

# *Dispersion of a passive scalar within and above an urban street network*

Article

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1 **Dispersion of a passive scalar within and above an**  
2 **urban street network**

3 **EV Goulart · O Coceal · SE Belcher**

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6 **Abstract** The transport of a passive scalar from a continuous point-source  
7 release in an urban street network is studied using direct numerical simulation  
8 (DNS). Dispersion through the network is characterized by evaluating  
9 horizontal fluxes of scalar within and above the urban canopy and vertical ex-  
10 change fluxes through the canopy top. The relative magnitude and balance of  
11 these fluxes are used to distinguish three different regions relative to the source  
12 location: a near-field region, a transition region and a far-field region. The par-  
13 titioning of each of these fluxes into mean and turbulent parts is computed.  
14 It is shown that within the canopy the horizontal turbulent flux in the street  
15 network is small, whereas above the canopy it comprises a significant fraction  
16 of the total flux. Vertical fluxes through the array top are predominantly tur-  
17 bulent. The mean and turbulent fluxes are respectively parametrized in terms  
18 of an advection velocity and a detrainment velocity and the parametrization  
19 incorporated into a simple box-network model. The model treats the coupled  
20 dispersion problem within and above the street network in a unified way and  
21 predictions of mean concentrations compare well with the DNS data. This  
22 demonstrates the usefulness of the box-network approach for process studies  
23 and interpretation of results from more detailed numerical simulations.

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24 **Keywords** Dispersion model · Street network · Urban dispersion

## 25 1 Introduction

26 Noteworthy studies of dispersion in urban areas include a number of detailed  
27 field and scaled model experiments (e.g., Davidson et al., 1995, 1996; Mac-  
28 donald et al., 1997, 1998; Yee and Biltoft, 2004; Yee et al., 2006; Hilderman  
29 et al., 2007; Carpentieri et al., 2009) as well as high-resolution numerical sim-  
30 ulations (e.g., Hanna et al., 2002; Milliez and Carissimo, 2007; Branford et al.,  
31 2011; Philips et al., 2013). These have provided insight into how the presence  
32 of buildings modifies concentration levels in urban areas and what flow and  
33 dispersion processes contribute to these differences. For a general context, we  
34 refer the reader to reviews included in Britter and Hanna (2003) and Branford  
35 et al. (2011). Against the apparent complexity of empirical results, it is helpful  
36 to ask whether a core set of robust dispersion processes can be identified that  
37 could be of practical use in building approaches to model dispersion in the  
38 urban environment.

39 The need for urban dispersion models suitable for operational air-quality  
40 and emergency-response applications in particular requires novel approaches  
41 that can represent potentially complex turbulent flow processes in a simpli-  
42 fied way. Recently Belcher et al. (2015) have proposed a simple approach for  
43 modelling dispersion in a *street network regime*, where the buildings are close  
44 enough that a distinct network of streets emerges. The methodology follows  
45 Soulhac (2000) who developed the governing equations for a family of network  
46 models, together with methods for estimating the model parameters, which  
47 then led to the development of an operational dispersion model, SIRANE  
48 (Soulhac et al., 2011, 2012, 2016). Hamlyn et al. (2007) constructed a much  
49 simpler network model for dispersion through an array of cubes, showing im-  
50 pressive agreement with measurements made in a water channel by Hilderman  
51 et al. (2007). Belcher (2005) and Belcher et al. (2015) developed an analytical  
52 model for the dispersion of a passive scalar within a regular street network,  
53 which showed that the concentration is given in a closed form solution that  
54 includes an explicit dependence on the basic geometrical and flow parameters,  
55 which combine into only three effective parameters. Despite the important the-  
56 oretical insight that this solution provides, the authors found that the solution  
57 is restricted to the so-called near-field regime, where the vertical dispersion is  
58 dominated by detrainment out of the street network into the flow above. Be-  
59 yond the near-field region the re-entrainment of material back into the street  
60 network needs to be taken into account; this cannot be handled analytically in  
61 a robust way, although Belcher et al. (2015) gained additional insight through  
62 the use of a toy model for re-entrainment. Moreover, the dispersion above the  
63 canopy must be modelled too, as the interaction between the canopy and the  
64 flow above is a two-way process.

65 Against this background, our study is motivated by the aim of developing  
66 a simple model based on a minimal set of processes that will produce reli-

able estimates of the mean concentration both in the near-field region and beyond. The requirement of simplicity stems from the need for viable approaches for modelling in emergency-response or regulatory contexts. The interest in a process-based approach ensures that the very design of the model rests upon sound physical insights. This requires that we have an understanding of which processes are the most important to include and how best to parametrize them. The objectives are therefore two-fold: (i) In order to better understand the dispersion processes both within and above the urban canopy and how they interact we propose to analyze data from a previously-performed direct numerical simulation (DNS) over an array of cubical buildings (Branford et al., 2011). (ii) We extend the model of Belcher et al. (2015) to treat the dispersion both within and above the street network in a coupled way; this then explicitly represents the re-entrainment of material into the street network beyond the near-field regime.

The paper is structured as follows: Sect. 2 outlines the DNS dataset and method of analysis adopted; Sect. 3 is devoted to reporting and discussing results from analysis of the DNS data on the horizontal and vertical transport within and above the street network. In Sect. 4 the main results are used to formulate a simple street network model and to perform numerical experiments and parameter sensitivity studies with it. Conclusions are given in Sect. 5.

## 2 Numerical data and analysis

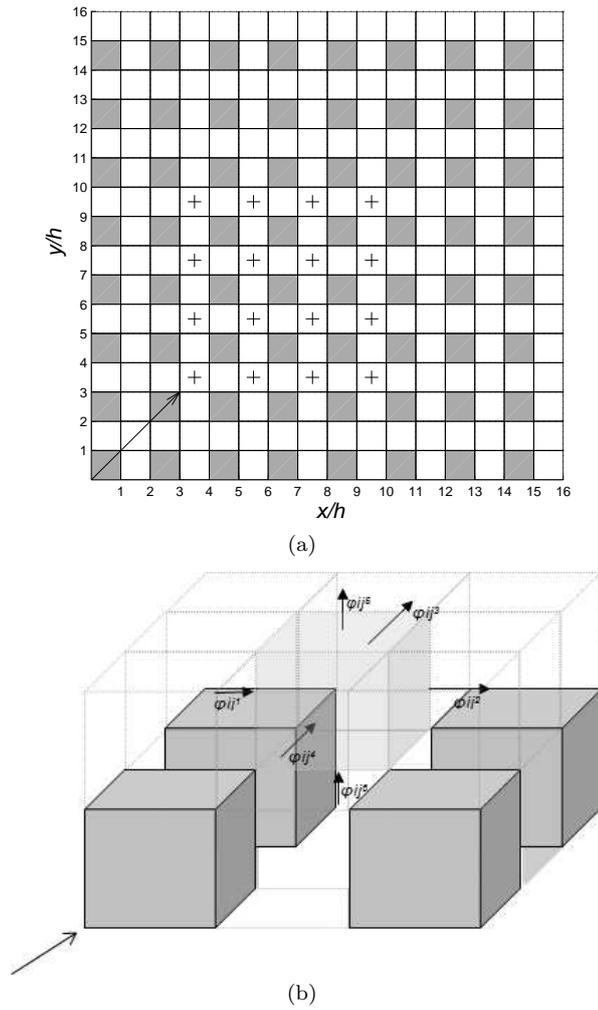
This section briefly outlines the DNS dataset and the method of analyzing the data.

### 2.1 Direct numerical simulations over a regular array

DNS data of Branford et al. (2011) are used here. The DNS models the dispersion of passive scalars by numerically solving the scalar equation,

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = D \nabla^2 c + S, \quad (1)$$

where  $c$  is the concentration of scalar,  $\mathbf{u}$  is the instantaneous velocity field vector,  $D$  is the molecular diffusivity and  $S$  is a source term. The instantaneous turbulent velocity field  $\mathbf{u}$  is a solution of the Navier-Stokes equations. The Schmidt number  $Sc \equiv \nu/D = 1$  in all the simulations. A steady point-source release near the ground was simulated, so that the source term is given by  $S = q \delta^3(\mathbf{x} - \mathbf{x}_s)$ , where  $q$  is a constant source emission rate,  $\mathbf{x}$  is the position vector,  $\mathbf{x}_s$  is the position vector of the source and  $\delta^3(\mathbf{x})$  is the Dirac delta function. In practice, the source is discretized as a Gaussian ball over a few grid points. The computational set-up, consisting of a regular array of cubes, allowed for multiple independent scalar fields to be modelled during each simulation. Figure 1a shows the computational domain and source locations, with



**Fig. 1** (a) Plan view of the computational domain in the DNS. Plus signs denote locations of the ground sources. (b) Schematic of fluxes through a box above an intersection.

102 a mean flow direction of  $45^\circ$  as indicated in the figure. We note that the flow  
 103 is symmetric with respect to the two horizontal components,  $u$  and  $v$ . Much  
 104 existing work in the literature has dealt with cases where the mean flow is  
 105 either aligned with or perpendicular to streets. However, these idealised cases  
 106 almost never occur under actual meteorological conditions; indeed they give  
 107 rise to somewhat artificial flow regimes. A mean flow oblique to the streets  
 108 constitutes a more realistic scenario.

109 The DNS employed dimensionless units, with lengths normalised by the  
 110 building height  $h$ , velocities normalised by the friction velocity  $u_\tau$  and with the

111 density of air  $\rho = 1$ . All quantities and parameters are given in corresponding  
 112 dimensionless units unless otherwise stated.

113 Time- and ensemble- averaged concentration statistics showed very good  
 114 agreement with experimental data (Branford et al., 2011). The data generated  
 115 from these simulations are here analyzed within a box-network framework,  
 116 described in the next section.

## 117 2.2 Analysis within a box-network framework

118 In the box-network framework an array of buildings is considered as forming  
 119 a network of ‘streets’ (here defined as the space between adjacent buildings)  
 120 joined at ‘intersections’; each of the streets and intersections can be thought of  
 121 as a box, through whose facets a scalar can enter or leave. Goulart et al. (2016)  
 122 showed that to a first approximation the scalar is generally well mixed in each  
 123 such box except near the source and the edges of the plume. Further layers of  
 124 boxes can be envisaged above the streets, intersections and buildings as shown  
 125 in Fig. 1b. The transport of scalars in such a street network can be analyzed  
 126 by considering the fluxes entering and leaving the boxes. Such an approach  
 127 forms the basis of a family of street network dispersion models (Soulhac, 2000;  
 128 Belcher, 2005; Hamlyn et al., 2007; Soulhac et al., 2011; Belcher et al., 2015),  
 129 a version of which will be presented in Sect. 4. To inform the development of  
 130 such a model, in Sect. 3 scalar fluxes over the facets of the boxes are computed  
 131 from the DNS data.

## 132 3 Scalar transport through a street network: results from DNS

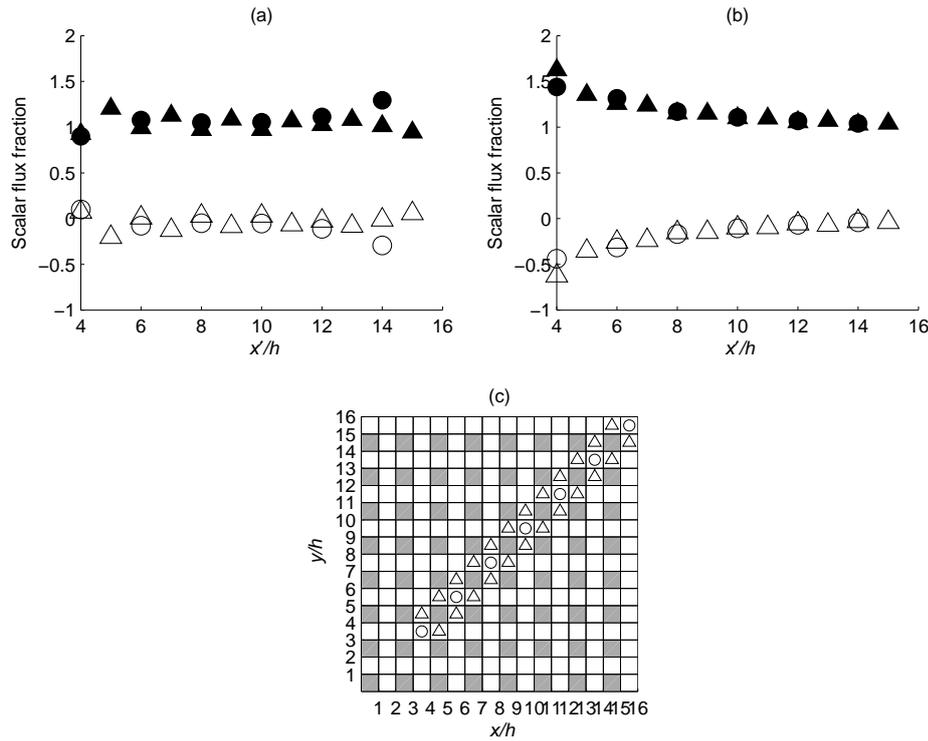
133 Dispersion of scalars through the street network is controlled by horizontal  
 134 fluxes within and above the urban canopy and by vertical exchange fluxes  
 135 through the canopy top linking these two regions. Each of these fluxes can be  
 136 formally decomposed into a mean and a turbulent component,

$$\langle \overline{cu_i} \rangle = \langle \bar{c} \bar{u}_i \rangle + \langle \overline{c' u_i'} \rangle, \quad (2)$$

137 where  $c$  is the instantaneous concentration and  $u_i$  is an instantaneous velocity  
 138 component perpendicular to the relevant facet. In Eq. 2 the overbar denotes  
 139 time-averaging and angled brackets denote spatial averaging over a facet. Hor-  
 140 izontal and vertical fluxes within and above the array and their mean and  
 141 turbulent components are computed from the DNS data. The results are then  
 142 applied in the configuration of the network model in Sect. 4.

### 143 3.1 Horizontal scalar fluxes within and above the canopy

144 Horizontal scalar fluxes within the canopy calculated from the DNS data are  
 145 plotted as a fraction of the total flux at different locations from the source in



**Fig. 2** Ratio of horizontal scalar fluxes, (a) within the canopy, (b) above the canopy. Filled symbols: ratio of mean to total flux  $\langle \bar{c} \bar{u} \rangle / \langle \bar{c} \bar{u} \rangle$ . Empty symbols: ratio of turbulent to total flux  $\langle \bar{c}' u' \rangle / \langle \bar{c} \bar{u} \rangle$ . (c) Sampling locations. Circles: intersections. Triangles: streets.  $x'$  represents the spanwise direction.

146 Fig. 2a. The locations of the boxes in which the fluxes were calculated lie along  
 147 three transects, as shown in Fig. 2c. We note that the middle transect involves  
 148 only intersections and the other two transects involve only streets. The results  
 149 for the latter two transects have been averaged. It is apparent from Fig. 2a  
 150 that, for both streets and intersections, the mean flux is much larger than the  
 151 turbulent flux irrespective of distance from the source. The average value of  
 152 the ratio of mean to total vertical flux in the canopy is  $\langle \bar{c} \bar{u} \rangle / \langle \bar{c} \bar{u} \rangle = 0.99$  (and  
 153 similarly for the  $v$  components, by symmetry). This ratio is relatively constant  
 154 throughout the array.

155 Figure 2b shows flux fractions along corresponding transects for the layer  
 156 of boxes just above the buildings. The most noteworthy difference is that the  
 157 turbulent fluxes are now negative and comprise a significant fraction of the  
 158 total flux (up to 0.5). The mean flux fraction is always larger than 1, with a  
 159 maximum value of about 1.5. The average value of the ratio of mean to total  
 160 vertical flux just above the canopy is  $\langle \bar{c} \bar{u} \rangle / \langle \bar{c} \bar{u} \rangle = 1.27$ . The corresponding  
 161 average turbulent flux ratio is therefore  $\langle \bar{c}' u' \rangle / \langle \bar{c} \bar{u} \rangle = -0.27$ . The occurrence

162 of this large counter-gradient turbulent flux ratio above the canopy contrasts  
163 with the small positive value of 0.01 within the canopy. The origin of these  
164 negative turbulent fluxes is unclear; a possible mechanism could involve ejections  
165 associated with coherent structures above the canopy (e.g., Coceal et al.,  
166 2007).

### 167 3.2 Vertical scalar fluxes through the canopy top

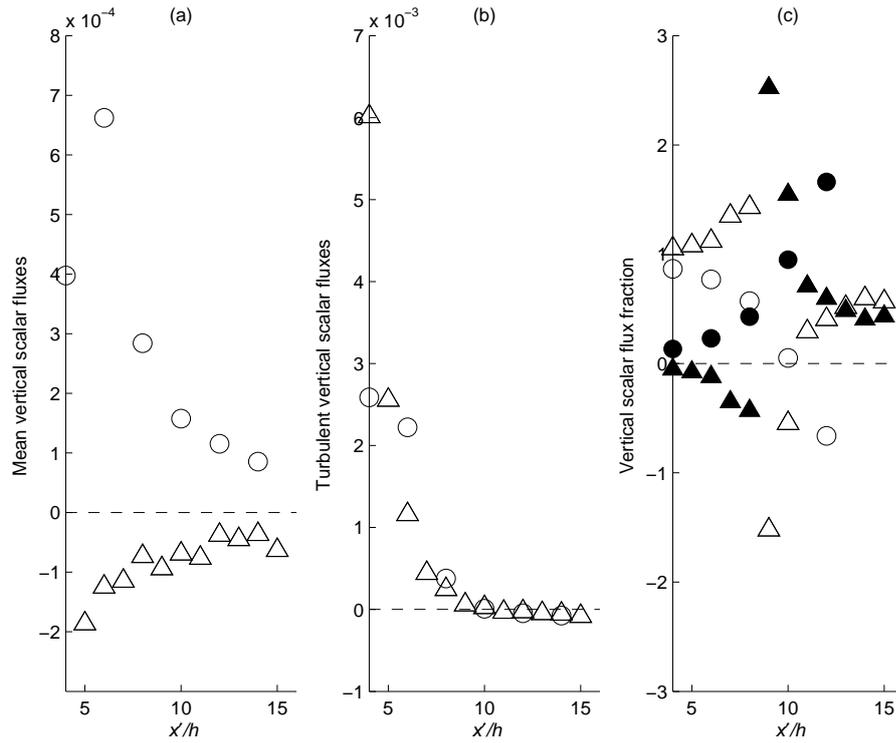
168 The mean and turbulent components of the vertical flux through the top of  
169 the array are shown in Fig. 3. The mean vertical flux  $\langle \bar{c} \bar{w} \rangle$  is always positive in  
170 the intersections and always negative in the streets (Fig. 3a). Since  $\bar{c}$  is always  
171 positive, the sign of  $\langle \bar{c} \bar{w} \rangle$  is determined by that of  $\bar{w}$ . Hence, the pattern of  
172 mean inflow or outflow is determined by the mean vertical velocity pattern.  
173 The vertical velocity averaged over the top facet of a street  $\langle \bar{w} \rangle$  is indeed  
174 downward, whereas it is upward over an intersection (not shown). We note  
175 that the mean vertical flux from the first intersection (which contains the  
176 source) is anomalously low; see below.

177 There is little difference between the turbulent fluxes  $\langle c'w' \rangle$  for streets and  
178 intersections (Fig. 3b). They are positive for both streets and intersections in  
179 the near-field region, but becomes slightly negative from the third intersection  
180 onwards. The maximum turbulent flux is about an order of magnitude larger  
181 than the maximum mean flux. The turbulent flux decays much quicker with  
182 distance from the source than the mean vertical flux. This may be because  
183 turbulent scalar exchanges take place in both directions, and hence tend to  
184 equalise quicker.

185 The ratios of the mean and turbulent vertical fluxes to the total vertical  
186 flux (Fig. 3c) reveal the following: (i) Up to a distance of about four building  
187 heights from the source the turbulent flux is the dominant component for  
188 both streets and intersections. (ii) However, far from the source (beyond a  
189 distance of about ten building heights) there is considerable scatter in the flux  
190 ratio. This is because both the turbulent and mean fluxes are small in the  
191 far-field region. The turbulent flux is slightly negative for intersections and  
192 both turbulent and mean flux are negative for streets.

### 193 3.3 Horizontal vs. vertical transport

194 The vertical flux through the canopy top exerts a strong control on how a  
195 plume spreads through a street network. Vertical detrainment from the canopy  
196 results in a reduction in the amount of material available to disperse horizon-  
197 tally through the canopy; this should cause a rapid fall-off of the concentration  
198 with distance from the source. However, material can also be re-entrained into  
199 the canopy further downstream. The balance between detrainment and re-  
200 entrainment is not the only factor that determines the subsequent horizontal  
201 fall-off. Equally important is the lateral spread through the canopy.



**Fig. 3** Vertical fluxes through the canopy top at the same sampling locations as in 2c. (a) mean, (b) turbulent, (c) ratio of mean (filled symbols) and turbulent (empty symbols) to total. Circles: intersections. Triangles: streets. Fluxes have been normalized using the release rate  $q$ .

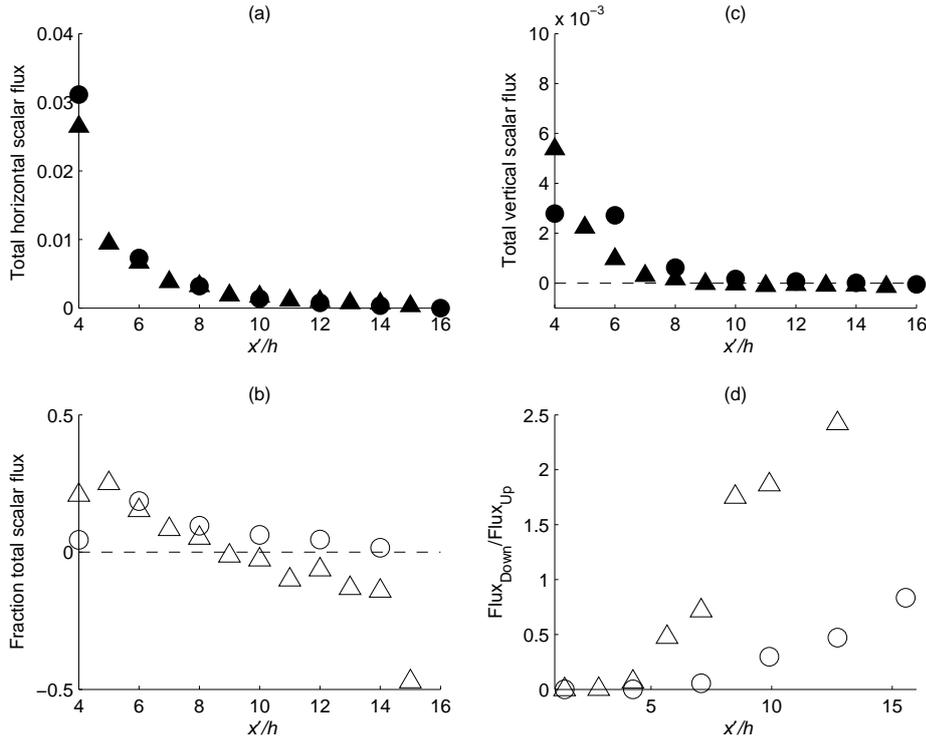
202 Figures 4a, 4b and 4c respectively show the horizontal scalar flux through  
 203 the canopy, vertical flux through the canopy top, and the ratio of vertical to  
 204 horizontal flux, as a function of distance from the source. There is a rapid  
 205 decrease in both the horizontal and vertical fluxes up to the third intersection  
 206 downstream, followed by a much more gradual decrease thereafter. The total  
 207 horizontal flux behaves in roughly the same way in streets and intersections.  
 208 In contrast, there is a clear difference between the vertical fluxes in the streets  
 209 and the intersections; the near-field and far-field behaviours also differ. The  
 210 vertical flux in the intersections is generally positive, so that material is nearly  
 211 always detrained out of the intersections into the air above. The vertical flux  
 212 in the streets is positive close to the source but changes sign between the  
 213 second and third intersections downstream. This implies that re-entrainment  
 214 begins to exceed detrainment very rapidly downstream of the release, at least  
 215 in the present set-up. The magnitude of the flux in the intersections is larger  
 216 than that from the streets in the near-field region (up to the third intersection  
 217 downstream of the release). As noted earlier, the vertical flux in the first  
 218 intersection (which contains the source) is anomalously low compared to that

219 in the streets immediately adjacent to it. This arises because material released  
220 in an intersection is rapidly swept to the next streets downstream, caught in the  
221 wakes of adjacent buildings and pushed upwards by a strong updraft (Coceal  
222 et al., 2014). This gives rise to ‘secondary wake sources’ (Vincent, 1978) in  
223 the relevant streets, which detrain material at a much higher rate than in the  
224 intersection where the source is located. Secondary sources were also observed  
225 in previous experimental studies, e.g. Davidson et al. (1995, 1996).

226 Fig. 4c shows that the magnitude of the vertical flux is generally less than a  
227 quarter of the horizontal flux, except at the location furthest from the source  
228 (where both fluxes are small). After an initial increase with distance from  
229 the source location this ratio decreases steadily up to the third intersection.  
230 Beyond this point there is a difference in the behaviour in intersections and  
231 streets. In intersections there is a continual slow decrease towards zero. In  
232 streets the ratio becomes negative because the vertical flux changes sign due  
233 to re-entrainment into the canopy.

234 It is instructive to decompose the vertical flux into an upward component  
235 (detrained flux) and a downward component (entrained flux). Figure 4d shows  
236 the ratio of the downward flux to the upward flux for the same intersections  
237 and streets. The downward flux is a small fraction (around 0.05) of the upward  
238 flux in the first intersection after the release location. This fraction then rises  
239 nearly linearly to a value of over 0.8 over the next three intersections. In  
240 the streets the downward flux comprises a larger fraction of the upward flux,  
241 starting at around 0.5 in the first street downwind of the release to over 2 over  
242 the next six streets.

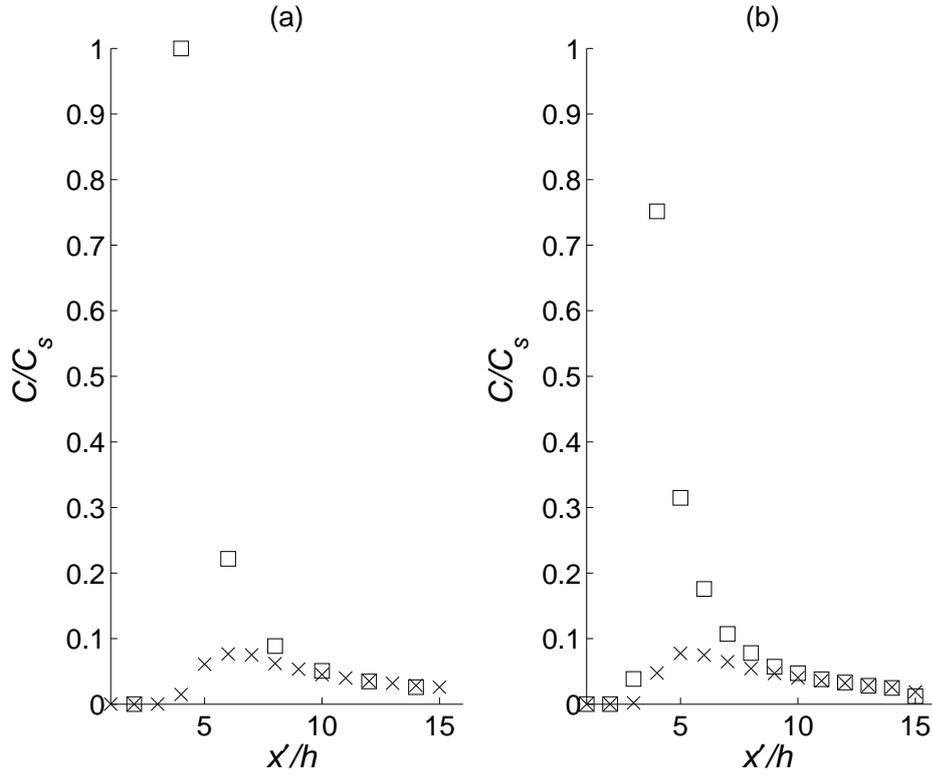
243 Based on these observations, it is possible to identify three different regimes  
244 based on distance downwind of the source. Very close to the source, the vertical  
245 upward flux is a substantial fraction (up to around 0.25) of the horizontal flux  
246 through the network. In the intermediate region the vertical flux consists of  
247 both an upward and a downward component of comparable magnitudes, so  
248 that the net vertical flux is a smaller fraction of the horizontal flux. Further  
249 downwind, there is a qualitative difference in the behaviour in streets and  
250 intersections. In intersections the ratio of downward to upward flux approaches  
251 (but does not exceed) 1; hence the ratio of the net vertical flux to the horizontal  
252 flux approaches 0. In streets the downward flux exceeds the upward flux and  
253 hence the net vertical flux becomes negative; it is a non-negligible fraction  
254 (around 0.15) of the horizontal flux. However, these differences are probably  
255 unimportant since the vertical fluxes are very small and the concentrations  
256 within the canopy and above are virtually the same at this distance. Indeed,  
257 the plume is vertically well-mixed both through the canopy and immediately  
258 above it beyond the third intersection from the source (Fig. 5).



**Fig. 4** Comparison between horizontal fluxes through the canopy and vertical fluxes out of the canopy top: (a) total horizontal flux, (b) total vertical flux, (c) ratio of vertical to horizontal flux, (d) ratio of downward flux to upward flux. Fluxes have been normalised using the release rate  $q$ .

#### 259 **4 A process-based model of dispersion within and above a street** 260 **network**

261 The results of the last section motivate an approach for modelling dispersion  
262 through a network of streets by considering the balance of fluxes through a coupled  
263 system of boxes representing each street and intersection in the network.  
264 This approach forms the basis of the SIRANE model (Soulhac, 2000; Soulhac  
265 et al., 2011, 2012), used operationally for air quality modelling. Belcher  
266 et al. (2015) recently developed an analytical model for regular street net-  
267 works, which demonstrated how the geometrical and flow parameters combine  
268 into a small number of non-dimensional effective parameters that control the  
269 dispersion in the network. We now generalize the analytical model developed  
270 by Belcher et al. (2015) to include dispersion above the street network. The  
271 resulting equations cannot be solved analytically, but can be readily modelled  
272 numerically. Our aim here is to develop a minimal model that is as simple as  
273 possible while still capturing the most important processes identified from the  
274 analysis presented in Sect. 3. In doing so we do not claim that the assump-



**Fig. 5** Variation of mean concentration with distance from the source, (a) intersections, (b) streets. Squares: within canopy. Crosses: above canopy. The concentration is normalized by the concentration in the source box,  $C_s$

275 tions made here have complete generality; indeed some of them will need to  
 276 be modified in other contexts.

#### 277 4.1 Governing equations

Following a rigorous formalism (Belcher et al., 2015), we represent each street and intersection as a box and take the volume- and ensemble-average of the scalar conservation equation over the volume  $V$  of the box to give

$$\frac{dC}{dt} + \frac{1}{V} \int_{\partial V} \overline{c\mathbf{u}} \cdot d\mathbf{S} = Q, \quad (3)$$

278 where  $C$  and  $Q$  are the ensemble- and volume-averaged concentration and  
 279 source emission rate in the box,  $\partial V$  is the surface area enclosing the box, and  
 280 the overline denotes an ensemble average.

The flux term can be separated into mean and turbulent scalar fluxes,

$$\int_{\partial V} \bar{c} \bar{\mathbf{u}} \cdot d\mathbf{S} = \int_{\partial V} \bar{c} \bar{\mathbf{u}} \cdot d\mathbf{S} + \int_{\partial V} \overline{c' \mathbf{u}'} \cdot d\mathbf{S}, \quad (4)$$

where primes denote fluctuations from the ensemble average. The mean and turbulent fluxes are each parametrized as described in the next section.

#### 4.2 Parametrization of the fluxes

Belcher et al. (2015) show that the mean flux density  $\bar{c} \bar{\mathbf{u}}$  can be written formally as the product  $\langle \bar{\mathbf{u}} \rangle_{\partial V}$  of the velocity averaged over the area  $\partial V$  and an average concentration  $C_a$ ,

$$\bar{c} \bar{\mathbf{u}} = C_a \langle \bar{\mathbf{u}} \rangle_{\partial V}. \quad (5)$$

In the next section, the facet-averaged mean velocity is computed from the DNS data. The formally undetermined average concentration  $C_a$  can be approximated as the volume-average concentration in each box, assuming that the scalar is well-mixed. Goulart (2012) and Belcher et al. (2015) demonstrate that this is a reasonable approximation for the current set-up.

Following Belcher et al. (2015), the turbulent flux density is parametrized assuming the gradient diffusion model,

$$\overline{c' \mathbf{u}'} = -\mathbf{K} \nabla \bar{c}, \quad (6)$$

where  $\mathbf{K} = \text{diag}(K_x, K_y, K_z)$  is a diagonal matrix with diagonal components equal to the eddy diffusivity coefficients  $K_x, K_y$  and  $K_z$  in the  $x, y$  and  $z$  directions respectively.

It is common to represent the scalar exchange between the canopy and the air above with a detrainment velocity  $E$ , defined as

$$E = \frac{K_z}{\Delta z}, \quad (7)$$

where  $\Delta z$  is an appropriate vertical distance, here taken to be the vertical separation between the centres of a box in the canopy and the one immediately above it.

We can generally neglect the horizontal turbulent flux within the canopy, except when the flow direction is closely aligned with one of the streets. Additionally, the mean vertical flux can be neglected in comparison with the turbulent vertical flux.

It is straightforward to discretize Eq. 3. A first-order scheme yields the following, for each box

$$\Delta C = \frac{\Delta t}{V} \left( \sum_{k=1}^n F^k + \sum_{k=1}^n f^k + Q \right), \quad (8)$$

where  $F^k$  and  $f^k$  are respectively the advective and diffusive scalar fluxes through each facet  $k$  of the box and  $n$  is the total number of facets enclosing the box.

### 4.3 Calculation of model parameters from DNS

For the current DNS set-up, with the flow at  $45^\circ$  to the regular cubical array, the horizontal facet-averaged advection velocity components  $\langle \bar{u} \rangle_k$  and  $\langle \bar{v} \rangle_k$  are approximately equal. Figure 6a shows the average of  $\langle \bar{u} \rangle_k$  and  $\langle \bar{v} \rangle_k$  computed for intersections and streets along the transects shown in Fig. 2c. The advection velocities in intersections (average value 1.13) are slightly lower than in streets (average value 1.18). The facet-averaged velocities in the boxes just above the array (around 3.4) are about three times those in the array (not shown). For comparison, Fig. 6a also shows corresponding values of ‘flux velocities’, defined as the ratio of  $\overline{c\mathbf{u}}$  and  $C_a$ . There is a difference of around 10 – 15% between the facet-averaged velocities and the flux velocities. This gives an indication of the margin of error involved in using the facet velocity as an input parameter in the model.

The detrainment velocity  $E$  characterizing vertical turbulent transfer out of the canopy top is computed as follows:

$$E = \frac{\langle c'w' \rangle}{(C_{in} - C_{abv})}, \quad (9)$$

where  $C_{in}$  and  $C_{abv}$  are the box-averaged mean concentration within and above the canopy respectively, and the facet average of the vertical flux (indicated by the angled brackets) is taken over the interface separating the two boxes.

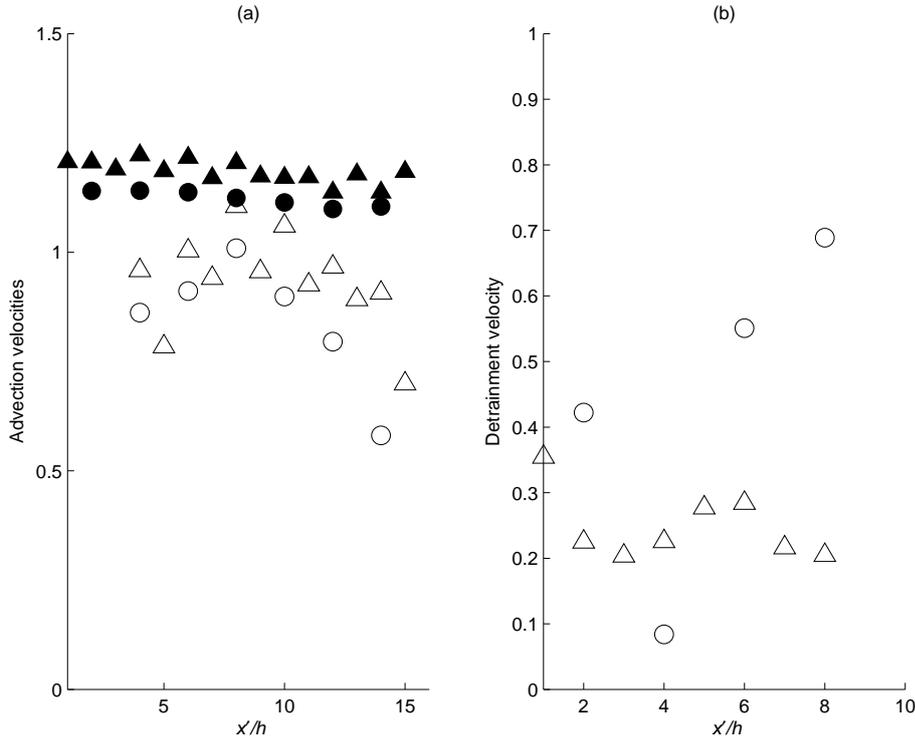
Figure 6b shows the detrainment velocity at the same locations in streets and intersections as in Fig. 6a. Values are plotted only up to a distance of  $8h$  from the source since both the vertical flux and concentration difference become tiny beyond this distance, giving indeterminate values for their ratio. The difference in detrainment velocity in streets and intersections is evident. Intersections have, on average, a detrainment velocity approximately 60% larger than streets. The average detrainment velocity for streets and intersections are:  $E_s = 0.3$  and  $E_i = 0.5$ .

Values for the diffusion coefficients  $K_x$ ,  $K_y$  and  $K_z$  can be computed from the DNS data, with  $K_x = K_y = 0.5$  and  $K_z = 0.3$  used here. These values are consistent with those used in the literature for rough surfaces (e.g. Pasquill, 1962).

## 5 Numerical experiments with the network model

The parameters calculated from the DNS data in the last section are summarized in Table 1. These values are used as input to configure a set of runs with the network model described in Sect. 4.

Figure 7 shows comparisons between the mean concentrations computed by the network model (indicated by triangles) and the DNS (indicated by circles) along the plume centreline and along lateral transects at different distances from the source. The network model generally captures well both the decay



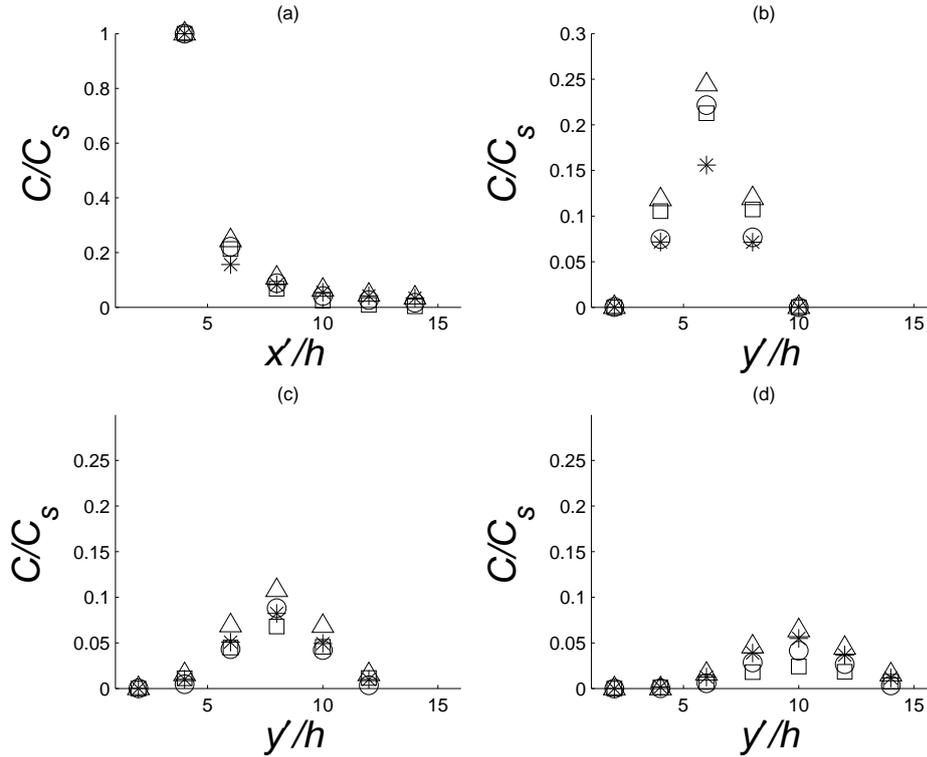
**Fig. 6** (a) Filled symbols: facet-averaged advection velocities within the canopy. Empty symbols: flux advection velocity. (b) Detrainment velocities. Circles: intersections. Triangles: streets. Locations correspond to Fig. 2c.

$U_i \approx V_i$	$U_s \approx V_s$	$E_i$	$E_s$	$U_{abv} \approx V_{abv}$	$K_x$	$K_y$	$K_z$
1.13	1.18	0.5	0.3	3.43	0.5	0.5	0.3

**Table 1** Non-dimensional input parameters for the network model. Here  $U$  and  $V$  denote horizontal facet-averaged velocity components in the  $x$  and  $y$  directions respectively. The subscripts  $i$  and  $s$  refer to intersections and streets respectively, while  $abv$  refers to the layer just above the canopy layer.

341 in the centreline concentration and the lateral spread of the plume. The values  
 342 predicted by the network model are generally within around 30% of the  
 343 DNS values. This is encouraging, given the extreme simplicity of the model  
 344 compared to the DNS.

345 Corresponding profiles in the layer just above the canopy are shown in  
 346 Fig. 8. The agreement with the DNS is even better than in the canopy. It  
 347 is especially good further from the source, from a distance of around  $6h\sqrt{2}$   
 348 onwards. Close to the source, at a distance of  $2h\sqrt{2}$ , the model underpredicts  
 349 the concentration above the canopy by up to around 30%. This is consistent  
 350 with an overprediction within the canopy by approximately the same amount.  
 351 This is likely a result of secondary wake sources in the streets close to the

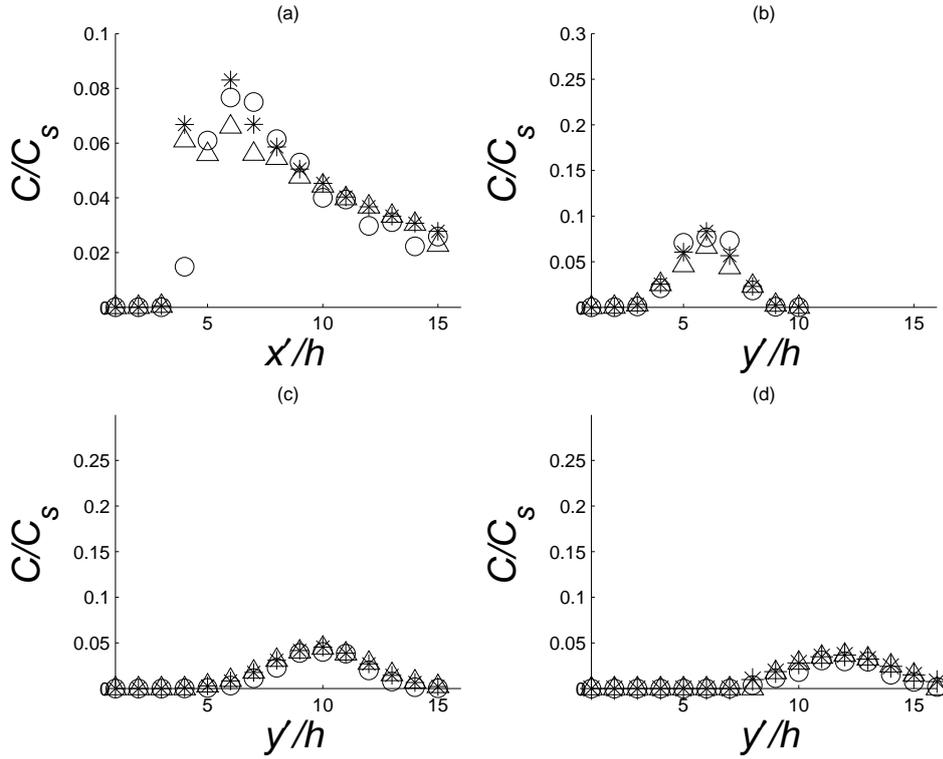


**Fig. 7** Comparison between in-canopy concentration computed from network model and DNS (a) Centreline. Lateral profiles at (b)  $2h\sqrt{2}$ , (c)  $4h\sqrt{2}$  and (d)  $6h\sqrt{2}$  from the source. Triangles: network model without secondary sources. Asterisks: network model with secondary sources. Squares: analytical solution of Belcher et al. (2015). Circles: DNS. Distances along and perpendicular to the plume centreline are denoted by  $x'$  and  $y'$  respectively.

352 release, which lead to an enhanced initial detrainment of material (Coccal et  
 353 al. 2014). The network model does not represent these secondary sources.

354 A crude way to investigate the possible effect of the secondary sources is  
 355 to simply increase the detrainment velocity in the relevant streets where they  
 356 occur. The star symbols in Figs. 7 and 8 show the effect of increasing  $E_s$  to 2,  
 357 which is approximately 6.7 times the value in other streets. This indeed leads to  
 358 closer correspondence with the DNS near the source, while the values further  
 359 away are much less affected. This shows that any enhanced initial detrainment  
 360 due to the secondary sources is compensated by greater re-entrainment further  
 361 afield.

362 The sensitivity of the predicted concentrations to the input parameters is  
 363 investigated by increasing and decreasing each parameter independently by  
 364 10%. The concentration is then averaged along the plume centreline over six  
 365 successive intersections, including the intersection in which the source is lo-  
 366 cated. The averaged concentration along a lateral transect at a distance of



**Fig. 8** Comparison between above-canopy concentration computed from network model and DNS. (a) Centreline. Lateral profiles at (b)  $2h\sqrt{2}$ , (c)  $6h\sqrt{2}$  and (d)  $8h\sqrt{2}$  from the source. Triangles: network model without secondary sources. Asterisks: network model with secondary sources. Circles: DNS.  $x'$  and  $y'$  are the streamwise and spanwise distance to the plume centreline, respectively.

367  $8h$  from the source is also computed. Similar computations are made for cor-  
 368 responding boxes just above the canopy layer. Table 2 shows the percentage  
 369 difference in the computed concentrations relative to the run performed with  
 370 the original input parameter values (as given in Table 1). The results show  
 371 that changing the parameters have different effects on the concentration aver-  
 372 aged along the centreline, and along the lateral transect. The effect on the  
 373 concentration below and above the canopy are also different. On the whole  
 374 the advection velocities within the canopy have the largest effect. The above-  
 375 canopy concentrations are especially sensitive to the advection velocities in  
 376 the intersections, but show little dependence on the advection velocities in the  
 377 streets. There is little dependence on the values of  $K_x$  and  $K_y$ , but a change in  
 378 the value of  $K_z$  of 10% changes the concentration above the canopy by about  
 379 30% on average.

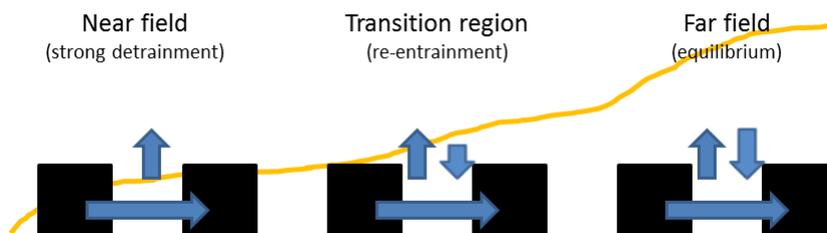
**Table 2** Network model sensitivity analysis.  $D1$  is the difference between the network model and DNS along the centerline of the plume within the canopy.  $D2$  is the difference between the network model and DNS along a lateral profile at  $8h\sqrt{2}$  from the source within the canopy.  $D3$  is the difference between the network model and DNS along the lateral profile at  $9h\sqrt{2}$  from the source above the canopy.

Variables	Increase of 10%			Decrease of 10%		
	D1	D2	D3	D1	D2	D3
$U_i, V_i$	-7	-12	-40	30	17	211
$U_s, V_s$	-20	-14	2	39	21	-3
$U_{abv}, V_{abv}$	13	10	71	-11	-9	-33
$E_i$	13	7	-13	-12	-7	16
$E_s$	21	11	0	-16	-10	0
$K_x, K_y$	1	0	5	-1	0	-5
$K_z$	5	4	38	-5	-4	-24

## 380 6 Conclusions

381 The dispersion from a localized source within an idealized street network has  
 382 been studied using DNS data. The dispersion characteristics within and above  
 383 the network were compared by evaluating horizontal and vertical fluxes and  
 384 their partitioning into mean and turbulent parts. The results show that the  
 385 horizontal flux within the canopy is almost exclusively comprised of the mean  
 386 flux, whereas above the canopy a significant counter-gradient turbulent part  
 387 exists. By contrast, the vertical flux through the canopy top is generally dom-  
 388 inated by the turbulent component. A fraction of the material originally re-  
 389 leased within the canopy and detrained into the air above is re-entrained re-  
 390 latively soon downstream. Based on the relative magnitude and balance of  
 391 the horizontal and vertical fluxes, three distinct regions have been delineated:  
 392 a near-field region, a transition region and a far-field region (summarized in  
 393 simplified form in Fig. 9).

394 The results from the DNS have been used to develop a minimal process-  
 395 based street network model that treats the dispersion within and above the  
 396 network in a unified way. The model incorporates a small set of key urban  
 397 dispersion processes including horizontal advection, vertical detrainment and  
 398 re-entrainment. A rigorous formulation based on volume-averaging the govern-  
 399 ing equations reduced the highly complicated original problem to an effective  
 400 model described by only a few parameters. Comparisons with DNS data show  
 401 that this highly simplified modelling approach still gives accurate quantitative  
 402 estimates of mean concentrations both within and above the street network.  
 403 This indicates that the processes included in the model are indeed the most  
 404 important ones and that the parametrizations on which it is based are vi-  
 405 able. The fact that the input parameters of the simpler model were deduced  
 406 from the DNS in the current exercise ensures consistency in the evaluation of  
 407 the approach. Naturally, if the model were to be used in a predictive mode,  
 408 it would need to be supplemented by methods to determine the parameters  
 409 independently.



**Fig. 9** Plume growth for a ground source release in an urban canopy, with arrows indicating relative magnitudes of horizontal and vertical fluxes in the near-field, the transition and the far-field regions.

410 The method can be readily generalized for other set-ups including non-  
 411 regular geometries, although some of the specific assumptions made here may  
 412 have to be modified in other scenarios. For example, the operational SIRANE  
 413 model (Soulhac et al., 2011, 2012) employs a different model for parametriz-  
 414 ing fluxes at intersections that does not assume well-mixed conditions. It also  
 415 treats above-roof dispersion as a series of point sources giving rise to Gaus-  
 416 sian plumes that are then superimposed. Moreover, as a self-contained opera-  
 417 tional model, the SIRANE model includes built-in methods for estimating  
 418 the model parameters such as advection and exchange velocities. This work  
 419 has focused on examining the conceptual and empirical basis of the under-  
 420 lying street-network approach, and to assess its performance when stripped  
 421 of as many specific modelling assumptions as possible. One noteworthy result  
 422 is that the basic street-network approach, as incorporated in a model much  
 423 simpler than even the SIRANE model, shows a promising performance. The  
 424 level of agreement obtained with the DNS data shows the predictive potential  
 425 of the approach, if used in conjunction with accurate methods of estimating  
 426 the model parameters. This implies that efforts to improve the SIRANE model  
 427 should focus on further developing and testing such methods.

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