Kb3° Qb2+" 3. Ka4° Qa1+" 4. Kb3 Qxa7' {*dtm* = 27m} 0-1.

KQNNKQ position QNN-Q2 - *dtm* = 4m, *dtz* = 3m, dtz_{50} = 4m:

SZ_{50}^+ – SZ⁻: 1. ... Qa3+'?? 2. Kd1" Qa1+" 3. Ke2° Qxh1" {*dtz* = 52m; White can 50m-draw} ½-½.

SZ_{50}^+ – SZ_{50}^-: 1. … Qe3+" 2. Kb1' Qb6+" 3. Kc1' Qb2+' 4. Kd1° Qd2#" 0-1.

KQRNKQ position QRN-Q – *dtm* = 42m, *dtz* = 2m, dtz_{50} = 34m:

SZ⁻ – SZ_{50}^+: 1. Nf5+'?? {unnecessary sac.} 1... Qxf5+" {*dtz* = 54m; Black can 50m-draw} ½-½.

SZ_{50}^- – SZ_{50}^+: 1. Kb3' Qd3+ 2. Kb4' Qd4+' 3. Kb5' Qd7+' 4. Ka6' Qd6+' 5. Ka5' Qa3+' 6. Kb5' Qd3+' 7. Kc6' Qc4+' 8. Kb7' Qf7+' 9. Kb6' Qf2+' 10. Kc6' Qf6+' 11. Kb5' Qe5+' 12. Ka4' Qd4+' 13. Rb4' Qa7+' 14. Kb3' Qe3+' 15. Ka2' Qe2+' 16. Rb2' Qa6+' 17. Kb1' Qd3+' 18. Rc2' Qb3+' 19. Kc1' Qa3+' 20. Kd2' Qd6+' 21. Kc3 Qc5+' 22. Kd3' Qd6+' 23. Kc4' Qa6+' 24. Kd5' Qb5+' 25. Rc5' Qb3+ 26. Kd6' Qb8+' 27. Kd7' Qa7+' 28. Rc7' Qd4+ 29. Ke6" Qe3+' 30. Kf7' Qb3+' 31. Kg7' Qb2+' 32. Kh7' Qb4 33. Qf3+' Kh2' 34. Qg2#' 1-0.

KRBBKQ position RBB-Q – *dtm* = 66m, *dtz* = 1m, dtz_{50} = 15m:

SZ_{50}^+ – SZ⁻: 1. ... Kxc1'?? {*dtz* = 55m; White can 50m-draw} ½-½.

SZ_{50}^+ – SZ_{50}^-: 1. … Qd3+" 2. Kf4' Qd6+" 3. Kf5' Qf8+' 4. Ke4 Qe8+' 5. Kd4' Qh8+" 6. Ke4' Qh7+' 7. Ke5' Qh2+' 8. Kd5' Qg2+" 9. Kc4' Qg8+' 10. Kc5' Qf8+' 11. Kc4' Qf7+' 12. Kb4' Qb7+" 13. Kc4' Qc7+' 14. Kd4' Qf4+' 15. Kd5 Kxc1' {*dtm* = 16m} 0-1.

KRNNKQ position RNN-Q - *dtm* = 63m, *dtz* = 12m, dtz_{50} = 33m:

SZ_{50}^+ – SZ⁻: 1. … Qg4+'?? 2. Kd2" Qg2+" 3. Kc1" Qh2" 4. Rf7" Qg3" 5. Kd1 Qd3+" 6. Ke1" Ke3' 7. Re7+" Kf3" 8. Nf7" Qb1+' 9. Kd2° Qb4+" 10. Kd3" Qxe7" {*dtz* = 52m; White can 50m-draw} ½-½.

SZ_{50}^+ – SZ_{50}^-: 1. ... Qe8+" 2. Kd2' Qe3+" 3. Kc2' Qc3+" 4. Kb1' Kd3' 5. Rd7+' Ke3" 6. Re7+' Kf2' 7. Rf7+' Kg1' 8. Nb7' Qd2' 9. Rg7+' Kf1' 10. Rc7' Qb4+" 11. Ka2 Qa4+' 12. Kb2' Qd4+' 13. Kc2' Qf2+' 14. Kd1' Qe2+' 15. Kc1° Qe5 16. Rf7+ Ke1 17. Kc2 Qe3 18. Kb2 Qd3 19. Ka2 Qc3 20. Re7+ Kd2 21. Rf7 Qb4 22. Rd7+ Ke2 23. Rc7 Qb6 24. Rc2+ Kd1 25. Rb2 Qa6+ 26. Kb1 Qd3+ 27. Ka2 Qc4+ 28. Ka1 Qa4+ 29. Kb1 Qe4+ 30. Ka2 Kc1 31. Ka3 Qd3+ 32. Rb3 Qa6+ 33. Kb4 Qxb7+ {*dtm* = 30m} 34. Ka4 Qa8+ 35. Kb4 Qxh8 {*dtc* = 24m} 0-1.