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Does the Representation of Flow Structure and Turbulence at a Cold Front Converge on Multi-scale Observations with Model Resolution?

Ben Harvey[1], John Methven[2], Chloe Eagle[3], and Humphrey Lean[3]

[1]National Centre for Atmospheric Science, University of Reading, UK
[2]Department of Meteorology, University of Reading, Reading, UK
[3]MetOffice@Reading, University of Reading, UK

Abstract

In situ aircraft observations are used to interrogate the ability of a numerical weather prediction model to represent flow structure and turbulence at a narrow cold front. Simulations are performed at a range of nested resolutions with grid spacings of 12 km down to 100 m and the convergence with resolution is investigated. The observations include the novel feature of a low-altitude circuit around the front that is closed in the frame of reference of the front, thus allowing the direct evaluation of area-average vorticity and divergence values from circuit integrals. As such, the observational strategy enables a comparison of flow structures over a broad range of spatial scales, from the size of the circuit itself (≈100 km) to small-scale turbulent fluctuations (≈10 m). It is found that many aspects of the resolved flow converge successfully towards the observations with resolution if sampling uncertainty is accounted for, including the area-average vorticity and divergence measures and the narrowest observed cross-frontal width. In addition, there is a gradual handover from parametrized to resolved turbulent fluxes of moisture and momentum as motions in the convective boundary layer behind the front become partially-resolved in the highest resolution simulations. In contrast, the parametrized turbulent fluxes associated with subgrid-scale shear-driven turbulence ahead of the front do not converge on the observations. The structure of frontal rainbands associated with a shear instability along the front also does not converge with resolution, indicating that the mechanism of the frontal instability may not be well represented in the simulations.

1 Introduction

Atmospheric frontal systems are associated with numerous high-impact weather phenomena. The majority of extreme precipitation events in mid-latitudes are associated with fronts (Catto and Pfahl, 2013) and intense wind gusts, including tornadoes, commonly occur near frontal rainbands (Clark and Parker, 2014). Despite being embedded within large-scale weather systems, often stretching over thousands of kilometers, their narrow cross-frontal scale coupled with often intense meso- and convective-scale circulations mean they continue to provide a challenge for numerical weather prediction (NWP) models.

In the idealised, frictionless semi-geostrophic limit, frontogenetic motions acting on a baroclinic zone cause the collapse of the cross-frontal scale to zero in a finite time (Hoskins and Bretherton, 1972). Whilst in reality this collapse is halted by other processes neglected in that model, albeit not fully-understood ones, observed frontal zones are often found to be narrower than can be resolved by current NWP models. Therefore the modeled frontal widths are typically set artificially by (implicit or explicit) numerical diffusive effects, rather than by resolved physical motions. The degree to which the prediction of associated high-impact weather is affected by this limitation is not understood.

Operational local-area forecast models are now approaching convection-permitting grid spacings of $O(1\text{ km})$ (Clark et al., 2016). In such models deep tropospheric convection is at least partially-resolved (Lean et al., 2008) and fronts with cross-frontal scales of several kilometers should be partially represented. However boundary-layer turbulence, which is known to strongly influence frontal structure and the representation of associated high-impact surface weather, remains poorly resolved at these resolutions (e.g. Williams, 1974; Sinclair and Keyser, 2015). Development is also underway at several forecasting centres on experimental local-area NWP models with sub-kilometer resolutions, down to $O(100\text{ m})$ grid spacings. In this case, shallow boundary-layer convective motions will also be partially-resolved, resulting in a reduced need for the parametrization of non-local boundary-layer mixing. Such models have been shown to have an improved representation of summertime UK convection (Stein et al., 2015; Hanley et al., 2015), cold-pooling in valleys (Vosper et al., 2013), the formation of marine stratocumulus (Boutle et al., 2014) and the formation of tornado-like structures in free-running simulations over the US Great Plains (Hanley et al., 2016). However, the validation of such models for fast-moving dynamical features such as fronts is problematic due to limitations on domain size and a lack of suitable observations.

In this study, the representation of a mature ana-type cold front in a high-resolution NWP model is interrogated across a range of model resolutions with gridspacings of 12 km down to 100 m. A key feature is the use of novel in situ aircraft observations which include a closed-circuit around the front within the boundary-layer. This observational strategy enables the direct evaluation of area-average vorticity and convergence at the front via circuit integral techniques. Since the front is fast-moving, the aircraft circuit is not closed in the Earth-relative frame of reference, but rather is designed with the aim of being approximately closed in the frame of reference moving with the front. Together with observations of local wind speeds and vertical turbulent fluxes these provide a detailed evaluation of the convergence of the model with resolution against reality at a range of spatial scales. A series of nested numerical simulations is employed, spanning the range from a traditional NWP model (12 km grid spacing) in which both tropospheric and boundary-layer convective mixing are performed by parametrization schemes, through convection-permitting resolutions in which the convection scheme is switched off (2.2 km and 1.5 km grid spacings) and down to sub-kilometer resolution models (500 m, 200 m and 100 m grid spacings) in which both the convection scheme and the non-local boundary-layer mixing scheme are switched off.

Ana-cold fronts typically exhibit sharp frontal transition zones in the boundary layer, accompanied by strong updraughts and a narrow band of relatively heavy precipitation called a narrow cold-frontal rainband (NCFR) (e.g. Browning and Harrold, 1970). Such rainbands are often observed to break up into line segments separated by gaps of weaker or no precipitation (James and Browning, 1979; Hobbs and Biswas, 1979). The mechanism of the break up is usually attributed to a horizontal shear instability, whereby the band of strong horizontal shear along the frontal transition zone is unstable and the resulting motion acts to wrap the strip of vorticity into a series of coherent vortices (Hobbs and Persson, 1982; Kawashima, 2011). In terms of surface impacts, the most intense precipitation along the front falls between the vortices, on narrow filaments of strong shear and temperature gradient, and tornadic structures, when they occur in the UK, typically also occur in the braids joining such vortices (Clark and Parker, 2014; Mulder and Schultz, 2015).

The presence of such rainband segments in the case studied here provides both opportunities and complications. By good fortune, the aircraft circuit crossed both a narrow filament of strong shear on one frontal transect and a coherent vortex structure on the other, thus enabling a comparison of both features within the model simulations. However, since the rainband segments are associated with strong along-front inhomogeneities, care is needed with the area-average vorticity and divergence observations to ensure they are representative of the front as a whole.

The paper is organised as follows. In Section 2 the flight track and instrumentation are summarized. The numerical model is also described including details of the dynamical core and physics parametrisations. In Section 3 a synoptic overview of the case is presented, highlighting the presence of segmented rainbands along the front. In Section 4 the convergence of the resolved flow with increased model resolution is addressed, and in Section 5 the representation of vertical turbulent fluxes in the boundary layer are considered. In Section 6 the structure of the frontal rainband segments in the model is discussed and the conclusions and further discussion are presented in Section 7.

2 Data and methods

2.1 Flight track and instrumentation

The case studied here is from 24 November 2009, during which an area of deep low pressure developed to the west of Scotland and the associated trailing cold front advanced from the west towards the UK and France. During the afternoon of that day the Facility for Airborne Atmospheric Measurements (FAAM) BAe146 aircraft undertook a research flight, as part of the T-NAWDEX Pilot campaign (Knippertz et al., 2010; Vaughan et al., 2015), over the southwest approaches of the UK to examine the structure of the front. There were two aims: to take detailed in situ measurements of the frontal structure at low-altitude (300 m), including the frontal circuit and measurements of turbulent fluxes, and to observe the vertical structure of the warm conveyor belt, including measurements of the associated moisture transport. The warm conveyor belt observations are discussed in Martínez-Alvarado et al. (2014); the focus of the present study is on the near-surface frontal structure and its representation in the numerical simulations.

Figure 1 depicts the route of the flight track together with the approximate position of the front at 1500 UTC 24 November 2009. The FAAM BAe-146 aircraft took off from Cranfield Airport, England, at 1311 UTC and flew southwestward at cruising altitude, reaching the vicinity of the front at 1439 UTC (point A). Two low-level maneuvers were then performed: a 100 km along-front leg (labeled AB) slightly ahead of the front at 40 m altitude (1439 UTC to 1456 UTC), and a front-crossing rectangular circuit (BCDEG) at 300 m altitude (1456 UTC to 1604 UTC) of size 80 km by 140 km.

Leg AB provides measurements of surface-layer turbulent fluxes in storm force winds, and estimated peak-to-trough ocean wave heights of 6-12 m. The circuit BCDEG was designed to be closed in a frame of reference moving with features on the front, but in practice the circuit was found to be best closed by a point between E and G, labeled F in Figure 1 (see Section 33.2). Subsequently the aircraft turned to cross the front again before ascending through the cold-sector boundary layer (1635 UTC to 1705 UTC) and finally crossing the front at high altitude to produce a vertical cross section of the front from dropsonde data (1705 UTC to 1742 UTC; see Figure 4 of Martínez-Alvarado et al. (2014)). An air-relative speed of 200 knots ($\approx$100 m s^{-1}) was maintained throughout the flight.

Full details of the instrumentation carried by the aircraft are given by Renfrew et al. (2008), Petersen and Renfrew (2009) and Vaughan et al. (2015), and the in situ observations used here are available from the Facility for Airborne Atmospheric Measurements (2014). Key to this study are a Rosemount 102BL temperature sensor and the FAAM five-port wind and turbulence probe, both of which report measurements at 32 Hz (approx 3 m spacing at science speed) with precision of $\pm$0.3 K and $\pm$0.25 m s^{-1} respectively. Use is also made of specific humidity measurements from a Lyman-alpha hygrometer which reports measurements at 64 Hz with an accuracy of $\pm$0.15 g kg^{-1}, although this data is re-sampled to 32 Hz to match the other variables. Turbulent fluxes are calculated following the methodology of Cook and Renfrew (2015), as follows. Each low-level leg of the flight track is split into straight and level runs of 2 minute duration (≈12 km). Run-average vertical fluxes of sensible heat, latent heat and momentum are then calculated as

$$SH = \overline{\rho} c_p \overline{w'\theta'} \tag{1}$$

$$LH = \overline{\rho} L_v \overline{w'q'}, \tag{2}$$

$$\tau = \overline{\rho}\sqrt{\overline{u'w'}^2 + \overline{v'w'}^2} \tag{3}$$

respectively, where u', v', w', θ' and q' are perturbations of the wind components, potential temperature and humidity for the run, detrended for each run, and bars denote run-average values. In addition, $\overline{\rho}$ is the run-average air density, $c_p = 1004$ Jkg^{-1} K^{-1} is the specific heat capacity for dry air and $L_v = 2.5 \times 10^6$ J kg^{-1}K^{-1} is the latent heat of vaporization. Cook and Renfrew (2015) apply a surface-layer correction to τ in order to obtain an estimate of the value at the surface; this correction is not applied here since the values are compared directly with model output at the aircraft altitude.

2.2 Numerical model

The simulations presented here are performed with the Met Office Unified Model (UM), version 8.4, which employs a non-hydrostatic, deep-atmosphere dynamical core with a semi-Lagrangian time stepping scheme (Davies et al., 2005). Six limited-area simulations are performed with resolutions ranging from 12 km gridspacing with 38 levels to 100 m gridspacing with 140 levels, as described in Table 1. The 12 km model takes its initial and boundary conditions from a global simulation with 40 km gridspacing, and each subsequent resolution is one-way nested from the previous. The model domains are shown in Figure 2. The presence of extreme strong winds and the fast propagation speed of the front itself provide a substantial computational challenge. To allow time for the spin-up of small scale features as the front enters each sub-domain, the domains are made as large as practically possible. In addition, the nesting of each sub model, which is achieved by passing boundary conditions from each parent model to each sub model at a predetermined updating frequency (see Table 1) is made as frequent as practically possible.

The global and 12 km limited-area models employ the following parametrization schemes: the radiation scheme of Edwards and Slingo (1996), the surface-layer scheme of Best et al. (2011), the mixed-phase cloud microphysics scheme of Wilson and Ballard (1999), the non-local boundary-layer scheme of Lock et al. (2000) and a convection scheme based on Gregory and Rowntree (1990). At resolutions of 2.2 km and below the convection scheme is switched off and an additional local subgrid turbulent-mixing scheme is used (Halliwell, 2007). Subgrid turbulent mixing is an essential component of NWP models with grid spacings of 0.1–1.5 km, since at these resolutions the boundary-layer inertial subrange is at best only partially resolved. The simulations presented here use a Smagorinsky-Lilly-type scheme which acts either just in the horizontal with vertical mixing provided by the non-local boundary layer scheme (2.2 km and 1.5 km simulations), or one which acts in all three spatial dimensions with the boundary layer scheme switched off (500 m, 200 m and 100 m simulations). In all cases the subgrid mixing length is set to 0.2 times the horizontal gridspacing. These configurations of the Met Office Unified Model have been developed during a number of previous studies, including Vosper et al. (2013), Hanley et al. (2015) and Stein et al. (2015).

2.3 Methodology for model-observations comparison

In order to perform circuit integrals in the front-relative frame of reference, the following change of coordinates is performed to the flight track. First, a reference time t_{ref} is chosen for which model output is available (for instance, t_{ref}=1500 UTC). Then, each location along the flight track $\mathbf{x}_{\text{obs}}(t)$ is shifted to its position relative to the front at time t_{ref} according to

$$\mathbf{x}_{\text{rel}}(t; t_{\text{ref}}) = \mathbf{x}_{\text{obs}}(t) + (t_{\text{ref}} - t)\mathbf{V}_{\text{f}} \quad (4)$$

where $\mathbf{x}_{\text{rel}}(t; t_{\text{ref}})$ is the position of the shifted flight track corresponding to the observation made at time t and $\mathbf{V}_{\text{f}}$ is the velocity of the front. Finally, the aircraft observations are compared to model output at time t_{ref} interpolated to the shifted flight track position $\mathbf{x}_{\text{rel}}(t; t_{\text{ref}})$. Since the circuit BCDEF took just over an hour to complete, all quantities presented in the following have been evaluated twice, using model output at t_{ref} = 1500 UTC and t_{ref} = 1600 UTC, and found to be similar.

The front velocity $\mathbf{V}_{\text{f}}$ is estimated from the model simulations. If the front were homogeneous in the along-front direction, only the component of the frontal velocity $\mathbf{V}_{\text{f}}$ perpendicular to the front would be required in Equation (4). However, the front considered here exhibits substantial along-front inhomogeneities (see below) so both along-front and cross-front components of $\mathbf{V}_{\text{f}}$ are used. These are estimated by manually tracking vorticity anomalies on the front in the model simulations, and found to be 14.2 ms^{-1} (along-front speed) and 9.4 ms^{-1} (cross-front speed). These values are largely consistent between the model simulations, and the results presented here are not sensitive to precise values used. Using the above stated values for $\mathbf{V}_{\text{f}}$ with t_{ref} = 15 UTC gives the shifted flight track position indicated by the grey dotted line in Figure 1.

3 The observed cold front

3.1 Synoptic situation

November 2009 was mild and exceptionally wet in the UK. An almost continuous chain of intense extratropical cyclones tracked across the North Atlantic towards the British Isles, resulting in strong winds and widespread heavy rainfall (Eden, 2010). Most notably, the storms of 18-20 November resulted in what was at the time the wettest 24 hour period ever recorded at a location in the UK, in Borrowdale in northern England (316 mm), and the subsequent flooding of the town of Cockermouth (Eden and Burt, 2010). The subject of the present paper is the cyclone which developed west of Ireland on 24 November and passed over Scotland on 25 November. A pressure minimum of 962 hPa was attained at 0000 UTC on 25 November, at which time its cold front was oriented SW-NE across the UK (see Figure 3(a)). Whilst less intense than its predecessors earlier in the month, the impact of the precipitation was still felt, particularly in northwest UK, due to the ground being saturated from the earlier events.

The radar-derived precipitation rate associated with the cyclone and cold front at 1900 UTC 24 November 2009 is shown in Figure 3(b). During the evening of 24 November the frontal precipitation advanced over Wales and England. It consisted of a broad band of moderate rainfall around 100km wide with a narrow cold-frontal rainband (NCFR) of intense rainfall, of order 10 km wide, at its leading edge. The NCFR is not continuous along the front, but rather is split into discrete segments of precipitation separated by gaps where precipitation rates are low.

3.2 In situ aircraft observations of frontal structure

Measurements of horizontal wind, temperature and humidity from the low-level legs A to G are shown in Figure 4. The track begins ahead of the front at point A from which it runs roughly parallel to the front at 40 m altitude before ascending to 300 m altitude at point B and turning towards the front, which it crosses at 1510 UTC (*transect BC*). At point C it turns to run parallel with the front in the cold sector. At point D it turns back towards the front, which it crosses around 1545 UTC (*transect DE*). Finally, at point E the circuit turns to run parallel with the front in the warm sector. Point F is the point along the leg EG which is closest to point B in the front-relative frame of reference (see Figure 1). Exact closure was not achieved because of the difficulty in forecasting the frontal velocity in real time, however the error is small.

Away from the front each of the four variables shown in Figure 4 are roughly constant: there is a 30 m s^{-1} south-southwesterly flow ahead of the front at 300 m and a 15 m s^{-1} westerly flow behind, there is a temperature difference of 4 K between the air masses and a specific humidity difference of 1.5 g kg^{-1}. Clark and Parker (2014) classify a series of NCFRs observed over the UK into three types, based on the magnitude of wind veer across the front and drop in wind speed. The strong reduction in windspeed across the front evident in Figure 4 suggests this is their 'Type B' NCFR, which they suggest is unlikely to be tornadic.

In the vicinity of the front there is a remarkable contrast between the frontal structure in the two transects. Transect BC exhibits a single sharp transition in wind velocity and temperature, with a change in the along-front wind of 14 m s^{-1} over a horizontal distance of 600 m, equating to a shear vorticity of 0.023 s^{-1}. In transect DE the wind direction and humidity fields both exhibit two distinct transition regions spaced a distance 12 km apart. Both transects also have spikes in the humidity field at the front crossings which are likely due to a combination of precipitation and the response time of the instruments.

4 Quantitative evaluation of frontal structure in the simulations

An overview of the low-level horizontal structure of the front in the model simulations is presented in Figure 5 which shows snapshots of the vertical component of relative vorticity and the horizontal divergence at 300 m altitude. These fields show clearly the presence of the front, the position and orientation of which is consistent between the simulations. Also evident in Figure 5 is the NCFR instability in which the vorticity band along the front rolls up into isolated coherent vortices, connected by thin filaments of high vorticity and convergence (i.e. negative divergence).

Whilst the front and the frontal instability are evident in each model simulation, there are systematic changes as model resolution is increased. Most notably, it appears that as the model resolution increases: the width of the front decreases, the maximum magnitudes of the vorticity and divergence fields increase, the size of the coherent vortices decreases, and their along-front spacing decreases. In addition, there is substantial resolved convective activity behind the front, but not ahead of it, in the higher-resolution simulations. For a complementary overview of the frontal structure in the model simulations, Figure 1 in the Supplementary Material shows corresponding plots for potential temperature and specific humidity, and the same qualitative conclusions can be drawn from those variables.

In this section, the resolved frontal structures are compared quantitatively between the model simulations and the aircraft observations in order to assess the extent to the model converges on the observed atmospheric characteristics with increasing resolution. First, the area-average vorticity and divergence across the front are calculated via circuit integrals. Whilst the width of the front decreases with resolution, the magnitude of the maxima in the vorticity and divergence fields increase. The expectation is that the area-average values are insensitive to resolution, even if the fine-scale structure of the front is not well resolved. Second, the local structure of the front is analyzed. Do the highest resolution model simulations capture the observed frontal width? Aspects relating to the turbulent activity seen Figure 5 are considered in Section 5 and characteristics of frontal instability are discussed in Section 6.

It is noted that Figure 5 also shows evidence of an influence from the domain boundary in the 200 m and 100 m simulations, in which roll-like structures emanate from the proximate in-flow (i.e. western) boundary (panels e and f). Such rolls are a common feature in $O(100\text{ m})$ models (Boutle et al., 2014). The rolls decay before reaching the area of the flight track in the 200 m simulation, but not the 100 m simulation, indicating that care is needed in drawing conclusions from that simulation. In addition, there are strong wave type features immediately ahead of the front in the 100 m simulation, perhaps indicating that the frontal structure is adjusting to smaller scales in the 100 m simulation and emitting pre-frontal gravity waves in the process (e.g. Shakespeare and Taylor, 2014). A strong pre-frontal gravity wave packet parallel to the front was observed from the aircraft, as described by Knippertz et al. (2010), but the similarity to the gravity waves evident in the 100 m simulation is not explored further here. Finally, it is also noted that the 100 m simulation appears to exhibit a numerical instability lying along the front from where it leaves the domain at the northern boundary. Together, these shortcomings of the 100 m simulation highlight current limitations of trying to attain high resolution simulations of non-stationary dynamic features such as fronts. Further 100 m simulations with a larger domain will prove useful for understanding these processes, but were not possible at the time of writing due to the computational cost.

4.1 Integral measures of front intensity

As discussed above, the design of the flight track as a closed circuit in a system-relative sense allows a direct evaluation of the area-average vorticity (VOR) and divergence (DIV) values within the circuit. Stokes' and Gauss' theorems give respectively:

$$VOR = -\frac{1}{A}\oint_{\mathcal{C}} \mathbf{V}\cdot\mathbf{s}\,\mathrm{d}l \tag{5}$$

and

$$DIV = \frac{1}{A}\oint_{\mathcal{C}} \mathbf{V}\cdot\mathbf{n}\,\mathrm{d}l, \tag{6}$$

where $\mathbf{V}$ is the horizontal wind, $\mathcal{C}$ is the shifted front-relative flight track, $\mathbf{s}$ and $\mathbf{n}$ are unit vectors pointing along and perpendicular to the left of the flight track respectively, A is the area enclosed by $\mathcal{C}$ and l is distance around the perimeter of $\mathcal{C}$ measured in the direction of the flight. Note that the shifted flight track is the appropriate contour for both the model data and the observations, despite the observations being taken along the unshifted flight track, because it is equivalent to viewing observations in a front-relative frame under the assumption that the fine-scale structure of the front is frozen in time. The negative sign in (5) arises because the aircraft flew clockwise around the circuit, whereas the mathematical convention is that circuit integrals are performed counter-clockwise.

Simpler bulk estimates for shear vorticity and divergence are commonly calculated from differences in the mean along-front and cross-front wind components ahead and behind the front. This calculation is also performed here to explore the impact of using the exact circuit integral expressions. Assuming that the flight track is parallel to the front along the segments CD and EF, which was the aim of the flight plan but cannot be verified exactly, these bulk estimates of vorticity and divergence take the form

$$VOR_{\mathrm{bulk}} = -\frac{1}{d}\left(\overline{\mathbf{V}\cdot\mathbf{s}}^{CD} + \overline{\mathbf{V}\cdot\mathbf{s}}^{EF}\right) \tag{7}$$

and

$$DIV_{\text{bulk}} = -\frac{1}{d}\left(\overline{\mathbf{V}\cdot\mathbf{n}}^{CD} + \overline{\mathbf{V}\cdot\mathbf{n}}^{EF}\right) \tag{8}$$

respectively, where d is the distance between the middle of the along-front segments CD and EF, and over bars indicate averages along the flight segments indicated. The similarity with the exact contour integral expressions (5)-(6) is apparent. The key differences are that the bulk estimates (i) assume the flight track is parallel to the front, and (ii) neglect the contributions from the cross-front transects BC and DE.

Figure 6 shows the contour integrals (5)-(6) (panel (a)) and the bulk estimates (7)-(8) (panel (b)), both calculated from the observations and all model simulations except for 100 m resolution since the flight track exceeds the limits of the simulation domain in that case. In the observations, both vorticity diagnostics are positive and both divergence diagnostics are negative, indicative of cyclonic shear and convergence at the front respectively. The magnitudes of all four values are very similar at around 1×10^{-4} s^{-1}. The fact the contour integrals and bulk estimates are similar suggests that the flight track is indeed parallel to the front. Note that these are smaller in magnitude than typical point values of vorticity and divergence at fronts because they represent an area-average over the flight circuit region, which is considerably broader than the front itself (see Figure 5).

The values have been calculated from both 1500 UTC and 1600 UTC model output and found to be largely similar. However, Figure 5 shows that there are substantial along-front inhomogeneities in the vorticity and divergence fields associated with the frontal instability in the model, and these can be expected to show up in the area-average diagnostics. If these features are realistic then the observed values will vary depending on the precise location of the flight track. Whilst the scale and structure of the frontal instability may be captured by the model simulations, the locations of the frontal segments are likely to have little predictability. Therefore the along-front inhomogeneity amounts to a sampling uncertainty in the observations. To estimate this uncertainty, the vorticity and divergence diagnostics (5)-(8) are recalculated for a sample of 20 alternative flight tracks, identical to the actual flight track except for a translation parallel to the front of up to 120 km, and the distribution of values obtained are indicated by the 'box-and-whisker' symbols in Figure 6. In total there are 42 values calculated (from the original circuit plus 20 alternative circuits at both 1500 UTC and 1600 UTC), except for the 200 m simulation where 12 of the alternative circuits exceed the limits of the simulation domain so are excluded, leaving a sample of 30 values. The locations of the 1500 UTC sample of shifted flight tracks are shown in Figure 5(a) for reference.

Both divergence diagnostics are consistent with the observations, in the sense that the observed values lie within the sample spread of the simulations, at all resolutions except 12 km. In that simulation the bulk estimate is larger in magnitude than the observation. The sampling uncertainty is substantially larger for the contour integrals than the bulk estimates, indicating that the cross-front transects BC and DE are contributing to the variability of the area-average diagnostics. In contrast, the vorticity diagnostics are less consistent between the simulations and the observations: the mean vorticity values from all model circuits is larger than the observations in all simulations, with only the lowest vorticity values from the 1.5 km and 2.2 km simulations encompassing the observed value. Interestingly, the mean of the sample does not vary much with resolution but the sample spread does, with the 1.5 km and 2.2 km simulations exhibiting the largest along-front inhomogeneity. This is consistent with the larger vortices exhibited by those two simulations in Figure 5.

The mismatch between the observed and simulated vorticity diagnostics in all but the 1.5 km and 2.2 km simulations could be due to errors in the initial and boundary conditions from the global model acting to produce a front that is too strong in all simulations. Alternatively, it may be that the along-front inhomogeneity is under-represented in the highest resolution simulations. It is clear from Figure 5 that the 2.2 km and 1.5 km simulations exhibit larger vortices than the other simulations, and this is consistent with the large sample spread evident in those simulations in Figure 6(a). We hypothesize that the low vorticity value in the observations relative to the model simulations may be the result of the chance positioning of the flight circuit relative to the along-front rainbands, leading to a value from near the low end of the sample spread, combined with the fact that the vortices are too small in the sub-kilometer simulations. This hypothesis is explored further below.

4.2 Cross-front transects

Although a full picture of the actual along-front structure cannot be inferred from the two observed transects alone, the difference between the frontal structure in the two observed transects (see Figure 4) provides strong evidence for the influence of NCFRs. It appears that transect BC passes through a narrow filament of strong vorticity and convergence of width 600 m, whereas transect DE crosses the core of a vortex with a diameter of at least 12 km, with the two transition regions located at the edges of the vortex. This interpretation is now examined further by comparison with the model simulations.

To compare the observed cross-front transects with the model simulations, the along-front and cross-front wind components are calculated as

$$U_{\mathrm{para}} = \mathbf{V} \cdot \overline{\mathbf{n}}^{CD} \tag{9}$$

and

$$U_{\mathrm{perp}} = \mathbf{V} \cdot \overline{\mathbf{s}}^{CD} \tag{10}$$

respectively, where $\overline{\mathbf{s}}^{CD}$ and $\overline{\mathbf{n}}^{CD}$ are the unit vectors along and perpendicular to the left of the flight track averaged along transect CD. Figure 7 shows profiles of U_{para} from the two observed transects (panel (a)) and the 42 sample circuits for each model simulation (panels (c)-(h)). The cross-front wind (U_{perp}) from the two observed transects is also shown (panel (b)). All of the profiles have been shifted so that they align where $U_{\mathrm{para}} = 25$ m s^{-1}, for clarity, since the position of the front relative to the circuits is not identical in each simulation.

The difference between the two observed transects in Figure 7a is striking, indicating that the double-step structure in the wind direction along transect DE is associated with a similar structure in the along-front wind speed. The two cross-front wind speed transects are more similar to each other, with most of the convergence at the leading edge of the front in both cases. In contrast to the observations, the 12 km simulation exhibits a relatively smooth transition across the front of along-front wind speed over a distance of around 20 km, which is larger than observed in either transect, with little variation along the front. As the resolution is increased the gradient of along-front wind speed at the front increases, with the three highest-resolution simulations capturing the width of the sharp transition observed in transect BC (see below). Out of all the model simulations, the presence of a double-step structure of separation 12 km is only captured by the 1.5 km simulation, and partially by the 2.2 km simulation, consistent with these simulations having the largest vortices present in Figure 5.

To test the hypothesis that the chance positioning of the observed circuit has led to a relatively low value of the vorticity integral, the sample circuit with the lowest vorticity value from the 1.5 km simulation is highlighted in panel (e). There is a strong similarity with the aircraft observation, with the first transect consisting of a single sharp jump in wind speed and the second transect exhibiting a double step structure. Whilst it is expected that the relationship between the transect wind speeds and the area-average vorticity values is non-trivial, the fact that the positioning of the sample circuit with the lowest vorticity values appears similar to the observed circuit provides evidence that the observed circuit may encompass lower vorticity than elsewhere along the front.

For completeness, Figure 2 in the supplementary material shows the corresponding transects of potential temperature from the observations and the model simulations. Interestingly, for transect DE the potential temperature structure is very different to the along-front wind but has similarities with the cross-front wind. As the model resolution increases the maximum potential temperature gradient at the front increases, and again the most realistic vortex structures are found in the 1.5 km simulation. Furthermore, the highlighted circuit in panel (e), which again corresponds to the model circuit with the smallest vorticity value, shows some similarity to the observed transects. The main difference is a temperature maximum at around -15 km in the second transect which is not present in the observations. Figure 1 in the supplementary material shows that there is substantial structure in the potential temperaure field associated with the vortices in the 1.5 km simulation, including temperature maxima in the centres of some of the vortices. The fact that the observed transect does not exhibit such a maxima is likely due to the precise positioning of the aircraft circuit relative to the vortex centre.

Also apparent in Figure 7 is the presence of overshoots in the along-front wind speed at the model-simulated fronts. These are particularly clear in the 1.5 km, 2.2 km and 12 km simulations (where $U_{\mathrm{para}} < 0$ for example), although they can be seen to some extent in all simulations. The overshoots are not present in the observed transects, suggesting that they are a numerical artifact. Such overshoots are common near regions of strong gradients in (non-monotonic) semi-Lagrangian advection schemes, and may indicate that the advection scheme numerics is playing a role in limiting the frontal collapse in the rather than other more physical processes. This hypothesis is discussed further in Section 7. The overshoots are less evident in the 500 m, 200 m and 100 m simulations, perhaps indicating that the frontal width is not being limited by the advection scheme numerics in those cases.

In order to infer further information about the variation of the frontal gradients with resolution, a simple measure of frontal width is defined as the minimum distance between points with $U_{para} = 12$ m s^{-1} and $U_{para} = 25$ m s^{-1} in Figure 7. This definition of frontal width is inversely proportional to the bulk wind shear at the front. The threshold values were chosen subjectively in order to capture the clear double-step feature in the observations. However, the results are not qualitatively sensitive to the precise values used, nor are they sensitive to using thresholds based on the temperature transects instead of wind speed. The frontal width values are shown in Figure 8 in units of physical distance in panel (a) and scaled by the model grid spacing in panel (b). As previously, the two contrasting observed values are indicated by the crosses and the 'box-and-whisker' symbols illustrate the distribution from the sample of circuits from each simulations. Following the discussion above, the lower end of the sample bars is interpreted as the scale of the narrowest PV filaments in each simulation whereas the upper end represents the width of the widest vortex. The three highest resolution simulations are consistent with the narrow observation (transect BC), but are some way off encompassing the wide observation (transect DE). Therefore whilst the high resolution simulations are able to capture the remarkably sharp gradients observed at the front, and indeed these appear to be well resolved in the 200 m and 100 m simulations, they do not produce vortices with core widths as large as observed by the research aircraft. Only the 1.5 km simulation comes close to encompassing both the high and low observed width values, and as such appears to have the most realistic representation of both the frontal width and the frontal shear instability.

5 Turbulent fluxes

Figure 5 shows that resolved turbulent activity becomes more active in the cold sector at high resolution. In contrast, there is much less resolved turbulent activity in the warm sector, even in the 100 m simulation, despite the observed wind speed in Figure 4 being most variable there. This highlights the difference in boundary-layer regimes in the two regions: there is intense shear-driven turbulence present in the warm sector ahead of the front but at scales too small to be resolved in the simulations, whereas the boundary layer in the cold sector is convectively unstable and there are overturning circulations present of a similar scale to the boundary-layer depth. This case therefore provides a challenging test for the turbulence parametrization in the model. In this section both the resolved and sub-grid scale parametrized turbulent fluxes from the model simulations are evaluated against the observations.

Values of the sensible and latent heat fluxes, windstress and turbulent kinetic energy (TKE) have been calculated from the aircraft observations, as described in Section 22.1. Timeseries of the observation-derived fluxes are shown in Figure 9, in which each dot represents a single straight and level two-minute leg. In the warm sector (legs AB and EG) there is a downwards sensible heat flux, an upwards wind stress and an upwards latent heat flux. These are as expected due to the advection of relatively warm air over a cooler ocean. In the cold sector (leg CD), the sensible heat flux and windstress are both much smaller, whereas the latent heat flux is similar in magnitude to its pre-frontal value. Typically, surface sensible and latent heat fluxes are large and positive in cold sector air. This is not the case here, presumably because the fluxes are measured at 300 m altitude rather than the surface. Apart from in the vicinity of the front itself, the values of all the fluxes are roughly constant. As such, for comparison with the model simulations attention is now restricted to the average values along the legs AB, CD and EG.

Turbulent fluxes in the model simulations are computed as the sum of the resolved and parametrized components. The resolved component is computed following the same method as the observations, by first interpolating variables onto the shifted flight track, and then using the same straight and level two-minute runs as the observations to compute covariance values. The parametrized component is taken directly from the relevant parametrisations scheme and is likewise interpolated onto each two-minute run. Figure 10 shows these values calculated for each model simulation. Indicated in the figure are the observations (crosses) as well as the resolved (blue), parametrized (red) and total (black) fluxes for each model simulation. The box-and-whisker symbols indicate the spread of leg-average model values from the sample of circuits, whereas the small crosses indicate the range of individual two-minute run values in the observations.

Considering first the cold sector leg CD, both the latent heat flux and the wind stress exhibit a gradual transition from being fully parametrized in the 12 km simulation to around 80% resolved at 100 m resolution. To within the sample spread, the sum of the resolved and parametrized components remains constant with varying resolution and is consistent with the observed values. The subgrid turbulence scheme is therefore successfully accounting for the partially-resolved eddies in this case. Likewise the TKE converges, although sampling variability becomes very large in the 100 m simulation. The sensible heat flux, in contrast, does not handover monotonically from parametrized to resolved with increasing resolution. The resolved component is essentially zero in all simulations except 100 m, whereas the parametrized component has largest magnitude in the 1.5 km simulations.

In the warm sector, the resolved fluxes are small in all model simulations at both 40 m (leg AB) and 300 m (leg EG) altitude, as anticipated from the lack of resolved turbulent activity in Figure 5. The simulations which apply the non-local 1-d boundary-layer scheme (those with grid spacing $\geq$1.5 km) show remarkably similar values for all three fluxes, and at both altitudes. However, in each case the magnitudes of all three fluxes are larger than observed. The TKE, in contrast, is smaller than the observed values and jumps between the 2.2 km and 12 km simulations. Switching from the 1-d boundary-layer scheme to the 3-d Smagorinsky scheme at 500 m resolution does not affect the fluxes much, but as the resolution is increased further the magnitudes of all fluxes decrease substantially. At 40 m altitude (leg AB) the effect is to move the model fluxes closer to the observed values, and the 100 m simulation agrees closely with the observed values, except perhaps sensible heat flux. However, the values do not appear to have converged by 100 m, and the large jump between the 100 m and 200 m values suggests they may continue to decrease, beyond the observed values, if the resolution is increased further. At 300 m altitude this is precisely what happens: the fluxes reduce in magnitude as the resolution is increased and end up smaller than the observed values in the 100 m simulation. In that case, the parametrized fluxes are closest to the observations in the 200 m simulation. It is of note that Stein et al. (2015) likewise found the 200 m configuration performed best when comparing the width of convective updrafts in this model with radar observations, however there is no clear reason for assuming that the results are related in the different dynamical regimes.

A possible explanation for the poor performance of the 3-d Smagorinsky scheme ahead of front is that the scheme assumes the presence of a partially-resolved inertial cascade. This is not the case here since the shear-driven turbulence is subgrid, even at 100 m grid spacing (see Figure 5). In effect, the scheme is appropriate in the surface layer, which encompasses the 40 m observations, but not at higher altitude in the boundary layer. Physically, the mixing length is chosen to be proportional to the grid length in these simulations, but since grid-scale eddies are not present ahead of the front, the magnitude of the resolved shear does not increase with resolution. Therefore the parametrized fluxes can be expected to decrease in line with the mixing length. The study of Boutle et al. (2014) introduced a pragmatic blending methodology in which a linear combination of the local and non-local mixing schemes is objectively selected based on environmental conditions, and it is expected that such a scheme may act to alleviate the unrealistic fluxes found ahead of the front here.

6 Frontal instability and structure of the NCFRs

Whilst the aircraft observations provide high-resolution information on the cross-frontal structure and turbulent fluxes, the along-front lengthscale of the NCFRs cannot be inferred from the aircraft observations alone. However, the NCFRs are clearly visible in the radar image of Figure 3, albeit at a later time than the aircraft observations, and their along-front lengthscale can be measured manually as the average distance between breaks in the NCFRs. This process has been repeated for each hour from the time the front enters the radar domain (1800 UTC). At each hour all of the NCFRs visible in the radar domain (around six) are used to compute the average wavelength. A similar process has been performed to the precipitation fields from the model simulations and the resulting wavelengths are summarized in Figure 11.

The radar images exhibit an average wavelength of 100 km, which remains roughly constant over time. In contrast, the 12 km simulation has wavelengths that are too long (around 300 km) and the sub-kilometer simulations exhibit wavelengths that are too short (around 40 km and 20 km respectively). The 2.2 km and 1.5 km, however, have similar wavelengths to those seen in the radar image (around 100 km). Taken together with the results of Section 44.2, in which it was shown that the width of the vortices in the sub-kilometer simulations are too small, Figure 11 provides evidence that the structure of the instability in the sub-kilometer models is indeed collapsing to a scale that is smaller than observed.

It is of note that Figure 11 shows the wavelength in the 500 m simulation increasing during the final 3 hours of the simulation, towards a value closer to that observed. The corresponding precipitation maps are shown in Figure 12 in which the small scale of the NCFRs at 1500 UTC in the 500 m simulation is evident (comparing panels (a) and (b)), together with the increase in scale at 2300 UTC (panel (f)). The reason for this increase in scale is not clear. One hypothesis is that a change in environmental conditions in the vicinity of the front over time leads to a change in the properties of the instability later in the simulation. Alternatively, the structure of the front may still be equilibrating to the higher resolution of the nested 500 m domain at 2300 UTC. It takes around 12 hours for the vortical structures to spin-up at the start of the 2.2 km simulation (not shown), which is similar to the time taken for the front to cross the 500 m domain.

Investigating further the dynamics of the NCFR instability in the sub-kilometer simulations is beyond the scope of this work. However, it is noted that other studies have been able to produce more realistic simulations of similar events using models of comparable resolution (e.g. Smart and Browning, 2009; Apsley et al., 2016), albeit on smaller domains and therefore with less time for the high-resolution dynamics to modify the initial state taken from the coarser resolution parent model. It is also noted that moist-frontal instability is studied in an idealised setting by Kawashima (2011) who suggests the nature of the instability is sensitive to the environmental conditions ahead of the front. They show that the NCFR instability can be stabilized by reducing the ambient cross-frontal shear, and if there is sufficient environmental CAPE then instead of NCFRs the fastest-growing instability is convective in nature with a much smaller along-front scale. This provides yet another hypothesis for the shift of the instability to small scales in the high-resolution simulations, that systematic biases in the turbulent fluxes ahead of the front impact the frontal instability via this mechanism. These aspects of the dynamics will be investigated in a future study.

7 Conclusions and discussion

In situ aircraft observations are used to interrogate the convergence with resolution of simulations of a narrow cold front. Simulations are performed with the MetUM at a range of nested resolutions from grid spacings of 12 km to 100 m. The observational strategy employed enables a comparison of flow structure over a broad range of scales, from the scale of the aircraft circuit (80 km by 140 km) to small-scale turbulent motions as measured by 32 Hz instruments (approx 3 m). Integral measures of the wind field are employed together with velocity gradients and turbulent fluxes to provide a comprehensive picture of the front across a wide range of scales.

The low-altitude horizontal divergence at the front successfully converges by 2.2 km resolution, consistent with the frontal convergence field being driven by the cross-frontal circulation and relatively insensitive to the small-scale details at the front itself. The simulated low-altitude vorticity is only consistent with the observations if the sampling uncertainty associated with the position of 100 km precipitation segments along the front is accounted for. The model also converges on the observed frontal width where it is narrowest (600 m) in the highest-resolution simulations. These results imply that the net ascent out of the boundary layer at the front is also well represented.

The presence of overshoots in the along-front wind behind the front is evidence that at lower resolution (1.5 km and coarser) implicit diffusion from the semi-Lagrangian advection scheme is limiting the frontal width. Repeated use of interpolation at back-trajectory departure points introduces diffusive-like effects, with the leading-order influence resembling ∇^2 for linear interpolation and ∇^4 for cubic interpolation (e.g. Harvey, 2011). Whereas ∇^2 diffusion acts to smooth a region of large gradient without exceeding the bounds of the initial field, hyper-diffusion of the form ∇^{2n} with $n > 1$ acts to smooth the regions of large gradient but can exceed the bounds of the initial field. In particular, applied to a step function (an approximation to the along-front wind field), such hyper-diffusion inevitably leads to overshoots (Mariotti et al., 1994). The horizontal scale of the overshoots should scale with the gridspacing, but the magnitude of the overshoots is independent of resolution, being a function only of the form of interpolation used. Their presence here suggests that the advection scheme numerics is playing a role in limiting the frontal collapse. The overshoots are less evident in the 500 m, 200 m and 100 m simulations, perhaps indicating that the frontal width is not being limited by the advection scheme numerics in those cases.

In the convective boundary layer on the cold side of the front, the sum of parametrized and resolved vertical eddy fluxes is approximately constant as resolution increases and parametrized fluxes hand over to resolved motions. The values are consistent with the aircraft observations indicating a good representation of the turbulent grey zone in the convective boundary layer regime. However, on the warm side of the front where the boundary layer is stably-stratified and the turbulence is shear-driven, the model fluxes are entirely subgrid scale and the values are only consistent with observations from a 40 m altitude flight leg. Poor performance is found from a 300 m altitude flight leg in the sub 1 km resolution simulations, perhaps due to a switch from a 1-d boundary-layer scheme to a 3-d turbulent mixing scheme more suited to Monin-Obukhov-type boundary layers.

Despite success in the simulation of frontal width and turbulent fluxes, the simulation of vortex roll-up along the front and the development of narrow cold-frontal rainbands does not converge. Large vortices consistent with the observations appear only in the 1.5 km simulation, and to some extent in the 2.2 km simulation, but not in the sub 1 km simulations. The reason for this is not clear, however one hypothesis is that the transition between two types of instability is sensitive to initial conditions and the model parametrisations, giving along-front instability low predictability. This result is unfortunate since such frontal roll-up can be associated with high impact weather events. For example, tornadoes typically occur in the British Isles in the "braids" joining the vortices (Clark and Parker, 2014; Mulder and Schultz, 2015). The prediction of the frontal instability is therefore a challenge for forecasting of high-impact weather, and the results presented here emphasize that model resolution alone is not sufficient for success. Future work will aim to understand the nature of the instability in more detail and the reasons for the collapse to smaller scales in the sub-kilometer simulations.

The results shown here also demonstrate the utility of the observational strategy employed. The use of a closed-circuit flight track in the frame of reference of the front enabled the accurate calculation of area-average vorticity and divergence values, although the substantial along-front inhomogeneities present in this case introduced a large sampling uncertainty. Looking forward, there is a clear need for more observational campaigns focused on measuring near-surface turbulent fluxes in active regions of the atmosphere, such as in and around fronts in a translating frame of reference, and in situ aircraft observations provide a means to achieve this.

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Table 1: Model configurations used in this study.

Resolution	Approximate domain size	Timestep	Convection scheme	Boundary layer scheme	Subgrid mixing scheme	Initialisation time	Boundary updating frequency
40 km, L70	Global	12 min	On	On	Off	0600 UTC 23 Nov	-
12 km, L38	10000 x 6100 km	5 min	On	On	Off	0600 UTC 23 Nov	3 hr
2.2 km, L70	3100 x 2700 km	75 s	Off	On	2-d Smag	1200 UTC 23 Nov	30 min
1.5 km, L70	1600 x 1500 km	50 s	Off	On	2-d Smag	1500 UTC 23 Nov	30 min
500 m, L140	850 x 600 km	10 s	Off	Off	3-d Smag	1800 UTC 23 Nov	15 min
200 m, L140	320 x 320 km	6 s	Off	Off	3-d Smag	1200 UTC 24 Nov	15 min
100 m, L140	150 x 150 km	3 s	Off	Off	3-d Smag	1200 UTC 24 Nov	15 min

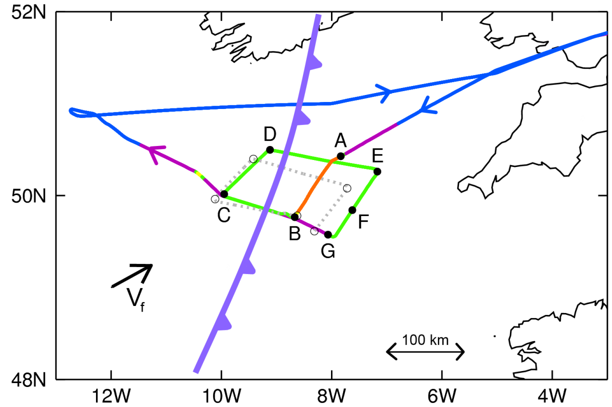

Figure 1: The flight track and the approximate position of the front at 1500 UTC. Colour indicates altitude, increasing from orange for altitudes below 200 m to green, purple and blue at levels above 200 m, 400 m and 4 km respectively. The black arrow shows the displacement of the front over 1 hr. The gray dotted line indicates the flight track shifted to a frame relative to the front at t_{ref}=1500 UTC (see Section 22.3). Black dots indicate the locations A to G on the flight track, and the open circles the corresponding points B to F on the shifted flight track. The timing of all the labelled points are shown in Figure 4.

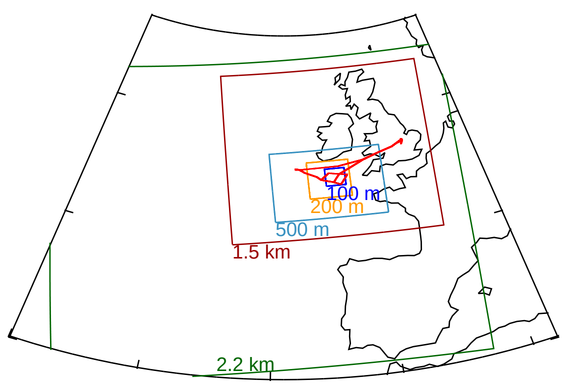

Figure 2: The model domains used for the 2.2 km, 1.5 km, 500 m, 200 m and 100 m simulations in this study. The domain of the 12 km simulation is much larger than the region shown, extending from 90W to 75E at the latitude of the flight track and 25N to 80N at the longitude of the flight track. The flight track is shown by the red line.

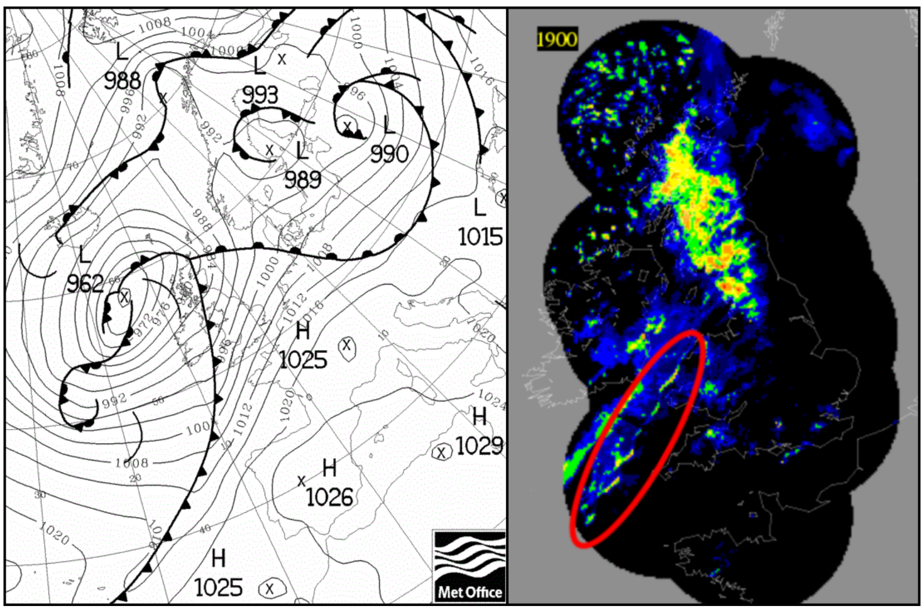

Figure 3: (a) Met Office surface analysis at 0000 UTC 25 November 2009 and (b) Met Office radar-derived precipitation rates at 1900 UTC 24 November 2009. The position of the narrow cold-frontal rainbands are highlighted in the radar image.

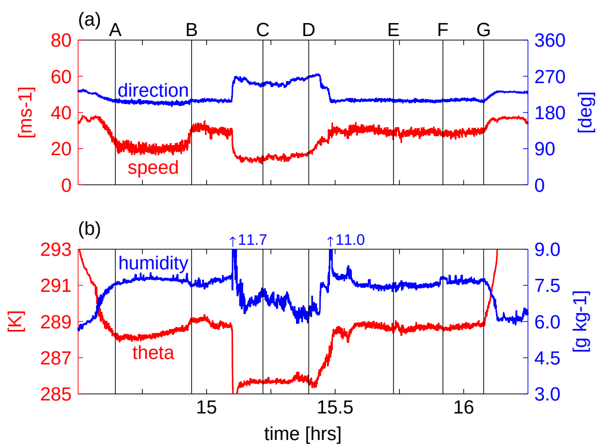

Figure 4: Time series of observed (a) wind speed and direction, and (b) potential temperature and specific humidity. The aircraft altitude was 40 m during leg AB and 300 m during circuit BG, with the ascent beginning one minute before point B. The locations of points A to G are indicated in Figure 1.

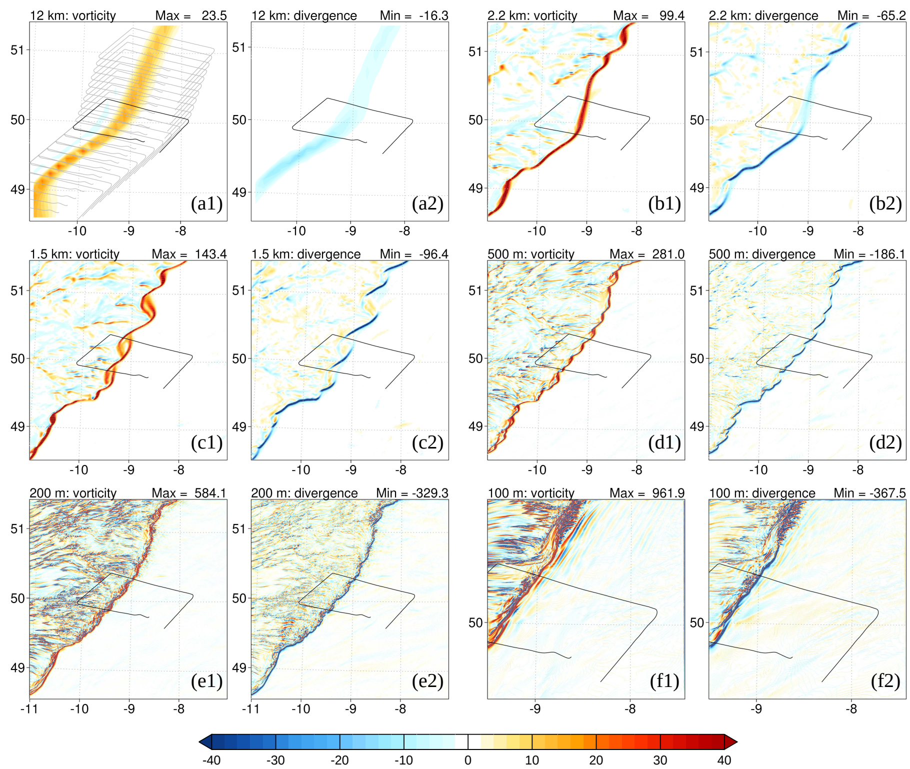

Figure 5: Simulated relative vorticity (1) and divergence (2) in the vicinity of the front at 1500 UTC, altitude 300 m (units: $10^{-4}\ s^{-1}$). The simulations shown are (a) 12 km, (b) 2.2 km, (c) 1.5 km, (d) 500 m, (e) 200 m and (f) 100 m. The 1500 UTC shifted flight track is indicated in all panels and panel (a1) also shows the sample of 20 alternative circuits described in Section 44.1. Note that panels (f1) and (f2) show a smaller area, equal to the full domain used for the 100 m simulations.

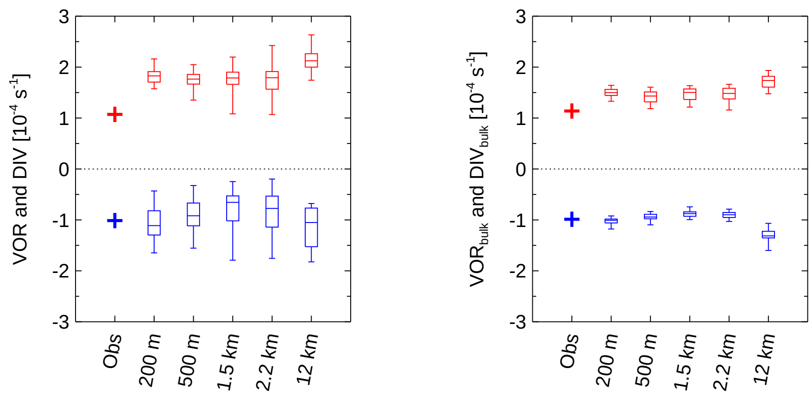

Figure 6: Area-average vorticity (red) and divergence (blue) values from (a) the exact expressions of Equations (5)-(6), and (b) the bulk estimates of Equations (7)-(8). The plus symbols indicate the observed values and the box-and-whisker symbols indicate the median, inter-quartile range and full range of the sample of 42 model circuits consisting of the original flight track and the 20 alternative circuits, all evaluated from both 1500 UTC and 1600 UTC data model output. The 100 m simulation is not included as the domain used does not cover the entire flight track region, and the 200 m simulation only has a sample of 30 circuits due to some lying outside of the domain.

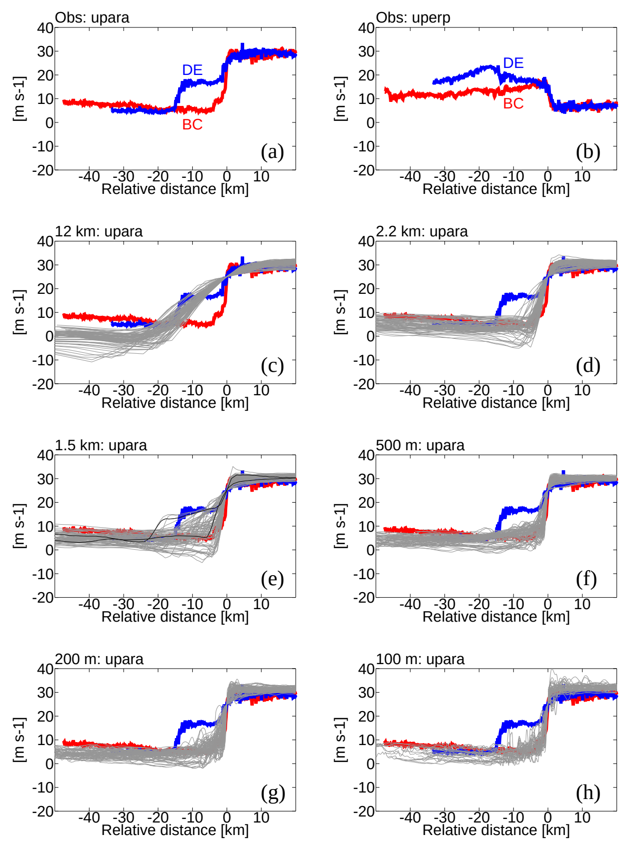

Figure 7: (a) Observed along-front wind profiles from flight legs BC (red) and DE (blue), (b) observed cross-front wind profiles from flight legs BC (red) and DE (blue), (c)-(h) model profiles of along-front wind from the simulations with each gray line showing one of the sample of model circuits (see text) and the red and blue lines are as in panel (a). In addition, in panel (e) the sample circuit exhibiting the lowest area-average vorticity value is highlighted in black. In all panels the profiles are shifted spatially to align the front as described in the text.

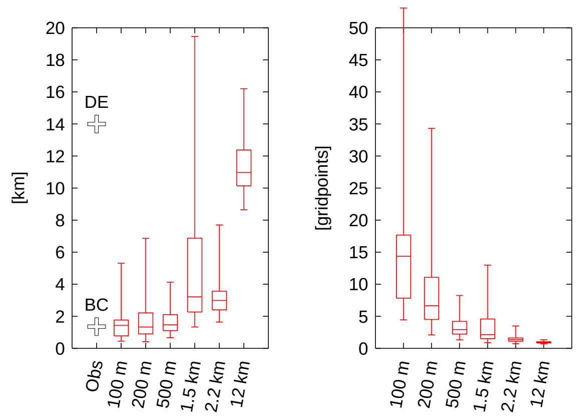

Figure 8: Frontal widths, calculated as described in the text, in units of (a) km and (b) gridpoints. The box-and-whisker symbols indicate the median, interquartile range and full range from the sample of circuits. The crosses in (a) are the observed values from the two transects, as indicated.

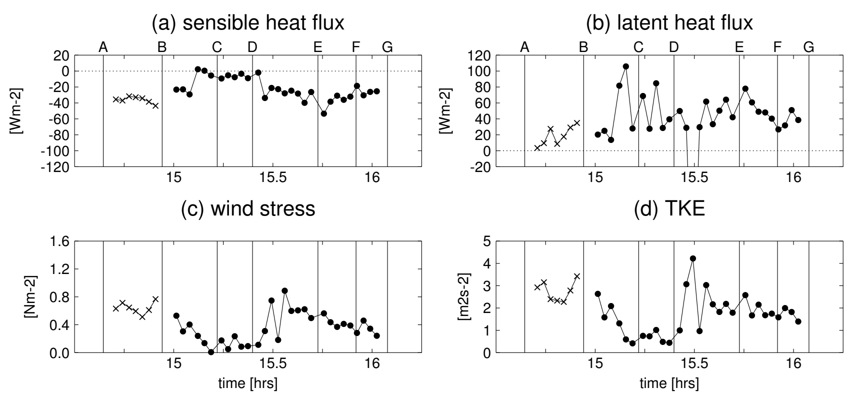

Figure 9: Timeseries of (a) sensible heat flux, (b) latent heat flux, (c) wind stress and (d) turbulent kinetic energy. Each dot (cross) represents the eddy covariances calculated from 32 Hz data over straight and level two-minute legs at 300 m (40 m) altitude, as described in the text.

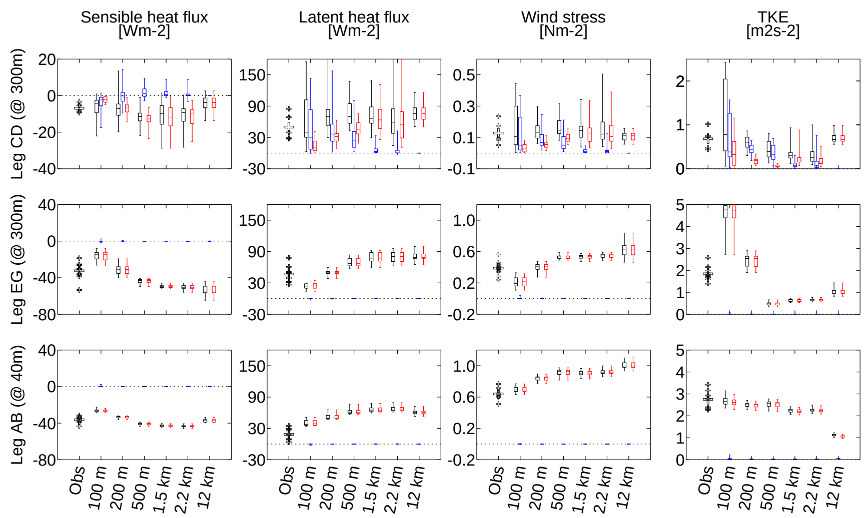

Figure 10: Summary of observed (crosses) and simulated (box-and-whisker) turbulent fluxes along: (top) cold side of front at 300 m, (mid) warm side of front at 300 m, and (bottom) warm side of front at 40 m. For the observations the large cross shows the leg average value and the small crosses show the values of the individual two-minute runs. For the model simulations, the three bars indicate the median, inter-quartile range and full range of leg average values from the sample of circuits for the parametrized (red), resolved (blue) and total (parametrized plus resolved; black) fluxes.

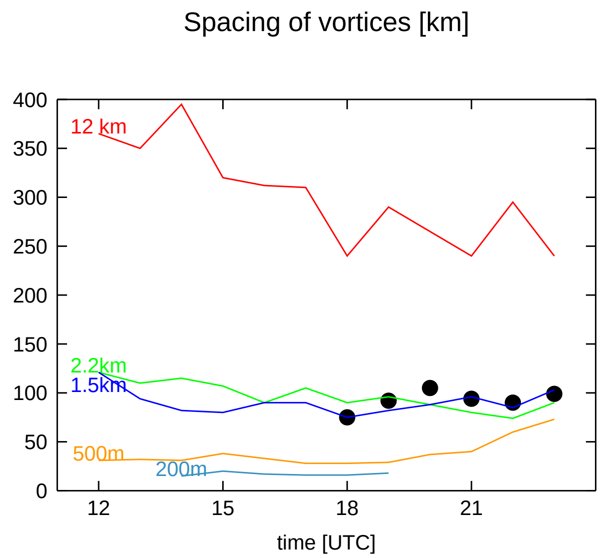

Figure 11: The evolution of narrow cold-frontal rainband spacing in the radar images (black dots) and the 5 simulations indicated (lines). The rainbands are outside of the radar range before 1800 UTC.

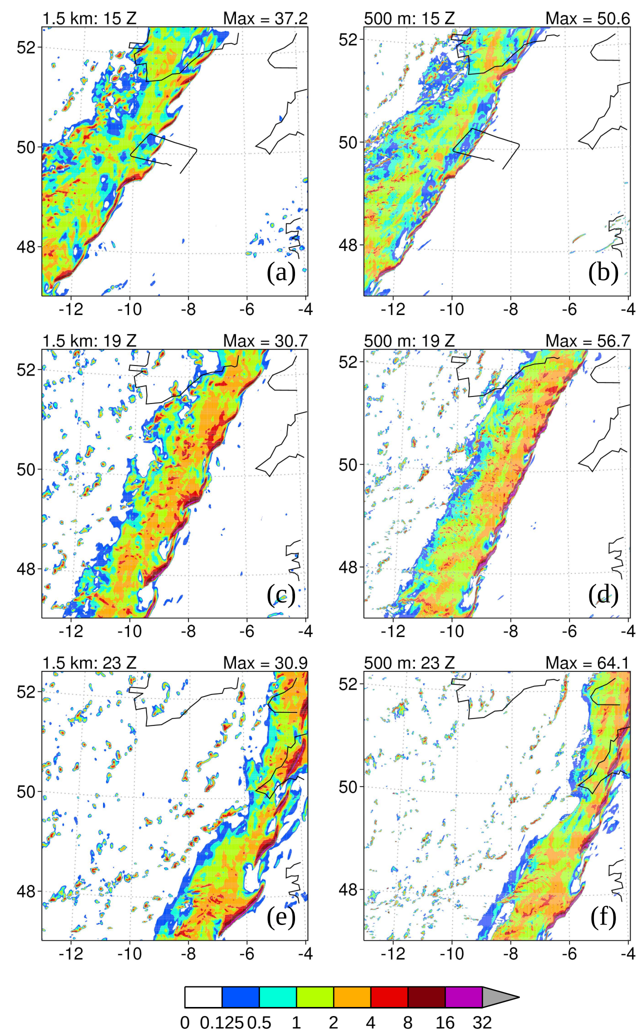

Figure 12: Precipitation rate (units: mm hr^{-1}) in the (a,c,e) 1.5 km and (b,d,f) 500 m simulations at 1500, 1900 and 2300 UTC respectively. The 1500 UTC shifted flight track is indicated in (a) and (b).