# International Reserves

# in Production Small Open Economies

**by**

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## Declaration I

I confirm that this is my own work and the use of all material from other sources has been properly and fully acknowledged.

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**Declaration II**

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# Abstract

Making three distinct contributions to the literature on international reserves for small open economies (SOEs), this thesis is composed of three main chapters. Chapters 1 and 2 contribute to the underlying theory of optimal international reserves, extending the Jeanne and Rancière (2011) endowment SOE model to a production economy, each using one of the most common technology specifications in Neoclassical growth theory. Chapter 3, then, examines empirically key aspects of reserve holdings as observed in a dataset of high and low middle income emerging market economies (EMEs) used in the calibration of the preceding theoretical chapters.

Chapter 1 explores the effects of investment and production on optimal reserves in SOE EMEs and derives an optimal reserves-to-output formula in the case where capital is the sole factor of production as in the AK model of endogenous growth. We refer to this version as the one-factor production SOE AK model, or simply the AK model (of endogenous growth). This version implies increasing returns to scale (IRS) and is justified on the grounds of the ability of the AK model to generate endogenously, via the influence of policy – such as subsidies or taxes on investment – on capital accummulation, sustained long-run growth observed in the data. We find that the endogenous growth AK model with IRS implies a negative relationship between the optimal reserve-to-output ratio and capital-augmenting (in fact, here sole-factor) technological progress. Depending on the calibration of the productivity parameter, the model quantifies the optimal ratio of reserves to output at 1.74% for SOEs.

Chapter 2 introduces labour, making the production function more general. More precisely, we switch to a conventional labour-augmenting Cobb-Douglas (CD) production function, which embodies alternative assumptions of constant returns to

scale (CRS) overall – but with diminishing returns to scale (DRS) for each of the two factors, capital and labour – and convergence to a balanced growth path (BGP) in the long run. In turn, this version is justified on the grounds of being consistent with a long-run BGP in Neoclassical models of exogenous growth and with sustained per capita income growth in these models. The second chapter thus focuses on the effects of labour-augmenting productivity on the optimal reserves-to-output ratio in a production SOE. Moreover, the alternative modelling of the production function, IRS AK versus CRS CD, and the type of growth, endogenous versus exogenous, allows us to compare the analytical results in chapter 1 (AK model) with those in chapter 2 (the CD model). Similarly to the endogenous growth AK model, we find that in the exogenous growth CD model along the BGP labour-augmenting technological progress decreases the optimal reserves-to-output ratio. Depending on the calibration of the labour-augmenting productivity parameter, the CRS CD model quantifies the optimal ratio of reserves to output at 5.5% in the richer two-factor production SOE model. This roughly three times higher ratio of optimal reserve holdings to output arises from the difference in the specification of technology in the production functions. This ratio is still quite lower than the corresponding one derived in the endowment SOE model of Jeanne and Rancière (2011), 9.1%. The main reason for optimally maintaining a lower ratio of international reserves to output in a production SOE with investment and productive capital relative to the endowment SOE benchmark is as follows. With the capital stock now accumulated via investment and potentially used as a pledge to external creditors in obtaining borrowing and therefore insuring better against sudden stops, the optimal reserve-to-output ratio is much lower relative to an otherwise similar endowment economy in the AK model. As depreciation depletes the existing capital stock, opposite to investment, the reversal of the relationship is not surprising in both the AK and CD models. Whereas the AK model can generate endogenously, via policy, persistent capital accummulation leading to sustained long-run growth, adding labour as a second factor in a CD production function, consistent with a long-run BGP in Neoclassical models of exogenous growth and with sustained per capita income growth, results in a roughly mid-point optimal reserve-to-output ratio of 5.5%.

Chapter 3, finally, takes a complementary, statistical approach and examines the key theoretically derived determinants of international reserves relative to output together with the most common empirically motivated determinants suggested in the literature as 'control variables' in a dataset of 26 high and low middle income economies. For this purpose, we initially estimate a pooled OLS benchmark and a panel data fixed effect model to analyse the relative importance of such empirically measured determinants of real-world reserve holdings as well as possible country specificities. We then use quantile regression techniques to examine the variation in these determinants across the reserve holdings distribution in our sample. We examine the uniformity of coefficients by several quantile regressions and the overall models. Our quantile regression results suggest that there is substantial variation in middle income countries in terms of the reserve holdings distribution. Our findings from inter-quantile regressions show that there are statistically significant differences in share of imports in GDP, investment share of GDP, and short term external debt to GDP.

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**Summary of Notation for Chapters 1 and 2**

$U_t$: Utility Function

$C_t$: Consumption

$Y_t$: Output

$I_t$: Investment

$L_t$: External Debt

$Z_t$: Government Net Transfers

$r$: Interest Rate

$\sigma$: Risk Aversion Parameter

$A_K$: Capital Productivity

$A_L$: Labour Productivity

$S$: Total Savings

$s$: Investment Rate

$K_t$: Capital Stock

$\delta$: Depreciation Rate of Capital

$g$: Growth Rate of the Economy

$g_k$: Growth Rate of Capital

$g_N$: Growth Rate of the Population

$\alpha_t$ : Time-Varying Parameter (capturing the warranty of next-period domestic output to external lenders)

$\gamma$: Output Loss

$\pi$: Probability of a Sudden Stop in Capital Inflows

$X_t$: Payment as an Insurance Premium against Sudden Stops

$R_t$: International Reserves

$\mu_t$: Marginal Utility of Funds for Investors

$p$: Relative Price of Non-Crisis Dollars

$\lambda$: External Debt to GDP Ratio

$y$: GDP per Worker

$k$: Capital Labour Ratio

# Introduction

This thesis is composed of three essays on international reserves, explicitly focusing on the role of investment, capital and labour in production SOE models featuring technology consistent with the most common endogenous (AK IRS) and exogenous (labour-augmenting CD CRS) Neoclassical growth specifications. The theoretical chapters 1 and 2 analyse the implications of these two conventional production functions from Neoclassical growth theory on the level of optimal reserves relative to output. Chapter 3 then extends this analytical work into the empirics of panel data and quantile regressions, notably adding typical control variables to the theory-derived key determinants.

Figure 1 shows a snapshot of the relationship between the ratios of investment and international reserves, respectively, to real GDP in the sample of 34 middle income EMEs used in Jeanne and Rancière (2011), referred to henceforth as JR. This figure could serve as a general motivation for the theoretical study undertaken in the first two chapters of the thesis. It is insightful to note that while there is a statistically significant (p-value of 0.006) positive (0.865) slope coefficient in the JR sample when regressing the reserve-to-output ratio on a constant and the investment-to-output ratio, just eliminating the two obvious outliers[1] (Botswana and China) results in statistical insignificance at all conventional levels (p-value of 0.14). This fact demonstrates the sensitivity of simple empirical regressions to the outliers in a sample and, hence, the importance of analytical results that can be derived in theoretical environments.

[1] We applied Grubbs' (1969) test in order to detect outliers of the JR sample using Stata. The test results imply that China's gross fixed capital formation to GDP series show an outlier influences in 2013. Moreover, Grubbs' test also implies Botswana's reserves-to-GDP ratio shows outlier values from 1985 to 2010.

We, therefore, focus on examining analytically the role that investment in production SEO models plays in optimal reserve holdings under alternative but commonly used technology specifications. In doing so, we make three distinct contributions to the literature on international reserves, specific to each of the chapters.

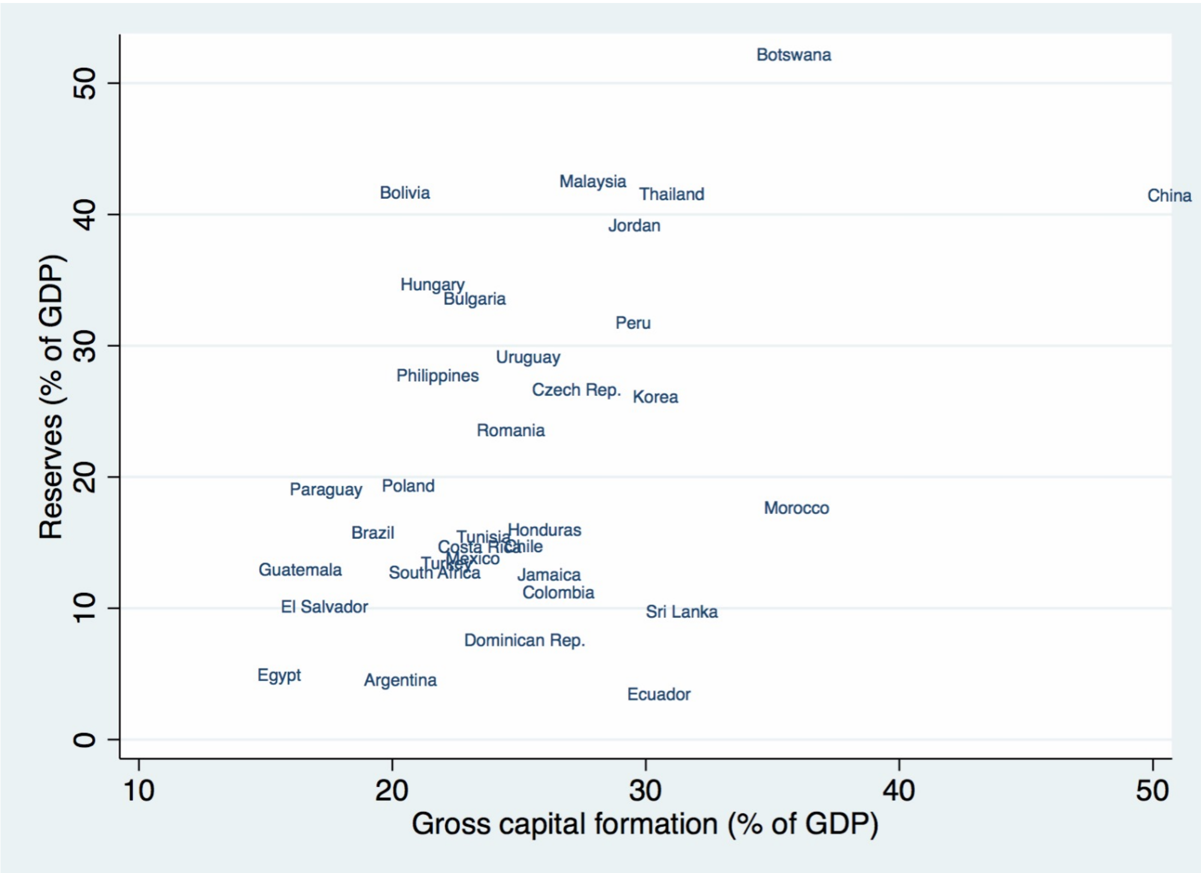
Our first contribution is to study the implications of investment and productive capital on optimal international reserves in SOE EMEs and to derive an optimal reserves-to-output formula that extends the endowment JR benchmark to the more realistic case where, initially, in Chapter 1, capital is the sole factor of production as in the AK model of endogenous growth. We refer to this version as the one-factor production SOE AK model, or simply the AK model (of endogenous growth). This version implies increasing returns to scale (IRS) and is justified on the grounds of the ability of the AK model to generate endogenously, via the influence of policy – such as subsidies or taxes on investment – on capital accummulation, sustained long-run growth observed in the data (Acemoglu, 2009, p. 55; Jones and Vollrath, 2013, p. 216). We find that the endogenous growth AK model with IRS implies a negative relationship between the optimal reserve-to-output ratio and either the investment-to-output ratio or capital-augmenting (in fact, here sole-factor) technological progress. Depending on the calibration of the productivity parameter, the model quantifies the optimal ratio of reserves to output at 1.74% for SOEs.

Our second contribution, in Chapter 2, is to introduce labour as a conventional second factor of production, making the specification of technology more general. In particular, we switch to a labour-augmenting Cobb-Douglas (CD) production function, which embodies alternative assumptions of constant returns to scale (CRS) overall – but with diminishing returns to scale (DRS) for each of the two factors, capital and labour – and convergence to a balanced growth path (BGP) in the long run. In turn, this version is justified on the grounds of being consistent with a long-run BGP in neoclassical models of exogenous growth (Acemoglu, 2009, p. 59) and with sustained per capita income growth in these models (Jones and Vollrath, 2013, pp. 36-37). The second chapter thus focuses on the effects of labour-augmenting productivity on the optimal reserves-to-output ratio in a production SOE. Moreover, the alternative modelling of the production function, IRS AK versus CRS CD, and the type of growth, endogenous versus exogenous, allows us to compare the analytical results in chapter 1 (the AK model) with those in chapter 2 (the CD model). Similarly to the endogenous growth AK model, we find that in the exogenous growth CD model along the BGP labour-augmenting technological progress decreases the optimal reserves-to-output ratio. Depending on the calibration of the labour-augmenting productivity parameter, the CRS CD model quantifies the optimal ratio of reserves to output at 5.5% in the richer two-factor production SOE model. This roughly three times higher ratio of optimal reserve holdings to output arises from the difference in the specification of technology in the respective production functions, emphasising the sensitivity of optimal reserves to the way technological progress is modelled. This ratio is still quite lower than the corresponding one derived in the endowment SOE benchmark of Jeanne and Rancière (2011), 9.1%. Our intuition for the result that the optimal ratio of international reserves to output in a production SOE with investment and productive capital is lower relative to the endowment SOE benchmark can be summarised as follows. With the capital stock now accumulated via investment and potentially used as a pledge to external creditors in obtaining borrowing and therefore insuring better against sudden stops, the optimal reserve-to-output ratio is much lower in the AK model relative to an otherwise similar endowment economy. As depreciation depletes the existing capital stock, opposite to investment, the reversal of the relationship is not surprising in both the AK and CD models. Whereas government policy in the AK model can generate endogenously persistent capital accummulation leading to sustained long-run growth, adding labour as a second factor in a CD production function, consistent with a long-run BGP in Neoclassical models of exogenous growth and with sustained per capita income growth, results in a roughly mid-point optimal reserve-to-output ratio of 5.5%.

Finally, our third contribution, in Chapter 3, consists in complementing the key theoretically derived determinants of international reserves relative to output with some of the most common empirically motivated determinants suggested in the literature as 'control variables' in panel data and quantile regression estimation based on a dataset of 26 high and low middle income economies. Our main aim is to show empirical behaviour of the key theoretical parameters of the first two chapters such as growth rate of GDP, the external-debt-to-GDP ratio and probability of a sudden stop and the investment rate that was missing from the earlier applied literature, in a broader concept including most typical empirical determinants of reserve holding such as export volatility, trade openness, the broad money-to-GDP ratio, the volatility of nominal effective exchange rate. For this purpose, we initially estimate a pooled OLS benchmark and a panel data fixed effect model to analyse the relative importance of such empirically measured determinants of real-world reserve holdings as well as possible country specificities. We then use quantile regression techniques to examine the variation in these determinants across the reserve holdings distribution in our sample. We examine the uniformity of coefficients by several quantile regressions and the overall models. Our quantile regression results suggest that there is substantial variation in middle income countries in terms of the reserve holdings distribution. Our findings from inter-quantile regressions show that there are statistically significant differences in import share of GDP, investment share of GDP, and short term debt to GDP.

Each chapter of this thesis can be read as a stand-alone paper. Hence, each individual chapter includes its own introduction and a review of the literature (although chapter 1 and 2 share mostly a common literature). The aim of this introductory chapter is to introduce the overall setting of this thesis. After the third chapter, we also offer overall concluding remarks to the thesis outlining some directions for future research.

**Figure 1:** Reserves and Investment Ratio




Source: World Bank, *World Development Indicators* (online), and authors' calculations. Data on reserves to GDP ratios and gross capital formation are for 2013 for the 34 middle income countries listed in Table 1.2 in Appendix 1.A.

# Chapter 1: The Optimal Level of International Reserves in a Production Small Open Economy: AK Model

**Abstract**

One of the most significant current discussions in international macroeconomics is the rapid increase in international reserves for emerging market economies (EMEs). Recent developments characterised by a perceived excess of reserve holdings in many EMEs have heightened the need to revisit analytically and quantitatively the optimal level of reserves in terms of real output. Moreover, while there is a huge literature on international reserves as an insurance against sudden stops of capital inflows, little is known regarding the role of optimal reserves as insurance when a production economy with investment is explicitly modelled. The aim of this chapter is to fill in this gap in the literature, by extending the endowment benchmark of JR to a production economy with endogenous growth based on AK production technology and deriving the corresponding formula for the optimal ratio of reserves to output in SOEs facing the risk of sudden stops. We find that in for production economies, the reserve-to-GDP ratio is negatively related to productive capital and investment and positively related to the depreciation rate of capital. With the capital stock now accumulated via investment and potentially used as a pledge to external creditors in obtaining borrowing and therefore insuring better against sudden stops, the optimal reserve-to-output ratio is much lower, 1.74%, relative to an otherwise similar endowment economy, 9.1% in JR. As depreciation depletes the existing capital stock, opposite to investment, the reversal of the relationship is not surprising.

**Keywords:** International Reserves, Sudden Stops, Capital Productivity, Investment, AK Technology
**JEL Classification:** F31, F32, F33, F41

## 1.1 Introduction

International reserves have increased substantially in recent years. Figure 1.1 shows that middle-income countries account for nearly a half of this increase. Consequently, the accumulation of international reserves in emerging market economies (EMEs) has become one of the most debated issues in open-economy macroeconomics (Chinn *et al.*, 1999; Aizenman and Marion, 2003; Dooley *et al.*, 2004; Caballero and Panageas, 2007; Alfaro and Kanczuk, 2009; Durdu *et al.*, 2009; Jeanne and Rancière, 2011; Calvo *et al.*, 2012; Dominguez *et al.*, 2012). Have many EMEs, in fact, accumulated excessive rather than adequate reserves? And what is an optimal ratio of reserves to output in a small open economy (SOE)? The recent literature offers contradictory explanations on these questions of immediate policy relevance: in particular, there is no consensus on whether the reserve accumulation is driven by self-insurance against abrupt reversals of capital flows or new mercantilism (Aizenman and Marion, 2004).

Basically, two main benefits of large reserve holdings have been emphasized: (i) international reserves provide liquidity to smooth consumption (Jeanne and Rancière, 2011); (ii) international reserves give a flexibility to manage sizable capital outflows in periods of crises (Aizenman *et al.*, 2007). Moreover, it has also been argued that reserve policies can help guard away an economy from a crisis or contribute to a recovery after a crisis (Aizenman and Marion, 2004; Dominguez *et al.*, 2012).

The issue has been discussed under two main approaches: (i) one of them rationalises why EMEs hold a high level of reserves as a form of self-insurance against 'sudden stops'[2] in capital inflows (Aizenman *et al.*, 2007; Aizenman and Marion, 2003; Aizenman and Marion, 2004; Chinn *et al.*, 1999; Dominguez *et al.*,

[2] Calvo (1998) seems to have coined and interpreted first this term.

2012; Dooley *et al.*, 2004; Eichengreen and Mathieson, 2000; Greenspan, 1999); (ii) the other examines what the determinants of reserve holdings are and, furthermore, what the optimal level of reserves is (Alfaro and Kanczuk, 2009; Caballero and Panageas, 2007; Calvo *et al.*, 2013; Durdu *et al.*, 2009; Jeanne and Ranciere, 2006; Jeanne and Rancière, 2011).

Jeanne and Rancière (2011), in particular, have considered the role of optimal international reserves as an insurance against sudden stops in capital inflows in an endowment SOE, abstracting from physical capital accumulation through investment. However, the literature has not yet analysed the same problem in a richer SOE set-up that models production and investment explicitly. Our contribution, thus, consists in filling in this gap. Indeed, most studies on international reserves have focused on other reserve-related issues, such as active reserve management (Aizenman and Marion, 2003) or the new type of monetary mercantilism (Aizenman *et al.*, 2007) and the optimal level of reserves (Jeanne and Ranciere, 2006; Jeanne and Rancière, 2011).

Our paper proposes an extension to a production economy of the endowment SOE model in Jeanne and Rancière (2011), hereafter JR. More fundamentally, in doing so we bring together two strands of literature that have developed independently and separately from each other over many years, namely neoclassical growth theory of the 1950s and 1960s and the open-economy theory of capital flows under the risk of sudden stops since the late 1990s. JR have developed an 'insurance model' of optimal international reserves where the representative consumer can smooth consumption during sudden stops if the central bank holds a stock of international reserves. The authors derive a closed-form expression for the optimal level of reserves relative to the level of output. They find results broadly consistent with the earlier literature: their model predicts a reserve-to-GDP ratio of 9%. Yet, JR suggest that their set-up can be extended in several ways, one of which is to incorporate productive capital and investment. In the present paper, we do so with the particular aim to explore the implications of such an extended, more realistic set-up for the optimal reserves-to-output ratio in EMEs.

More precisely, to study the effects of investment and production on optimal reserves in EMEs, we develop a theoretical framework for a production SOE. In this chapter, we derive optimal reserves-to-output where capital is the sole factor of production, and we refer to this version as the one-factor production SOE AK model, or simply the AK model (of endogenous growth); this version implies increasing returns to scale (IRS) and is justified on the grounds of the ability of the AK-model to generate endogenously, via the influence of policy – such as subsidies or taxes on investment – on capital accummulation, sustained long-run growth (Acemoglu, 2009, p. 55; Jones and Vollrath, 2013, p. 216).

We find that the AK model with IRS suggests a negative relationship between the optimal reserve-to-output ratio and capital-augmenting (in fact, sole-factor) technological progress. Our calibration implies that the model quantifies the optimal ratio of reserves to output as 1.74 % in one-factor production SOEs of the EME type. Following JR, our sample consists of 34 middle income countries over the period 1975 to 2014, and the calibration is based on the mean values for each country across years and then the average of all countries. Thus, we do not only present an extension of the optimal reserves-to-output formula to an AK production SOE, but also update the time range of the dataset up to 2014.

The rest of the chapter is structured as follows. Section 1.2 gives an overview of the literature on the optimal level of reserves. Section 1.3 presents an extended version of the JR endowment economy model of optimal reserve holding, namely the

AK model for a production economy. The results of the calibration are provided in section 1.4. Section 1.5 concludes.

## 1.2 Literature Review

There is a comprehensive literature on international reserves since at least the 1960s. It mainly focuses on two approaches. The first approach studies the determinants of reserve holdings, examining the reasons behind reserve accumulation. The second approach is concerned with the management of high levels of international reserves. The present chapter contributes to the first approach, and we therefore only review this strand of the literature.

### 1.2.1 Earlier Literature on the Determinants of Reserves

In the earlier literature, international reserves were seen essentially as a buffer stock, and the relationship between reserves and liquidity was in the centre of interest (Balogh, 1960; Caves, 1964). Balogh (1960) proposes an economic theory of reserve holdings. According to his view, the level of international reserves depends on the objective of economic policy and provides liquidity to the economy. Caves (1964) defines the liquidity problem as financing the United States (US) deficit, and analyses the role of international reserves in potential issues related to fixed exchange-rate regimes. Even though both papers try to answer why nations hold reserves, they do not explicitly present motives behind holding reserves.

Such motives were given by Heller (1966), by analogy with the motives for holding money in the Keynesian tradition: (i) a transaction motive, (ii) a precautionary motive and (iii) a speculative motive. Furthermore, he was the first to propose an optimal reserve function in taking adjustment cost and opportunity cost into account. It depends on the marginal propensity to import, the opportunity cost of reserves and the balance of payments (BoP) volatility.

It is widely accepted in the subsequent literature that the above three motives are the ‘traditional factors’ of the optimal reserve function. Clark (1970), Kelly (1970), and Hamada and Ueda (1977) developed this ‘traditional view’ in terms of modifying some assumptions, but their key result remains consistent with Heller (1966): the optimal reserve level increases with BoP volatility and decreases with the propensity to import and the opportunity cost.

Frenkel and Jovanovic (1981) proposed a buffer stock model for the optimal reserve level based on the same two types of costs. The first one is the opportunity cost, which can be defined as a comparison of alternative investment returns. The second one is the cost of adjustment, which is a cost of reserve depletion. In order to determine the optimal level of reserves, both costs are minimized. However, there is a negative relationship between them, so the opportunity cost increases when reserves are at a high level, whereas a high level of reserves is associated with a lower adjustment cost.

With increasing trade and financial liberalization in the course of the 1990s, precautionary demand of holding international reserves has gained more importance in international reserve analysis. Ben-Bassat and Gottlieb (1992) introduce the effect of sovereign risk on the precautionary demand for holding international reserves. They discuss the cost of decreasing the reserve level, which might be a signal of an external payment problem for a country with external debt. The authors also point out that past defaults are important. If a country had experienced a default in the past, it would require holding more reserves to keep its international credibility.

In the wake of the East Asian financial crisis of 1997-1998, researchers and policymakers have also become concerned with the issue of 'reserve adequacy', allowing a safeguard for a country from a sudden stop of capital inflows. Some simple policy rules have been proposed in order to provide insurance for economies which have a risk of vulnerability from such episodes or crises. Feldstein (1999) argues that an accumulation of foreign reserves is an insurance against sudden stops of capital inflows and capital outflows in EMEs. Greenspan (1999) similarly suggests a measure of the 'optimal' level of reserves, according to which a country's reserve level should be equal to its short-term external debt (STED). Chinn *et al.* (1999) compare the Latin American countries and the East Asian countries in terms of an insurance model for a currency crisis. Eichengreen and Mathieson (2000) analyse the importance of the currency composition of international reserves for EMEs along the proposed concepts by Feldstein (1999) and Greenspan (1999).

The accumulation of reserves in the Asian economies after the 1997-1998 financial crisis, in an attempt to prevent future occurrences of similar major disturbances, led to further attention to the potential vulnerabilities of EMEs and the role of reserve holdings to mitigate them. One of the main common features of Asian EMEs after the 1997-1998 crisis is that most of them have been generating current account surpluses. Dooley *et al.* (2004) develop a theory of the determinants of international reserves by also considering the current account (CA). The CA surpluses may lead to an appreciation of the domestic currency. Furthermore, a relationship between the CA surplus and the demand for reserves could be either negative or positive. If a central bank keeps on buying foreign reserves during the current account surplus period, it obviously increases the country's reserve level.

On the other hand, a negative relationship between the CA surplus and international reserves is presented by Aizenman *et al.* (2007). According to them the CA surplus may be a signal that a country is less exposed to external shocks, and this might be a reason for a decrease in reserve levels. If a country runs a CA deficit, it is expected that the central bank sells their reserves to purchase domestic currency, which causes a decrease in international reserves.

### 1.2.2 More Recent Literature on the Optimal Level of Reserves

As just outlined, most studies in the field of international reserves have traditionally focused on the determinants of reserves and the fundamental trade-off between the benefits of holding reserves and its costs. Only a few recent papers have examined the optimal level of international reserves.

Caballero and Panageas (2007) examine the relationship between reserve accumulation and sudden stops of capital inflows in a dynamic general equilibrium model for EMEs. Given the fact that EMEs run persistent CA deficits to smooth consumption intertemporally, they need capital inflows from foreign countries, but these inflows are subject to the risk of a sudden stop. The authors calibrate the model under this condition and find that insurance strategies aiming at perfect as well as imperfect risk sharing may both lead to a high reduction of reserve accumulation.

Durdu *et al.* (2009) propose a dynamic stochastic general equilibrium (DSGE) model in order to implement a quantitative assessment of the ‘new mercantilism’ under two theoretical models; a one-sector endowment economy and a two-sector production economy. In their analysis, three key factors are changes in the business cycle volatility of output, financial globalization, and self-insurance against a sudden stop. They derive a formula for the optimal level of reserves and find that financial globalization and the risk of a sudden stop are the main reasons behind reserve accumulation. These authors also find that CA surpluses and undervalued exchange rates are two important factors of the large build-up of reserves in response to financial globalization or sudden stop risk.

Based on a different DSGE SOE model, Alfaro and Kanczuk (2009) argue that the optimal level of reserves is zero, since a country can protect itself by defaulting on its external debt instead of accumulating reserves. There is a large volume of published studies on international reserve holdings that take the level of external debt as given. In order to examine the implications of the joint decision on holding reserves and sovereign debt, the authors suggest that an alternative option is to decrease the level of sovereign debt and hence reduce the probability of, and the negative effect of, a potential crisis.

The issue of the optimal international reserve level has also been explored by Calvo *et al.* (2012) within a statistical model where reserves affect the probability of a sudden stop and output costs. The global financial environment is a key factor in their analysis, as the expected return from reserve holdings is conditional on it. In addition, the opportunity cost of reserve holdings is calculated as the spread of public-sector bonds over the interest earned from holding reserves. The optimal level of reserves is then determined as the one that maximizes expected return net of cost, given global financial conditions. One of the main contributions of the paper is that the authors endogenize the probability of sudden stops and the costs of a crisis.

JR (2011) present an 'insurance model against sudden stops' for an endowment SOE. The optimal level of reserves depends on the key determinants of reserve holdings, such as the probability and size of sudden stops, consumers' risk aversion, the opportunity cost of reserves, and potential output growth. Their calibration results show that the optimal level of reserves relative to GDP is 9% for 34 middle-income countries over the period of 1975-2003. However, depending on parameters, the optimal level of reserves could be larger or smaller for individual cases. The JR model differs from the above mentioned models of optimal reserves in several aspects. Firstly, JR provide a closed-form formula for the optimal level of reserves in terms of the level of output, whereas Caballero and Panageas (2007) and Durdu *et al.* (2009) solve their models numerically. Secondly, JR analyse the optimal level of reserves as an insurance against sudden stops rather than precautionary savings or the mercantilist motive.

### 1.2.3 Literature on Capital, Productivity, Growth and Reserves

The earlier literature (Heller, 1966; Hamada and Ueda, 1977; Frenkel and Jovanovic, 1981; and Ben-Bassat and Gottlieb, 1992) derives optimal reserve formulas using a cost-benefit approach. Moreover, the opportunity cost of holding reserves is described as a difference between the return on capital and on reserves. In other words, the marginal productivity of capital enters the optimal reserve formula through the definition of the opportunity cost of holding international reserves. Edwards (1985) discusses the issue and defines the opportunity cost of holding reserves alternatively, as a difference between the interest rate on the debt of a country and the return on reserves. Ben-Bassat and Gottlieb (1992) accept this idea only when the marginal borrowing rate equals or exceeds the marginal productivity of capital. These authors claim that in reality the marginal productivity of capital exceeds the borrowing cost because of market imperfections. However, with increasing financial globalization, the opportunity cost of reserve holdings is better measured as in Edwards (1985), by the difference between the interest rate paid on external liabilities of a country and the lower return on its reserve holdings (García and Soto, 2004; Rodrik, 2006; JR, 2011). Accordingly, many recent studies on international reserves optimality ignore the relationship between capital productivity and reserve holdings.

However, another strand of research has indeed been trying to explain the relationship between productivity, capital accumulation, growth and international reserves. Bonfiglioli (2008) analyses the effects of financial globalization on economic growth in terms of total factor productivity (TFP) and capital accumulation. He shows that financial globalization has a positive effect on productivity. However, his empirical study also finds no direct relationship between financial globalization and capital accumulation. Gourinchas and Jeanne (2006) argue that an improvement in productivity gains larger than increasing capital accumulation when analysing the relationship between capital accumulation, TFP and financial openness. Kose *et al.* (2009) examine the relationship between financial openness and TFP growth. The authors find that capital account openness has a positive effect on TFP growth.

Mourmouras and Russel (2009) investigate the wisdom behind the large reserve holding in terms of investment, capital liquidation and short-term liabilities for a SOE. The authors suggest that capital liquidation and short-term debt are good for economies in terms of increasing investment and higher real wages[3] for workers in good times. On the other hand, capital inflows may cause more financial instability when a country is hit by a sudden stop. In order to prevent a country against a sudden stop, the authors suggest to increase international reserves. By accumulating a high level of reserves, central banks can eliminate or decrease the negative effect of capital liquidation on wage variability and workers' welfare.

[3] Mourmouras and Russel (2009) explain the reason of higher real wages in terms of a higher capital intensity ratio.

Cheng (2015) studies a dynamic open economy model in order to analyse the role of domestic financial underdevelopment in the accumulation of reserves. Showing the needs of domestic saving instruments for emerging market firms which have an external credit constraint, he describes the role of central banks as a financial intermediary: central banks provide liquidity to firms relaxing the credit constraint. In order to decrease the level of reserves he suggests increasing financial market deepness domestically. His paper is based on three stylized facts of EMEs. Firstly, these economies experienced fast economic growth and accumulated a high level of reserves. The positive relationship between the rate of economic growth and the level of international reserves can be seen as part of a 'catch-up' strategy for EMEs. Secondly, these economies have underdeveloped domestic financial markets. Therefore, they require external financing. Lastly, there is a big persistent difference between gross domestic savings and domestic loans in some EMEs. To shed light on these facts, Cheng (2015) argues that foreign reserves affect economic growth via fixed capital formation. Using Granger causality tests, he finds evidence of the relationship between the growth of reserves and gross fixed capital information.

Benigno and Fornaro (2012) present a model for fast growing EMEs which run CA surpluses, hold a high level of reserves and experience capital inflows. The authors analyse the joint behaviour of private and public capital inflows in EMEs indicating differences between the tradable and non-tradable sectors. They suggest that by holding a high level of reserves governments have an important instrument for growth strategies in relation to growth externalities and financial stabilization. The key mechanism is that an increase in reserves leads to real depreciation[4] and to a reallocation of production towards the tradable sector, which increases the use of

[4] Rodrik (2006) provides evidence on this mechanism. He shows that real depreciations stimulate economic growth in developing countries.

imported inputs, the absorption of foreign knowledge and productivity growth. However, this mechanism depends on the imperfect substitutability of private and public capital flows.

Gourinchas and Jeanne (2013) examine the neoclassical framework of the growth model which implies that higher productivity growth attracts more foreign capital inflows. However, the authors find the opposite relationship to hold empirically in the data for developing countries. They call this the 'allocation puzzle'[5] and propose a solution, involving the interaction of growth, saving and international reserve accumulation. They show that the allocation puzzle is much more related to saving and the behaviour of capital flows (generally, the accumulation of reserves) than to investment.

## 1.3 Optimal Level of Reserves with Deterministic AK Technology: Sustained Endogenous Growth

In this subsection, an AK type growth model is employed to examine the effect of productive capital and investment on international reserve holding. All the assumptions of the JR model are maintained, but now an AK production function is added to the model. The two main contributions of such an extension are that the AK technology allows to consider: (i) physical capital accumulation; and (ii) the influence of productivity in the modified optimal reserves formula.

[5] Gourinchas and Jeanne (2013) argue that public capital flows and the accumulation of reserves play an important role in creating the allocation puzzle.

### 1.3.1 Assumptions on the Optimal Level of Reserves under the AK Model

Following JR, we focus on the optimal level of reserves relative to the level of output that is perceived as insurance for a SOE against losing access to the international credit market. A representative domestic agent, or a private sector, is assumed, as well as a domestic government. There is also an international representative agent, referred to as foreign insurers or the rest of the world (RoW), who provide international reserves to the country. The representative domestic agent in the SOE produces a single (composite) good, which is consumed or invested as physical capital domestically as well as consumed abroad (as SOE exports). The model is set out in discrete time with infinite horizon, using the subscript $t$ = 0, 1, 2, … Apart from the risk of sudden stops in capital inflows, there is no other source of uncertainty. In that sense, the country faces a risk of international liquidity problems.

As in JR, the domestic private sector consists of a continuum of atomistic and identical infinitely-lived consumers. Their intertemporal utility $U_t$ is written as

$$U_t = E_t\left[\sum_{i=0,\dots,+\infty} (1+r)^{-i} u(C_{t+i})\right] \tag{1}$$

where $r$ denotes the constant (world) interest rate, the period utility function $u(C_{t+i})$ is assumed to be of the constant relative risk aversion (CRRA) type, with CRRA parameter $\sigma \geq 0$ and $C$ being aggregate consumption,

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma}, \qquad \sigma \neq 1 \tag{2}$$

with $u(C) = \log(C)$ for $\sigma = 1$.

Consumers maximise their current consumption subject to the budget constraint which now includes investment in physical capital,

$$C_t = Y_t - I_t + L_t - (1+r)L_{t-1} + Z_t \tag{3}$$

where $Y_t$ is domestic output, $I_t$ is investment in physical capital domestically in order to increase the capital stock and next-period output, $L_t$ is newly-contracted external debt in *t* with a one-period maturity only and $Z_t$ is a net transfer from the government in *t*. As in JR, external debt accumulated in *t-1* has to be repaid in *t* at $r$, $(1+r)L_{t-1}$, and default in paying back external debt as well as foreign lending by the SOE are assumed away. Differently from JR, investment in physical capital provides a third channel of saving in any period *t*, in addition to the net indebtedness of the SOE to the RoW, $L_t - (1+r)L_{t-1}$, and to the domestic government (or the public sector), entering via the net transfer, $Z_t$. It is perhaps easier to see the implications of our extension to a production SOE by writing disposable income of the domestic private sector in *t*, $DY_t$, compactly as:

$$DY_t \equiv Y_t + L_t - (1+r)L_{t-1} + Z_t.$$

Then, the SOE private-sector budget constraint (3) can now be re-written as

$$C_t = DY_t - I_t; \tag{3'}$$

and, hence, the SOE private-sector saving in physical capital is defined, as standard, by

$$I_t = S_t \equiv DY_t - C_t. \tag{4}$$

As in neoclassical growth theory, it is common to assume that all firms have an identical production function. With AK technology, the aggregate production function is

$$Y_t = F(K_t) = AK_t. \tag{5}$$

As in the JR model, there are two states in the economy: the normal state (denoted by a superscript $n$) or a crisis state interpreted as a sudden stop (denoted by superscript

$s$). Furthermore, following neoclassical growth theory, let: (i) investment in physical capital be a constant proportion of total output in normal times,

$$\frac{I_t^{\ n}}{Y_t^{\ n}} = \mathrm{s}, \tag{6}$$

where *s* is a constant saving rate; and (ii) the increase in the physical capital stock (net investment) in any current period *t* equals the difference between new investment and depreciated capital,

$$\Delta K_{t+1} = I_t^{\ n} - \delta K_t, \tag{7}$$

where a constant proportion of the capital stock $\delta$ is assumed to depreciate each period.

In this first production SOE model version featuring endogenous growth under AK technology in the present chapter, we assume that there is no population growth. From (6), using (7) to write investment and (5) to write output, we can express the (constant) domestic saving rate in physical capital as,

$$s = \frac{\Delta K_{t+1} + \delta K_t}{AK_t} \tag{8}$$

In line with the AK model, we further assume that: (i) capital grows at a constant net rate $g_k$,

$$K_{t+1} = (1 + g_k)K_t; \tag{9}$$

and (ii) that the growth rate of the economy equals the growth rate of capital, $g = g_k$, The latter assumption is shown to be the condition for sustainable growth in the AK technology model.

Using the AK technology to replace output and equation (7) and (9), we obtain

$$K_{t+1} = sAK_t + (1-\delta)K_t \tag{9'}$$

Then, the gross growth rate of the capital stock is, as standard in neoclassical growth theory,

$$\frac{K_{t+1}}{K_t} = 1 + g_k = 1 + sA - \delta \tag{10}$$

And, therefore, the net growth rate of the capital stock is,

$$g_k = sA - \delta \tag{11}$$

We assume that the capital stock does not grow in sudden stop episodes. Hence, $g_k = 0$, so that $K^s_{t+1}: K^s_t = K_t$, which implies that in crisis times in the AK model, $sA = \delta$. Then, we obtain a condition for investment in sudden stop episodes,

$$I^s_t = \delta K^s_t \tag{12}$$

In this model, one of the critical assumptions is related to newly-contracted one-period ahead external debt, $L_t$. How much can a SOE borrow from foreign lenders? There should be some limit on the amount of output that can be guaranteed by the domestic private sector to foreign creditors. In the JR model, this restriction is given by the condition that the external debt must be completely paid back in the next period, which requires:

$$(1+r)L_t \le \alpha_t F(K_{t+1})^{\,n} \tag{13}$$

where $F(K_{t+1})^{\,n}$ is trend output in period *t+1* and $\alpha_t$ is a time-varying parameter. The economy can only borrow according to this rule; hence $\alpha_t$ indicates the warranty of next-period domestic output to external lenders. Considering that the agents know the value of $\alpha_t$ and $F(K_{t+1})^{\,n}$ in any current period t, condition (13) states that external debt in period t is default-free as long as (13) fulfilled. We follow JR in assuming that the time-varying parameter $\alpha_t$ as an exogenous variable: because of the possibility of sudden stops, the rigidity of the consumer's external debt borrowing constraint can fluctuate over time and $\alpha_t$ can be seen as a penalty for domestic agents if they default on their debts.

In the non-crisis state, output increases by a fixed rate $g$ and the economy can guarantee a constant portion of the output,

$$F(K_t)^{\,n} = (1+g)^t F(K_{t-1}) \tag{14}$$

$$\alpha_t^n = \alpha \tag{15}$$

On the other hand, when the economy faces a sudden stop, domestic output decreases by a constant fraction $\gamma$ below its long-run growth path, and guaranteed output goes down to zero:

$$F(K_t)^{\,s} = (1-\gamma)F(K_t)^{\,n} \tag{16}$$

$$\alpha_t^s = 0 \tag{17}$$

Due to normalization, the guaranteed output does not drop below a positive level. The sum of the time-varying parameter and the output loss parameter is assumed lower than unity, $\alpha + \gamma < 1$, in order to secure that the domestic private sector does not have difficulty to pay back all the debt during the crisis. The interest rate on external debt repayment is assumed to be higher than the growth rate of SOE's output (itself equal to the growth rate of physical capital), $r > g$, to hold the private sector's intertemporal income limited as in JR.

We follow JR in also assuming that after a sudden stop the capital inflow converges to its pre-crisis pattern within a certain number of periods, $\upsilon$. Moreover, the country returns to the normal state, $n$, in period $t+\upsilon+1$. In reality, a country would gain access to international liquidity as in its pre-crisis level in more than one year, if a sudden stop hits the economy in the current period $t$. Therefore a 'sudden stop phase' can be defined as the length $[t, t + \upsilon]$, as in the JR model. In other words, matching the various times of a crisis stage $s_t = s^0, s^1, \ldots. s^\upsilon$, in a specific period $t$ the country might be either in the non-crisis state, $s_t = n$, or in the crisis state of $\upsilon + 1$, which are the substates of $\upsilon + 2$ phases.

As in the JR model, the dynamics of external debt depends on the dynamics of output. However, in our extension here output is determined by capital accumulation rather than given as endowment. Therefore, when the dynamics of domestic output during the sudden stop is described, the dynamics of the external debt is also defined:

$$F(k_{t+\tau})^{\,s} \quad = [1 - \gamma(\tau)]F(k_{t+\tau})^{\,n}, \tag{18}$$

$$\alpha_{t+\tau}^{s} = \alpha(\tau), \tag{19}$$

where $\tau = 0,1, \ldots, \theta$. In both equations (18) and (19), $\gamma(\tau)$ and $\alpha(\tau)$ are exogenously determined since they depend on $\tau$. Recalling equations (16) and (17), we know that $\gamma(0) = \gamma$ and $\alpha(0) = 0$ for $\tau = 0$, as in the JR model. Furthermore, it is assumed that the SOE converges to its pre-crisis pattern monotonously, because $\gamma(\tau)$, is non-negative, $\gamma(\tau) \geq 0$ and decline in $\tau$, whereas $\alpha(\tau)$ is also non-negative, $\alpha(\tau) \geq 0$, but increasing in $\tau$. When the crisis ends, the private sector can be financed by international liquidity as in pre-crisis periods, so there will be no restriction to access foreign markets, hence, $\alpha(\upsilon) = \alpha$, as in JR.

In our model version with physical capital and AK technology outlined thus far, sudden stops have negative effects on consumption and investment decisions of domestic consumers, and therefore reduce their welfare. Economic crises reduce trend consumption because consumers' elasticity of intertemporal substitution in consumption is bounded. Moreover, it causes a reduction of domestic output which implies a decrease the consumers' intertemporal income (see Jeanne and Rancière, 2011). It is obvious that a reduction in domestic output and an abrupt fall in capital inflows lead to a strong decrease in economic activity during the crisis state and so consumption goes down sharply (as in JR).

Eventually, consumption increases as foreign capital flows return into the economy after the sudden stop. However, investment continues to decrease after the sudden stop. Figure 1.2 illustrates this in five-year event periods. There might be many possible explanations for the persistent effect of sudden stops on investment, such as increasing costs of investment, difficulty to find foreign funds for investment, or the preference to invest in more stable economies.

The second domestic agent of the economy is the government – or, equivalently, the monetary(-fiscal) authority – of the SOE, which plays a critical role in the JR model and in our extensions. The task of the government in this set-up is to provide smooth domestic consumption between normal and crisis states. To implement such policy, the government has as a tool what JR term 'reserve insurance contracts'. Introducing investment in physical capital in our extensions does not affect the government, and we therefore keep all assumptions related to it and its transfers as in JR. Yet, for completeness, we briefly describe the behaviour of the government next.

Following the JR model, a reserve insurance contract is a simple contract between the government and foreign insurers. The aim of the government is to protect domestic agents from the case of a sudden reversal in capital flows; therefore, the government forgoes some funds today in order to gain capital access during the crisis.[6] In this sense, reserve insurance contracts embody the trade-offs in reserve management, and the mechanism is as follows. Firstly, the government announces a

[6] This could be seen as the cost of reserves and JR show that this kind of insurance should be financed by long-term liabilities.

settlement with external creditors in period *0*. Then, the external fund providers receive a payment $X_t$ from the monetary authority in period *t*. This process continues until a crisis occurs. Once the crisis started at time *t*, the economy obtains a fund $R_t$. After the sudden stop occurs, the monetary authority might sign a new reserve insurance deal with foreign insurers when the sudden stop phase ends.[7]

The government's role can be seen in the budget constraint (3) since it shifts the funds coming from the agreement with foreign investors to the private sector as follows; if the country is in the non-crisis stage,

$$Z_t^n = -X_t; \tag{20}$$

however, if a sudden stop arises, the government secures a payment in the form of,

$$Z_t^s = R_t - X_t. \tag{21}$$

Equation (21) shows the government gain during the sudden stop of capital inflows. The economy earns $R_t$ from foreign insurers, but should also effect the last payment of the reserve insurance contract, $X_t$, within the duration of the sudden stop. Thus, the transmission of the government access to international liquidity is captured by the difference between $R_t$ and $X_t$, as in JR.

There is no change either in foreign insurers' participation condition once we incorporate physical capital and investment. Therefore, all assumptions regarding foreign insurers are kept as in JR. For completeness, we briefly describe their behaviour next.

The role of external creditors is to supply international liquidity to the economy during the sudden stop via the reserve contracts. This definition requires a

[7] Since the time of the crisis is unknown, an insurance contract signed in period *0* must be specified as an infinite sequence of conditional payments $(X_t, R_t)_{t=1,\dots,+\infty}$ (see JR).

condition that foreign creditors should agree on the price of the government contracts. This is a critical parameter in the JR model which shows the condition of foreign insurers' participation. The marginal utility of funds for the investors at date $t$ is denoted by $\mu_t$. As in JR, it is more expensive in the crisis than in the normal state:

$$\mu_t^s \geq \mu_t^n \tag{22}$$

The price of insurance depends on the ratio between $\mu_t^s$ and $\mu_t^n$. For simplicity, the JR model assumes that the price parity of funds in normal times to funds in the sudden stop episode is fixed and equal to or less than one, which we follow:

$$p = \frac{\mu_t^n}{\mu_t^s} \leq 1 \tag{23}$$

The JR model considers external investors as being perfectly competitive and as sharing the same time discount rate with the domestic private sector. Under these assumptions foreign insurers supply any 'reserve insurance contract' $(X_t, R_t)_{t=1,\dots,+\infty}$ whose present discounted value is non-negative, of the form,

$$\sum_{t=1}^{+\infty} \beta^t (1-\pi)^{t-1}[(1-\pi)X_t\mu_t^n - \pi(R_t - X_t)\mu_t^s] \geq 0 \tag{24}$$

### 1.3.2 A Formula for the Optimal Level of Reserves under the AK Model

The production SOE relies on self-insurance against sudden stops by choosing the right amount of international reserves. The advantage of the intentional parsimony of the AK model version introduced thus far is that it allows for a closed-form solution for optimal reserves as a ratio of output and related analytical insights, provided that the borrowing constraint (13) is binding.[8]

Due to the above assumption, the economy's short-term debt to output ratio is constant in non-crisis time. The short-term external debt (STED) to GDP ratio is denoted by $\lambda$

$$\lambda = \frac{L_t^n}{AK_t^n} = \frac{1 + sA - \delta}{1 + r}\ \alpha \tag{25}$$

To provide smooth consumption, the government maximises private sector's intertemporal utility (1) subject to constraints (3), (6), (12), (20), (21), the binding credit constraint (13) and external creditors' participation condition (24).

$$L = \sum_{t=1}^{+\infty} \beta^t (1-\pi)^t \{(1-\pi)u(C_t^n) + \pi u(C_t^s) + v[(1-\pi)X_t\mu_t^n - \pi(R_t - X_t)\mu_t^s]\} \tag{26}$$

where $v$ is the shadow cost of constraint (24), and the normal state consumption is given by,

$$C_t^n = AK_t - sAK_t + \lambda AK_t - \alpha AK_t - X_t$$

$$C_t^n = \left[1 - s - \lambda \frac{r - (sA - \delta)}{1 + (sA - \delta)}\right] AK_t - X_t \tag{27}$$

while the sudden stop episode consumption is given by

$$C_t^s = (1-\gamma)AK_t - \delta K_t - \alpha AK_t + R_t - X_t \tag{28}$$

[8] If the constraint is not binding, a closed-form solution is not possible. Moreover, condition (13) implies that there is no precautionary savings in the model, since the reserve insurance contract plays a substitution role to the precautionary savings.

$$C_t^s = \left[-\gamma - \frac{\delta}{A} - \lambda \frac{r - (sA - \delta)}{1 + (sA - \delta)}\right] AK_t + R_t - X_t$$

The first-order conditions imply

$$u'(C_t^n) = pu'(C_t^s) \tag{29}$$

Equation (29) shows that the domestic consumption can be substituted at the same rate between the normal and crisis state by the private sector and external creditors, as in the JR model. If we simply rewrite (29) and the external creditors' binding condition (13), we can describe the government transfers $X_t$, in form of,

$$X_t = \frac{\pi}{\pi + p(1-\pi)} R_t \tag{30}$$

Now we can solve the first-order condition if the borrowing constraint (13) is always binding. Assuming these conditions meet, we can express the optimal level of international reserves relative to output under the AK production economy as the ratio $\rho \equiv R_t / F(K_t)^n$, in form of,

$$\rho^* = \frac{R_t}{AK_t} = \frac{\gamma + \lambda - \left(1 - p^{\frac{1}{\sigma}}\right)\left[1 - \lambda \frac{r - (sA - \delta)}{1 + (sA - \delta)}\right] + \frac{\delta}{A} - p^{\frac{1}{\sigma}} s}{1 - \frac{\pi}{\pi + p(1-\pi)}\left(1 - p^{\frac{1}{\sigma}}\right)} \tag{31}$$

where, $\gamma$ is the output loss in the first period of capital outflows, $\lambda$ is the STED to GDP ratio, $p$ is the price ratio of funds in different states (normal times and sudden stop episodes), $r$ is the interest rate, $s$ is the saving rate of the economy, $\delta$ is the depreciation rate of physical capital, $A$ is the technology level of the economy, $\pi$ is a crisis probability, and $\sigma$ is the CRRA.

The optimal level of international reserves in terms of output for a SOE derived under an AK production function, equation (31), features some determinants that are common with the original JR endowment SOE model. For example, it is positively related to the STED-GDP ratio, $\lambda$; to the output cost of sudden stops, $\gamma$; to the probability of a sudden stop, $\pi$.

In addition to these determinants, we have three new ones which are related to the production structure of the economy. One such new determinant under an AK technology is the investment rate of economy, $s$. It affects negatively the optimal reserves-output ratio. A second additional determinant is the depreciation rate of capital, $\delta$. It influences positively the reserve ratio. The third additional determinant under the AK technology is the productivity level, $A$, which influences the optimal reserve ratio negatively. As we discussed in the model assumptions, the growth rate of economy is equal to the growth rate of capital stock, and equation (11) gives the parameters of the growth rate of capital stock where it equals investment rate of economy times capital productivity minus the depreciation rate of physical capital. Therefore, if we follow the joint sign of these parameters, we can see that the endogenous growth rate of AK technology is also negatively related with the optimal reserves-output ratio.

In order to compare our extended SOE model with AK technology to the original JR endowment SOE model, we can rewrite our formula for the optimal level of reserves in terms of GDP as follows;

$$\gamma + \lambda - \rho^* = \frac{\left(1-p^{\frac{1}{\sigma}}\right)}{1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)}\left[1-\alpha-\gamma-\frac{\delta}{A}+p^{\frac{1}{\sigma}}s+(\gamma+\lambda)\left(\frac{p(1-\pi)}{\pi+p(1-\pi)}\right)\right] \qquad (32)$$

By inspection of equation (32), we can see lots of similarities with the analogous expression in JR, their equation (19); yet, our extended model derives an additional term, $\left(\frac{\delta}{A} + p^{\frac{1}{\sigma}} s\right)$, which results from of adding investment and productive capital under our AK technology assumption.

To begin with the similarities, first, the left hand side (LHS) of equation (32) is the same as in the JR model. Secondly, the case of $p = 1$, which implies that external insurers do not have preferences between the crisis and non-crisis states, collapses our equation (32) in the same way as it collapses equation (19) in JR, making the right hand side (RHS) zero. In this special case of full insurance, the economy's reserve ratio is equal to the aggregate size of the output loss and the STED-GDP ratio, $(\gamma + \lambda\,)$. Thirdly, the influence of risk aversion is the same in both models because optimal reserves depend positively on $\sigma$. A fourth similarity is the response to the Greenspan-Guidotti rule, which states that reserves should cover the STED, i.e., $\rho = \lambda$. So, the only case when $p = 1$ and there is no loss in output, i.e., $\gamma = 0$, implies $\rho = \lambda$ (similarly to JR).

The key difference between the JR endowment benchmark and our AK production model lies in the role of investment and productive capital, captured in (32) by the depreciation rate of physical capital, $\delta$, the investment rate of economy, $s$ , and the productivity level, $A$. Firstly, a higher depreciation rate in SOEs requires a quicker replacement of the capital stock. Then, these economies (as production mostly depends on imported goods) might need to finance their production and, hence, need access to external borrowing. In order to provide insurance for the private sector during the sudden stop, a higher depreciation rate implies a higher reserve to GDP ratio.

Secondly, an increase in the investment rate $s$ leads to a decrease in optimal reserve holding. Since the investment rate equals the saving rate and if investment is financed by a higher proportion of domestic savings, then the economy needs less foreign capital to insure itself against a sudden stop of capital flows. This theoretical result also attests that investment still could be thought as an opportunity cost of reserves (see Rodrik, 2006).

Thirdly, a higher productivity of capital $A$ leads to a lower reserve to GDP ratio. Therefore, not only investment decreases the reserve to GDP ratio, but also productivity (of capital, in the AK model) plays an important role in reducing optimal reserve holding for SOEs relative to the endowment benchmark in JR. We could think of a mechanism working in the opposite direction of capital stock depletion via the depreciation rate. As long as the private sector employs a sufficiently productive AK technology, this might decrease the need for international borrowing compared to a less productive AK technology or to the JR endowment benchmark.

## 1.4 Calibration

In this section, we analyse some quantitative implications of our AK production SOE model using data for 34 middle income countries[9] from 1975 to 2014. In order to show the overall behaviour of the model parameters, domestic consumption[10] is defined in terms of domestic output, financial account, investment, income transfers and change in reserves,

[9] In order to be able to make a direct comparison between our AK production SOE model and the endowment benchmark of Jeanne and Rancière (2011), we used the same sample of 34 countries, extending the original dataset to 2014. They classified these as middle income countries according to the World Bank's classification. However, this classification has changed (the sample includes 7 high income countries, i.e., Argentina, Korea, Hungary, Poland, Chile, the Czech Republic and Uruguay) after the publication of their paper. Following JR, we also exclude major oil-producing countries from our dataset.
[10] Equation (33) can also be interpreted as domestic absorption since domestic absorption equals the sum of domestic consumption and investment, $D_t = C_t + I_t$.

$$C_t = Y_t - I_t + FA_t + IT_t - \Delta R_t \tag{33}$$

where $FA_t$ is the financial account, $IT_t$ is the income and transfers from abroad and $\Delta R_t$ shows the change in reserves. A sudden stop is defined as an unexpected decrease in financial account which leads a decrease in domestic consumption and it might cause a decrease in domestic output or it can be adjusted by international reserves (see Jeanne and Rancière, 2011). Equation (33) shows the link with our budget constraint in the sudden stop as,

$$\underbrace{C_t^s}_{C_t} = \underbrace{(1-\gamma)Y_t^n}_{Y_t} - \underbrace{\delta K_t^s}_{I_t} + \underbrace{(-L_{t-1})}_{FA_t} + \underbrace{[-rL_{t-1} - (\pi + \omega)R_t]}_{IT_t} - \underbrace{(-R_t)}_{\Delta R_t} \tag{34}$$

where $\omega$ denotes a pure risk premium[11] and might be interpreted as an opportunity of holding reserves.

Our extended model to AK production allows us to describe the dynamics of output with investment during sudden stop episodes. Figure 1.2 illustrates a novel feature of this output dynamics driven obviously by investment dynamics relative to the JR endowment benchmark abstracting from investment. The average behaviour of equation (34) in a five-year event window is depicted in the figure, where the middle observation '0' labels a sudden stop year. A sudden stop is identified as a more than 5% decrease in the ratio of capital inflows to GDP, $KA_t/Y_t$, relative to the preceding year, following Guidotti *et al.* (2004) and Jeanne and Rancière (2011). Although all components of equation (34) display a similar pattern with the JR model, investment adds inertia in its own adjustment and, hence, in the adjustment of output. Both investment and output in our AK model continue to decrease after the sudden stop

[11] Because it has no role in affecting productivity and investment, the opportunity cost of holding reserves is not described in this model. However, in order to make a comparison between our AK production model and the JR endowment benchmark, we follow their methodology in expressing $X_t = (\pi + \omega)R_t$.

period, featuring higher persistence, whereas all other components of equation (34) start recovery after period ‘0’, as is the case with output when investment and capital are not modelled in JR. The difficulties in accessing international borrowing facilities after the sudden stop and the capital outflows during the crisis make the private sector vulnerable, and this affects investment decisions. Therefore, a recovery may not be seen in investment and output in the first year of the sudden stop.

The countries in our sample and the years in which they had a sudden stop[12] are presented in Table 1.2. Even though we use the same sample of countries as JR (2011), our sudden stop years were defined applying their methodology to our updated dataset, and therefore some minor differences in the sudden stop episodes by country are observed. Moreover, when we calculate capital inflows in our dataset mostly World Bank’s *World Development Indicators* (WDI, online) was used, whereas JR relied on IMF’s *International Financial Statistics* (IFS).

Our calibration of the key parameters entering the optimal reserve-to-output ratio is given in Table 1.3. We used equations (31), (33) and (34) above. Furthermore, we recomputed some of the JR model parameters in our updated sample (such as the output loss, the size of the sudden stop, the crisis probability) since they play similar roles in our AK technology model. We did not change some JR parameters (such as the interest rate, the price of non-crisis dollar and the CRRA) as they have no distinct novel role in our model but are necessary for a comparison between both models.

[12]Capital inflows-capital account over GDP ratio- measured as a ratio of the current account deficit minus reserve accumulation to GDP as in JR. Jordan and Poland show most noticeable differences with JR’s sudden stop years, because of data limitations, since IFS 2016, and WDI do not have data for these countries in those years when a sudden stop can be observed. In JR, Poland has two sudden stop years but Jordan has more than 5 years. Therefore, our calibration might show 1 or 2 % differences. In order to make better comparison between our model and JR model, we tried to calibrate our parameters similar to JR as much as we can.

The unconditional probability of a crisis, $\pi$, is 9.8% per year which is rounded to 10%. Our calibration of $\pi$ is consistent with JR (2011), as they found $\pi$=0.1 (with a range between a minimum of 0 and a maximum of 0.24).

The STED-GDP ratio, interpreted by JR as the size of a sudden stop, $\lambda$, is calibrated at the average level of the ratio of capital inflows to GDP, $\left[\left(\frac{KA_t}{Y_t}\right) - \left(\frac{KA_{t-1}}{Y_{t-1}}\right)\right]$, over our sample of crisis episodes, and is almost 9.9%, which is rounded to 10%. JR also set $\lambda$ to 10% (range 0 to 0.30).

Output loss, $\gamma$, was calibrated at the average difference between the GDP growth rate one period before the crisis and the growth rate in the first year of the capital outflows. We observed a 2% decrease in GDP growth rates on average in the first year of capital outflows and a 4% decrease when we restrict the sample to countries that suffered an output reduction; however, it shows large variation across countries. JR assume[13] $\gamma$= 0.065 and we use their calibration in order to make more consistent comparison of our model with theirs.

The risk free short term interest rate, $r$, the risk aversion parameter, $\sigma$, and the price ratio of funds in dollars,[14] $p$, are calibrated as in Jeanne and Rancière (2011) at 5%, 2, 0.855, respectively.

The role played by the investment rate, $s$, and the depreciation rate of physical capital, $\delta$, constitute our main contribution to extending the optimal reserve formula in JR to a production SOE. We found the average level of the investment rate to be equal to 24%, which is the sample average of the investment share in total income in our data from the Penn World Tables (PWT, 7.0). Therefore, we calibrated the

[13] JR calculate output decreases by 4% on average in the first year of sudden stops and by 9% when they only focus attention on subset of the countries in which output fell. Then they take the average of two estimates and set output loss to 6.5%.
[14] Which is based on the calculation of the opportunity cost of reserves from JR.

investment rate at 24%. Following the growth accounting literature, we set the depreciation rate of physical capital to 6% per annum (Caselli, 2005; Gourinchas and Jeanne, 2013).

In addition to the determinants of optimal reserves in the JR endowment benchmark, our SOE model extended to investment and production also includes a key technology parameter, $A$, which is calibrated based on model assumptions, as described next. For simplification, and following Caselli (2005), we denote $\frac{y}{k} = A_K$ in the AK model, where $y$ is equal to GDP per worker in the data and $k$ is capital per worker in the data. Our sample shows average GDP per worker equal $15141 from PWT (7.0). In line with Caselli (2005), we found capital per worker, $k$, is 2.49 times higher than GDP per worker. Then, we calculated $A_K$ to be equal to 0.4.

Our extended model highlighting an AK technology results in a lower optimal reserve-to-output ratio, 1.74%, relative to the JR endowment benchmark, 9.1% and commonly accepted reserve adequacy indicators[15] (i.e. international reserves as % of total external debt, broad money as % of international reserves and international). Thus, when taking into account investment and productive capital, countries would need less reserves. Our model implies that the optimal level of international reserves to GDP is a decreasing function of the investment rate. Furthermore, higher productivity (of capital, here in the AK model) implies a lower reserve-to-output ratio too. In other words, capital productivity decreases the need of higher international reserves relative to the JR endowment benchmark.

Figure 1.3 illustrates the relationship between the optimal level of international reserves and its determinants for the AK model. This figure also shows the sensitivity of our results to the key determinants of the optimal reserve-to-output

[15] Figures 1.4, 1.5 and 1.6 show commonly accepted reserve adequacy indicators for our sample according to their continents in appendix 1.A.1.

ratio. It is based on the optimal reserve formula (31) and the reported calibrations. As can be seen in the respective panels of Figure 1.3, our results suggest a positive relationship between the reserve-to-GDP ratio and some of its key determinants, such as the size of sudden stop, the output cost of a sudden stop, the probability of sudden stops, the interest rate, the coefficient of relative risk aversion and – in our extension to an AK production – also on the depreciation rate of capital. On the other hand, and perhaps most importantly given the aims of the present chapter, we were able to uncover the novel findings that the optimal reserve-to-GDP ratio depends negatively on other determinants, notably those that arise when modelling a production economy, here under an AK technology, namely the investment rate of economy and capital productivity. Figure 1.3 also shows the negative relationship between the endogenous growth rate and optimal reserve-to-output ratio. Although it does not appear in the optimal reserve formula (31) explicitly, it can be easily derived from model assumptions and equation (11).

## 1.5 Concluding Remarks

This chapter argues that investment, depreciation and productive capital can all play an important role as key determinants of the optimal international reserves relative to output in SOEs when production is modelled explicitly. When extended to an AK technology, the JR endowment SOE benchmark implies a richer formula for optimal reserves-to-GDP[16]. Our extended model shows analytically and quantitatively, given calibration using the JR sample of 34 countries updated to 2014, that a higher capital productivity and a higher investment rate decrease the ratio of optimal reserves-to-GDP, while a higher depreciation rate increases it.

[16] An analytical comparison of optimal reserves-to-output formula in the JR endowment SOE benchmark and in our extension to AK production model can be seen in Figure 1.7.

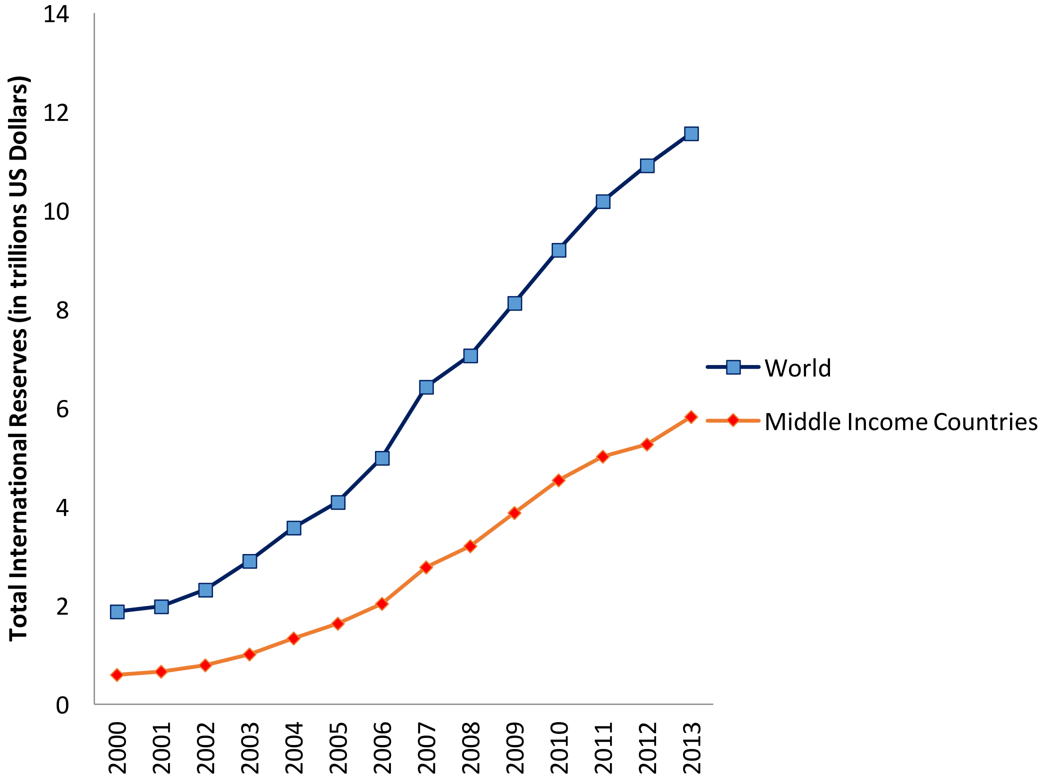
In the AK production model, we found the optimal ratio of international reserves to output to be much lower, 1.74%, than the corresponding one derived in the endowment SOE benchmark of Jeanne and Rancière (2011), 9.1%. We would outline our intuition regarding this main result in the following way. When the capital stock is accumulated through investment and potentially used as a pledge to external creditors in obtaining borrowing and therefore insuring better against sudden stops, the optimal reserve-to-output ratio can optimally be much lower in the AK model relative to an otherwise similar endowment economy. On the other hand, depreciation depletes the existing capital stock, an opposite effect to that of net investment; hence, the optimal reverses ratio also depends positively on depreciation of capital in the AK model. Furthermore, government policy in the AK model can generate endogenously persistent capital accummulation leading to sustained long-run growth throug tax and subsidy instruments.

While our extension in chapter 1 implies increasing returns to scale (IRS) and is justified on the grounds of the ability of the AK model to generate endogenously capital accummulation and sustained long-run growth observed in the data (Acemoglu, 2009, p. 55; Jones and Vollrath, 2013, p. 216), it abstracts from labour as a second input into the production function. Therefore, in chapter 2 we proceed to add labour and switch to a labour-augmenting Cobb-Douglass (CD) production function, justified on the grounds of being consistent with a long-run balanced growth path (BGP) in neoclassical models of exogenous growth (Acemoglu, 2009, p. 59) and with sustained per capita income growth in these models (Jones and Vollrath, 2013, pp. 36-37). We shall see how such an alternative and plausible technology specification would modify the optimal reserves-to-output formula.

# Appendix to Chapter 1

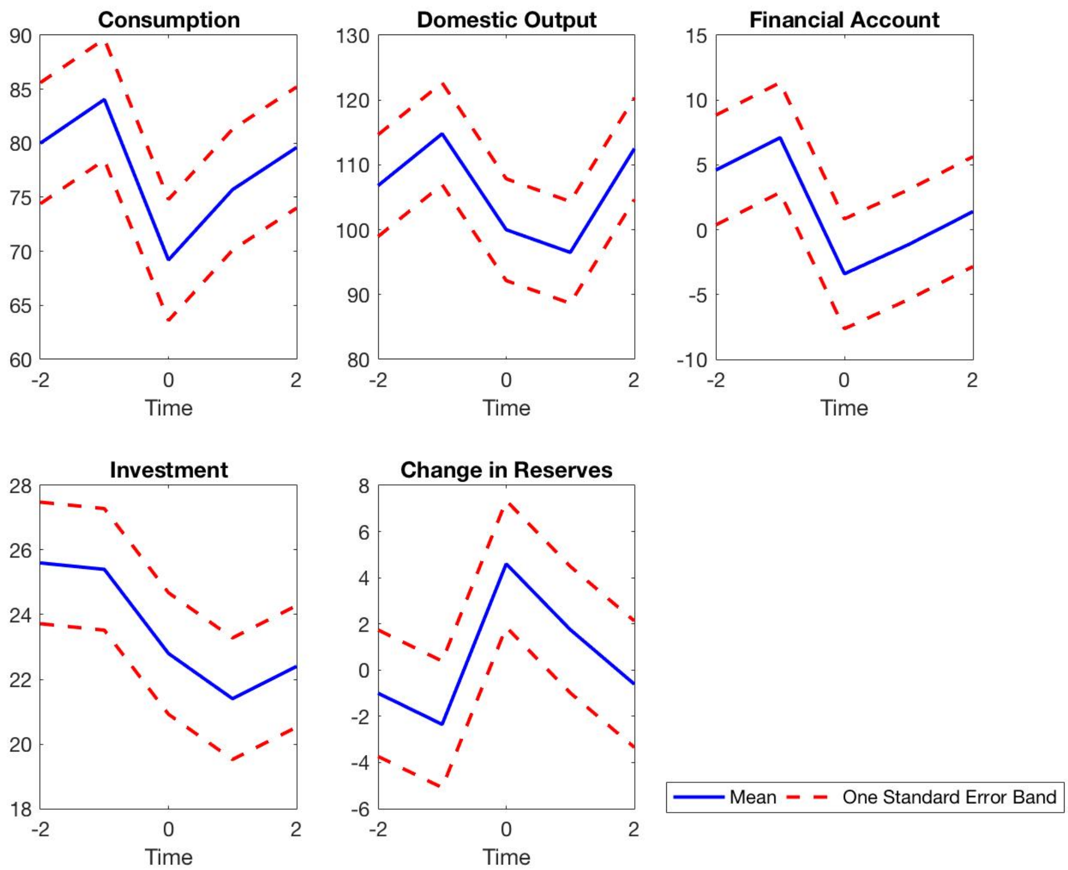
## 1.A.1 Figures

**Figure 1.1: Global International Reserves (in trillions US Dollars)**



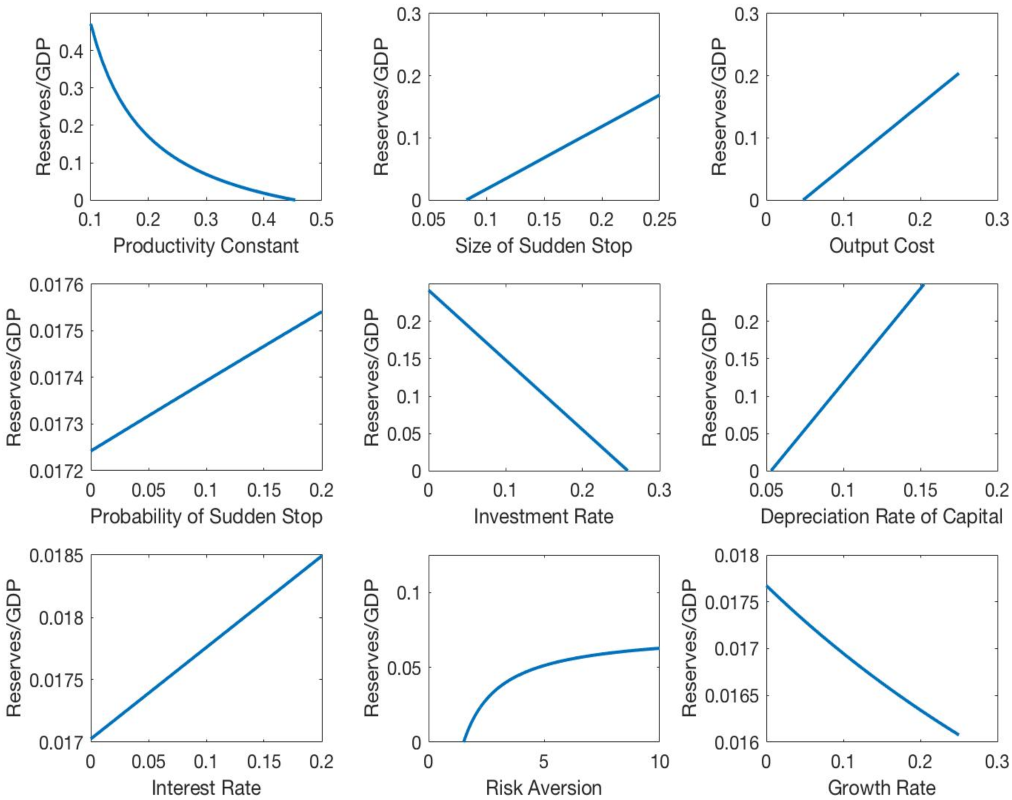

Source: World Bank, *World Development Indicators* (online). Data on internatioanl reserves less gold for the 34 middle income countries (listed in Table 1.2 in Appendix 1.A) and the world.

**Figure 1.2:** Average Dynamics of Key Model Variables in Sudden Stops



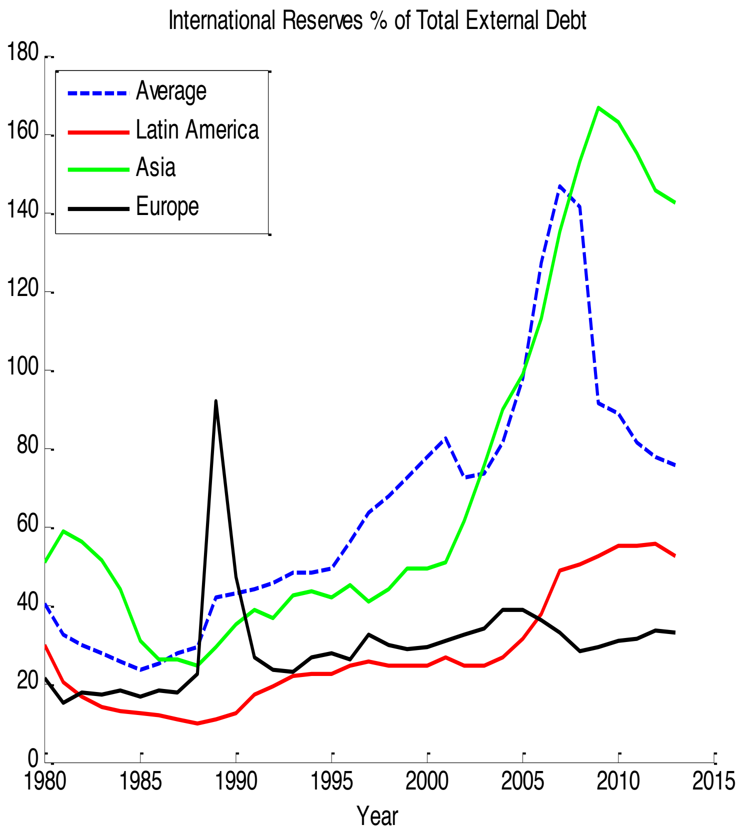

Source: Author's calculation, using data from IMF's *International Financial Statistics*, and World Bank's *World Development Indicators*.

**Figure 1.3:** Optimal Reserves-to-GDP Ratio as a Function of Its Key Determinants in the AK model



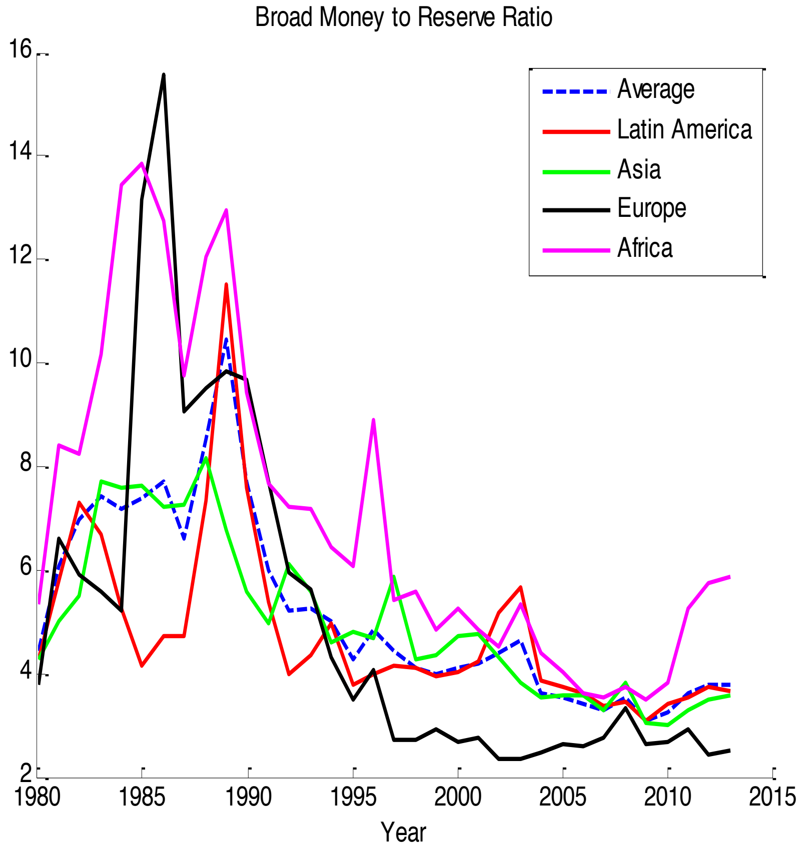

Source: Author's calculation, using data from IMF's *International Financial Statistics*, Penn World Table 7.0 and World Bank's *World Development Indicators*.

**Figure 1.4:** International Reserves as % of Total External Debt by World Regions



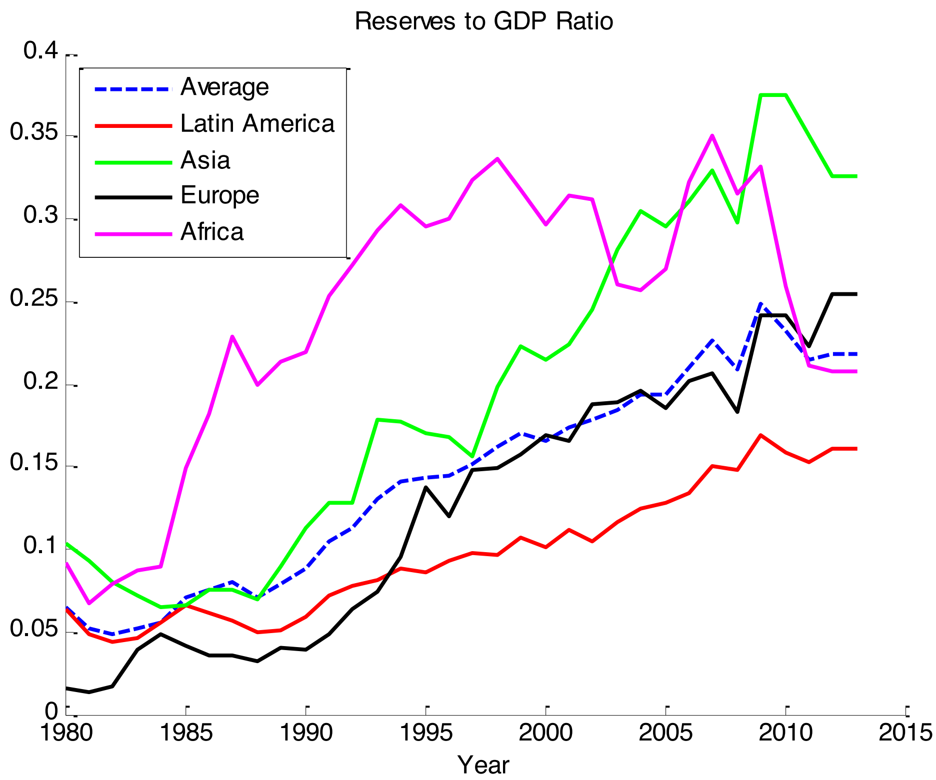

Note: Figures 1.4, 1.5 and 1.6 (that follow) show commonly accepted reserve adequacy indicators for the 34 middle income countries in our/JR sample. All data is from IMF's *International Financial Statistics* and World Bank's *World Development Indicators*. Countries are grouped according to their continent.

**Figure 1.5:** Broad Money as % of International Reserves by World Regions




Note: See the note to Figure 1.5.

**Figure 1.6:** International Reserves to GDP by World Regions




Note: See the note to Figure 1.5.

**Figure 1.7:** Optimal Reserves-to-Output Formula in the JR Endowment SOE Benchmark and in Our Extension to AK Production: Analytical Comparison

**<u>JR Endowment SOE Model</u>**

$$\rho^* = \frac{\gamma + \lambda - \left(1 - \frac{r-g}{1+g}\lambda\right)\left(1 - p^{\frac{1}{\sigma}}\right)}{1 - \frac{\pi}{\pi + p(1-\pi)}\left(1 - p^{1/\sigma}\right)}$$

**Our Extended SOE Model with <u>AK Production</u>**

$$\rho^* = \frac{R_t}{AK_t} = \frac{\gamma + \lambda - \left(1 - p^{\frac{1}{\sigma}}\right)\left[1 - \lambda \frac{r - (sA - \delta)}{1 + (sA - \delta)}\right] + \frac{\delta}{A} - p^{\frac{1}{\sigma}} s}{1 - \frac{\pi}{\pi + p(1-\pi)}\left(1 - p^{\frac{1}{\sigma}}\right)}$$

## 1.A.2 Tables

**Table 1.1:** Countries and Years of Sudden Stops

| *Country* | *Dates of Sudden Stops* |
|---|---|
| Argentina | 1989, 1994, 2001, 2002, 2008 |
| Bolivia | 1980, 1982, 1983, 1985, 2000, 2003, 2006 |
| Botswana | 1977, 1987, 1993, 2001 |
| Brazil | 2008 |
| Bulgaria | 1989, 1990, 1994, 1996, 2008 |
| Chile | 1982, 1983, 1998, 2007 |
| China | |
| Colombia | |
| Costa Rica | 1981 |
| Czech Republic | 1996, 2003 |
| Dominican Rep. | 2002 |
| Ecuador | 1983, 1999, 2000, 2006 |
| Egypt | 1987, 1990, 1999, 2006 |
| El Salvador | 2004, 2007 |
| Guatemala | |
| Honduras | |
| Hungary | 1994, 1996 |
| Jamaica | 1985, 1986, 2002, 2003 |
| Jordan | 1976, 1979, 1980, 1984, 1989, 1992, 1993, 1998, 2001, 2003 |
| Korea | 1997, 2008 |
| Malaysia | 1987, 1994, 1996, 1997, 1998, 2005, 2008 |
| Mexico | 1982, 1995 |
| Morocco | 1978, 1995 |
| Paraguay | 1985, 1988 |
| Peru | 1983, 1998 |
| Philippines | 1983, 1997, 1998, 2008 |
| Poland | 1994 |
| Romania | 2008 |
| South Africa | |
| Sri Lanka | |
| Thailand | 1998, 2007 |
| Tunisia | |
| Turkey | 1994, 2001 |
| Uruguay | 1983, 2002, 2004 |

Note: A sudden stop is defined if the ratio of capital inflows to gross domestic product (GDP) decreases by more than 5% relative to the preceding year. Source: IMF, *International Financial Statistics* and World Bank, *World Development Indicators* (online).

**Table 1.2:** Calibration of Key Parameters in the AK Model

| Parameters | AK Model | Range of Variation |
|---|---|---|
| Technology | $A_K = 0.40$ | |
| Size of a Sudden Stop | $\lambda = 0.10$ | [0, 0.30] |
| Probability of a Sudden Stop | $\pi = 0.10$ | [0, 0.24] |
| Output Loss | $\gamma = 0.065$ | [0,0.2] |
| Price of a Non-Crisis Dollar | $p = 0.855$ | |
| Depreciation Rate of Capital | $\delta = 0.06$ | [0, 1] |
| Risk Free Rate | $r = 0.05$ | |
| Coefficient of Risk Aversion | $\sigma = 2$ | [1, 10] |
| Capital-Labour Ratio | $k = 37701$ | |
| GDP per Worker | $y = 15141$ | |
| Investment Rate | 0.24 | [0, 0.48] |

Source: Author's calculation, using data from IMF's *International Financial Statistics*, Penn World Table 7.0 and World Bank's *World Development Indicators*.

## 1.B Model Derivation

### Foreign Insurers' Contract

The foreign insurers' contract is given by

$$\sum_{t=1}^{+\infty} \beta^t (1-\pi)^{t-1}[(1-\pi)X_t\mu_t^n - \pi(R_t - X_t)\mu_t^s] \geq 0,$$

and if it is binding, as assumed, it holds with equality so that one can write

$$(1-\pi)X_t\mu_t^n = \pi(R_t - X_t)\mu_t^s$$

$$p = \frac{\mu_t^n}{\mu_t^s} = \frac{\pi(R_t - X_t)}{(1-\pi)X_t}$$

$$p = \frac{\pi(R_t - X_t)}{(1-\pi)X_t}$$

$$\frac{p(1-\pi)}{\pi} = \frac{(R_t - X_t)}{X_t}$$

$$\frac{p(1-\pi)}{\pi} = \frac{R_t}{X_t} - 1$$

$$\frac{R_t}{X_t} = \frac{p(1-\pi)}{\pi} + 1$$

$$X_t = \frac{\pi}{\pi + p(1-\pi)} R_t.$$

**First Order Condition from Maximizing the Lagrangian Function**

$$u'(C_t^n) = pu'(C_t^s)$$

$$p = \frac{u'(C_t^n)}{u'(C_t^s)}.$$

With the assumed isoelastic, or CRRA, period utility,

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma},$$

we further obtain

$$p = \frac{(C_t^n)^{-\sigma}}{(C_t^s)^{-\sigma}} \text{ or } p = \left(\frac{C_t^n}{C_t^s}\right)^{-\sigma} \text{ and finally}$$

$$p^{\frac{1}{\sigma}}(C_t^n) = C_t^s$$

**Budget Constraint in Normal Times and Sudden Stop Episodes**

AK Technology (assuming constant population irrelevant to technology)

With the AK technology, output is given by

$$Y_t = F(K_t; A) = AK_t.$$

The 'normal time' budget constraint (superscript *n*) of the private sector is given by

$$C_t^n = Y_t^n(\cdot) - I_t^n + L_t^n - (1+r)L_{t-1}^n + Z_t^n$$

and investment by

$$I_t^n = sY_t^n(\cdot),$$

with capital accumulation written as

$$K_{t+1}^n\colon\ K_{t+1} = I_t^n + (1-\delta)K_t.$$

Using the AK technology to replace $Y_t^n(\cdot) = F(K_t; A) = AK_t$ in the assumption for normal-time investment above, we obtain

$$K_{t+1} = sAK_t + (1-\delta)K_t.$$

Then, the gross growth rate of the capital stock is, as standard in neoclassical growth theory,

$$1 + g_K = \frac{K_{t+1}}{K_t} = 1 + sA - \delta.$$

And, therefore, this is also the gross growth rate of the output in the AK model.

Note that the short-term external debt (STED) ratio to output remains constant as in JR but is now given by

$$\lambda \equiv \frac{1 + (sA - \delta)}{1 + r}\,\alpha.$$

Replacing investment, we can write the normal-time budget constraint of the private sector as

$$C_t^n = Y_t^n(\cdot) - sY_t^n(\cdot) + L_t^n - (1+r)L_{t-1}^n + Z_t^n.$$

With AK technology, output is given as $Y_t^n = F(K_t; A) = AK_t$, and using $L_t^n = \frac{1+(sA-\delta)}{1+r}\alpha Y_t^n(\cdot) = \lambda Y_t^n(\cdot)$ and $Z_t^n = -X_t$, we further obtain (successively):

$$C_t^n = AK_t - sAK_t + \frac{1+sA-\delta}{1+r}\alpha AK_t - (1+sA-\delta)\alpha AK_{t-1} - X_t$$

$$C_t^n = AK_t - sAK_t + \lambda AK_t - \alpha AK_t - X_t$$

$$C_t^n = AK_t - sAK_t + \lambda AK_t - \lambda\frac{1+r}{1+sA-\delta}AK_t - X_t$$

$$C_t^n = \left[1 - s + \lambda\left(1 - \frac{1+r}{1+sA-\delta}\right)\right]AK_t - X_t$$

$$C_t^n = \left[1 - s + \lambda\frac{(sA-\delta)-r}{1+(sA-\delta)}\right]AK_t - X_t$$

$$C_t^n = \left[1 - s - \lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]AK_t - X_t.$$

The ‘sudden stop’ budget constraint (superscript *s*) of the private sector is given by

$$C_t^s = (1-\gamma)Y_t^n(\cdot) - I_t^s - (1+r)L_{t-1}^s + Z_t^s,$$

and we assume that the capital stock does not grow in sudden stops, $g_K = 0$, so that

$$K_{t+1}^s\colon K_{t+1} = K_t,$$

which implies, in the AK model,

$$1 + g_K = \frac{K_{t+1}}{K_t} = 1 + sA - \delta = 0$$

so that

$$sA = \delta$$

and

$$I_t^s = \delta K_t.$$

Note that in the AK model capital grows only if $sA > \delta$.

Replacing sudden-stop investment, we can also write the above budget constraint as

$$C_t^s = (1-\gamma)Y_t^n(\cdot) - \delta K_t + L_t^s - (1+r)L_{t-1}^s + Z_t^s.$$

With AK technology, output is given by $Y_t = F(K_t; A) = AK_t$, and using $L_t^s = 0$, $L_{t-1}^s = \frac{1+(sA-\delta)}{1+r} \alpha Y_{t-1}^n(\cdot) = \lambda Y_{t-1}^n(\cdot)$ and $Z_t^s = R_t - X_t$, we further obtain (successively):

$$C_t^s = (1-\gamma)AK_t - \delta K_t - (1 + sA - \delta)\alpha AK_{t-1} + R_t - X_t$$

$$C_t^s = (1-\gamma)AK_t - \delta K_t - \alpha AK_t + R_t - X_t$$

$$C_t^s = (1-\gamma)AK_t - \delta K_t - \frac{1+r}{1+sA-\delta}\lambda\, AK_t + R_t - X_t$$

$$C_t^s = \left[-\gamma - \frac{\delta}{A} + 1 - \frac{1+r}{1+sA-\delta}\lambda\right] AK_t + R_t - X_t$$

$$C_t^s = \left[-\gamma - \frac{\delta}{A} + \frac{1+sA-\delta-1-r}{1+sA-\delta}\lambda\right] AK_t + R_t - X_t$$

$$C_t^s = \left[-\gamma - \frac{\delta}{A} + \frac{(sA-\delta)-r}{1+(sA-\delta)}\lambda\right] AK_t + R_t - X_t$$

$$C_t^s = \left[-\gamma - \frac{\delta}{A} - \lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right] AK_t + R_t - X_t.$$

Therefore, from the first order condition, the optimal level of reserves as a ratio to output can be expressed as:

$$p^{\frac{1}{\sigma}}\left\{\left[1-s-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]AK_t-X_t\right\}$$

$$=\left[-\gamma-\frac{\delta}{A}-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]AK_t+R_t-X_t$$

$$p^{\frac{1}{\sigma}}\left[1-s-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]AK_t-p^{\frac{1}{\sigma}}X_t$$

$$=\left[-\gamma-\frac{\delta}{A}-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]AK_t+R_t-X_t$$

$$p^{\frac{1}{\sigma}}\left[1-s-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]AK_t-\left[-\gamma-\frac{\delta}{A}-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]AK_t$$

$$=R_t-X_t+p^{\frac{1}{\sigma}}X_t$$

$$\left[p^{\frac{1}{\sigma}}(1-s)-p^{\frac{1}{\sigma}}\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}+\gamma+\frac{\delta}{A}+\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]AK_t=R_t-\left(1-p^{\frac{1}{\sigma}}\right)X_t$$

$$\left[p^{\frac{1}{\sigma}}(1-s)+\left(1-p^{\frac{1}{\sigma}}\right)\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}+\gamma+\frac{\delta}{A}\right]AK_t$$

$$=R_t-\left(1-p^{\frac{1}{\sigma}}\right)\frac{\pi}{\pi+p(1-\pi)}R_t$$

$$\left[p^{\frac{1}{\sigma}}(1-s)+\left(1-p^{\frac{1}{\sigma}}\right)\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}+\gamma+\frac{\delta}{A}\right]AK_t$$

$$=\left[1-\left(1-p^{\frac{1}{\sigma}}\right)\frac{\pi}{\pi+p(1-\pi)}\right]R_t$$

And, finally, we obtain the optimal reserves-to-output ratio, $\rho^*$, under the AK technology case in the production SOE we analysed:

$$\rho^* = \frac{R_t}{AK_t} = \frac{\gamma + \frac{\delta}{A} + p^{\frac{1}{\sigma}}(1-s) + \lambda \frac{r-(sA-\delta)}{1+(sA-\delta)}\left(1-p^{\frac{1}{\sigma}}\right)}{1 - \frac{\pi}{\pi + p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)},$$

An alternative equivalent expression where the parameter $\lambda$ appears also additively as in the original JR optimal reserves expression can be obtained, as follows.

Since $C_t^s = (1-\gamma)AK_t - \delta K_t - \frac{1+r}{1+sA-\delta}\lambda\, AK_t + R_t - X_t$

we could write

$$C_t^s = \left[(1-\gamma) - \frac{\delta}{A} - \frac{1+r}{1+sA-\delta}\lambda\right] AK_t + R_t - X_t$$

$$C_t^s = \left[(1-\gamma) - \frac{\delta}{A} - \frac{1+r+1+(sA-\delta)-1-(sA-\delta)}{1+(sA-\delta)}\lambda\right] AK_t + R_t - X_t$$

$$C_t^s = \left[(1-\gamma) - \frac{\delta}{A} - \frac{1+(sA-\delta)}{1+(sA-\delta)}\lambda - \frac{1+r-1-(sA-\delta)}{1+(sA-\delta)}\lambda\right] AK_t + R_t - X_t$$

$$C_t^s = \left[(1-\gamma) - \frac{\delta}{A} - \lambda - \frac{r-(sA-\delta)}{1+(sA-\delta)}\lambda\right] AK_t + R_t - X_t.$$

Then, optimal reserves-to-output can be obtained:

$$p^{\frac{1}{\sigma}}\left\{\left[1-s-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]AK_t-X_t\right\}=\left[(1-\gamma)-\frac{\delta}{A}-\lambda-\frac{r-(sA-\delta)}{1+(sA-\delta)}\lambda\right]AK_t+R_t-X_t$$

$$p^{\frac{1}{\sigma}}\left[1-s-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]AK_t-p^{\frac{1}{\sigma}}X_t=\left[(1-\gamma)-\frac{\delta}{A}-\lambda-\frac{r-(sA-\delta)}{1+(sA-\delta)}\lambda\right]AK_t+R_t-X_t$$

$$p^{\frac{1}{\sigma}}\left[1-s-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]AK_t-\left[(1-\gamma)-\frac{\delta}{A}-\lambda-\frac{r-(sA-\delta)}{1+(sA-\delta)}\lambda\right]AK_t=R_t-X_t+p^{\frac{1}{\sigma}}X_t$$

$$\left[p^{\frac{1}{\sigma}}(1-s)-p^{\frac{1}{\sigma}}\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}-(1-\gamma)+\lambda+\frac{\delta}{A}+\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]AK_t=R_t-\left(1-p^{\frac{1}{\sigma}}\right)X_t$$

$$\left[p^{\frac{1}{\sigma}}-p^{\frac{1}{\sigma}}s+\left(1-p^{\frac{1}{\sigma}}\right)\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}-1+\gamma+\lambda+\frac{\delta}{A}\right]AK_t=R_t-\left(1-p^{\frac{1}{\sigma}}\right)\frac{\pi}{\pi+p(1-\pi)}R_t$$

$$\left[-\left(1-p^{\frac{1}{\sigma}}\right)-p^{\frac{1}{\sigma}}s+\left(1-p^{\frac{1}{\sigma}}\right)\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}+\gamma+\lambda+\frac{\delta}{A}\right]AK_t=R_t-\left(1-p^{\frac{1}{\sigma}}\right)\frac{\pi}{\pi+p(1-\pi)}R_t$$

$$\left[\gamma+\lambda+\frac{\delta}{A}-p^{\frac{1}{\sigma}}s-\left(1-p^{\frac{1}{\sigma}}\right)\left(1-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right)\right]AK_t=\left[1-\left(1-p^{\frac{1}{\sigma}}\right)\frac{\pi}{\pi+p(1-\pi)}\right]R_t$$

$$\rho^*=\frac{R_t}{AK_t}=\frac{\gamma+\lambda-\left(1-p^{\frac{1}{\sigma}}\right)\left[1-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]+\frac{\delta}{A}-p^{\frac{1}{\sigma}}s}{1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)}.$$

$sA>\delta$ is the condition for the AK economy to increase its capital stock, and hence to grow, over time. Note that $(sA-\delta)>0$ and $r-(sA-\delta)>0$ by assumption – and, then $0 < \lambda \frac{r-(sA-\delta)}{1+(sA-\delta)} < 1$ so that $\left(1 - \lambda \frac{r-(sA-\delta)}{1+(sA-\delta)}\right) > 0$ and $\rho^* > 0$ as long as $\gamma + \lambda + \frac{\delta}{A} > \left(1 - p^{\frac{1}{\sigma}}\right)\left(1 - \lambda \frac{r-(sA-\delta)}{1+(sA-\delta)}\right) + p^{\frac{1}{\sigma}}s]$

We can manipulate our equation (31) in order to get equation (32) as follows:

$$\rho^* = \frac{\gamma + \lambda - \left(1 - p^{\frac{1}{\sigma}}\right)\left[1 - \lambda \frac{r - (sA - \delta)}{1 + (sA - \delta)}\right] + \frac{\delta}{A} - p^{\frac{1}{\sigma}}s}{1 - \frac{\pi}{\pi + p(1 - \pi)}\left(1 - p^{\frac{1}{\sigma}}\right)}$$

$$\rho^* \left(1 - \frac{\pi}{\pi+p(1-\pi)}\left(1 - p^{\frac{1}{\sigma}}\right)\right) = \gamma + \lambda - \left(1 - p^{\frac{1}{\sigma}}\right)\left[1 - \lambda \frac{1-(sA-\delta)}{1+(sA-\delta)}\right] + \frac{\delta}{A} - p^{\frac{1}{\sigma}}s$$

$$\rho^* - \rho^* \frac{\pi}{\pi+p(1-\pi)}\left(1 - p^{\frac{1}{\sigma}}\right) = \gamma + \lambda - \left(1 - p^{\frac{1}{\sigma}}\right)\left[1 - \lambda \frac{r-(sA-\delta)}{1+(sA-\delta)}\right] + \frac{\delta}{A} - p^{\frac{1}{\sigma}}s$$

$$\gamma + \lambda - \rho^* = \left(1 - p^{\frac{1}{\sigma}}\right)\left[1 - \lambda \frac{r-(sA-\delta)}{1+(sA-\delta)}\right] + \frac{\delta}{A} - p^{\frac{1}{\sigma}}s - \rho^* \frac{\pi}{\pi+p(1-\pi)}\left(1 - p^{\frac{1}{\sigma}}\right)$$

$$\begin{aligned} &\gamma + \lambda - \rho^* \\ &= \left(1 - p^{\frac{1}{\sigma}}\right)\left[1 - \lambda \frac{r - (sA - \delta)}{1 + (sA - \delta)}\right] + \frac{\delta}{A} - p^{\frac{1}{\sigma}}s \\ &- (\frac{\gamma + \lambda - \left(1 - p^{\frac{1}{\sigma}}\right)\left[1 - \lambda \frac{r - (sA - \delta)}{1 + (sA - \delta)}\right] + \frac{\delta}{A} - p^{\frac{1}{\sigma}}s}{1 - \frac{\pi}{\pi + p(1 - \pi)}\left(1 - p^{\frac{1}{\sigma}}\right)}) \frac{\pi}{\pi + p(1 - \pi)}\left(1 - p^{\frac{1}{\sigma}}\right) \end{aligned}$$

$$\gamma + \lambda - \rho^*$$

$$= \frac{\left(\left(1 - p^{\frac{1}{\sigma}}\right)\left[1 - \lambda \frac{r - (sA - \delta)}{1 + (sA - \delta)}\right] + \frac{\delta}{A} - p^{\frac{1}{\sigma}} s\right)}{1 - \frac{\pi}{\pi + p(1 - \pi)}\left(1 - p^{\frac{1}{\sigma}}\right)}\left(1 - \frac{\pi}{\pi + p(1 - \pi)}\left(1 - p^{\frac{1}{\sigma}}\right)\right)$$

$$- \left(\frac{\gamma + \lambda - \left(1 - p^{\frac{1}{\sigma}}\right)\left[1 - \lambda \frac{r - (sA - \delta)}{1 + (sA - \delta)}\right] + \frac{\delta}{A} - p^{\frac{1}{\sigma}} s}{1 - \frac{\pi}{\pi + p(1 - \pi)}\left(1 - p^{\frac{1}{\sigma}}\right)}\right)\frac{\pi}{\pi + p(1 - \pi)}\left(1 - p^{\frac{1}{\sigma}}\right)$$

$$\gamma + \lambda - \rho^* =$$

$$\frac{\left(\left(1-p^{\frac{1}{\sigma}}\right)\left[1-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]+\frac{\delta}{A}-p^{\frac{1}{\sigma}}s\right)\left(1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)\right)-\left(\gamma+\lambda-\left(1-p^{\frac{1}{\sigma}}\right)\left[1-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]+\frac{\delta}{A}-p^{\frac{1}{\sigma}}s\right)\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)}{1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)}$$

$$\gamma + \lambda - \rho^* =$$

$$\frac{\left(\left(1-p^{\frac{1}{\sigma}}\right)\left[1-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]+\frac{\delta}{A}-p^{\frac{1}{\sigma}}s\right)\left(1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)\right)-\left(\gamma+\lambda-\left(1-p^{\frac{1}{\sigma}}\right)\left[1-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]+\frac{\delta}{A}-p^{\frac{1}{\sigma}}s\right)\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)}{1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)}$$

$$=$$

$$\frac{\left(\left(1-p^{\frac{1}{\sigma}}\right)-\left(1-p^{\frac{1}{\sigma}}\right)\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}+\frac{\delta}{A}-p^{\frac{1}{\sigma}}s\right)\left(1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)\right)-\left((\gamma+\lambda)\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)-\left(\left(1-p^{\frac{1}{\sigma}}\right)\left[1-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]+\frac{\delta}{A}-p^{\frac{1}{\sigma}}s\right)\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)\right)}{1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)}$$

$$=$$

$$\frac{\left(\left(1-p^{\frac{1}{\sigma}}\right)\left(1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)\right)-\left(\left(1-p^{\frac{1}{\sigma}}\right)\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}+\frac{\delta}{A}-p^{\frac{1}{\sigma}}s\right)\left(1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)\right)\right)-\left((\gamma+\lambda)\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)-\left(\left(1-p^{\frac{1}{\sigma}}\right)\left[1-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]+\frac{\delta}{A}-p^{\frac{1}{\sigma}}s\right)\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)\right)}{1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)}$$

$$
= \frac{\left(1-p^{\frac{1}{\sigma}}\right)}{1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)}\left[1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}-\frac{\delta}{A}\right.
$$

$$
+p^{\frac{1}{\sigma}}s\left(1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)\right)-(\gamma+\lambda)\frac{\pi}{\pi+p(1-\pi)}
$$

$$
\left.+\left(\left[1-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]+\frac{\delta}{A}-p^{\frac{1}{\sigma}}s\right)\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)\right]
$$

$$
\left[1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}-\frac{\delta}{A}\right.
$$

$$
+p^{\frac{1}{\sigma}}s\left(1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)\right)-(\gamma+\lambda)\frac{\pi}{\pi+p(1-\pi)}
$$

$$
+\left(\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)\right)\left[-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]+\frac{\delta}{A}
$$

$$
\left.-p^{\frac{1}{\sigma}}s\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)\right]
$$

$$
1-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}-\frac{\delta}{A}+p^{\frac{1}{\sigma}}s+\left(\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}-\frac{\delta}{A}+p^{\frac{1}{\sigma}}s\right)\left(\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)\right)-
$$

$$
(\gamma+\lambda)\frac{\pi}{\pi+p(1-\pi)}+\left[-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}\right]+\frac{\delta}{A}-p^{\frac{1}{\sigma}}s\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)
$$

$$
\gamma+\lambda-\rho^{*}=\frac{\left(1-p^{\frac{1}{\sigma}}\right)}{1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)}\left[1-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}-\frac{\delta}{A}+p^{\frac{1}{\sigma}}s-(\gamma+\lambda)\frac{\pi}{\pi+p(1-\pi)}\right]
$$

$$
\left[1-\lambda\frac{r-(sA-\delta)}{1+(sA-\delta)}-\frac{\delta}{A}+p^{\frac{1}{\sigma}}s-(\gamma+\lambda)\frac{\pi}{\pi+p(1-\pi)}+(\gamma+\lambda)-(\gamma+\lambda)\right]
$$

$$\left[1-\lambda\left(1-\frac{r-(sA-\delta)}{1+(sA-\delta)}\right)-\frac{\delta}{A}+p^{\frac{1}{\sigma}}s+(\gamma+\lambda)\left(1-\frac{\pi}{\pi+p(1-\pi)}\right)-\gamma\right]$$

$$\left[1-\lambda\left(\frac{1-(sA-\delta)+r+(sA-\delta)}{1+(sA-\delta)}\right)-\frac{\delta}{A}+p^{\frac{1}{\sigma}}s+(\gamma+\lambda)\left(\frac{\pi+p(1-\pi)-\pi}{\pi+p(1-\pi)}\right)\right.$$
$$\left.-\gamma\right]$$

$$\left[1-\lambda\left(\frac{1+r}{1+(sA-\delta)}\right)-\frac{\delta}{A}+p^{\frac{1}{\sigma}}s+(\gamma+\lambda)\left(\frac{p(1-\pi)}{\pi+p(1-\pi)}\right)-\gamma\right]$$

$$\gamma+\lambda-\rho^{*}=\frac{\left(1-p^{\frac{1}{\sigma}}\right)}{1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)}\left[1-\alpha-\gamma-\frac{\delta}{A}+p^{\frac{1}{\sigma}}s\right.$$
$$\left.+(\gamma+\lambda)\left(\frac{p(1-\pi)}{\pi+p(1-\pi)}\right)\right]$$

# Chapter 2: The Optimal Level of International Reserves in a Production Small Open Economy: Cobb-Douglas Model


### Abstract

This chapter revisits the role of investment and production on the optimal level of international reserves in terms of output for small open economies (SOEs), now using a more general production function in order to incorporate labour input and population growth in the model of the preceding chapter. In particular, consistent with neoclassical growth theory, we employ a labour-augmenting constant returns to scale (CRS) Cobb-Douglas (CD) production function. We find, as in chapter 1, that the optimal reserve-to-GDP formula decreases with the investment rate and the growth rate of capital and increases with the depreciation rate but now, along the balanced growth path (BGP) in chapter 2, it also increases with population growth and the capital-labour ratio and decreases with labour productivity and the capital share in output. Depending on the calibration of the labour-augmenting productivity parameter, the CRS CD model quantifies the optimal ratio of reserves to output at 5.5% in the richer two-factor production SOE model, i.e., roughly three times higher than the AK model of chapter 1 but at the same time 60% lower than the endowment SOE model of Jeanne and Rancière (2011). As we suggested in chapter 1 already, with the capital stock now accumulated via investment and potentially used as a pledge to external creditors in obtaining borrowing and therefore insuring better against sudden stops, the optimal reserve-to-output ratio is lower relative to an otherwise similar endowment economy in the endogenous growth AK model. Differently from chapter 1, however, adding labour and constant population growth, consistent with a long-run BGP in neoclassical models of exogenous growth and with sustained per capita income growth, results in a lower per capita pledge and, hence, a higher optimal reserve-to-output ratio in the CD model relative to the AK model.



**Keywords:** International Reserves, Sudden Stops, Labour-Augmenting Productivity, Investment, Cobb-Douglas Technology

**JEL Classification:** F31, F32, F33, F41

## 2.1 Introduction

The role of investment in determining the optimal level of reserves for a small open production economy has been discussed in the preceding chapter in an AK model version. In it, capital was the only factor of production and, by assumption, technology enhanced the productivity capital. However, we have not considered yet the role of labour as a second production input, labour-augmenting technology and population growth in determining optimal reserves-to-GDP in SOEs. Therefore, the aim of this chapter is to extend the preceding one by investigating the importance of these additional key macro-variables typical in neoclassical growth theory relative to that of the established determinants of optimal reserves in the AK production model and the JR endowment benchmark.

We derive a reserve-to-GDP ratio in a two-factor production model where labour is also included, and we refer to this version as the two-factor production SOE Cobb-Douglas (CD) model with labour-augmenting technological progress and exogenous population growth, or simply the CD model (of exogenous growth); in turn, this version implies constant returns to scale (CRS) and is justified on the grounds of being consistent with a long-run balanced growth path (BGP) in exogenous growth models (Acemoglu, 2009, p. 59) and with sustained per capita income growth in these models (Jones and Vollrath, 2013, pp. 36-37).

More precisely, in chapter 2 we switch to a labour-augmenting CD production function relative to the AK production function in chapter 1 in order to compare our results across this alternative technology specification typical in neoclassical growth theory. The CD model in the present chapter embodies as well an alternative assumption of CRS, but with diminishing returns to scale (DRS) for each of the two factors, capital and labour, convergence to a balanced growth path (BGP) in the long run, with 'catching up' of countries where capital-to-output is initially relatively low. This version of the model thus focuses on the effects of labour-augmenting productivity and population growth on the optimal reserves-to-output ratio in a production SOE.

We find, as in chapter 1, that the optimal reserve-to-GDP formula decreases with the investment rate and the growth rate of capital and increases with the depreciation rate. In addition, along the BGP in chapter 2, it also increases with population growth and the capital-labour ratio and decreases with labour productivity and the capital share in output. Depending on the calibration of the labour-augmenting productivity parameter, the CRS CD model quantifies the optimal ratio of reserves to output at 5.5% in the richer two-factor production SOE model, i.e., roughly three times higher than the AK model of chapter 1 but at the same time 60% lower than the JR endowment SOE model. As we argued in chapter 1, with the capital stock now accumulated via investment and potentially used as a pledge to external creditors in obtaining borrowing and therefore insuring better against sudden stops, the optimal reserve-to-output ratio is lower relative to an otherwise similar endowment economy in the endogenous growth AK model. Differently from chapter 1, however, adding labour and constant population growth, consistent with a long-run BGP in neoclassical models of exogenous growth and with sustained per capita income growth, results in a lower per capita pledge and, hence, a higher optimal reserve-to-output ratio in the CD model relative to the AK model.

To be able to make a consistent comparison between the three theoretical results deriving the optimal ratio of international reserves to output, the JR endowment SOE model, our AK IRS endogenous growth SOE model in chapter 1 and our CD CRS exogenous growth SOE model of chapter 2), we follow most of the assumptions in the JR benchmark and in our AK technology extension. Consequently, we use same dataset as in the first chapter.

The rest of the paper is structured as follows. Section 2.2 presents our extension to the JR endowment economy and the AK IRS model for a production economy to a CD CRS SOE model version. The results of the calibration are provided in section 2.3. Section 2.4 concludes.

## 2.2 Optimal Level of Reserves with Deterministic CRS Labour-Augmenting Technology: Balanced Growth Path

In the previous chapter, labour was normalised and productive capital was the only variable of interest, featuring the AK model as one of the prominent endogenous growth models in neoclassical theory. In this subsection, we also introduce labour and examine the effect of a more general production function on optimal reserve holdings under exogenous population growth, in the tradition of neoclassical growth theory. More precisely, we now employ a CRS labour-augmenting CD production function. We employ this particular production function rather than the common alternatives such as Hicks-neutral technology and Solow-neutral technology, because the Harrod-neutral technology we choose is the only one that is consistent with a solution for the BGP in the long run (Acemoglu, 2009, p. 59) and with sustained per capita income growth in these models (Jones and Vollrath, 2013, pp. 36-37). It also allows us to check robustness of the results we obtained for the production SOE under AK technology implying IRS and perpetual endogenous growth with those under an alternative CD technology implying CRS and exogenous BGP.

All assumptions of the AK model hold through, except that the production function hereafter takes a different form. The latter requires a few additional assumptions, which are first discussed. Then, a corresponding formula for the optimal level of international reserves in terms of GDP is derived within the CD model version of our production SOE.

### 2.2.1 Assumptions under Labour-Augmenting CD Technology

This model version differs from the one with AK technology in that it introduces labour, as a second factor of production. Assuming that all firms in this economy have an identical production function for the final goods, the aggregate production function is then[17]

$$Y_t = F(K_t, AN_t) = K_t^{\theta}(AN)_t^{1-\theta}. \tag{1}$$

$Y_t$ is the total amount of output of the final good at time $t$, $K_t$ is the capital stock, $N_t$ is total employment and $A$ is now a parameter interpreted in the neoclassical tradition as labour-augmenting technology. As is conventional with CD production, $0 < \theta < 1$ measures the contribution of the capital stock to output and is proxied by the capital share in national income; $1 - \theta$ then measures the contribution of labour services to output and is proxied by the labour share in national income; furthermore, CD typically assumed CRS, as implied by the domain of $\theta$. Following neoclassical growth theory and, more recently in similar model contexts Jeanne and Rancière (2011) and Gourinchas and Jeanne (2013), we also assume: (i) a unitary labour force

[17] The same labour-augmenting CD production function is employed by Gourinchas and Jeanne (2013) in a similar set-up establishing what they term 'the allocation puzzle' in capital flows to developing countries.

participation rate, i.e., $N_t$ denotes both the size of the population in period $t$ as well as the size of workers; (ii) perfect competition in factor markets implying that each factor of production is paid its marginal product.

The well-known standard features of this production function are assumed: continuity, twice-differentiability with respect to each argument, positive diminishing returns to each factor and constant returns to scale to both factors – see, e.g., Acemoglu (2009), p. 29, Assumption 1.

Introducing labour, $N_t$, into the production SOE model requires a description of population growth, assumed to be exogenously given, as in the neoclassical growth theory and, more recently, in Gourinchas and Jeanne (2013),

$$\frac{\Delta N_{t+1}}{N_t} = g_N \; ; \; N_t = (1 + g_N)^t N_o, \tag{2}$$

where $N_o$ is the population level in a base period and $g_N$ is the constant population net growth rate.

There is no change in the definition of the budget constraint relative to chapter 1, but domestic output is now replaced by equation (1). As in neoclassical growth theory, we can express the production function in % terms, that is, in growth rates, by taking natural logarithms from both sides in period $t$ and *t-1*, and then subtracting to form the respective first log-differences:

$$lnY_t = (1-\theta)lnA + lnN_t + \theta lnK_t - \theta lnN_t \tag{3}$$

$$g_Y = (1-\theta)g_N + \theta g_K$$

which decomposes the growth rate of output, $g_Y$, as a weighted average of the growth rates of the population, $g_N$, and the capital stock, $g_K$, with the weights defined by the respective contributions of the two productive factors used as inputs, $\theta$ for capital and $1-\theta$ for labour, to final output.

As in neoclassical growth theory, along a balanced growth path (BGP) for the population and the capital stock, defined as standard by $g_N = g_K$ so that $g_Y = g_N = g_K$ too, and thus $k_t = \frac{K_t}{N_t} = k = const$, hence, $k_t = k$ is a steady state (SS) for $k_t$ or, equivalently, $g_k = 0$. Assuming again that the saving-to-output ratio is constant, *s*, we can now define the rate of growth of capital per capita,

$$g_k = \frac{\Delta k_{t+1}}{k_t} = \frac{\Delta K_{t+1}}{K_t} - \frac{\Delta N_{t+1}}{N_t} = s\frac{y_t}{k_t} - \delta - g_N = s\frac{Y_t}{K_t} - \delta - g_N = 0 \qquad (4)$$

and which – according to neoclassical growth theory (see, e.g., Jones and Vollrath, 2013, p. 28) – implies capital *widening*: namely, capital per worker does not change, $g_k = 0$, but the capital stock grows at the *same* rate as the population,

$$g_K = s\frac{y_t}{k_t} - \delta = s\frac{Y_t}{K_t} - \delta = g_N.$$

If – by contrast – we allow for growth in the capital-labour ratio, $g_k = \frac{\Delta k_{t+1}}{k_t} > 0$, denoted in the literature as capital *deepening*, then

$$g_k = \frac{\Delta k_{t+1}}{k_t} = \frac{\Delta K_{t+1}}{K_t} - \frac{\Delta N_{t+1}}{N_t} = s\frac{y_t}{k_t} - \delta - g_N > 0$$

Note that

$$g_k = \frac{\Delta k_{t+1}}{k_t} = \frac{\Delta K_{t+1}}{K_t} - \frac{\Delta N_{t+1}}{N_t} = g_K - g_N,$$

so that

$$g_K = g_k + g_N = s\frac{Y_t}{K_t} - \delta - g_N + g_N = s\frac{Y_t}{K_t} - \delta > g_N.$$

In the *normal* state, we assume that $g_K$ is (marginally) higher than $g_N$ so that $g_k$ grows but (very) slowly (and therefore the capital-labour ratio does not explode in a longer-run perspective – due to the deterministic nature of the model in this simplest version). That is:

$$g_k = \frac{\Delta k_{t+1}}{k_t} > 0;$$

then, we can rewrite equation (3),

$$g_Y = dlnY_t = (1-\theta)g_N + \theta\left(s\frac{Y_t}{K_t} - \delta\right) \tag{5}$$

As assumed, domestic private sector saving occurs through investment in physical capital and is a constant fraction of output in the normal state:

$$s = \frac{S_t}{Y_t} = \frac{I_t}{Y_t} \tag{6}$$

Then, as in neoclassical growth theory (see Appendix 2.B), we can re-write equation (4) as,

$$\frac{K_t}{Y_t} = \frac{s}{g_K + \delta + g_N} = const\ (along\ BGP) \tag{7}$$

Equation (7) implies that the capital-output ratio is constant along the BGP and it equals the domestic investment rate, $s$, over the sum of the growth rate of per capita capital, $g_k$, the depreciation rate, $\delta$, and the growth rate of labour, $g_N$. From equation (7), investment in normal times is,

$$I_t^n = sY_t = (g_K + \delta + g_N)K_t \tag{8}$$

If we replace output by the production function in order to see the effect of its components on the optimal level of reserves in normal times, we obtain

$$K_t^{\theta}(AN_t)^{1-\theta} = \frac{g_K + \delta + g_N}{s} K_t \tag{9}$$

where output is proportional to the capital stock.

As conventional in such model contexts, the economy follows a BGP, where all key variables grow at the same rate $g$,

$$g = g_K = g_N = g_A \tag{10}$$

Now the budget constraint of the consumers in normal times can be written as,

$$C_t^n = Y_t^n(\cdot) - sY_t^n(\cdot) + L_t^n - (1+r)L_{t-1}^n + Z_t^n. \tag{11}$$

where $L_t^n$ and $Z_t^n$ denote external debt and government net transfers, respectively, as in chapter 1.

We assume that the capital-labour ratio (or per capita capital) does not grow in sudden stops, equivalent to writing:

$$k_{t+1}^s = k_t^s$$

$$g_k = \frac{\Delta k_{t+1}}{k_t} = 0 \text{ results in } s\frac{y_t}{k_t} - \delta - g_N = 0,$$

so that investment in the CD model is given by,

$$I_t^s = sY_t = (\delta + g_N)K_t, \tag{12}$$

As in chapter 1, replacing investment, we can write the sudden-stop budget constraint of the private sector as,

$$C_t^s = (1-\gamma)Y_t^n(\cdot) - (\delta + g_N)K_t + L_t^s - (1+r)L_{t-1}^s + Z_t^s. \tag{13}$$

Following Jeanne and Rancière (2011), in this subsection, the economy can be again in two states, normal times and crisis state, as in the previous chapter. So, the assumptions for output remain valid, $F(K_t)^{\,n} = (1+g)^t F(K_{t-1})$ and $F(K_t)^{\,s} = (1-\gamma)F(K_t)^{\,n}$. The economy still follows the restriction for borrowing since there is no change on the pledgeable output, $(1+r)L_t \leq \alpha_t F(K_{t+1})^{\,n}$. The role of the monetary authority and the participation condition for external creditors are same as in the JR endowment SOE benchmark, i.e., $Z_t^n = -X_t$; and $Z_t^s = R_t - X_t$; and $\sum_{t=1}^{+\infty} \beta^t (1-\pi)^{t-1}[(1-\pi)X_t\mu_t^n - \pi(R_t - X_t)\mu_t^s] \geq 0$.

### 2.2.2 A Formula for the Optimal Level of Reserves under the CD Model

As in the JR model, we continue to assume that $(1+r)L_t \leq \alpha_t F(K_{t+1})^{\,n}$ is always binding, which allows for a closed-form solution of this simple insurance problem of the SOE, now with labour-augmenting CD production and exogenous population growth. However, the parameter $\lambda$, denoting as before the short term external debt (STED) to output ratio, now takes a slightly different form, namely

$$\lambda = \frac{L_t^n}{K_t^{\theta}(AN_t)^{1-\theta}} \equiv \frac{1 + [(1-\theta)g_N + \theta g_K]}{1+r}\,\alpha \tag{14}$$

since the country keeps a constant STED-to-output ratio when the external credit constraint is always binding. $\lambda$ denotes the same parameter as in the AK production SOE model version of chapter 1, but here in chapter 2 the definition of output in the denominator of (14) has changed, and therefore the expression for $\lambda$ too.

Using the CD production function, consumption in the non-crisis state can be written as

$$C_t^n = \left\{1 - s - \lambda \frac{r - [(1-\theta)g_N + \theta g_K]}{1 + [(1-\theta)g_N + \theta g_K]}\right\} K_t^{\theta}(AN_t)^{1-\theta} - X_t \tag{15}$$

By analogy, consumption in the sudden stop episode can be written as,

$$C_t^s = \left[-\gamma - \frac{(\delta + g_N)K_t}{K_t^{\theta}(AN_t)^{1-\theta}} - \lambda \frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\right] K_t^{\theta}(AN_t)^{1-\theta} + R_t - X_t \tag{16}$$

Then, as in JR and chapter 1, we set a similar Lagrangian optimisation problem[18]; since there is no change in the role of the monetary authority, it enters a reserve insurance contract as described above in order to maximize the private sector's utility subject to the constraints (8), (11), (12), (13), the external borrowing constraint and the external creditors' participation condition.

The optimal reserves-to-output ratio, $\rho^*$, is then constant, as in chapter 1, but is now given by a different expression

$$\rho^* = \frac{R_t}{K_t^{\theta}(AN_t)^{1-\theta}} = \frac{\gamma + \lambda - \left(1 - \lambda \frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\right)\left(1 - p^{\frac{1}{\sigma}}\right) + (\delta + g_N)\left(\frac{k}{A}\right)^{1-\theta} - p^{\frac{1}{\sigma}} s}{1 - \frac{\pi}{\pi + p(1-\pi)}\left(1 - p^{\frac{1}{\sigma}}\right)} \tag{17}$$

As formula (17) demonstrates, the optimal level of reserves in terms of GDP with deterministic CD CRS production function has many common determinants with

[18] Hence; this results in a similar first order condition, $u'(C_t^n) = pu'(C_t^s)$, which implies that domestic consumption can be substituted at the same rate between normal times and sudden stop episodes by the private sector and external creditors as in the first chapter. Similarly, this model also yields the same expression for government net transfers, $X_t = \frac{\pi}{\pi + p(1-\pi)} R_t$.

the AK model in chapter 1, such as: $\gamma$ is the output loss in the first period of capital outflows, $\lambda$ is the STED-to-output ratio, $p$ is the price ratio of funds in different states (normal times and sudden stop episodes), $r$ is the world interest rate, $s$ is the constant investment rate of the domestic SOE, $\delta$ is the depreciation rate of physical capital, $\pi$ is the crisis probability, and $\sigma$ is the coefficient of relative risk aversion (CRRA). However, differently from the analogous optimal reserves-to-GDP formula in chapter 1, several additional determinants enter into consideration, such as: the (net) growth rate of labour, $g_N$; the (net) growth rate of the capital stock, $g_K$; the capital-labour ratio, $k$; the capital share in income, $\theta$; and the technology level of the CD economy[19] where $A$ is now labour-augmenting technology.

**Theorem:** Given that the capital share of income is between zero and one,$0 < \theta < 1$, the partial derivative of reserve-to-output ratio with respect to the capital share of income of less than zero.

$$\frac{\partial \rho^*}{\partial \theta} < 0$$

**Proof:** Take the first derivative of equation (17) with respect to capital share of income, $\theta$ which gives,

$$\frac{\partial \rho^*}{\partial \theta} = \frac{-\left(1-p^{\frac{1}{\sigma}}\right)\left[\left(\lambda\frac{(g_K-g_N)(r-(1-\theta)g_N-\theta g_K)}{(1+(1-\theta)g_N+\theta g_K)^2}\right)+\left(\lambda\frac{(g_K-g_N)}{1+(1-\theta)g_N+\theta g_K}\right)\right]-\left(\frac{k}{A}\right)^{1-\theta}(\delta+g_N)\ln\left(\frac{k}{A}\right)}{1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)}$$

Given that the sign of numerator is negative, then the sign of partial derivative is negative.

[19] We use $A_N$ for the CD CRS exogenous growth model version in the present chapter and $A_K$ for the AK IRS endogenous growth model in the preceding chapter when we compare the two theoretically derived expressions for the optimal reserves-to-output ratio.

Equation (17) implies some determinants of optimal reserves in terms of GDP that are the same as those in the AK production model version[20] discussed in chapter 1. For example, the optimal reserve ratio is a positive function of: (i) the output cost of a sudden stop, $\gamma$; (ii) the level of short term debt, $\lambda$; (iii) the probability of a sudden stop, $\pi$; (iv) the depreciation rate, $\delta$; (v) the world interest rate, $r$; and (vi) risk aversion, $\sigma$; whereas it is a negative function of the investment rate of the economy, $s$.

Differently from the AK model, the additional determinants in this CD production version influence the optimal reserves-to-output ratio as follows: (i) population growth, $g_N$, positively, as expected[21]; (ii) the capital share in income, $\theta$, negatively; (iii) the capital-labour ratio, $k$, positively; (iv) labour-augmenting productivity, $A_N$, negatively; (v) the growth rate of capital, $g_k$, negatively.

The differences in the AK IRS model of endogenous growth versus the CD CRS model of exogenous growth arise from the modelling of investment and production functions. A first difference is the growth rate of the economy. Unlike the JR endowment SOE model, in both the previous chapter 1 and present chapter 2 we analyse components of the growth rate of the economy rather than a simple parameter $g$. However, our two production SOE versions implied a different relation between the growth rate of the economy and the component growth rates: (i) $sA - \delta$ in the AK model and (ii) $(1-\theta)g_N + \theta g_K$ in the CD model. Consequently, we need the growth rate of labour, $g_N$, and physical capital, $g_K$, in the CD model of chapter 2. Secondly, the CD model assumes production in a richer context than the AK model since it includes labour input and population growth. Therefore, we have the additional determinants of optimal reserves-to-output: the capital-labour ratio, $k$ and the capital

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[20] And also with the original JR endowment SOE model.

[21] The earlier literature on the optimal level of reserves assumes that population is a scale variable and shows a positive relationship with the level of reserves – see, e.g., Heller (1966), Clark (1970), Kelly (1970), Hamada and Ueda (1977), Frenkel and Jovanovic (1981).

share in income, $\theta$. A final difference relates to the definition of productivity, $A$. In each model version, AK and CD, productivity (no matter whether it is capital productivity, $A_K$, or labour-augmenting productivity, $A_N$) decreases the reserve-to-GDP ratio. However, this difference leads to different productivity results in magnitude for the respective model versions of our production SOE. We discuss this in the calibration part. In Figure 2.2, we present the original JR formula together with our two extended versions for a clear analytical comparison.

In order to compare the CD model in this chapter and the JR endowment benchmark, the optimal reserve formula can be written as,

$$\gamma + \lambda - \rho^* = \frac{\left(1 - p^{\frac{1}{\sigma}}\right)}{1 - \frac{\pi}{\pi + p(1-\pi)}\left(1 - p^{\frac{1}{\sigma}}\right)}\left[1 - \alpha - \gamma - (\delta + g_N)\left(\frac{k}{A}\right)^{1-\theta} + p^{\frac{1}{\sigma}} s + (\gamma + \lambda)\left(\frac{p(1-\pi)}{\pi + p(1-\pi)}\right)\right] \quad (18)$$

It can easily be seen that if the terms $(\delta + g_N)\left(\frac{k}{A}\right)^{1-\theta} + p^{\frac{1}{\sigma}} s$ are ignored, the CD model would reduce to the JR model. Therefore, both models include many similar determinants, such as the terms in the LHS of equation (18), the case of $p = 1$, the role of risk aversion parameter, and the link to the Greenspan-Guidotti rule as discussed in the previous chapter.

## 2.3 Calibration

In order to make direct comparisons between the production SOE model versions and the JR endowment benchmark, we use the same dataset and the same calibration strategy as in chapter 1. The investment rate is somewhat different in the present chapter,

$$\underbrace{C_t^s}_{C_t} = \underbrace{(1-\gamma)Y_t^n}_{Y_t} - \left\{ \underbrace{(\delta + g_N)K_t}_{I_t} + \underbrace{(-L_{t-1})}_{FA_t} + \underbrace{[-rL_{t-1} - (\pi + \omega)R_t]}_{IT_t} - \underbrace{(-R_t)}_{\Delta R_t} \right. \quad (19)$$

While in the AK model investment in sudden stops was described as $\delta K_t$, it now includes dependence on labour via population growth in the CD version of the present chapter 2. Hence, investment in crisis episodes is now described as $(\delta + g_N)K_t$.

As in the first chapter and the JR model, we follow Guidotti *et al.* (2004) approach for sudden stops. Therefore, we have the same sudden stop years as in the previous chapter. Hence, we use Table 1.2 with the same dataset and same period coverage (34 middle income countries over 1975-2014).

Our calibration parameters are given in Table 2.1. Our CD model version has some common parameters with our AK model version, which are calibrated by reference to the same parameters as in the previous chapter. Following a similar calibration methodology as in chapter 1, we used our formulas (17), (18) and our benchmark equation (19) in this CD version of the production SOE model.

We use the calibration values from chapter 1 for the size of the sudden stop, $\lambda$; the crisis probability, $\pi$; the output loss, $\gamma$; GDP per worker, $y$; the investment rate of economy, $s$; the depreciation of physical capital[22], $\delta$; capital per worker, $k$. Moreover, we used the JR calibration results for the price of the non-crisis dollar, $p$; the risk-free world interest rate, $r$; and risk aversion, $\sigma$.

The CD model includes the technology parameter, $A$, which is calculated by the implied model assumptions and following again the methodology in Caselli (2005), as in chapter 1. To distinguish across the different definition and

[22] Following the calibration methodology in Caselli (2005), as described in chapter 1. This methodology to calibrate depreciation is also employed by Gourinchas and Jeanne (2013).

interpretation of the technology parameter $A$, we denote it henceforth by $A_N$ for the labour-augmenting technology in the CD CRS model. Following Caselli (2005), in calibrating $A_N$ we assume that $\left(\frac{y}{k^\theta}\right)^{\frac{1}{1-\theta}} = A_N$, where $y$ is GDP per worker and $k$ is capital per worker. As in chapter 1, the average GDP per worker is equal to \$15141 for our dataset from PWT (7.0) and capital per worker, $k$, is 2.49 times higher than GDP per worker. Then we calculate $A_N$ to be equal to 10241.

The average growth rate of population, $g_N$, is found to be 1.5% in our dataset, from 1975 to 2014. The capital share of income, $\theta$, is set to 0.3 (as in Gourinchas and Jeanne, 2013). For the 34 middle income countries in the JR and our sample, the average real GDP growth rate is 4% between 1975 and 2014. We used it in calculation of $g_K$ (which is equal to 9.8%), since $g_Y = (1-\theta)g_N + \theta g_K$.

Based on our optimal reserves formulas (equations (17) and (18)), our CD CRS model quantifies the optimal ratio of reserves to output at 5.5% in the richer two-factor production SOE model, i.e., roughly three times higher than the AK model of chapter 1 but at the same time 60% lower than the JR endowment SOE model. As we argued in chapter 1, with the capital stock now accumulated via investment and potentially used as a pledge to external creditors in obtaining borrowing and therefore insuring better against sudden stops, the optimal reserve-to-output ratio is lower relative to an otherwise similar endowment economy in the endogenous growth AK model. Differently from chapter 1, however, adding labour and constant population growth, consistent with a long-run BGP in neoclassical models of exogenous growth and with sustained per capita income growth, results in a lower per capita pledge and, hence, a higher optimal reserve-to-output ratio in the CD model relative to the AK model.

Figure 2.1 illustrates the relationship between the optimal level of international reserves and its determinants for the CD model.[23] Similarly to the corresponding Figure 1.3 in chapter 1, this figure also shows the sensitivity of our results to the key determinants of the optimal reserve-to-output ratio. It is based on the optimal reserve formula (17) and the reported calibrations. As can be seen in the respective panels of Figure 2.1, our results suggest that the optimal reserve-to-GDP ratio depends positively on some of its key determinants, as was in fact in chapter 1 and the JR endowment SOE benchmark, such as: (i) the size of the sudden stop; (ii) the output cost of a sudden stop; (iii) the probability of sudden stops; (iv) the world interest rate; (v) the coefficient of relative risk aversion; (vi) the depreciation rate of capital and – in our extension to a CD production in the present chapter 2 – also on (vii) population growth and (viii) the capital-labour ratio. On the other hand, and notably given the objective of the present chapter, our analysis revealed some novel results, as follows: the optimal reserve-to-GDP ratio depends negatively on: (i) the investment rate, as in chapter 1; and now on the additional determinants highlighted by the CD production SOE model of the present chapter 2, namely; (ii) labour-augmenting technology; (iii) the growth rate of capital; (iv) the capital share in output.

## 2.4 Concluding Remarks

This chapter revisits the role of investment and production on optimal international reserves in SOEs. We derive an optimal reserve-to-output ratio in a two-factor CD production model where labour and population growth is included as well as labour-augmenting productivity in addition to investment in physical capital. We, consequently, focus on the importance of these additional determinants of optimal

[23] A summary of the JR model and its extensions can be seen in Figure 2.2.

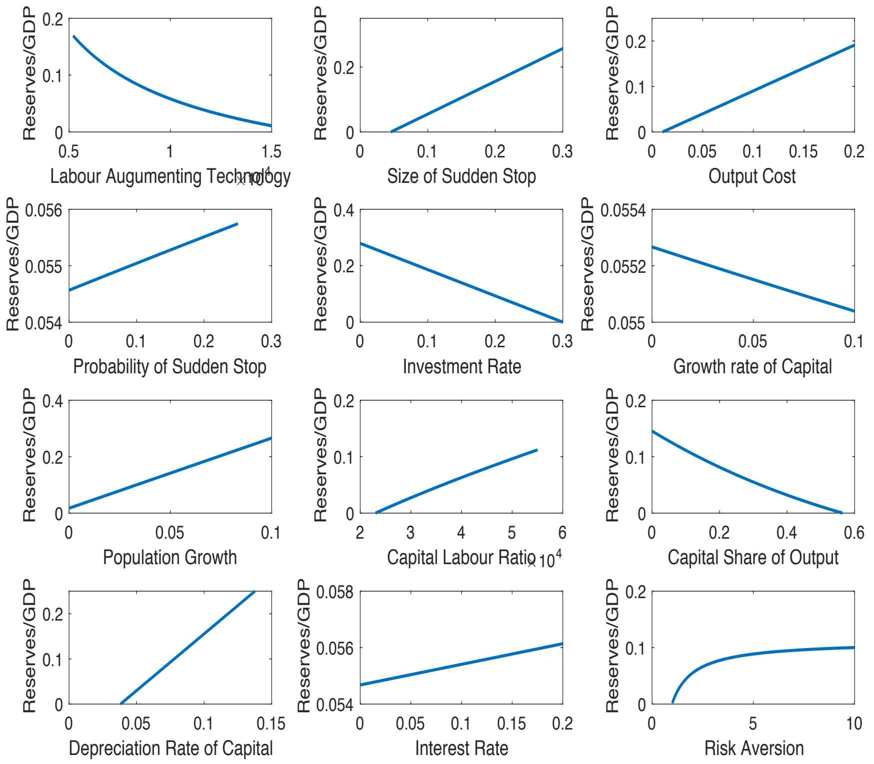
reserves, comparing them to AK technology model and the JR endowment benchmark.

As in chapter 1, the optimal reserve-to-output ratio decreases with the investment rate and the growth rate of capital but increases with the depreciation rate. However now, along the balanced growth path (BGP) in chapter 2, it also increases with population growth and the capital-labour ratio but decreases with labour productivity and the capital share in output. Given our plausible calibration of the labour-augmenting productivity parameter following the methodology proposed in Caselli (2005), the CRS CD model quantifies the optimal ratio of reserves to output at 5.5% in the richer two-factor production SOE model, i.e., roughly three times higher than the AK model of chapter 1. Yet, this value is at the same time 60% lower than that found in the endowment SOE model by Jeanne and Rancière (2011). Along the interpretation we proposed in chapter 1, our intuition here again is that with the capital stock now accumulated via investment and potentially used as a pledge to external creditors in obtaining borrowing and therefore insuring better against sudden stops, the optimal reserve-to-output ratio is lower relative to an otherwise similar endowment economy in the endogenous growth AK model. But now differently from chapter 1, further adding labour and constant population growth, consistent with a long-run BGP in neoclassical models of exogenous growth and with sustained per capita income growth, results in a lower per capita pledge and, hence, a higher optimal reserve-to-output ratio in the CD model relative to the AK model. This is, essentially, because capital accumulation has to make up for not only deprecyaition of capital but also population growth in the CD exogenous BGP model version of our production SOE relative to its AK endogenous perpetual growth model version.

# Appendix to Chapter 2

## 2.A.1 Figures

**Figure 2.1:** Optimal Reserves-to-GDP Ratio as a Function of Its Key Determinants in the CRS-Labour Augmented Cobb-Douglas Model




Source: Author's calculation, using data from IMF's *International Financial Statistics*, Penn World Table 7.0 and World Bank's *World Development Indicators*.

**Figure 2.2:** Optimal Reserves-to-Output Formula in the JR Endowment SOE Benchmark and in Our Extension to CD Production: Analytical Comparison

**<u>JR Endowment SOE Model</u>**

$$\rho^* = \frac{\gamma + \lambda - (1 - \frac{r-g}{1+g}\lambda)(1 - p^{\frac{1}{\sigma}})}{1 - \frac{\pi}{\pi + p(1-\pi)}(1 - p^{1/\sigma})}$$

**Our Extended SOE Model with <u>AK Production</u>**

$$\rho^* = \frac{R_t}{AK_t} = \frac{\gamma + \lambda - \left(1 - p^{\frac{1}{\sigma}}\right)\left[1 - \lambda \frac{r - (sA - \delta)}{1 + (sA - \delta)}\right] + \frac{\delta}{A} - p^{\frac{1}{\sigma}} s}{1 - \frac{\pi}{\pi + p(1-\pi)}\left(1 - p^{\frac{1}{\sigma}}\right)}$$

**<u>Our Extended SOE Model with Labour-Augmenting CD Production</u>**

$$\rho^* = \frac{R_t}{K_t^{\theta}(AN_t)^{1-\theta}} = \frac{\gamma + \lambda - \left(1 - \lambda \frac{r - [(1-\theta)g_N + \theta g_K]}{1 + [(1-\theta)g_N + \theta g_K]}\right)\left(1 - p^{\frac{1}{\sigma}}\right) + (\delta + g_N)\left(\frac{k}{A}\right)^{1-\theta} - p^{\frac{1}{\sigma}} s}{1 - \frac{\pi}{\pi + p(1-\pi)}\left(1 - p^{\frac{1}{\sigma}}\right)}$$

### 2.A.2 Tables

**Table 2.1:** Calibration Parameters in CD Model

| Parameters | CRS-Labour-Augmenting Cobb-Douglas | Range of Variation |
|---|---|---|
| Technology | $A_N = 10241$ | |
| Size of Sudden Stop | $\lambda = 0.10$ | [0, 0.30] |
| Probability of a Sudden Stop | $\pi = 0.10$ | [0, 0.24] |
| Output Loss | $\gamma = 0.0.65$ | [0,0.2] |
| Price of a Non-Crisis Dollar | $p = 0.855$ | |
| Potential Output Growth | $g = 0.04$ | [0, 0.25] |
| Depreciation Rate | $\delta = 0.06$ | [0, 1] |
| Risk Free Rate | $r = 0.05$ | |
| Risk Aversion | $\sigma = 2$ | [1, 10] |
| Growth Rate of Population | $g_N = 0.015$ | [0, 0.11] |
| Growth rate of Capital | $g_K = 0.098$ | |
| Capital-Labour Ratio | $k = 37701$ | |
| GDP per Worker | $y = 15141$ | |
| Capital Share of Output | $\theta = 0.3$ | [0,1] |
| Investment Rate | 0.24 | [0, 0.48] |

Source: Authors calculation, using data from International Financial Statistics, Penn World Table 7.0 and World Bank Development Indicators.

## 2.B Model Derivation

With the CD labour-augmenting technology, output is given by

$$Y_t = F(K_t, AN_t; \theta) = K_t^{\theta}(AN_t)^{1-\theta},$$

which can be written alternatively in terms of output per capita (or output per worker, provided the usual implicit assumption in neoclassical growth theory that the labour force participation rate is constant and unitary that we used in the main text), $y_t \equiv \frac{Y_t}{N_t}$, or also in terms of capital per worker or the capital-labour ratio, $k_t \equiv \frac{K_t}{N_t}$:

$$y_t = f(k_t, AN_t; \theta) = A^{1-\theta}\frac{K_t^{\theta}}{N_t^{\theta}} = A^{1-\theta}k_t^{\theta}.$$

In this model version, exogenous constant population growth at net rate $g_N$ is assumed:[24]

$$\frac{\Delta N_{t+1}}{N_t} = g_N \quad \text{and } N_t = (1+g_N)^t N_0.$$

To express the production function in % terms, that is, in growth rates, take natural logarithms from both sides in period *t* and *t-1*, and then subtract to form the respective first log-differences:

$$lnY_t = (1-\theta)lnA + lnN_t + \theta lnK_t - \theta lnN_t$$

[24] $N_0$ is the population level in some base period.

$$lnY_{t-1} = (1-\theta)lnA + lnN_{t-1} + \theta lnK_{t-1} - \theta lnN_{t-1}$$

$$lnY_t - lnY_{t-1} = (lnN_t - lnN_{t-1}) + \theta(lnK_t - lnK_{t-1}) - \theta(lnN_t - lnN_{t-1})$$

$$lnY_t - lnY_{t-1} = (1-\theta)(lnN_t - lnN_{t-1}) + \theta(lnK_t - lnK_{t-1})$$

$$dlnY_t = (1-\theta)dlnN_t + \theta dlnK_t$$

$$g_Y = (1-\theta)g_N + \theta g_K,$$

$$g_Y = dlnY_t = (1-\theta)g_N + \theta g_K$$

or, equivalently,

$$g_Y = dlnY_t = (1-\theta)g_N + \theta\left(s\frac{Y_t}{K_t} - \delta\right).$$

The 'normal time' budget constraint (superscript *n*) of the private sector is given by

$$C_t^n = Y_t^n(\cdot) - I_t^n + L_t^n - (1+r)L_{t-1}^n + Z_t^n,$$

and

$$I_t^n = sY_t^n(\cdot).$$

The rate of growth of capital per capita, $k_t = \frac{K_t}{N_t}$, is then

$$g_k = \frac{\Delta k_{t+1}}{k_t} = \frac{\Delta K_{t+1}}{K_t} - \frac{\Delta N_{t+1}}{N_t} = s\frac{y_t}{k_t} - \delta - g_N,$$

hence

$$\frac{y_t}{k_t} = \frac{\frac{Y_t}{N_t}}{\frac{K_t}{N_t}} = \frac{Y_t}{K_t} = \frac{g_k + \delta + g_N}{s}.$$

From the last equality above, investment in normal times is

$$sY_t = (g_k + \delta + g_N)K_t.$$

Note that the definition of STED now implies

$$\lambda \equiv \frac{1 + [(1-\theta)g_N + \theta g_K]}{1+r}\ \alpha.$$

Replacing investment, we can write the normal-time budget constraint of the private sector as

$$C_t^n = Y_t^n(\cdot) - sY_t^n(\cdot) + L_t^n - (1+r)L_{t-1}^n + Z_t^n.$$

With CRS Cobb-Douglas technology, output is given by $Y_t^n(\cdot) = F(K_t, AN_t; \theta) = K_t^\theta (AN_t)^{1-\theta}$ and using $L_t^n = \frac{1+[(1-\theta)g_N + \theta g_K]}{1+r}\alpha Y_t^n(\cdot) = \lambda Y_t^n(\cdot)$ and $Z_t^n = -X_t$, we further obtain (successively):

$$C_t^n = K_t^\theta (AN_t)^{1-\theta} - sK_t^\theta (AN_t)^{1-\theta} + \frac{1 + [(1-\theta)g_N + \theta g_K]}{1+r}\alpha K_t^\theta (AN_t)^{1-\theta}$$

$$-\{1+[(1-\theta)g_N+\theta g_K]\}\alpha K_{t-1}^{\theta}(AN_{t-1})^{1-\theta}-X_t$$

$$C_t^n=K_t^{\theta}(AN_t)^{1-\theta}-sK_t^{\theta}(AN_t)^{1-\theta}+\lambda K_t^{\theta}(AN_t)^{1-\theta}-\alpha K_t^{\theta}(AN_t)^{1-\theta}-X_t$$

$$C_t^n=K_t^{\theta}(AN_t)^{1-\theta}-sK_t^{\theta}(AN_t)^{1-\theta}+\lambda K_t^{\theta}(AN_t)^{1-\theta}$$
$$-\lambda\frac{1+r}{1+[(1-\theta)g_N+\theta g_K]}K_t^{\theta}(AN_t)^{1-\theta}-X_t$$

$$C_t^n=\left[1-s+\lambda\left(1-\frac{1+r}{1+[(1-\theta)g_N+\theta g_K]}\right)\right]K_t^{\theta}(AN_t)^{1-\theta}-X_t$$

$$C_t^n=\left\{1-s+\lambda\frac{[(1-\theta)g_N+\theta g_K]-r}{1+[(1-\theta)g_N+\theta g_K]}\right\}K_t^{\theta}(AN_t)^{1-\theta}-X_t$$

$$C_t^n=\left\{1-s-\lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\right\}K_t^{\theta}(AN_t)^{1-\theta}-X_t.$$

The ‘sudden stop’ budget constraint (superscript *s*) of the private sector is given by

$$C_t^s=(1-\gamma)Y_t^n(\cdot)-I_t^s-(1+r)L_{t-1}^s+Z_t^s,$$

and we assume that the capital-labour ratio (or per capita capital) does not grow in sudden stops, equivalent to writing:

$$k_{t+1}^s=k_t^s$$

$g_k=\frac{\Delta k_{t+1}}{k_t}=0$ results in $s\frac{y_t}{k_t}-\delta-g_N=0$,

$$\text{that is, } s\frac{\frac{Y_t}{N_t}}{\frac{K_t}{N_t}} = \delta + g_N$$

$$s\frac{Y_t}{K_t} = \delta + g_N$$

so that

$$sY_t = (\delta + g_N)K_t,$$

which implies, in the CD model,

$$I_t^s = (\delta + g_N)K_t.$$

Replacing investment, we can write the sudden-stop budget constraint of the private sector as

$$C_t^s = (1-\gamma)Y_t^n(\cdot) - (\delta + g_N)K_t + L_t^s - (1+r)L_{t-1}^s + Z_t^s.$$

With CRS Cobb-Douglas technology, output is given by $Y_t^n(\cdot) = F(K_t, AN_t; \theta) = K_t^\theta (AN_t)^{1-\theta}$ and using $L_t^s = 0$, $L_{t-1}^s = \frac{1+[(1-\theta)g_N + \theta g_K]}{1+r}\alpha Y_{t-1}^n(\cdot) = \lambda Y_{t-1}^n(\cdot)$ and $Z_t^s = R_t - X_t$, we further obtain (successively):

$$C_t^s = (1-\gamma)K_t^\theta (AN_t)^{1-\theta} - (\delta + g_N)K_t - (1 + [(1-\theta)g_N + \theta g_K])\alpha K_{t-1}^\theta (AN_{t-1})^{1-\theta} + R_t - X_t$$

$$C_t^s = (1-\gamma)K_t^\theta(AN_t)^{1-\theta} - (\delta + g_N)K_t - \alpha K_t^\theta(AN_t)^{1-\theta} + R_t - X_t$$

$$C_t^s = (1-\gamma)K_t^\theta(AN_t)^{1-\theta} - (\delta + g_N)K_t - \frac{1+r}{1+[(1-\theta)g_N + \theta g_K]}\lambda\, K_t^\theta(AN_t)^{1-\theta} + R_t - X_t$$

$$C_t^s = \left[-\gamma - \frac{(\delta + g_N)K_t}{K_t^\theta(AN_t)^{1-\theta}} + 1 - \frac{1+r}{1+[(1-\theta)g_N + \theta g_K]}\lambda\right]K_t^\theta(AN_t)^{1-\theta} + R_t - X_t$$

$$C_t^s = \left[-\gamma - \frac{(\delta + g_N)K_t}{K_t^\theta(AN_t)^{1-\theta}} + \frac{1+[(1-\theta)g_N + \theta g_K] - 1 - r}{1+[(1-\theta)g_N + \theta g_K]}\lambda\right]K_t^\theta(AN_t)^{1-\theta} + R_t - X_t$$

$$C_t^s = \left[-\gamma - \frac{(\delta + g_N)K_t}{K_t^\theta(AN_t)^{1-\theta}} + \frac{[(1-\theta)g_N + \theta g_K] - r}{1+[(1-\theta)g_N + \theta g_K]}\lambda\right]K_t^\theta(AN_t)^{1-\theta} + R_t - X_t$$

$$C_t^s = \left[-\gamma - \frac{(\delta + g_N)K_t}{K_t^\theta(AN_t)^{1-\theta}} - \lambda\frac{r - [(1-\theta)g_N + \theta g_K]}{1+[(1-\theta)g_N + \theta g_K]}\right]K_t^\theta(AN_t)^{1-\theta} + R_t - X_t$$

Therefore, from first order conditions, the optimal level of reserves as a ratio of output can be expressed as:

$$p^{\frac{1}{\sigma}}\left\{\left[1 - s - \lambda\frac{r - [(1-\theta)g_N + \theta g_K]}{1+[(1-\theta)g_N + \theta g_K]}\right]K_t^\theta(AN_t)^{1-\theta} - X_t\right\} = \left[-\gamma - \frac{(\delta + g_N)K_t}{K_t^\theta(AN_t)^{1-\theta}} - \lambda\frac{r - [(1-\theta)g_N + \theta g_K]}{1+[(1-\theta)g_N + \theta g_K]}\right]K_t^\theta(AN_t)^{1-\theta} + R_t - X_t$$

$$p^{\frac{1}{\sigma}}\left[1-s-\lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\right]K_t^{\theta}(AN_t)^{1-\theta}$$

$$-\left[-\gamma-\frac{(\delta+g_N)K_t}{K_t^{\theta}(AN_t)^{1-\theta}}-\lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\right]K_t^{\theta}(AN_t)^{1-\theta}$$

$$=R_t-X_t+p^{\frac{1}{\sigma}}X_t$$

$$\left[p^{\frac{1}{\sigma}}(1-s)-p^{\frac{1}{\sigma}}\lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}+\gamma+\frac{(\delta+g_N)K_t}{K_t^{\theta}(AN_t)^{1-\theta}}\right.$$

$$\left.+\lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\right]K_t^{\theta}(AN_t)^{1-\theta}=R_t-\left(1-p^{\frac{1}{\sigma}}\right)X_t$$

$$\left[p^{\frac{1}{\sigma}}(1-s)+\left(1-p^{\frac{1}{\sigma}}\right)\lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}+\gamma+(\delta\right.$$

$$\left.+g_N)\left(\frac{K_t}{AN_t}\right)^{1-\theta}\right]K_t^{\theta}(AN_t)^{1-\theta}=R_t-\left(1-p^{\frac{1}{\sigma}}\right)\frac{\pi}{\pi+p(1-\pi)}R_t$$

$$\left[p^{\frac{1}{\sigma}}(1-s)+\left(1-p^{\frac{1}{\sigma}}\right)\lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}+\gamma+(\delta\right.$$

$$\left.+g_N)\left(\frac{k_t^S}{A}\right)^{1-\theta}\right]K_t^{\theta}(AN_t)^{1-\theta}=\left[1-\left(1-p^{\frac{1}{\sigma}}\right)\frac{\pi}{\pi+p(1-\pi)}\right]R_t$$

And, finally, we obtain the optimal reserves-to-output ratio, $\rho^*$, under the CD technology case in the production SOE we analysed:

$$\rho^* = \frac{R_t}{K_t^{\theta}(AN_t)^{1-\theta}}$$

$$= \frac{\gamma + \lambda \frac{r - [(1-\theta)g_N + \theta g_K]}{1 + [(1-\theta)g_N + \theta g_K]}\left(1 - p^{\frac{1}{\sigma}}\right) + p^{\frac{1}{\sigma}}(1-s) + (\delta + g_N)\left(\frac{k}{A}\right)^{1-\theta}}{1 - \frac{\pi}{\pi + p(1-\pi)}\left(1 - p^{\frac{1}{\sigma}}\right)},$$

An alternative equivalent expression where the parameter $\lambda$ appears also additively as in the original JR optimal reserves expression can be obtained, as follows.

Since,

$$C_t^s = (1-\gamma)K_t^{\theta}(AN_t)^{1-\theta} - (\delta + g_N)K_t - \frac{1+r}{1+[(1-\theta)g_N+\theta g_K]}\lambda\, K_t^{\theta}(AN_t)^{1-\theta} + R_t - X_t,$$

we could write

$$C_t^s = \left[(1-\gamma) - (\delta + g_N)\left(\frac{K_t}{AN_t}\right)^{1-\theta} - \frac{1+r}{1+[(1-\theta)g_N+\theta g_K]}\lambda\right]K_t^{\theta}(AN_t)^{1-\theta} + R_t - X_t$$

$$C_t^s = \left[(1-\gamma) - (\delta + g_N)\left(\frac{k_t^s}{A}\right)^{1-\theta} - \frac{1+r+[1+(1-\theta)g_N+\theta g_K]-[1+(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\lambda\right]K_t^{\theta}(AN_t)^{1-\theta} + R_t - X_t$$

$$C_t^s = \left[(1-\gamma) - (\delta + g_N)\left(\frac{k}{A}\right)^{1-\theta} - \frac{1+[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\lambda - \frac{1+r-1-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\lambda\right]K_t^{\theta}(AN_t)^{1-\theta} + R_t - X_t$$

$$C_t^s = \left[(1-\gamma) - (\delta + g_N)\left(\frac{k}{A}\right)^{1-\theta} - \lambda - \frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\lambda\right] K_t^{\theta}(AN_t)^{1-\theta} + R_t - X_t.$$

Then, the optimal level of reserves in terms of output can be derived as follows:

$$p^{\frac{1}{\sigma}}\left\{\left[1 - s - \lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\right]K_t^{\theta}(AN_t)^{1-\theta} - X_t\right\}$$

$$= \left[(1-\gamma) - (\delta + g_N)\left(\frac{k}{A}\right)^{1-\theta} - \lambda\right.$$

$$\left. - \frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\lambda\right] K_t^{\theta}(AN_t)^{1-\theta} + R_t - X_t$$

$$p^{\frac{1}{\sigma}}\left[1 - s - \lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\right]K_t^{\theta}(AN_t)^{1-\theta} - p^{\frac{1}{\sigma}}X_t$$

$$= \left[(1-\gamma) - (\delta + g_N)\left(\frac{k}{A}\right)^{1-\theta} - \lambda\right.$$

$$\left. - \frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\lambda\right] K_t^{\theta}(AN_t)^{1-\theta} + R_t - X_t$$

$$p^{\frac{1}{\sigma}}\left[1 - s - \lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\right]K_t^{\theta}(AN_t)^{1-\theta}$$

$$- \left[(1-\gamma) - (\delta + g_N)\left(\frac{k}{A}\right)^{1-\theta} - \lambda\right.$$

$$\left. - \frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\lambda\right] K_t^{\theta}(AN_t)^{1-\theta} = R_t - X_t + p^{\frac{1}{\sigma}}X_t$$

$$\left[p^{\frac{1}{\sigma}}(1-s) - p^{\frac{1}{\sigma}}\lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]} - (1-\gamma) + \lambda + (\delta + g_N)\left(\frac{k}{A}\right)^{1-\theta}\right.$$

$$\left. + \lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\right] K_t^{\theta}(AN_t)^{1-\theta} = R_t - \left(1 - p^{\frac{1}{\sigma}}\right)X_t$$

$$\left[p^{\frac{1}{\sigma}}-p^{\frac{1}{\sigma}}s+\left(1-p^{\frac{1}{\sigma}}\right)\lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}-1+\gamma+\lambda+(\delta+g_N)\left(\frac{k}{A}\right)^{1-\theta}\right]K_t^{\theta}(AN_t)^{1-\theta}=R_t-\left(1-p^{\frac{1}{\sigma}}\right)\frac{\pi}{\pi+p(1-\pi)}R_t$$

$$\left[-\left(1-p^{\frac{1}{\sigma}}\right)-p^{\frac{1}{\sigma}}s+\left(1-p^{\frac{1}{\sigma}}\right)\lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}+\gamma+\lambda+(\delta+g_N)\left(\frac{k}{A}\right)^{1-\theta}\right]K_t^{\theta}(AN_t)^{1-\theta}=R_t-\left(1-p^{\frac{1}{\sigma}}\right)\frac{\pi}{\pi+p(1-\pi)}R_t$$

$$\left[\gamma+\lambda+(\delta+g_N)\left(\frac{k}{A}\right)^{1-\theta}-p^{\frac{1}{\sigma}}s-\left(1-p^{\frac{1}{\sigma}}\right)\left(1-\lambda\frac{r-(1-\theta)g_N+\theta g_K}{1+(1-\theta)g_N+\theta g_K}\right)\right]K_t^{\theta}(AN_t)^{1-\theta}=R_t-\left(1-p^{\frac{1}{\sigma}}\right)\frac{\pi}{\pi+p(1-\pi)}R_t$$

$$\left[\gamma+\lambda+(\delta+g_N)\left(\frac{k}{A}\right)^{1-\theta}-p^{\frac{1}{\sigma}}s-\left(1-p^{\frac{1}{\sigma}}\right)\left(1-\lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\right)\right]K_t^{\theta}(AN_t)^{1-\theta}=\left[1-\left(1-p^{\frac{1}{\sigma}}\right)\frac{\pi}{\pi+p(1-\pi)}\right]R_t$$

$$\rho^*=\frac{R_t}{K_t^{\theta}(AN_t)^{1-\theta}}=\frac{\gamma+\lambda-\left(1-\lambda\frac{r-[(1-\theta)g_N+\theta g_K]}{1+[(1-\theta)g_N+\theta g_K]}\right)\left(1-p^{\frac{1}{\sigma}}\right)+(\delta+g_N)\left(\frac{k}{A}\right)^{1-\theta}-p^{\frac{1}{\sigma}}s}{1-\frac{\pi}{\pi+p(1-\pi)}\left(1-p^{\frac{1}{\sigma}}\right)}.$$

# Chapter 3: Heterogeneity across the Empirical Distribution of International Reserves in Small Open Economies


### Abstract

This chapter examines empirically the key determinants of international reserves in a panel of 26 middle income economies over 1970-2014, with a primary interest to uncover potential common and idiosyncratic characteristics. To this end, we first estimate a pooled OLS regression and a panel data fixed effects model. Secondly, we use quantile regression techniques to analyse the variation of reserve holding determinants across the reserve holdings distribution in our sample. In particular, we apply F-tests to check the uniformity of coefficients in several inter-quantile regressions and we reject null hypothesis that the models for different quantiles of the reserve distribution in our sample were similar. This shows that the empirical models estimating the relative contribution of each of the key determinants of reserve holdings should consider the country specific features of middle income economies. Moreover, our empirical work uncovers a significant positive effect of the investment rate on actual reserve holdings, while the theory in chapters 1 and 2 derived analytically a negative effect of the same determinant but now to optimal reserves. While this may seem surprising, Gourinchas and Jeanne (2013) also find empirically opposite results to these predicted by neoclassical growth theory under a labour augmenting CD technology as in our chapter 2. Finally, while our theoretical chapters 1 and 2 derive the ‘optimal’ level of international reserves as a ratio to output, the dataset really measures the *de facto* level of international reserves-to-GDP prevailing in the countries of our sample. As the optimal reserve ratio quantified at the level of 5.5% of GDP in chapter 2 is found near the bottom of the empirical distribution of actual reserves in our dataset (Figure 1), this fact may well indicate that actual reserves are excessive relative to GDP for many middle income countries, as claimed notably by Aizenman and Marion (2004) and Alfaro and Kanczuk (2009).



**JEL Classification Numbers**: C3, F31, F32, F37, F41, O57

**Keywords:** Reserves, Quantile Regression, Emerging Markets, Sudden Stops, Debt, Investment.

## 3.1 Introduction

The reasons behind the rapid accumulation of reserves have been one of the most debatable and attractive research areas for the past 20 years. The main question is how much an economy should rely on international reserves. Even there is no single answer to that question, reserve holdings depend on their determinants. In this chapter, we add some control variables that are common in the applied literature to our key theoretically derived determinants to analyse empirically the reasons behind accumulating reserves. In effect, we find a high degree of heterogeneity in reserve holdings in middle income countries.

In line with our theoretical chapters, the main contribution of this empirical chapter consists in testing the relative importance of the central determinants of reserves derived in our production SOE model versions, such as investment, GDP growth, the external debt-to-GDP ratio and a sudden stop dummy. To do this, we add these theoretically relevant variables, notably the investment rate that was missing from earlier applied work, to the typical explanatory variables in the standard empirical models of reserve accumulation (Frenkel and Jovanovic, 1981). In the previous two chapters, we show analytically and quantitatively that the investment rate is a factor that reduces the reserves-to-GDP ratio. In addition, as control variables, we examine the role of common reserve holding indicators, such as export volatility, trade openness, the broad money-to-GDP ratio, the volatility of the nominal effective exchange rate (NEER). We also examine some country-specific factors coming from the political economy literature, such as political stability and corruption (see, e.g., Aizenman and Marion, 2004). As a first pass, we use pooled OLS and fixed-effects panel data techniques. Then, in order to check heterogeneity in the reserve holding distribution in our sample of 26 middle income countries, we use quantile-regression analysis.

To the best of our knowledge, this is the first paper that presents the unobserved cross-country preferences on international reserves holdings that incorporates the role played by the investment rate in middle income economies on explicitly derived (in the preceding two chapters) theoretical grounds.[25] As in Frenkel and Jovanovic (1981) and Mwase (2012), we estimate a “buffer-stock model” using pooled OLS and fixed-effects panel data regressions.

This empirical chapter relates to similar work by Sula (2011), Ghosh *et. al*. (2012) and Mwase (2012), since these authors also emphasize the heterogeneity in reserve holdings across emerging market economies (EMEs). All these papers stress the advantages of using quantile regression over the usual pooled OLS estimation in the context of middle income countries.

Mwase (2012) analyses the determinants of reserves by comparing EMEs and small islands. The author finds a wide difference across EMEs in terms of their estimated slope coefficients in a standard OLS regression. Therefore, the paper suggests to use quantile regression to avoid this problem.[26] We use the same methodology, but our view is broader in that we include the theoretically derived investment rate as a key determinant to the standard empirical reserve determinants, focusing exclusively on middle income countries. Moreover, in running these

[25] Rodrik (2006) also attempts to explain the relationship between reserves and investment, but in a purely descriptive policy-oriented paper. He claims that high level of debt might require better risk sharing, financial system and higher domestic investment, though he finds that the link between gross capital formation and short term capital flows is unclear. In this perspective, he shows external financing is overrated as in Aizenman (2006). However, external financing is an important factor, especially, for the countries that do not accumulate foreign reserves via current account surpluses. If a country increases reserves when it runs a current account deficit, its production is highly dependent on external financing.

[26] The quantile regression techniques have been developed by Koenker and Bassett (1978) in order to solve the potential sample selection bias in OLS. Koenker and Hallock (2001) explain why quantile regressions are preferable to OLS in subsamples (Ghosh *et. al*., 2012).

regressions we also test empirically some of the other theoretically derived reserves determinants that, differently from investment, have already been included in some empirical work.

The analogous empirical approach employed by Ghosh *et. al.* (2012) is more related to the motives behind reserve holdings rather than their determinants. They find that reserve holding motives have changed over time from current account shocks in the 1980s to sudden stops of capital inflows in EMEs. In addition, they state that the motives behind reserve holdings depend on where a particular country stands in the empirical reserve distribution. The present chapter does not focus on the motives behind reserve holdings; instead, we are interested in the determinants of reserves, and in potentially uncovering heterogeneity across the middle income countries in our sample.

Sula (2011) tests the determinants of reserves for 108 EMEs and reveals remarkable differences. He finds that standard OLS regressions lead to statistically insignificant results at different quantiles of the reserve holdings distribution. His dataset is broader than our dataset in terms of the number of countries, since in this empirical chapter we restricted our sample to a subset of 26 middle income economies out of the original JR sample of 34 such economies, by excluding the high-income countries from the JR sample, as well as Botswana as an outlier.

In line with Mwase (2012), we present testing of slopes of the quantile regressions in order to determine whether there are any statistically significant differences between lower quantile economies and higher quantile economies in terms of reserve accumulation variables. This is a particularly important question since slopes have effect on the respective elasticities in the regression. If there are differences between quantiles, the potential heterogeneity across the international reserve distribution can be hidden by simply running standard OLS regressions.

Our results, using a 'buffer stock model' in explaining reserve holdings, suggest that, in particular, the investment rate is an important positive determinant of reserve holdings in middle income countries, together with financial depth and the import share in GDP. Having added investment vulnerabilities to the standard empirical literature and taking the share of gross fixed capital formation in GDP as a proxy for the investment rate, we find the latter to have significant and positive effects on reserve holdings. Current account balances in terms of GDP also play a positive role in building up reserves; in order to provide consumption smoothing, countries tend to hold more reserves. In other words, current account adjustments in response to shocks and financial depth are significant factors in reserve holdings. We also find that NEER volatility requires a lower level of international reserves, as intervention in foreign exchange markets is not as much needed to stabilise the NEER and financial markets. The broad money-to-GDP ratio is another important factor affecting positively reserve holdings in most cases: countries with a higher ratio tend to have higher reserves. This can be seen as a stylized fact of reserve holdings, in line with the literature (Mwase, 2012).

Using quantile regression techniques, our results suggest that there are significant differences across middle income countries in terms of the reserve holdings distribution. Our findings from inter-quantile regressions show there are significant differences in the investment rate, the import share in GDP, and short term external debt (STED)-to-GDP. Using inter-quantile differences, F-test results imply that we can reject the null hypothesis of constant coefficients along the distribution.

We only found an insignificant F-test result for 25th-50th inter-quantile regression while all remaining inter-quantiles show significant F-test results.

The rest of the paper is organized as follows: Section 3.2 examines empirically what are the main determinants of international reserves in EMEs. Section 3.3 reports our data analysis, the empirical methodology and our testing of cross-country differences with pooled OLS, fixed-effects panel data and quantile regression. Section 3.4 presents our main findings from the regressions and section 3.5 concludes.

## 3.2 Empirical Determinants of International Reserves

As we saw in the theoretical chapters already, the determinants of international reserves are debatable in the huge extant literature. Even if there is a big consensus on the most important variables such as GDP, openness, probability of sudden stops, recent theories are still far away from convincingly explaining the rapid accumulation of international reserves observed after the East Asian crisis of 1997-1998 (Jeanne and Rancière, 2011). In this section, we discuss the main determinants of reserve holdings for middle income countries uncovered in the large empirical literature.

It is hard to understand why countries hold high level of reserves without understanding the distinction between the benefits and opportunity cost of reserves (see Feldstein, 1999; Aizenman, 2006; De Beaufort Wijnholds and Sondergaard, 2007). EMEs hold a high level of reserves since they have experienced sudden stops of capital flows and possible negative welfare effects (i.e., reducing output and consumption) of losing access to financial liquidity in that crisis period. The motives behind reserve holdings can be precautionary (Jeanne and Rancière, 2008; Jeanne and Rancière, 2011), mercantilist (Aizenman and Lee, 2007) or country-specific requirements. The main discussion is about why some countries hold a high level of reserves and what are their main determinants. To address these questions empirically, our dependent variable naturally is the reserve-to-GDP ratio; but what should the independent variables be?

The most traditional variables in the literature explaining reserve holdings, starting from Heller's (1976) seminal paper and up to most recent work (as outlined below), are 'current account', 'capital account', 'exchange rate regime', 'opportunity costs of reserves', 'economic size', 'experiences from previous crisis'. Furthermore, there is a growing literature on 'the role of institutions', 'political variables' and 'economic growth' (Aizenman and Marion, 2004; Gourinchas and Jeanne, 2006; Bonfiglioli, 2008; Cheung and Ito, 2009; Benigno and Fornaro, 2012; Cheng, 2015). In terms of testing the role of investment in reserve holdings - the centre of our interest in this third, empirical chapter, arising from the two preceding theoretical chapters - we use the investment share in GDP (also defined as gross fixed capital formation) as a proxy for the investment rate (which is also done by Rodrik, 2006 in his policy-oriented analysis).

Current account shocks are key factors in reserve holdings. In that view, international reserves provide smooth consumption intertemporally. Imports and exports are mostly used as proxies in order to capture shocks to current account (Mwase, 2012). The literature[27] converges to a positive correlation between reserves and current account shocks. We used share of imports in GDP, exports volatility and the degree of trade openness as proxies for current account shocks.

Another important determinant coming out of the literature is capital account shocks. Radelet and Sachs (2000) shows the importance of capital inflows since most of the economic activity depends on foreign liquidity in EMEs. Hence, any sudden decrease in capital flows might lead to enormous negative effects on the real

[27] See Flood and Marion (2001); Aizenman and Marion (2004); Aizenman *et al.* (2007).

economy. Aizenman *et. al.* (2007) have discussed the role of shocks on capital account in a financial crisis, in particular with reference to the East Asian crisis. By reducing output and consumption, sudden stops of capital flows caused a negative welfare effects in the East Asian economies between the mid-1990s and the beginning of 2000s.

Even if there is no consensus on how to capture 'capital account shocks', the literature mostly focuses on three key variables, such as broad money and short-term debt to reserves ratios and capital flows. First of all, broad money might be a signal of potential risks on the pressure of reserve holdings via currency mismatches, bank deposits depletion and capital outflows (Mwase, 2012). Showing evidence of a 'tequila crisis', Calvo (1996) states that the broad money-to-reserves ratio is one of the key indicators of financial vulnerability. Wijnholds and Kapteyn (2001) also emphasize the role of the ratio in reserve adequacy. Obstfeld *et. al.* (2010) use the broad money-to-GDP ratio as a proxy for financial development and find significant results. Secondly, short term debt plays a critical role. If the ratio of short term debt to reserves is high, it might be an indicator of more vulnerability to economic crisis (Sachs *et. al.* 1996). Lastly, capital flows such as FDI and portfolio flows, carry importance in the literature,[28] although the results depend on the type of capital flows (Mwase, 2012).

The exchange rate is another highly used variable in empirical work on reserve holdings. In the literature, it affects the reserve holdings in two ways, namely, via the exchange-rate regime and via the volatility of the exchange rate. The earlier literature on reserve holdings suggests that, if an economy follows a pegged exchange-rate regime and in order to defend the value of its domestic currency, a

[28] See, e.g., Feldstein (1999); Aizenman *et. al.* (2007); Gourinchas and Jeanne (2013).

country needs a larger reserve stock than a country following a flexible exchange-rate regime (see Heller, 1966; Clark, 1970; Kelly, 1970; Frenkel, 1974; Edwards, 1985). However, Calvo and Reinhart (2002) state that the distinction between a floating exchange-rate regime and pure fixed exchange-rate regime is not very clear in reality, since some countries have described themselves as operating freely flexible exchange regime economies (*de jure*), while actually (*de facto*) they are not. This problem leads to a kind of measurement problem (Mwase, 2012). Another aspect of the exchange rate is its volatility. Flood and Marion (2001) and Aizenman and Marion (2003) have discussed that even countries with flexible exchange-rate regimes still keep a high level of reserves in order to reduce exchange rate volatility. Therefore, the nominal effective exchange rate (NEER) volatility could be used as a proxy to show the relationship between reserves and exchange rate.

The opportunity cost of reserve holdings is also another important component of the determinants of international reserves. It has been particularly discussed in the earlier literature (Heller, 1966; Clark, 1970; Kelly, 1970; Frenkel, 1974), which calculates the opportunity cost of reserve holdings in real terms, such as via the marginal productivity of capital, but with increasing financial globalisation, the opportunity cost has been calculated more and more frequently in financial terms (Bahmani-Oskooee and Brown, 2002; Jeanne and Rancière, 2006; Jeanne and Rancière, 2011). However, both approaches in the reserve literature reach a consensus on the relationship between the opportunity cost of reserve holdings[29] and the level of international reserves, agreeing on an expected negative relationship.

The international reserve literature uses 'economic size' as a scaling variable

[29] Given the overall consensus in the literature, and differences in calculation of opportunity cost, in this study we excluded this variable in the regressions.

in order to make better comparisons among countries. The traditional approach[30] uses 'population' as a proxy for 'economic size'. The more recent studies (e.g., Flood and Marion, 2001; Choi *et. al*., 2007; Delatte and Fouquau, 2011; Mwase, 2012) employ 'the volume of international transactions' as a proxy for economic size rather than population. Both approaches imply a positive relationship between economic size and reserve holdings.

Following financial liberalisation, many countries faced economic crises and experienced a loss of access to international liquidity with sudden reversals of capital flows. These experiences made most EMEs more risk averse. Therefore, economies with previous sudden stop episodes usually hold more reserves than their counterparts (Jeanne and Rancière, 2011). The literature investigates the effect of such a crisis on reserve holdings and takes into account a dummy variable using either the exchange market pressure (EMP) index (Eichengreen *et. al.*,1996; Mwase, 2012) or the Guidotti *et al.* (2004) approach (Jeanne and Rancière, 2006; Jeanne and Rancière, 2011). We use the second methodology since our analysis is based on the JR sample. Therefore, as in the preceding chapters, we define sudden stops as a more than 5% decrease in the ratio of capital flows to GDP relative to the previous year.

Institutions or governments (political credibility) are also commonly employed explanatory variables in the recent reserve holdings literature (Aizenman and Marion, 2004; Cheung and Ito, 2009). This literature explains that economies need a higher level of international reserves if they have weaker institutions or government. The reason behind this is that such a country needs to gain international reputation, credibility and confidence. Corruption is also another issue for that kind of

[30] This approach starts from Heller (1966) and almost all papers in the earlier literature use population as a proxy for economic size. However, it can still be seen in the more recent literature on reserve holding (e.g., Aizenman and Marion, 2003).

economies and the literature shows that less corrupted countries need less reserves (Aizenman and Marion, 2004; Cheung and Ito, 2009; Mwase, 2012).

The recent literature also focuses on the role of economic growth dynamics on reserve holdings (Bonfiglioli, 2008; Kramer, 2010; Benigno and Fornaro, 2012; Cheng, 2015). But this is at the same time a debatable factor, since economic growth has been found to generally have an ambiguous effect on reserve holdings.

## 3.3 Data and Methodology

We examine, initially, the determinants of reserves for middle income countries using pooled OLS and panel data fixed effects looking at several alternative regressions. We, then, benefited from the 'quantile regression method' in order to test the differences in coefficients at different quantiles of the reserve holdings distribution across our sample.

Firstly, we construct an unbalanced panel data model with fixed effect[31] including 26 middle income economies.[32] The data are annual and selected from 26 middle income countries from the World Bank classification, and covering the period from 1970 to 2014. We, thus, work with 26 individual countries over 45 years. However, not all variables are available in this time span. Therefore, due to lack of data we have missing variables. The description of our regression variables and our

[31] We employed the Hausman (1978) test in order to decide which model is more suitable, fixed effects or random effects. The results from this test indicate that our regression should be estimated by panel data fixed effects rather than random effects. Therefore, we only present the fixed effects results. Although we present the pooled OLS results in table 3.3, the pooled OLS method gives inconsistent estimators if the true model is the fixed effects (Cameron and Trivedi, 2005, p. 702-703).

[32] The original JR (2011) sample consists of 34 countries, and we used it for the calibration in the theoretical chapters 1 and 2. In this empirical chapter 3, we excluded 7 high income countries from the sample. We also excluded Botswana since it has an outlier influence -based on Grubbs' (1969) test- in reserves series from 1985 to 2010 in the JR dataset. We, therefore, only studied 26 high and low middle income countries in the present chapter 3.

sample can be seen in Table 3.1 and 3.2, respectively. Our data is mostly calculated from the World Bank's *World Development Indicators* (WDI), IMF's *International Financial Statistics* (IFS) and *World Economic Outlook* (WOE).

We consider the 'reserve-to-GDP' ratio as a dependent variable, defined as

$$\rho_{i,t} = \frac{R_{i,t}}{Y_{i,t}}$$

where $R_{i,t}$ is international reserves minus gold and $Y_{i,t}$ is gross domestic product (GDP) of for country $i$ in period $t$. International reserves and GDP are both measured in nominal US dollars.

In line with Mwase (2012), our baseline empirical specification is given by,

$$\rho_{i,t} = \beta_0 + \beta_1 \left(\frac{IM}{Y}\right)_{i,t} + \beta_2 \left(\frac{I}{Y}\right)_{i,t} + \beta_3 \left(d\frac{M2}{Y}\right)_{i,t} + \beta_4 \left(\frac{srDebt}{Y}\right)_{i,t} + \beta_5 (NEERv)_{i,t} + \beta_6 CD + t + \varepsilon_{i,t} \quad (1)$$

where $IM$ is imports, $I$ is investment, $M2$ is broad money, $d$ is first difference, $srDebt$ is short-term external debt (STED), $NEERv$ is nominal effective exchange rate (NEER) volatility, $CD$ is a dummy variable for sudden stops for $i = 1, \dots, N$ and $t = 1, \dots, T$, where $N$ and $T$ display the cross-section and the time dimensions of the panel. Our dummy variable is equal to 1 if a sudden stop hits the country and 0 otherwise. $t$ represents the time dummies and $\varepsilon_{i,t}$ is the error term.

We also estimate an alternative panel models including additional variables[33] such as government effectiveness, political stability, the growth rate of real GDP per capita, current account balance, openness, and export volatility, in order to check their importance in reserve holdings.

[33] Since we have an unbalanced panel dataset, we focused on two main approaches in the literature, namely, Im-Pesaran-Shin unit root test and Fischer-type unit root test. Our unit root test results indicate that our variables such as volatility in the exchange rate, the growth rate of GDP per capita, the current account balance to GDP, export volatility, government effectiveness and political stability are all stationary. Moreover, the reserves-to-GDP ratio, the share of imports in GDP, the STED-to-GDP ratio and trade openness are trend stationary. These results are available upon request.

Based on our baseline specification (1), we first estimated a panel data model from 1970 to 2014, and then we estimated several alternative panel regressions in order to test the effects of each variable as a determinant of reserve holdings. Doing this, we are able to compare the alternative specifications.

Secondly, in order to empirically investigate heterogeneity in reserve holdings in our sample of middle income countries, we applied quantile regression methods developed by Koenker and Bassett (1978). Since the OLS regression does not allow changes across the conditional distribution of the dependent variable, the effect of the independent variables varies along the conditional distribution and can be analysed by the quantile regression method.[34]

The latter is applicable to reserve holdings since we would like to compare the respective coefficients at different quantiles of the reserve distribution as in Sula (2011), Mwase (2012) and Ghosh *et. al.* (2012). To do this, we have to allow different elasticities, and so we re-write our equation (1) as follows,

$$\rho_{i,t} = \beta_0(\gamma) + \beta_1(\gamma)\left(\frac{IM}{Y}\right)_{i,t} + \beta_2(\gamma)\left(\frac{I}{Y}\right)_{i,t} + \beta_3(\gamma)\left(d\frac{M2}{Y}\right)_{i,t} + \beta_4(\gamma)\left(\frac{srDebt}{Y}\right)_{i,t} +$$
$$\beta_5(\gamma)(NEERv)_{i,t} + \beta_6(\gamma)CD + t + \varepsilon_{i,t} \qquad (2)$$

where $\gamma$ is the weight given to the reserve level and all slope coefficients are a function of $\gamma$. It can be assumed that a higher level of reserves is linked with greater $\gamma$, *ceteris paribus* (Mwase, 2012).

[34] See appendix 3.A for a brief explanation of the quantile and inter-quantile regression approach.

## 3.4 Empirical Results

In this section, we present first our estimation results of the standard pooled OLS and the panel data fixed effects models. Secondly, we report the empirical results from the 'quantile' and 'inter-quantile' regressions. We estimate the models using the Stata and use 'robust' option to obtain heteroscedasticity-robust standard errors, namely the Huber/White standard errors (also known as 'sandwich estimators').

### 3.4.1 Empirical Results from Pooled OLS and Fixed Effects Regressions

Table 3.3 presents our results for the pooled OLS and the fixed effects regressions based on our sample of 26 middle income countries. The number of observations are different in each regression since our dataset covers 45 (from 1970 to 2014) years and not all variables are available in this time period. Therefore, due to lack of data we have missing variables in some particular years in the dataset. Our findings are mostly consistent with the literature (e.g., Aizenman and Marion, 2003; Mwase, 2012).

We analyse our findings under four main groupings, 'current account vulnerabilities', 'capital account vulnerabilities', 'credibility of a country', and 'investment vulnerability'.

#### 3.4.1.1 Current Account Vulnerabilities

We estimate the effects of current account vulnerabilities in reserve holdings using proxies such as 'share of imports in GDP', 'current account balance to GDP ratio', 'export volatility' and 'trade openness'.

Our first explanatory variable is the 'share of imports in GDP', which captures current account vulnerabilities. We used this variable in all alternative specifications since it is a main determinant of reserves in the above-mentioned literature. A one unit increase in the ‘share of imports in GDP’ ratio leads to a 0.187 unit increase in the reserves-to-GDP ratio, and the coefficient is statistically significant in the pooled OLS model. In the fixed effects models, we have mixed findings since the coefficient is not significant or positive in some specifications. The coefficient varies from -0.835 to 0.225 in alternative specifications.

The remaining three proxies for current account shocks added to the fixed effects models 4, 5 and 6, are ‘current account balance to GDP’, ‘export volatility’ and ‘trade openness’, respectively. All of them are significant and positive. However, export volatility has a very minimal effect on the reserve-to-GDP ratio, whereas trade openness and current account to GDP have larger effects. The reserve-to-GDP ratio increases by 0.54 unit following a one unit increase in trade openness, while it increases by 0.74 unit due to a one unit increase in current account to GDP ratio. We also see the influences of ‘current account balance to GDP’ and ‘export volatility’ in model 9 where we include all model variables excluding ‘trade openness’. Both coefficients support findings from models 4 and 5, since they are still significant and positive in model 9.

#### 3.4.1.2 Capital Account Vulnerabilities

Capital account vulnerabilities are captured by the ‘change in broad money to GDP ratio’ and ‘STED-to-GDP’ ratio. Both variables are important determinants of reserves-to-GDP for our sample.

The first ratio is significant in models 5, 6, 7, 8 and 9, and has a positive effect on the reserves-to-GDP ratio in all specifications. However, it yields an insignificant result in the pooled OLS model. In the fixed effects specifications, the coefficient is always positive and varies from 0.12 to 0.27 and, mostly, it is statistically significant. The ratio is insignificant in models 2, 3 and 4.

The second ratio, STED-to-GDP, indicates a positive and mostly significant relationship with the reserves-to-GDP ratio in our sample. In the pooled OLS model, a one unit increase in the ratio leads to a 0.41 unit increase in the reserve-to-GDP ratio. In the fixed effects specifications, the coefficient varies from 0.17 to 0.27. The literature (Choi *et. al.*, 2007; Mwase, 2012) presents similar results.

#### 3.4.1.3 Credibility of Country, Exchange Rate Arrangements and Economic Growth

Real GDP growth per capita, which is a proxy for real earnings growth, comes out as statistically insignificant from zero in model 3, but it is statistically significant and positive in model 9. In model 9, one unit increase in the growth rate of the economy leads a 0.334 unit increase in reserve-to-GDP ratio.

Another important determinant of reserve holdings is exchange rate flexibility. In line with Mwase, (2012), we used volatility in the NEER as a proxy to determine this effect. We find a significant negative coefficient for it in the pooled OLS model. A one unit increase in volatility of NEER decreases the reserve-to-GDP ratio by 0.0147. We find mostly insignificant results in most of the specifications in the fixed effects models. Only our model 4 and 9 give significant results in the fixed effect models.

In terms of the power of institutions, we used two proxies: government effectiveness and political stability. Cheung and Ito (2009) approach the same issue and conclude that more powerful institutions in the economy can be seen as higher credibility of a country and, therefore, it leads to a lower reserve-to-output ratio. However, both these proxies, when included, as in models 7, 8 and 9, yield insignificant estimates in our sample.

#### 3.4.1.4 Investment Vulnerability

In the pooled OLS model, the investment rate – which is of central theoretical and empirical interest in this PhD thesis – comes out with a significant positive coefficient, which does not support our analytical results in chapters 1 and 2. While surprising, this empirical finding is not inconsistent with similar results in empirical regressions which do not confirm theoretical implications of some neoclassical growth models (see, e.g., Gourinchas and Jeanne, 2013). In our sample, a one unit increase in the investment rate leads to an increase in the reserve-to-GDP ratio by 0.506 unit. However, this coefficient is not robust across all specifications. Although the coefficient mostly yields insignificant coefficients in the fixed effect models, it is only positive and significant in model 9. The reserve-to-GDP ratio increases by 0.35 unit following a one unit increase in investment rate in the model 9.

### 3.4.2 Empirical Results from Quantile Regressions

The estimation results from the quantile and inter-quantile regressions for our sample can be seen in tables 3.4 and 3.5, respectively. In both cases, we use only specification (1), where the reserve-to-GDP ratio is the dependent variable and the change in broad money-to-GDP, share of imports in GDP, investment rate, volatility in the NEER, and STED-to-GDP are the independent variables. In line with Mwase, (2012), we estimated the model at the 5$^{th}$, 25$^{th}$, 50$^{th}$, 75$^{th}$, and 95$^{th}$ quantiles for our 26 countries, see Table 3.4. Table 3.5 reports, in turn, the results from the comparison of respective pairs of quantiles. Figure 3.1 shows the 'box plots' of our variables and indicates that almost each variable has 'outliers' in the corresponding distribution of the variable. Figure 3.2 presents the 'quantile distribution plots' of our variables and almost all of them show their distributions are skewed right.

The change in the coefficients of model 1 can be seen in Figure 3.3 as the quantile increases from 5% to 95%. The green line shows the point estimates from the quantile regression from 5% to 95% percentile of the distribution. The blue line shows the estimates from the OLS regression. The red dotted lines above and below represent the 95% confidence interval.

We can see obvious differences along the reserve holding distribution between middle income countries. Our results thus imply very different results in each variable comparing with the pooled OLS model.

STED-to-GDP has a significant and strongly positive effect across reserve holding. It reaches its peak for the countries at 5$^{th}$ percentile and intersects its OLS coefficient almost at 75$^{th}$ quantile. However, it gives lower estimate for 75$^{th}$ and 95$^{th}$ quantiles.

The change in the broad money-to-GDP ratio gives insignificant results in all quantiles except the 25$^{th}$; but it reaches its pooled OLS estimate for the countries located around the 75$^{th}$ quantile, where it has its maximum along the distribution. However, it gives very long estimates for the countries at the tails.

The share of imports in GDP shows significant positive results alongside the distribution. It reaches its peak for the countries located at the 95$^{th}$ quantile and crosses its pooled OLS estimate between the 50$^{th}$ and 75$^{th}$ quantiles. Along the distribution, it increases as the quantile increases.

The coefficient to the investment rate shows mostly significant positive values along the reserve holding distribution. It has its minimum for the countries located at the 25$^{th}$ quantile, whereas it is only insignificant at the 5$^{th}$ quantile. It reaches its pooled OLS estimate almost at the 50$^{th}$ quantile, and its peak is at the 75$^{th}$ quantile.

The volatility in the nominal effective exchange rate yields mostly insignificant estimates along the reserve distribution; it is only significantly negative at 50$^{th}$ quantile.

The dummy variable for sudden stops results in mixed findings. It only yields significant estimates for the countries located at the 5$^{th}$ and 75$^{th}$ quantiles, but it has a negative value at 5$^{th}$ quantile, whereas it has a positive value for the 75$^{th}$ quantile (reaching its maximum). An increase in the sudden stop dummy decreases reserve-to-GDP ratio for poorer countries in terms of reserve holding, while it leads an increase for relatively richer countries for reserve holding. It can be interpreted that the countries in the upper level of reserve-to-GDP distribution holds reserves because of an increase in sudden stop probability as a result of precautionary savings. However, lower level of reserve-to-GDP distribution shows contradictory results, since any increase in the sudden stop probability decreases reserve-to-GDP ratio. This might also show a weakness of poorer countries across the sudden stop of capital inflows. Their reserve stock could easily deplete during the sudden stops of capital inflows and any increase in the probability of sudden stop would lead difficulties in external borrowing.

Table 3.5 shows that the inter-quantile estimates are not constant along the different quantiles of the reserve holding distribution. Significant differences, particularly between the 5$^{th}$ and the 95$^{th}$ quantiles for the share of imports in GDP, can be observed. The investment rate displays highly significant differences in almost all quantiles between 5$^{th}$ and 25$^{th}$, 75$^{th}$ and 95$^{th}$, and finally 5$^{th}$ and 95$^{th}$. It only gives insignificant difference between the 25$^{th}$ and 50$^{th}$ quantiles. The STED-to-GDP ratio shows significant differences between 25$^{th}$ and 50$^{th}$, 50$^{th}$ and 75$^{th}$, 75$^{th}$ and 95$^{th}$, 5$^{th}$ and 95$^{th}$ quantile. It also gives insignificant difference between 25$^{th}$ and 50$^{th}$ quantiles.

As the coefficient of our main interest is the ‘investment rate’, the quantile regression specifications reveal a significant positive estimate for it between all inter-quantiles (including $5^{th}$-$95^{th}$ range) except for $75^{th}$-$95^{th}$ quantiles, where it is significant and negative. Although we cannot see a negative relationship in our sample, in light of the theoretically derived negative influence of the investment rate on the optimal reserves-to-GDP ratio in chapters 1 and 2, this last finding can be interpreted as indicating that the investment rate affects positively and increasingly in terms of magnitude the reserve-to-GDP ratio until the $75^{th}$ quantile, but then the positive magnitude of this coefficient decreases. In other words, only for the top-end of the middle income country distribution, i.e., after a specific threshold of economic development, investment increases the empirical holdings of reserves in a decreasing way (with the coefficient still positive) relative to lower quantiles.

As in Mwase (2012), middle income countries are found to manifest significant differences across the quantiles in our sample. Table 3.5 and Figure 3.3 confirm these differences. There is huge variation in statistical significance and the magnitude of the estimates, as one moves along the quantiles of the reserve holdings distribution. Our results are in line with the literature on the precautionary motive, according to which countries with high level of short-term external debt accumulate international reserves in order to smooth consumption in case of losing access to international liquidity.

### 3.4.3. Comparison of Our Empirical Findings with the Recent Literature

We can directly compare our regression results with Mwase (2012) since we use a similar methodology in the empirical specifications and the inclusion of explanatory variables, but Mwase's dataset is larger than ours as we only focus on the 'middle income countries' in the JR sample. On the other hand, we increased the annual coverage of the sample size. While Mwase (2012) only analyses data from 1999-2010, our unbalanced panel dataset covers a period from 1970 to 2014. Despite these differences, our models give mostly consistent results with Mwase (2012). In order to reflect on sample selection, we also check our regressions reducing our data range as Mwase (2012). We run same regressions restricting our data. Tables 3.6, 3.7, and 3.8, show our empirical findings from reserve demand regressions, quantile and inter-quantile regression for our dataset between 1999 and 2010. We can also compare our empirical estimates with Aizenman and Marion (2003), Ghosh *et. al.* (2012) and Sula (2011).

In this subsection, we first present a comparison of the pooled OLS and the panel data fixed-effects specifications with the recent literature, and then a comparison of the 'quantile regressions' is provided.

#### 3.4.3.1 Comparison of the Pooled OLS and Fixed-Effects Results with the Recent Literature

In the pooled OLS model, our variables, 'volatility in nominal effective exchange rates', 'share of imports in GDP', 'STED-to-GDP', all give significant results as in Mwase (2012) except change in broad money to GDP ratio which is significant in Mwase's model, with a coefficient of 0.33, while it is insignificant in our sample. However, when we restrict our sample between 1999 and 2010, we find a significant and positive coefficient as in Mwase (2012). 'Share of imports in GDP' gives very similar and significant positive estimates, in both studies approximately 0.18. We find slightly higher significant positive coefficient in restricted sample at 0.22. Unlike Mwase (2012), our pooled OLS model implies almost 3 times larger significant positive coefficient for 'STED-to-GDP' ratio. Our reduced sample also supports our findings for 'STED-to-GDP' ratio. In terms of $R^2$, our pooled OLS specification including 'investment share in GDP' gives significantly higher model fit than Mwase (2012). Our specification results in $R^2$ of 0.51 for the 'middle income countries' sample, whereas Mwase has it at 0.22 for his EMEs sample. We have similar result in $R^2$ of 0.53 when we reduce our sample.

These results are also supported by Ghosh *et. al.* (2012). The authors find a significant positive relationship between reserve holdings and 'share of imports in GDP' as well as 'STED-to-GDP'. They also report a mostly significant negative relationship between the volatility in the NEER and reserves holding. However, Sula (2011) only finds a significant and positive relationship for 'shares of imports in GDP', with insignificant coefficients on 'exchange rate volatility' and 'exports volatility' in the pooled OLS specification.

This chapter covers a larger range of fixed-effects specifications than Mwase (2012). We analyse 'current account balance to GDP', 'real GDP per capita growth rate', 'export volatility', 'trade openness', 'government effectiveness' and 'political stability' as well as the variables in the pooled OLS model, while Mwase (2012) only focusses on 'government effectiveness' in the fixed effects specifications[35] for EMEs. Therefore, we can only compare our models 3 and 7 with Mwase's paper.

In model 2, we only focus on the same parameters as in the 'pooled OLS specification' without any additional variables. Our 'fixed effects' specification in model 2 yields only one significant coefficient, to the 'STED-to-GDP' for the middle income countries in the sample, estimated at 0.18. Mwase (2012) reports similar results, but the author finds two more significant variables, namely 'difference in broad money to reserve ratio' and 'share of imports'. However, there is much difference in our findings and Mwase's findings in terms of estimated coefficients.

[35] Mwase (2012) focusses those variables (such as, growth rate, export volatility, trade openness) for 'small islands' rather than emerging economies.

Mwase (2012) reports fixed-effects significant coefficients of 'change in broad money to GPD ratio', 'shares of import in GDP'[36] and 'external debt to GDP' for his EMEs sample at 0.17, 0.20 and 0.15, respectively. In our reduced sample model also gives contradictory results with Mwase (2012). We only find a significant coefficient for 'change in broad money to reserve ratio'. In model 7, once we include 'government effectiveness' in the fixed-effects specifications, we found statistically insignificant results. Likewise, Mwase (2012) reports an insignificant coefficient for 'government effectiveness'. Our reduced sample also supports our finding for 'government effectiveness'.

Ghosh *et. al.* (2012) also mostly support these findings. The authors find a significant positive relationship between reserve holdings and 'share of imports in GDP' and 'STED-to-GDP'. They also report an insignificant (negative) relationship between volatility in the nominal exchange rate and reserve holdings in most of their alternative models.

As in the pooled OLS specifications, our fixed-effect specifications give significantly higher $R^2$ values than Mwase (2012). Our specifications show a range between 0.40 and 0.51, whereas Mwase's range was 0.294-0.296 for his EME sample. The reduced sample versions of the models also give significantly higher $R^2$ values than Mwase (2012) since the range is between 0.40 and 0.53.

The findings in the fixed-effects specifications are also supported by Aizenman and Marion (2003). In their fixed-effect specification, the authors report a significant positive estimate for 'imports share in GDP'. Aizenman and Marion (2003) also support our findings for 'real GDP per capita', 'volatility in nominal exchange rate' and 'export volatility'. Aizenman and Marion (2003) state that the

[36] Sula (2011) also finds a significant and positive relationship for the 'share of imports in GDP'.

volatility in export receipts should be positively correlated with country's reserve holdings if they are planned to help cushion the economy, and they find a significant positive coefficient for 'export volatility'. As expected, we found a significant and positive estimate for 'export volatility' in our sample too. Aizenman and Marion (2003) also report negative relationship between 'volatility in nominal exchange rates and reserve holdings' since higher flexibility in exchange rates could lead to a decrease in reserve holdings (as the need for reserves declines if countries do not have to manage fixed exchange rates). In model 4, we find a significant negative relationship for 'volatility in nominal exchange rates' for our sample of 26 middle income countries too.

### 3.4.3.2 Comparison of Our Quantile Regression Results with the Recent Literature

In our 'quantile regressions', we found insignificant coefficients from mostly 5$^{th}$ quantile to 95$^{th}$ quantile for 'broad money to GDP ratio' for middle income countries whereas Mwase (2012) found mixed results in terms of the significance of this variable in his EMEs sample. The author finds a significant value at 25$^{th}$, 50$^{th}$ and 75$^{th}$ quantiles, while the coefficient is insignificant at 5$^{th}$ and 95$^{th}$ quantiles. We find only significant coefficient at the 25$^{th}$ quantile. In our reduced version of the model, the coefficient only gives a significant result for 95$^{th}$ quantile. However, Ghosh *et. al.* (2012) report a significant positive value at all quantiles for 'broad money to GDP ratio'.

We found a statistically significant and increasing coefficient for the 'share of import in GDP' across all quantiles. The coefficient increases considerably from 0.13 to 0.29 from quantiles 5$^{th}$ to 95$^{th}$. Mwase (2012) indicates an insignificant coefficient at the 5$^{th}$ quantile, but significant coefficients as the quantile increases. However, the coefficient follows a mixed pattern since it is relatively low at the 25th and 95th quantiles, while it reaches higher values at the 50th and 75th quantiles. Our reduced sample also gives a significant positive coefficient for all quantiles and the coefficient increases from 25th quantile to 95th quantile. Ghosh *et. al.* (2012) find a significant and positive coefficient for all quantiles, but it decreases from the 25th quantile to 90th quantile. Sula (2011) also supports these findings for 'shares of imports in GDP', since the coefficient is significant and positive for all quantiles.

Nominal effective exchange rate[37] volatility gives insignificant results across almost all quantiles in our models, except at the 50th quantile, as well as in Mwase (2012)[38]. This finding is also supported by Ghosh *et. al.* (2012). They found a weak significant and negative coefficient for the 25th and 50th quantiles, but the coefficient is always insignificant (and negative) in the upper quantiles.

The short-term external debt to GDP ratio yields a significant value across all quantiles. The range for this coefficient varies from 0.17 to 0.62 across the quartiles. We thus find a considerably higher coefficient for the 'external debt to GDP ratio' than Mwase (2012). He reports a significant coefficient for almost all quantiles except the 5th quantile, and the range is between 0.11 and 0.27. Our restricted model also supports our findings with a higher ratio than Mwase (2012) since the coefficient is significant and varies from 0.15 to 0.60. Ghosh *et. al.* (2012) report mixed findings for the 'external debt to GDP' ratio. Even if the coefficient decreases from the 25th quantile to the 90th quantile, it yields insignificant estimates in the upper quantiles.

[37] Sula (2011) presents a significant coefficient after the 25th quantile; however, the author uses volatility of the real effective exchange rate (REER), rather than nominal effective exchange rate (NEER).

[38] In Mwase (2012), the coefficient only gets a significant value in the 10-percent confidence interval at the 50th quantile. However, our restricted sample shows a significant coefficient for all quantiles for nominal exchange rate.

We find a significant coefficient for the 'share of imports in GDP' at 5th-95th quantiles, whereas Mwase (2012) finds significant coefficients for 5th-25th, 25th-75th and 50th-75th quantiles. Our restricted model also shows a significant coefficient for the 'share of imports in GDP' at 5th-95th and 75th-95th quantiles. Sula (2011) reports different results than our findings. The author finds a negative relationship between 'reserve holdings' and 'shares of import' across all inter-quantiles except 5th-25th quantile and the coefficient only shows a significant value in 25th-75th inter-quantile range.

'Nominal effective exchange rate volatility' implies an insignificant coefficient for all inter-quantile ranges in our specifications as well as in Mwase (2012). Our restricted model also supports our findings with an insignificant coefficient for all inter-quantiles. Sula (2011) finds mixed results, with significant range in 5th-25th and 75th-95th quantiles. We found a significant result at 5th -25th, 50th-75th, 75th-95th and 5th-95th inter-quantiles for 'external debt to GDP ratio', whereas Mwase (2012) only shows a significant value for the 5th -25th and 5th-95th inter-quantiles. The restricted model supports our findings since the coefficient is significant at 5th -25th, 50th-75th, 75th-95th and 5th-95th inter-quantiles.

## 3.5 Conclusion

Using panel data and quantile regression techniques, this chapter examines the determinants of international reserves from a broader perspective than usual studies in the literature and employing a sample of 26 middle income countries. We find many interpretable significant relationships between reserve holdings and their empirical determinants. Moreover, our results also imply considerable heterogeneity of the estimates across the reserve holdings distribution for our sample.

Based on our analytical results in chapters 1 and 2, we essentially added the investment share in GDP to the standard 'buffer stock' model explanatory variables in the literature in this last, applied chapter 3 of the thesis. Our pooled OLS results show that almost all variables have significant expected effects on reserve holdings in terms of sign of coefficients, except the 'change in broad money to GDP' which comes out as not significant in our sample and the investment rate, which is significant but positive. In that latter sense, the empirical findings with regard to this central determinant of interest here, the investment rate, are not supportive of the analytical results in chapters 1 and 2. These theoretical chapters derived analytically a negative influence of the investment rate on the optimal level of reserves in terms of output. Note however that, firstly, Gourinchas and Jeanne (2013) state the following: "The textbook neoclassical growth model predicts that countries with faster productivity growth should invest more and attract more foreign capital. We show that the allocation of capital flows across developing countries is the opposite to this prediction: capital does not flow more to countries that invest and grow more." (p. 1484, abstract). Insofar these authors employ exactly the same labour-augmenting CD production function specification as we do in chapter 2, our lack of empirical support for theoretical results found within the same neoclassical growth framework and common assumptions may not be that surprising. Then, secondly, while our theoretical chapters 1 and 2 derive the 'optimal' level of international reserves as a ratio to output, what the dataset measures indeed is the *de facto* level of international reserves-to-GDP prevailing in the countries of our sample. As the optimal reserve ratio quantified at the level of 5.5% of GDP in chapter 2 with labour-augmenting CD technology is found near the bottom of the empirical distribution of actual reserves in our dataset (see Figure 1), this fact may well indicate that actual reserves are excessive relative to GDP for many middle income countries, which has also been claimed, notably, by Aizenman and Marion (2004) and Alfaro and Kanczuk (2009). A final point of precision to the above findings requires to emphasise that our quantile and inter-quantile regression results reveal a nuance in the sense that at the top-end of the empirical reserves distribution the positive coefficient on the investment rate starts to decline in magnitude relative to the lower quantiles. In other words, even if we cannot see a negative relationship between the reserves-to-output ratio and the investment rate in our dataset, the coefficient to the investment rate is relatively lower for the richest middle income countries compared to the poorer ones.

Overall, our quantile and inter quantile regression results suggest mixed findings along the reserve holdings distribution. Our F-test results comparing the inter quantiles reject the null hypothesis that our models for different quantiles of the reserve holding distribution of the middle income countries in the sample were similar. In other words, it suggests that there are significant differences for the countries along the reserve holdings distribution, particularly for some variables such as, the 'share of imports in GDP', 'investment rate' and 'short term external debt to GDP'.

Our analysis can be developed in several ways. First of all, the sample can be extended for emerging market economies rather than only focusing on middle income countries. Secondly, exchange rate regimes can be another dummy variable, in addition to sudden stops: then, we could possibly judge about the effect of exchange-rate regimes in a better way. Lastly, we followed only determinants which are related to precautionary motives of reserve holding rather than mercantilist motives, as some studies claim to be observed for East Asian economies. These variables can be added to the usual determinants of international reserves.

# Appendix to Chapter 3

## 3.A Quantile Regression Methodology

Since the OLS regression does not allow for changes across the conditional distribution of the dependent variable, the effect of the independent variables that vary along the conditional distribution can be analysed by the quantile regression method (Koenker and Bassett 1978; Koenker and Hallock 2001).

The original model can be presented as follows,

$$\rho = X'\beta + \epsilon$$

where $\rho$ is the dependent variable (reserves to GDP ratio), and $X$ is a matrix of explanatory variables and $\epsilon$ is the error term.

Then for the $\theta^{th}$ quantile of the dependent variable conditional on the value of the independent variables describe as,

$$Quantile_{\theta}(\rho|X = x) = x'\beta(\theta) \quad and \quad 0 < \theta < 1$$

where $Quantile_{\theta}(\rho|X = x)$ indicates the $\theta^{th}$ quantile of $\rho$ conditional on $X = x$. $\beta$ is a vector of coefficients for each $\theta^{th}$ quantile with between 0 and 1. The error term, $\varepsilon$, is specified as an independent and identically distributed (i.i.d) process and it satisfies the quantile restriction,

$$Quantile_{\theta}(\varepsilon|X = x) = 0$$

Then, in order to show the $\theta^{th}$ regression quantile estimate $\hat{\beta}(\theta)$, we need to minimize the sum of absolute deviations. Therefore, the minimization problem can be written as,

$$\min_{\beta \epsilon R^K} \sum_{\rho \geq X'\beta} \theta|\rho - X'\beta| + \sum_{\rho \leq X'\beta} (1-\theta)|\rho - X'\beta|$$

Increasing $\theta$ from 0 to 1, we can follow the distribution of dependent variable, $\rho$, conditional on explanatory variables, $X$. We followed this process for $\theta = 0.05$, $\theta = 0.25$, $\theta = 0.5$, $\theta = 0.75$ and $\theta = 0.95$.

The standard errors for the quantile estimates can be obtained by bootstrapping with 1000 replications.

Once we have the distribution of dependent variable for different quantiles, an inter-quantile regression can also be estimated by taking the difference of the coefficients from any specified quantiles. This method allows to test the significance of the difference among quantile coefficients. For any given two quantiles $\theta_1$ and $\theta_2$, the inter-quantile difference gives an estimate of,

$$\rho(\theta_2) - \rho(\theta_1) = x'\beta(\theta_2) - x'\beta(\theta_1)$$

where the values of the depended and independent variables at the different quantiles are provided by bootstrap (Davino et al., 2014 p. 81).

### 3.B.1 Tables

**Table 3.1:** Description of Variables

| **Variables** | **Rationale** | **Variable Description** | **Data Source** |
|---|---|---|---|
| Reserves | Dependent variable | Reserves minus Gold as a share of GDP | WDI, WDI |
| Export Volatility | Captures current account vulnerability | Export to GDP | WDI |
| GDP Growth | Proxy for real earnings growth | Real GDP per capita growth | WDI |
| Share of Imports in GDP | Proxy for current account vulnerability | Imports as a share of GDP | WDI |
| Investment rate | Proxy for investment vulnerability | Gross capital (fixed) formation as a share of GDP | WDI |
| Openness | Proxy for current account vulnerability | Imports plus exports as a share of GDP | WDI |
| External Debt to GDP | Proxy for capital account vulnerability | Short-term external debt as a share of GDP | WDI |
| Money-to-GDP | Proxy for financial depth; captures capital account vulnerability | Broad money as a share of GDP | WDI |
| Interest Rate | Proxy for the opportunity cost of holding reserves | Interest differential with the US. | WEO, IFS, IMF, WDI |
| Government Effectiveness | Proxy for institutions | Proxy for institutions, focusing on governance | World Bank, Kaufmann and Kraay governance indicators |
| Political Stability | Proxy for institutions | Proxy for institutions, focusing on governance | World Bank, Kaufmann and Kraay governance indicators |
| NEER Volatility | Exchange rate volatility | Volatility of the nominal effective exchange rate | Bruegel |
| Crisis Dummy | Crisis effect | Is 1 crisis, 0 otherwise. Constructed using sudden stops of capital flows | Author’s calculations |

**Table 3.2:** Country List

| Country | Low Middle Income | High Middle Income | High Income |
|---|---|---|---|
| Argentina | 0 | 0 | 1 |
| Bolivia | 1 | 0 | 0 |
| Botswana | 0 | 1 | 0 |
| Brazil | 0 | 1 | 0 |
| Bulgaria | 0 | 1 | 0 |
| Chile | 0 | 0 | 1 |
| China | 0 | 1 | 0 |
| Colombia | 0 | 1 | 0 |
| Costa Rica | 0 | 1 | 0 |
| Czech Republic | 0 | 0 | 1 |
| Dominican Rep. | 0 | 1 | 0 |
| Ecuador | 0 | 1 | 0 |
| Egypt | 1 | 0 | 0 |
| El Salvador | 1 | 0 | 0 |
| Guatemala | 1 | 0 | 0 |
| Honduras | 1 | 0 | 0 |
| Hungary | 0 | 0 | 1 |
| Jamaica | 0 | 1 | 0 |
| Jordan | 0 | 1 | 0 |
| Korea | 0 | 0 | 1 |
| Malaysia | 0 | 1 | 0 |
| Mexico | 0 | 1 | 0 |
| Morocco | 1 | 0 | 0 |
| Paraguay | 0 | 1 | 0 |
| Peru | 0 | 1 | 0 |
| Philippines | 1 | 0 | 0 |
| Poland | 0 | 0 | 1 |
| Romania | 0 | 1 | 0 |
| South Africa | 0 | 1 | 0 |
| Sri Lanka | 1 | 0 | 0 |
| Thailand | 0 | 1 | 0 |
| Tunisia | 0 | 1 | 0 |
| Turkey | 0 | 1 | 0 |
| Uruguay | 0 | 0 | 1 |

Note: The countries are classified according to the World Bank's classification.

**Table 3.3:** Emerging Markets Reserve Demand Regression

| VARIABLES | (1) Pooled OLS | (2) Model 2 | (3) Model 3 | (4) Model 4 | (5) Model 5 | (6) Model 6 | (7) Model 7 | (8) Model 8 | (9) Model 9 |
|---|---|---|---|---|---|---|---|---|---|
| Δ(Broad Money to GDP) | 0.160 | 0.115 | 0.126 | 0.0682 | 0.146** | 0.117* | 0.271** | 0.272** | 0.226*** |
| | (0.105) | (0.0748) | (0.0795) | (0.0564) | (0.0629) | (0.0668) | (0.116) | (0.112) | (0.0736) |
| Share of Imports in GDP | 0.187*** | 0.156 | 0.159* | 0.225** | 0.152 | -0.835** | 0.148 | 0.146 | 0.203** |
| | (0.0293) | (0.0950) | (0.0929) | (0.0990) | (0.0912) | (0.353) | (0.0961) | (0.102) | (0.0972) |
| Investment Rate | 0.506*** | 0.0290 | -0.00977 | 0.408 | -0.0468 | 0.310 | 0.124 | 0.137 | 0.354* |
| | (0.0847) | (0.245) | (0.250) | (0.247) | (0.167) | (0.241) | (0.210) | (0.216) | (0.175) |
| NEER Volatility | -0.0147** | -0.0114 | -0.0125 | -0.0172** | -0.0171 | -0.0186 | -0.0133 | -0.0154 | -0.0209*** |
| | (0.00696) | (0.0171) | (0.0172) | (0.00785) | (0.0171) | (0.0172) | (0.0147) | (0.0145) | (0.00686) |
| STED-to-GDP | 0.410*** | 0.184** | 0.186*** | 0.241* | 0.182** | 0.272*** | 0.152 | 0.153 | 0.178* |
| | (0.0495) | (0.0711) | (0.0659) | (0.121) | (0.0700) | (0.0943) | (0.109) | (0.109) | (0.0957) |
| Sudden Stop Dummy | 0.464 | -1.918** | -1.522* | -2.426** | -2.060** | -2.476*** | -2.137 | -2.227* | -2.613** |
| | (1.580) | (0.886) | (0.814) | (0.874) | (0.853) | (0.813) | (1.372) | (1.252) | (1.019) |
| Growth Rate | | | 0.204 | | | | | | 0.334* |
| | | | (0.128) | | | | | | (0.162) |
| Current Account Balance to GDP | | | | 0.735*** | | | | | 0.583** |
| | | | | (0.242) | | | | | (0.226) |
| Export Volatility | | | | | 1.78e-09*** | | | | 1.24e-09*** |
| | | | | | (1.68e-10) | | | | (2.13e-10) |
| Trade Openness | | | | | | 0.541** | | | |
| | | | | | | (0.201) | | | |
| Government Effectiveness | | | | | | | 3.459 | | 0.779 |
| | | | | | | | (4.510) | | (3.238) |
| Political Stability | | | | | | | | 0.335 | 0.189 |
| | | | | | | | | (2.154) | (1.641) |
| Constant | -10.25*** | 3.625 | 3.799 | -5.433 | 5.347 | -4.192 | 2.523 | 2.049 | -4.005 |
| | (2.191) | (5.286) | (5.307) | (6.070) | (3.787) | (6.713) | (4.824) | (4.905) | (5.402) |
| Observations | 512 | 512 | 512 | 501 | 512 | 512 | 410 | 410 | 403 |
| R-squared | 0.510 | 0.406 | 0.411 | 0.513 | 0.502 | 0.493 | 0.410 | 0.404 | 0.561 |
| Year dummies | YES | YES | YES | YES | YES | YES | YES | YES | YES |
| Number of country | | 26 | 26 | 26 | 26 | 26 | 26 | 26 | 26 |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

**Table 3.4:** Quantile Regression

| VARIABLES | (1) Pooled OLS | (2) Model 2 | (3) 5th Quantile | (4) 25th Quantile | (5) 50th Quantile | (6) 75th Quantile | (7) 95th Quantile |
|---|---|---|---|---|---|---|---|
| Δ(Broad Money to GDP) | 0.160 | 0.115 | 0.0450 | 0.101* | 0.0926 | 0.166 | 0.0387 |
| | (0.105) | (0.0748) | (0.0384) | (0.0527) | (0.0632) | (0.108) | (0.191) |
| Share of Imports in GDP | 0.187*** | 0.156 | 0.138*** | 0.141*** | 0.152*** | 0.205*** | 0.291*** |
| | (0.0293) | (0.0950) | (0.0172) | (0.0140) | (0.0240) | (0.0377) | (0.0451) |
| Investment Rate | 0.506*** | 0.0290 | 0.0306 | 0.376*** | 0.502*** | 0.757*** | 0.468*** |
| | (0.0847) | (0.245) | (0.0266) | (0.0581) | (0.0874) | (0.121) | (0.153) |
| NEER Volatility | -0.0147** | -0.0114 | 0.00298 | -0.00154 | -0.0183*** | -0.0109 | -0.0380 |
| | (0.00696) | (0.0171) | (0.0140) | (0.0202) | (0.00659) | (0.00974) | (0.0670) |
| STED-to-GDP | 0.410*** | 0.184** | 0.624*** | 0.476*** | 0.508*** | 0.354*** | 0.174* |
| | (0.0495) | (0.0711) | (0.0274) | (0.0337) | (0.0267) | (0.0788) | (0.0962) |
| Sudden Stop Dummy | 0.464 | -1.918** | -2.761*** | 0.142 | 1.491 | 3.525* | 2.559 |
| | (1.580) | (0.886) | (0.658) | (0.664) | (2.428) | (1.802) | (2.698) |
| Constant | -10.25*** | 3.625 | -6.060*** | -10.27*** | -9.771*** | -13.01*** | -3.132 |
| | (2.191) | (5.286) | (0.616) | (1.450) | (1.904) | (2.646) | (11.32) |
| Observations | 512 | 512 | 512 | 512 | 512 | 512 | 512 |
| R-squared | 0.510 | 0.406 | | | | | |
| Year dummies | YES | YES | YES | YES | YES | YES | YES |
| Number of country | | 26 | | | | | |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

**Table 3.5:** Inter-Quantile Regression

| VARIABLES | (1)<br>5th-25th Quantile | (2)<br>25th-50th Quantile | (3)<br>50th-75th Quantile | (4)<br>75th-95th Quantile | (5)<br>5th-95th Quantile |
|---|---|---|---|---|---|
| Δ(Broad Money to GDP) | 0.0556 | -0.00804 | 0.0734 | -0.127 | -0.00632 |
| | (0.0972) | (0.120) | (0.118) | (0.188) | (0.167) |
| Share of Imports in GDP | 0.00292 | 0.0116 | 0.0526 | 0.0860 | 0.153*** |
| | (0.0408) | (0.0177) | (0.0476) | (0.0525) | (0.0494) |
| Investment Rate | 0.346*** | 0.125 | 0.256** | -0.289* | 0.437*** |
| | (0.109) | (0.117) | (0.113) | (0.169) | (0.166) |
| NEER Volatility | -0.00452 | -0.0167 | 0.00737 | -0.0271 | -0.0410 |
| | (0.0706) | (0.0254) | (0.0194) | (0.0179) | (0.0458) |
| STED-to-GDP | -0.148* | 0.0318 | -0.154* | -0.180** | -0.450*** |
| | (0.0860) | (0.0666) | (0.0910) | (0.0866) | (0.119) |
| Sudden Stop Dummy | 2.902* | 1.349 | 2.034 | -0.967 | 5.319 |
| | (1.625) | (2.313) | (2.392) | (3.004) | (3.568) |
| Constant | -4.209 | 0.498 | -3.235 | 9.873 | 2.927 |
| | (3.086) | (2.409) | (3.045) | (6.836) | (7.837) |
| Observations | 512 | 512 | 512 | 512 | 512 |
| Year dummies | YES | YES | YES | YES | YES |
| F-test | 2.431** | 0.870 | 2.512** | 3.742*** | 6.379*** |
| Prob>F | 0.0342 | 0.501 | 0.0293 | 0.00247 | 9.42e-06 |

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

**Table 3.6:** Emerging Markets Reserve Demand Regression between 1999-2010

| VARIABLES | (1)<br>Pooled OLS | (2)<br>Model 2 | (3)<br>Model 3 | (4)<br>Model 4 | (5)<br>Model 5 | (6)<br>Model 6 | (7)<br>Model 7 | (8)<br>Model 8 |
|---|---|---|---|---|---|---|---|---|
| Δ(Broad Money to GDP) | 0.271** | 0.209*** | 0.229*** | 0.145** | 0.204** | 0.188*** | 0.248** | 0.246** |
| | (0.123) | (0.0739) | (0.0776) | (0.0537) | (0.0741) | (0.0598) | (0.0996) | (0.102) |
| Share of Imports in GDP | 0.227*** | 0.104 | 0.105 | 0.231*** | 0.132 | -1.116*** | 0.0856 | 0.0880 |
| | (0.0413) | (0.0844) | (0.0828) | (0.0816) | (0.0833) | (0.338) | (0.0816) | (0.0855) |
| Investment Rate | 0.572*** | 0.0179 | -0.0432 | 0.455* | -0.0469 | 0.333 | 0.0277 | 0.0300 |
| | (0.125) | (0.210) | (0.201) | (0.261) | (0.157) | (0.240) | (0.190) | (0.194) |
| NEER Volatility | -0.291** | 0.0369 | 0.0791 | -0.111 | 0.000330 | -0.108 | -0.00792 | 0.00111 |
| | (0.136) | (0.128) | (0.144) | (0.117) | (0.125) | (0.113) | (0.157) | (0.170) |
| STED-to-GDP | 0.348*** | 0.135 | 0.165 | 0.199 | 0.0677 | 0.138 | 0.0986 | 0.113 |
| | (0.0584) | (0.121) | (0.106) | (0.152) | (0.103) | (0.162) | (0.129) | (0.128) |
| Sudden Stop Dummy | 1.836 | -0.422 | -0.417 | -0.927 | -0.622 | -0.968 | -0.434 | -0.445 |
| | (2.177) | (1.314) | (1.264) | (1.134) | (1.226) | (0.922) | (1.513) | (1.574) |
| Growth Rate | | | 0.327 | | | | | |
| | | | (0.194) | | | | | |
| Current Account Balance to GDP | | | | 0.742*** | | | | |
| | | | | (0.256) | | | | |
| Export Volatility | | | | | 1.80e-09*** | | | |
| | | | | | (1.64e-10) | | | |
| Trade Openness | | | | | | 0.686*** | | |
| | | | | | | (0.195) | | |
| Government Effectiveness | | | | | | | -4.186 | |
| | | | | | | | (5.432) | |
| Political Stability | | | | | | | | -0.297 |
| | | | | | | | | (1.899) |
| Constant | -9.455*** | 7.367* | 7.390* | -5.162 | 7.774** | -2.146 | 7.898* | 7.946* |
| | (2.834) | (4.258) | (4.104) | (6.937) | (3.092) | (6.220) | (4.167) | (3.934) |
| Observations | 284 | 284 | 284 | 279 | 284 | 284 | 259 | 259 |
| R-squared | 0.538 | 0.401 | 0.415 | 0.522 | 0.513 | 0.523 | 0.388 | 0.382 |
| Year dummies | YES | YES | YES | YES | YES | YES | YES | YES |
| Number of country | | 26 | 26 | 26 | 26 | 26 | 26 | 26 |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

**Table 3.7:** Quantile Regression between 1999-2010

| VARIABLES | (1) Pooled OLS | (2) Model 2 | (3) 5th Quantile | (4) 25th Quantile | (5) 50th Quantile | (6) 75th Quantile | (7) 95th Quantile |
|---|---|---|---|---|---|---|---|
| Δ(Broad Money to GDP) | 0.271** | 0.209*** | 0.0317 | 0.0848 | 0.169 | 0.251 | 0.264* |
| | (0.123) | (0.0739) | (0.0825) | (0.0794) | (0.106) | (0.191) | (0.134) |
| Share of Imports in GDP | 0.227*** | 0.104 | 0.182*** | 0.157*** | 0.170*** | 0.191*** | 0.304*** |
| | (0.0413) | (0.0844) | (0.0143) | (0.0259) | (0.0281) | (0.0685) | (0.0451) |
| Investment Rate | 0.572*** | 0.0179 | 0.0917** | 0.442*** | 0.579*** | 0.809*** | 0.602*** |
| | (0.125) | (0.210) | (0.0388) | (0.0658) | (0.106) | (0.228) | (0.132) |
| NEER Volatility | -0.291** | 0.0369 | -0.156 | -0.286*** | -0.313** | -0.521*** | -0.428*** |
| | (0.136) | (0.128) | (0.123) | (0.0555) | (0.122) | (0.107) | (0.127) |
| STED-to-GDP | 0.348*** | 0.135 | 0.599*** | 0.480*** | 0.470*** | 0.337*** | 0.154* |
| | (0.0584) | (0.121) | (0.0322) | (0.0386) | (0.0580) | (0.118) | (0.0925) |
| Sudden Stop Dummy | 1.836 | -0.422 | -2.360** | -1.532 | 3.157 | 7.792*** | 5.649*** |
| | (2.177) | (1.314) | (1.098) | (0.960) | (4.532) | (2.776) | (1.815) |
| Constant | -9.455*** | 7.367* | -7.188*** | -7.990*** | -8.585*** | -7.575 | -1.408 |
| | (2.834) | (4.258) | (2.463) | (1.539) | (2.015) | (4.953) | (2.668) |
| Observations | 284 | 284 | 284 | 284 | 284 | 284 | 284 |
| R-squared | 0.538 | 0.401 | | | | | |
| Year dummies | YES | YES | YES | YES | YES | YES | YES |
| Number of country | | 26 | | | | | |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

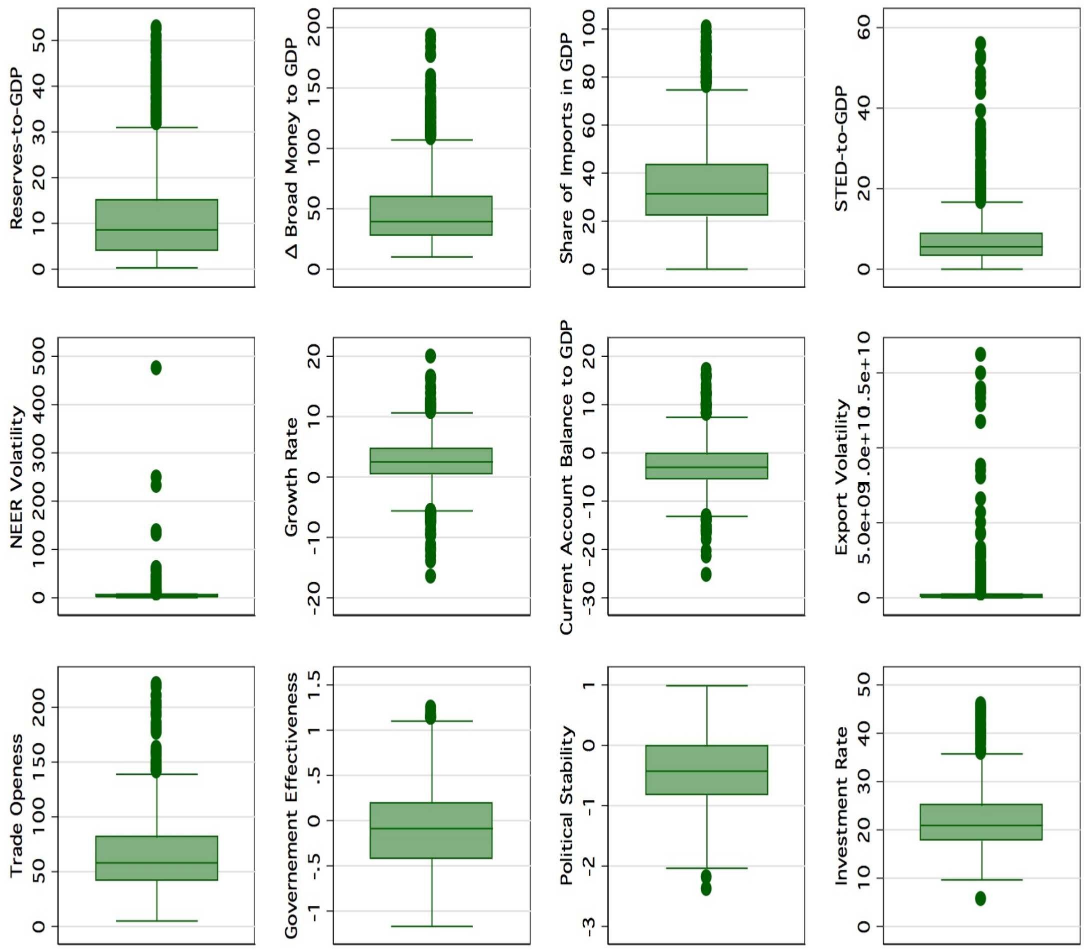
**Table 3.8:** Inter-Quantile Regression between 1999-2010

| VARIABLES | (1)<br>5th-25th Quantile | (2)<br>25th-50th Quantile | (3)<br>50th-75th Quantile | (4)<br>75th-95th Quantile | (5)<br>5th-95th Quantile |
|---|---|---|---|---|---|
| Δ(Broad Money to GDP) | 0.0531 | 0.0839 | 0.0828 | 0.0122 | 0.232 |
| | (0.152) | (0.176) | (0.201) | (0.187) | (0.295) |
| Share of Imports in GDP | -0.0253 | 0.0135 | 0.0202 | 0.113* | 0.121* |
| | (0.0442) | (0.0381) | (0.0566) | (0.0648) | (0.0660) |
| Investment Rate | 0.351*** | 0.137 | 0.230 | -0.207 | 0.510** |
| | (0.125) | (0.132) | (0.166) | (0.170) | (0.225) |
| NEER Volatility | -0.130 | -0.0269 | -0.208 | 0.0928 | -0.272 |
| | (0.211) | (0.160) | (0.214) | (0.289) | (0.221) |
| STED-to-GDP | -0.119** | -0.0104 | -0.133** | -0.183* | -0.446*** |
| | (0.0606) | (0.0614) | (0.0627) | (0.0941) | (0.111) |
| Sudden Stop Dummy | 0.828 | 4.688 | 4.635 | -2.143 | 8.008** |
| | (2.442) | (4.184) | (2.921) | (6.554) | (3.708) |
| Constant | -0.803 | -0.595 | 1.010 | 6.168 | 5.780 |
| | (4.194) | (2.744) | (4.365) | (3.918) | (4.514) |
| Observations | 284 | 284 | 284 | 284 | 284 |
| Year dummies | YES | YES | YES | YES | YES |
| F-test | 3.976 | 0.539 | 4.118 | 1.072 | 4.500 |
| Prob>F | 0.00170 | 0.747 | 0.00128 | 0.376 | 0.000591 |

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

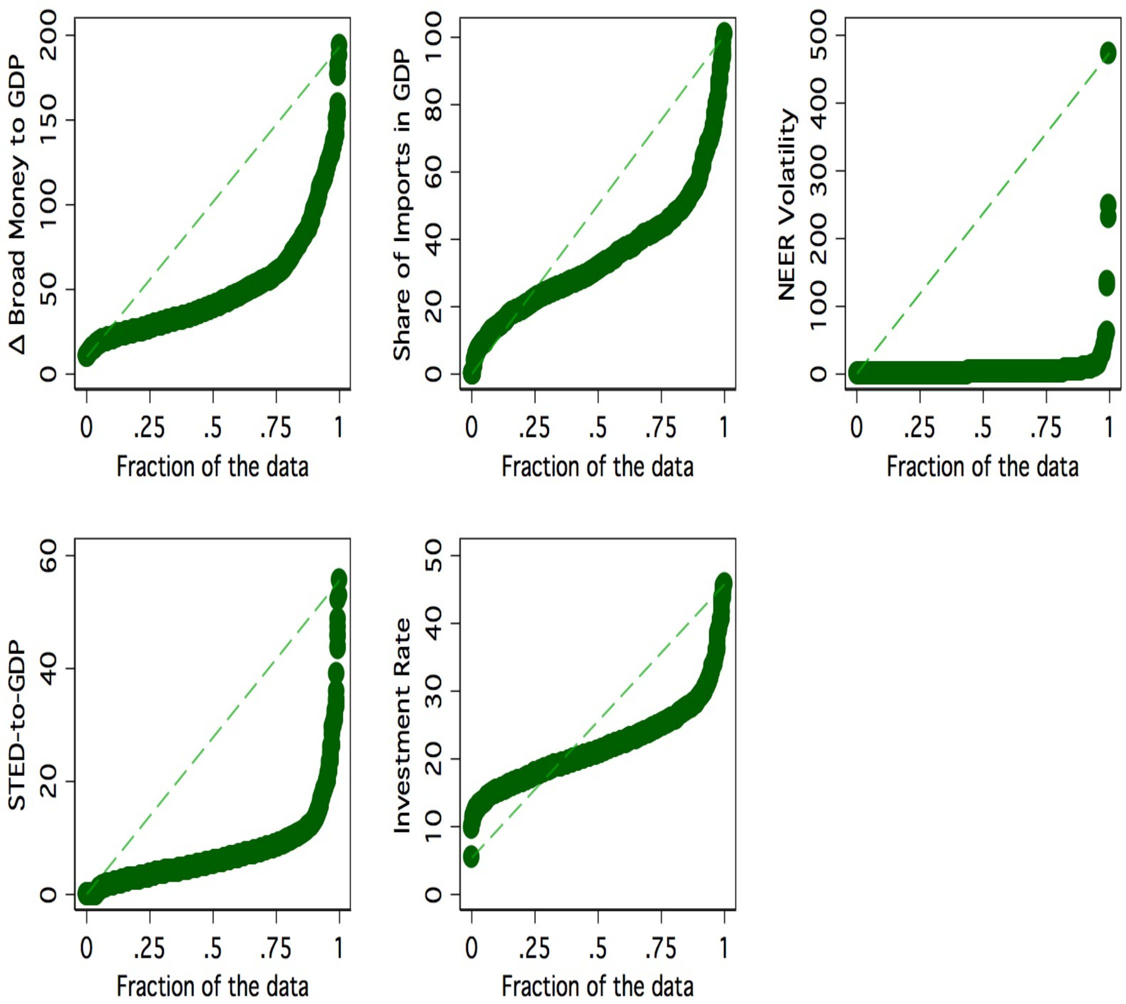
### 3.B.2 Figures

**Figure 3.1:** Box Plots



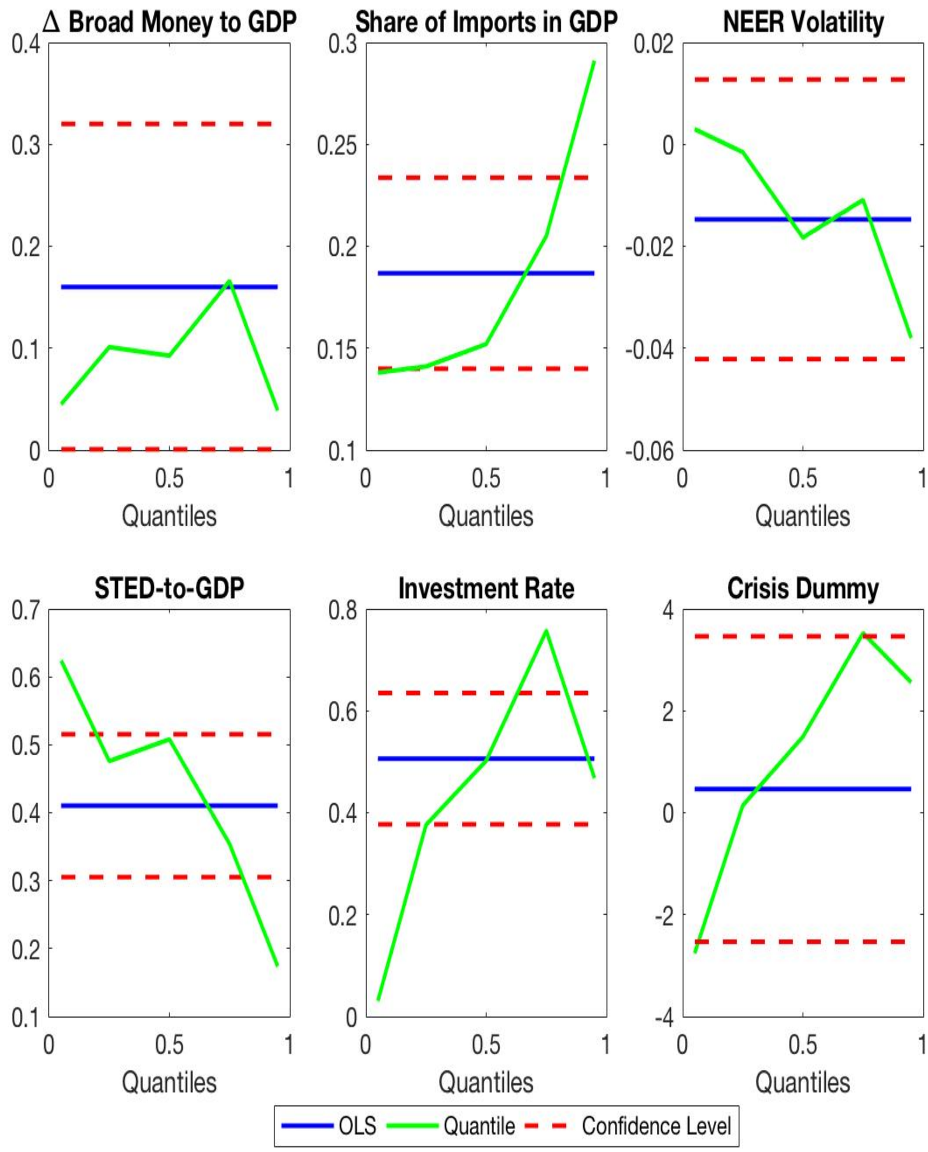

Note: Panels shows the distribution of the dataset. Although almost all panels include outliers, our dataset is positively skewed for: i) the reserve to GDP ratio; ii) broad money to GDP; iii) the share of imports in GDP; iv) the external debt to GDP ratio, v) trade openness; vi) the investment rate; and negatively skewed for government effectiveness. Furthermore, the growth rate of economy, the current account balance to GDP ratio and political stability are mostly normally distributed.

**Figure 3.2:** Quantile Distribution Plots




Note: These are the variables used in the quantile regressions. Each value of the variable is plotted against the fraction of their corresponding data that have values less than that fraction. The diagonal line is a reference line. It is obvious that all the points for each variable are below the reference line. Hence, we know that the distributions of all variables are skewed right.

**Figure 3.3:** Comparison of OLS and Quantile Regression Coefficient Estimates




Note: Figure shows, the change in the coefficients of model 1 as the quantile increases from 5% to 95%. The green line shows the point estimates from the quantile regression from 5% to 95% percentile of the distribution. The blue line shows the estimates from the OLS regression. The red dotted lines above and below represent the 95% confidence interval.

**Figure 3.4:** International Reserves as % of GDP by Country


Source: Author's calculations, using data from IMF's *International Financial Statistics*, and World Bank's *World Development Indicators*.

# Conclusion and Directions for Future Research

This PhD thesis contributed to the literature on international reserves, analysing in particular the role of key neoclassical growth theory variables such as investment, capital, labour and population growth on the optimal holdings of reserves relative to output in production small open economies (SOEs). Featuring technology consistent with the most common endogenous (AK IRS) and exogenous (labour-augmenting CD CRS) neoclassical growth specifications, our theoretical chapters 1 and 2 derived and quantified using calibration based on our dataset, respectively, two operational formulas of the optimal reserves-to-GDP ratio for production SOEs that are seeking to insure themselves against sudden stops of capital inflows. The last, empirical chapter 3 took a statistical approach to the distribution of actual international reserves in our sample of middle income SOEs, and essentially found much heterogeneity. To reveal this heterogeneity, it employed the better suited 'quantile regressions', in addition to 'pooled OLS' and fixed-effects panel data methods, to analyse our sample of middle income countries, considering idiosyncratic as well as common features across them.

Chapter 1 examined the effects of investment and production on optimal reserves in SOE EMEs and derived an optimal reserves-to-output formula in the case where capital is the only factor of production as in the AK model of endogenous growth. We found that the endogenous growth AK model with IRS implies a negative relationship between the optimal reserve-to-output ratio and capital-augmenting (in fact, here sole-factor) technological progress. Depending on the calibration of the productivity parameter, the model of chapter 1 quantified the optimal ratio of reserves to output at 1.74% for production SOEs.

Chapter 2 introduced labour, making the production function more general and employing a labour-augmenting Cobb-Douglas (CD) production function. The latter embodied the alternative assumptions of constant returns to scale (CRS) overall – but with diminishing returns to scale (DRS) for each of the two factors, capital and labour – and convergence to a balanced growth path (BGP) in the long run. Hence, the second chapter focused on the effects of labour-augmenting productivity and population growth on the optimal reserves-to-output ratio in a production SOE. Moreover, the alternative modelling of the production function, IRS AK versus CRS CD, allowed a comparison of the analytical results in chapter 1 (AK model) with those in chapter 2 (the CD model). As in the endogenous growth AK model, we found analytically that in the exogenous growth CD model along the BGP labour-augmenting technological progress decreases the optimal reserves-to-output ratio. Depending on the calibration of the labour-augmenting productivity parameter, the CRS CD model quantified the optimal ratio of reserves to output at 5.5% in the richer two-factor production SOE model. This is almost three times higher ratio of optimal reserve holdings to output relative to the AK IRS model.

However, both these ratios are still quite lower than the corresponding one derived in the endowment SOE model of Jeanne and Rancière (2011), 9.1% that we extended to production. Our intuition for the optimally lower ratios in chapter 1 and 2 can be outlined in the following way. With the capital stock now accumulated via investment and potentially used as a pledge to external creditors in obtaining borrowing and therefore insuring better against sudden stops, the optimal reserve-to-output ratio is much lower in the AK endogenous growth model relative to the otherwise similar endowment economy model of JR. As depreciation depletes the installed capital stock, thus acting in the opposite direction to net investment, the reversal of the relationship is not surprising in both the AK and CD models.

Applying innovatively appropriate econometric techniques to investigate the distribution of actual international reserves based on a dataset of 26 middle income economies, chapter 3 studied the key theory-derived determinants of reserves relative to output highlighted in chapters 1 and 2 together with the most common empirically motivated determinants suggested in the literature as ‘control variables’. For this purpose, we initially estimated a pooled OLS benchmark and a panel data fixed-effects model to analyse the relative importance of such empirically measured determinants of real-world reserve holdings as well as possible country specificities. We then used quantile regression techniques to explore the variation in these determinants across the reserve holdings distribution in our sample. We examined the uniformity of coefficients by several quantile regressions and the overall models. Our quantile regression results suggest that there are substantial variations in middle income countries in terms of the reserve holdings distribution. We found many interpretable significant relationships between reserve holdings and their empirical determinants. Moreover, our results also implied considerable heterogeneity of the estimates across the reserve holdings distribution for our sample.

In terms of potential implications for future research, we would like to emphasise that the neoclassical growth assumptions we employed may well be too abstract and unrealistic when it comes to empirical support in the data. This is also recently established by other similar studies, notably Gourinchas and Jeanne (2013). This implies that potential extensions to our modelling work in chapters 1 and 2 may need to relax some restrictive neoclassical assumptions, most obviously the constant saving rate equivalent to the investment rate. Moreover, it may be worth going into richer, two-sector open economy models and into richer structure of financial instruments and markets. In our case, the theoretical chapters 1 and 2 derived analytically a negative influence of the investment rate on the optimal level of reserves in terms of output, while the empirical chapter 3 mostly found a significant positive effect.

In terms of policy implications, we think that the main insight from this PhD thesis relates to the debate of excessive reserve levels relative to GDP as an insurance device against capital flow reversals kept by many real-world economies. Our theoretical chapters 1 and 2 derived the 'optimal' level of international reserves as a ratio to output, and quantified it at 5.5% of GDP in chapter 2 with labour-augmenting CD technology. This order of magnitude places the optimal reserve-to-GDP ratio near the bottom of the empirical distribution of actual reserves in our dataset (illustrated in Figure 1). This finding, highlighting a considerable discrepancy between the theoretically derived optimal reserves-to-output ratio and the much higher real-world ratios now involving not optimal but actual reserves, leads us to conclude in favour of excessive reserves relative to GDP held in most of the middle income countries in our sample. Such a conclusion is in line with recent claims that actually held reserves are excessive by Aizenman and Marion (2004) and Alfaro and Kanczuk (2009), among many other authors.

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