

# *Revisiting the hysteresis effect in surface energy budgets*

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# Revisiting the hysteresis effect in surface energy budgets

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[1] The hysteresis effect in diurnal cycles of net radiation  $R_n$  and ground heat flux  $G_0$  has been observed in many studies, while the governing mechanism remains vague. In this study, we link the phenomenology of hysteresis loops to the wave phase difference between the diurnal evolutions of various terms in the surface energy balance.  $R_n$  and  $G_0$  are parameterized with the incoming solar radiation and the surface temperature as two control parameters of the surface energy partitioning. The theoretical analysis shows that the vertical water flux  $W$  and the scaled ratio  $A_s^*/A_T^*$  (net shortwave radiation to outgoing longwave radiation) play crucial roles in shaping hysteresis loops of  $R_n$  and  $G_0$ . Comparisons to field measurements indicate that hysteresis loops for different land covers can be well captured by the theoretical model, which is also consistent with Camuffo-Bernadi formula. This study provides insight into the surface partitioning and temporal evolution of the energy budget at the land surface. **Citation:** Sun, T., Z.-H. Wang, and G.-H. Ni (2013), Revisiting the hysteresis effect in surface energy budgets, *Geophys. Res. Lett.*, 40, 1741–1747, doi:10.1002/grl.50385.

## 1. Introduction

[2] An integrated earth system model must be able to physically resolve the transfer of energy, water, and tracers across the land-atmosphere interface [Liang *et al.*, 1994; Sellers *et al.*, 1997; Katul *et al.*, 2012]. Partitioning of solar energy at the land surface provides the lower boundary conditions for the atmospheric dynamics of energy and water cycles, and dictates the land-atmospheric interactions [McCumber and Pielke, 1981; Chen and Dudhia, 2001]. The surface energy balance (SEB) equation is given by:

$$R_n - G_0 = H + LE, \quad (1)$$

where  $R_n$ ,  $G_0$ ,  $H$ , and  $LE$  are the net radiation, ground, sensible, and latent heat fluxes, respectively. The left-hand side of equation (1) denotes the available energy received at the land surface, while the right-hand side is the atmospheric energy dispersion through turbulent transport.

[3] While the net radiation and turbulent (sensible and latent) fluxes can be readily measured in the atmospheric boundary layer using standard radiometry and eddy-

covariance technology, respectively, accurate determination of the ground heat flux is more challenging. Historically,  $G_0$  is parameterized as a constant fraction of  $R_n$  during a diurnal cycle [Humes *et al.*, 1994]. However, the assumed linear proportionality between  $R_n$  and  $G_0$  is not satisfied due to the existence of the hysteresis loop between the two energy fluxes, as observed by numerous researchers in field measurements [Taesler, 1980; Camuffo and Bernardi, 1982; Doll *et al.*, 1985; Kustas and Daughtry, 1990; Yoshida *et al.*, 1990; Grimmond *et al.*, 1991; Asaeda and Ca, 1993; Asaeda *et al.*, 1996; Taha, 1997; Anandakumar, 1999; Meyn and Oke, 2009]. The hysteresis effect is essentially due to the existence of phase difference between the diurnal variations of  $R_n$  and  $G_0$ , which is  $\pi/4$  for dry soils. The Camuffo-Bernadi formula was first proposed to characterize the diurnal evolution of  $G_0$  as a function of  $R_n$  [Camuffo and Bernardi, 1982]:

$$G_0(t) = a_1 R_n(t) + a_2 \frac{\partial R_n(t)}{\partial t} + a_3, \quad (2)$$

where  $a_1$ ,  $a_2$ , and  $a_3$  are site-specific empirical coefficients. Note that the time derivative of net radiation in equation (2) introduces the phase difference between  $G_0$  and  $R_n$ , albeit empirically. Santanello and Friedl [2003] discussed the phase lag between  $G_0$  and  $T_s$  and found the value varies with soil moisture conditions and equals  $\pi/4$  for dry soils. By solving the one-dimensional advection-diffusion equation of coupled heat and liquid water transport, Gao *et al.* [2003, 2010] concluded that the vertical water flux plays a crucial role in regulating the phase lag between  $G_0$  and  $T_s$ . Relating the evolution of  $R_n$  to that of  $T_s$  without phase lag partially explains the hysteresis effect, with  $G_0$  always leading in phase as compared to  $R_n$ , which contradicts field observations under certain conditions [e.g., Camuffo and Bernardi, 1982; Anandakumar, 1999].

[4] In this letter, we revisit the physical mechanisms governing the hysteresis effect between the net radiation and the ground heat flux by focusing on their diurnal wave phase evolution. We propose theoretical parameterization schemes for  $R_n$  and  $G_0$  and test them using field experiment data from various land use land cover (LULC) types. A linkage between the proposed scheme and the empirically based Camuffo-Bernadi formula is also established.

## 2. Theoretical Analysis

### 2.1. Parameterization of $R_n$

[5] The net radiation  $R_n$  is the sum of incoming and outgoing shortwave and longwave components and can be written as:

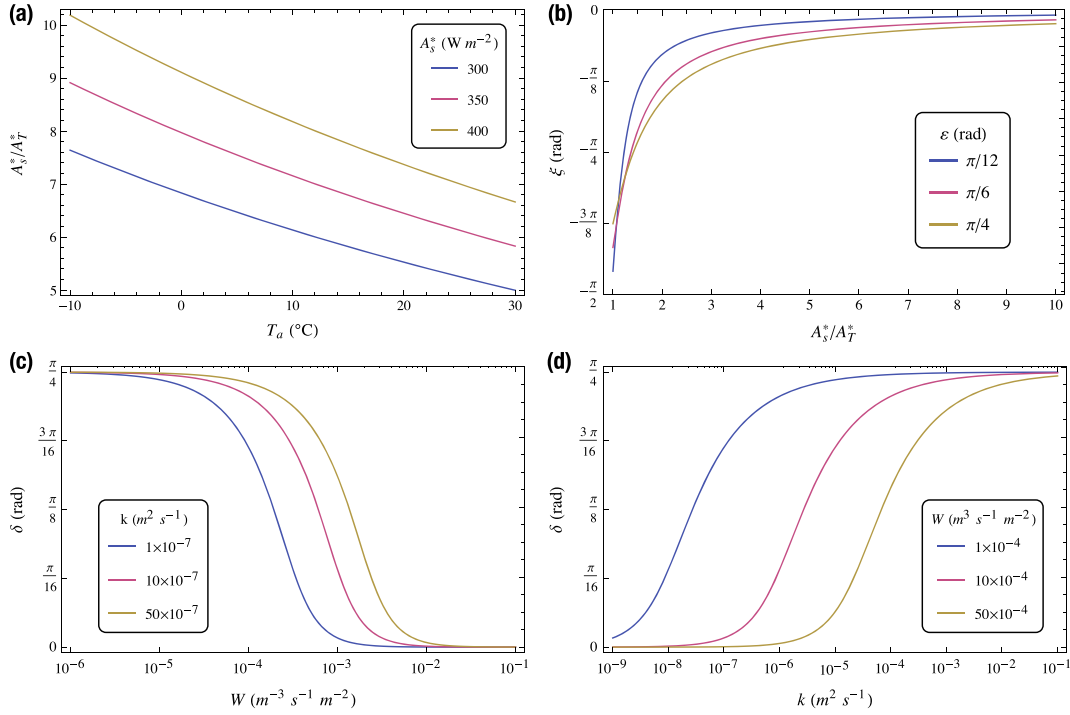
$$R_n = (1 - \alpha) S_d + \varepsilon_a \sigma T_a^4 - \varepsilon_s \sigma T_s^4, \quad (3)$$

where  $\alpha$  is the albedo of the ground surface,  $S_d$  the incoming shortwave radiation,  $\varepsilon_a$  the effective emissivity of the

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**Figure 1.** Variation of (a) scale ratio of radiative forcing  $A_s^*/A_T^*$  as a function of air temperature  $T_a$ , (b) phase lag  $\xi$  as a function of scale ratio of  $A_s^*/A_T^*$ , (c) phase lag  $\delta$  as a function of water flux density  $W$  with different thermal diffusivity  $k$ , and (d) phase lag  $\delta$  as a function of thermal diffusivity  $k$  with different water flux density  $W$ .

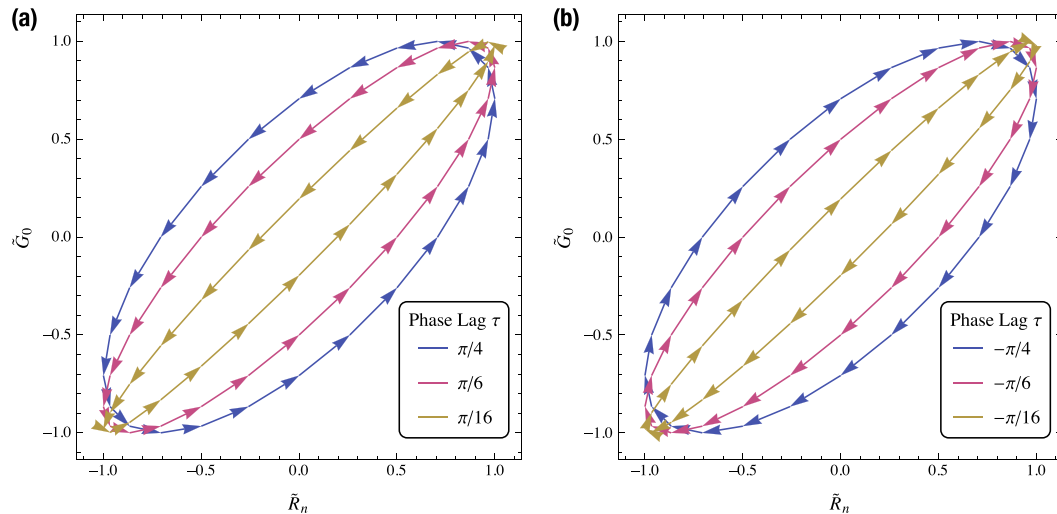
atmosphere,  $\varepsilon_s$  the emissivity of the land surface,  $\sigma = 5.67 \times 10^{-8}$  (W m<sup>-2</sup> K<sup>-4</sup>) the Stefan-Boltzmann constant,  $T_a$  the air temperature, and  $T_s$  the surface temperature. The incoming longwave radiation is parameterized using the atmospheric temperature [Brutsaert, 1975].

[6] During a clear day, the ground first responds to the incoming solar energy with increasing surface temperature  $T_s$ , while the response of overlying atmosphere is indirect due to surface heating. Taking  $S_d$  and  $T_s$  as the control

parameters of SEB and using Taylor expansion, equation (3) can be linearized in terms of  $S_d$  and  $T_s$  [Bateni and Entekhabi, 2012], as

$$R_n = (1 - \alpha)S_d + (\varepsilon_a + 3\varepsilon_s)\sigma T_a^4 - 4\varepsilon_s\sigma T_a^3 T_s. \quad (4)$$

By setting  $t=0$  at sunrise, we first parameterize  $S_d$  and  $T_s$  using sinusoidal functions occurring at a principle diurnal frequency of the Earth's rotation  $\omega = 2\pi/24$  (in rad/h) as:



**Figure 2.** Hysteresis loops between the normalized net radiation  $\tilde{R}_n$  and the normalized ground heat flux  $\tilde{G}_0$ , evolving in (a) counterclockwise ( $\tau > 0$ ), and (b) clockwise ( $\tau < 0$ ) directions, respectively. The width of hysteresis loops is regulated by the wave phase difference  $\tau$  between  $\tilde{R}_n$  and  $\tilde{G}_0$ .

$$S_d(t) = A_s \sin(\omega t) + \bar{S}_d, \quad (5)$$

$$T_s(t) = A_T \sin(\omega t - \varepsilon) + \bar{T}_s, \quad (6)$$

where  $A_s$  and  $A_T$  are amplitudes of daily  $S_d$  and  $T_s$  variation, respectively,  $\bar{S}_d$  the daily mean solar radiation,  $\bar{T}_s$  the daily mean temperature, and  $\varepsilon \geq 0$  the phase lag between  $T_s$  and  $S_d$  indicating the response time of ground surface to solar heating. With a simple harmonic at the principle diurnal frequency, it is analogous to the force-restore method [Bhumralkar, 1975]. In practice, sinusoidal variation mimics actual evolutions of  $T_s$  and  $S_d$  for clear days with reasonable accuracy [Gao *et al.*, 2003]. For better representation of diurnal cycles of  $T_s$  and  $S_d$ , Fourier series including more harmonic functions with higher frequencies can be used [Gentine *et al.*, 2010; 2011; 2012] in equations (5) and (6), without qualitatively altering our subsequent analysis. Substituting equations (5) and (6) into equation (4), we have

$$R_n(t) = A_R \sin(\omega t - \xi) + R'_n, \quad (7)$$

where  $A_R$  is the amplitude of net radiation,  $\xi$  the phase lag between  $R_n$  and  $S_d$ , and  $R'_n$  the residual term. The three terms in equation (7) are given by

$$A_R = [(1 - \alpha)^2 A_s^2 - 8(1 - \alpha) \varepsilon_s \sigma T_a^3 A_s A_T \cos(\varepsilon) + 16 \varepsilon_s^2 \sigma^2 T_a^6 A_T^2]^{-1/2}, \quad (8)$$

$$\xi = \arctan \left[ \frac{4 \varepsilon_s \sigma T_a^3 A_T \sin(\varepsilon)}{4 \varepsilon_s \sigma T_a^3 A_T \cos(\varepsilon) - (1 - \alpha) A_s} \right], \quad (9)$$

and

$$R'_n = (1 - \alpha) \bar{S}_d + (\varepsilon_a + 3 \varepsilon_s) \sigma T_a^4 - 4 \varepsilon_s \sigma T_a^3 \bar{T}_s, \quad (10)$$

respectively. Excluding  $R'_n$  from equation (7) and successively normalizing  $R_n$  by the amplitude  $A_R$ , we have the normalized net radiation  $\tilde{R}_n$  as:

$$\tilde{R}_n(t) = \sin(\omega t - \xi). \quad (11)$$

## 2.2. Parameterization of $G_0$

[7] Following Gao *et al.* [2003, 2010], we consider the one-dimensional advection-diffusion equation in the soil:

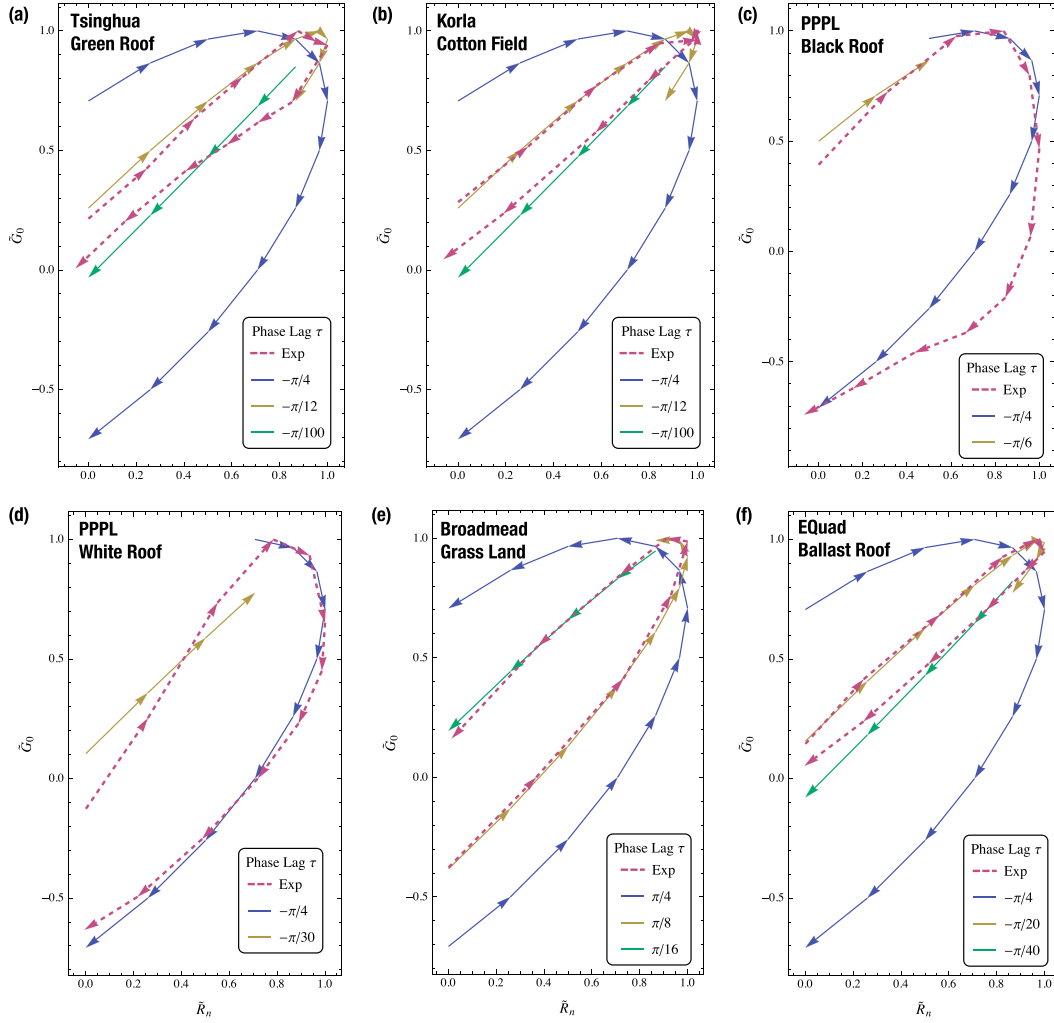
$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} + W \frac{\partial T}{\partial z}, \quad (12)$$

where  $T$  is the soil temperature at a reference depth  $z$  (positive downward),  $k$  is the soil thermal diffusivity, and  $W = \partial k / \partial z - (C_W / C_g) w \varphi$  is the soil water flux density [Ren *et al.*, 2000] with  $C_W$  the volumetric heat capacity of water,  $C_g$  the volumetric heat capacity of soil,  $w$  the pore water velocity, and  $\varphi$  the volumetric soil water content. Taking the surface temperature given by equation (6) as the upper boundary condition, solutions of soil temperature and heat flux of equation (12) are

**Table 1.** Description of the Experiment Sites

Surface Type	Surface Wetness	Period	Site Name	Location (Coordinate)	$G_0$ Calculation	Additional Information <sup>a</sup>
Green Roof	Wet	July 2012	Tsinghua Green Roof [Sun <i>et al.</i> , 2013]	Beijing, China (40.00 N, 116.33 E)	Heat Storage	$C_g = 1.0 \times 10^6$ $z = 0.02$ $h = 5.0$
Natural Soil Black Roof	Moderate Dry	May 2012 12–13 Sept. 2012	Korla Cotton Field PPPL Black Roof	Xinjiang, China (41.58 N, 86.16 E) New Jersey, U.S. (40.35 E, 74.64 W)	Energy Residual Heat Storage	$C_g = 2.0 \times 10^6$ $z = 0.01$
White Roof	Dry	12–13 Sept. 2012	PPPL White Roof	New Jersey, U.S. (40.35 N, 74.60 W)	Heat Storage	$C_g = 2.0 \times 10^6$ $z = 0.01$
Grass Land	Wet	Oct. 2010	Broadmead Grass Land [Wang and Bou-Zeid, 2012]	New Jersey, U.S. (40.35 N, 74.60 W)	Heat Storage	$C_g = 1.3 \times 10^6$ $z = 0.08$
Suburban Area	Moderate	Oct. 2010	EQuad Ballast Roof [Wang <i>et al.</i> , 2011]	New Jersey, U.S. (40.35 N, 74.65 W)	Energy Residual	$h = 23.2$

<sup>a</sup>The dry soil volumetric heat capacity  $C_g$  (in  $\text{J m}^{-3} \text{K}^{-1}$ ) and the depth  $z$  (in m) at which soil heat fluxes  $G_z$  were measured are provided for sites using heat storage method. For sites where the energy residual method is used, the height of eddy-covariance instruments  $h$  (in m) is reported.



**Figure 3.** Daytime hysteresis loops between  $\tilde{R}_n$  and  $\tilde{G}_0$  from field measurements at: (a) Tsinghua Green Roof site, (b) Korla Cotton Field site, (c) PPPL Black Roof site, (d) PPPL White Roof site, (e) Broadmead Grass Land site, and (f) EQuad Ballast Roof. Measurement datasets are plotted in dashed lines and the theoretical ones in solid lines. The arrows indicate loop directionality. The information of field measurement sites and sampling dates is provided in Table 1.

$$T(z, t) = A_T \exp(-z/M) \sin(\omega t - \varepsilon - z/N) + \bar{T}_s, \quad (13)$$

$$\tilde{G}_0(t) = \sin(\omega t - \varepsilon + \delta). \quad (17)$$

$$\begin{aligned} c(z, t) &\equiv -\lambda \frac{\partial T}{\partial z} \\ &= \lambda A_T \frac{\sqrt{M^2 + N^2}}{MN} \exp(-z/M) \sin(\omega t - \varepsilon - z/N + \delta), \end{aligned} \quad (14)$$

where  $\lambda$  is the soil thermal conductivity,  $\Delta = \sqrt{(W^2 + \sqrt{W^4 + 16\kappa^2\omega^2})/2}$ ,  $N = \Delta/\omega$ , and  $M = 2\kappa/(\Delta + W)$ . In equation (14),

$$\delta = \arctan\left(\frac{M}{N}\right) = \arctan\left[\frac{2k\omega}{(\Delta + W)\Delta}\right], \quad (15)$$

is the phase lag of the soil temperature response to the heat flux forcing, which holds at any depth  $z$ . In particular, at the surface  $z=0$ , the ground heat flux  $G_0$  is given by,

$$G_0(t) = \lambda A_T \frac{\sqrt{M^2 + N^2}}{MN} \sin(\omega t - \varepsilon + \delta), \quad (16)$$

which can be normalized as

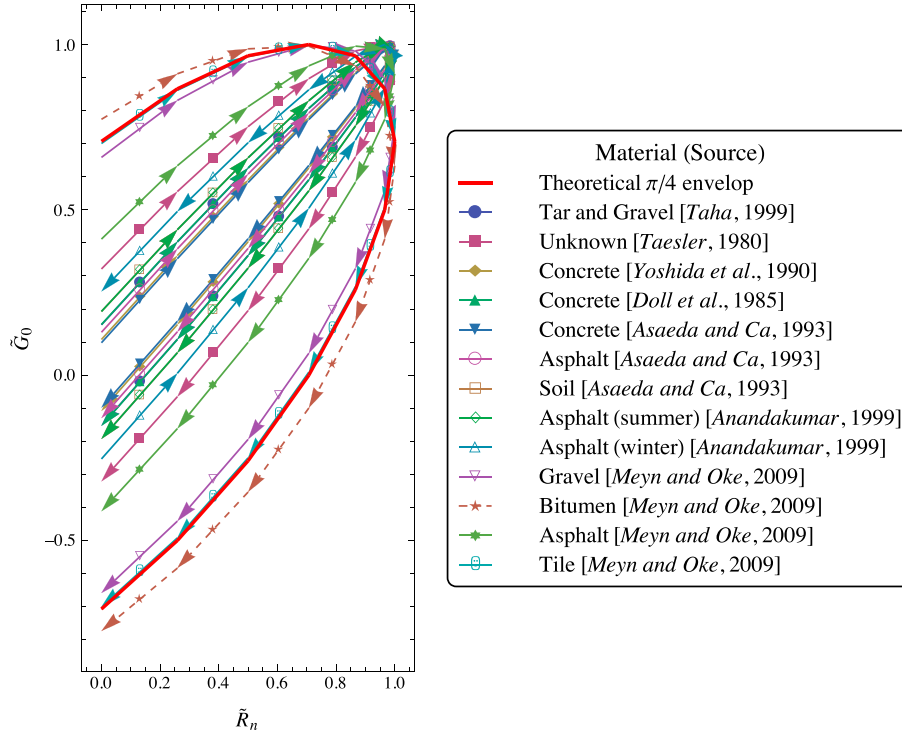
### 2.3. Analyzing the Phase Lag Between $R_n$ and $G_0$

[8] Comparing equations (11) and (17), the phase lag  $\tau$  between  $R_n$  and  $G_0$  is

$$\tau = -(\xi + \delta) + \varepsilon. \quad (18)$$

$\tau > 0$  implies the phase evolution of  $\tilde{G}_0$  falls behind that of  $\tilde{R}_n$  and vice versa. Of the three contributors to  $\tau$ ,  $\xi$  is the phase difference between  $R_n$  and  $S_d$ ;  $\delta$  is the phase difference between  $T_s$  and  $G_0$ ; and  $\varepsilon$  is the phase difference between two control parameters  $T_s$  and  $S_d$ . The three contributors to  $\tau$  are analyzed term by term as follows:

[9] 1.  $\xi$ : By introducing two scaling variables,  $A_s^* = (1 - \alpha)A_s$  and  $A_T^* = 4\varepsilon_s\sigma T_a^3 A_T$  for the scaled amplitudes of the net shortwave radiation and the outgoing longwave radiation, respectively, equation (9) can be simplified as



**Figure 4.** Daytime hysteresis loops between  $\tilde{R}_n$  and  $\tilde{G}_0$  occurred on surfaces of various materials retrieved from the literature. The bold solid red line denotes the theoretical enveloping loop with the limit  $\tau = \pm \pi/4$ . All hysteresis loops except one reported in the literature fall within the theoretical limit.

$$\xi = \arctan\left(\frac{\sin \varepsilon}{\cos \varepsilon - A_s^*/A_T^*}\right). \quad (19)$$

The ratio  $A_s^*/A_T^*$  is a function of  $T_a$  with magnitude greater than unity, as shown in Figure 1a. The phase lag  $\xi$  varies in the range of

$$-\pi/2 < \xi < 0, \quad (20)$$

according to equation (19) as a function of  $A_s^*/A_T^* > 1$ , and is plotted in Figure 1b.

[10] 2.  $\delta$ : From equation (15) [Gao et al., 2003, 2010],  $\delta$  varies in the range of

$$0 < \delta \leq \pi/4, \quad (21)$$

depending on the soil water flux density  $W$ , as shown in Figure 1c. The limiting case  $\delta = \pi/4$  is attained for dry soils with water flux  $W = 0$ . For a given water flux density  $W$ , the phase lag  $\delta$  increases with the thermal diffusivity  $k$  as shown in Figure 1d.

[11] 3.  $\varepsilon$ : As  $S_d$  directly drives the SEB system and  $T_s$  responds to  $S_d$  relatively quickly through surface heating, it is reasonable to assume that the response time of  $T_s$  to  $S_d$  (represented by  $\varepsilon$ ) is insignificant as compared to the one between  $S_d$  and  $R_n$  (represented by  $\xi$  as indirect response through atmospheric heating), which leads to  $\varepsilon \ll \xi$ . In addition, the response of  $T_s$  to  $G_0$  (represented by  $\delta$  through soil heating) is also much slower than that of  $T_s$  to  $S_d$  (represented by  $\varepsilon$ ), leading to  $\varepsilon \ll \delta$ .

[12] From Figure 1, it is clear that both the scaled ratio  $A_s^*/A_T^*$  that indicates ambient forcing intensity, and the water flux density  $W$  that is associated with the soil water transport of heat, play crucial roles in dictating the total phase lag  $\tau$  between  $\tilde{R}_n$  and  $\tilde{G}_0$ . In the limiting cases: (a) with strong radiative forcing as  $A_s^*/A_T^* \rightarrow \infty$ ,  $\xi$  approaches 0; (b) with weak ambient forcing as  $A_s^*/A_T^* \rightarrow 1$ ,  $\xi$  decreases to  $-\pi/2$ . Meanwhile, as most available solar energy  $A_s^*$  is dissipated by the outgoing longwave radiation  $A_T^*$ , the evaporation dominated by the atmospheric demand will be weak, leading to  $W \rightarrow 0$  and  $\delta \rightarrow \pi/4$ . This is the likely scenario encountered in most field observations especially under clear-sky conditions, which, however, does not exclude the theoretical possibility that evaporation can still occur under the condition  $A_s^*/A_T^* \rightarrow 1$  in more general settings (e.g., rain on hot pavement surfaces). Applying these limiting cases, and combining equations (18), (20), and (21), we then have a physical range of  $\tau$  variation, as

$$-\pi/4 < \tau < \pi/4. \quad (22)$$

[13] Given this range, Figure 2 shows that, for a positive  $\tau$ , the hysteresis loop between diurnal  $\tilde{R}_n$  and  $\tilde{G}_0$  evolves in a counterclockwise direction, and vice versa. Physically, the directionality of hysteresis loops indicates the relative strength of radiative forcing and the soil wetness acting in opposite directions: wider counterclockwise hysteresis loops for stronger radiative forcing and clockwise ones for drier soils. In addition, note that the diurnal variation of soil water flux (or evaporation) is not considered in this study, which

may contribute to the actual phase evolution of surface energy budgets as well.

### 3. Validation by Field Measurements

[14] To validate our theoretical quantification of the phase lag between  $\tilde{R}_n$  and  $\tilde{G}_0$ , we selected field experiment data from six sites with different LULC types (i.e., natural and artificial) and wetness conditions (i.e., wet, moderate, and dry) for comparisons. Note that cloud cover has significant impact on the actual phase evolution of all energy budgets [Camuffo and Bernardi, 1982]. Shallow cumulus clouds generate more high frequencies and perturb the main daily harmonic [Gentine et al., 2012]. In this study, we only include data periods with clear days to exclude impacts of cloud cover and precipitation. Due to different instrumentation at each site, we use either energy residual or heat storage method to obtain the ground heat flux  $G_0$ . The energy residual is a straightforward way by taking  $G_0$  as the residual of other SEB terms, while the heat storage method estimates  $G_0$  by adding the heat storage above a heat flux plate measurement [Anandakumar, 1999]. The magnitude of  $G_0$  computed as the energy residual is inaccurate due to the SEB closure problem [Foken, 2008], whereas we assume that the phase evolution is less susceptible. The description of the data sources is given in Table 1.

[15] Notice that actual evolutions of  $T_s$  and  $S_d$  mimic sinusoidal variation with principle diurnal frequency only during daytime [Gao et al., 2003]. Thus, for field datasets, we rescale  $R_n$  and  $G_0$  in the following way: after obtaining the hourly series of  $R_n$  and  $G_0$ , we first set the daily peak value as the upper limit, and the value observed 5 h prior to the peak as the lower limit. The diurnal series are then normalized by the upper and lower limits, resulting in a rescaled series ranging in  $[0, 1]$  (with 0 and 1 corresponding to the lower and upper limits, respectively).

[16] Figure 3 shows normalized daytime hysteresis loops between  $R_n$  and  $G_0$  from field measurements, with comparisons to theoretical predictions. All field measurements, except the one in Figure 3e, exhibit clockwise hysteresis loop, indicating  $G_0$  is leading in phase. An explanation of the fact that  $G_0$  is usually leading in phase as compared to  $R_n$  is given by Gentine et al. [2011]. The hysteresis effect between  $R_n$  and  $G_0$ , conventionally described using three empirical coefficients, is completely characterized by a single theoretically-derived parameter, viz. the phase difference  $\tau$ . In particular, for artificial (dry) materials, the theoretical  $-\pi/4$  phase difference is successfully recovered in the afternoon loop as shown in Figures 3c and 3d. It is noteworthy that in Figure 3, the morning segments differ from the afternoon ones, which is likely due to the impact of soil water flux uptake. For engineered roofs in Figures 3c and 3d, the morning-afternoon difference can be attributed to evaporation of dew formed on roof surfaces in the morning.

[17] It is also noteworthy that distinguished hysteresis patterns in two adjacent sites (i.e., Broadmead Grass Land and EQuad Ballast Roof sites) are observed in Figures 3e and 3f, respectively. Given the same meteorological conditions at these two sites, the difference in loop patterns, in terms of both directionality and magnitude, reveals the importance of soil water advection in regulating  $G_0$ . At Broadmead site (grass land), there is a much stronger water flux density  $W$ , as compared to that at EQuad Roof

site (suburban area with  $W \rightarrow 0$ ). As a result, the positive phase lag  $\delta$  (nearly  $\pi/4$  for dry surfaces) at EQuad site effectively offsets the slightly negative phase lag  $\xi$ , leading to  $\tau = -\xi - \delta < 0$ , while at Broadmead site  $\tau \approx - > 0$  remains positive as  $\delta \rightarrow 0$  due to strong soil water flux advection.

### 4. Linkage to Camuffo-Bernadi Formula

[18] Here we revisit the Camuffo-Bernadi formula in equation (2) originated from empirical analysis. By substituting equation (7) into equation (2), we have

$$G_0(t) = A_R[a_1 \sin(\omega t - \xi) + a_2 \omega \cos(\omega t - \xi)] + a_3 + a_1 R'_n \quad (23)$$

Following the previous procedure of normalization, we obtain

$$\tilde{G}_0(t) = \sin(\omega t - \xi - \tau'), \quad (24)$$

where

$$\tau' = \arctan(-a_2 \omega / a_1). \quad (25)$$

It is apparent that  $\tau'$  in equation (24) is equivalent to the formulation of  $\tau$  in equation (18), implying  $-\pi/4 < \tau' < \pi/4$ . With empirical coefficients  $a_1$  and  $a_2$  reported by numerous researchers in the literature, hysteresis loops between normalized  $R_n$  and  $G_0$  for land surfaces of various materials are plotted in Figure 4. It is clear that the derived range of  $\tau'$  from Camuffo-Bernadi formula in equation (24) falls within the theoretical range of  $[-\pi/4, \pi/4]$ , for all loops except one.

[19] In practice, for any given site, the physical basis of these empirical coefficients  $a_1$  and  $a_2$  can now be characterized to elucidate the phase lag between the two energy budget terms by relating equations (24) and (18), in the light of the present analysis under clear-sky conditions. The remaining coefficient  $a_3$  is characterized as the intercept of a  $G_0$  versus  $R_n$  plot (ignored in our analysis through normalization), which is independent of the relative phase evolution of  $R_n$  and  $G_0$ . The physical interpretation of  $a_3$  therefore requires further investigation.

### 5. Concluding Remarks

[20] This study provides insight into the governing mechanisms of the hysteresis effect between the ground heat flux and net radiation, based on theoretical characterization of the diurnal phase difference between  $G_0$  and  $R_n$ . Our model captures the bi-directional (i.e., clockwise and counterclockwise) patterns of experimentally observed hysteresis loops well. The evaporation-driven soil water flux density  $W$  and the radiative forcing ratio  $A_s^*/A_T^*$  (net shortwave radiation to outgoing longwave radiation) essentially dictate the shape and directionality of hysteresis loops. The theoretical analysis is validated against field measurements over a wide variety of LULC types. The empirical coefficients in the classic Camuffo-Bernadi model admit physical interpretations in the light of current analysis. Following a similar methodology, the analysis in this study can be potentially extended to develop novel parameterization schemes for computing sensible and latent heat fluxes in the SEB system.



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