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Scoring rules and abnormally low bids criteria in construction tenders: a taxonomic review

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In the global construction context, the best value or most economically advantageous tender is becoming a widespread approach for contractor selection, as an alternative to other traditional awarding criteria such as the lowest price. In these multi-attribute tenders, the owner or auctioneer solicits proposals containing both a price bid and additional technical features. Once the proposals are received, each bidder's price bid is given an economic score according to a scoring rule, generally called an economic scoring formula (ESF) and a technical score according to pre-specified criteria. Eventually, the contract is awarded to the bidder with the highest weighted overall score (economic + technical). However, economic scoring formula selection by auctioneers is invariably and paradoxically a highly intuitive process in practice, involving few theoretical or empirical considerations, despite having been considered traditionally and mistakenly as objective, due to its mathematical nature. This paper provides a taxonomic classification of a wide variety of ESFs and abnormally low bids criteria (ALBC) gathered in several countries with different tendering approaches. Practical implications concern the optimal design of price scoring rules in construction contract tenders, as well as future analyses of the effects of the ESF and ALBC on competitive bidding behaviour.

Keywords: Bidding; competitiveness; international comparison; scoring rule; tendering

Introduction

Competitive tendering[1] is the conventional method for procuring major construction projects such as building, infrastructure and shipbuilding. The need to guarantee transparency, publicity and equal opportunity in public procurement demands clear procedures to be followed by bidders (de Boer *et al.*, 2001; Falagario *et al.*, 2012) in order to reduce the risk of unfair bias or corruption (Celentani and Ganuza, 2002; Auriol, 2006).

The simplest, most transparent and effective means of doing this is by what is usually termed the traditional method, in which the contract is awarded to the lowest bidder (Waara and Bröchner, 2006; Wang *et al.*, 2006). This method provides the best motivation for project cost reduction (Bajari and Tadelis, 2001) and predominates in both public and private sectors in the United States (Art Chaovalitwongse *et al.*, 2012), Europe (Rocha de Gouveia, 2002; Bergman and Lundberg, 2013) and many other countries worldwide.

Despite its widespread use, however, the traditional lowest bid method is considered by many to be a recipe for trouble (Holt *et al.*, 1994a; Williams, 2003), especially in an oversupplied market (Hatush and Skitmore, 1998; Oviedo-Haito *et al.*, 2014). Factors such as shortage of contracts, difficulties in prescribing and measuring the quality of work, uncertainty of future costs and potential for claims, encourage a situation where the lowest bid is often not the best bid in terms of price (Hatush and Skitmore, 1998; Wong *et al.*,

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2001), time (Shr and Chen, 2003; Lambropoulos, 2007) and quality (Molenaar and Johnson, 2003; Asker and Cantillon, 2008).

In contrast with the construction industry's devotion to the traditional method (Palaneeswaran and Kumaraswamy, 2000; Wang *et al.*, 2006), selection of the best *price-quality* bidder has been promoted for a long time, with early work dating back to 1968 (Simmonds, 1968). This involves also taking non-price or *technical/quality factors* into consideration in obtaining an optimum outcome for the contracting authority, the owner or the auctioneer (Wang *et al.*, 2013), i.e. the best value for money (Holt *et al.*, 1995). For this, the auctioneer seeks to maximize the owner's value for a certain budget (price). Generally, this change of paradigm is named best value (BV) in the US (Molenaar and Johnson, 2003) and the most economically advantageous tender (MEAT or EMAT) in the EU and other parts of the world (Bergman and Lundberg, 2013).

In short, the implementation of this awarding approach requires the technical/quality and economic proposals of bidders to be scored and weighed to allow the auctioneer to rank them and identify the most economically advantageous tender. The problem lies in knowing how the economic scoring affects the bidders' aggressive/conservative behaviour (Ballesteros-Pérez, González-Cruz, Pastor-Ferrando *et al.*, 2012), the bias or unfairness of bidder ranking, or how it even facilitates collusion among competitors (Dini *et al.*, 2006). However, no attempts have been made to date to propose a unified classification of the current economic scoring rules (named here as an economic scoring formula, or ESF) that affect the bid price, to differentiate them from the technical/quality bid factors that are also scored and weighed in order to award a contract (not addressed in this study).

A clear ESF classification or taxonomy is generally a first-order requirement to homogenize ongoing research and allow future developments in almost any discipline, but most likely the countries' different paradigms concerning bidding and awarding criteria and the traditional common belief considering these rules as 'given' and 'immutable' might have had a strong influence in keeping such a unified ESF taxonomy from being effectively developed (Ballesteros-Pérez and Skitmore, 2014). Therefore, a taxonomic review is presented of the mathematical expressions for the ESF used in many countries to convert the economic component (bid price) of proposals into scores. In order to do this, a comprehensive review of several countries' bidding practices is analysed and their common features summarized into a single parametric model that includes both the ESFs themselves along with the abnormally low bids criteria (ALBC) responsible for setting a price threshold for identifying unrealistically low bids. The findings of this research will contribute to improved ESF and ALBC selection by auctioneers in the future and to expand new research, raising awareness about aspects that still need to be treated in scoring rule bidding.

In order to achieve this goal, the paper is organized as follows. The next section provides a literature review structured into two subsections. The first subsection introduces the weighted scoring method, while the second deals with the different components that comprise the scoring rules. In the following section, two important tender aspects are highlighted: the difference between the ranking and scoring rules, and the difference between capped and uncapped tenders. Later, a conceptual framework is proposed in the form of a taxonomic classification, taking into consideration the scoring parameters actually implemented by the ESF; ALBC are also analysed at this point. Finally, a discussion of the results is then included, where an effect deeply related to the ESF mathematical configuration, named *apparent or phony economic bid weighting*, is also highlighted and studied.

Literature review

Weighted scoring method

Under different denominations, most public international procurement laws and guidelines (e.g. European Union, 2004; United Nations, 2006, 2011; World Bank, 2011; EuropeAID, 2014) provide two main contract awarding approaches, namely: a price-only (lowest price) criterion or weighted multiple criteria (MEAT or BV) (Dini *et al.*, 2006). Generally, the lowest price is recommended for procurement, where the technical specifications or statement of works, as well as bill of quantities, are clear (Dini *et al.*, 2006). On the other hand, a weighted multiple criteria approach is used for more complex procurement where the evaluation requires a number of criteria other than price to be considered and balanced in order to ensure best value for money and where there are different types of scales to be used for the various elements of the offer (Dini *et al.*, 2006). For this reason, these auctions or tenders are often called multi-attribute or multidimensional.

The need for weighting and scoring economic criteria or price-related factors (e.g. life cycle costs, cost of maintenance, decommission costs) along with technical criteria (e.g. compliance, time, availability, quality) is because they are part of a mathematical expression that determines (theoretically) the best return on investment of the procurement of goods, works or services for the owner (Asker and Cantillon, 2010). Whenever a weighted scoring method is implemented, the owner, contracting authority or auctioneer must specify beforehand in the tender specifications both the criteria and the weights with which the bidders' proposals will be evaluated. As a general rule, weighted scoring methods can be expressed as:

$$O_i = \{W_e \cdot S_i + W_t \cdot T_i\}\, \delta_i \quad (1)$$

where:

O_i is the overall score achieved by bidder i (with $i = 1, 2, \ldots, N$ bidders) in a tender.

W_e is the weight of the economic criteria for tenders for similar projects. In general, W_e is pre-set by the auctioneer within $0 \le W_e \le 1$. When $W_e = 1$, the tender is awarded to the lowest price bidder.

S_i is bidder i's economic score that is calculated according to bidder i's submitted economic bid and by means of the ESF pre-set by the auctioneer. For the sake of simplicity, it is assumed here that $0 \le S_i \le 1$, but this variable is also usually expressed by a score, for example $0 \le S_i \le 100$ points.

W_t is the weight of the technical criteria. In general, W_t is also pre-set by the auctioneer and, since whenever there are no special tender requirements $W_t = 1 - W_e$, it is also the case that $0 \le W_t \le 1$. Analogously, when $W_t = 1$ the tender is awarded exclusively according to the technical criteria; these tenders are sometimes called *beauty contests* (Bergman and Lundberg, 2013).

T_i is bidder i's technical score that is calculated according to a set of rules, scales or rates for the different attributes that interest the owner or auctioneer. Again, it is assumed that $0 \le T_i \le 1$, but this variable can also be expressed as the sum of several technical and/or quality aspects that are also usually scored in points.

δ_i is an abnormality index that equals 1 when bidder i's bid is above (more expensive) than the threshold defined by the ALBC, allowing the bidder to compete, and which equals 0 if this condition is not fulfilled. Whenever $\delta_i = 0$, bidder i's bid is cheaper than the ALBC or, in other words, unrealistically low and, therefore, disqualified. δ_i is calculated according to another mathematical expression, named the ALBC, which is generally independent of the ESF.

Components of the scoring rules

Having defined mathematically the weighted scoring methods, there are four aspects that can be analysed: (a) the way the economic score is calculated (variable S_i, i.e., the ESF); (b) the way the technical score is calculated (variable T_i); (c) the way the weights are set (relative importance of variables W_e and W_t to each other or even the sub-weights within each economic and technical proposal); and, finally, (d) how the ALBC are defined (variable δ_i). This study will focus later only on the ESF and ALBC (variables S_i and δ_i).

To date, many researchers have dealt with defining the technical factors, T_i, to be taken into consideration in BV/MEAT selection (e.g. Holt *et al.*, 1994a, 1994b; Palaneeswaran and Kumaraswamy, 2000; Shen *et al.*, 2004; Waara and Bröchner, 2006).

With regard to the economic and technical weight values (variables W_e and W_t), the most common approach is the linear weighting method (European Union, 2004), where the auctioneer assigns a weight to each criterion in advance. Considered in this way, the issue then becomes one of solving a multi-criteria decision-making problem concerning the weights of several factors (Holt *et al.*, 1994c; Hatush and Skitmore, 1998; Pongpeng and Liston, 2003; Wang *et al.*, 2013). Furthermore, Jennings and Holt (1998) define multi-criteria decision-making as a 'selection based on evaluation of tender submissions against criteria predetermined by auctioneers and considered important by them in terms of achieving successful project completion'. Additional techniques have been applied by other researchers, including multi-attribute analysis (Holt *et al.*, 1994b, 1994c), the analytic hierarchy process (Pastor-Ferrando *et al.*, 2010; Wang *et al.*, 2013), fuzzy sets (Nieto-Morote and Ruz-Vila, 2012), case-based reasoning models (Dikmen *et al.*, 2007), neural networks (Art Chaovalitwongse *et al.*, 2012), and data envelopment analysis (Falagario *et al.*, 2012).

However, despite the extensive scientific literature focused on ensuring the best balance of economic and technical weights, the weights to be disclosed in requests for proposals are still currently based on subjective judgments (Lorentziadis, 2010). Fixed criterion weights ensure objectivity and reduce the risk of unfairness and corruption in the evaluation of bidders' proposals, but only provided they accurately reflect the relative importance of the evaluation factors to the owner. However, it is still possible to create an unfair evaluation system in which too much emphasis is placed on particular evaluation factors, thus favouring (intentionally or unintentionally) those bidders that score highly in the corresponding factors (Lorentziadis, 2010). When weights are subjectively set and fixed before the bid process, the evaluation system is said to correspond to a pre-subjective input model (Pongpeng and Liston, 2003).

Two multi-attribute auction variables remain to be addressed: the economic scoring formula (ESF, variable S_i) and the abnormally low bids criteria (ALBC, variable δ_i) which are the main concern of this paper for, as will be seen later, they can also significantly influence previous variables (T_i, W_e and W_t).

The ESF, as already mentioned, is used to translate the bid prices proposed by the bidders into economic scores (Ballesteros-Pérez, González-Cruz, and Cañavate-Grimal, 2012). Auctioneers tend to use similar or identical ESFs for all their projects but different auctioneers use different ESFs. ESFs also differ between countries. Waara and Bröchner (2006) and Fuentes-Bargues *et al.* (2014), for example, report a variety of different ESFs used by Swedish municipalities and Spanish public agencies. Nevertheless, in the highly competitive world of construction bidding, the ESF chosen is likely to have significant consequences on the outcome of the auction in terms of aggressiveness (very low bids to win the auction) or conservativeness (higher bids to avoid being disqualified as being unrealistic) of bidders and the outcome of the project (Palaneeswaran and Kumaraswamy, 2001).

However, very little is known of the relationship between ESFs and other multiple aspects of bidding behaviour. Consequently, ESF selection by auctioneers in practice is invariably a highly intuitive and subjective process (Holt *et al.*, 1994b, 1994c), involving few theoretical or empirical considerations. This produces scoring rules in practice that are often poorly designed (Bergman and Lundberg, 2013) and affected by internal consistency and validity problems (Borcherding *et al.*, 1991); this situation is unfortunately shared with other tender documents and leads to cost estimate inaccuracy, claims and disputes (Laryea, 2011).

Therefore, despite the extensive research on competitive bidding over the years (Holt, 2010; Oo *et al.*, 2010), ESF selection is a relatively unresearched area. With very few exceptions, such as Asker and Cantillon (2008, 2010), there is a paucity of research that bridges the gap between the theoretical analyses of abstract scoring rules and their practical application in procurement practice (Bergman and Lundberg, 2013).

Likewise, unrealistically low bids have also received very little attention in the literature to date (Chao and Liou, 2007; Ballesteros-Pérez *et al.*, 2013b). However, when we refer to abnormally low bids criteria (ALBC), we are not focusing on analysing the reason or even the features of bids considered too low to be acceptable. Instead, we refer to how the auctioneer defines mathematically, before receiving the bids, the value below which every bidder will be *objectively* disqualified when submitting a cheaper bid (Ballesteros-Pérez, González-Cruz, Pastor-Ferrando *et al.*, 2012). For example, some countries define abnormally low bids by the arithmetic deviation from the average bid (International Chamber of Commerce, 2000), even though there is no assurance that such methods accurately identify an actually unrealistically low bid (European Union, 2002; Chotibhongs and Arditi, 2012).

On the other hand, many attempts have been made to propose objective statistical methods to determine the threshold below (or above) which a bid is considered to be abnormal. The problem is that all these methods are useful *ex post* (after the tender deadline, and therefore not included in the tender specifications). Since everyone acknowledges that statistical methods are open to error and distortion, no successful (objective and indisputable) solution has been found so far (European Union, 2002).

Therefore, the definition of ALBC here only attempts to draw a line that will disqualify low bids; it does not intend to deal with auction rules to discourage collusion, as discussed in depth in the scientific literature (Che and Kim, 2006, 2009; Chowdhury, 2008). ALBC are not always present, but the narrower they are, the more conservative the bids become in order to avoid being disqualified (Ballesteros-Pérez, González-Cruz, and Cañavate-Grimal, 2012; Ballesteros-Pérez *et al.*, 2013a). According to the specifications and procurement guidelines studied, the largest difference between countries lies in ALBC values used.

Therefore, in addressing the problem of ESF and ALBC selection, a conceptual framework in the form of a taxonomic classification for both variables in construction auctions is first proposed, followed by some insights into its use. It is anticipated, therefore, that the findings of this research will contribute to improved ESF and ALBC selection by auctioneers in the future and to expand new research, raising awareness of the aspects still in need of treatment in the bidding scoring rule domain.

Economic scoring formula (ESF) taxonomy

In order to create an ESF taxonomy, several notation and methodological aspects need to be addressed to homogenize current knowledge of these scoring rules.

First, a clear difference between a ranking and a scoring rule needs to be established. Ranking rules are used whenever the only awarding criterion is the price, whereas scoring rules are required in multi-attribute tenders to be able to combine their technical and economic components. Mathematical expressions are necessary for the latter kind of rules when it comes to converting the bid values into scores, which is the reason the approach taken is eminently mathematical.

Second, the difference between capped and uncapped tenders needs to be recognized. This involves the setting (capped) or not (uncapped) of a maximum price for bids. It is important to distinguish between these two common bidding approaches as bidders behave differently in each of them, mainly because the ESFs and ALBC are also mathematically different.

Third, a brief explanation is given just before the ESF taxonomy proposal about the international tender sources that allowed the study and review of a varied array of tender specifications, as well as national and international public procurement economic scoring methods. This aims to show that both the ESF and ALBC taxonomies are not arbitrary, but based on real-life and representative samples.

Fourth, a taxonomy is finally proposed in terms of the variables contained in their mathematical expressions, the so-called scoring parameters (SPs), as these are the only common trait shared across ESFs and ALBC.

Fifth, the interrelationships among SPs in capped and uncapped tenders need to be studied, in order to understand why differences in subsequent bidding behaviour are likely to be due to the implementation of different combinations of SPs in the ESFs and ALBC.

Finally, a brief note is given on how ESFs and ALBC can be represented and that some of their features are better understood graphically when expressed as a function of one of their SPs.

Ranking versus scoring rules

When price is the sole criterion in awarding a contract, there is no need to score the bids, since the auctioneer is only interested in ordering or ranking the bids received in terms of their value. There are many *ranking rules*, including:

- Lowest price, which is the most common in construction procurement (Palaneeswaran and Kumaraswamy, 2000).
- Average bid method, in which the awarded bid is the closest to the average bid of all the bid prices for a project (Rocha de Gouveia, 2002).
- Below-average bid method, where the closest to but less than the average bid wins the project (Ioannou and Awwad, 2010).
- Truncated average bid or bid-spread method, where the winning bid is defined as the closest to the average computed after excluding outliers (Waara and Bröchner, 2006).

However, a rank is not enough whenever bid prices are combined with technical criteria, and an ESF is needed to translate a bid price into a numerical score. These latter mathematical expressions form the basis of the taxonomy.

Capped versus uncapped tenders

In general, two dominant approaches concerning the price boundaries are identified: capped and uncapped tenders. In uncapped tenders, a bidder i submits an economic bid (b_i) which can range from 0 to + ∞, unless ALBC are implemented. Conversely, in capped tenders, a bidder i submits a bid that is upper bounded (in price) by the auctioneer and therefore has no option but to equal or underbid this pre-set tender amount (A). Bids can therefore range from 0 to A, unless ALBC are implemented. Capped tenders also exhibit the property that bids can be expressed in discounts or drops (d_i) off A, i.e. a bidder i's bid can be expressed as:

$$d_i = 1 - \frac{b_i}{A} \quad \text{or} \quad b_i = (1 - d_i)A \tag{2}$$

Therefore, these discounts or drops can range from 0 to 1 in capped tenders. In addition, for clarification, the pre-set maximum economic tender amount (A), is sometimes called the *ceiling price* in the literature, whereas the term *reserve price* is identified with ALBC only if stated in the tender specifications (Chowdhury, 2008). Finally, as will be emphasized later, the most important difference between capped and uncapped tenders, beyond the way the bids are expressed, is that their respective 'scoring parameters' (variables to be introduced later that configure the ESF and ALBC mathematical expressions) behave in different ways.

Existing tender practices

The main goal of the current study is to propose an ESF and ALBC taxonomy, as both ESFs and ALBC constitute the two major components of the economic bid score (variables S_i and δ_i). In order to achieve this, a wide range of ESFs and ALBC in current practice are needed to identify their common features. However, the economic and technical bid weightings that are normally used with ESFs and ALBC (W_e and W_t respectively) are also available for use in identifying shared bidding behaviour trends across countries, and from which the *apparent or phony bid weighting* phenomenon was deduced as explained later in the Discussion section.

Therefore, in the first instance, a thorough review of tender specifications and national and international public procurement methods was made. This review consisted primarily of the compilation of ESFs and ALBC implemented by contracting authorities or supranational entities (EU and some multilateral agencies) in various countries since, by registering those mathematical criteria it was possible to find common traits, especially among the scoring parameters.

Discipline-related books, several international agencies commissioned reports as well as specific scientific publications also provided very valuable information and these were supplemented by real tendering data provided by multiple international construction contractors working in a wide range of countries.

In terms of books and reports, Ballesteros-Pérez and Skitmore (2014) provide a wide survey of ESFs used in Spain. Waara and Bröchner (2006) and Fuentes-Bargues *et al.* (2014) cover Swedish and Spanish ESFs respectively currently in use by contracting authorities. Del Caño-Gochi *et al.* (2008) analyse and compile the most common procurement approaches and awarding criteria in France, the United States, United Kingdom and Japan. Palaneeswaran and Kumaraswamy (2000) describe a range of different economic factors and systems still in use by public agencies in the United States, Canada and Hong Kong. The European Union (2002) sets a common framework with examples of how each country has customized ESFs and ALBC according to its needs. Furthermore, multilateral agencies' procurement guidelines, such as those of the World Bank (2011), United Nations (2006), EuropeAID (2014) and the Organisation for Economic Co-operation and Development (2009) were reviewed.

Finally, we obtained a variety of examples of datasets of tender specifications and results from several international construction contractors in countries as diverse as Mexico, Chile, Peru, Colombia, Argentina, Algiers, Morocco, Oman, Egypt, Turkey, Romania, Bulgaria, Australia, New Zealand and China. These tender specifications and bidding results also served the secondary purpose of the study, which was to determine the extent to which particular ESF and ALBC configurations forced bidders to behave in predictable ways.

ESF taxonomy proposal

The ESFs are mathematical expressions used to assign numerical scores (S_i) to each bidder i's bid price. However, these mathematical expressions commonly make use of other sub-variables for converting the price into a score. These sub-variables or scoring parameters (SPs) are usually calculated as a function of the final distribution of bids (Ballesteros-Pérez, González-Cruz, Pastor-Ferrando *et al.*, 2012).

In uncapped tenders, the primary SPs are: the minimum bid ($b_{\min}$), which corresponds to the lowest bid; the maximum bid ($b_{\max}$), which corresponds to the highest bid, the average bid (b_m), which corresponds to the average of all bids submitted, and, even though it is uncommon to find it as a variable within an ESF, the number of bidders (N) (Ballesteros-Pérez and Skitmore, 2014). As an example, an ESF that gives the maximum score (1) to the lowest bidder, i.e. $S_{(1)} = 1$, and the minimum score (generally 0) to the most expensive bidder, i.e. $S_{(N)} = 0$, would be written as:

$$S_i = \frac{b_{\max} - b_i}{b_{\max} - b_{\min}}$$

In capped tenders, the primary SPs are the same, but expressed in discounts or drops, that is: the maximum drop ($d_{\max}$) corresponds to the lowest bid; the minimum drop ($d_{\min}$) corresponds to the highest bid; the average drop (d_m) corresponds to the average of all bids (expressed in drops) submitted; and, again, the number of bidders (N). The ESF example above can therefore be equally expressed in drops whenever there is a tender amount (A) as

$$S_i = \frac{d_i - d_{\min}}{d_{\max} - d_{\min}}$$

Apart from the primary SPs, other frequently used measures include the standard deviation of the bids/-drops (s in uncapped tenders and σ in capped tenders) (Ballesteros-Pérez, González-Cruz, Pastor-Ferrando *et al.*, 2012).

As a result, although ESFs may or may not use a SP, in most cases they use at least one SP. Many ESFs were identified in the aforementioned tender specifications and national and international public procurement review. Classifying all these ESFs is similar to classifying different kinds of equations found in mathematics. Therefore, it was considered that the best way to create the taxonomic review was to classify the ESFs according to the SP they actually implemented. The result is shown in Figure 1.

As can be seen, full, dotted and dashed lines represent many combinations of specific mathematical expressions that ESFs may use to assign economic scores to bids. As will also be noted later, the selection of the SPs to be used by each ESF is not trivial and has immediate repercussions on bidders' competitiveness.

Scoring parameter (SP) relationships

To understand how an ESF may produce effects on competitive behaviour, it is necessary to first understand how the SPs actually behave and how they are interconnected. In doing this, several studies have recently made significant advances. Of these, Ballesteros-Pérez, González-Cruz, and Cañavate-Grimal (2012) first proposed a set of equations (specified later in Table 2) that relate each SP to each other in capped tenders with average curve shapes depicted at the bottom in Figure 2 as a function of the SP mean drop d_m. These curved trajectories seem quite logical, taking into account the two boundary price conditions of capped tenders (represented with symbol ◎ in the graph). These types of tenders are upper-limited by A and below by 0, so that, if expressed in drops, bids are $0 \le d_i \le 1$. These particular boundaries force the SP to coincide at points $(0, 0)$ and $(1, 1)$, with the exception of σ at $(1, 0)$.

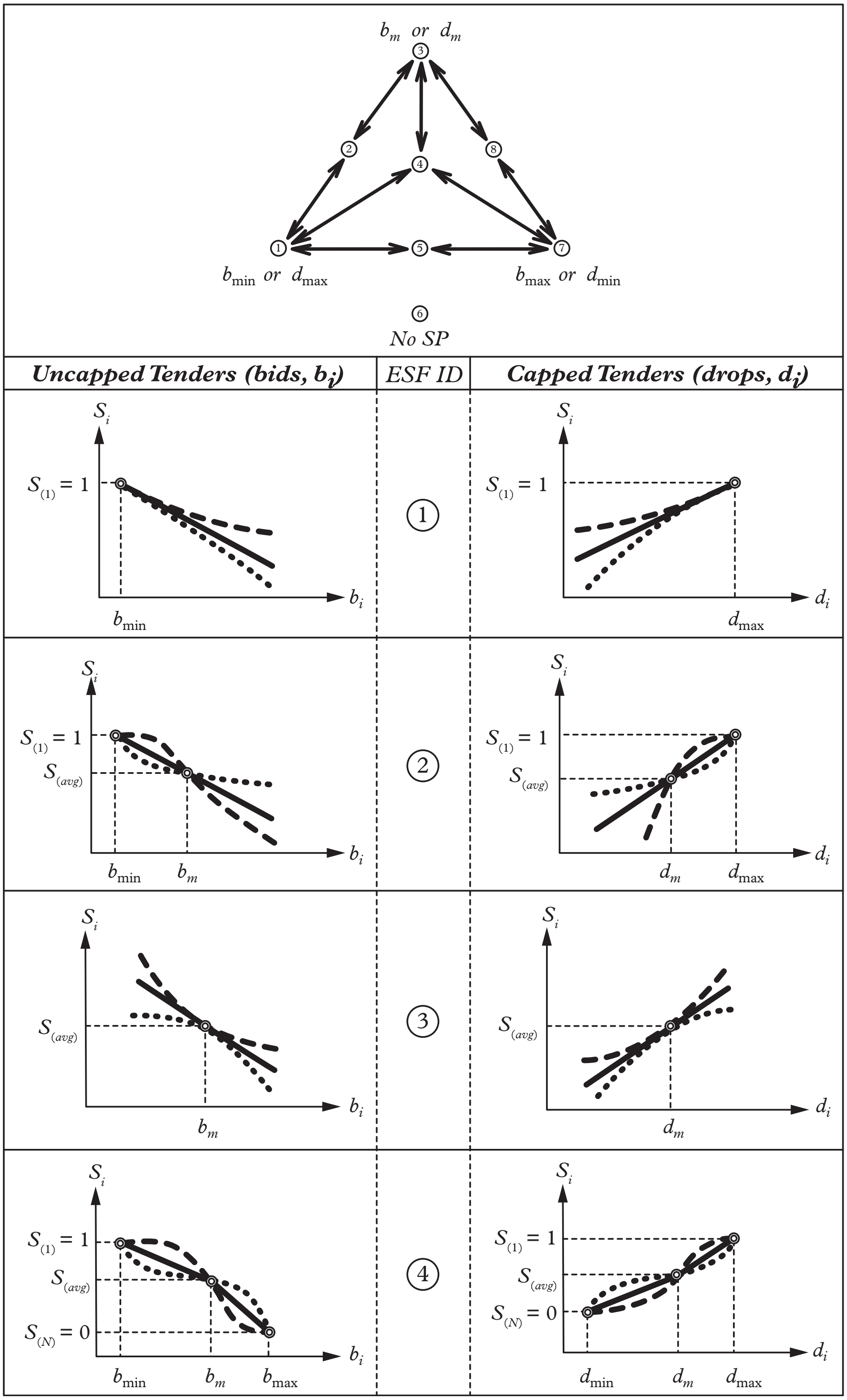
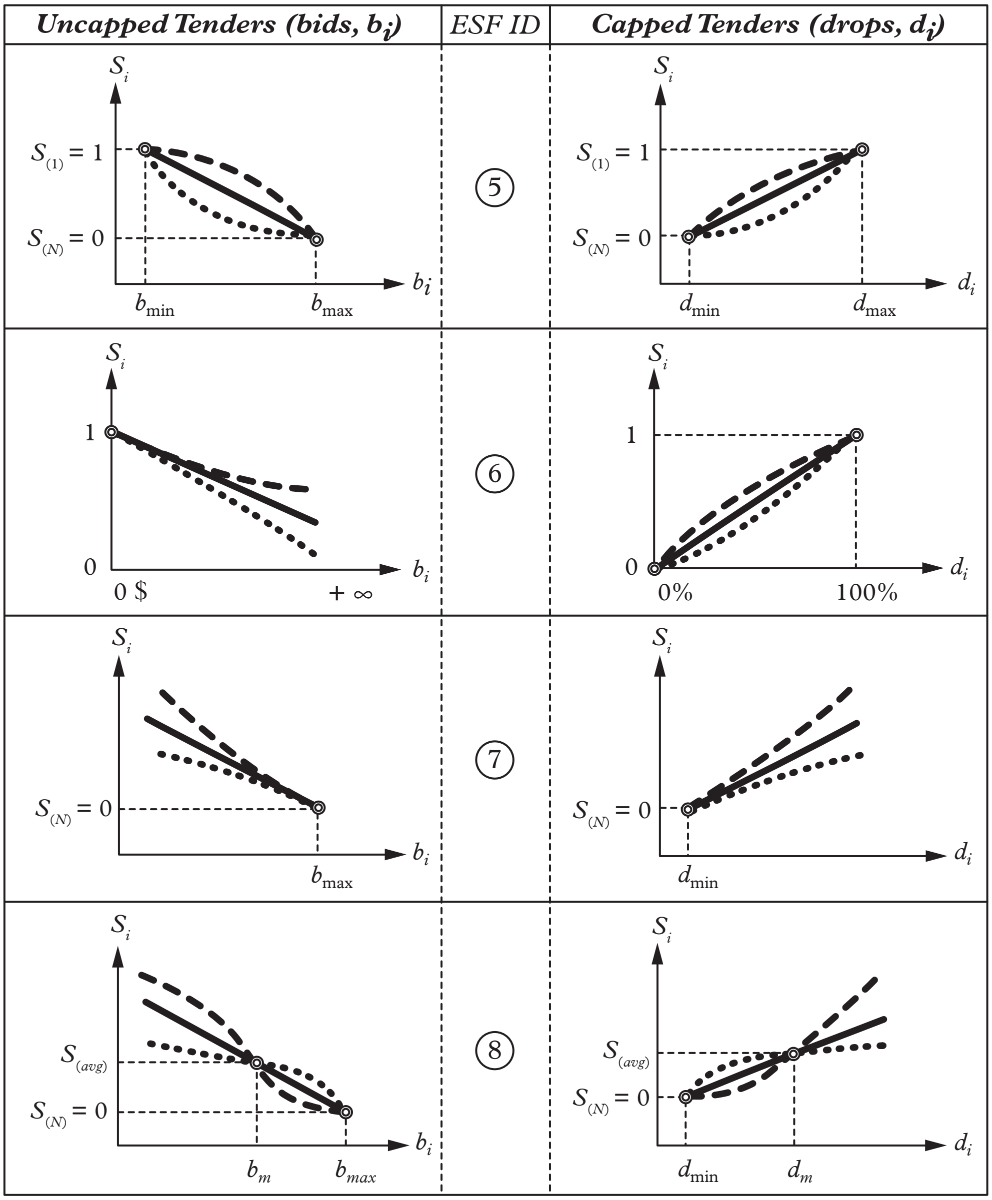


Figure 1 Economic scoring formulae taxonomy as a function of their scoring parameters



Primary Scoring Parameters (*SPs*)

- b_{min} : Minimum bid (lowest bid)
- b_m : Mean bid (average bid)
- b_{max} : Maximum bid (highest bid)
- d_{max} : Maximum drop (lowest bid)
- d_m : Mean drop (average bid)
- d_{min} : Minimum drop (highest bid)

Secondary SPs, such as **N and **s** or σ, have not been considered for the sake of simplicity*

Independent variables (*X-axis*)

- b_i : Monetary bid (in uncapped tenders)
- d_i : Per-unit drop(in capped tenders)

Dependent variables (*Y-axis*)

- S_i : Bidder *i*'s economic score calculated by the Economic Scoring Formula
- $S_{(1)}$: Maximum Score (generally assigned to the lowest bidder and equals 1)
- $S_{(avg)}$: Score assigned to the mean bid b_m (generally not equals 0.5)
- $S_{(N)}$: Minimum Score (generally assigned to the highest bidder and equals 0)

Economic Scoring Rule Curves

- (solid line) Linear / bi-linear criteria
- (dashed line) / (dotted line) Curvilinear criteria

Figure 1 (*Continued*)

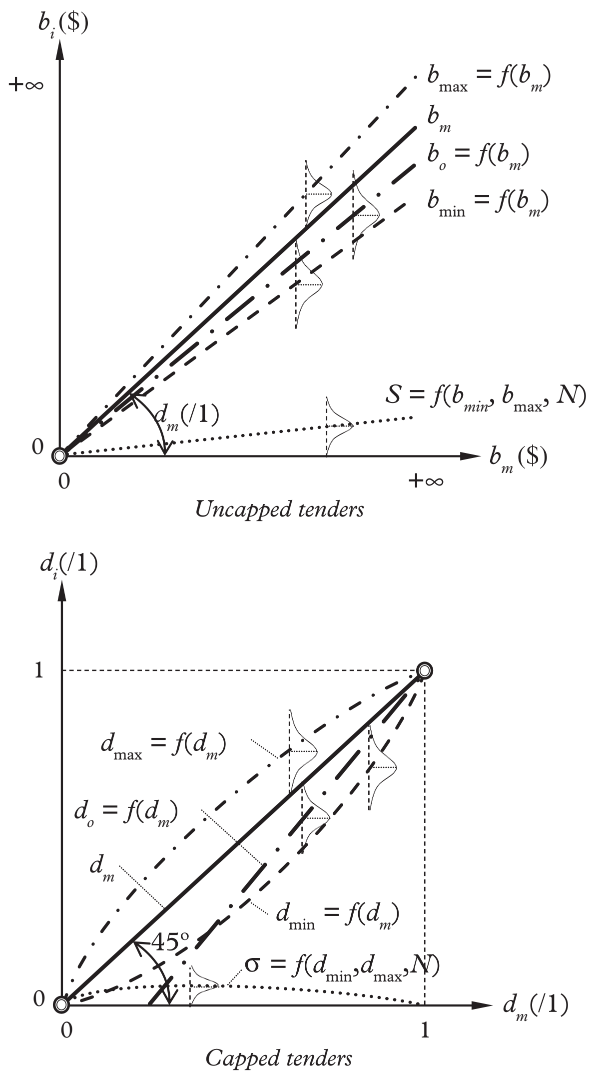


Figure 2 Major scoring parameter (SP) relationships in capped and uncapped tenders

By understanding the capped SP relationships, it is easy to obtain the uncapped SP relationships too by means of the graph at the top. This is of course a simpler case with only one boundary condition, which is shared with the graph as represented by symbol ◎. Therefore the SP in uncapped tenders should follow the linear relationships depicted at the top of Figure 2. These relationships are not deterministic since SPs have statistical variation around their average curves.

However, despite seeming logical, the uncapped SP relationships inferred require a demonstration. In order to do so, Ballesteros-Pérez's (2010) actual uncapped construction tender database is used. This dataset comprises 45 tenders of design, build and operation of waste water treatment plants and sewage systems contracts from northern Spain awarded between 2007 and 2008. The dataset includes all bidders' bids from which calculating the SPs mean bid (b_m), minimum bid ($b_{\min}$), maximum bid ($b_{\max}$) and the standard deviation of bids (s) is straightforward. The dataset also includes one bidder's cost estimates (b_o) for 14 tenders.

The most representative results of the SP curve calculations can be seen in Figure 3 and Table 1 along with the coefficients of determination (R^2). R^2 values close to 1 confirm that the SPs' relationships deduced from the capped tender case point in the right direction.

Furthermore, it is emphasized that the curves depicted in Figures 2 and 3 are expressed as a function of some regression parameters: named a, b and c in uncapped tenders, and α, β and γ in capped tenders. Therefore, by analysing the variation of these regression parameters over time, it is possible to study how aggressively or conservatively the bidders bid in a particular context: with the same ESF and ALBC for instance, or even according to a country's particular economic situation.

Additional details of how these regression parameters are calculated when a number of n tenders is analysed for capped tenders can be found in Ballesteros-Pérez and Skitmore (2014) and summarized for the first time for both capped and uncapped tenders in Table 2.

This Table, despite representing a collective model (i.e., not taking into account the bidders' identities), provides an important step towards understanding both the ESF and the way bidders behave in a particular tender.

ESF graphical representation

In order to finish describing the most representative features of an ESF it is worth mentioning that ESFs can be represented in several different ways. The first, which could be called the classic way, consists of representing the ESF variation in a graph with axes expressed in bids b_i or drops d_i (X-axis) and score S_i (Y-axis). This is the kind of representation chosen for the 16 graphs shown in Figure 1.

Another recent approach to represent an ESF is by iso-Score Curve Graphs (iSCG) (Ballesteros-Pérez, González-Cruz, Pastor-Ferrando *et al.*, 2012) in which the X-axis usually represents one of the SPs, the Y-axis represents any bidder's bid or drop (b_i or d_i), while the curves represent the combination of (X, Y) points in which the ESF provides the same level of score to a bidder's bid or drop.

These iSCG have the advantage of showing the whole picture of how any ESF reacts as a function of both the SPs themselves and as a function of the bidders' past encounters, which suggests applications in competitive bidding issues again and a new way to interpret ESF effects on bidding behaviour.

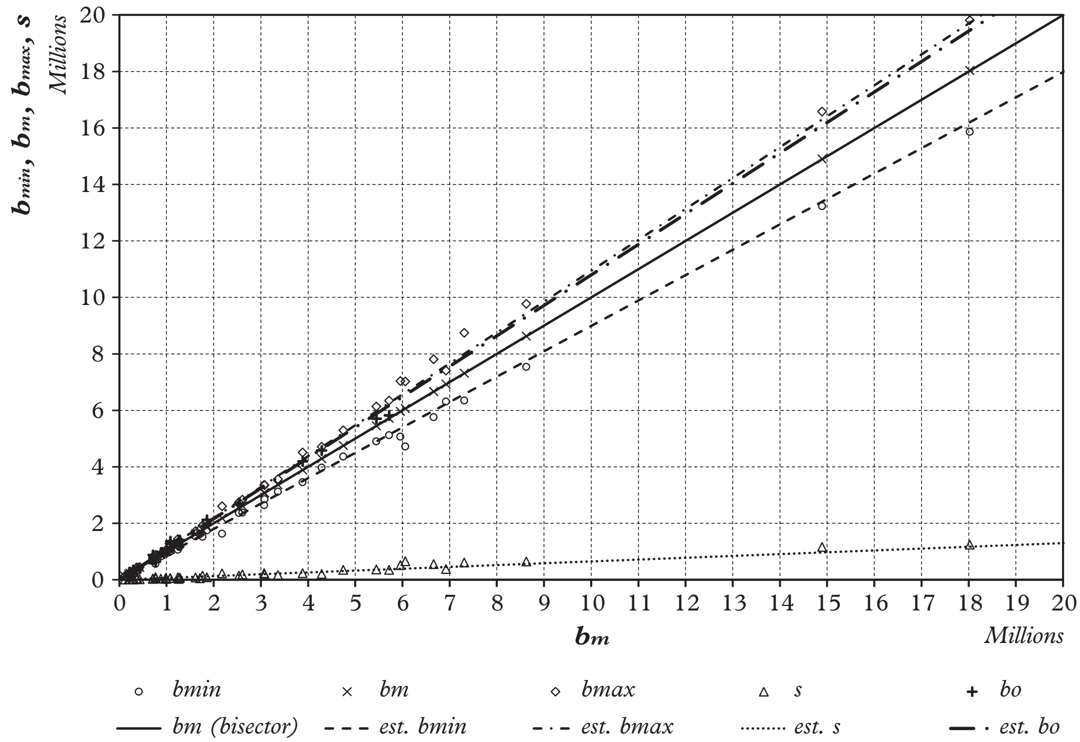


Figure 3 Scoring parameter (SP) relationships in an uncapped construction tender dataset

Abnormally low bids criteria (ALBC) taxonomy

In parallel with reviewing the prominent features of the ESF and its parameters, the ALBC expressions were also analysed. ALBC have the task of setting a cut-off bid (b_{abn}) or drop (d_{abn}) that disqualifies any bidder whose bid is cheaper (unless the bidder is capable of justifying this price (European Union, 2004)).

There are several existing systems in use by many countries that are intended to detect abnormally low bids. The most recurring example essentially consists of arithmetic systems that measure the deviation of a particular bid from the average of all bids submitted, with minor differences in the percentage and/or calculation of the average (for instance Belgium, France, Italy, Portugal, Spain and Greece use ranges mostly varying between 10% and 15%) (European Union, 1999). However, as the EU Commission reports (European Union, 1999), there is to date no systematic method that enables the effective evaluation of ALBC in EMAT or BV auctions, since the systems currently in use are recognized to be of limited efficacy.

Of the tender specifications analysed, six generic ALBC were identified. Some are applicable to capped tenders only and others apply to both capped and uncapped tenders. Basically, there are two groups of ALBC: those that make use of a SP (only cases of b_m/d_m, s/σ and N have been found), and those that do not make use of any SP and, therefore, the cut-off limit does not depend on the final bid distribution. In these ALBC, the cut-off economic limit can be known in advance, that is, before the tender deadline. This also happens with the ESF: whenever no SP is used (case 6 in Figure 1), the ESF is totally predictable and unmovable, no matter what final bids are submitted.

The six ALBC, the first four of which are expressed as a function of one SP and the last two as a function of no SP whatsoever, are then:

$b_{abn} = (1 - \varepsilon)\, b_m$ Possible in both capped and uncapped tenders. Basically, it is the most common criterion in EU countries, with a parameter ε that is usually set between 0.05 and 0.20. Any bid that fulfils the condition $b_i < b_{abn}$ will be ruled out as inadmissible

$d_{abn} = (1 + \theta)\, d_m$ Possible in capped tenders only. This uses a multiple of the average drop such that all bidders with a higher drop ($d_i > d_{abn}$) will be not considered. Parameter θ also usually ranges between 0.05

Table 1 Scoring parameter relationship calculations for the uncapped construction tender dataset

Actual scoring parameter (SP) values from the original 45 tenders							*Table 2 (2nd column)*			*Calculated according to Table 2 (1st column)*				
ID	b_o	b_{min}	b_m	b_{max}	s	N	a	b	c	**ID**	est. b_{min}	est. b_{max}	est. s	est. b_o
1	5702262	4897082	5450186	6131465	361268	13	0.899	1.125	1.046	1	4901406	5965979	415725	5884521
2	714225	724993	775167	818247	48387	4	0.935	1.056	0.921	2	697116	848527	44867	836942
3		264537	287515	307227	14807	14	0.920	1.069		3	258565	314724	14219	
4		115979	122657	126270	5790	3	0.946	1.029		4	110306	134265	5942	
5	1365910	1012084	1082862	1170446	57705	11	0.935	1.081	1.261	5	973828	1185342	54858	1169157
6	879023	685135	713584	744935	27504	5	0.960	1.044	1.232	6	641733	781115	25894	770450
7	965533	820075	883144	916683	36662	6	0.929	1.038	1.093	7	794220	966722	39044	953523
8	4190581	3449084	3885370	4501876	233182	22	0.888	1.159	1.079	8	3494151	4253072	332859	4195002
9		2845414	3080428	3347952	191483	10	0.924	1.087		9	2770259	3371952	177308	
10	1276480	989040	1077819	1187773	60353	25	0.918	1.102	1.184	10	969293	1179821	62150	1163712
11		196663	229253	248081	16976	11	0.858	1.082		11	206170	250949	17812	
12		5066621	5959673	7030823	517085	43	0.850	1.180		12	5359592	6523682	594017	
13		5752980	6664934	7803728	552845	43	0.863	1.171		13	5993840	7295687	620190	
14		13229417	14894721	16578018	1158373	12	0.888	1.113		14	13394968	16304321	1142414	
15		15854287	18027863	19811686	1239358	23	0.879	1.099		15	16212633	19733976	1246257	
16		7532621	8625632	9768072	648213	39	0.873	1.132		16	7757115	9441941	679283	
17		323376	364765	416617	24482	18	0.887	1.142		17	328037	399285	30083	
18	2119811	1739817	1856797	2045096	86955	14	0.937	1.101	1.142	18	1669836	2032520	101684	2004768
19		3115326	3363910	3556584	166633	6	0.926	1.057		19	3025197	3682262	178333	
20		4715326	6062617	7016838	655983	45	0.778	1.157		20	5452171	6636369	694589	
21	1398605	1185908	1286376	1363499	63557	7	0.922	1.060	1.087	21	1156850	1408115	68355	1388889
22		1153250	1275801	1379826	60650	19	0.904	1.082		22	1147341	1396540	72674	
23	1391729	1220847	1260234	1296154	31301	4	0.969	1.029	1.104	23	1133341	1379500	36232	1360665
24	4578381	3963763	4289341	4711201	200837	22	0.924	1.098	1.067	24	3857446	4695274	236316	4631166
25		1626951	2179868	2602736	224859	25	0.746	1.194		25	1960377	2386166	305158	
26		402918	433576	449665	14387	9	0.929	1.037		26	389919	474608	16868	
27	5822694	5114302	5719436	6343885	347329	16	0.894	1.109	1.018	27	5143545	6260710	402277	6175228
28		6310437	6927068	7406163	374692	10	0.911	1.069		28	6229580	7582628	386600	
29		4361422	4749403	5291885	348740	9	0.918	1.114		29	4271185	5198875	335752	
30		6344781	7312009	8743060	608123	27	0.868	1.196		30	6575761	8004000	745579	
31		918054	987694	1043243	63773	3	0.929	1.056		31	888242	1081166	72278	
32		1534762	1616923	1718210	75920	6	0.949	1.063		32	1454115	1769945	74140	
33	2676422	2357470	2536349	2744372	145192	9	0.929	1.082	1.055	33	2280964	2776383	139612	2738476
34	636369	555494	771586	878669	91272	9	0.720	1.139	0.825	34	693895	844607	116616	833075
35		263641	277774	287344	8700	5	0.949	1.034		35	249805	304062	10263	
36		2634847	3071591	3359343	232485	9	0.858	1.094		36	2762312	3362279	261430	
37		975812	1002202	1036119	30850	3	0.974	1.034		37	901290	1097048	34818	
38		305270	380479	432013	39886	12	0.802	1.135		38	342169	416487	43240	

(Continued)

Table 1 (Continued)

Actual scoring parameter (SP) values from the original 45 tenders						*Table 2 (2nd column)*			*Calculated according to Table 2 (1st column)*				
ID	b_o	b_{min}	b_m	b_{max}	s	N	a	b	c	**ID**	est. b_{min}	est. b_{max}	est. s	est. b_o
39		192669	201402	209590	5992	5	0.957	1.041		39	181123	220463	7327	
40		653260	713573	816069	45191	12	0.915	1.144		40	641724	781104	55544	
41		1612924	1708144	1778198	48210	8	0.944	1.041		41	1536150	1869798	61342	
42		1059343	1243280	1439521	89446	20	0.852	1.158		42	1118094	1360941	121300	
43		1520757	1764662	1928426	144439	11	0.862	1.093		43	1586978	1931665	141221	
44		289842	308259	323608	15109	4	0.940	1.050		44	277221	337432	16246	
45		2378881	2612412	2831442	189587	5	0.911	1.084		45	2349368	2859645	195965	
					Average=	14.133	0.899	1.095	1.080	***R=***	**0.998**	**0.999**	**0.989**	**0.996**

and 0.20. Perhaps, as found many times in the literature, it is interesting to point out that, whenever the expression of d_{abn} comes from the translation of the previous ALBC as a function of ε, then $d_{abn} = 1 - (1 + \varepsilon)(1 - d_m)$

$b_{abn} = b_m - \lambda s$ — Possible in both uncapped (under the expression on the left) and uncapped tenders (under the translated expression $d_{abn} = d_m + \lambda \sigma$). It sets a threshold in bid or drop standard deviation multiples, beyond which all bidders are disqualified. Parameter λ usually ranges between 0.5 and 2

$N_{abn} = (1 - \mu)\,\frac{N}{2}$ — Possible for capped and uncapped tenders. Basically, this criterion directly eliminates a proportion μ of bidders just for being located at the extremes (in one or in both extremes lowest and highest). μ usually ranges between 0.05 and 0.25. Finally, there is another variation of this criterion by which a pre-set number of bidders ($N_{abn} = \eta$) is disqualified (no matter how many bidders are actually competing)

$b_{abn} = \omega$ — Useful for capped and uncapped tenders. This makes no use of SP so it is a deterministic cut-off limit for a particular economic amount the auctioneer considers too low to be acceptable. As happens with the rest of ALBC expressions, this limit has to be included in the tender specifications, otherwise it does not comply with the principles of transparency, publicity and equality of opportunity. Parameter ω is generally chosen depending on the particular tender economic volume and/or the engineer's estimate

$d_{abn} = \delta$ — Similar to the previous ALBC, but only applicable for capped tenders. This sets a drop value above which any bidder's drop will be disqualified. Parameter δ is generally set within the range 0.10 to 0.30

Table 2 Mathematical relationships of scoring parameters (SP) in capped and uncapped tenders

Uncapped Tenders		*Capped Tenders*	
Mean (average) bid b_m $0 \le b_m \le +\infty$	$b_i = (1 - d_i) A$ (whenever A exists) *Regression coefficients*	Mean (average) drop d_m $0 \le d_m \le 1$	$d_i = 1 - \frac{b_i}{A}$ *Regression coefficients*
Minimum (lowest) bid $b_{\min}$ $est\ b_{\min} = a \cdot b_m$	$a = \frac{1}{n} \sum_{k=1}^{k=n} \frac{b_{\min k}}{b_{mk}}$ $0 \le a \le 1$ (bid aggressiveness bid conservativeness) (bid dispersion bid concentration)	Maximum drop (lowest bid) $d_{\max}$ $est\ d_{\max} = b_m^{\alpha}$ (potential expression) (Parabolic relationship expressions are also found in Ballesteros-Pérez et al. (2012a))	$\alpha = \frac{1}{n} \sum_{k=1}^{k=n} \frac{LN\ d_{\max k}}{LN\ d_{mk}}$ $0 \le \alpha \le 1$ (bid aggressiveness bid conservativeness) (bid dispersion bid concentration)
Maximum (highest) bid $b_{\max}$ $est\ b_{\max} = b \cdot b_m$	$b = \frac{1}{n} \sum_{k=1}^{k=n} \frac{b_{\max k}}{b_{mk}}$ $1 \le b \le +\infty$ (bid concentration bid dispersion)	Minimum drop (highest bid) $d_{\min}$ $est\ d_{\min} = b_m^{\beta}$ (potential expression) (Parabolic relationship expressions are also found in Ballesteros-Pérez et al. (2012a))	$\beta = \frac{1}{n} \sum_{k=1}^{k=n} \frac{LN\ d_{\min k}}{LN\ d_{mk}}$ $1 \le \beta \le +\infty$ (bid concentration bid dispersion)
Bid standard deviation s $est\ s = \frac{N+1}{N-1} \cdot \frac{b_{\max} - b_{\min}}{2\sqrt{3}}$	$N = \frac{1}{n} \sum_{k=1}^{k=n} N_k$ (N is the average of the participating bidders in the n past tenders analyzed) (b_{min} and b_{max} are obtained as above)	Drop standard deviation σ $est\ \sigma = \frac{N+1}{N-1} \cdot \frac{d_{\max} - d_{\min}}{2\sqrt{3}}$	$N = \frac{1}{n} \sum_{k=1}^{k=n} N_k$ (N is the average of the participating bidders in the n past tenders analyzed) (d_{min} and d_{max} are obtained as above)
Estimated cost bid b_o $est\ b_o = c \cdot b_m$ (This expression is commonly used the other way around, i.e., as a function of b_o which actually is the Forecasting Parameter)	$c = \frac{1}{n} \sum_{k=1}^{k=n} \frac{b_{ok}}{b_{mk}}$ $0 \le c \le +\infty$ (bid conservativeness bid aggressiveness)	Estimated cost drop d_o $est\ d_o = 1 + \gamma(d_m - 1)$ (This expression is also commonly used the other way around, i.e., as a function of d_o)	$\gamma = \frac{1}{n} \sum_{k=1}^{k=n} \frac{d_{ok} - 1}{d_{mk} - 1}$ $0 \le \gamma \le +\infty$ (bid conservativeness bid aggressiveness)

All these ALBC are interconnected, that is, it is possible to find a mathematical equivalency between the proportion of bidders disqualified in the first four ALBC (the ones that use a SP) and between the last two ALBC (the ones that make no use of any SP). This equivalency has been proposed in the Appendix by means of Tables A1 and A2, respectively. However those calculations require knowing the exact bid probability distribution function, which has been an unsolved and ongoing research bidding topic over the years. In this connection Skitmore (2013) reports some of the most common found in the scientific literature, such as uniform, normal, lognormal, gamma and Weibull. For a first approach, however, Tables A1 and A2 assume a simple uniform distribution.

A last relevant practical note concerning ALBC found during the tender review was that, when competing with ALBC mathematical expressions that make no use of SPs (expressions $b_{abn} = \omega$ and $d_{abn} = \delta$), most bids tend to be close to the cut-off limit (ω or δ), apparently sacrificing bigger profits.

Mathematically this can be simply explained for uncapped tenders as, when $b_m \to \omega$, therefore, $b_{\min} \to \omega$ (otherwise the lowest bidder is directly disqualified) and the maximum bid has no option but $b_{\max} \to \omega$. Then, since $b_{\max} - b_{\min} \to 0$, so does the standard deviation $s \to 0$. Analogously, $N_{abn} \to 0$ (because everyone knows where the cut-off limit is located), therefore, b_m is stuck near ω making it impossible to establish a statistical relationship with the rest of ALBC which make use of SPs (first four shown in this section).

In capped tenders, a similar reasoning process may arise: $d_m \to \delta$ and so do $d_{\max} \to \delta$ and $d_{\min} \to \delta$, forcing $\sigma \to 0$, whereas $N_{abn} \to 0$ as well.

This situation has immediate practical repercussions since it constitutes the first empirical proof that when bidders can accurately calculate the risk of being disqualified (because they know in advance where exactly ω or δ are), most will place their bids just before crossing that extreme. In this way, bidders avoid losing as much economic score as possible, despite frequently relinquishing more profits compared to situations in which the ALBC depend on a SP and the final position of the cut-off limit is not known in advance.

Discussion

In addition to the review of tender specifications, literature and public procurement methods allowing the ESF and ALBC taxonomies to be created, several other interesting issues on bidding behaviour have emerged. For example, how SPs relate to each other (summarized in Table 2), how bid distribution concentrates near the cut-off limit when the ALBC make no use of SP, and how the ALBC are mathematically interconnected (shown in the Appendix). Another recurrent effect of *apparent or phony economic bid weighting* takes place whenever a percentage of the economic score (S_i) is either never achievable or always awarded.

To introduce this phenomenon, suppose the economic and technical bid weightings in a tender are balanced ($W_e = W_t = 0.5$) and that the tender specifications adopt an ESF that gives away 0.30 (out of the total 1.00) no matter the bid or drop the bidder is submitting. An example of this ESF would be:

$$S_i = 0.30 + 0.70\frac{b_{\max} - b_i}{b_{\max} - b_{\min}} \quad \text{or}$$
$$S_i = 0.30 + 0.70\frac{d_i - d_{\min}}{d_{\max} - d_{\min}}$$

In this case, bidders can only compete to achieve an economic score from 0.30 to 1.00. In other terms, the following fraction of the overall score, O_i, $0.30 \cdot W_e = 0.30 \cdot 0.5 = 0.15$ is not disputed. If this happens, the *true economic bid weighting* (W_e^*) is not now 0.5, but $W_e(1-0.30)$ out of the overall possible score $W_e(1-0.30)+W_t$; that is, $\frac{W_e(1-0.30)}{W_e(1-0.30)+W_t} = \frac{0.5\,(1-0.30)}{0.5\,(1-0.30)+0.5} == \frac{0.35}{0.35+0.5} \approx 0.412$, which forces the *true technical bid weighting* (W_t^*) to be $1 - W_e^* \approx 1 - 0.412 = 0.588$, instead of 0.5. This is a significant deviation from the situation in which the weightings were intended to be balanced.

This phenomenon can be generalized, even for the technical bid weighting, and takes place not only whenever a fraction of the economic score (Q) is given away by the ESF, but also when a fraction of the score is unreachable mathematically or at least unreachable (undisputed) in normal conditions of competitiveness. In these cases, the general expression for calculating the *true economic bid weighting* is:

$$W_e^* = \frac{W_e(1-Q)}{W_t + W_e(1-Q)} \tag{3}$$

If $W_t = 1 - W_e$ then,

$$W_e^* = \frac{W_e(1-Q)}{(1-W_e) + W_e(1-Q)} = \frac{(1-Q)\,W_e}{1-QW_e} \tag{4}$$

where:

W_e: is the original economic bid weighting (in per-unit values) stated in the tender specifications.

W_e^*: is the true economic bid weighting (in per-unit values) with $W_e^* \le W_e$ always.

Q: is the fraction of the economic score either rarely or almost always achievable (in per-unit values).

W_t: is the original technical bid weighting (in per-unit values) stated in the tender specifications.

W_t^*: is the true technical bid weighting (in per-unit values) with $W_t^* = 1 - W_e^*$.

A representation of Equation 4 can be found in Figure 4 for all the intervening variables.

Using the diagram above is quite simple. Generally, the user must enter by the lower X-axis through analysing the ESF and estimating Q, then select the curve W_e corresponding to the value stated in the tender specifications and find the position of the vertical intersection with which to obtain the true economic (W_e^*) and technical ($W_t^* = 1 - W_e^*$) bid weighting values on the left and right, respectively.

Practical implications of both Equation 4 and Figure 4 are evident. If tender specifications implement ESFs with mathematical expressions that do not allow awarding the whole range of economic scores (from 0 to 1) to the competing bidders, the economic and technical bid weightings will become increasingly reversed (W_e will lose actual weight in favour of the technical bid weighting W_t) as the fraction of undisputed economic score increases. This situation could mislead bidders' strategies, or even be used (intentionally or unintentionally) by the contracting authorities to give the appearance of applying some economic and technical bid weightings while actually applying different ones.

However, perhaps, the most difficult issue is to estimate Q, since not all ESFs are as simple as the one provided in the example. For this purpose, the bidders or contracting authorities can make use of the SP estimated cost bid (b_o) or drop (d_o) from a future tender for forecasting the rest of SP (by means of Table 2) and, with these values, calculate the final ESF curve, with which observing Q is trivial.

In general, any owner or auctioneer, when designing and implementing a new ESF for future tender specifications should bear in mind that the 'whole range' of possible scores (from 0.00 to 1.00) must always be actually achievable by the bidders in normal conditions of competitiveness. Nonetheless, strictly speaking, this can only be possible by implementing an ESF under cases 4 or 5 of the ESF taxonomy in Figure 1, since they are the only ones that award the maximum score ($S_{\max} = 1$) to the lowest bidder (that is, to SP $b_{\min}$ or $d_{\max}$) and the minimum score ($S_{\min} = 0$) to the highest bidder (that is, to $b_{\max}$ or $d_{\min}$). From this last statement, it is clear that specific ESFs that make no use of any SP (case 6 in Figure 1) are the most vulnerable to apparent economic bid weighting.

However, the problem with cases 4 and 5 is that these ESFs are the most vulnerable to collusion, particularly cover-bidding, in which bidding rings can greatly condition the final economic scores (by submitting extremely high and/or low bids for pushing the rest of the bidders' scores towards the average, thus also paradoxically diminishing the economic bid weighting).

In this sense, all the combinations of SPs from Figure 1 would actually require ALBC to be implemented for both the high and lower extremes of the bid distribution with the simultaneous aim of avoiding

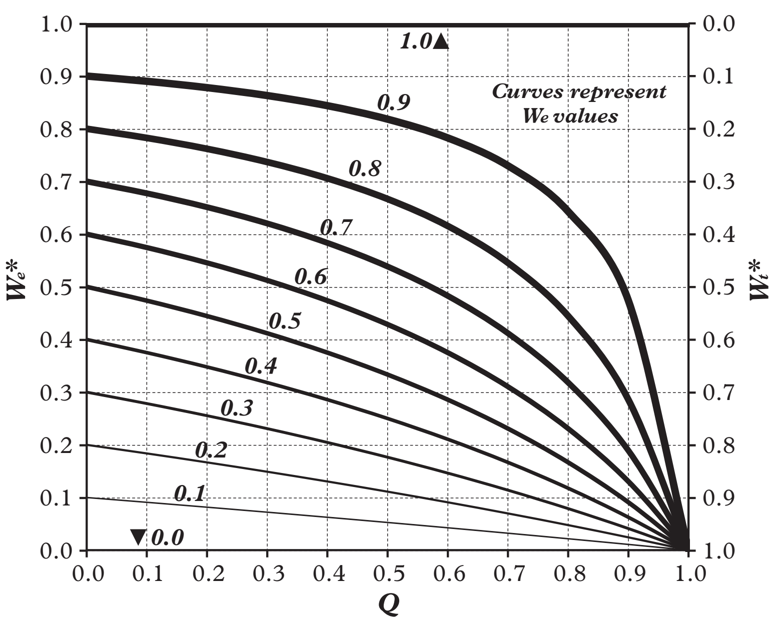


Figure 4 Apparent economic bid weighting variation

bid-covering. The key is how to set the right ALBC width: narrow enough to make collusion difficult, but not so narrow so as to reject bids that are actually competitive and truthful. Obviously, the problem of reaching the perfect configuration and combination of ESF and ALBC still requires further research, but has now acquired a new dimension by highlighting how apparent or phony bid weighting is also an important effect to be considered in seeking a solution.

Conclusions

Whenever there is need for converting price bids into scores for combination with technical proposal attributes, such as quality or client's preferences (like MEAT and BV), mathematical criteria need to be included in the tender specifications. The classification of these mathematical criteria, named economic scoring formulas (ESFs) and abnormally low bids criteria (ALBC), constituted the main aim of the present study.

By going through their taxonomies it is clear that there are many ESFs and ALBC currently in use for evaluating price bid proposals in construction auctions and they affect bidding behaviour in profound ways, most of which are little understood. As a result, their design in practice is invariably a highly intuitive process, involving few theoretical or empirical considerations.

In this paper, several outcomes relating to ESFs and ALBC have been considered and analysed. After a wide but thorough review of international tender specifications along with multiple other sources such as international public procurement guidelines and scientific articles and books on the topic, new ESF and ALBC taxonomies have been proposed. These taxonomies will enable expanding research in the near future while establishing a reasonable degree of homogeneity concerning nomenclature and denominations.

Furthermore, because of classifying the ESF and ALBC according to their scoring parameters (SPs) actually used, their relationships have now been adduced for uncapped tenders (tenders without an upper-price limitation). This will be useful for analysing changes or habits in bidding behaviour in upcoming research since they can accurately depict recurring statistical information on tenders.

Additionally, several other results derived from the ESF and ALBC taxonomies have been obtained. For example, it has been explained how bid distribution concentrates near the cut-off limit when the ALBC makes no use of a SP, as well as how ALBC are actually mathematically interrelated whenever a SP is used.

Finally, apparent or phony economic bid weighting explains how the economic bid weighting is actually overestimated whenever an ESF does not assign the whole range of scores to all the participating bidders. This phenomenon is quite common in ESF in real practice and has to be avoided when designing both ESFs and ALBC.

From the several examples provided in the paper, it is clear that previous research on auction design is still very far from incorporating important practical issues, some of which have been described here. The main contribution here is a compilation and perhaps a first step towards a new approach in bidding analysis useful to both auctioneers and bidders. This is especially the case with the former when designing or selecting a particular combination of ESF and ALBC for the tender specifications. However, the present analysis is mostly restricted to providing a general qualitative picture. The next logical research step will be the development of a quantitative means for determining, and hence controlling, the effect of small variations in the ESF and/or ALBC mathematical expressions on, for instance, the level of bidders' aggressiveness/conservativeness in a future tender. Taken together with the risk attitudes of the individuals involved, a new door is opened for the possibility of personalized optimal price scoring rules in construction auction design.

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Note

1. To avoid confusion, the terms 'auction' and 'tender' will be used here as synonymous, as well as 'auctioneer', 'client', 'owner' and 'contracting authority'. Strictly speaking, the words 'construction auctions' in this study do not refer to 'classical auctions' where the highest bidder often wins, but actually refer to 'procurement auctions' or 'reverse auctions', which are a common type of auction in which the roles of the buyer (client, owner, auctioneer or contracting authority) and the seller (bidders or tenderers) are reversed with the primary objective to drive purchase prices downward.

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Appendix

The following tables allow the conversion of one criterion of the ALBC to another assuming the bid distribution follows a uniform distribution, which constitutes a simplification of the reality. Depending on which is the known ALBC expression, locate that column and go down until reaching the same row where the text 'independent variable' can be read. The rest of the ALBC expressions into which the initial known ALBC expression can be translated will remain in the same row in adjacent cells.

For the interested reader, mathematical proofs (❶–⓮) of Tables A1 and A2 can be found as Supplemental online material.

Table A1 Mathematical relationships among abnormally low bids criteria (ALBC) with scoring parameters (SPs)

$b_{abn} = (1-\varepsilon) b_m$	$d_{abn} = (1+\theta) d_m$	$b_{abn} = b_m - \lambda s$ $d_{abn} = d_m + \lambda \sigma$	$N_{abn} = (1-\mu)\frac{N}{2}$
Generally: $0 \le \varepsilon \le 1$ *(from tougher to softer)*	Generally: $0 \le \theta \le \frac{1-d_m}{d_m}$ *(from tougher to softer)*	Generally: $0 \le \lambda \le \frac{b_m}{s}$ *or* $\frac{1-d_m}{\sigma}$ *(from tougher to softer)* For uncapped and capped tenders, respectively: $\frac{b_m}{s} = \frac{N-1}{N+1} \cdot \frac{2\sqrt{3}}{b-a}$ $\frac{1-d_m}{\sigma} = 2\sqrt{3}\frac{N-1}{N+1} \cdot \frac{1-d_m}{d_m{}^{\alpha} - d_m{}^{\beta}}$	Generally: $0 \le \mu \le 1$ *(from tougher to softer)* If N_{abn} was defined as a Natural number η, this would be the equivalency: $\mu = 1 - \frac{2\eta}{N}$
Actual mathematical limits: $-\infty < \varepsilon \le 1$ *ε=0* disqualifies *N/2* bidders	Actual mathematical limits: $-1 < \theta \le \frac{1-d_m}{d_m}$ *θ=0* disqualifies *N/2* bidders	Actual mathematical limits: $-\infty < \lambda \le \frac{b_m}{s}$ $-\frac{d_m}{\sigma} < \lambda \le \frac{1-d_m}{\sigma}$ *λ=0* disqualifies *N/2* bidders	Actual mathematical limits: $-1 < \mu \le 1$ *μ=-1* disqualif. the *N* bidders *μ=0* disqualifies *N/2* bidders *μ=1* disqualifies no bidder
ε *(independent variable)*	$\theta = \frac{b_m - \varepsilon \cdot A}{T - b_m}$ ❶	$\lambda = \frac{N-1}{N+1} \cdot \frac{2\varepsilon\sqrt{3}}{b-a}$ ❷	$\mu = \frac{N-1}{N+1} \cdot \frac{2\varepsilon}{b-a}$ ❸ $N_{abn} = 0$ *if* $\varepsilon \ge \frac{N+1}{N-1} \cdot \frac{b-a}{2\sqrt{3}}$ $0 < N_{abn} \le \frac{N}{2}$ *if* $0 \le \varepsilon < \uparrow$
$\varepsilon = \frac{A-b_m}{b_m} \cdot \theta$ ❹	θ *(independent variable)*	$\lambda = \frac{N-1}{N+1} \cdot \frac{2\theta d_m \sqrt{3}}{d_m{}^{\alpha} - d_m{}^{\beta}}$ ❺	$\mu = \frac{N-1}{N+1} \cdot \frac{2\theta d_m}{d_m{}^{\alpha} - d_m{}^{\beta}}$ ❻ $N_{abn} = 0$ *if* $\theta \ge \frac{N+1}{N-1} \cdot \frac{d_m{}^{\alpha} - d_m{}^{\beta}}{2d_m}$ $0 < N_{abn} \le \frac{N}{2}$ *if* $0 \le \theta < \uparrow$
$\varepsilon = \frac{N+1}{N-1} \cdot \frac{b-a}{2\sqrt{3}} \cdot \lambda$ ❼	$\theta = \frac{N+1}{N-1} \cdot \frac{d_m{}^{\alpha} - d_m{}^{\beta}}{d_m} \cdot \frac{\lambda}{2\sqrt{3}}$ ❽	λ *(independent variable)*	$\mu = \frac{\lambda}{\sqrt{3}}$ ❾ $N_{abn} = 0$ *if* $\lambda \ge \sqrt{3}$ $0 < N_{abn} \le \frac{N}{2}$ *if* $0 \le \lambda < \sqrt{3}$
$\varepsilon = \frac{N+1}{N-1} \cdot \frac{b-a}{2} \cdot \mu$ ❿	$\theta = \frac{N+1}{N-1} \cdot \frac{d_m{}^{\alpha} - d_m{}^{\beta}}{2d_m} \cdot \mu$ ⓫	$\lambda = \mu\sqrt{3}$ ⓬	μ *(independent variable)*

Table A2 Mathematical relationships among abnormally low bids criteria (ALBC) without scoring parameters (SPs)

$b_{abn} = \omega$	$d_{abn} = \delta$
$0 \le \omega \le +\infty$ *(from softer to tougher)*	$0 \le \delta \le 1$ *(from tougher to softer)*
ω *(independent variable)*	$\delta = 1 - \frac{\omega}{A}$ ⓭
$\omega = (1 - \delta) A$ ⓮	δ *(independent variable)*