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1 **Entropy Production and Coarse Graining of the Climate Fields in**
2 **a General Circulation Model**

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Abstract

9
10 We extend the analysis of the thermodynamics of the climate system by investigating the role
11 played by processes taking place at various spatial and temporal scales through a procedure of
12 coarse graining. We show that the coarser is the graining of the climatic fields, the lower is
13 the resulting estimate of the material entropy production. In other terms, all the spatial and
14 temporal scales of variability of the thermodynamic fields provide a positive contribution to the
15 material entropy production. This may be interpreted also as that, at all scales, the temperature
16 fields and the heating fields resulting from the convergence of turbulent fluxes have a negative
17 correlation, while the opposite holds between the temperature fields and the radiative heating
18 fields. Moreover, we obtain that the latter correlations are stronger, which confirms that radiation
19 acts as primary driver for the climatic processes, while the material fluxes dampen the resulting
20 fluctuations through dissipative processes. We also show, using specific coarse-graining procedures,
21 how one can separate the various contributions to the material entropy production coming from
22 the dissipation of kinetic energy, the vertical sensible and latent heat fluxes, and the large scale
23 horizontal fluxes, without resorting to the full three-dimensional time dependent fields. We find that
24 most of the entropy production is associated to irreversible exchanges occurring along the vertical
25 direction, and that neglecting the horizontal and time variability of the fields has a relatively small
26 impact on the estimate of the material entropy production. The approach presented here seems
27 promising for testing climate models, for assessing the impact of changing their parametrizations
28 and their resolution, as well as for investigating the atmosphere of exoplanets, because it allows for
29 evaluating the error in the estimate of their thermodynamical properties due to the lack of high-
30 resolution data. The findings on the impact of coarse graining on the thermodynamic fields on
31 the estimate of the material entropy production deserve to be explored in a more general context,
32 because they provide a way for understanding the relationship between forced fluctuations and
33 dissipative processes in continuum systems.

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34 I. INTRODUCTION

35 Along the lines of the theoretical construction due to Lorenz [25, 26] of energy cycle of the
36 atmosphere, the climate can be seen as a non-equilibrium multi-scale system, which generates
37 entropy through a variety of irreversible processes [8, 30, 39–41], and transforms moist
38 static energy into mechanical energy, as it features a positive spatio-temporal correlation
39 between heating and temperature patterns, so that it can be represented schematically as a
40 heat engine with a given efficiency [14, 28]. For a given value of the external and internal
41 parameters, the climate system achieves a steady state by balancing the input and output
42 of energy and entropy with the surrounding environment [42]. The large scale motions of
43 the geophysical fluids are at the same time the result of the mechanical work produced by
44 the climatic engine, and contribute to reducing the temperature gradients which make the
45 energy conversion possible [41, 48]. Obtaining a closure to this problem would be equivalent
46 to developing a self-consistent theory of climate dynamics.

47 Developing a comprehensive theory of climate dynamics is one of the grand contemporary
48 scientific challenges, also for its obvious environmental, social and economical relevance,
49 and it is far from being an accomplished task [11, 45]. In recent years, the extraordinary
50 developments of planetary sciences coming from the discovery of extra-solar planets and the
51 ensuing need for understanding the properties of atmospheric circulations realized under
52 physical and chemical conditions very different from those of the Earth and of the other
53 solar planets has provided further stimulation in this direction [46].

54 While the thermodynamic interpretation of the baroclinic disturbances, which provide
55 the dominant contributions to the low-to-high latitudes heat transport, lies at the core of
56 dynamical meteorology [13], and thermodynamics provides indeed the best framework for
57 studying strong meteorological features like hurricanes [6], recent results suggest that the
58 structural properties of the climate system [44] and in particular its tipping points [23, 35]
59 can be effectively analyzed using the thermodynamic indicators developed in [28], with the
60 efficiency and the entropy production providing the most interesting indicators [1, 31, 32].
61 Moreover, recent studies have underlined that it is instead possible to define generalized
62 climate sensitivities able to describe quite accurately the responses of thermodynamic quan-
63 tities to changes in CO₂ concentration [29].

64 Despite its relevance at theoretical level [2, 20], traditionally, entropy production is not

65 one of the first physical quantities climate modelers investigate when assessing the perfor-
66 mance of a global climate model (GCM) or the response of the climate system to forcings.
67 One should note that the attention towards entropy production in the climate system and
68 in climate modeling has been revived when several authors started proposing it as target
69 function to maximize when tuning free or empirical parameters of approximate numerical
70 models [17, 22, 27] or for getting good first order approximations of the climate state without
71 resorting to long integrations [12, 35] This is the *weak* or *pragmatic* version of the so-called
72 maximum entropy production principle (MEPP) [18], which, in its *strong* form, proposes
73 that any non-equilibrium systems adjust itself in order to maximize the entropy production;
74 see [18, 34]. The MEPP theoretical foundations [3, 4] have been criticized both at theoretical
75 level [10] and in terms of its geophysical applications [9, 37], so that weaker formulations
76 are now mostly preferred [5]. Recently, some authors have turned their attention on testing
77 whether it is possible to propose a a variational principle for applies to another index of the
78 irreversibility of the system, namely the rate of dissipation of kinetic energy [19, 38].

79 In this paper, we wish to investigate the entropy production of a climate model for
80 studying, instead of large scale balances, its fluctuations at different temporal and spatial
81 scales. Climate is a multi-scale system where dynamics takes place on vast range of inter-
82 acting scales. The definition of parametrizations for unresolved scales is a major challenge
83 of climate modeling and the proposal of closure theories connecting small and large scale
84 properties is a major part of any attempt at formulating approximate theories for climate
85 dynamics. The issue of understanding the impact of small scales on large scales and vice-
86 versa, and of performing properly the upscaling and downscaling of a model's output is
87 of great relevance also for intercomparing the performances of various versions of a given
88 numerical model, or of a set of numerical model simulating the same system, differing for
89 the adopted spatial and temporal resolution, and for comparing model data to observations.

90 Our goal is manifold. One one side, we want to introduce a way to evaluate how the
91 different scales of motion contribute to the overall entropy production of the climate system.
92 This investigation, therefore, complements the investigation of how much energy is contained
93 in the various scales of motion and of the energy fluxes across these scales. In order to achieve
94 this goal, we consider the entropy budget of the FAMOUS GCM [15] in standard, present
95 climate configuration, taking advantage of the fact that it is one of the very few climate
96 models where the entropy production diagnostics has been implemented and throughly tested

97 [36, 37]. Starting from the output fields at the highest possible resolution given by the
98 model (temporal resolution of one time step, and same spatial resolution of the actual
99 numerical model), we perform a coarse graining in space and in time to the dynamical and
100 thermodynamical fields appearing in the terms describing the entropy production of the
101 system, and we test how the estimate of the entropy production changes when different
102 coarse graining are applied to the data. We anticipate that we obtain that the coarser is
103 the graining procedure, the lower is the estimate of the entropy production one obtains, as
104 somehow intuitive. One must note that this is, in fact an obvious result when considering
105 simple diffusive system, but not so obvious when fully nonlinear, multiphase systems are
106 considered. We also obtain similar results when considering different degrees of longitudinal
107 averaging of the fields, up to considering zonally averaged fields only. Our findings provide
108 a way to assess how having low-resolution information about the dynamics of turbulent
109 systems affects our ability to reconstruct its thermodynamical properties. Moreover, the
110 procedure discussed in this paper allows to put on firmer ground the results proposed in
111 [30] on the possibility of separating vertical and horizontal exchange processes as far as
112 entropy production is concerned. Finally, we can study in detail the relationship between
113 two apparently equivalent ways of computing the entropy production proposed in [8].

114 This paper is structured as follows. In section II we briefly recapitulate some definitions
115 and equations relevant for setting the problem of computing the entropy production of the
116 climate system we explain how to perform such a calculation in a climate model. In section
117 III we explain what we mean precisely by coarse graining of the data and describe how it
118 is actually implemented in the model's output. We also provide some conjectures what will
119 be discussed in later in the paper. In section IV we present our results. We first describe
120 the impact of performing coarse graining on time alone, thus exploring the range between
121 time step data and long term averaged data, and then we extend our analysis to the space
122 domain, showing how performing zonal, horizontal, and mass-weighted averaging over the
123 output data impacts the obtained estimate of the entropy production. In section V we
124 present our conclusions and perspective for future works. In appendix A we present some
125 theoretical arguments on a simple diffusive system for clarifying the meaning of the results
126 obtained from the data analysis.

127 II. CLIMATE ENTROPY BUDGET

128 Following [2, 20], for any system it is possible to decompose the rate of change of its
129 entropy dS/dt as $dS/dt = d_e S/dt + d_i S/dt$, where the first term is called the external and
130 the second term is the internal contribution to the entropy budget. The external contribution
131 corresponds to the entropy flux through the boundaries of the system whereas the internal
132 entropy production is associated with the irreversible processes taking place in the system.
133 The second law of thermodynamics imposes that the internal entropy production has to
134 be nonnegative at all instants, so that $d_i S/dt \geq 0$. When a statistically steady state is
135 achieved, the external and internal entropy production have to balance each other so that
136 the total rate of entropy change is zero: we have $\overline{dS/dt} = 0 \rightarrow \overline{d_i S/dt} = -\overline{d_e S/dt} \geq 0$, where
137 the overline indicates averaging over a long time interval compared to the internal scales of
138 the system. The previous expression means that a non-equilibrium system generates on
139 the average a positive amount of entropy through irreversible processes, and such excess
140 of entropy is expelled at the boundaries. Non-equilibrium is maintained if the system is
141 in contact with more than one reservoir with given temperature and/or chemical potential
142 [7]. Of course, if the system is at equilibrium, the previous inequality becomes an equality,
143 as in the long run the system reaches an homogeneous state of maximum entropy and no
144 additional entropy is generated.

145 In the climate system two rather different set of processes contribute to the total entropy
146 production [8, 41]. The first set of processes are responsible for the irreversible thermaliza-
147 tion of the photons emitted near the Sun's corona at roughly 5700 K at the much lower
148 temperatures, typical of the Earth's climate. This contributes for about 95% of the total
149 average rate of entropy production for our planet, which is about 0.90 W m^{-2} [8, 41]. The
150 remaining contribution is due to the processes responsible for mixing and diffusion inside the
151 fluid component of the Earth system, and for the dissipation of kinetic energy due to viscous
152 processes. This constitutes the so-called material entropy production, and is considered to
153 be the entropy related quantity of main interest as far as the properties of the climate system
154 are concerned. See [37] for an extensive discussion of this issue a careful estimate of its value
155 in two climate models, including the one used in this study.

156 When separating the entropy budget for radiation and for the fluid part of the climate

157 system, and taking long term averages, one can derive the following equation [8, 14]:

$$\int_V d^3\mathbf{x} \left[\overline{\left(\frac{\dot{q}_{rad}}{T}\right)} + \overline{\dot{s}_{mat}} \right] = 0 \quad (1)$$

158 where the integral is over the whole volume V of the climate system, \dot{q}_{rad} is the radiative
 159 heating rate, \dot{s}_{mat} is the instantaneous specific rate of material entropy production due to
 160 irreversible processes involving the climatic fluid, and T the temperature field. The following
 161 expression is usually adopted for \dot{s}_{mat} [14, 16, 28, 41]:

$$\dot{s}_{mat} = \frac{\epsilon^2}{T} + \mathbf{F}_{SH} \cdot \nabla \left(\frac{1}{T} \right) + \mathbf{F}_{LH} \cdot \nabla \left(\frac{1}{T} \right) \quad (2)$$

162 where ϵ^2 the specific dissipation rate of kinetic energy, \mathbf{F}_{SH} the turbulent sensible heat
 163 flux, \mathbf{F}_{LH} the turbulent latent heat flux, where by turbulent we mean not related to large
 164 scale advection due to winds, which is in principle reversible. Romps [43] refers to the
 165 representation of the entropy production given by Eq. (2) as resulting from the bulk heating
 166 budget, because water is treated mainly as a passive substance, while processes such as
 167 irreversible mixing of water vapor are altogether ignored. More detailed description of
 168 the moist atmosphere have led to a consistent treatment of the entropy generated by the
 169 various processes accounting for hydrological cycle these processes [8, 39, 40, 43]. Apparently,
 170 though, the overall effect of hydrological cycle-related entropy production is captured quite
 171 well using Eq. (2) [8, 30, 36].

172 Integrating the term \dot{s}_{mat} in Eq. (1) over the volume V of the climate system and taking
 173 a long-term average, we obtain the average rate of material entropy production:

$$\overline{\dot{S}_{mat}} = \int_V d^3\mathbf{x} \overline{\dot{s}_{mat}} = \overline{\dot{S}_{mat}^{dir}}, \quad (3)$$

174 which gives the so called *direct formula* for the material entropy production. Using Eq. (1),
 175 we derive an equivalent expression involving radiative heating rates only:

$$\overline{\dot{S}_{mat}} = - \int_V d^3\mathbf{x} \overline{\left(\frac{\dot{q}_{rad}}{T}\right)} = \overline{\dot{S}_{mat}^{ind}}, \quad (4)$$

176 which is the *indirect formula* for computing the average rate of entropy production, where
 177 obviously $\overline{\dot{S}_{mat}^{dir}} = \overline{\dot{S}_{mat}^{ind}} = \overline{\dot{S}_{mat}}$. Equation (1) provides an intimate link between the radiative
 178 fields and the material flow properties inside the climate system. Moreover, Eq. (4) is very
 179 powerful because it permits to work out the average rate of material entropy production

180 by considering only the optical properties of the fluid. Pascale et al. [36] showed using the
 181 climate model FAMOUS adopted in this study that $\overline{\dot{S}_{mat}^{dir}}$ and $\overline{\dot{S}_{mat}^{ind}}$ agree up to an excellent
 182 degree of precision (within 1%). Since Pascale et al. [36] used the approximate expression
 183 (2) for the specific material entropy production, they also confirmed that, indeed, at all
 184 practical purposes using such simplified representation of the irreversibility associated to
 185 the hydrological cycle is appropriate.

186 A. Entropy diagnostics in Climate Models

187 In an actual climate model the implementation of entropy diagnostics faces some difficul-
 188 ties, both at theoretical level and in terms of practical implementation of the entropy-related
 189 diagnostics. A theoretical difficulty is that, as evidenced in [33], many state-of-the-art cli-
 190 mate models features an inconsistent energetics, such that when all parameters are held
 191 fixed and the system reaches a steady state, the long-term average of the energy budget
 192 at the top of the atmosphere (TOA), which is the only boundary of the climate system, is
 193 unexpectedly biased with respect to the vanishing long-term average one should expect to
 194 observe. Interestingly, all biased models feature a positive energy budget at TOA, which
 195 implies that the time averaged outgoing long wave radiative flux is smaller than the net
 196 incoming shortwave flux. This fact implies that there must be a positive definite spurious
 197 sink of energy somewhere inside the system. More specific analyses make clear that such
 198 spurious sinks are related to the imperfect closure of the hydrological cycle [24] and to the
 199 inconsistent treatment of the dissipation of kinetic energy, which is not entirely (or at all)
 200 fed back into the system as thermal energy [33]. Such inconsistencies at smallspatial and
 201 temporal scales impact large scale, long term climatic properties. As a result, climate models
 202 are biased cold, taking into consideration that the Earth emits approximately as black body,
 203 or feature negative biases in the planetary albedo, or both. Moreover, since the biases are
 204 related to climate processes, they are climate-dependent, and so hard to control a posteriori
 205 via removal of anomalies. In terms of entropy production, an energy bias of the order of
 206 1 W m^{-2} causes a bias in the entropy production of about $4 \times 10^{-3} \text{ W m}^{-2} \text{ K}^{-1}$, which
 207 is comparable with the range of estimates of material entropy production given by various
 208 climate models [30, 36]. The FAMOUS model we use in this study features minor inconsis-
 209 tencies in terms of closure of the energy budget (the bias is smaller than 0.1 W m^{-2} , so that

210 the problem exposed here does not affect significantly our results (see discussion later).

211 Moreover, in a climate model it is hard to deal directly with Eq. (2) because material
 212 turbulent fluxes are evaluated through parametrizations of unresolved processes. The cor-
 213 responding routines in the numerical code do not give as outputs heat fluxes. On the other
 214 hand the heating rates (i.e. the divergence of the heat fluxes) are easily diagnosed for these
 215 unresolved processes. Neglecting the geothermal flux from the inner Earth and noting that
 216 at the top-of-the-atmosphere we have only radiative fields, using Gauss' theorem, the mate-
 217 rial entropy production can be worked out by considering all the material diabatic heating
 218 rates, as shown in [36]:

$$\overline{\dot{S}_{mat}^{dir}} = \int_V d^3\mathbf{x} \left(\overline{\left(\frac{\epsilon^2}{T} \right)} - \overline{\left(\frac{\nabla \cdot \mathbf{F}_{SH}}{T} \right)} - \overline{\left(\frac{\nabla \cdot \mathbf{F}_{LH}}{T} \right)} \right) = \int_V d^3\mathbf{x} \overline{\left(\frac{\dot{q}_{mat}}{T} \right)} \quad (5)$$

219 In this paper we refer to the entropy budget of the FAMOUS GCM [15] which has been
 220 studied in detail by [36, 37]. Lets first focus on the evaluation of $\overline{\dot{S}_{mat}^{dir}}$. Different pro-
 221 cesses contribute to the entropy production terms described in Eq.(2): the heating rates
 222 are calculated as output of many different parametrization routines describing the unre-
 223 solved processes in the various subdomains of the climate system (atmosphere, ocean, soil,
 224 cryosphere):

- 225 • Entropy production due to dissipation of kinetic energy, $\overline{\dot{S}_{KE}}$, defined as:

$$\overline{\dot{S}_{KE}} = \int d^3\mathbf{x} \overline{\left(\frac{\epsilon^2}{T} \right)}. \quad (6)$$

226 In FAMOUS and, in general, in most climate models, the kinetic energy is dissipated
 227 mainly through four parametrized processes: the turbulent stresses occurring at the
 228 boundary layer, which extract kinetic energy from the free atmosphere, the gravity
 229 wave drag, which dissipates kinetic energy in the upper atmosphere, atmospheric con-
 230 vective processes, and small scale turbulence, which is represented by the horizontal
 231 momentum hyperdiffusion (which serves also the purpose of increasing the numerical
 232 stability of the model). In FAMOUS only the atmosphere contributes to this part of
 233 the entropy production. This is a reasonable approximation because the dissipation
 234 of kinetic energy occurring in the atmosphere is about two orders of magnitude larger
 235 than that occurring in the ocean [41, 49].

- 236 • Entropy production due to irreversible transfer of sensible and latent heat via turbulent

237 fluxes, $\overline{\dot{S}_{heat}} = \overline{\dot{S}_{SH}} + \overline{\dot{S}_{LH}}$, defined as:

$$\overline{\dot{S}_{heat}} = \int d^3\mathbf{x} \left[-\overline{\left(\frac{\nabla \cdot \mathbf{F}_{SH}}{T}\right)} - \overline{\left(\frac{\nabla \cdot \mathbf{F}_{LH}}{T}\right)} \right] = \overline{\dot{S}_{SH}} + \overline{\dot{S}_{LH}} \quad (7)$$

238 The boundary layer scheme contributes to the entropy production due to irreversible
 239 sensible and latent heat transfer in the four subdomains of the climate system, as it
 240 couples them through exchanges of sensible heat and water vapour; other parametrized
 241 processes contributing to $\overline{\dot{S}_{SH}}$ and $\overline{\dot{S}_{LH}}$ are atmospheric convection and the conden-
 242 sation and evaporation of water in the atmosphere, as determined by the clouds and
 243 precipitation parametrization schemes. Instead, processes contributing only to $\overline{\dot{S}_{SH}}$
 244 are the oceanic convection, the small scale turbulent mixing of temperature described
 245 by hyperdiffusion, and the mixing occurring inside the ocean associated to small scale
 246 eddies and in the mixed layer.

247 Table I provides a synthetic outline of which routines describing unresolved processes con-
 248 tribute to the various terms of the material entropy production in each climatic subdomain.
 249 Therefore, in practice, we compute $\overline{\dot{S}_{mat}^{dir}}$ as follows:

$$\overline{\dot{S}_{mat}^{dir}} = \sum_k \sum_c \int_{V_c} d^3\mathbf{x} \overline{\left(\frac{\dot{q}_k^c}{T}\right)} \quad (8)$$

250 where \dot{q}_k^c is the local instantaneous heating rate occurring in the subdomain V_c due to the
 251 process k .

252 The evaluation of $\overline{\dot{S}_{mat}^{ind}}$ is much easier because the heating rates are readily available from
 253 the radiation scheme, which affects all the subdomains c of the climate system:

$$\overline{\dot{S}_{mat}^{ind}} = - \sum_c \int_{V_c} d^3\mathbf{x} \left[\overline{\left(\frac{\dot{q}_{sw}^c}{T}\right)} + \overline{\left(\frac{\dot{q}_{lw}^c}{T}\right)} \right] \quad (9)$$

254 where we have divided the contribution \dot{q}_{sw} coming from the shortwave radiation, which
 255 is only absorbed (and scattered), inside the climate systems, so that $\dot{q}_{sw} \geq 0$, from the
 256 contribution \dot{q}_{lw} coming from the longwave radiation, which instead is scattered, absorbed,
 257 and emitted, and is the sole responsible for the radiative cooling.

258 **III. COARSE-GRAINING OF THE ENTROPY PRODUCTION TERMS: DEFINITIONS AND SOME CONJECTURES**
 259

260 The entropy budget is estimated using space and time integrals of the ratio between
 261 the local heating term and the local temperature. In many cases, either because we need
 262 to compress data or because climatological database only contain certain time or spatially
 263 averaged data, we have to deal with coarse grained data for the heating rate $\langle q(\mathbf{x}, t) \rangle_v^\tau$, and
 264 for the temperature $\langle T(\mathbf{x}, t) \rangle_v^\tau$, where τ refers to the time scale of the temporal averaging
 265 operation, and v refers to the set of stencil regions $v(x)$ centered over x over which (mass-
 266 weighted) spatial averaging is performed:

$$\langle X(\mathbf{x}, t) \rangle_v^\tau = \frac{1}{\tau \mu(v, t)} \int_v d^3 \mathbf{y} \int_{-\tau/2}^{\tau/2} d\sigma X(\mathbf{x} + \mathbf{y}, t + \sigma) \quad (10)$$

where $\mu(v(x), t) = \int_{v(x)} d^3 \mathbf{x}$ is the mass contained in the stencil $v(x)$ at time t . Mass-weighting is the natural choice in climate models as hydrostatic approximation is almost invariably used and vertical coordinates are expressed to a very good approximation in terms of pressure levels. Since the integrands in Eqs. (4) and (5) are nonlinear, we obviously have that for every τ and v :

$$\overline{\dot{S}_{mat}^{dir}} = \int_V d^3 \mathbf{x} \overline{\left(\frac{\dot{q}_{mat}}{T} \right)} \neq \int_V d^3 \mathbf{x} \overline{\left(\frac{\langle \dot{q}_{mat} \rangle_v^\tau}{\langle T \rangle_v^\tau} \right)} = \overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau}, \quad (11)$$

$$\overline{\dot{S}_{mat}^{ind}} = - \int_V d^3 \mathbf{x} \overline{\left(\frac{\dot{q}_{rad}}{T} \right)} \neq - \int_V d^3 \mathbf{x} \overline{\left(\frac{\langle \dot{q}_{rad} \rangle_v^\tau}{\langle T \rangle_v^\tau} \right)} = \overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau}. \quad (12)$$

Moreover, while as discussed before $\overline{\dot{S}_{mat}^{dir}} = \overline{\dot{S}_{mat}^{ind}}$, there is no *a priori* reason to expect that $\overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau}$ and $\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau}$ have the same value. Finally, we have that up to first order:

$$\overline{\dot{S}_{mat}^{dir}} - \overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau} = \Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]_v^\tau \simeq - \int_V d^3 \mathbf{x} \frac{\Delta [\dot{q}_{mat}]_v^\tau \Delta [T]_v^\tau}{[\langle T \rangle_v^\tau]^2} \quad (13)$$

$$\overline{\dot{S}_{mat}^{ind}} - \overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau} = \Delta \left[\overline{\dot{S}_{mat}^{ind}} \right]_v^\tau \simeq \int_V d^3 \mathbf{x} \frac{\Delta [\dot{q}_{rad}]_v^\tau \Delta [T]_v^\tau}{[\langle T \rangle_v^\tau]^2} \quad (14)$$

267 where $\Delta [X]_v^\tau = X - \langle X \rangle_v^\tau$. It is natural to interpret $\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau}$, $\overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau}$ as the contribution
 268 to the entropy production due to irreversible processes occurring on scales large than what
 269 described by τ and v . Consequently, $\Delta \left[\overline{\dot{S}_{mat}^{ind}} \right]_v^\tau$, $\Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]_v^\tau$ in Eqs. (13)-(14) can be inter-
 270 preted as the contributions to the entropy production given by the material flows (Eq. (13))
 271 and radiative fluxes (Eq. (14)) with variability confined below the spatial scale given by v
 272 and by the time scale given by τ .

273 Equations (11)-(14) address practical questions such as: what is the error related to
 274 remapping the output of a climate model to a new resolution in space and time? How do
 275 diurnal, seasonal and interannual variability and how different spatial structures (midlatitude
 276 cyclones, equator-pole contrasts, longitudinal asymmetries due to ocean-land contrasts, etc)
 277 affect the entropy budget? How should we proceed to compare the estimates of material
 278 entropy production from models with different resolutions? Moreover, we need to understand
 279 whether it is more accurate to obtain estimates of the material entropy production from
 280 coarse grained fields of the radiative heating rates or of the material heating rates, which
 281 can be used for the indirect or direct formula for the entropy production, respectively. These
 282 issues may be sequentially investigated by filtering $q(\mathbf{x}, t)$ and $T(\mathbf{x}, t)$ over the associated
 283 time- and space- scales.

284 When we perform the coarse graining given in Eq. (10) to the thermodynamic variables
 285 and estimate the entropy production, we discount for the mixing processes occurring below
 286 the chosen spatial and time scales. Therefore, one expects that $\overline{\dot{S}_{mat}^{ind}/v}^\tau, \overline{\dot{S}_{mat}^{dir}}^\tau \geq 0$ and
 287 $\Delta \left[\dot{S}_{mat}^{ind} \right]_v^\tau, \Delta \left[\dot{S}_{mat}^{dir} \right]_v^\tau \geq 0$ for all choices of τ and v . Moreover, it seems natural to conjecture
 288 that if, given a model output, we choose a coarser graining, we should obtain a lower estimate
 289 of the entropy production, because we neglect the impact of a larger set of irreversible
 290 processes. In other terms, we should have that $\Delta \left[\dot{S}_{mat}^{dir} \right]_{v_1}^{\tau_1} \geq \Delta \left[\dot{S}_{mat}^{dir} \right]_{v_2}^{\tau_2}$ (or $\overline{\dot{S}_{mat}^{dir}}_{v_1}^{\tau_1} \leq$
 291 $\overline{\dot{S}_{mat}^{dir}}_{v_2}^{\tau_2}$) and $\Delta \left[\dot{S}_{mat}^{ind} \right]_{v_1}^{\tau_1} \geq \Delta \left[\dot{S}_{mat}^{ind} \right]_{v_2}^{\tau_2}$ (or $\overline{\dot{S}_{mat}^{ind}}_{v_1}^{\tau_1} \leq \overline{\dot{S}_{mat}^{ind}}_{v_2}^{\tau_2}$) if $\tau_2 \leq \tau_1$ and $v_2 \subset v_1$.
 292 Let's see how to interpret the inequalities $\Delta \left[\dot{S}_{mat}^{ind} \right]_v^\tau, \Delta \left[\dot{S}_{mat}^{dir} \right]_v^\tau \geq 0$ using the r.h.s. of Eqs.
 293 (13)-(14):

- 294 • The inequality $\Delta \left[\dot{S}_{mat}^{ind} \right]_v^\tau \geq 0$ can be interpreted as the fact that at all time and space
 295 scales, there is on the global average a *positive* correlation between the anomalies of
 296 radiative heating and the anomalies of temperature. This expresses the basic fact that
 297 the climate system is driven by radiative forcings, in the first place. Hence, this term
 298 refers to the *response of the system to the external forcing*. Note that the inequality
 299 holds despite the the strong negative correlation between temperature anomalies and
 300 long wave heating rate anomalies due to the Boltzmann feedback.
- 301 • The other inequality $\Delta \left[\dot{S}_{mat}^{dir} \right]_v^\tau \geq 0$, instead, implies that at all time and space scales
 302 on the average there is a *negative* correlation between the anomalies of heating due
 303 to convergence of material heat fluxes and anomalies of temperatures. This relation,

304 instead, expresses the fact that temperature anomalies are damped by the geophysical
 305 flows, and this terms refers to the *dissipation* occurring inside the system at all scales.

306 In other terms, these conjectured inequalities correspond to the well-known fact that the
 307 climate is 1. forced by anomalies in the radiative forcing, and 2. the atmospheric and oceanic
 308 circulations reduce the resulting temperature gradients. Climate processes are related in
 309 such a way at all scales, *in the average*, i.e. when space and time averages are considered.
 310 Obviously, locally in space and/or in time one can get, e.g. positive temperature fluctuations
 311 and, at the same time, a positive heating due to latent heat release and sensible heat
 312 convergence (e.g. tropical troposphere). Such processes can be positively correlated in time
 313 for some locations, but this must come at the expenses of negative correlations dominating
 314 elsewhere in the globe.

As the primary driving of climate is indeed the radiative forcing, while the fluid flows
 tend to dampen the resulting temperature gradients through instabilities, Therefore, one
 expects that the correlations between temperature and heating fields are stronger when
 considering the radiative fields as sources of heating. In other terms, the convergence of
 heat due to geophysical flows are neither strong nor fast enough to counter exactly the
 radiative forcing at all scales. As an example, one may consider the fact that the radiative-
 convective equilibrium is typically baroclinically unstable in the mid-latitudes, and, indeed,
 baroclinic disturbances reduce the North-South temperature gradient by transporting heat
 from South to North, but cannot reduce it to zero. Taking into consideration Eqs. (13)-(14),
 the different role - forcings vs. dampening - of the convergence of the radiative fluxes vs.
 material turbulent fluxes leads us to proposing an additional inequality. We conjecture that

$$\Delta \left[\overline{S_{mat}^{ind}} \right]_v^\tau \geq \Delta \left[\overline{S_{mat}^{dir}} \right]_v^\tau \quad \forall \tau, v,$$

from which, since $\overline{S_{mat}^{dir}} = \overline{S_{mat}^{ind}}$, we derive the following inequality

$$\langle \overline{S_{mat}^{dir}} \rangle_v^\tau \geq \langle \overline{S_{mat}^{ind}} \rangle_v^\tau, \quad \forall \tau, v.$$

315 IV. RESULTS

316 We first discuss briefly how the coarse graining operation is performed in practice. Let
 317 us consider a steady-state climate simulation lasting for a time period L (in our case $L = 50$

318 years), which we divide it in N sub-intervals $\tau = L/N$, where $\tau = M \times dt$, where dt is the
319 model's time step (1 h in our case). The horizontal resolution is specified by regular grids
320 with angle resolution of $5^\circ \times 7.5^\circ$ lat-lon), while in the vertical we have 11 levels for the
321 atmosphere, 20 oceanic levels, and 3 land surface levels [15]. Therefore, we subdivide the
322 domain V of integration into Q subdomains v_q , $q = 1, \dots, Q$, each containing (in the bulk of
323 the model's domain) R grid points. Given an intensive thermodynamic field $X(\mathbf{x}_k, t_j)$, for
324 $n = 1, \dots, N$ and $q = 1, \dots, Q$, we define its coarse grained version as:

$$\langle X(q, n) \rangle_v^\tau = \frac{1}{\tau \mu(v_q, n)} \sum_{j=R(q-1)+1}^{Rq} \sum_{k=M(n-1)+1}^{Mn} dt \mu(\mathbf{x}_j, \sigma_k) X(\mathbf{x}_j, \sigma_k), \quad (15)$$

325 where $\mu(\mathbf{x}_j, \sigma_k) = \nu(\mathbf{x}_j) \rho(\mathbf{x}_j, \sigma_k)$ is the mass contained in the grid box centered around \mathbf{x}_j of
326 volume $\nu(\mathbf{x}_j)$ at time σ_k and $\mu(v_q, n)$, correspondingly, is the time averaged (for time ranging
327 from $t_{M(n-1)+1}$ and t_{Mn}) mass contained in the domain v_q of volume $\nu(v_q)$. Therefore, our
328 estimate of the coarse grained value of the material entropy production is:

$$\overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau} = \frac{1}{N} \sum_{i=1}^Q \sum_{l=1}^N \nu(v_i) \frac{\langle \dot{q}_{mat}(i, l) \rangle_v^\tau}{\langle T(i, l) \rangle_v^\tau} \quad (16)$$

329 for the so-called direct formula, and:

$$\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau} = -\frac{1}{N} \sum_{i=1}^Q \sum_{l=1}^N \nu(v_i) \frac{\langle \dot{q}_{rad}(i, l) \rangle_v^\tau}{\langle T(i, l) \rangle_v^\tau} \quad (17)$$

330 for the so-called indirect formula. These formulas are the discretized versions of Eq. (11)
331 and Eq. (12), respectively. Obviously, the discrete versions of the exact formulas for the
332 material entropy production $\overline{\dot{S}_{mat}^{dir}}$ and $\overline{\dot{S}_{mat}^{ind}}$ are obtained by setting in Eq. (15) $R = M = 1$,
333 i.e., taking the model outputs at the highest possible resolution. The processes occurring
334 in the interior of the ocean and below the first soil level, as these contributions have been
335 shown to be entirely negligible in terms of entropy production [37], and so are discarded.

336 If we choose a given spatial resolution of our data and we consider different values of
337 M , we test how applying temporal coarse graining impacts the estimates of the material
338 entropy production. Instead, if we change the shape of the stencil v and/or the number of
339 points R while keeping M fixed, we investigate the impact of changing the spatial coarse
340 graining scheme. Obviously, we cannot capture the contributions to the material entropy
341 production due to irreversible processes taking place over timescale shorter than the model
342 timestep and over space scales smaller than the model resolution. It is not clear, given a

343 specific model’s settings, how relevant these could be, and, indeed, the only way to find this
 344 out is to alter the model’s resolution. This procedure may have relevance in terms of model
 345 tuning, as one could decide to change a model’s parameter when altering its resolution in
 346 such a way to keep the entropy production constant.

347 **A. Temporal coarse graining**

348 We start our investigation by performing coarse graining exclusively on time. We then
 349 analyze a long, steady state model’s run lasting 50 years with a model’s timestep of 1 hour,
 350 and consider 1 year as long-term averaging time. We use the following values for τ : 1 hour
 351 (model timestep, $N = 1$), 6 hours ($N = 6$), 12 hours ($N = 12$), 1 day ($N = 24$), 2 days
 352 ($N = 48$), 5 days ($N = 120$), 10 days ($N = 240$), 15 ($N = 360$) days, 1 month ($N = 720$),
 353 3 months ($N = 2160$), 6 months ($N = 4320$), 1 year ($N = 8640$). We then collect the
 354 50 1-year averaged value of the coarse grained material entropy production and compute
 355 the mean and standard deviation for the 50 data we have. Moreover, we consider longer
 356 averaging periods - 5 years, 10 years, and 50 years, and take in these cases τ equal to the
 357 averaging time, so that $N = 43200$, $N = 86400$, and $N = 432000$ in the $\tau = 5$, 10, and
 358 50 years case, respectively, thus spanning in total more than 5 orders of magnitude for N .
 359 We then compute for the coarse-grained estimates of the material entropy production the
 360 ten 5–year averages and the five 10–year averages, and compute the mean and standard
 361 deviation, plus the unique value referred to the 50– year average. The statistics for such
 362 large values of τ are extremely stable.

363 In Fig.1(a) we report the estimates of the material entropy production obtained through
 364 the direct formula $\overline{\langle \dot{S}_{mat}^{dir} \rangle_\tau}$ and the indirect formula $\overline{\langle \dot{S}_{ind}^{dir} \rangle_\tau}$, respectively, where we have
 365 dropped the lower index v because we do not perform any spatial coarse graining. The
 366 vertical bars indicate the uncertainty due to the long term variability.

367 The computed values (worked out at each timestep) of $\overline{\dot{S}_{mat}^{ind}} \approx 53.1 \text{ mW m}^{-2} \text{ K}^{-1}$
 368 ($1 \text{ mW} = 10^{-3} \text{ W}$) and $\overline{\dot{S}_{mat}^{dir}} \approx 53.5 \text{ mW m}^{-2} \text{ K}^{-1}$ have a difference of about 0.4 mW
 369 $\text{m}^{-2} \text{ K}^{-1}$, so that Eq. (1) is verified with great accuracy. The discrepancy term between the
 370 two estimates is due to the extremely small spurious radiative imbalance at TOA of about
 371 0.1 W m^{-2} (see [30]) and to numerical inaccuracies. Moreover, as discussed in [30, 37],
 372 these estimates are in good agreement with what found in climate models of higher degree

373 of complexity.

374 As conjectured, we find that the estimates of the coarse grained entropy production
375 decrease with increasing τ from these reference values obtained with no temporal coarse
376 graining. The bias resulting from the use of the indirect formula is larger for all values of τ .
377 In Fig. 1(a) we see that if we consider values of τ up to 6 hours, the impact of coarse graining
378 is extremely small. This implies that such small time scales the irreversible processes are
379 negligible; this matches well with the fact that convection, which is the dominating fast
380 process in the climate system, is parametrized with an instantaneous adjustment. This
381 immediately points to an unwelcome spurious effects of climate parametrizations.

382 The effect of coarse graining becomes more relevant when $\tau \geq 1$ day. Figures 1(b) and
383 1(c) present the values of $\Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]^\tau$ and $\Delta \left[\overline{\dot{S}_{mat}^{ind}} \right]^\tau$ as a function of τ . For $\tau \sim 1$ day, the
384 difference between the true and the coarse grained value of the entropy production is about
385 $\approx 0.6 \text{ mW m}^{-2} \text{ K}^{-1}$ if we use the direct formula, and $\approx 2 \text{ mW m}^{-2} \text{ K}^{-1}$ is we use the
386 indirect formula. Such biases are due to neglecting the mixing occurring on the time scale
387 of the day, mostly due related to the daily cycle of incoming radiation. When considering
388 the direct formula, it is interesting to note that $\Delta \left[\overline{\dot{S}_{KE}^{dir}} \right]^\tau$ is basically zero for all values of
389 τ (not shown), meaning that there is no time correlation between the dissipation of kinetic
390 energy and the temperature field. The coarse graining, instead, impacts the contribution to
391 entropy production due to the hydrological cycle. We can substantiate this statement by
392 observing that $\Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]^\tau \sim \Delta \left[\overline{\dot{S}_{heat}^{dir}} \right]^\tau$ (see definition of the latter in Eq. 8), as can be seen
393 by comparing Figs. 1(c) and 2(a).

394 The second timescale worth discussing is the one corresponding to 1 year ($\sim 3 \times 10^7$ s).
395 The use of annual means instead of time-step data introduces a bias of about $4 \text{ mW m}^{-2} \text{ K}^{-1}$
396 when using the indirect formula, which corresponds to neglecting the correlation between the
397 seasonal cycle of the radiation budget and that of the radiation temperature field. Similarly,
398 considering the direct formula, we obtain $\Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]^\tau \sim 1.5 \text{ mW m}^{-2} \text{ K}^{-1}$, which measures
399 the effect of neglecting the correlation of the seasonal cycle of the atmospheric and oceanic
400 transport and dissipation and of the temperature field. One must note that a considerable
401 contribution to the value of $\Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]^\tau$ for $\tau \geq 1$ year is given by the atmospheric temperature
402 hyperdiffusion, which in FAMOUS is implemented as a eight-order laplacian operator and
403 applied after the advection to the model prognostic variables. Hyperdiffusion is generally
404 introduced in dynamic cores for numerical reasons in order to smooth variables and avoid

405 local divergences. However it may thought as a way to represent turbulent dissipation and
 406 mixing at subgrid scale. We discover that a traditional numerical *trick* used in the climate
 407 modeling community for avoiding computational instabilities impacts a global scale physical
 408 properties of the system, as observed in [33] when looking at energy budgets.

409 We also observe that there is no clear signature emerging in the functions $\Delta \left[\overline{S_{mat}^{dir}} \right]^\tau$ and
 410 $\Delta \left[\overline{S_{mat}^{ind}} \right]^\tau$ for values of τ to weeks (synoptic waves) or monthly (low frequency variability)
 411 time scales while a relatively smooth transitions is found going from daily to yearly averages.
 412 This supports the idea that it is possible to look at weather disturbances as parts of a macro-
 413 turbulent cascade.

414 Estimating the entropy production via either the direct or the indirect formula using
 415 long term averages (but full spatial resolution) leads to underestimate the exact value of
 416 the entropy production by less than 10%. This suggests that long term averages of the
 417 climatic fields one can obtain from the climate repositories are enough to get a good idea
 418 of the properties of the climate system. As we shall see in the next section, things change
 419 drastically when the coarse graining impacts the spatial features of the climatic fields.

420 We conclude this section with a note on the oceanic processes, which we do not treat
 421 in this paper as they contribute negligibly to the overall entropy production in the climate
 422 system. in Fig. 2(a) we show the dependence of the entropy production due to the oceanic
 423 mixing on the temporal coarse graining (dashed) line. We discover that its exact value,
 424 computed at time step, is about $1 \text{ mW m}^{-2} \text{ K}^{-1}$, as in [37], and its coarse grained value
 425 does not noticeably decrease up to $\tau \sim 1$ year, above which the coarse grained estimate is
 426 roughly halved. The dash-dotted line in Fig. 2(a) gives the contribution due to the vertical
 427 mixing in the interior of the ocean, which is a very slow process and is, in fact, weakly
 428 affected by the temporal coarse graining. The other contribution to the entropy production
 429 in the ocean comes from the mixing occurring in the mixed layer. The mixing layer scheme
 430 [21] parametrizes the convection due to heating at depth and cooling at the surface as well
 431 as the mechanical stirring due to wind and is introduced in ocean models in order to account
 432 for the seasonal thermocline variations. The coarse grained value of this term goes virtually
 433 to zero for $\tau \geq 1$ because, when considering such an averaging, we discount for the impact
 434 of the seasonal cycle in the upper portion of the ocean.

435 B. Spatial and Temporal coarse-graining

436 In this section we analyze the combined effect of coarse graining the heating rates and
437 temperature fields in space and time by using extensively Eqs. 16 and 17. Of course, there
438 are many ways to perform coarse graining, boiling down to the selection of the stencil v
439 introduced before. Summarizing, we proceed as follows:

- 440 1. longitudinal averaging: the stencils v are given by arcs of varying length in the zonal
441 direction;
- 442 2. areal averaging: the stencils v are given by portions of varying size of the spherical
443 surface;
- 444 3. mass averaging: the stencils v are given by same-mass portions of the atmospheric
445 spherical shell obtained by thickening in the vertical direction the stencils described
446 in 2.;

447 in all cases we perform also temporal coarse graining by selecting the same averaging times
448 τ described in the previous subsection.

449 It is important to note that the averaging as in points 1. and 2 is performed at constant x_3 .
450 FAMOUS (and HadCM3) uses hybrid vertical coordinates, *i.e.* a coordinate system which
451 changes smoothly from a terrain-following specification near the lower boundary (σ coords.)
452 to a isobaric definition (p coords.) in the medium-upper troposphere and stratosphere. As
453 clear from Eqs. 15-17, the result of any coarse graining performed at constant value of
454 the vertical coordinates depends on the vertical coordinate considered. In order to avoid
455 the spurious effects of remapping the thermodynamic fields to a new coordinate system,
456 we choose coarse grained grid boxes that respect as much as possible the original model's
457 resolution.

458 We also remark that given the heavy computational burden of the operation, we restrict
459 our analysis to only one of the fifty year of available data. We have tested that

460 1. Longitudinal Averaging

461 We first investigate the effect of coarse graining on the estimate of the material entropy
462 production by averaging longitudinally the thermodynamic fields, up to the point of con-

463 sidering zonally averaged only fields, and by degrading their temporal resolution by using
 464 the averaging times τ described above. The estimates of $\overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau}$ and $\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau}$ are given in
 465 Fig. 3(a) and Fig. 3(c), respectively. The corresponding values of $\Delta \left[\dot{S}_{mat}^{dir} \right]_v^\tau$ and $\Delta \left[\dot{S}_{mat}^{ind} \right]_v^\tau$
 466 are reported in Fig. 3(b) and Fig. 3(d), respectively, and some specific results are given in
 467 Table II.

468 In all figures, the value of τ is reported in the abscissae, while in the ordinates the value of
 469 size of the spatial stencil, ranging from 7.5° (no coarse graining) to 360° (zonal averaging) is
 470 shown. In both figures, the lower left corner corresponds to the best estimate of the entropy
 471 production; the upper left corner corresponds to the material entropy production due to the
 472 longitudinally averaged, high-temporal resolution fields. the lower right corner corresponds
 473 to long-time averaged, high-resolution spatial case, and, eventually, the upper right corner
 474 corresponds to the highest degree of coarse graining: it represent the entropy production due
 475 to the long-term averaged, longitudinally averaged fields, and features the lowest value of
 476 $\overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau}$ and $\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau}$. We remark that the values reported at the border of the domain given
 477 by the lowest value of the ordinates coincide, obviously, with what shown in Fig. 1(a). As a
 478 general fact, we observe that the coarse grained estimates of the entropy production decrease
 479 (or remain virtually unchanged) as we perform coarser and coarser graining procedure, in
 480 time or in space, and that $\overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau} \geq \overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau}$.

481 The strongest dependence of the $\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau}$ is on τ : temporal coarse graining appears to
 482 be the dominating influence, while the effect of spatial coarse graining is apparent only
 483 for $\tau \leq 1$ day and for considerable longitudinal averaging, such that features below 60°
 484 are smeared out. In other terms, longitudinal averaging starts to matter only when we
 485 lose information on the alternating pattern continents/oceans. The total effect of removing
 486 totally the spatial structure is similar to that of performing a time-averaging of one day.
 487 When considering $\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau}$ the picture is partially different: first, the influence of the spatial
 488 averaging is relatively strong at all scales for $\tau \leq 1$ day. The coupling between the spatial
 489 and temporal scales indicates that using low pass filter and space and time we remove the
 490 fast traveling synoptic waves of the mid-latitudes. As opposed to the case of the coarse
 491 grained indirect estimate of the entropy production, spatial averaging plays a role also for 1
 492 day $\leq \tau \leq 3$ months. This is probably the signature of the relevance of low-frequency, large
 493 scale features of the tropical circulation, which are sustained by longitudinal gradients (and
 494 tend to reduce them), which are smeared out when extreme coarse graining is applied.

495 Concluding, we remark that neglecting information on the longitudinal fluctuations and
 496 temporal fluctuations of the thermodynamic fields does not bias considerably (in the worst
 497 case, by about 10%) the estimates of the entropy production one would obtain by retaining
 498 the full information. This agrees with the fact that, in first approximation, in our planet
 499 longitudinal gradients and longitudinal heat fluxes are relatively small [41]. Moreover, using
 500 either the direct or indirect formula we obtain rather similar results, with a bias of maximum
 501 5%. As expected, the direct formula gives more accurate estimates for all considered coarse
 502 graining procedures.

503 2. Areal averaging

504 As a second step for understanding the role of spatial and temporal coarse graining of
 505 the estimate of the material entropy production, we combine time averaging of the ther-
 506 modynamic fields with areal averaging along horizontal surfaces. This operation allows us
 507 to explore how the two-dimensional spatial covariance of heating and temperature fields
 508 contributes to entropy production at all time scales. In order to keep coherence with the
 509 previous coarse graining procedure, we proceed as follows. We divide the spherical surface in
 510 coarse grained grid boxes defined by intervals (in degrees in latitude and longitude) $(\Delta\lambda\Delta\phi)$
 511 such that $\Delta\phi/\Delta\lambda = 1.5$, which is consistent with the model's resolution of $5^\circ\text{lat} \times 7.5^\circ\text{lon}$.
 512 We then increase $\Delta\phi$ from 7.5° up to 90° , thus decreasing progressively the number of coarse
 513 grained grids from 1728 to 12. In order to complete the coarse graining, we select as two
 514 coarsest resolutions $(\Delta\lambda; \Delta\phi) = (90^\circ, 180^\circ)$ (four quadrants) and $(\Delta\lambda; \Delta\phi) = (180^\circ, 360^\circ)$
 515 (full spherical surface).

516 The estimates of $\overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau}$ and $\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau}$ are given in Fig. 4(a) and Fig. 4(c), respectively. The
 517 corresponding values of $\Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]_v^\tau$ and $\Delta \left[\overline{\dot{S}_{mat}^{ind}} \right]_v^\tau$ are reported in Fig. 4(b) and Fig. 4(d),
 518 respectively, and some specific results are given in Table II. We discover that, as opposed
 519 to the previous case, the impact of selecting coarser and coarser graining in space reduces
 520 considerably the value of $\overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau}$ for all values of τ , because such an averaging progressively
 521 removes the strong meridional dependence of the thermodynamic fields, up to the extreme
 522 case of v being the whole Earth's surface. In this case, the estimate of the entropy production
 523 $\overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau} \sim 47.2 \text{ mWm}^{-2}\text{K}^{-1}$. Note that when considering very strong spatial averaging the
 524 effect of changing τ is negligible, because the spatial averaging alone reduces the temporal

525 correlations by mixing areas of the planet experiencing, e.g., different seasons. The τ -
 526 dependence of $\overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau}$ is relevant only for $\tau \leq 1$ day and spatial scales smaller than \sim
 527 $3 - 4 \times 10^6$ m, which is, like in the previous case, hints at the fact that when averaging
 528 over large spatial and temporal scales, we remove the variability corresponding to synoptic
 529 waves.

530 The function $\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau}$ has a qualitatively similar but quantitatively stronger dependence
 531 on $\Delta\phi$ and τ with respect to $\overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau}$: the bias in the estimate production is larger for all
 532 the considered coarse graining. Additionally, the indirect formula is more strongly affected
 533 by averaging over long time scales τ , similar to what seen in Fig. 3(c), because the coupling
 534 between the seasonal cycle of the radiative budget and the temperature fields is very strong.
 535 Note that when we consider global or quasi-global spatial coarse graining, such effect disap-
 536 pears, because averaging such large scales already removes a large part of the season cycle
 537 signal. This is different from what reported in Fig. 3(c), because zonal averaging, obviously,
 538 cannot remove the asymmetry between northern and southern hemisphere.

539 3. Mass Averaging

540 Finally, we perform spatial averaging along the horizontal and vertical direction and
 541 for different values of τ for the direct and indirect formula of entropy production. In this
 542 way, we are able to ascertain the relevance of the processes involving irreversible fluxes
 543 across temperature gradients along the vertical direction. Results are reported in Fig. 5
 544 and Fig. 6, respectively, , and some specific results are given in Table III. Since we are
 545 now dealing with three variables describing the coarse graining - the amplitude in latitude
 546 $\Delta\phi$, the number of levels n , and τ , we present two cross sections obtained for $\tau = 1$ day
 547 (virtually indistinguishable from $\tau = 1$ hour) and $\tau = 1$ year, reported as panels (a) and (c),
 548 respectively, in both Figs. 5 and 6. In panels (b) and (d) of these two figures, we report,
 549 instead, $\Delta \left[\overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau} \right]$ and $\Delta \left[\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau} \right]$, respectively. The inequality $\Delta \left[\overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau} \right] < \Delta \left[\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau} \right]$ is
 550 clearly obeyed.

551 As we know from the previous discussions, the function $\overline{\langle \dot{S}_{mat}^{dir} \rangle_v^\tau}$ is relatively weakly af-
 552 fected by coarse graining along the horizontal directions and along the time axis. The modest
 553 importance of time averaging is confirmed in this more complete analysis, as Figs. 5(a), and
 554 5(c) are hard to distinguish. Instead, we find that averaging the thermodynamic fields along

555 the vertical reduces very severely the correlation between the temperature and heating fields,
 556 so that the estimate of the entropy production obtained from the coarse grained fields is
 557 much smaller than its true value $\overline{\dot{S}_{mat}}$. The results emphasize that vertical mixing is the
 558 dominant effect contributing to the material entropy production.

559 When considering the indirect formula, we can draw roughly the same conclusions as
 560 above, with the difference that $\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau}$ is more strongly affected by averaging along the
 561 horizontal surface and along the time axis. Therefore, Figs. 6(a) and 6(c) feature clear
 562 differences in terms of mean values, and, in each of them, the impact of performing very
 563 coarse graining along the horizontal direction is more pronounced with respect to what
 564 reported in Fig. 5. The effect of horizontal coarse graining become noticeable already when
 565 grid boxes with side of $(\Delta\lambda, \Delta\phi) \sim (20^\circ, 30^\circ)$ are considered. This implies that the spatial-
 566 temporal correlation between radiative heating and temperature fields is relevant for larger
 567 range of scales than in the case of described above. These results put in firmer ground the
 568 results given in [30].

569 Comparing Figs. 5 and 6, one can verify that in all cases $\Delta \left[\overline{\dot{S}_{mat}^{ind}} \right]_v^\tau > \Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]_v^\tau$.
 570 Moreover, one discovers that when considering the coarse possible graining, one obtains
 571 that $\overline{\langle \dot{S}_{mat}^{dir} \rangle_{v_{M,v,h}}^{\tau_M}} \sim 16mWm^{-2}K^{-1} > \overline{\langle \dot{S}_{mat}^{ind} \rangle_{v_{M,v,h}}^{\tau_M}} \sim 0$. In order to interpret some these
 572 results, we shall consider some limiting cases for Eqs. (11)-(12). We first take as averaging
 573 volume at each point at surface $v = v_{M,v}$ the vertical column ranging from the bottom of
 574 the fluid component of the climate system to the top of the atmosphere, and we consider a
 575 long averaging time $\tau = \tau_M \gg 1 y$, so that all temporal dependencies are removed. In other
 576 terms, we look at the thermodynamic properties of the *climatological fields*.

577 We start with the expression relevant for the indirect formula for estimating the material
 578 entropy production:

$$\overline{\langle \dot{S}_{mat}^{ind} \rangle_{v_{M,v}}^{\tau_M}} = - \int_V d^3\mathbf{x} \left(\overline{\left(\frac{\langle \dot{q}_{rad} \rangle_v^\tau}{\langle T \rangle_v^\tau} \right)} \right) = - \int_\Sigma dx_1 dx_2 \frac{F_{TOA}(x_1, x_2)}{T_{cli}(x_1, x_2)}. \quad (18)$$

579 where given the choice of τ , the time averaging operation given by the overbar in Eq. 18
 580 is immaterial. In Eq. 18 $F_{TOA}(x_1, x_2) = F_{TOA}^{SW}(x_1, x_2) - F_{TOA}^{LW}(x_1, x_2)$ is the climatological
 581 average of the net radiative wave flux at the top of the atmosphere (positive when there is
 582 net incoming radiation towards the planet), while the lower indices *LW* and *SW* indicate
 583 the long wave and shortwave components, respectively. $T_{cli}(x_1, x_2)$ is the long term mean
 584 of the vertical average of the fluid temperature. Such quantity can be closely approximated

585 by the emission temperature $T_E(x_1, x_2) = (F_{TOA}^{LW}(x_1, x_2)/\sigma)^{1/4}$, where σ is the Boltzmann's
 586 constant. Note that since F_{TOA} and T_E are positively correlated (regions having a net posi-
 587 tive incoming radiation are warmer), we have that F_{TOA} and $1/T_E$ are negatively correlated.
 588 Since $\int_{\Sigma} dx_1 dx_2 F_{TOA} = 0$, we derive that $\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau}$ in Eq. (18) is positive, as expected.

Equation (18) can also be given a different interpretation. We have that $F_{TOA}(x_1, x_2)$ is equal to the divergence of enthalpy transport due to the large scale climatological atmospheric and oceanic flow, so that

$$\begin{aligned} F_{TOA}(x_1, x_2) &= \nabla_2 \cdot \int_{z_{surf}}^{TOA} dx_3 [\mathbf{J}_{lat}(\mathbf{x}) + \mathbf{J}_{dry}(\mathbf{x})] \\ &= \nabla_2 \cdot [\tilde{\mathbf{J}}_{lat}(x_1, x_2) + \tilde{\mathbf{J}}_{dry}(x_1, x_2)]. \end{aligned} \quad (19)$$

589 In the previous equation, we have indicated with $\mathbf{J}_{lat}(\mathbf{x}) = L_w q(\mathbf{x}) \{v_1(\mathbf{x}), v_2(\mathbf{x})\}$ and
 590 $\mathbf{J}_{dry}(\mathbf{x}) = [C_p T(\mathbf{x}) + g x_3] \{v_1(\mathbf{x}), v_2(\mathbf{x})\}$ the large scale, advective horizontal fluxes of la-
 591 tent heat and of dry static energy, respectively, where L_w is the latent heat of evaporation
 592 of water (taken as a constant for simplicity), q is the specific humidity, and C_p is the heat
 593 capacity of air at constant pressure, and v_1 and v_2 indicate the two components of the hor-
 594 izontal velocity field. Finally, the $\tilde{}$ sign refers to the vertically integrated fluxes. Inserting
 595 the right hand side of Eq. (19) in Eq. (18), we conclude that Eq. (18) gives the entropy
 596 produced by the large scale horizontal transport of the geophysical flows and approximates
 597 the climate system as a purely 2D system featuring irreversible heat transport from warm
 598 to cold regions. Looking in the bottom right corner of Fig. 6(c) (which corresponds to the
 599 last entry in Table III), we obtain a value of $\sim 6.5 \text{ mWm}^{-2}\text{K}^{-1}$ (note that the result is
 600 virtually unaltered when averaging over any $\tau \geq 1y$).

601 We can bring the previous example to a more extreme case. If the spatial stencil v is the
 602 whole climate domain $v_{M,h,v}$, it is easy to derive that:

$$\overline{\langle \dot{S}_{mat}^{ind} \rangle_{v=M}^{\tau_M}} = \mu(v) \frac{\overline{\langle \dot{q}_{rad} \rangle_v^\tau}}{\overline{\langle T \rangle_v^\tau}} = -\mu(v) \frac{\int_{\Sigma} dx_1 dx_2 F_{TOA}(x_1, x_2)}{\overline{\langle T \rangle_v^\tau}} = 0, \quad (20)$$

603 because we have reduced the climate to a zero-dimensional system with a unique temperature
 604 where absorbed and emitted radiation are equal. Such a system is at equilibrium, cannot do
 605 any work, and cannot sustain any irreversible process. See Fig. 6(c) and penultimate entry
 606 in Table III.

Let's now repeat the same coarse graining operations for the direct formula. We first consider $v = v_{M,v}$. In each location, when integrating vertically, the surface sensible heat fluxes

cancel out with the heating rates associated to sensible heat fluxes in the atmosphere, so that for this choice of coarse graining their contribution to the entropy production vanishes.

We obtain:

$$\begin{aligned} \overline{\langle \dot{S}_{mat}^{dir} \rangle_{v_{M,v}}^{\tau M}} &= \int_V d^3\mathbf{x} \overline{\left(\frac{\langle \dot{q}_{mat} \rangle_v^\tau}{\langle T \rangle_v^\tau} \right)} = \int_\Sigma dx_1 dx_2 \left[\frac{L_w [P(x_1, x_2) - E(x_1, x_2)]}{T_{cli}(x_1, x_2)} + \frac{\tilde{\epsilon}^2(x_1, x_2)}{T_{cli}(x_1, x_2)} \right] \\ &= \int_\Sigma dx_1 dx_2 \left[\frac{-\nabla_2 \cdot \tilde{J}_{lat}(x_1, x_2)}{T_{cli}(x_1, x_2)} + \frac{\tilde{\epsilon}^2(x_1, x_2)}{T_{cli}(x_1, x_2)} \right]. \end{aligned} \quad (21)$$

607 where $L_w [P(x_1, x_2) - E(x_1, x_2)] = -\nabla_2 \cdot \tilde{J}_{lat}(x_1, x_2)$ as imposed by conservation of water
608 mass, with $P(x_1, x_2)$ and $E(x_1, x_2)$ time-averaged values of precipitation and evaporation,
609 respectively [41]. Furthermore, we indicate with $\tilde{\epsilon}^2(x_1, x_2)$ the vertically integrated kinetic
610 energy dissipation rate, and we choose, as a first approximation $T_{cli}(x_1, x_2)$ as characteristic
611 temperature defined as before. Therefore, Eq. (21) suggests that the bottom right corner
612 of Fig. 5(c) corresponds to the sum of entropy produced by large scale transport of latent
613 heat plus the entropy produced by the dissipation of kinetic energy. We find a value of
614 $\sim 18 \text{ mWm}^{-2}\text{K}^{-1}$. Considering that the entropy production due to large scale transport
615 of sensible heat is much smaller than the corresponding contribution due to latent heat
616 transport (from the precise calculation we get a factor of about 5 as ratio between the
617 two terms) we can derive that the dissipation of kinetic energy contributes for about ~ 13
618 $\text{mWm}^{-2}\text{K}^{-1}$ to the total material entropy.

619 Interestingly, if we compute $\overline{\langle \dot{S}_{mat}^{dir} \rangle_{v_{M,h,v}}^{\tau M}}$, we do not obtain a vanishing result. While the
620 contribution to entropy production due to heat fluxes is eliminated, the contribution coming
621 from the dissipation of kinetic energy is not removed by the operation of coarse graining:

$$\overline{\langle \dot{S}_{mat}^{dir} \rangle_{v=V}^\tau} = \mu(v) \frac{\overline{\langle \dot{q}_{mat} \rangle_v^\tau}}{\langle T \rangle_v^\tau} = \mu(v) \frac{\int_\Sigma dx_1 dx_2 \tilde{\epsilon}^2(x_1, x_2)}{\langle T \rangle_v^\tau} > 0. \quad (22)$$

622 Equation (22) gives, to a good degree of approximation, the *minimum* value of the entropy
623 production compatible with the presence of a total dissipation $\int_V d^3\mathbf{x} \overline{\epsilon^2(\mathbf{x}, t)}$ [28, 30]. The
624 second entry of Table III reports for the contribution given in Eq. 22 a value of about 16
625 $\text{mWm}^{-2}\text{K}^{-1}$. This value agrees well with what derived using Eqs. 20-21 and with what
626 obtained by direct estimate of $\overline{\dot{S}_{KE}} \sim 13.5 \text{ mWm}^{-2}\text{K}^{-1}$.

627 These results imply that we can obtain an extremely good estimate of the true value
628 of the material entropy production even using climatological, horizontally global averages
629 of the thermodynamics fields: in such a worst case scenario, we get a bias of about 10%.

630 Using few selected coarse grained estimates for the entropy production, one can derive that
631 it is possible to split the total value of $52.5 \text{ mWm}^{-2}\text{K}^{-1}$ as follows: $\sim 6 \text{ mWm}^{-2}\text{K}^{-1}$
632 can be attributed to large scale transports of latent and sensible heat; $\sim 13 \text{ mWm}^{-2}\text{K}^{-1}$
633 can be attributed to the entropy produced by dissipation of kinetic energy; the remaining
634 $\sim 33.5 \text{ mWm}^{-2}\text{K}^{-1}$ can be attributed to vertical transports of sensible and latent heat
635 (basically, convection). These estimates agree quite accurately with what obtained in [30]
636 using scaling analysis. Also, one obtains, in agreement with the inequality proposed in [30],
637 that the entropy produced by dissipation of kinetic energy is larger than that due to large
638 scale energy transport.

639 V. CONCLUSIONS

640 The investigation of the climate system using tools borrowed from non-equilibrium ther-
641 modynamics [2, 20] is a very active interdisciplinary research, which allows for connecting
642 concepts of great relevance for climate dynamics, such as large scale heat transports and
643 the Lorenz energy cycle [42], to basic thermodynamical concepts used in the investigation
644 of general non-equilibrium systems [16, 28]. Such a theoretical framework seems relevant
645 especially in the context of the growing field focusing on the study of the atmospheres of
646 exoplanets [1, 32], for which detailed measurements are hardly available. In fact, ther-
647 modynamical methods allow for defining inequalities and deducing apparently unexpected
648 relations between different physical quantities [30].

649 In this paper we have focused on understanding where, in the Fourier space, dissipative
650 and irreversible processes are dominant. This has been accomplished by computing how
651 different spatial and temporal scales contribute to the material entropy production in the
652 climate system. We have considered the output coming from a 50 *y* run under steady state
653 conditions performed with the FAMOUS climate model [15], and have used the entropy
654 diagnostics developed and tested in [36]. We have considered both the direct and the indirect
655 formulas for material entropy production [8]: the former estimates the material entropy
656 production using the heating rates associated to the dissipation of kinetic energy and the
657 convergence of material heat fluxes, the latter uses, instead, the heating rates associated to
658 radiative fluxes.

659 Our strategy has been the following: we have considered the estimates of the entropy

660 production coming from the coarse grained outputs of the heating and temperature fields.
661 The coarse-graining has been performed both in time and in space. The temporal coarse
662 graining ranges from hourly (timestep of the model) to yearly time scale, while the spatial
663 coarse grained has been performed in three different modalities: 1) performing longitudinal
664 averages; 2) performing averages along horizontal surfaces; 3) performing mass-weighted
665 averages along the horizontal and vertical directions. We have conjectured and then verified
666 numerically that the coarser the graining of the data, the lower is the resulting estimate of
667 the material entropy production, both in the case of the direct and of the indirect formula
668 for estimating the material entropy production. This implies that at all scales there is a
669 negative correlation between heating rates related to flow (kinetic energy dissipation, sensible
670 and latent heat fluxes) and the temperature field, and a positive correlation between the
671 heating rate due to radiation and the temperature field. In other terms, at all scales, the
672 climate systems results to be forced by radiation, while the resulting forced fluctuations are
673 dissipated by the material fluxes. In agreement with this interpretation, we have conjectured
674 that at all scales the correlation between the radiative heating and the temperature field is
675 stronger than the correlation between the temperature field and the material heating rates.
676 If the two correlation were equal, the climate system would be able to adjust, instantly and
677 locally, to (spatial and temporal) variations in the radiative heating. The numerical results
678 have provided support for this conjecture.

679 Considering various special cases of coarse graining, and using the basic thermodynamic
680 equations, we have been able to estimate in a consistent way the contributions to material
681 entropy productions coming from large scale horizontal heat transport ($\sim 6 \text{ mWm}^2\text{-K}^{-1}$),
682 dissipation of kinetic energy ($\sim 13 \text{ mWm}^2\text{-K}^{-1}$), and vertical processes of sensible and
683 latent heat exchanges (i.e. convection, $\sim 33.5 \text{ mWm}^2\text{-K}^{-1}$). This suggest that, as first
684 approximation, the climate system can be seen in terms of dissipative processes as a collection
685 of weakly coupled vertical columns featuring turbulent exchanges and dissipation. This
686 confirms the ideas presented in [30].

687 Note that one could use the quantitative information on the various contributions to the
688 material entropy production to derive some basic properties of the climate system without
689 resorting to the full three dimensional, time dependent fields, In particular, one can derive a
690 good estimate of the intensity of the Lorenz energy cycle by multiplying the estimated value
691 of the contribution of the dissipation to the kinetic energy to the material entropy production

692 times a characteristic temperature of the system, obtaining an estimate of $\sim 3 \text{ Wm}^{-2}$, which
693 is in good agreement with what obtained after processing of the high-resolution data [36].

694 The fact that the estimate of the material entropy production of the climate system
695 decreases when a coarser graining is considered is in qualitative agreement with what derived
696 in the Appendix A for the simple case of continuum systems featuring generalized flux-
697 gradient relations. This obviously does not imply that the climate system behaves as a
698 diffusive system, yet share with a diffusive system this interesting property. The fact that at
699 all spatial and temporal scales the system has a positive definite value of material entropy
700 production, when global averages are considered. This is not in contradiction with the well-
701 known phenomena of so-called *negative diffusion*, first noted by Starr [47], who observed
702 that in certain portions of the atmosphere - namely, near the storm track - the fluxes of
703 momentum transport momentum from low to high momentum regions. While this process
704 - a crucial element of the general circulation of the atmosphere, observed in our model
705 runs as well - seems to oppose the second law of thermodynamics, it is instead a local but
706 macroscopic phenomenon, where the creation of organized structures (thanks to long-range
707 correlations due to wave propagation), is, as we understand in this paper, over-compensated
708 by large entropy production at the same scales elsewhere in the atmosphere.

709 Apart from providing insights on the properties of forced fluctuations and irreversible
710 dissipative processes in the climate system at various spatial and temporal scales, this paper
711 deals with the relationship between a model, its output, and the chosen observables, by
712 providing information on what is the impact of being able to access data at lower resolution
713 with respect to the model which has generated them. We have learnt that this lack of
714 information always biases negatively our estimate of the entropy production, and that the
715 bias is serious only if we miss information describing the vertical structure of thermodynamic
716 fields.

717 Since performing time averages up to the yearly time scale does not bias substantially
718 the estimates of the material entropy production, we have that it is possible to intercom-
719 pare robustly the state-of-the-art climate models and assess on each of them the impact of
720 climate change on the entropy production by resorting to the output data provided in the
721 freely accessible PCMDI/CMIP3 (http://www-pcmdi.llnl.gov/ipcc/about_ipcc.php)
722 and PCMDI/CMIP5 (<http://cmip-pcmdi.llnl.gov/cmip5/>) repositories, where long cli-
723 mate runs outputs are typically stored in the form of monthly averaged data.

724 On a different note, the approach presented here seems promising specifically for the
725 investigation of the atmosphere of exoplanets, because it allows for evaluating the error in
726 the estimate of their thermodynamical properties due to the lack of high-resolution data.

727 In this paper, we have worked on the post-processing of data. The analysis presented
728 here should be complemented with an additional investigation of how changing the reso-
729 lution of a model impacts the estimate of its material entropy production, in total, and
730 process by process. Using the material entropy production as cost function for addressing
731 the interplay between the respective role of changes in the resolution of a model and of
732 changes in the coarse graining of the post-processed data seems promising in the tantalizing
733 quest for understanding what is a good model of a geophysical fluid and what is a robust
734 parametrization. The results obtained here seem to have a much more general validity than
735 for the specific case of the present Earth's climate. Therefore, we plan to extend the present
736 analysis, by studying the combined effect of changes in the resolution of the model and in
737 the effective resolution of the post-processed in a simpler geophysical fluid dynamical system
738 like an Aquaplanet. We believe that the conjectures presented on the effect of coarse grain-
739 ing thermodynamic fields on the estimate of the material entropy production is of general
740 validity for a vast range of systems that can be described by continuum mechanics.

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749 **Appendix A: Spectral Analysis of the Impact of Coarse Graining on the Entropy**
750 **Production for Diffusive Systems**

751 We wish to provide a simple outlook on how to interpret the results shown in this paper
752 taking a spectral point of view. This is relevant in practical terms because many numerical
753 models of geophysical fluids are actually implemented in spectral coordinates. We restrict
754 ourselves to the contributions to the material entropy production coming from the presence
755 of material fluxes transporting heat across temperature gradients. So, we do not consider
756 here the term responsible for the dissipation of kinetic energy (which is weakly affected by
757 coarse graining) nor the radiative terms contributing to the indirect formula. We consider
758 the case of a much simpler simpler 3D continuum physical system, where heat transport
759 obeys a generalized diffusive behavior. We make this choice not because we believe that
760 the climate system is, in any real sense, diffusive, but because we wish to show that the ob-
761 served dependence of the material entropy production on the coarse graining is in qualitative
762 agreement with what would be obtained for a diffusive system.

763 Let's assume that the contribution to the average rate of entropy production coming from
764 the transport of heat due to the flow J across the temperature field T can be computed as:

$$\bar{S}_{mat}^J = \frac{1}{V} \frac{1}{T} \int_V d^3\mathbf{x} \int_0^T dt \vec{J} \cdot \vec{\nabla} \frac{1}{T} = -\frac{1}{V} \frac{1}{T} \int_V d^3\mathbf{x} \int_0^T dt \frac{\vec{J} \cdot \vec{\nabla} T}{T^2} = -\frac{1}{V} \frac{1}{T} \int_V d^3\mathbf{x} \int_0^T dt \frac{\vec{\nabla} \cdot \vec{J}}{T} \quad (\text{A1})$$

765 Let's now make the simplifying diffusive-like assumption that $\vec{J} = -\vec{\nabla} G(T)$, where $dG/dT >$
766 0 , so that the flux is always opposed in verse to the temperature gradient; in the usual linear
767 flux-gradient approximation we have $G(T) = \kappa T$, $\kappa > 0$. We derive:

$$\bar{S}_{mat}^J = \frac{1}{V} \frac{1}{T} \int_V d^3\mathbf{x} \int_0^T dt \frac{G'(T)}{T^2} |\vec{\nabla} T|^2 = \frac{1}{V} \frac{1}{T} \int_V d^3\mathbf{x} \int_0^T dt |\vec{\nabla} \Psi(T)|^2 > 0 \quad (\text{A2})$$

768 where $\Psi(T) = \int dT \sqrt{G'(T)/T^2}$. For sake of simplicity - but without loss of generality -
769 we assume that our domain Σ is a parallelepiped of sides L_x , L_y , and L_z . Using Parseval's
770 theorem, we derive that the rate of entropy production can be written as:

$$\bar{S}_{mat}^J = \sum_{p,q,r,s} (k_p^2 + k_q^2 + k_r^2) |\Psi_{p,q,r,s}|^2 = \sum_{p,q,r,s} S_{p,q,r,s}^{J,mat} \quad (\text{A3})$$

771 where

$$\Psi_{p,q,r,s} = \frac{1}{V} \frac{1}{T} \int_V d^3\mathbf{x} \int_0^T dt \Psi \exp[2\pi i(p/L_x x + q/L_y y + r/L_z z - s/Tt)]. \quad (\text{A4})$$

772 Performing a spatio-temporal coarse graining to the field $\Psi \rightarrow \tilde{\Psi}$ can be reframed as applying
773 a linear filter in Fourier space, as $\Psi_{p,q,r,s} \rightarrow \tilde{\Psi}_{p,q,r,s} = \Phi_{p,q,r,s} \Psi_{p,q,r,s}$, where $|\Phi_{p,q,r,s}| \leq 1$
774 $\forall p, q, r, s$ plus the usual complex conjugacy properties. Note that in our case we would like
775 to be able to apply the filtering to the T field and to the heating field $-\vec{\nabla} \cdot \vec{J}$ and not to the
776 function Ψ constructed here. Nonetheless, assuming that the relative change of T across the
777 domain (see also Eq. (13)) is small, the conclusions are virtually unaltered.

778 Equation A3 has the remarkable property that all of terms of the summation $S_{p,q,r,s}^{J,mat}$ are
779 positive. We can interpret $S_{p,q,r,s}^{J,mat}$ as the entropy produced by processes occurring at the
780 scales described by the indices, in this case $\lambda_x = L_x/p$, $\lambda_y = L_y/q$, $\lambda_z = L_z/r$, and $\tau = T/s$.
781 Therefore, indicating as \tilde{S}_{mat}^J the value of the entropy production for the coarse grained
782 fields, we obtain:

$$\tilde{S}_{mat}^J = \sum_{p,q,r,s} (k_p^2 + k_q^2 + k_r^2) |\Phi_{p,q,r,s}|^2 |\Psi_{p,q,r,s}|^2 = \sum_{p,q,r,s} |\Phi_{p,q,r,s}|^2 S_{p,q,r,s}^{J,mat} \leq \bar{S}_{mat}^J. \quad (\text{A5})$$

783 In particular, we can associate a coarse graining on the scales Λ_x , Λ_y , Λ_z , and τ (referred to
784 the x -, y -, z -directions and time, respectively) to a filter of the form $\Phi_{p,q,r,s} = 0$ if $p > L_x/\Lambda_x$,
785 or $q > L_y/\Lambda_y$, or $r > L_z/\Lambda_z$, or $s > T/\tau$ and $\Phi_{p,q,r,s} = 1$ otherwise. Slightly different ways
786 of doing the coarse graining will result into different filters, which will be, nonetheless,
787 asymptotically equivalent if the involved scales are the same.

788 The main conceptual point behind this result is independent of the shape of the of the
789 domain of integration: the natural orthogonal expansion for atmospheric fields defined in
790 an (approximately) spherical thin shell is given by spherical harmonics in for the latitudinal
791 and longitudinal dependence and the usual Fourier expansion for the vertical direction. In
792 the case of a thin spherical shell of thickness L_z situated at distance R from the center of
793 the sphere, Eq. A3 can be rewritten as:

$$\bar{S}_{mat}^J = \sum_n \sum_{l \geq 0} \sum_{m=-l}^l \sum_s (k_n^2 + l(l+1)/R^2) |\Psi_{n,l,m,s}|^2 \quad (\text{A6})$$

794 with

$$\Psi_{n,l,m,s} = \frac{1}{L_z} \frac{1}{4\pi} \frac{1}{T} \int_{\Omega} d\Omega \int_0^T \Psi(z, \theta, \phi, t) \exp[2\pi i(n/L_z z - s/Tt)] * Y(\theta, \phi)_n^{m*}. \quad (\text{A7})$$

795 where $Y(\theta, \phi)_n^m$ are the usual spherical harmonics and Ω refers to the solid angle. In this
796 case, performing a spatio-temporal coarse graining to the field $\Psi \rightarrow \tilde{\Psi}$ results in reduced

797 value of the estimate of the entropy production:

$$\tilde{S}_{mat}^J = \sum_n \sum_{l \geq 0} \sum_{m=-l}^l \sum_s (k_n^2 + l(l+1)/R^2) |\Phi_{n,l,m,s}|^2 |\Psi_{n,l,m,s}|^2 \leq \bar{S}_{mat}^J. \quad (\text{A8})$$

798 It is now easy to relate the expression of $|\Phi_{n,l,m,s}|^2$ to common averaging operation performed
 799 on climate data. A coarse graining on the vertical scale Λ_z , on a temporal scale τ and on a
 800 horizontal surface area σ (or angular resolution σ/R^2) amounts to setting $|\Phi_{n,l,m,s}|^2 = 0$ if
 801 $l \geq \sqrt{8\pi R^2/\sigma}$ (corresponding approximately to a triangular truncation $T(K)$, where K is
 802 the integer closest to $\sqrt{8\pi R^2/\sigma}$), or $n > L_z/\Lambda_z$, or $s > T/\tau$, and $|\Phi_{n,l,m,s}|^2 = 1$ otherwise.
 803 Instead, performing zonal averages corresponds to setting, $|\Phi_{n,l,m,s}|^2 = 0$ if $m \neq 0$.

804 The bottom line of the previous considerations is that adopting a coarser graining cor-
 805 responds to increasing the involved scales determining the spectral cutoff. if we assume a
 806 flux-gradient relationship which is consistent with the second law of thermodynamics (even
 807 if it is not the usual Fickian, linear relation), Eqs. A5 and A8 imply that as the graining
 808 becomes coarser, the estimate of the entropy production becomes smaller, because we the
 809 summation is performed over fewer terms, all of them positive. This behavior is independent
 810 of the physical domain under consideration. Moreover, the previous considerations qualita-
 811 tively apply - even if results are somewhat more cumbersome - if the relationship between flux
 812 and gradient is more general than what previously assumed, e.g. if $J_i = -\partial_i G_i(T)$ (where
 813 the Einstein summation convention is not taken), under the condition that $dG_i/dT < 0 \forall i$.

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TABLE I. List of the main FAMOUS' parametrization routines for unresolved processes and their impact in term of heating rates \dot{q}_k^c on the various terms contributing to \dot{s}_{mat} . Codes: BL=Boundary Layer; AC=Atmospheric Convection; HD=Hyperdiffusion; OC=Oceanic Convection; D=Diffusion; GW=Gravity Waves; C/E=Condensation/Evaporation; ML=Mixed Layer

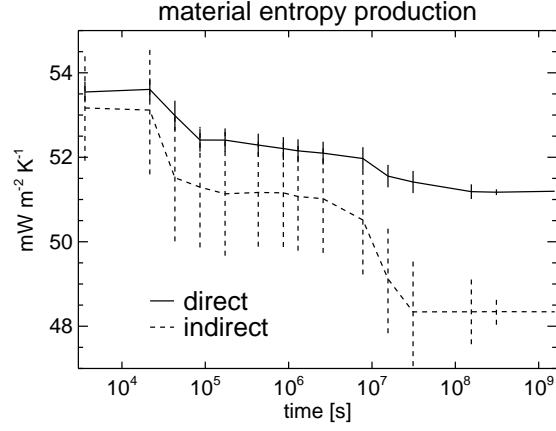
Contribution to \dot{s}_{mat}	Atmosphere	Ocean	Soil	Cryosphere
$\frac{\epsilon^2}{T}$	BL, C, GW, HD			
$-\frac{\nabla \cdot \mathbf{F}_{LH}}{T}$	BL, AC, C/E,	BL	BL	BL
$-\frac{\nabla \cdot \mathbf{F}_{SH}}{T}$	BL, AC, HD	BL, ML, OC, D	BL	BL

TABLE II. Values of $\Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]_v^\tau$ and $\Delta \left[\overline{\dot{S}_{mat}^{ind}} \right]_v^\tau$ obtained when considering the coarsest resolution in space (lower index: v_M), in time (lower index: τ_M), or both. Only 2D horizontal averagings are considered here. Reference values for highest resolution data: $\overline{\dot{S}_{mat}^{dir}} = 52.5 \text{ mWm}^{-2}\text{K}^{-1}$ and $\overline{\dot{S}_{mat}^{ind}} = 52.1 \text{ mWm}^{-2}\text{K}^{-1}$. All values are in units of $\text{mWm}^{-2}\text{K}^{-1}$.

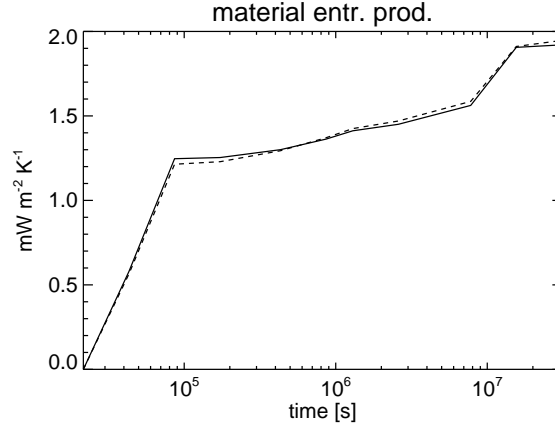
Averaging	$\Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]_{v_M}^\tau$	$\Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]_{v_M}^{\tau_M}$	$\Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]_{v_M}^{\tau_M}$	$\Delta \left[\overline{\dot{S}_{mat}^{ind}} \right]_{v_M}^\tau$	$\Delta \left[\overline{\dot{S}_{mat}^{ind}} \right]_{v_M}^{\tau_M}$	$\Delta \left[\overline{\dot{S}_{mat}^{ind}} \right]_{v_M}^{\tau_M}$
Longitudinal	2.1	2.2	2.1	2.2	5.0	5.0
Surface	5.3	5.3	2.2	12.3	12.8	5.0

TABLE III. Values of $\Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]_v^\tau$ and $\Delta \left[\overline{\dot{S}_{mat}^{ind}} \right]_v^\tau$ obtained when considering the coarsest resolution either in horizontal direction (lower index: $v_{M,h}$ or vertical direction (lower index: $v_{M,v}$), or in both (lower index: $v_{M,h,v}$). τ is set to 1 y. Reference values for highest resolution data: $\overline{\dot{S}_{mat}^{dir}} = 52.5 \text{ mWm}^{-2}\text{K}^{-1}$ and $\overline{\dot{S}_{mat}^{ind}} = 52.1 \text{ mWm}^{-2}\text{K}^{-1}$. All values are in units of $\text{mWm}^{-2}\text{K}^{-1}$.

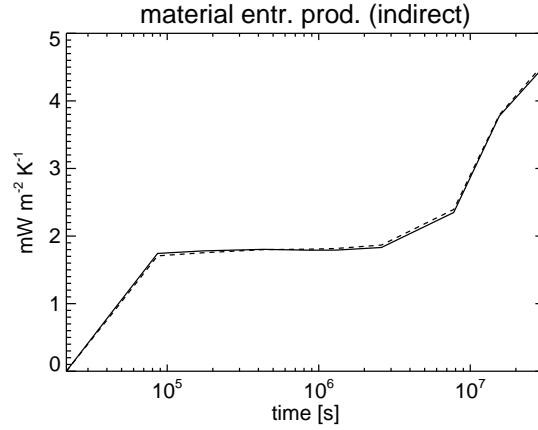
Averaging	$\Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]_{v_{M,h}}^{\tau_M}$	$\Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]_{v_{M,h,v}}^{\tau_M}$	$\Delta \left[\overline{\dot{S}_{mat}^{dir}} \right]_{v_{M,v}}^{\tau_M}$	$\Delta \left[\overline{\dot{S}_{mat}^{ind}} \right]_{v_{M,h}}^{\tau_M}$	$\Delta \left[\overline{\dot{S}_{mat}^{ind}} \right]_{v_{M,h,v}}^{\tau_M}$	$\Delta \left[\overline{\dot{S}_{mat}^{ind}} \right]_{v_{M,v}}^{\tau_M}$
Mass weighted	5.3	36.5	34.5	12.8	52.1	45.6



(a)

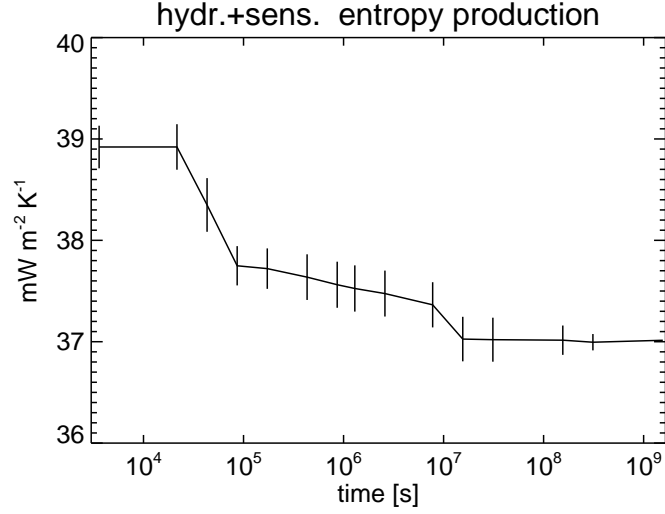


(b)

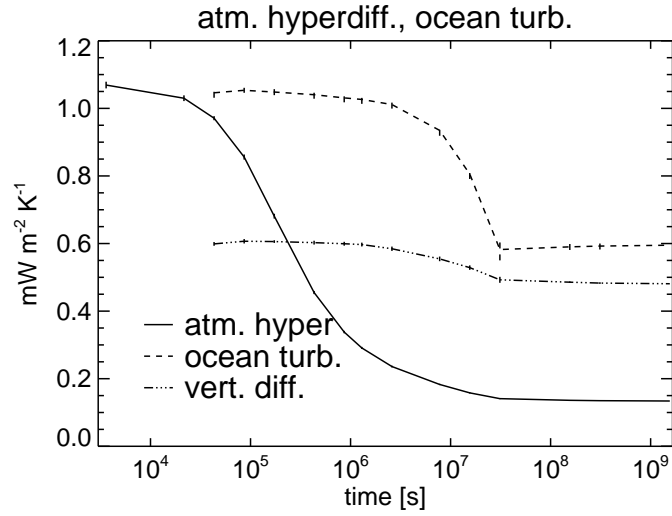


(c)

FIG. 1. (a) Time-scale dependence of the total material entropy production (direct and indirect estimates); τ ranges between 3600 s (model timestep) to 1.5×10^9 s (50 years); (b) Differences between the exact and timeocrase gained material entropy production $\Delta \left[\dot{S}_{mat}^{dir} \right]^\tau$ for $3600 \text{ s} \leq \tau \leq 1 \text{ year}$ (c); as in (b) but for $\Delta \left[\dot{S}_{mat}^{ind} \right]^\tau$. The dashed lines represent the correlation terms given in Eqs (14)-(15).



(a)



(b)

FIG. 2. (a) Material entropy production due to hydrological cycle and heat diffusion. Here $L = 50$ years. (b) Material entropy production due to atmospheric small-scale temperature diffusion (continuous line). The material entropy production due to ocean turbulence (vertical and horizontal diffusion, mixed layer physics and convection is also reported, see [36] for details). We also show the vertical diffusion contribution (dotted-dashed line) to the total turbulent material entropy production. Note that the oceanic processes have a 12-hour timestep and have not been considered in the total entropy budget .

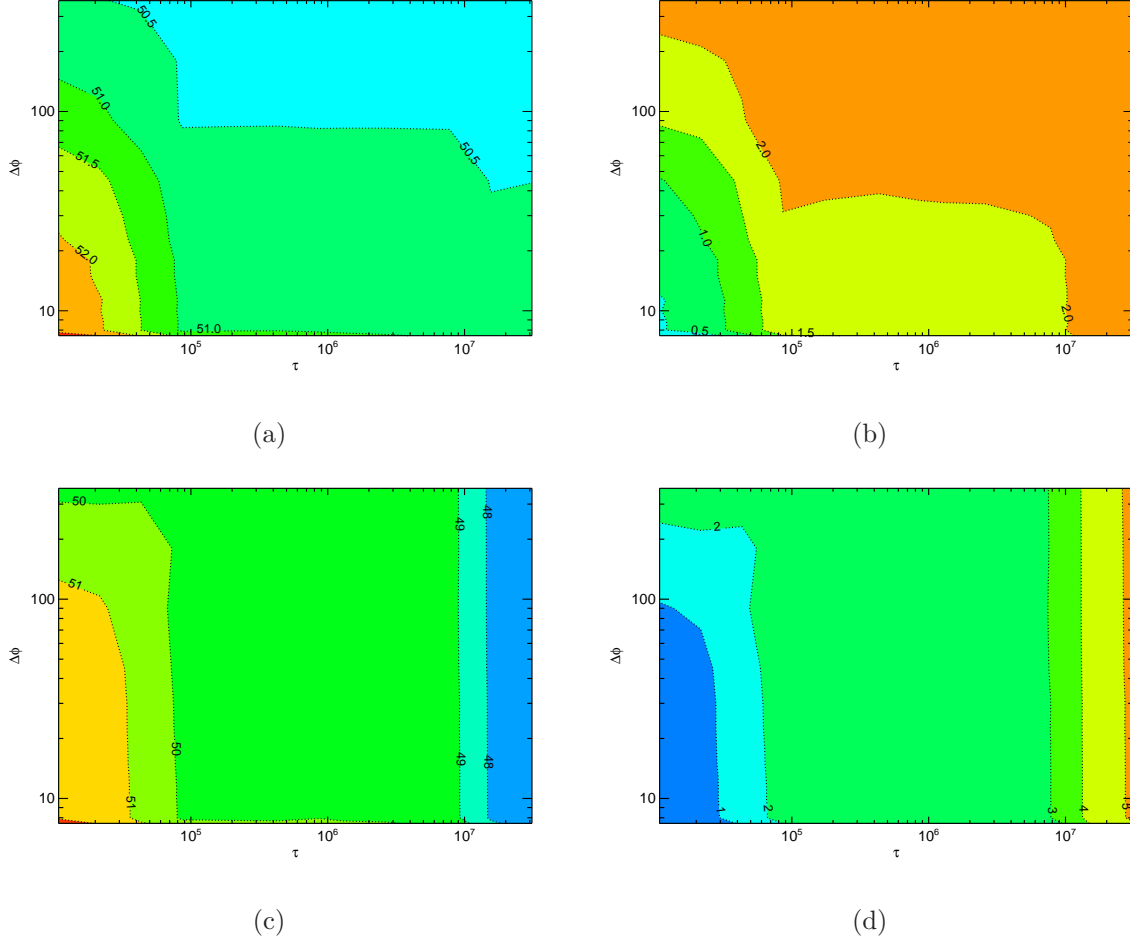
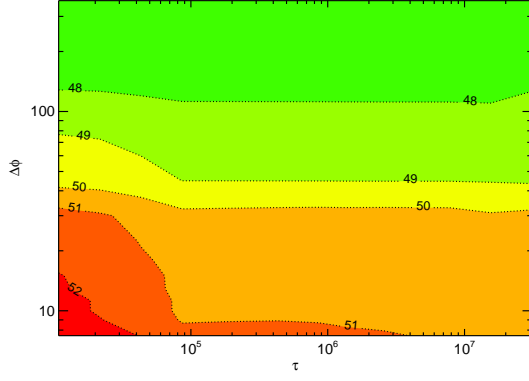
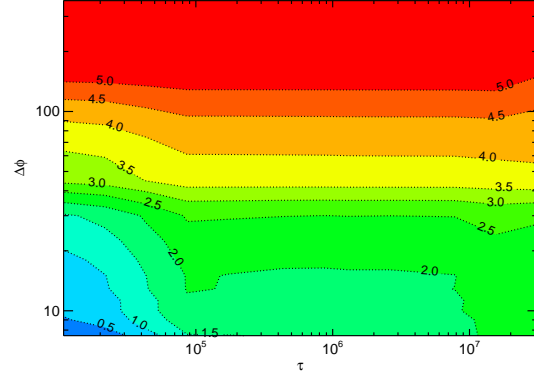


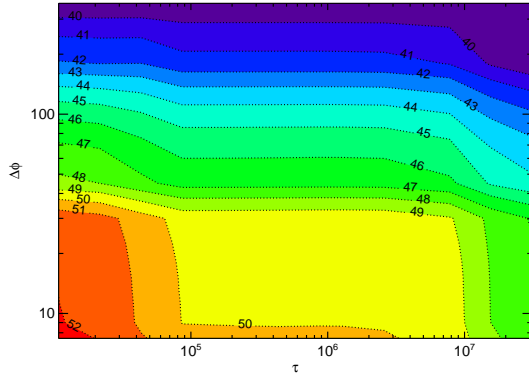
FIG. 3. Estimates of $\overline{\dot{S}_{mat}^{dir}}_{\tau}$ (a), $\Delta \left[\dot{S}_{mat}^{dir} \right]_{\tau}$ (b), $\overline{\dot{S}_{mat}^{ind}}_{\tau}$ (c), and $\Delta \left[\dot{S}_{mat}^{ind} \right]_{\tau}$ (d). The spatial averaging considered here is given by longitudinal averages on horizontal surface. The x -axis reports τ , the y -axis describes the extent $\Delta\phi$ of the averaging in $^{\circ}$.



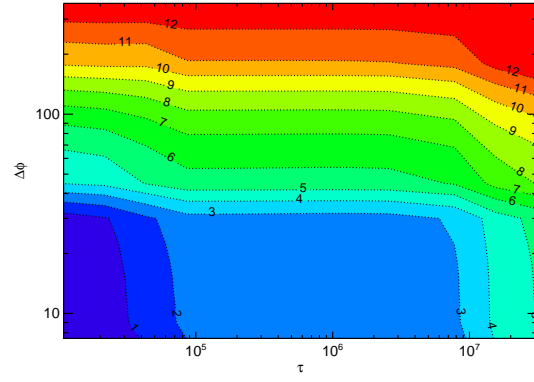
(a)



(b)



(c)



(d)

FIG. 4. Estimates of $\overline{\langle \dot{S}_{mat}^{dir} \rangle_{\tau}}_v$ (a), $\Delta \left[\overline{\langle \dot{S}_{mat}^{dir} \rangle_{\tau}} \right]_v^{\tau}$ (b), $\overline{\langle \dot{S}_{mat}^{ind} \rangle_{\tau}}_v$ (c), and $\Delta \left[\overline{\langle \dot{S}_{mat}^{ind} \rangle_{\tau}} \right]_v^{\tau}$ (d). The spatial averaging considered here is given by areal averages along horizontal surfaces. The x -axis reports τ , the y -axis describes the longitudinal extent $\Delta\phi$ of the coarse grained grid boxes. Details are given in the text.

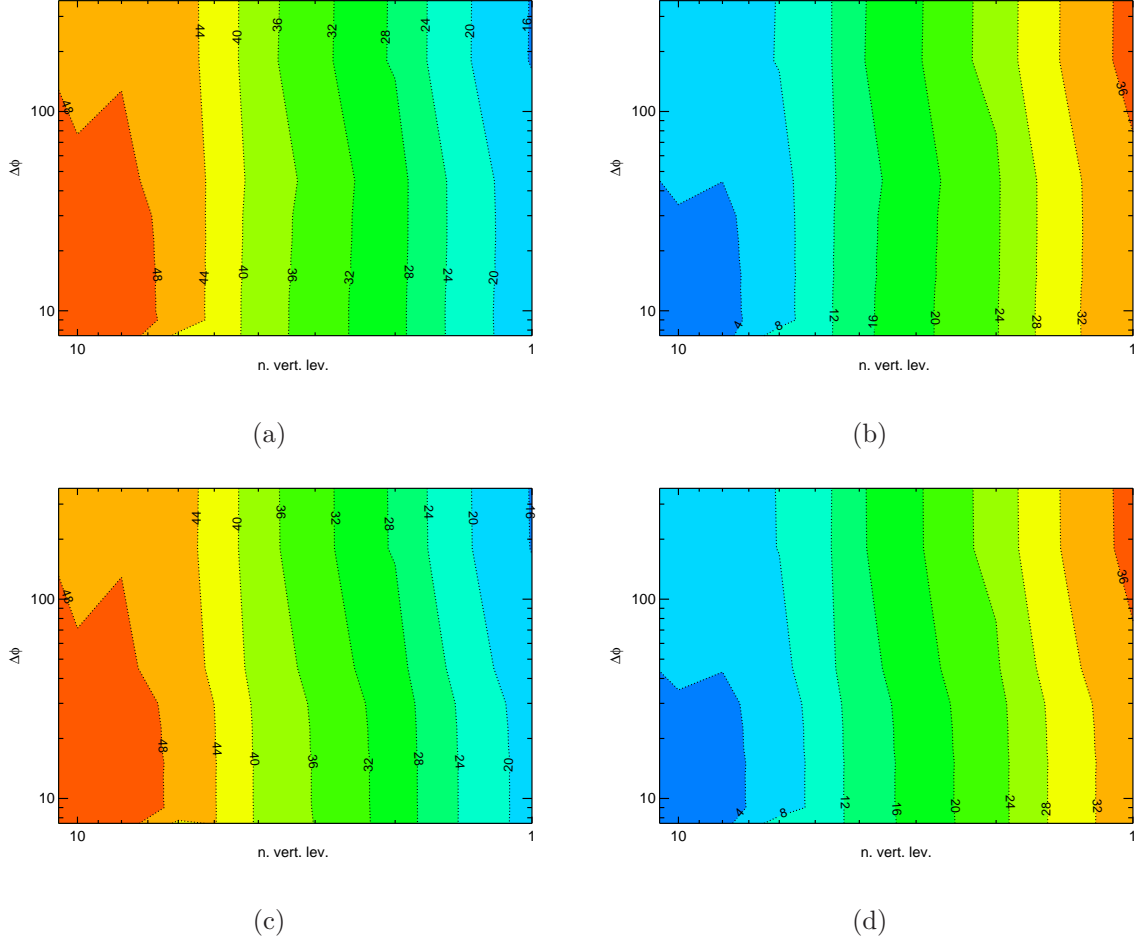
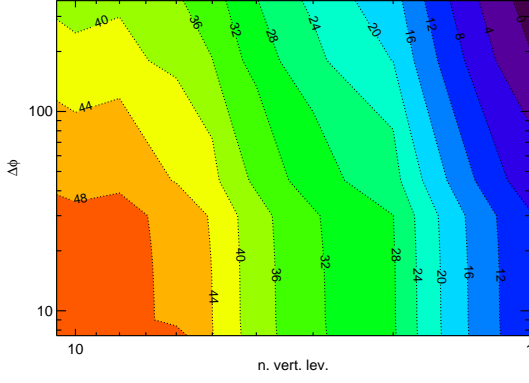
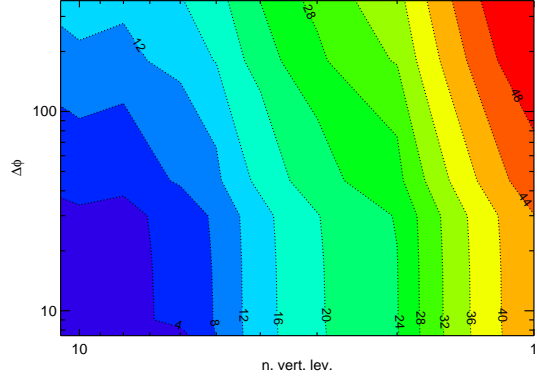


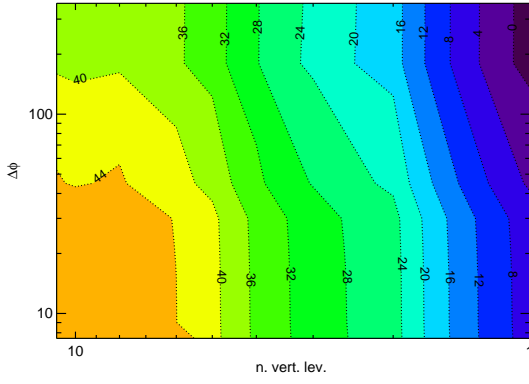
FIG. 5. Estimates of $\overline{\langle \dot{S}_{mat}^{dir} \rangle_v}^\tau$ (a), and $\Delta \left[\dot{S}_{mat}^{dir} \right]_v^\tau$ (b) for $\tau = 1$ day, and of $\overline{\langle \dot{S}_{mat}^{dir} \rangle_v}^\tau$ (c), and $\Delta \left[\dot{S}_{mat}^{dir} \right]_v^\tau$ (d) for $\tau = 1$ year. The spatial averaging considered here is given by areal averages along horizontal surfaces and vertical averages along columns. The x -axis reports the numbers of vertical levels involved in the averaging, the y -axis describes the longitudinal extent $\Delta\phi$ of the coarse grained grid boxes. Details are given in the text.



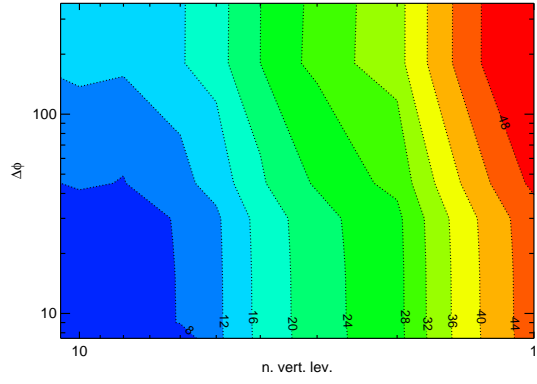
(a)



(b)



(c)



(d)

FIG. 6. Estimates of $\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau}$ (a), and $\Delta \left[\dot{S}_{mat}^{ind} \right]_v^\tau$ (b) for $\tau = 1$ day, and of $\overline{\langle \dot{S}_{mat}^{ind} \rangle_v^\tau}$ (c), and $\Delta \left[\dot{S}_{mat}^{ind} \right]_v^\tau$ (d) for $\tau = 1$ year. The spatial averaging considered here is given by areal averages along horizontal surfaces and vertical averages along columns. The x -axis reports the numbers of vertical levels involved in the averaging, the y -axis describes the longitudinal extent $\Delta\phi$ of the coarse grained grid boxes. Details are given in the text.